Probing ultralight dark matter with Interferometers

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What can ***GW interferometers*** tell us about
the nature of dark matter (more specifically ***ultralight dark matter***)?

Ultralight dark matter (ULDM)

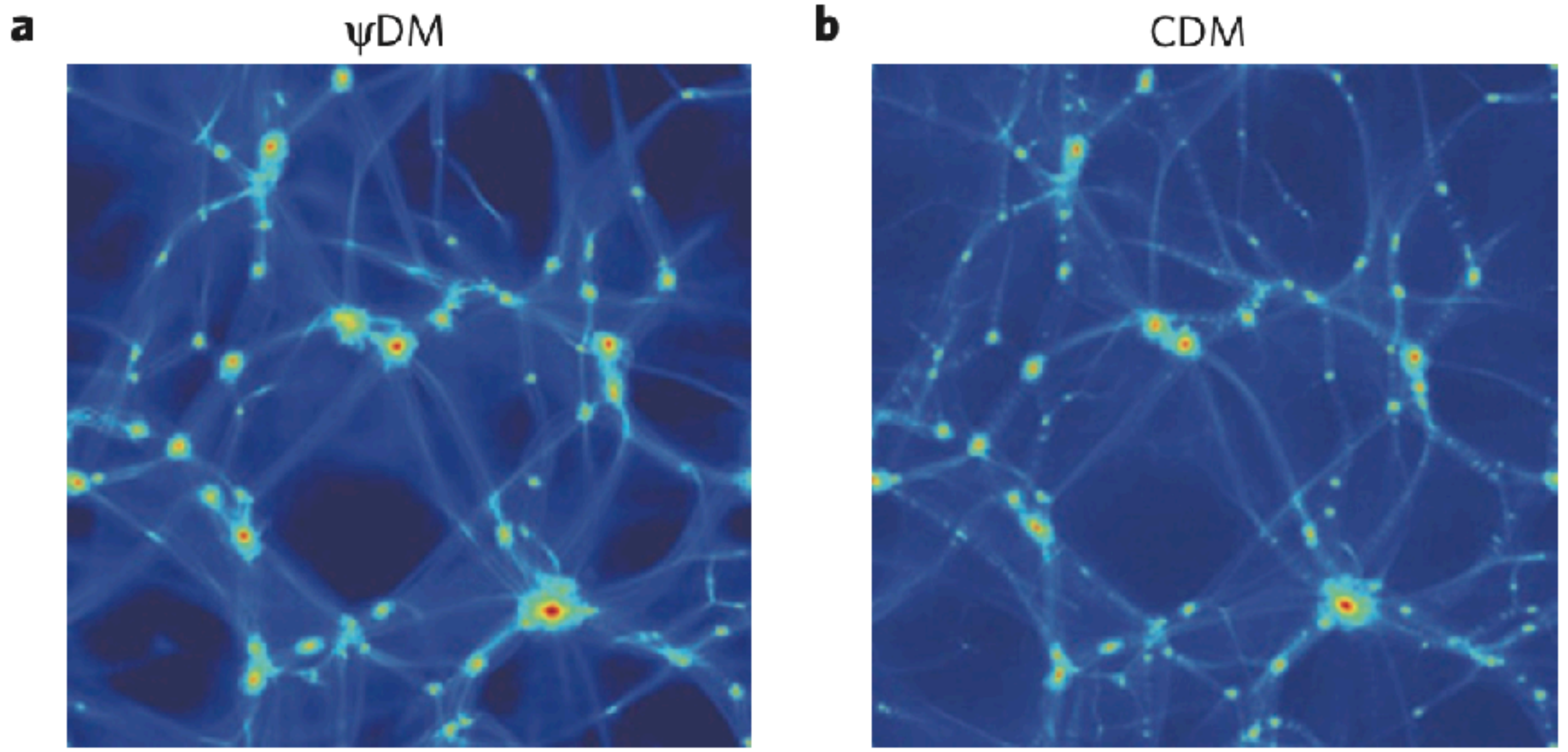
Terminology:

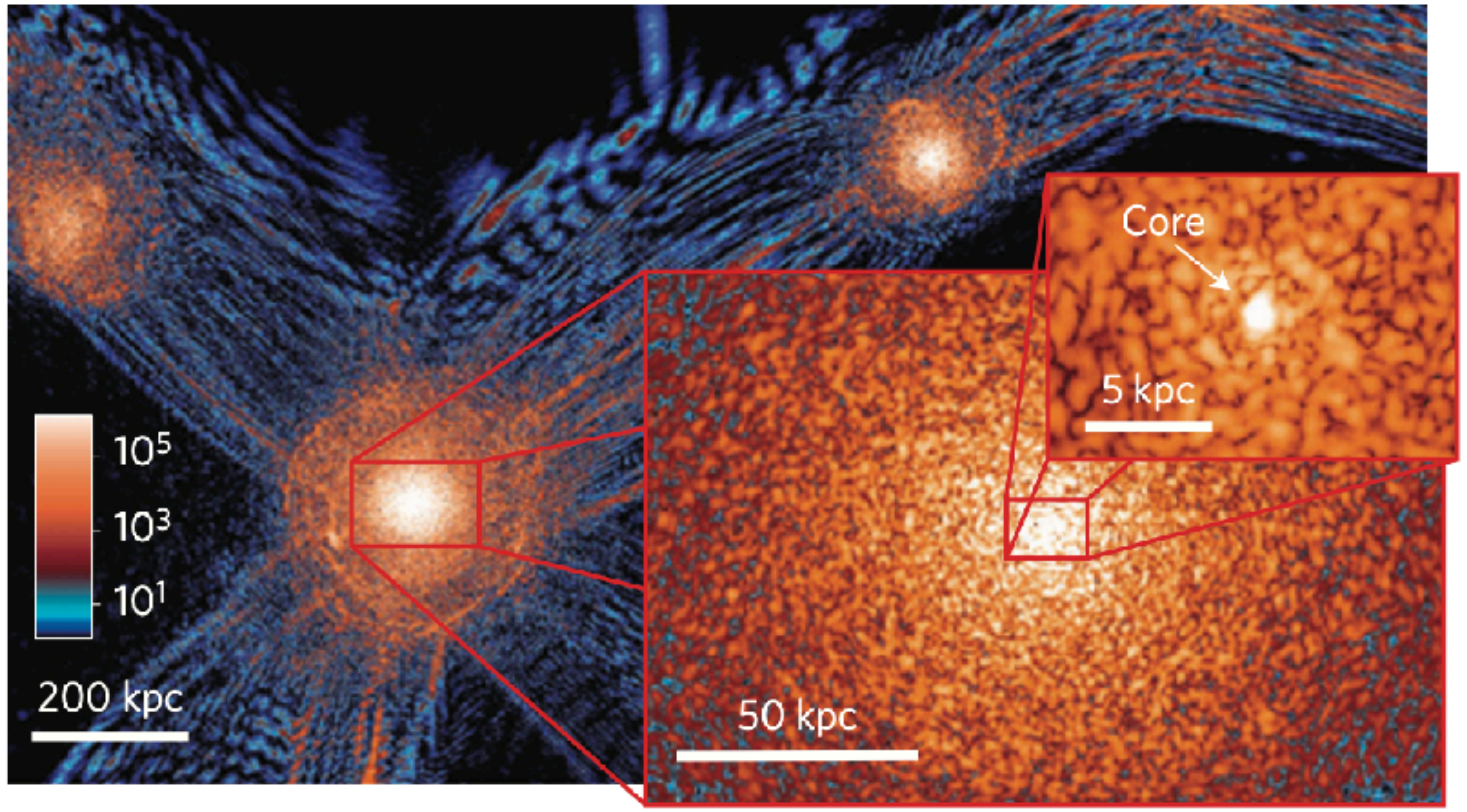
Ultralight (wave) dark matter

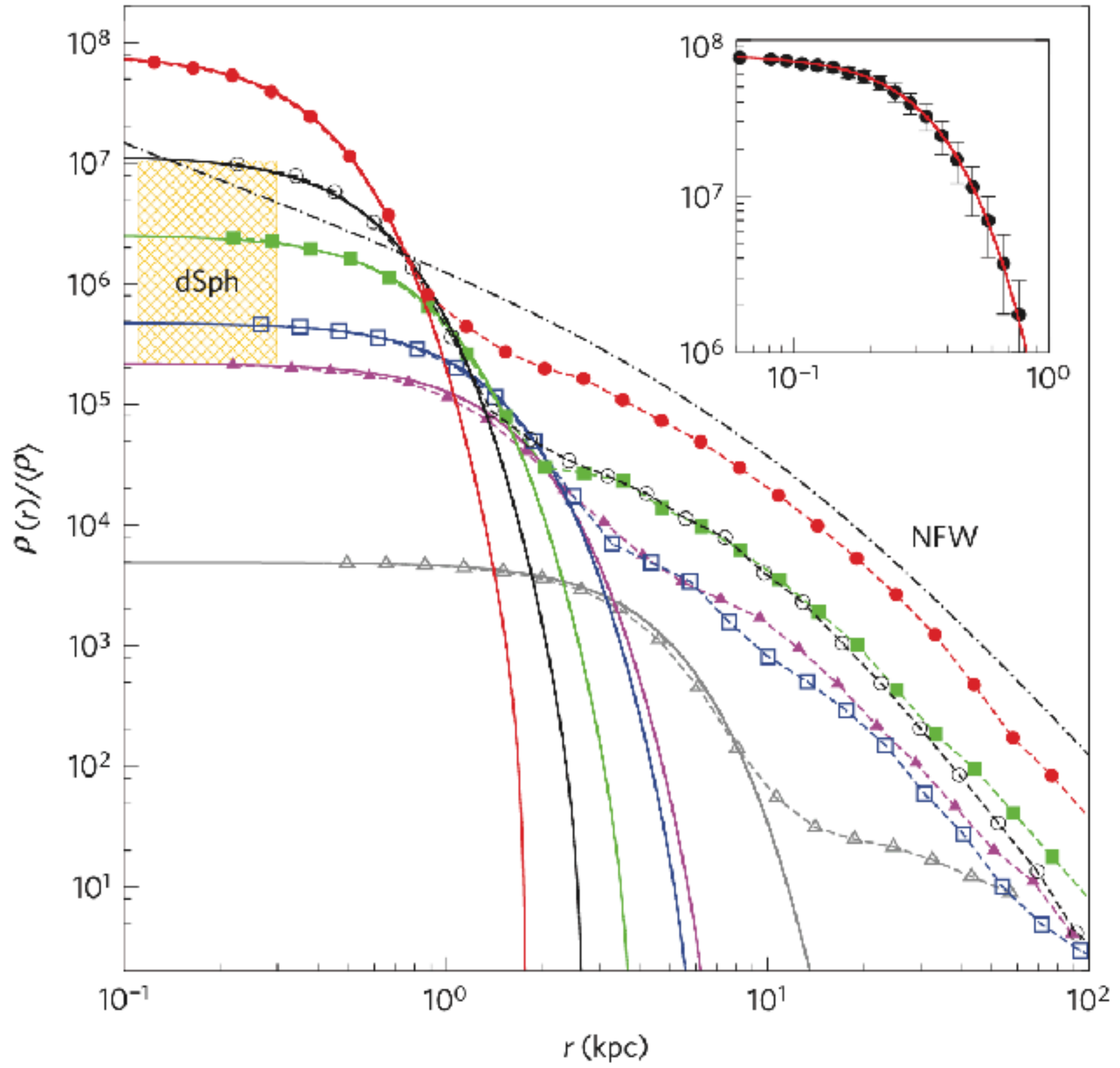
$$m \lesssim 10\,\mathrm{eV}$$

$$N_{\mathrm{occ}} \sim n_{\mathrm{dm}} \lambda^3 \sim \left(\frac{10\,\mathrm{eV}}{m}\right)^4$$

Schive, Chiueh, Broadhurst (14)

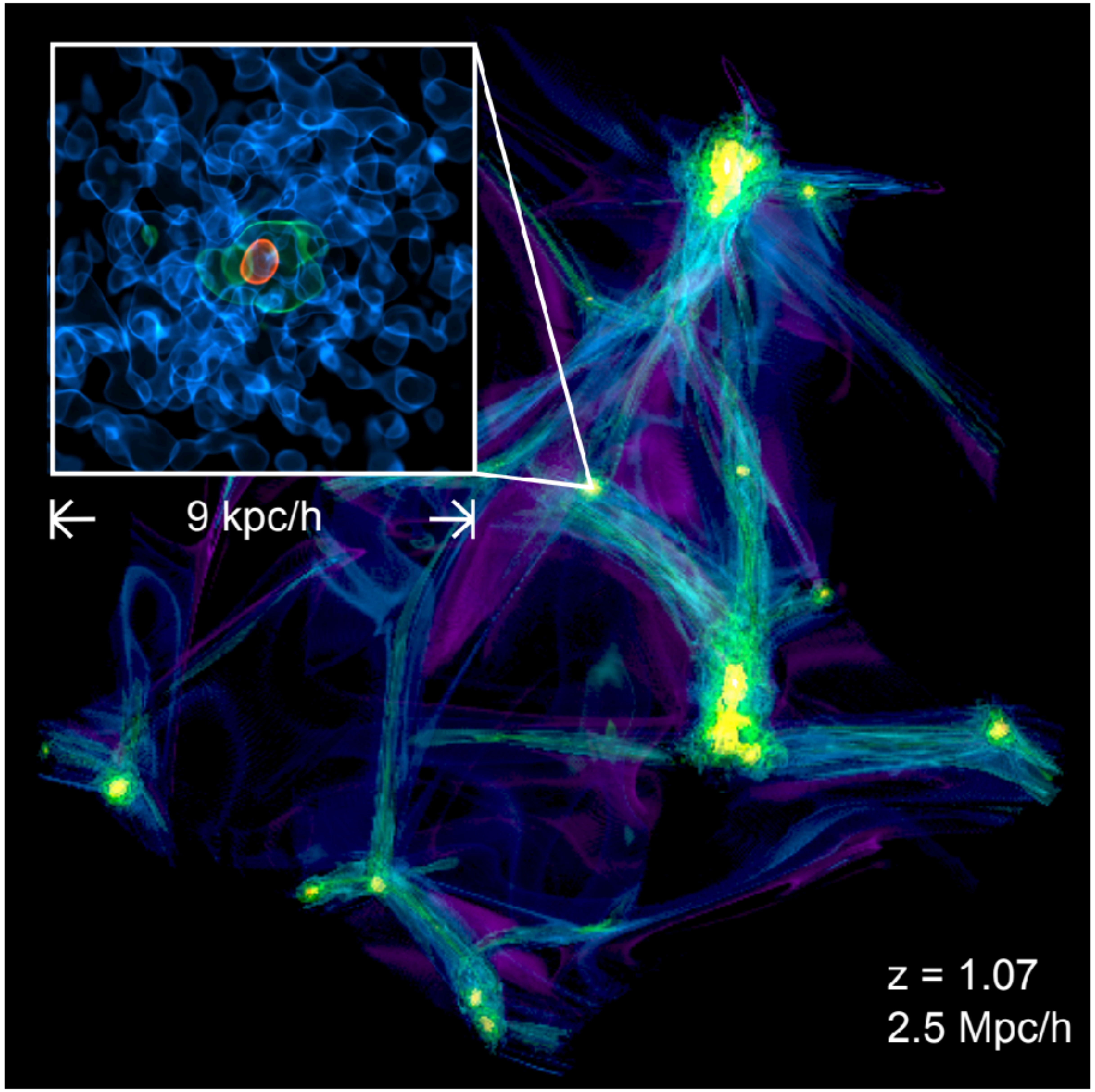






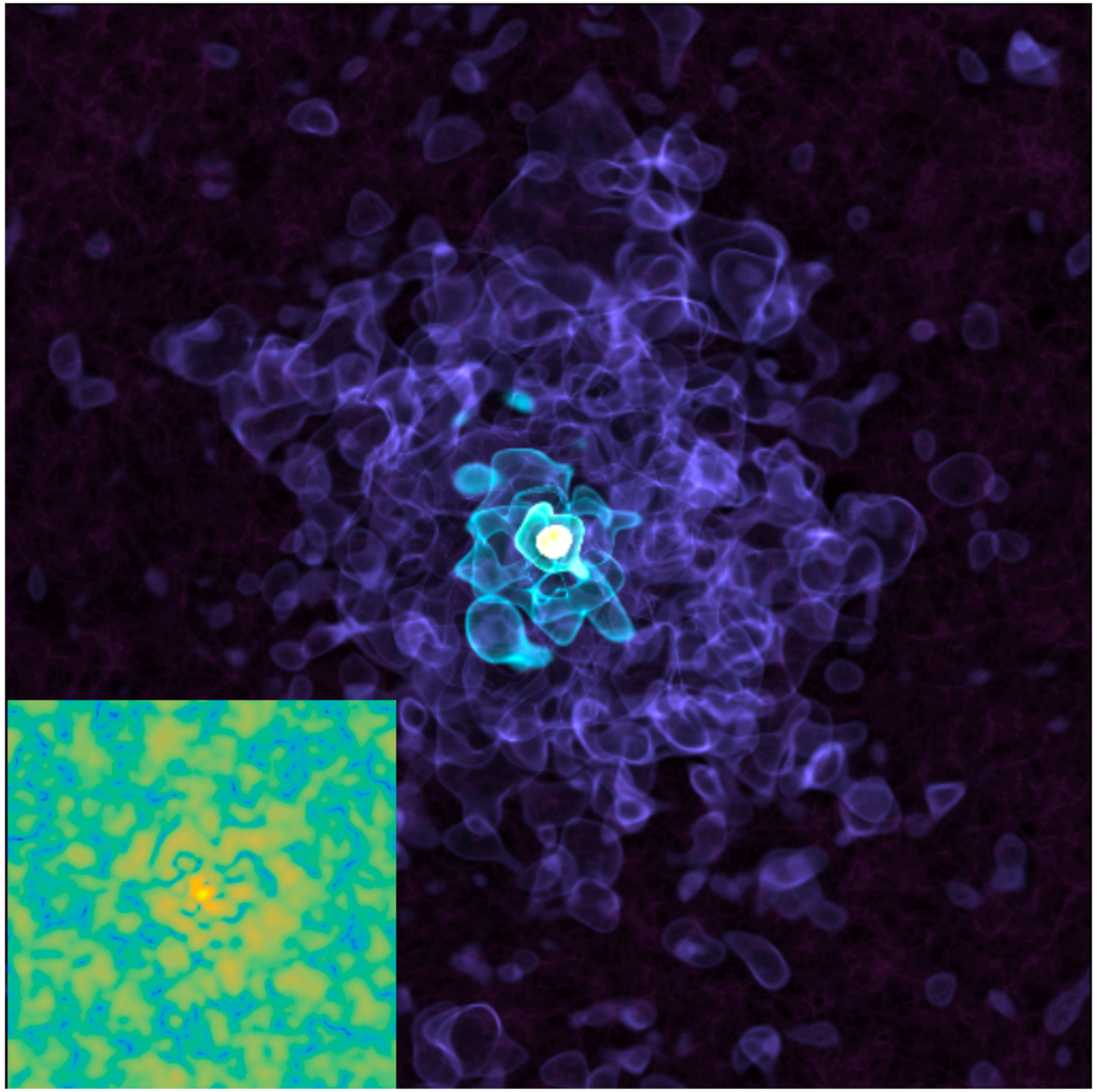
* characteristic *soliton* at the center has been observed

* small scale structures are erased



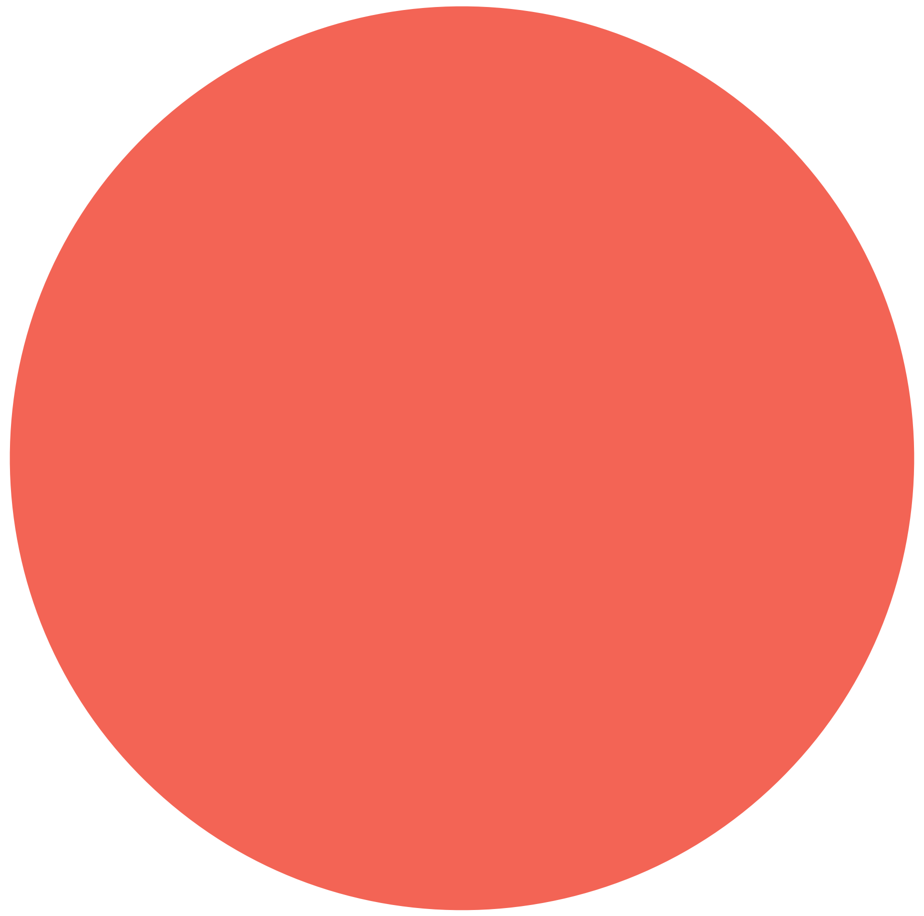
Veltmaat, Niemeyer, Schwabe (18)

Mocz et al (17)



An intuitive understanding of the granule structure:

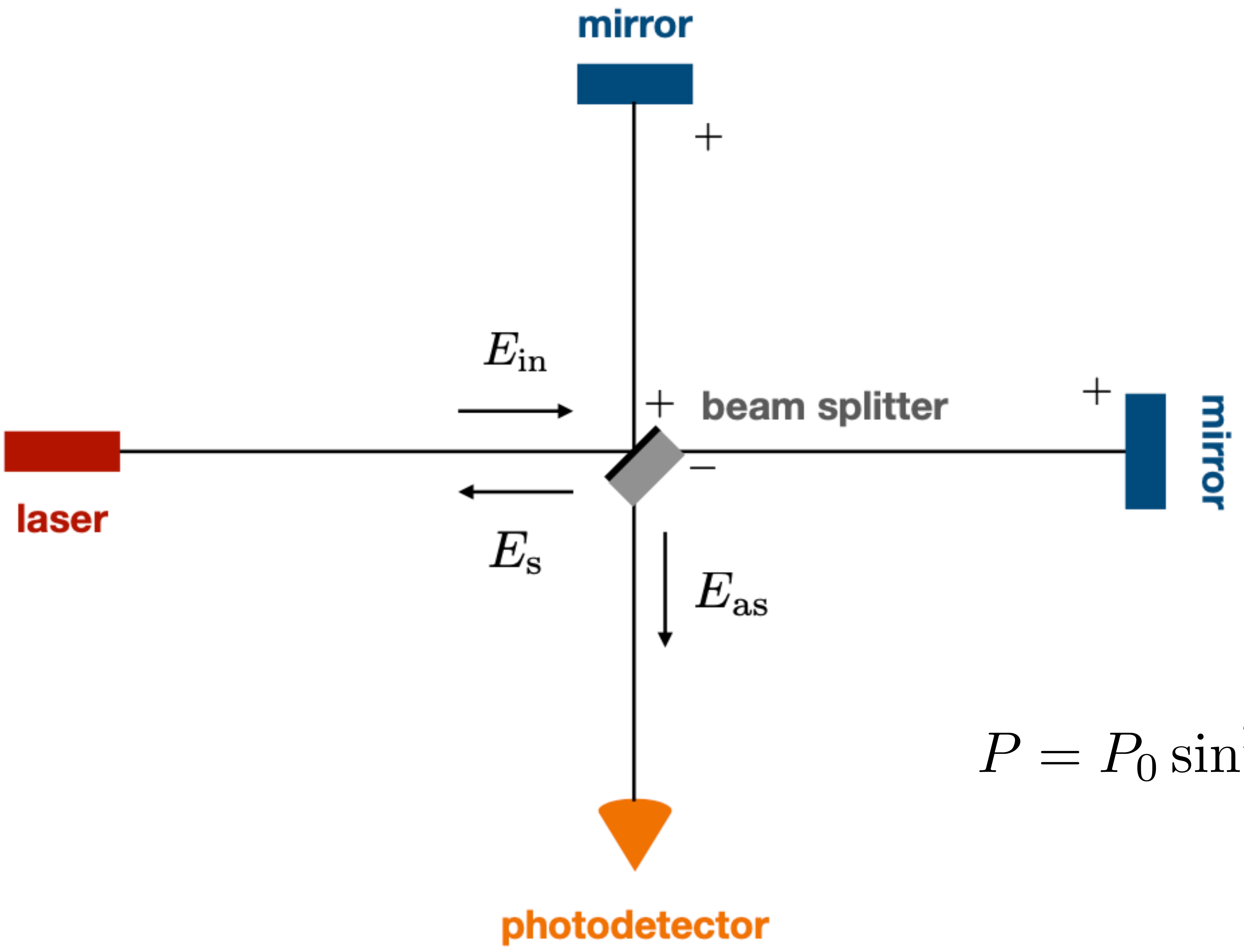
Quasiparticle



$$\ell \sim \lambda = \frac{1}{mv}$$

$$m_{\rm eff} \sim \rho_{\rm DM} \ell^3$$

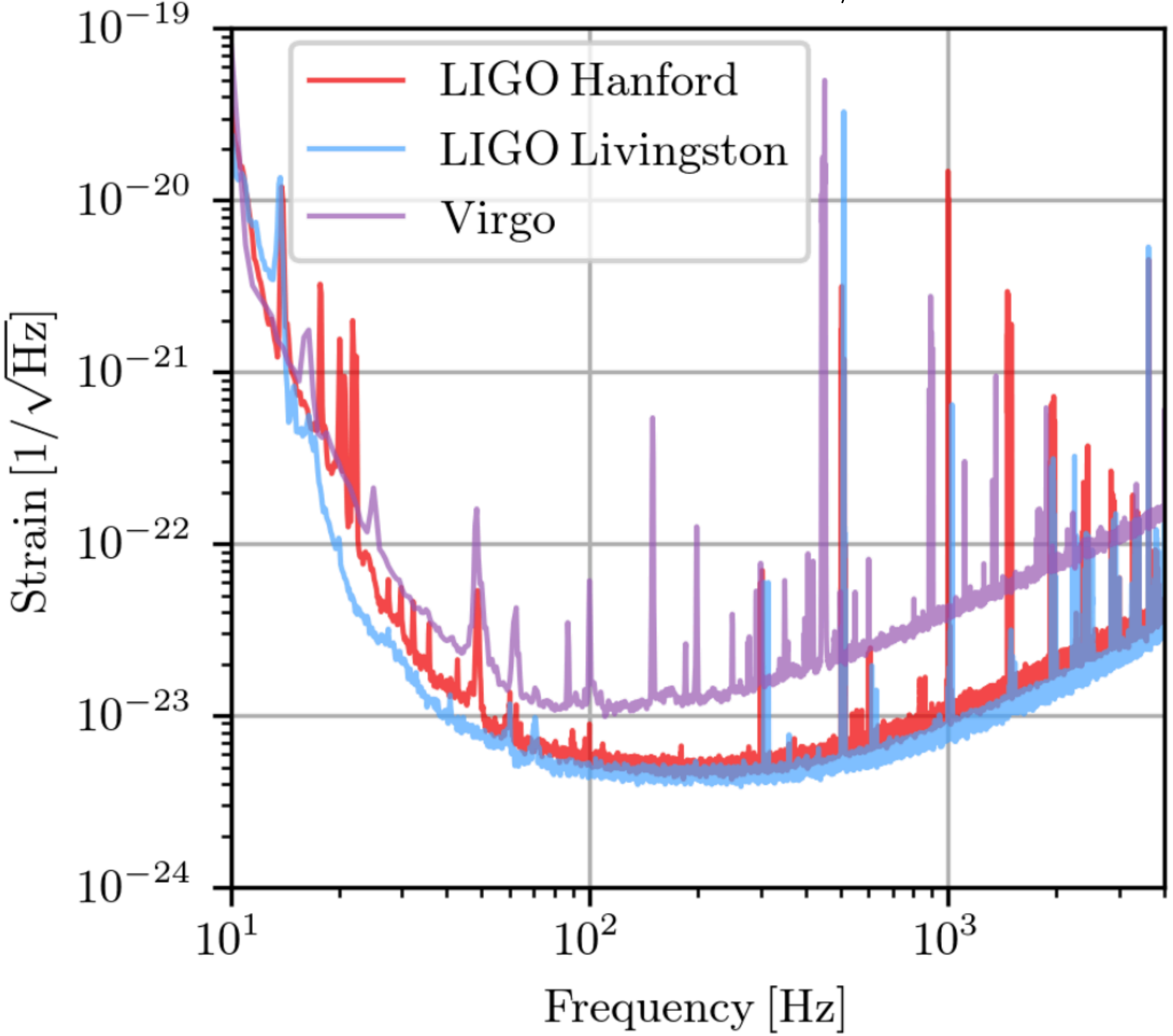
GW interferometers



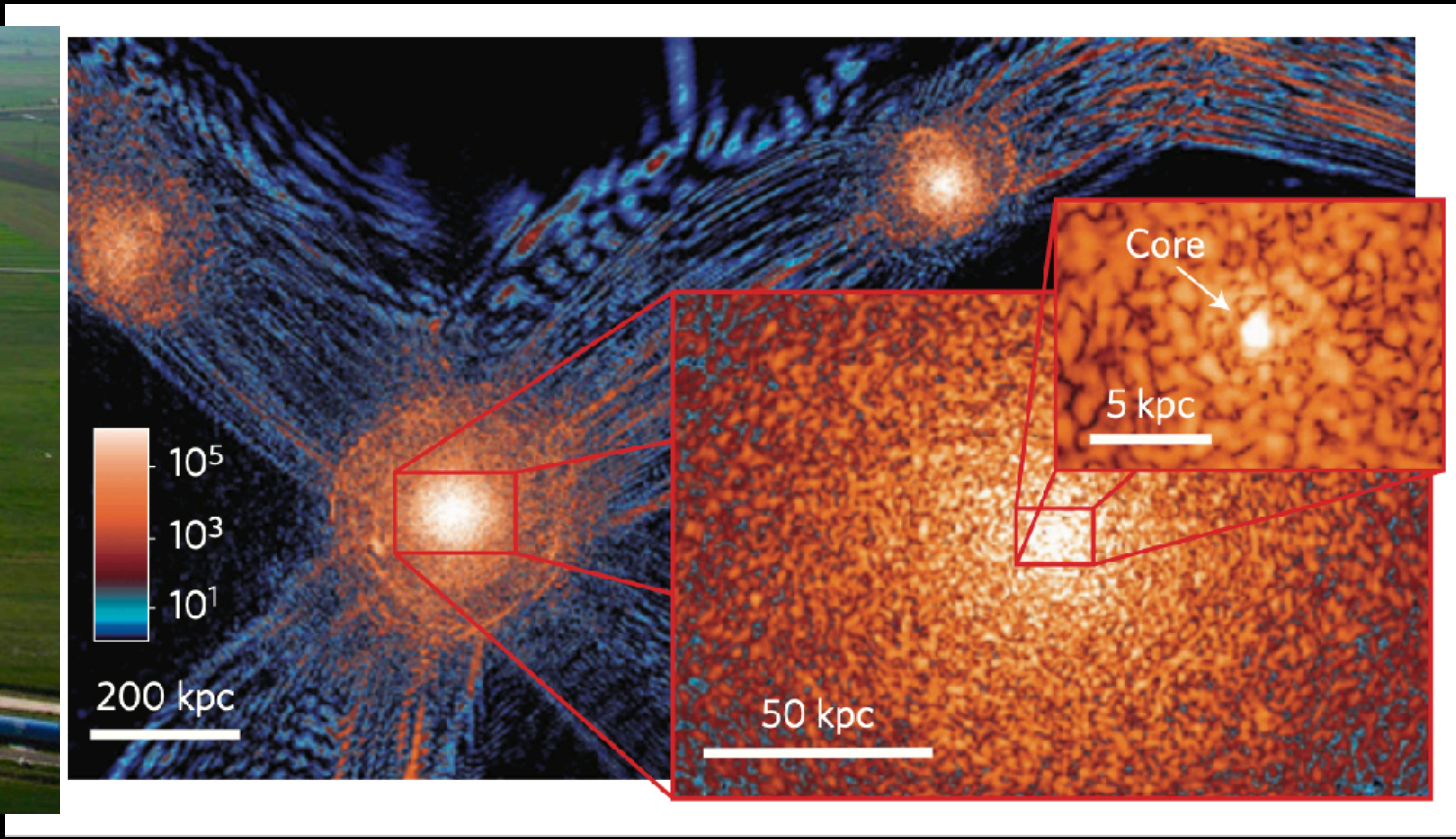
$$P = P_0 \sin^2(k\Delta L)$$

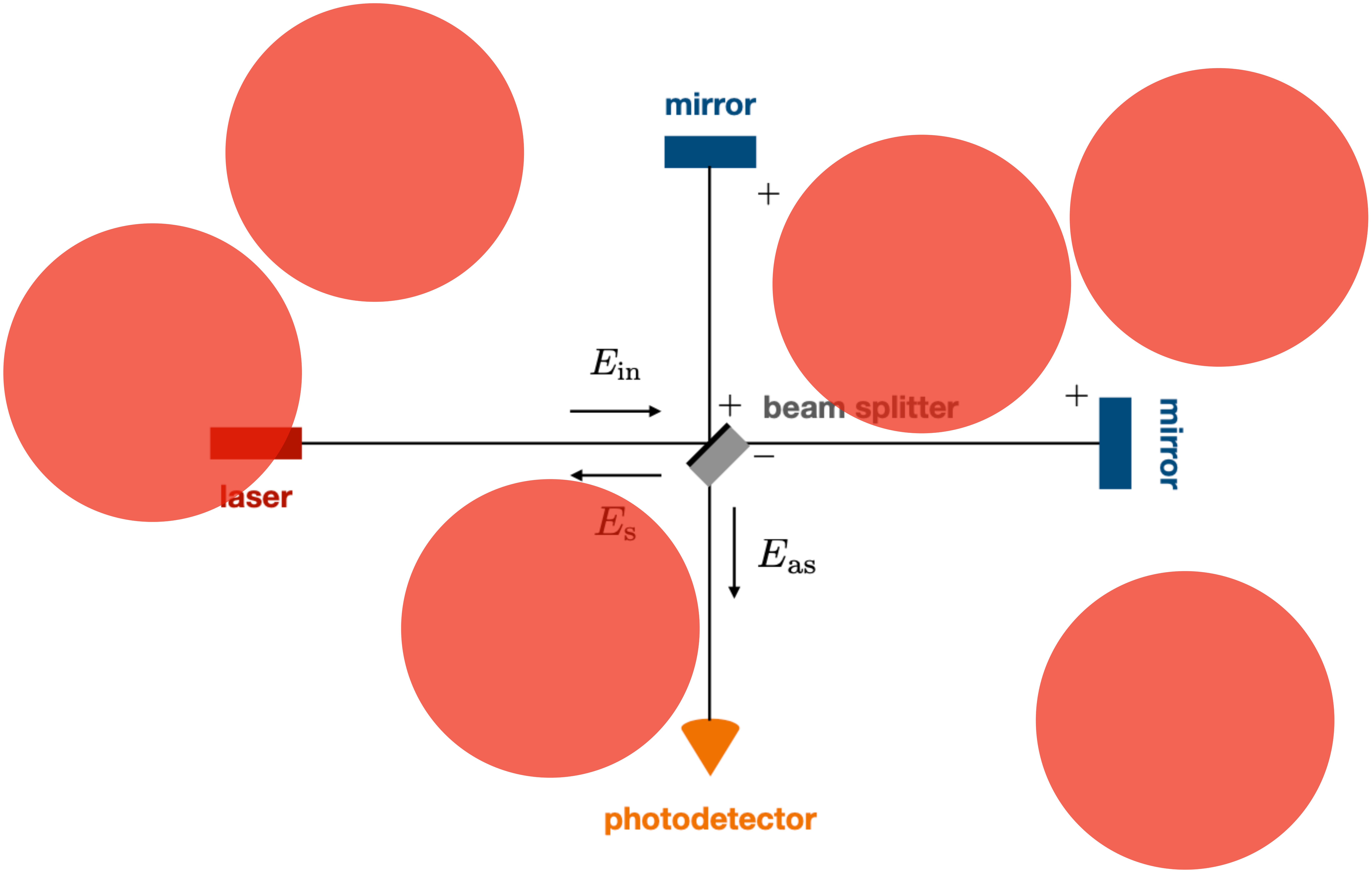
$$S_n^{1/2}(f) \sim S_{\Delta L/L}^{1/2}(f)$$

$$\langle x^2 \rangle = \int df \, S_x(f)$$



Imagine placing these sensitive GW interferometers in the sea of ultralight dark matter




mirror
+
$E_{\rm in}$
+
beam splitter
+
−
mirror
laser
$E_{\rm s}$
$E_{\rm as}$
photodetector

Two questions:

*1. Do we have to worry about **ULDM-induced noise***
in the current and future gravitational waves?

2. Can current and future GW interferometers
probe ultralight dark matter gravitationally?

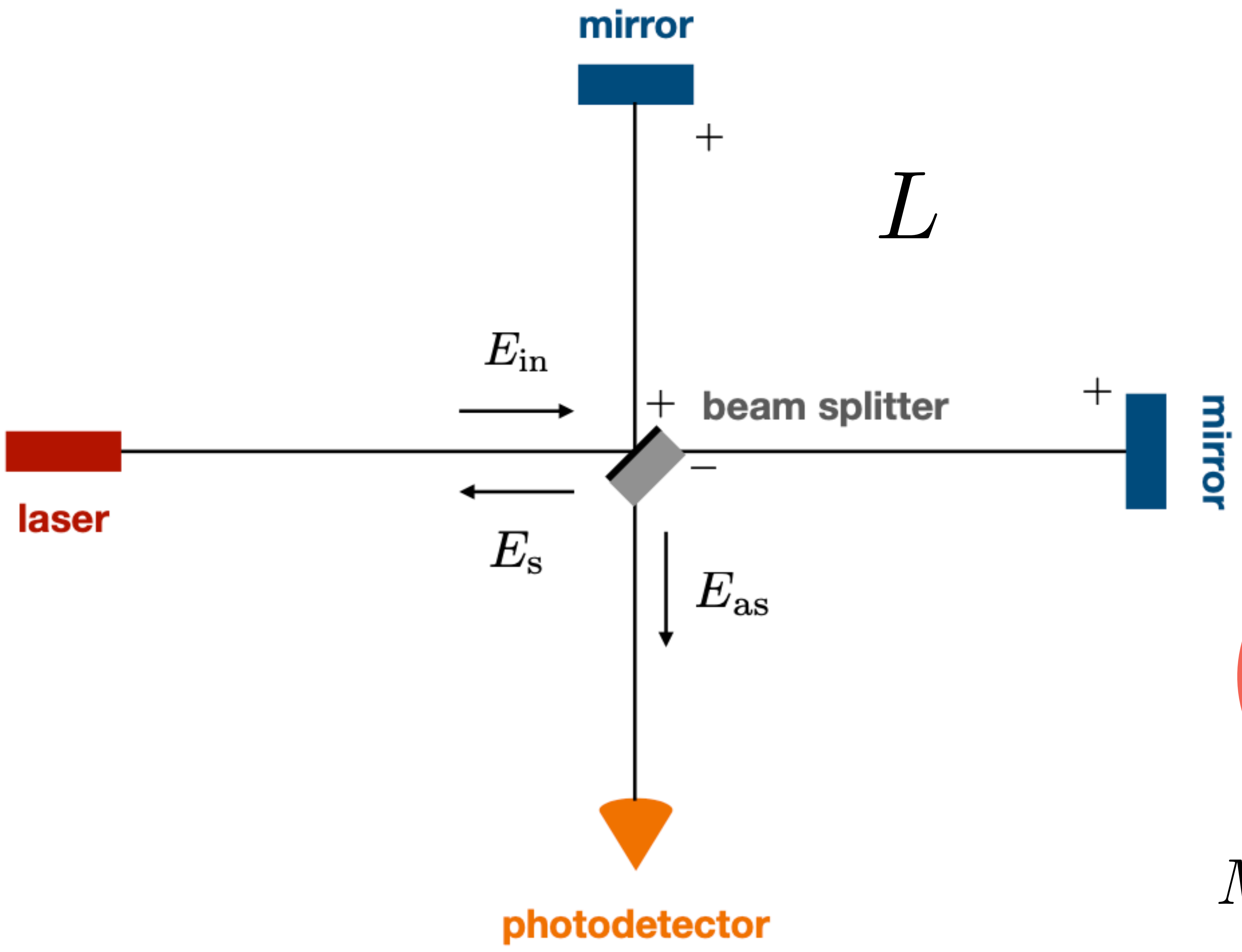
[For ULDM-SM interaction with GW interferometer
Grote & Stadnik 19]

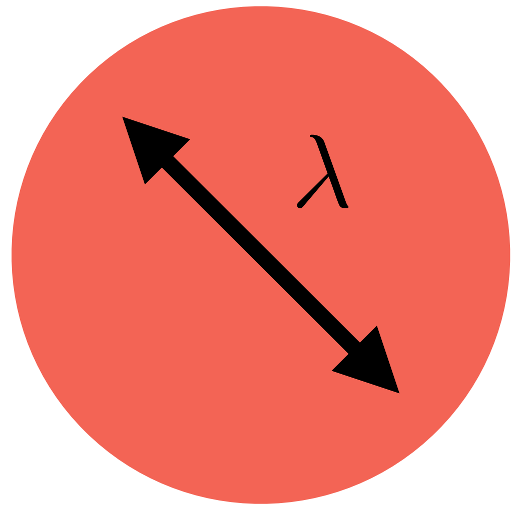
$\lambda = 1/mv$

Back-of-envelope estimation

$$\Delta a = a_1 - a_2 = \frac{GM_{\text{eff}}}{(L+\lambda)^2} - \frac{GM_{\text{eff}}}{\lambda^2}$$

$$\simeq \frac{GM_{\text{eff}}}{\lambda^2} = G\rho\lambda$$





$M_{\text{eff}} = \rho\lambda^3$

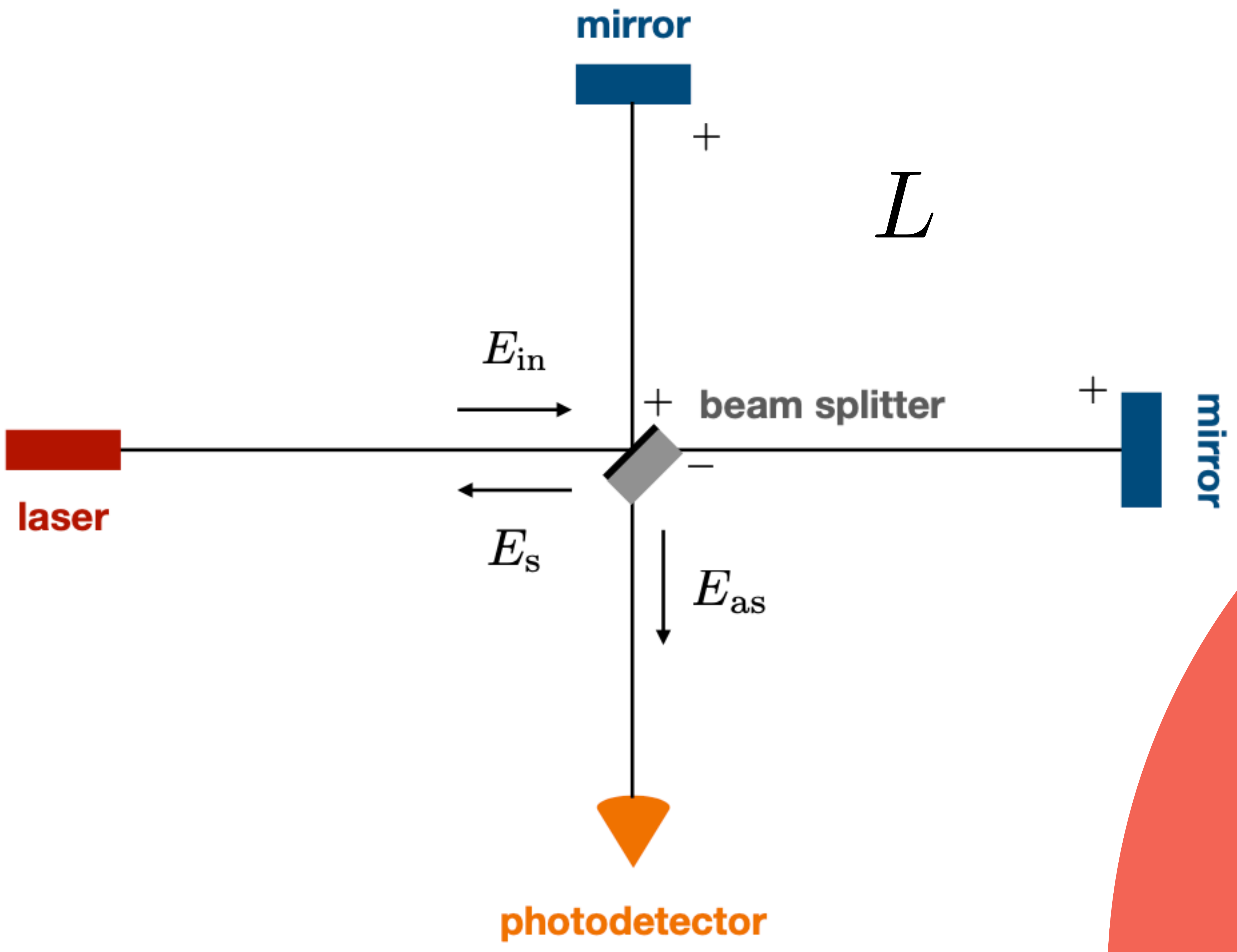
$\lambda = 1/mv$

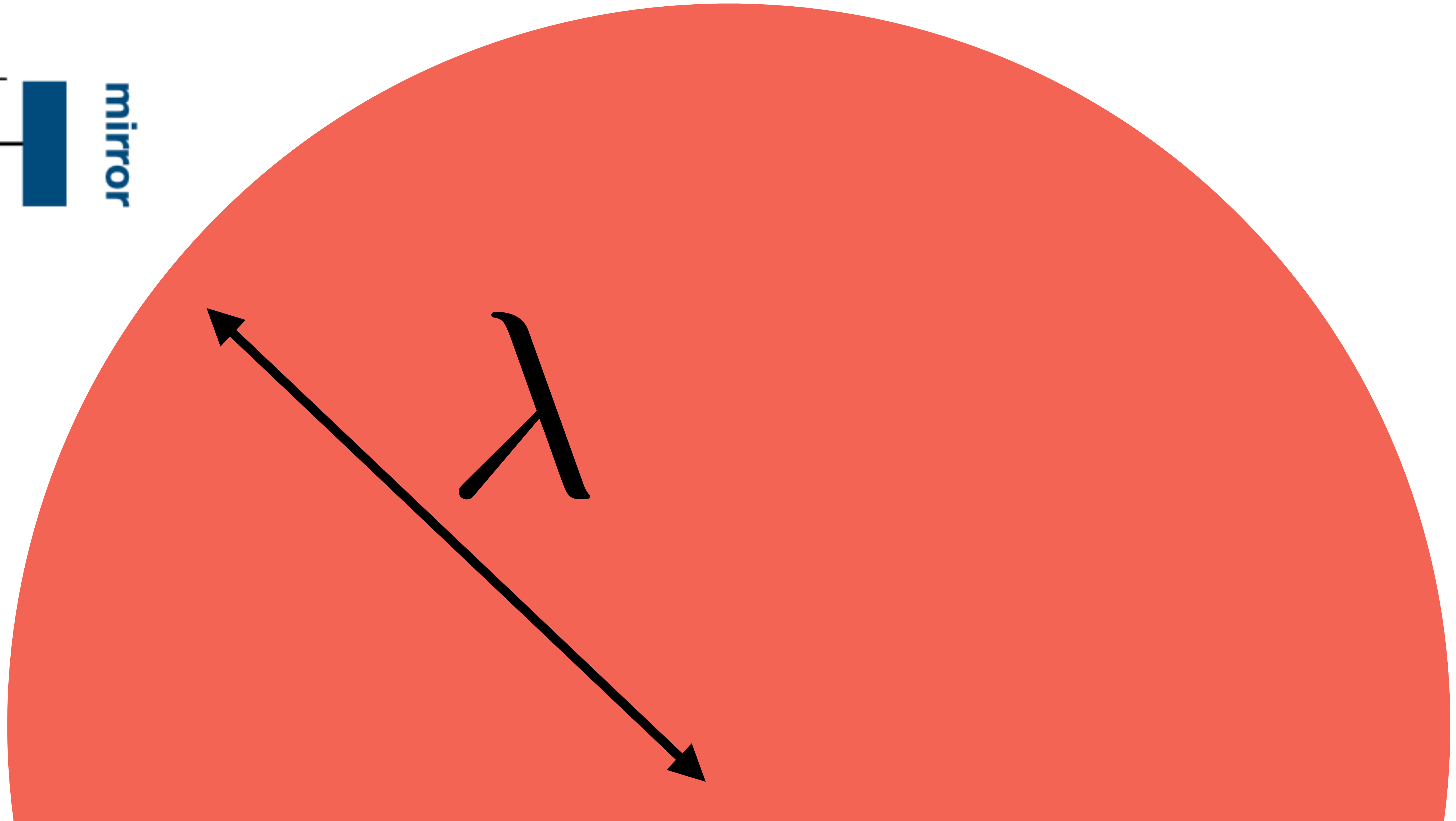
Back-of-envelope estimation

$$\Delta a = a_1 - a_2 = \frac{GM_{\rm eff}}{(L+\lambda)^2} - \frac{GM_{\rm eff}}{\lambda^2}$$

$$\simeq \frac{GM_{\rm eff}}{\lambda^2} = G\rho\lambda$$

$$\simeq \frac{GM_{\rm eff}}{\lambda^2}\frac{L}{\lambda} = G\rho L$$





$M_{\rm eff} = \rho\lambda^3$

Back-of-envelope estimation

$$\Delta a \simeq G\rho L \quad \simeq 10^{-28}\,\mathrm{m\,s^{-2}}$$

$L = 4\,\mathrm{km}$

LIGO VIRGO …

$$\simeq 10^{-22}\,\mathrm{m\,s^{-2}}$$

$L = 2.5\mathrm{M\,km}$

LISA

$$\simeq 10^{-20}\,\mathrm{m\,s^{-2}}$$

$L = 400\mathrm{M\,km}$

μAres or Asteroid?

[Sesana et al (19)]
[Fedderke et al (21)]

$$\Delta a \sim [S_h(2\pi f)^4 L^2 \Delta f]^{1/2}$$

$\sim 10^{-14}\,\mathrm{m\,s^{-2}}$ LIGO (~50 Hz)

$\sim 10^{-16}\,\mathrm{m\,s^{-2}}$ LISA (~0.1 mHz)

$\sim 10^{-18}\,\mathrm{m\,s^{-2}}$ μAres strawman mission concept (~ μHz) *to completely answer our questions, we need*

detailed computation for noise power spectrum from ultralight dark matter

Some statistical properties of ULDM

$$\phi = \sum_i \frac{1}{\sqrt{2mV}} \left[\alpha_i e^{-ikx} + \alpha_i^* e^{ikx} \right]$$

random #

Wave func.

where each of them is distributed according to the following p.d.f.

$$p(\alpha_i) = \frac{1}{\pi f_i} \exp\left[-\frac{|\alpha_i|^2}{f_i} \right]$$

$$dP = \prod_i p(\alpha_i) d^2\alpha_i$$

[Derevianko 18]

[Foster, Rodd, Safdi 18]

[Center et al 20]

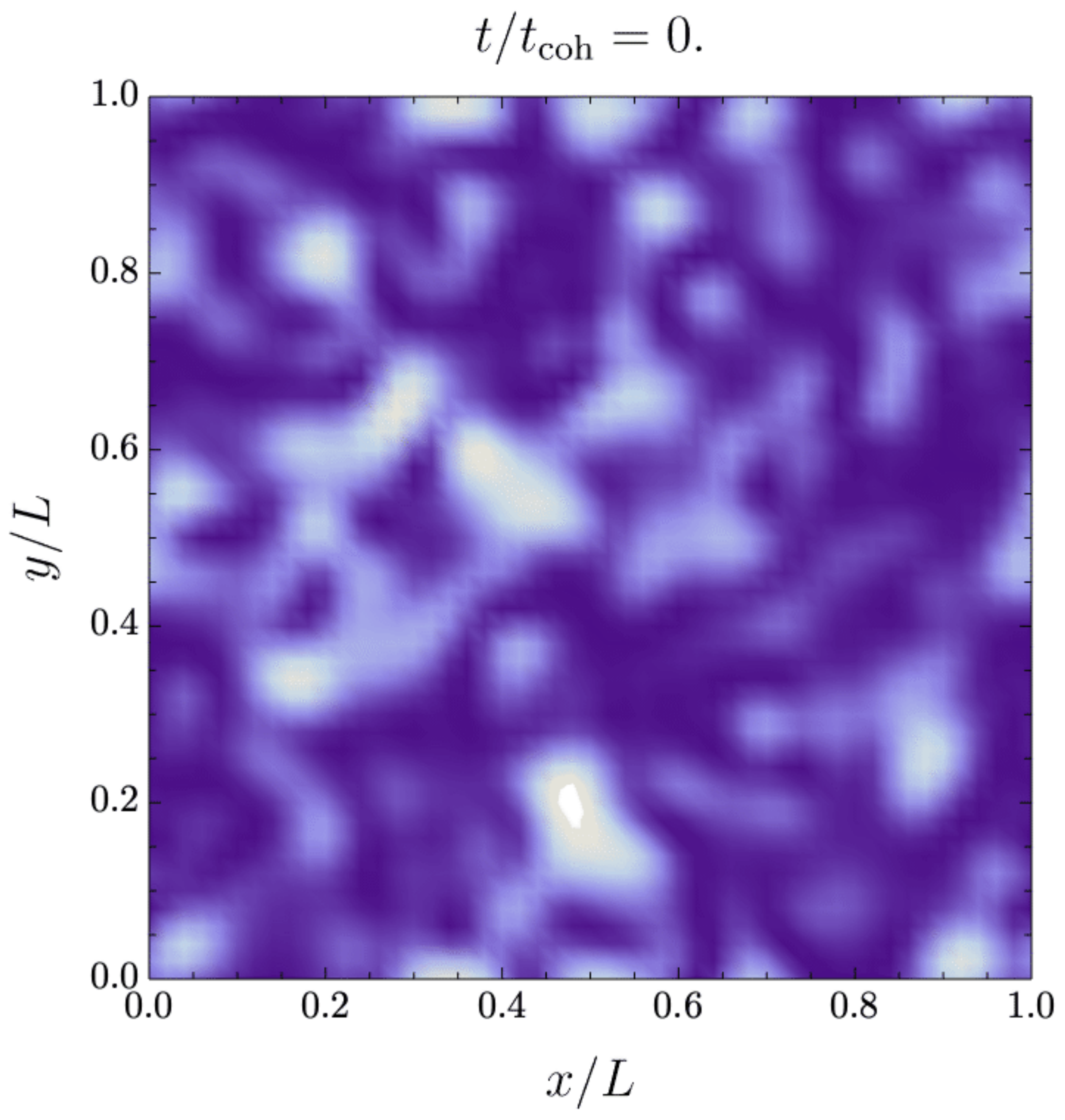
correlation function can be obtained by

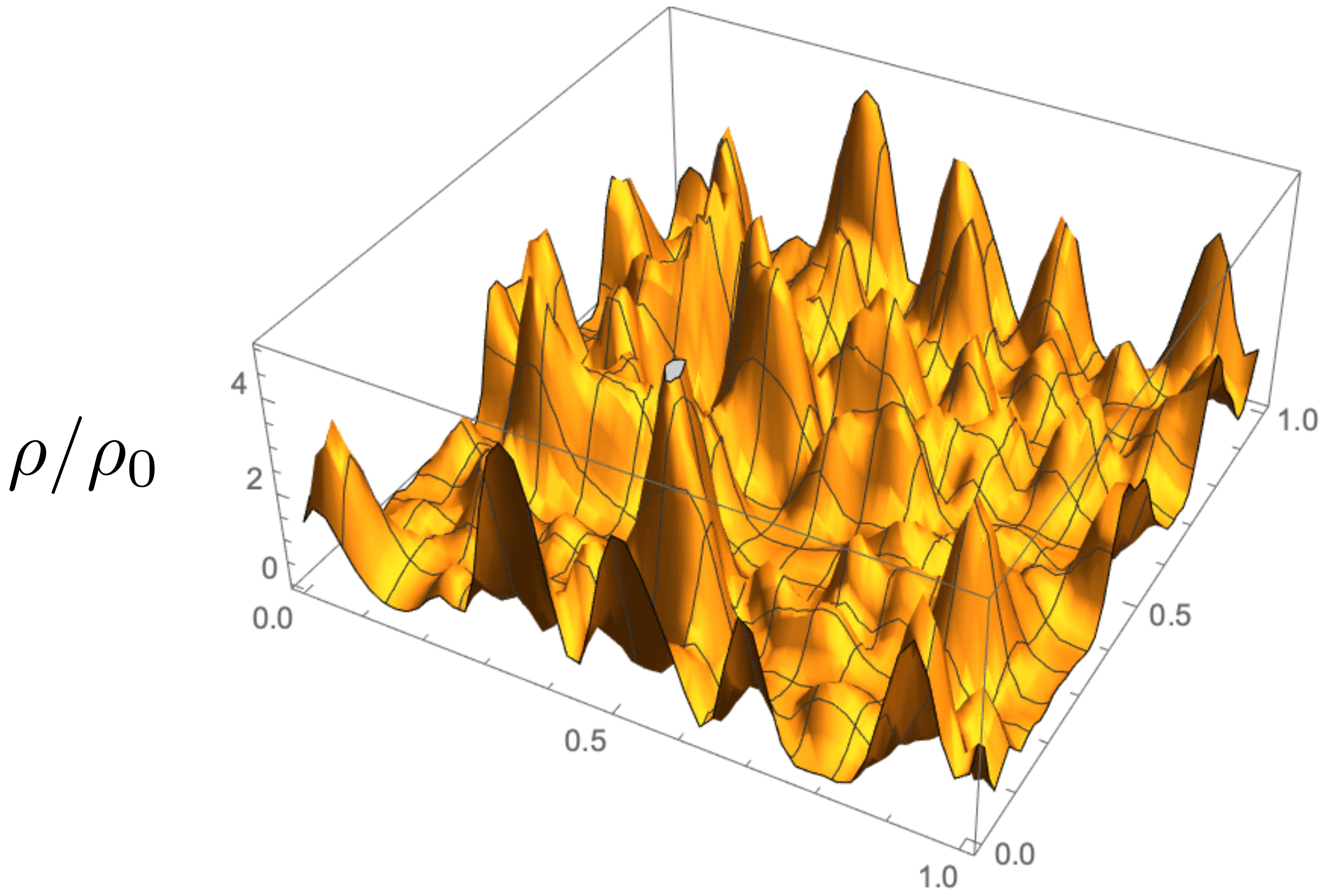
$$\langle \mathcal{O} \rangle = \int dP \, \mathcal{O}$$

From field theory
w. density operator

[Kim, Lenoci 21]

Once we obtain one particular realization of the field and compute the density of the field it would look like the following



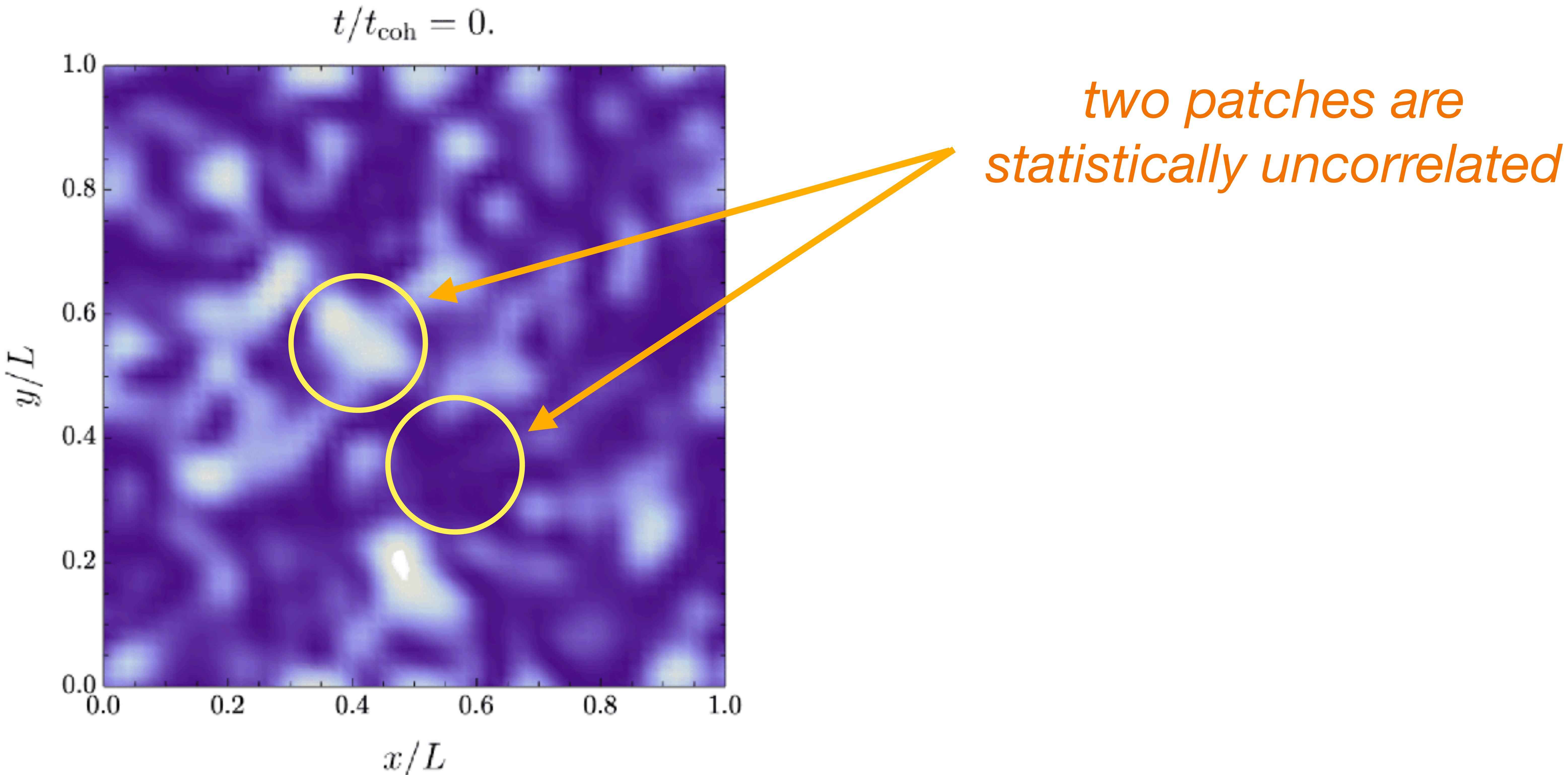


an order-one density fluctuation is very common

The statistical properties of these density clumps can be analytically investigated

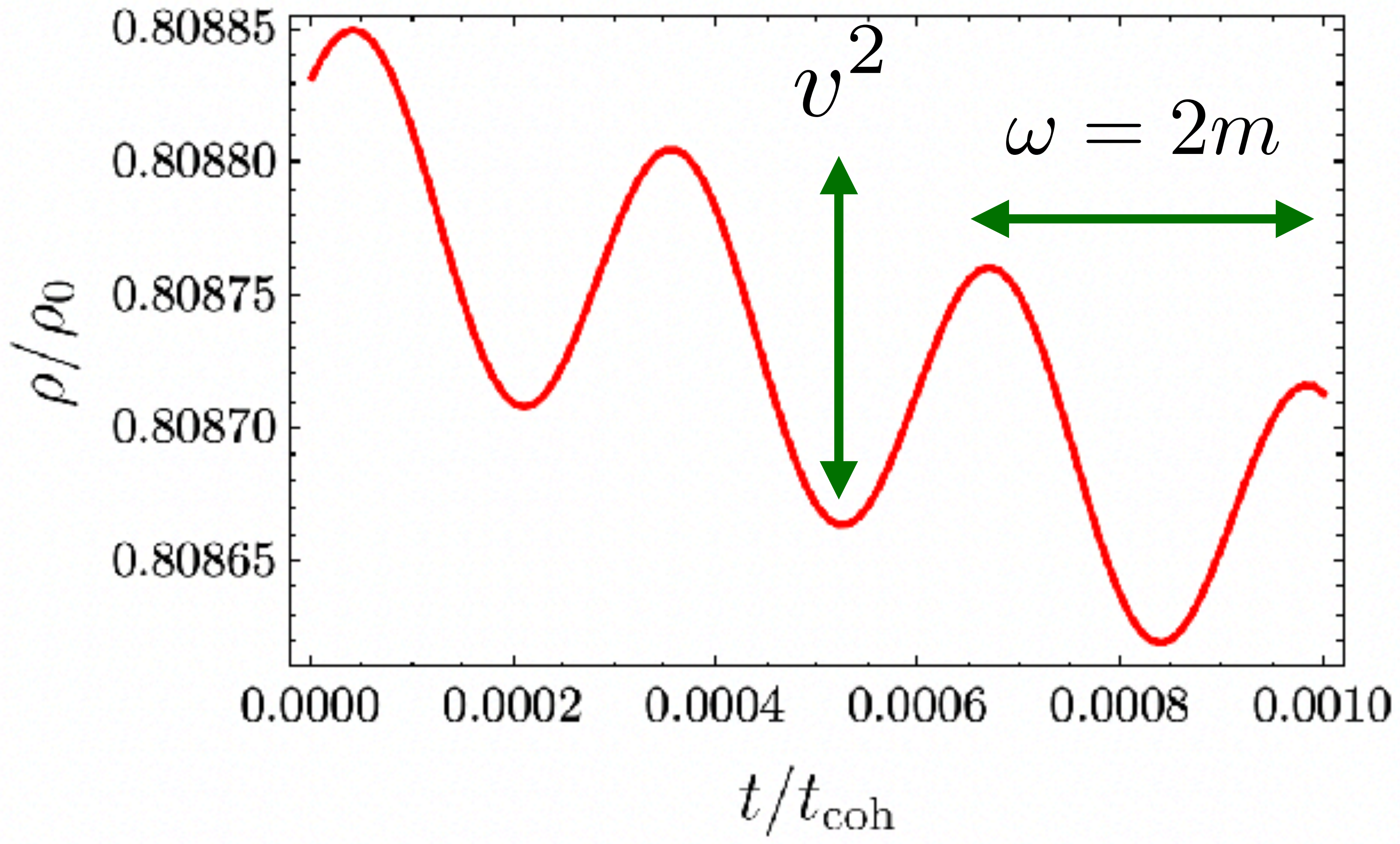
For instance
the density-density correlator of space-like separation is

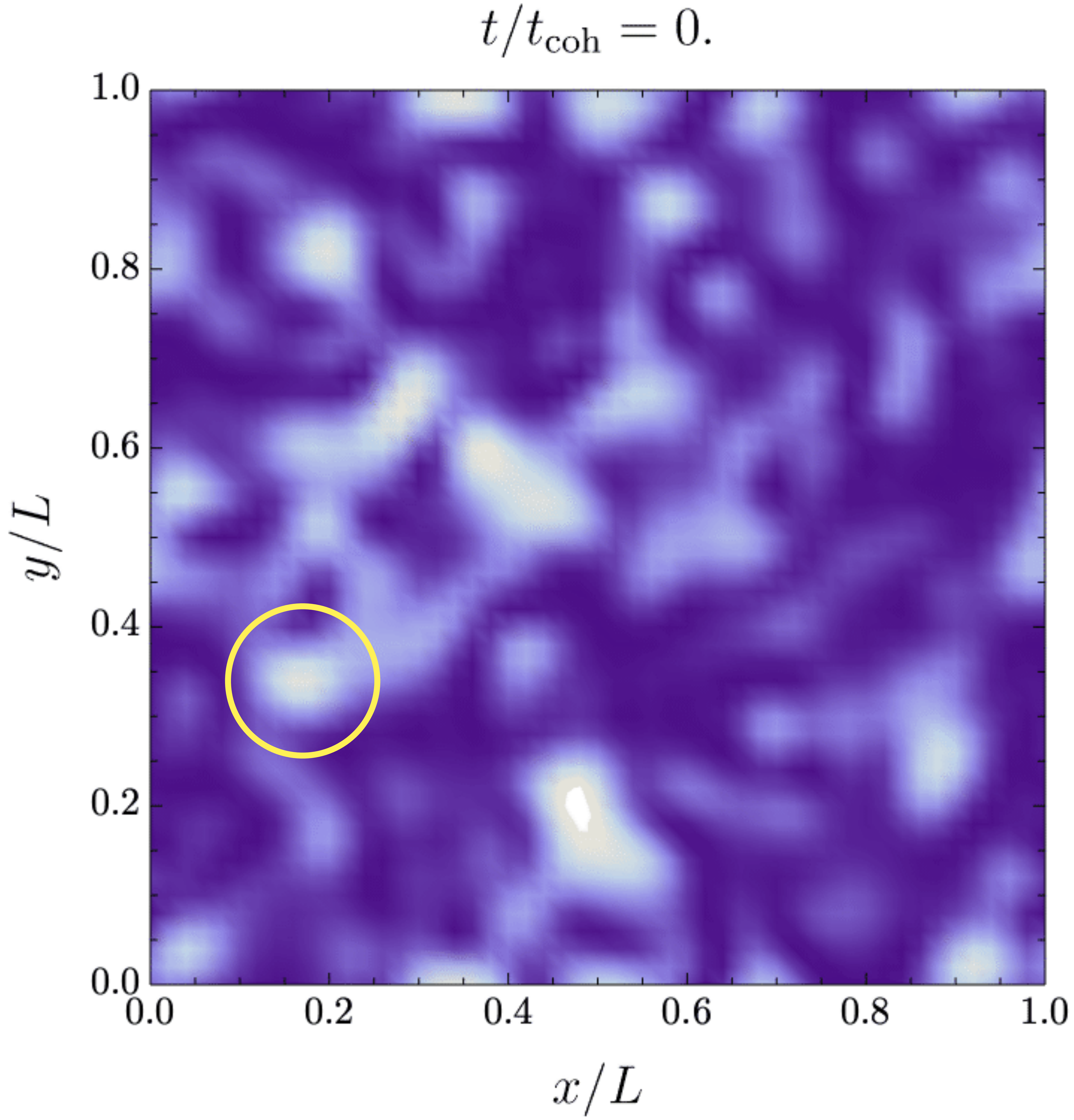
$$\langle \delta(x)\delta(y) \rangle \propto \exp\left[-|\Delta x|^2/\lambda^2\right]$$

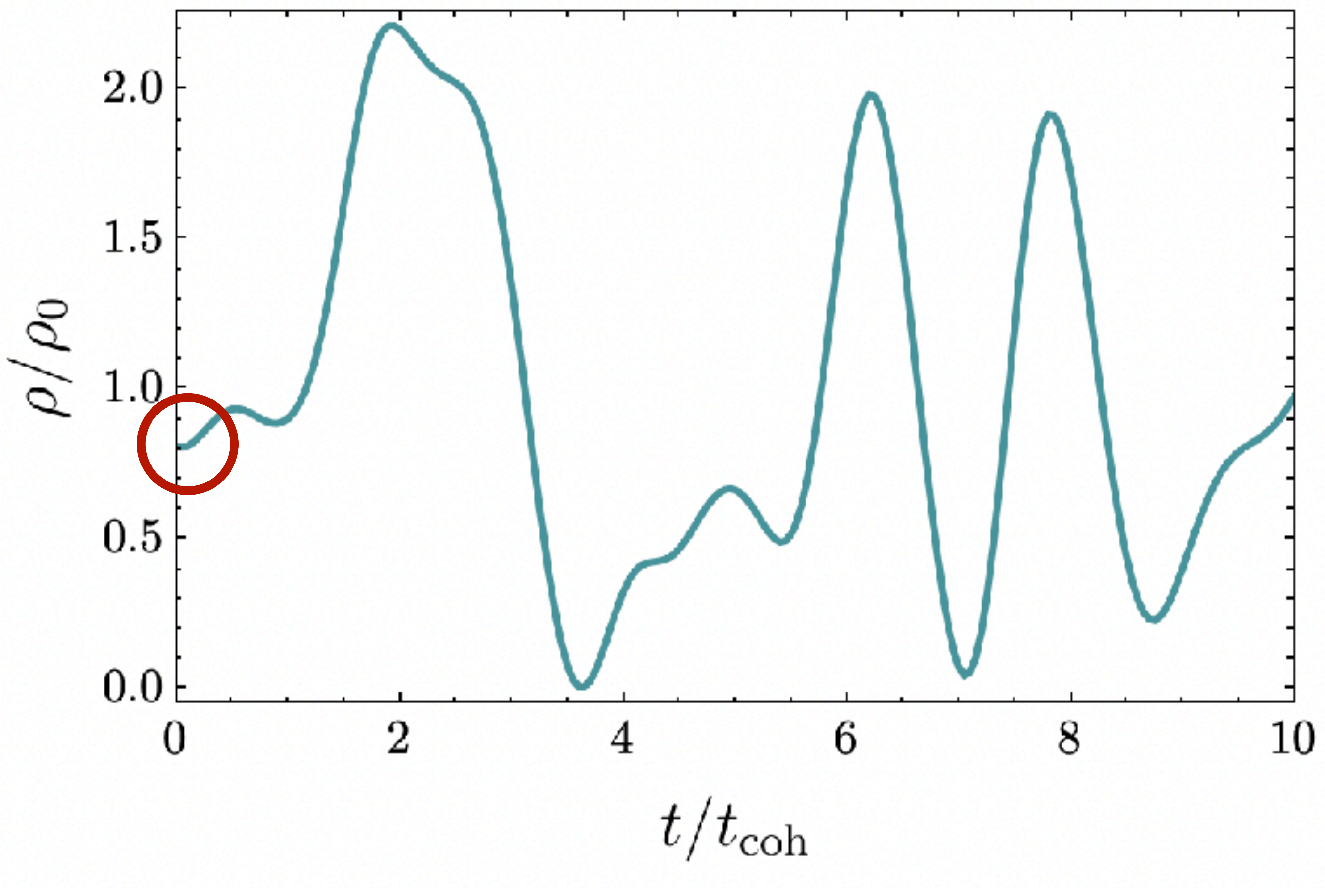


the density-density correlator at the same position is

$$\langle \delta(x)\delta(x) \rangle = \int \frac{d\omega}{2\pi} S_\delta(\omega)$$







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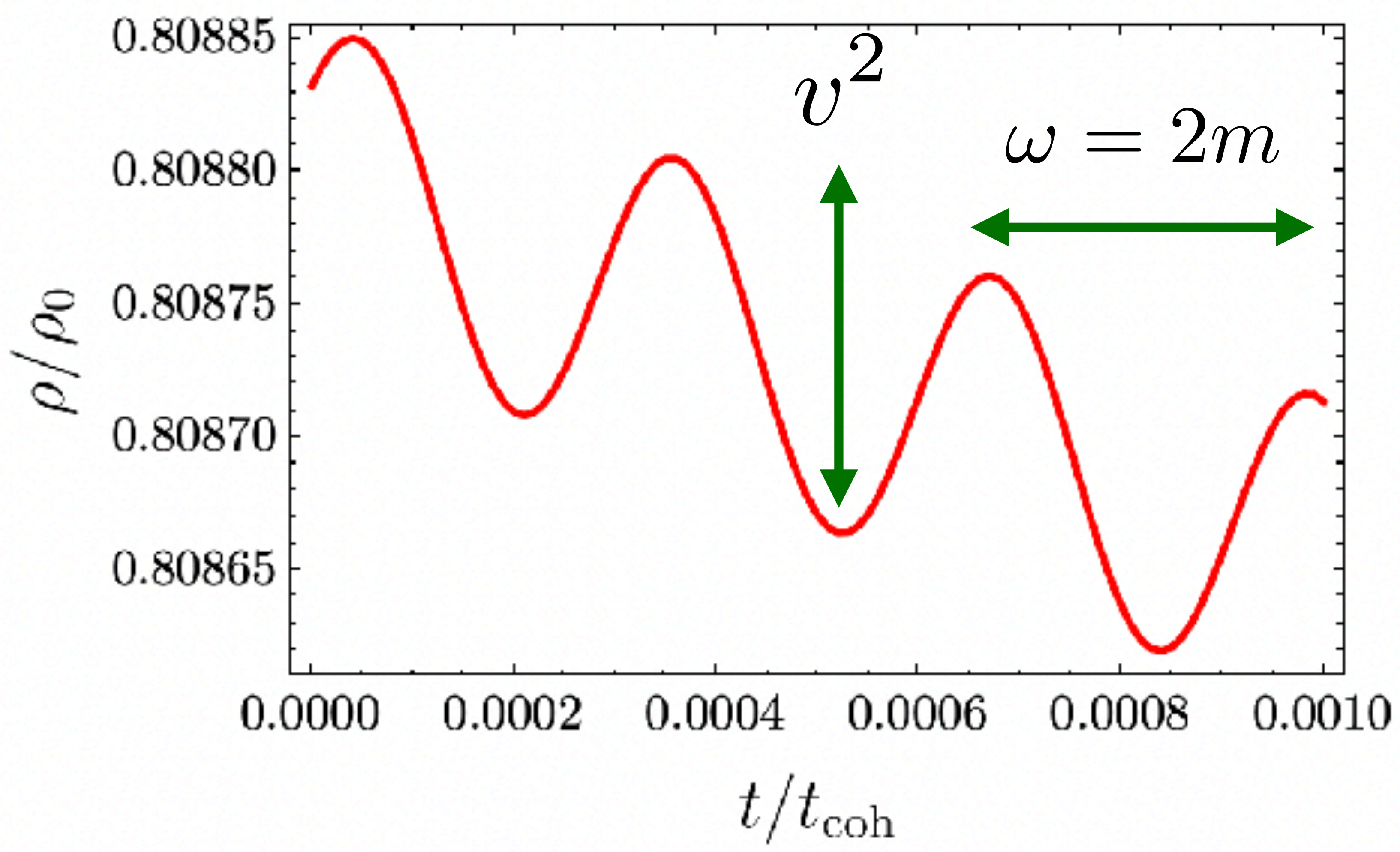


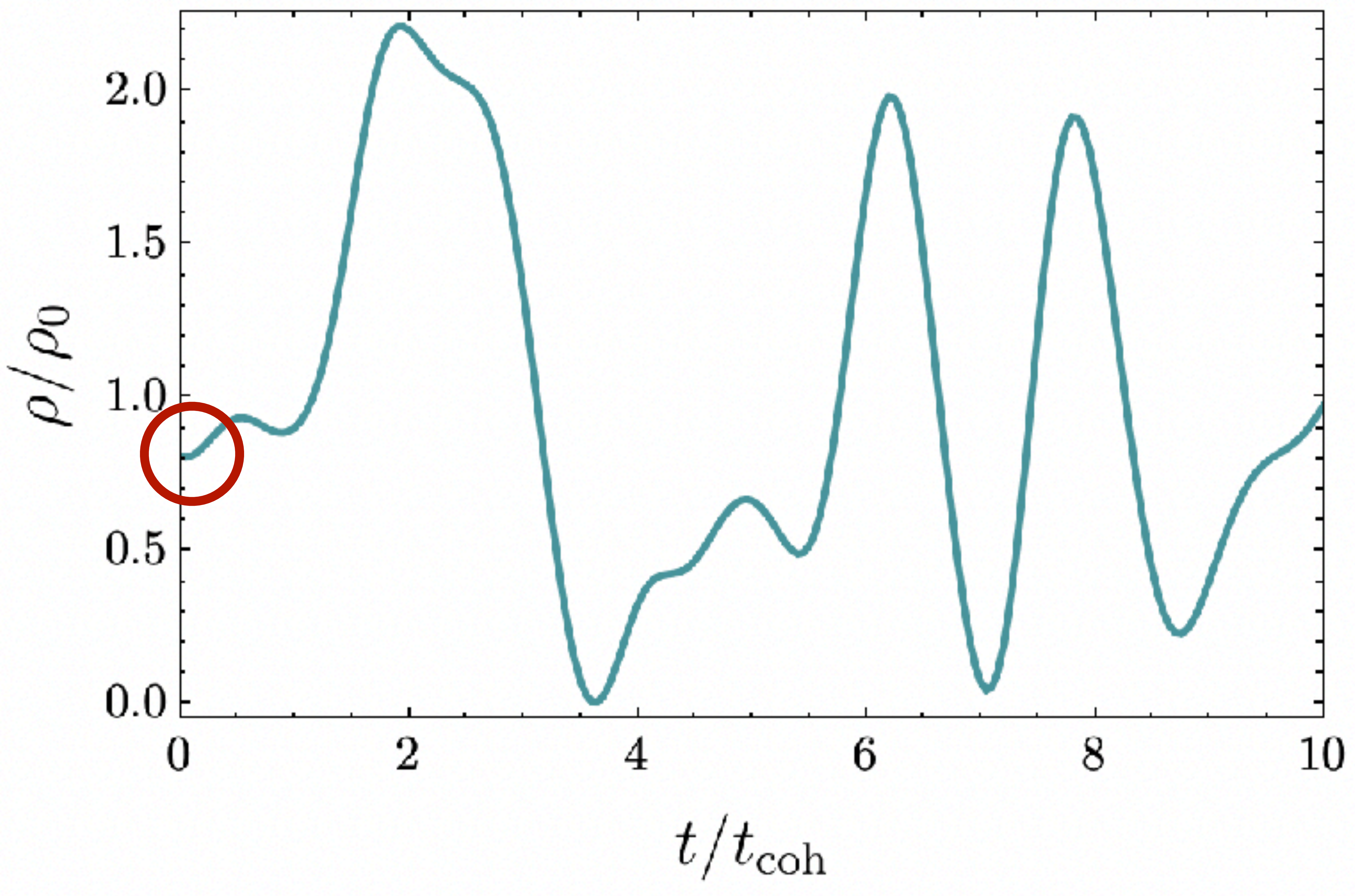
for a single mode

$$\phi = \phi_0 \cos(\omega t - kx)$$

the energy density is

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2$$

$$\simeq \rho_0[1 - v^2\cos(2(mt - kx))]$$



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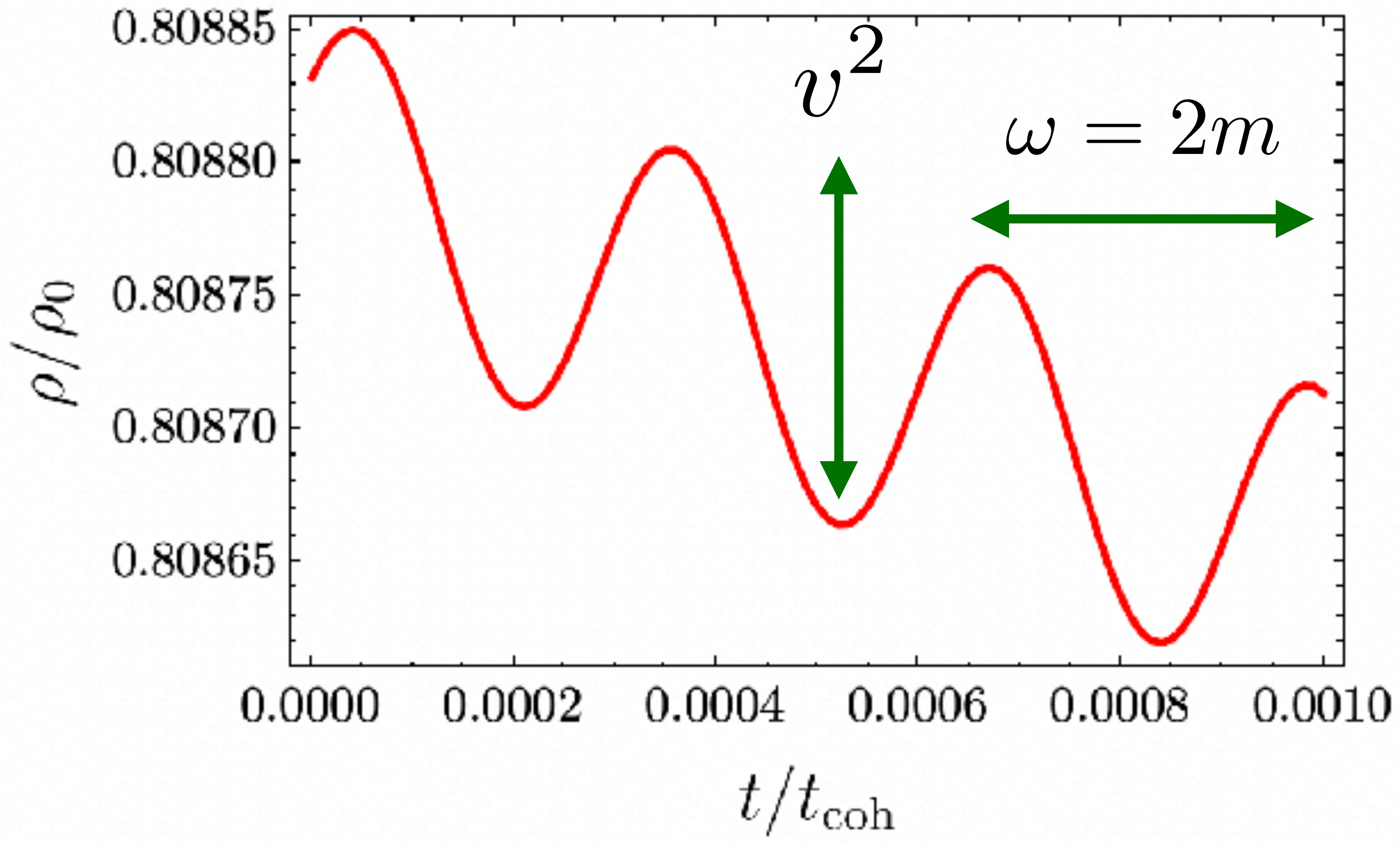
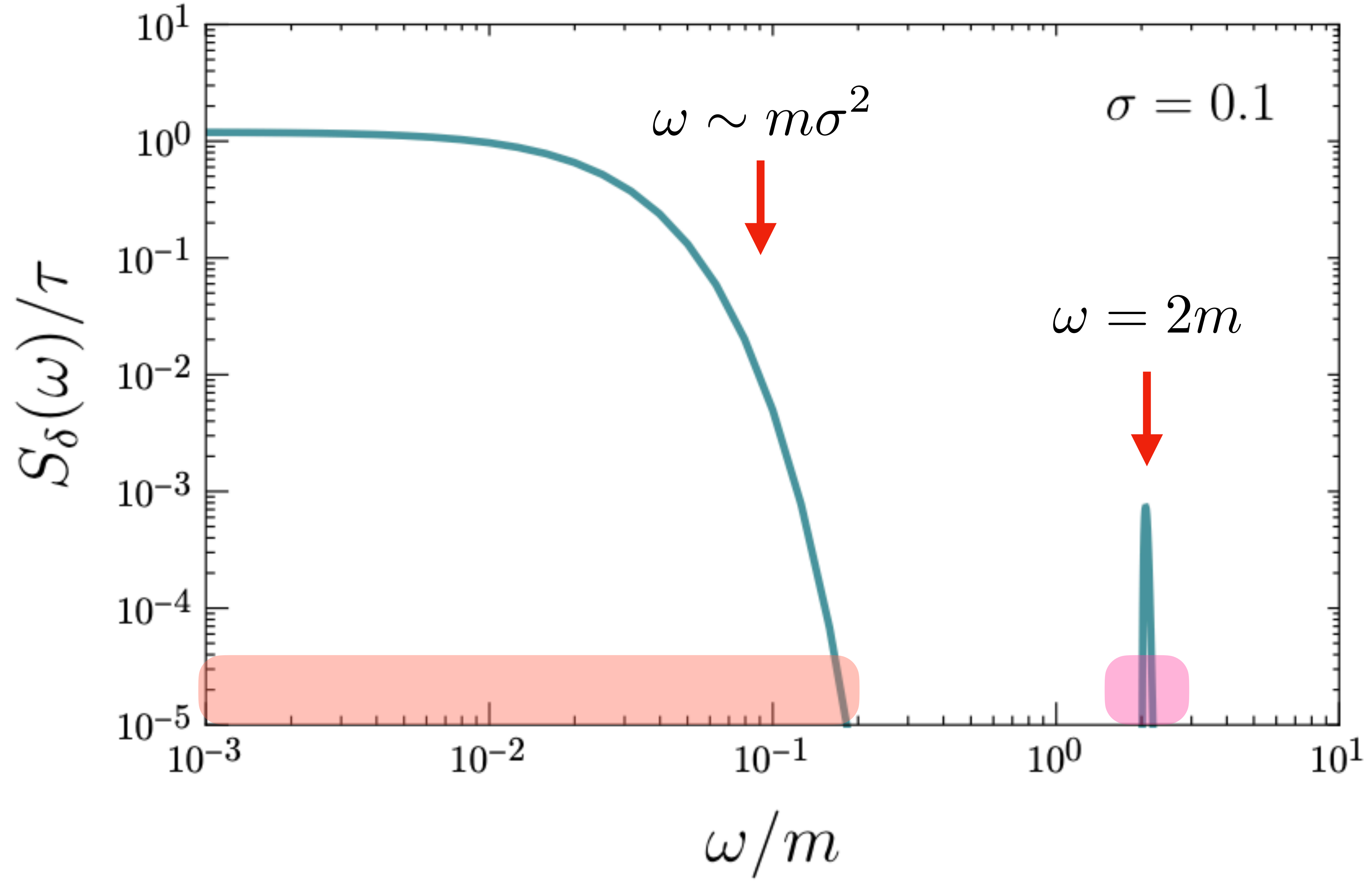
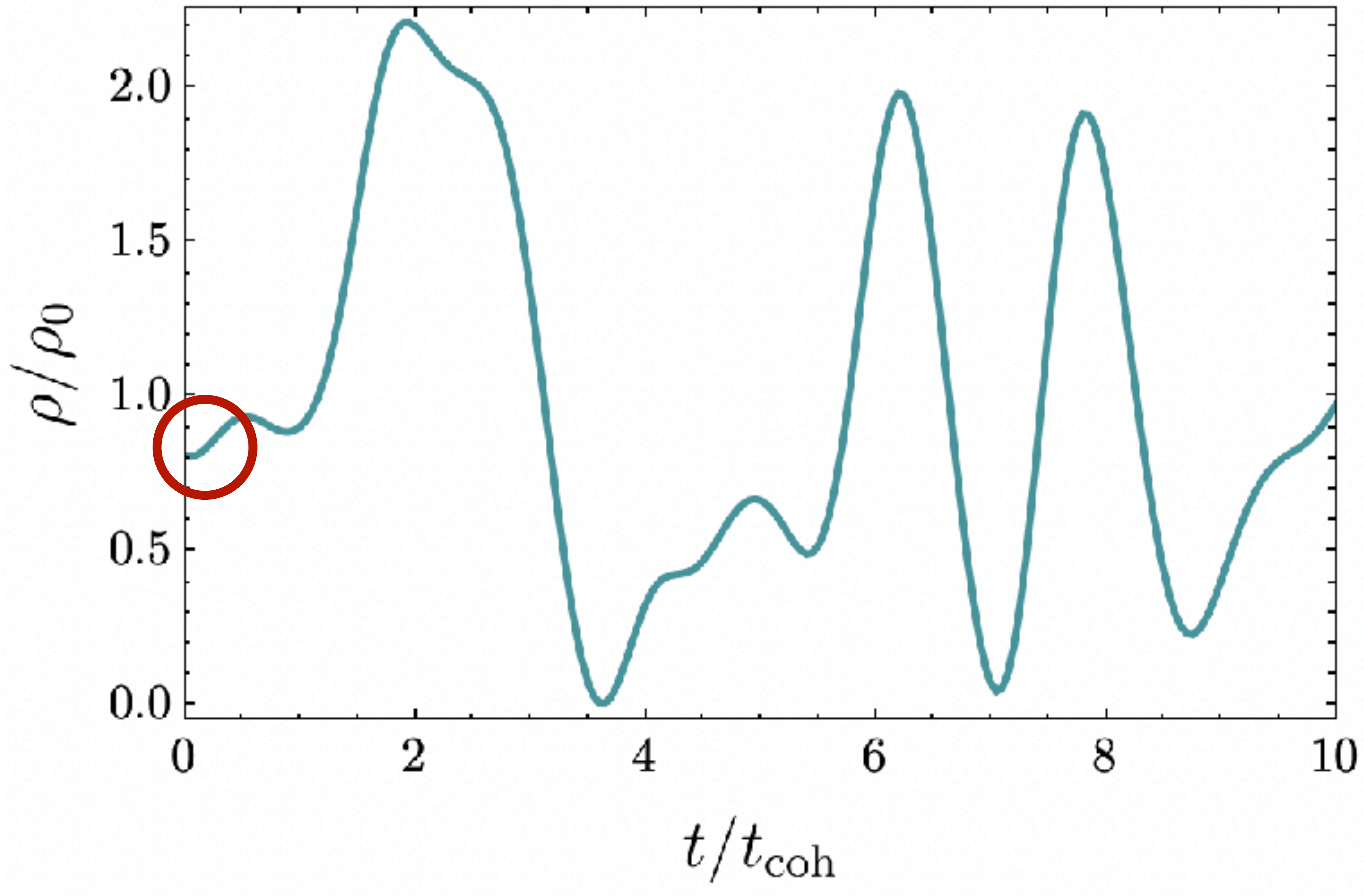
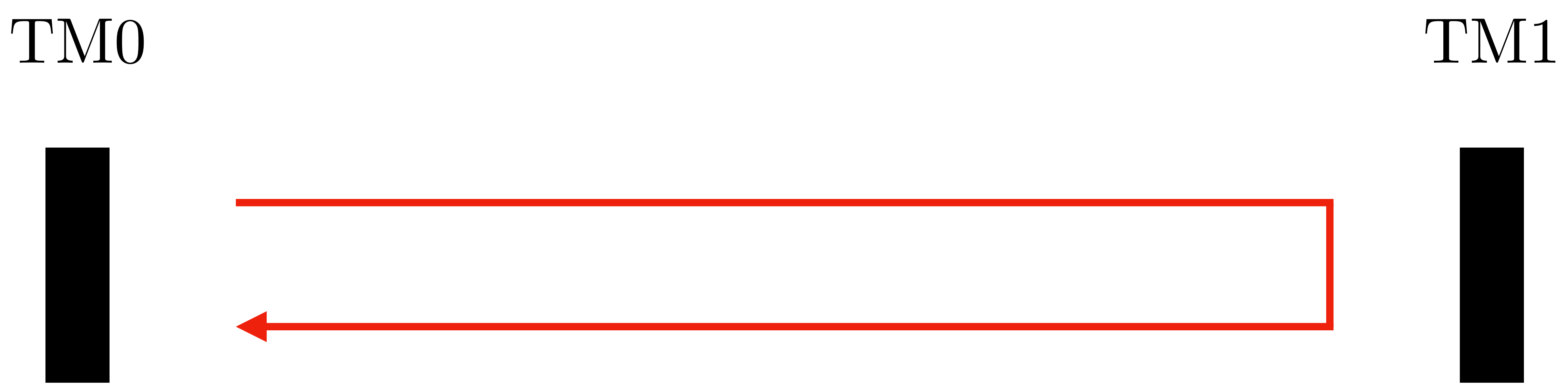


$$S_\delta(\omega) = \tau[\sigma^4 A_\delta(\omega) + B_\delta(\omega)]$$



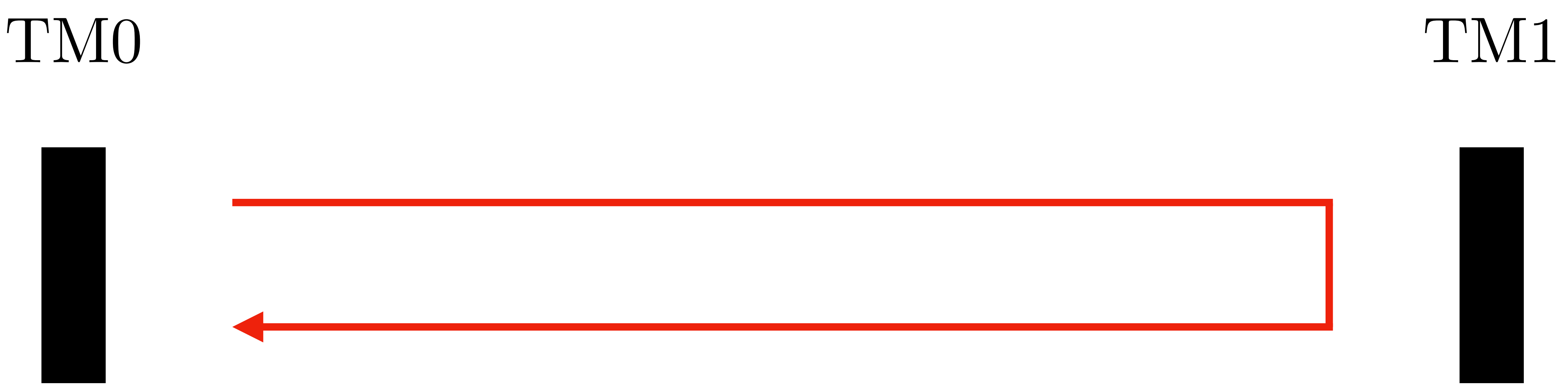


this is enough to study the response of interferometry w.r.t. ULDM
specifically let us consider



because of ULDM

*laser acquires an **additional phase***



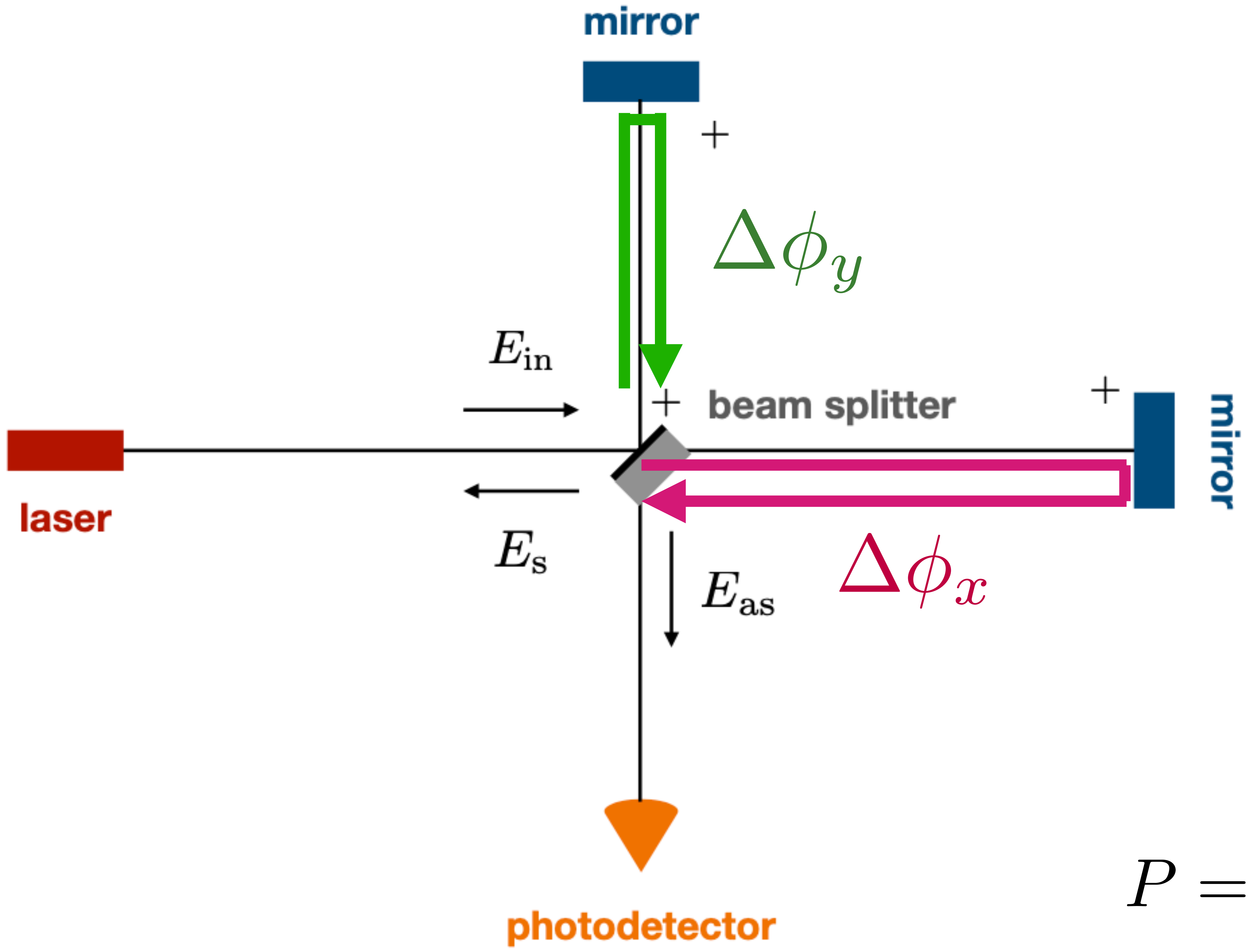
$$e^{2ik_L L} e^{i\Delta\phi_x}$$

$$\Delta\phi_x = 2\omega_L(\Delta t + \Delta L) \quad \propto \int \frac{d^4k}{(2\pi)^4} e^{-ikx}(i\vec{k}\cdot\vec{D})\tilde{\delta}(k)$$

ΔL; the change in positions of TM (and the distance between them)

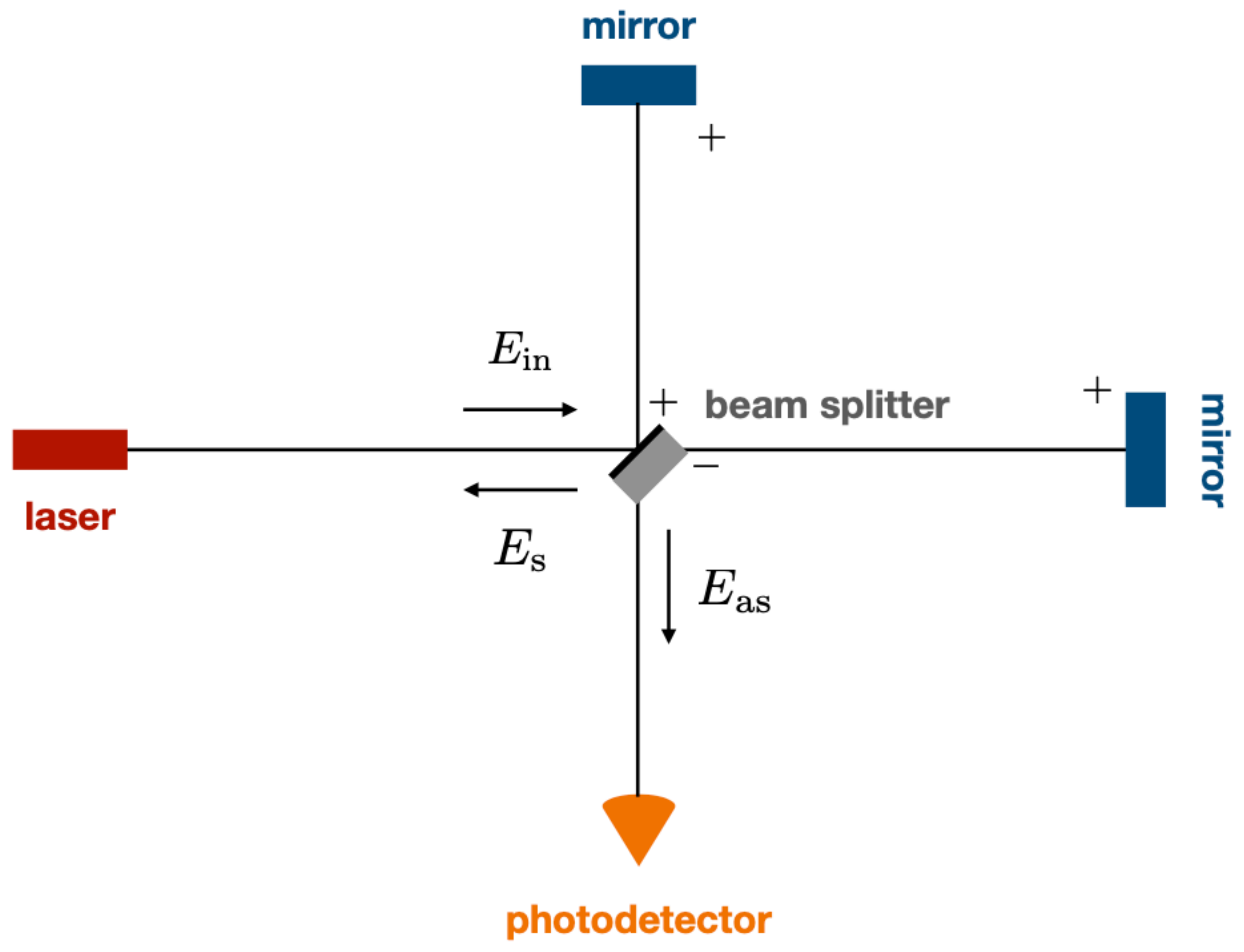
Δt; time delay due to the scalar metric perturbations induced by ULDM

In the case of Michelson interferometer (LIGO, VIRGO, etc)



$$P = P_0 \sin^2(k_L \Delta L + \Delta\phi)$$

$$\Delta\phi = \frac{1}{2}(\Delta\phi_x - \Delta\phi_y)$$



$$\bar{a} = \frac{G m_{\rm eff}}{\lambda^2}$$

$$\tau = \frac{1}{m\sigma^2}$$

Combining the effects along the two arms one finds

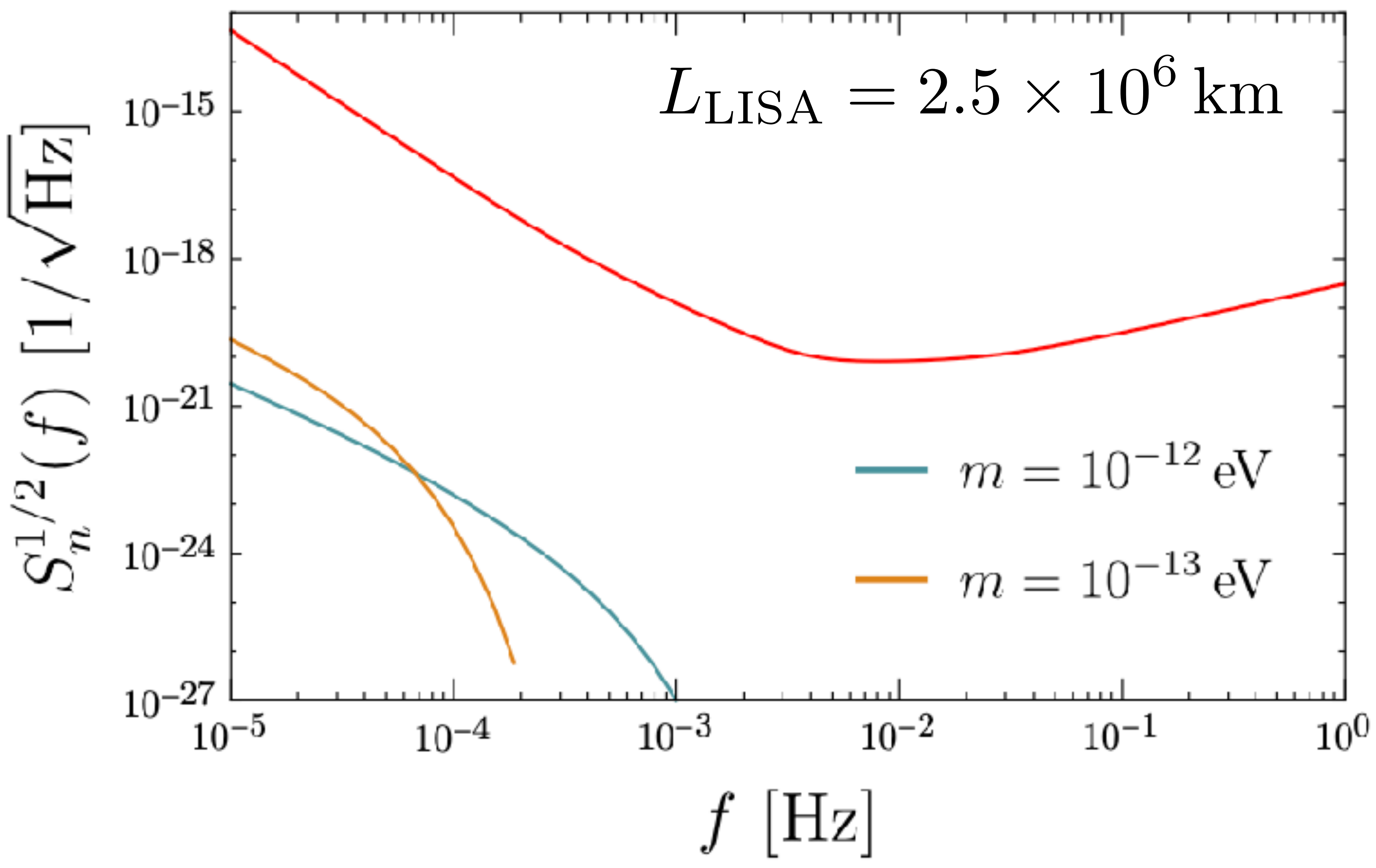
response of Mich. Interferometer
B ~ $(L/\lambda)^2$ if L << λ (tidal limit)

$$S_{\Delta L/L}(f) = \frac{\bar{a}^2\tau}{(2\pi f)^4 L^2}[\sigma^4 A_{\rm Mich}(f) + B_{\rm Mich}(f)]$$

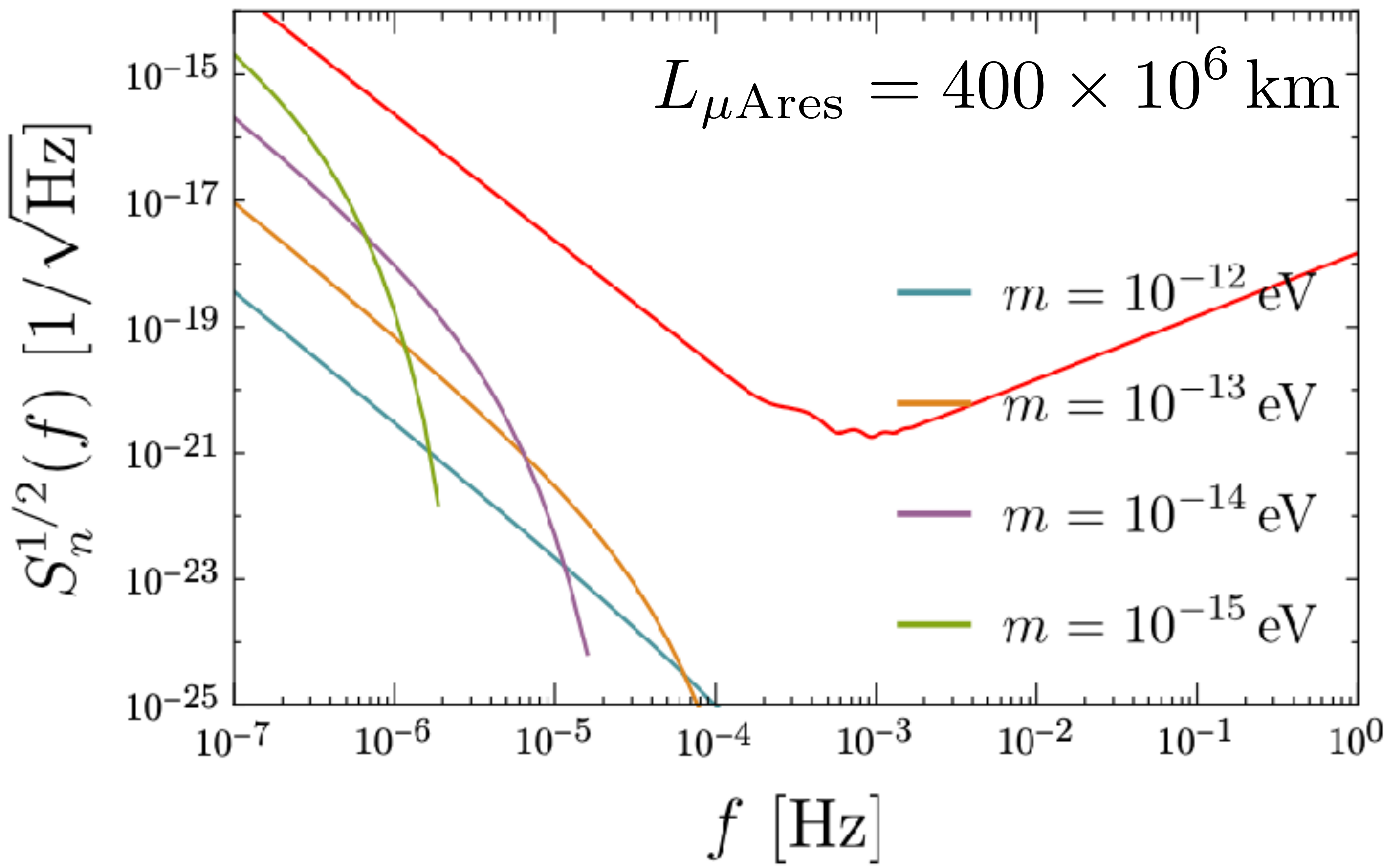
rms fluctuation of a single test mass $(\Delta L/L)^2_{rms}$ over Δt~1/f *when ULDM signal is translated into strain power spectrum*

[Kim, in preparation]

Preliminary LISA



Preliminary μAres



Even for space-borne interferometers, ULDM-induced noise are subdominant

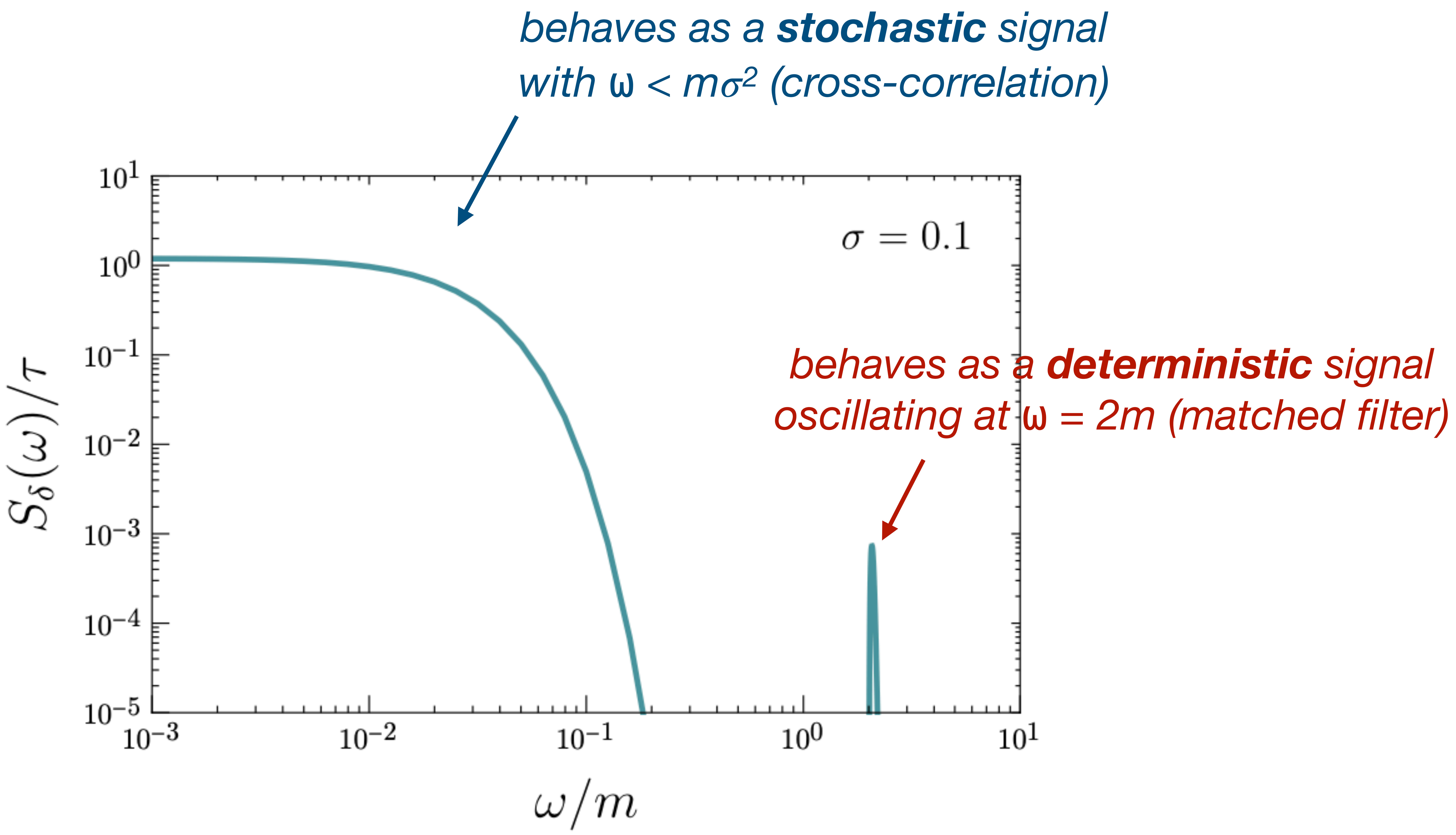
Two questions:

*1. Do we have to worry about **ULDM-induced noise***
in the current and future gravitational waves?

for current and future GW intereferometers
gravitational interaction of ULDM leaves subdominant noise

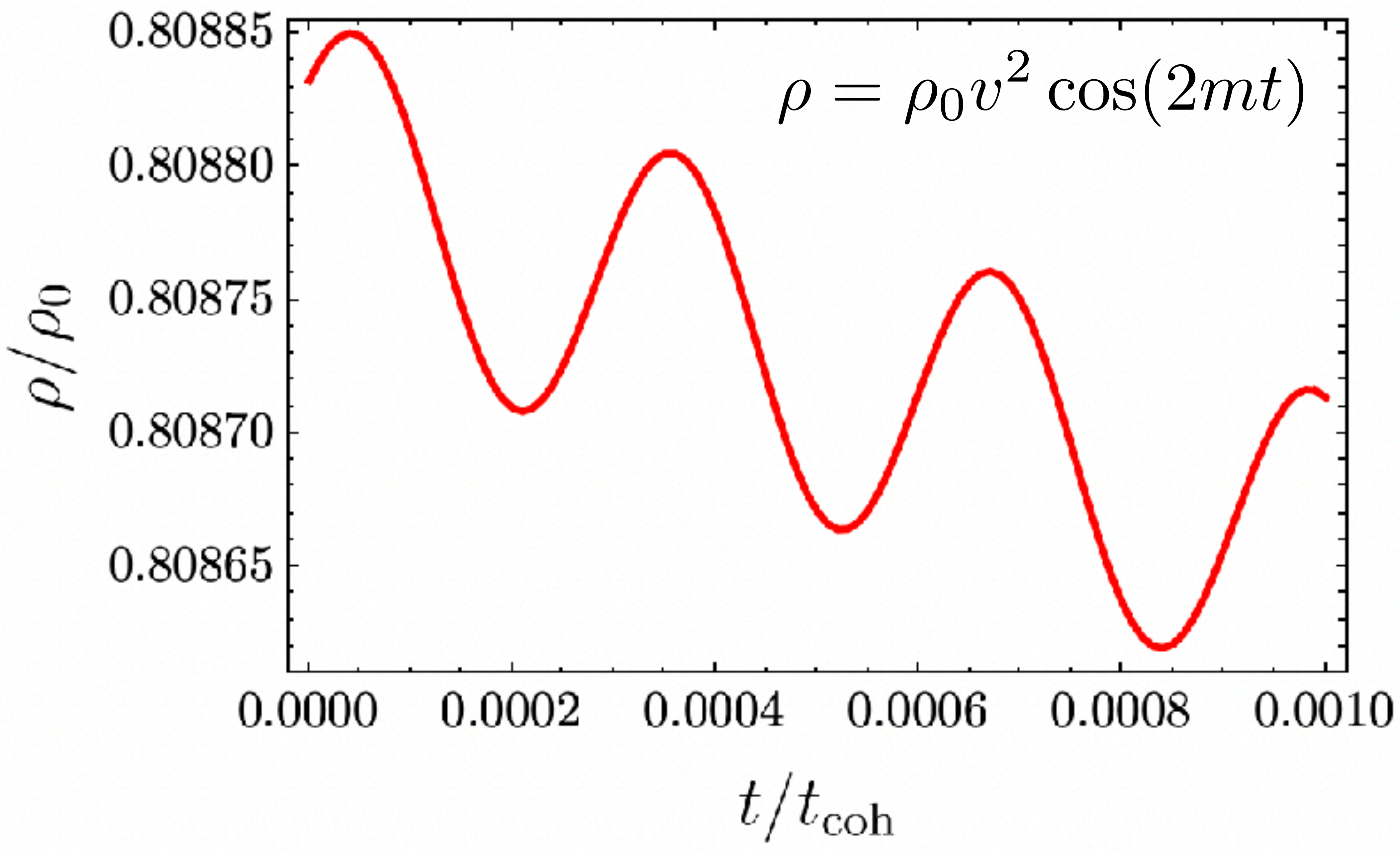
2. Can current and future GW interferometers
probe ultralight dark matter gravitationally?

more specifically
can we constrain dark matter density in the solar system
through gravitational interaction with GW interferometers?


behaves as a **stochastic** signal
with ω < mσ² (cross-correlation)
σ = 0.1
behaves as a **deterministic** signal
oscillating at ω = 2m (matched filter)
$S_\delta(\omega)/\tau$
ω/m
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}

the deterministic signal

we have seen coherently oscillating mode in ρ



the position of the test mass will also behave in a similar way

$$\Delta L/L \propto \cos(2mt)$$

we can 'filter' the detector output to maximize the signal by choosing the optimal filter K(t)

$$\int dt\, d(t) K(t)$$

$$d(t) = s(t) + n(t)$$

$$K(t) \propto \cos(2mt)$$

the signal is coherently added up while the noise is added incoherently

$$\frac{S}{N} = \left[T \int_{-\infty}^{\infty} df \, \frac{S_s(f)}{S_n(f)} \right]^{1/2}$$

coherent addition of signal

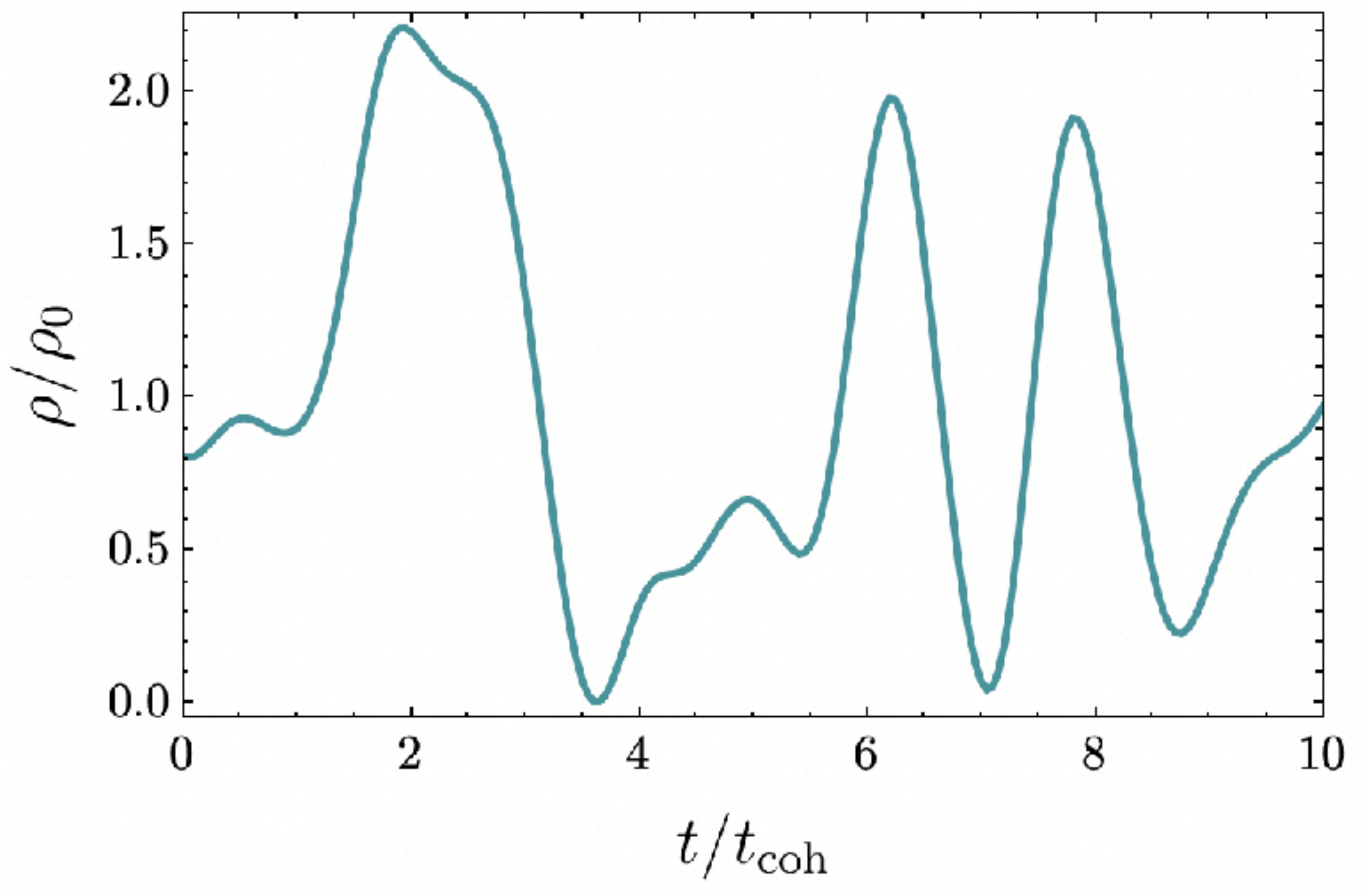
$(\Delta L/L)_{rms}$ over $\Delta t_m \sim 1/m$ due to ULDM

$(\Delta L/L)_{rms}$ due to detector noise

$$\sim \frac{[\bar{a}\sigma^2/(2\pi f_m)^2 L]}{[S_n(f_m)/\tau]^{1/2}} \left[T \int df \, A_{\rm Mich} \right]^{1/2}$$

the stochastic signal

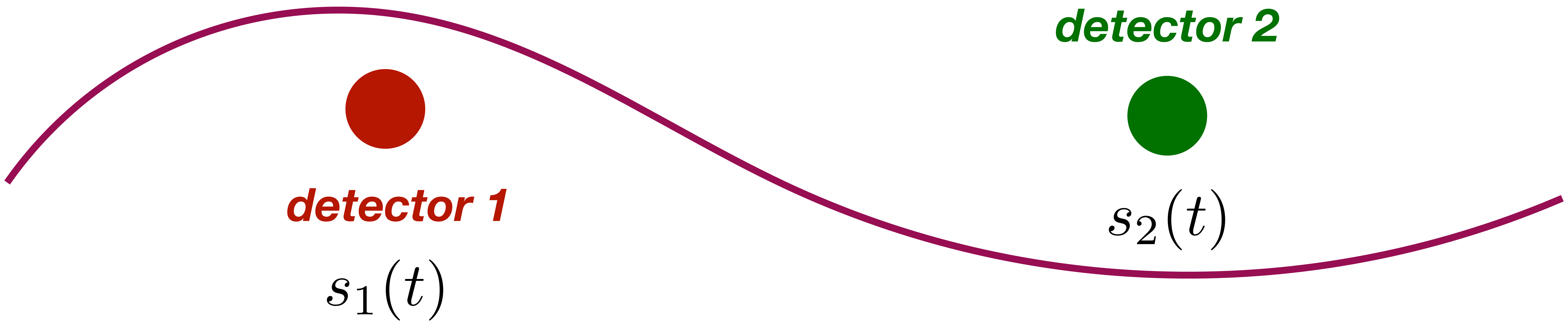
we have seen random changes in ρ over coherent time scale t_{coh}



the form of the signal is unknown, and hence, matched filter cannot be used *the stochastic signal*

if we have more than two detectors
we can cross-correlate the signals

$$Y = \int dt \int dt' s_1(t) s_2(t') Q(t - t')$$



the noise is expected to be uncorrelated
the correlated signal can be picked up by choosing an optimal filter Q(t)

the stochastic signal

if we have more than two detectors
we can cross-correlate the signals

$$\frac{S}{N} = \left[T \int_{-\infty}^{\infty} df \, \frac{|S_{12}(f)|^2}{S_n^2(f)} \right]^{1/2}$$

coherent addition of signal

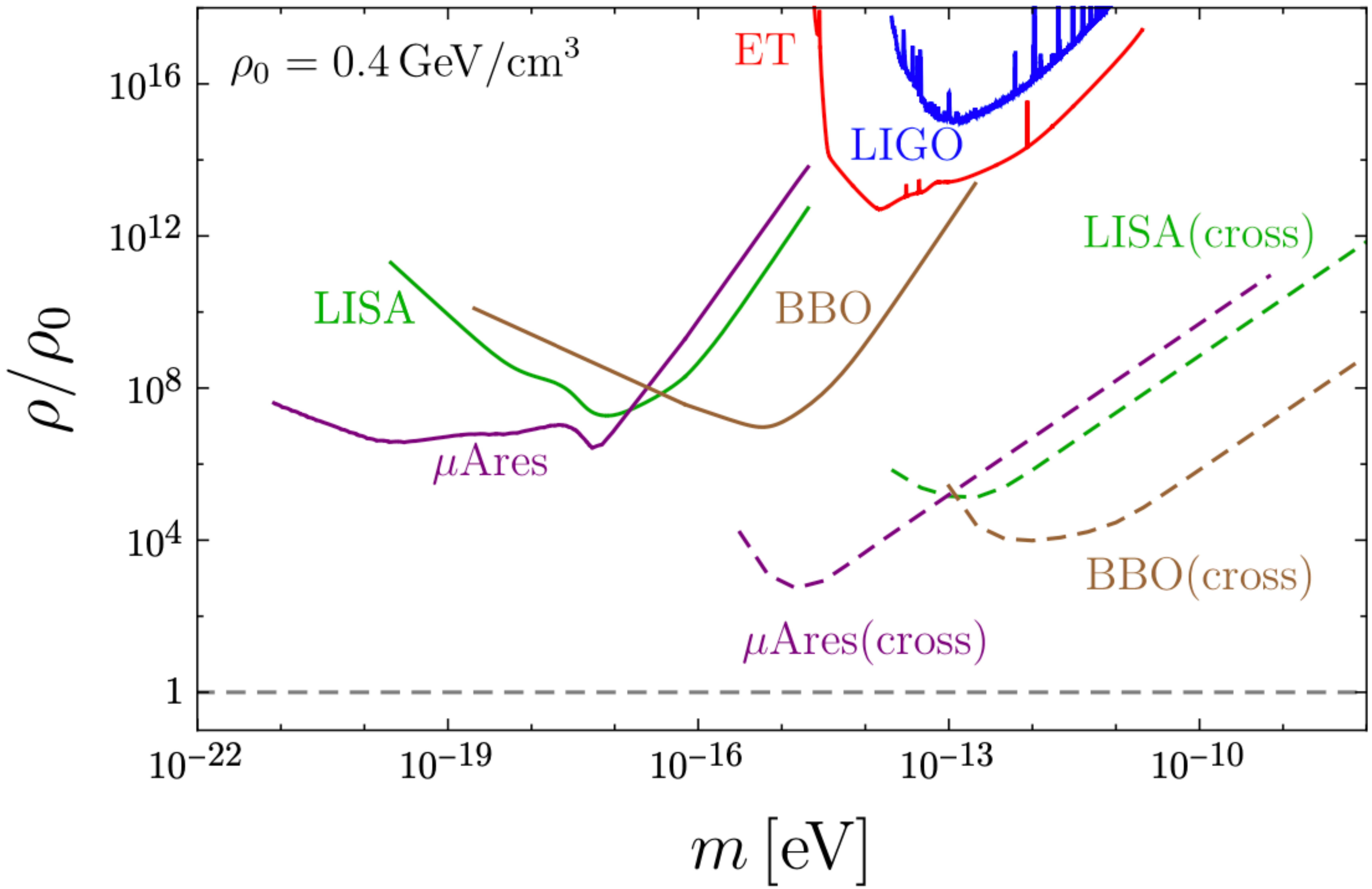
$(\Delta L/L)^2_{rms}$ over $\Delta t \sim t_{coh}$ due to ULDM

$$\sim \frac{\bar{a}^2}{(2\pi f_0)^4 L^2} \left[T \int df \, \frac{(f_0/f)^8 |B_{\rm cross}|^2}{f_0^2 S^2_{\Delta L/L}(f)} \right]^{1/2}$$

$(\Delta L/L)^4_{rms}$ due to detector noise

the noise is expected to be uncorrelated
the correlated signal can be picked up by choosing an optimal filter Q(t)

[Kim in preparation]
Preliminary



A few remarks on local dark matter density

$$\rho_0 = 0.4\,\mathrm{GeV}/\mathrm{cm}^3$$

is a measured value over the volume V > [O(10^2) pc]^3

see reviews e.g. [Read (14)]; [de Salas, Widmark (20)]

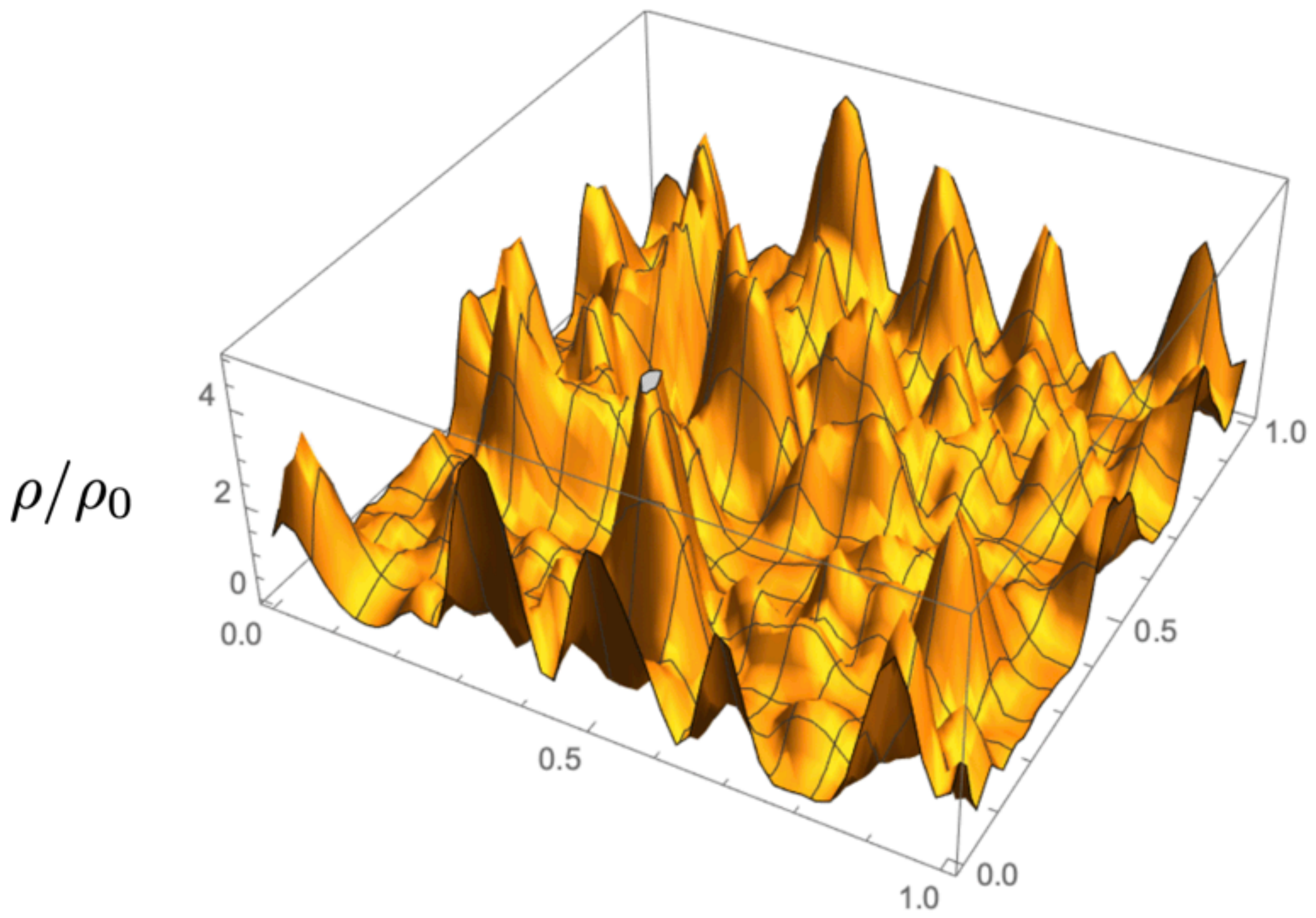
currently no direct measurement of dark matter density in the solar system but only constraints exist

$$\rho/\rho_0 \lesssim 10^4$$

From solar system ephemerides [Pitjev, Pitjeva (13)]

$$\rho/\rho_0 \lesssim 10^{11}$$

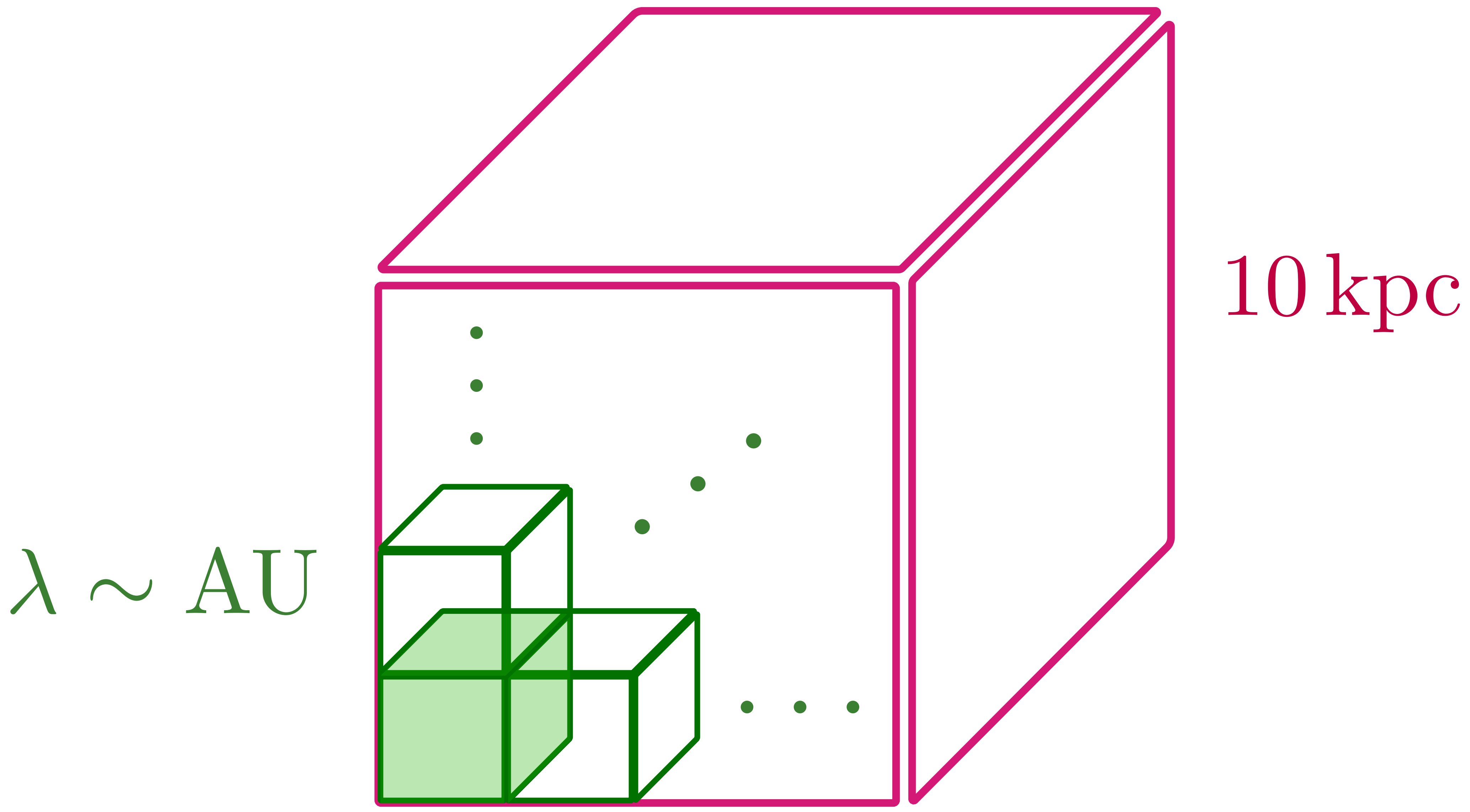
From geodetic satellite and LLR [Adler (08)] *in addition we have seen that there could be easily O(1) density fluctuation in the wave DM halo*



$$p(\rho)d\rho = \frac{1}{\rho_0}\exp\left[-\frac{\rho}{\rho_0}\right]d\rho$$

$$P(\rho > \rho_c) = e^{-\rho_c/\rho_0}$$

consider e.g. $m\sim10^{-15}$ eV where the wavelength is ~AU scale
in the volume of $V = (10\ \mathrm{kpc})^3$ there are 10^{28} AU-sized patches



statistically speaking
there will be ~ 100 patches
in this volume with $\rho > 60\ \rho_0$!

$$N_{\text{patches}} = (10\,\mathrm{kpc}/\mathrm{AU})^3 \simeq 10^{28}$$

$$P(\rho > 60\rho_0) = e^{-60} \simeq 10^{-26}$$