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New Axion Isocurvature

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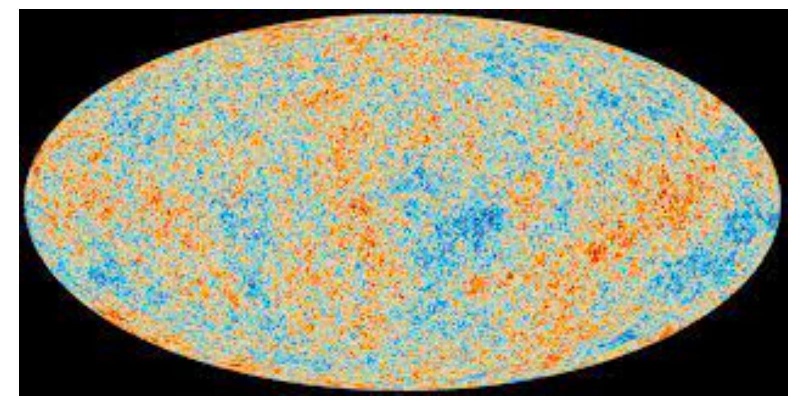
Adiabatic Perturbations

- The primordial perturbations are measured by the CMB to be adiabatic to good precision.

$$\delta f_i(x,t) = \frac{df_i(t)}{dt}\delta t(x,t)$$

Any thermodynamic quantity

Perturbation of the common clock



- Any perturbations that doesn't satisfy this is "isocurvature perturbation".

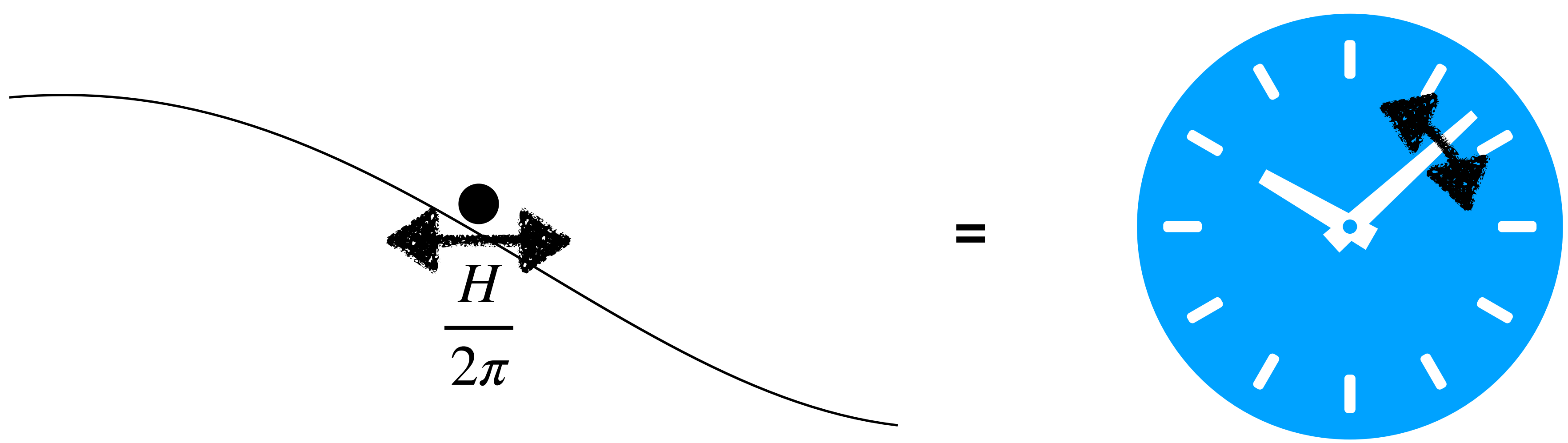
$$S = \frac{\delta\left(n_a/s\right)}{n_a/s}$$

(Late times)

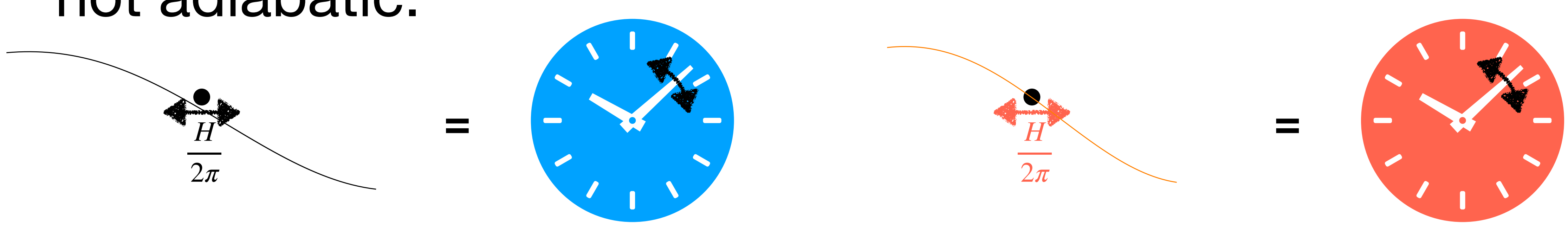
- CMB puts a constraint on the power spectrum of S.

Production of Isocurvature

- For a single field, slow roll inflation, only adiabatic modes are produced.



- For multi-field inflation, any perturbation of a single field is not adiabatic.



Adiabatic mode: $\frac{\delta\varphi_1}{\dot{\varphi}_1} = \frac{\delta\varphi_2}{\dot{\varphi}_2} = .. = \delta t$ **= Moving both clocks together**

Axion Isocurvature

- Standard Axion Isocurvature: during inflation, axions get dS-fluctuations

$$\left\langle \delta\phi_a, \delta\phi_a \right\rangle = \frac{H^2}{4\pi^2}$$

- These fluctuations cannot be removed by the perturbations of the clock.

- In other words:

$$\underbrace{\frac{\delta\phi_a}{\dot{\phi}_a}}_{\textbf{The axion}} \neq \underbrace{\frac{\delta\phi_i}{\dot{\phi}_i}}_{\textbf{The inflation}}$$

Axion Isocurvature

- We can estimate the isocurvature

$$S = \frac{\delta\left(n_a/s\right)}{n_a/s} \approx 2\frac{\delta a}{a} \sim \frac{H_{inf}}{f}$$

- From CMB

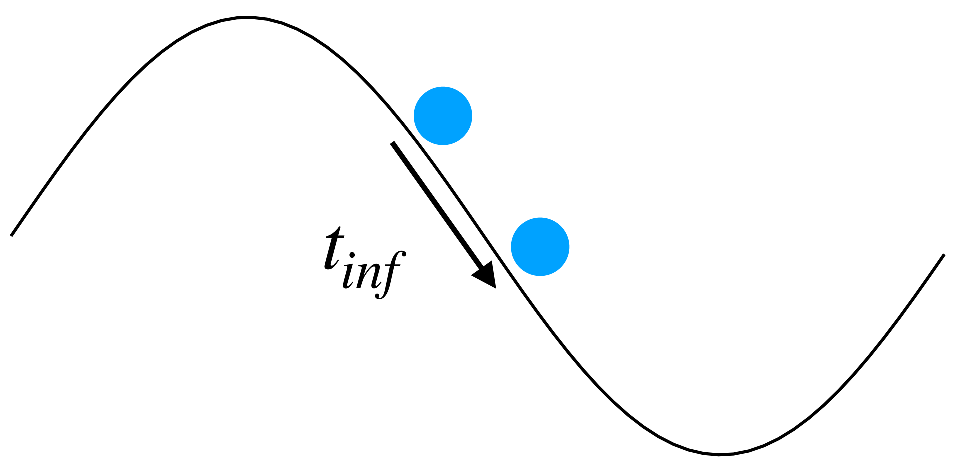
$$\alpha \sim \frac{S^2}{R^2} \lesssim 10^{-2} \qquad \textbf{(uncorrelated)}$$

- Bound on high scale inflation

$$H_{inf} \lesssim 10^{-6} f$$

Our Idea

- Neglect the axion field perturbations
- Assume that the axion can move a bit during inflation:



- The length of inflation is given by the inflation field, which has perturbations, i.e. t_{inf} is inhomogeneous
- **Result**: The axion at the end of inflation is inhomogeneous.

Estimating the Effect

- The distance the axion moves is

$$\Delta\phi_a = \frac{d\phi_a}{dN} N_{inf}$$

- The adiabatic perturbation can be written as

$$R = \delta N_{inf}$$

- Therefore the axion perturbation is

$$\delta\phi_a a = \frac{d\phi_a}{dN}\delta N_{inf} \longrightarrow S = 2\frac{\delta\phi_a}{\phi_a} \sim \frac{m^2}{H_{inf}^2} R$$

Low Scale Inflation

- Isocurvature perturbation:

$$S \sim \frac{m^2}{H_{inf}^2} R$$

- The CMB is sensitive to

$$\alpha \sim \frac{S^2}{R^2} < 10^{-3} \longrightarrow \begin{array}{l} S \sim 0.03R \\ H_{inf} \sim 5m_a \end{array}$$

(anti-correlated)

- Very low scale of inflation,

$$H_{inf} \lesssim \mathrm{e}V \longrightarrow \Lambda_{inf} \lesssim 100\ \mathrm{T}eV$$

Isocurvature Constraint

- Doing the calculation with ~~no~~ less squiggly lines

$$\alpha \equiv \frac{\langle|\mathcal{S}(k)|^2\rangle}{\langle|\mathcal{S}(k)|^2\rangle + \langle|\mathcal{R}(k)|^2\rangle} \simeq \frac{4\,V'^2_{ak}}{9\,H_k^4\phi_{ak}^2}$$

- Constraint

$$\boxed{\begin{aligned} H &\gtrsim 4.6\,\mathrm{eV}\left(\frac{m_a}{\mathrm{eV}}\right)\left(\frac{10^{-3}}{\alpha}\right)^{1/4} \\ \Lambda_{\rm inf} &= \left(\frac{3H^2M_{\rm pl}^2}{8\pi}\right)^{1/4} \gtrsim 140\,\mathrm{TeV}\left(\frac{m_a}{\mathrm{eV}}\right)^{1/2}\left(\frac{10^{-3}}{\alpha}\right)^{1/8} \end{aligned}}$$


That's all Folks!

Wait…
Where are the equations?

Solving the Perturbations during inflation

- During inflation, taking the newtonian gauge

$$ds^2 = (1+2\Phi)dt^2 - a(t)^2(1-2\Psi)d\vec{x}\cdot d\vec{x}$$

- For all the fields during inflation:

$$\dot{\Phi} + H\Phi = 4\pi G \sum_{j=1}^{N} \dot{\phi}_j \delta\phi_j,$$

$$\delta\ddot{\phi}_j + 3H\delta\dot{\phi}_j + \left(\frac{k^2}{a^2} + V_j''\right)\delta\phi_j = 4\dot{\phi}_j\dot{\Phi} - 2V_j'\Phi,$$

- Assuming slow roll and superhorizon

$$\Phi = -C_1\frac{\dot{H}}{H^2} - H\frac{d}{dt}\left(\frac{\sum_j d_j V_j}{\sum_j V_j}\right),$$

$$\frac{\delta\phi_i}{\dot{\phi}_i} = \frac{C_1}{H} - 2H\left(\frac{\sum_j d_j V_j}{\sum_j V_j} - d_i\right), \quad i,j = 1,\ldots,\tilde{N}.$$

Starobinsky and Polarski, 94’

Adiabatic and Isocurvature in Inflation

- The solution can be written as

$$\frac{\delta\phi_i}{\dot{\phi}_i} = \frac{C_1}{H} + 2\,HC_3\frac{V_a}{V_a + V_i}$$

$$\frac{\delta\phi_a}{\dot{\phi}_a} = \frac{C_1}{H} - 2\,HC_3\frac{V_i}{V_a + V_i}$$

$$\Phi = -C_1\frac{\dot{H}}{H^2} + \frac{C_3}{3}\frac{V_i V_a'^2 + V_a V'^2}{(V_a + V_i)^2}$$

- Where the constants are determined by initial conditions (Bunch-Davies)

$$\delta\phi_j(k,t) = \frac{H(t_k)}{\sqrt{2k^3}}e_j(\mathbf{k})$$

$$\left\langle e_j(\mathbf{k})e^*_{j'}(\mathbf{k}')\right\rangle = \delta_{jj'}\delta^{(3)}(\mathbf{k}-\mathbf{k}')$$

$$\xrightarrow[\substack{V_a' \gg V_i' \\ V_i \gg V_a}]{}$$

$$C_1 \simeq -\frac{8\pi G H_k}{\sqrt{2k^3}}\frac{V_i}{V_i'}e_i(\mathbf{k}) = -\frac{3H_k^3}{\sqrt{2k^3}V_i'}e_i(\mathbf{k})$$

$$C_3 \simeq -\frac{3H_k}{2\sqrt{2k^3}V_i'}e_i(\mathbf{k}),$$

Transition to Hot Big Bang

- After inflation, the C_3 contribution to Φ vanishes. And so

$$\boxed{C_1 = \text{adiabatic, } C_3 = \text{isocurvature}}$$

(Assumes axion vacuum energy is subdominant during inflation)

- Together the two modes give

Adiabatic

$$\begin{aligned} \delta_\gamma &= -2\Phi, \\ \delta_b &= \frac{3}{2}\delta_\nu = \frac{3}{4}\delta_\gamma \\ \delta_a &= S + \frac{3}{4}\delta_\gamma, \end{aligned}$$

Isocurvature

Primordial Initial Conditions

- For the CMB calculation, S and Φ are the initial conditions.

- Adiabatic mode

$$\frac{1}{4}\delta_\gamma = \frac{1}{4}\delta_\nu = \frac{1}{4}\delta_b = \frac{1}{3}\delta_a = R \simeq C_1 \qquad (R \simeq const)$$

$$\mathcal{R} \simeq -\frac{8\pi G H_k}{\sqrt{2k^3}} \sum_j \frac{V_j}{V_j'} e_j(\mathbf{k}) \simeq -\frac{3H_k^3}{\sqrt{2k^3}V_i'} e_i(\mathbf{k})$$

- Isocurvature mode

$$S = 2\left.\frac{\delta\phi_a}{\phi_a}\right|_{C_1\to 0} = \frac{4}{3}\frac{V'(\phi_a)_k C_3}{\phi_{ak}} = -2\frac{V'(\phi_a)_k}{\phi_{ak}}\frac{H_k}{\sqrt{2k^3}V'(\phi_i)_k} e_i(\mathbf{k}) \qquad \left(\frac{\delta\phi_a}{\phi_a} \simeq const\right)$$

Results

Power Spectra

- Power spectra

$$\Delta^2_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\left\langle \mathcal{R}^2 \right\rangle = \frac{H_k^2}{8M_{Pl}^2\pi^2\epsilon_k}$$

$$\Delta^2_{S}(k) = \frac{k^3}{2\pi^2}\left\langle S^2 \right\rangle = \frac{V'(\phi_a)_k^2 H_k^2}{\pi^2 V'(\phi_i)_k^2 \phi_{ak}^2} = \frac{4}{9}\frac{V'(\phi_a)_k^2}{H_k^4\phi_{ak}^2}\,\Delta^2_{\mathcal{R}}$$

$$\Delta^2_{S\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\left\langle S\,\mathcal{R} \right\rangle = \frac{3H_k^4 V'(\phi_a)_k}{2\pi^2 V'(\phi_i)_k^2\phi_a} = \frac{2V'(\phi_a)_k}{3H_k^2\phi_{ak}}\Delta^2_{\mathcal{R}}$$

- Fully correlated! (Not surprising - only inflaton perturbation)

Spectral Indices

- Definition

$$\Delta^2_{\mathcal{R}}(k) = A^2 \left(\frac{k}{k_0}\right)^{n_{ad}-1}$$

$$\Delta^2_{S}(k) = B^2 \left(\frac{k}{k_0}\right)^{n_{iso}-1}$$

- Results

$$\begin{aligned} n_{\rm ad} - 1 &= \frac{k_0}{A^2}\,\frac{d\Delta^2_{\mathcal{R}}(k)}{dk}\bigg|_{k=k_0} \\ &= \frac{k_0}{A^2}\frac{1}{8M^2_{\rm Pl}\pi^2}\,\frac{d(H^2_k/\epsilon_k))}{dk}\bigg|_{k=k_0} \\ &\simeq -6\,\epsilon_i + 2\,\eta_i - 4\epsilon_a \simeq 2\,\eta_i, \end{aligned}$$

$$\begin{aligned} n_{\rm iso} - 1 &= \frac{k_0}{B^2}\,\frac{d\Delta^2_{S}(k)}{dk}\bigg|_{k=k_0} \\ &= \frac{k_0}{\pi^2 B^2}\,\frac{d(H^2_k V'(\phi_a)_k/V'(\phi_i)^2_k\phi^2_a))}{dk}\bigg|_{k=k_0} \\ &\simeq 2\Big(\eta_i - \epsilon_a - \epsilon_i\Big) \sim (n_{\rm ad}-1), \end{aligned}$$

Constraints

- The isocurvature ratio

$$\alpha \equiv \frac{\langle |\mathcal{S}(k)|^2 \rangle}{\langle |\mathcal{S}(k)|^2 \rangle + \langle |\mathcal{R}(k)|^2 \rangle} \simeq \frac{4\, V_{ak}'^2}{9\, H_k^4 \phi_{ak}^2}$$

- The bounds on (anti-)correlated isocurvature from Planck

$$\alpha_{\mathrm{low-scale}} \lesssim 10^{-3}$$

$$H \gtrsim 4.6\,\mathrm{eV} \left(\frac{m_a}{\mathrm{eV}}\right) \left(\frac{10^{-3}}{\alpha}\right)^{1/4}$$

$$\Lambda_{\mathrm{inf}} = \left(\frac{3H^2 M_{\mathrm{pl}}^2}{8\pi}\right)^{1/4} \gtrsim 140\,\mathrm{TeV} \left(\frac{m_a}{\mathrm{eV}}\right)^{1/2} \left(\frac{10^{-3}}{\alpha}\right)^{1/8}$$

Relic Abundance

- There is a second bound: if the axion moves - it can roll to the bottom!
- Model dependent - how long inflation lasts before the CMB modes exit the horizon.
- Taking the conservative approach of the shortest possible inflation:

$$\theta_{\mathrm{I}} = \theta_i e^{\frac{-m_a^2 N}{3H^2}}$$

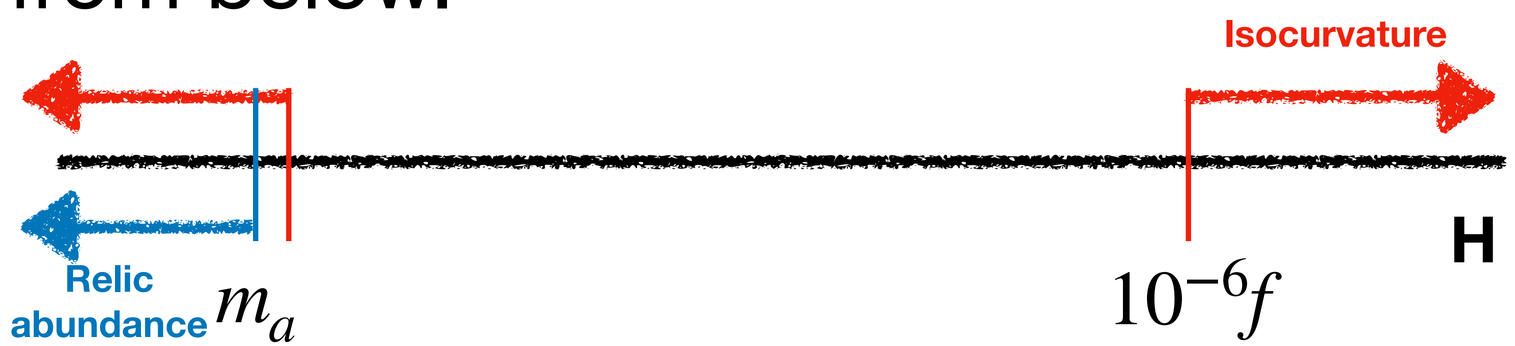
- And so we get a bound around

$$H \gtrsim 1.2\, m_a \left(\frac{N}{30}\right)^{1/2}$$

- Slightly weaker, but the isocurvature bound can be improved in the future!

Conclusion

- Bounds on the scale of inflation not only from above, but also from below.



- A new source of fully anti-correlated axion isocurvature

- The mechanism is general for any production mechanism before inflation.