On the dynamical origin of the η' potential and the axion mass

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Cornell University

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work in progress

with C. Csáki, R. d'Agnolo, R. Gupta, E. Kuflik *and* T. Roy

Axion and η' Mass in QCD

What generates the axion and η' mass in ordinary QCD?

Instantons!

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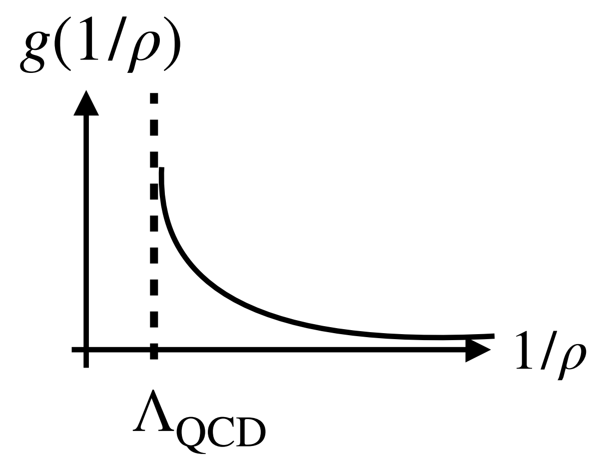

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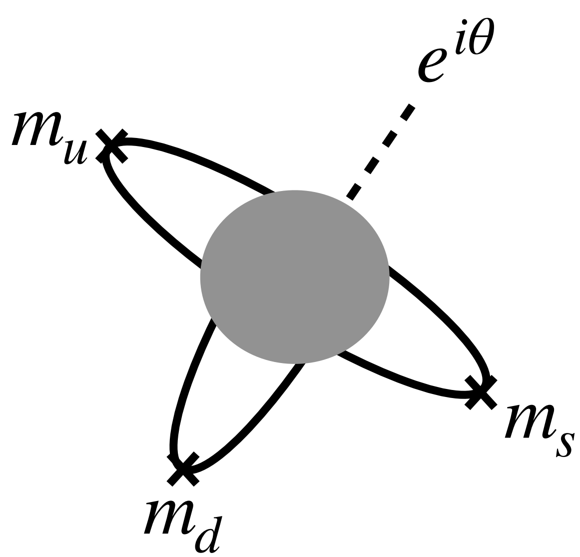

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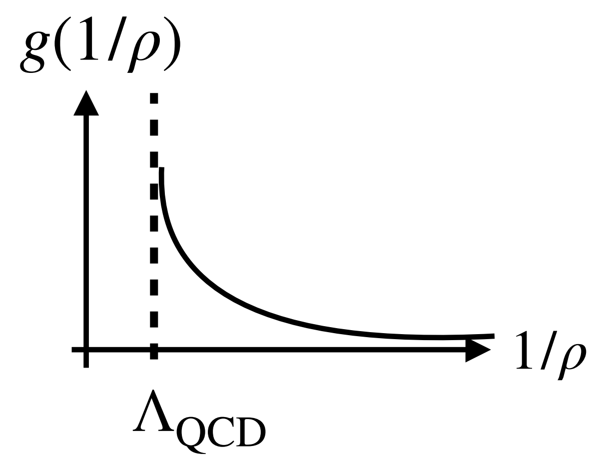

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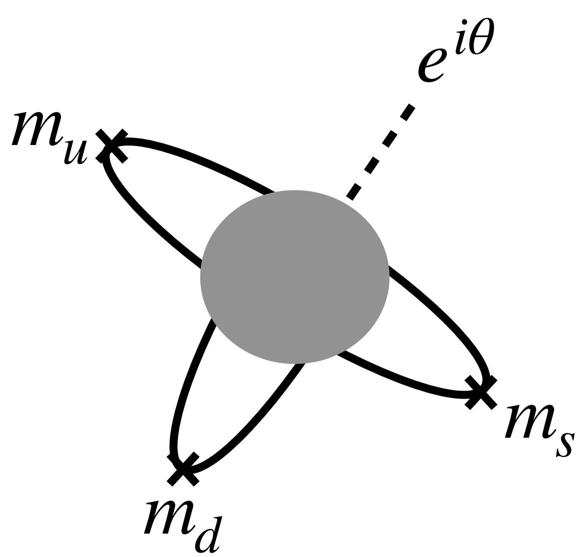

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What is IR cutoff $\Lambda_{\rm IR}$?

What role does confinement dynamics play?

Are instantons the main contribution to η' potential?

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Witten '79

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What happens in finite N and F QCD?

η' in SUSY QCD with AMSB

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η' in SUSY QCD with AMSB

- QCD is non-perturbative in IR ➡ cannot check explicitly what is going on

- **BUT:** can check in QCD-like theory

➡ softly-broken SUSY QCD is (fully) calculable

➡ SUSY QCD with AMSB has vacuum with chiral symmetry breaking for $F \leq 3N$

Murayama '21
Csaki et al '22

➡ ordinary QCD would be recovered in large SUSY breaking limit

Can give valuable lessons for ordinary QCD!

Outline

Part 1 (η': Standard lore in ordinary QCD):

- Chiral Lagrangian and instanton effects
- Insights from large N QCD

Part 2 (Lessons from SUSY QCD with AMSB):

- Review of SUSY QCD and AMSB
- Chiral Lagrangian in SUSY QCD with AMSB
- Spontaneous CP breaking at $\bar{\theta} = \pi$

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Part 1: η' standard lore in ordinary QCD

Chiral Lagrangian

- F flavor QCD with N colors exhibits SSB at low energies

$$U(F)_L \times U(F)_R \to U(F)_d = U(1)_B \times SU(F)_d$$

- **But:** $U(1)_A$ is anomalous $\qquad \partial_\mu j^\mu_A = F\frac{g^2}{8\pi^2}\mathrm{Tr}G\tilde{G}$

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- Parameterize GBs and would-be GB for $U(1)_A$ in GB matrix

$$U = e^{i\eta'} e^{i\pi^a T^a}$$

decay constants absorbed in η' and π^a

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- Lagrangian at 2-derivatives

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[(\partial_\mu U)^\dagger \partial^\mu U\right] + \alpha\Lambda f_\pi^2\left(\mathrm{Tr}\left[m_Q U\right] + \mathrm{h.c.}\right)$$

quark masses break $U(F)_A$ explicitly

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$$\mathcal{L}_{\text{inst}} = b\Lambda^2 f_\pi^2 e^{-i\theta} \det U + \text{h.c.}$$

implicitly assumes / consistent with instanton origin

instanton is always proportional to $e^{in\theta}$

η' Potential

- Potential $$V_{\eta'} = -2b\Lambda^2 f_\pi^2 \cos(\theta - F\eta')$$

→ η' mass is $O(\Lambda)$: $m_{\eta'}^2 \propto bF\Lambda^2$

non-canonical kinetic term

$$\mathcal{L}_{\text{kin}} = (2F)\frac{1}{2}\partial_\mu\eta'\partial^\mu\eta'$$

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  → if one $m_i = 0$, can absorb $\bar\theta$ into pion VEVs

  → if all $m_i \neq 0$, need extra DOF to eliminate $\bar\theta$: $\bar\theta \to \bar\theta + n\,a$

axion

anomaly coefficient

Large N limit

- Anomaly vanishes in $N \to \infty$ limit

$$\partial_\mu j_A^\mu \sim F \frac{g^2}{8\pi^2} \mathrm{Tr} G\tilde{G} \sim \frac{\lambda}{8\pi^2} \frac{F}{N} \mathrm{Tr} G\tilde{G} \to 0$$

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Witten '79

- Veneziano-Witten formula for η' mass

$\theta \to \theta - F\eta'$

$$m^2_{\eta'} = \frac{2F}{f_\pi^2} \frac{d^2 E}{d\theta^2}\Big|^{\text{pure QCD}}_{\theta=0} \propto \frac{1}{N}$$

Witten '79
Veneziano '79

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$m_{\eta'}^2 \sim 1/N$

$\eta'^4 \sim 1/N^4$
as expected from large N

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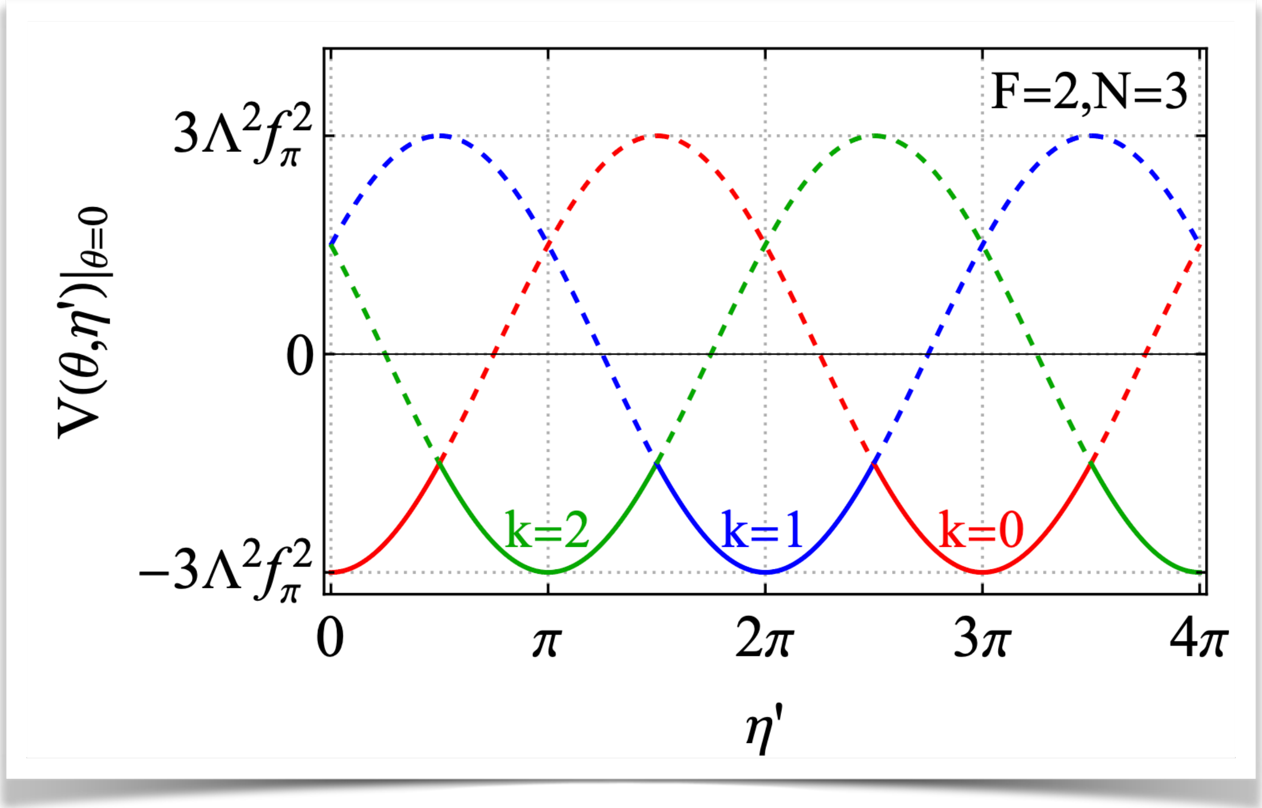

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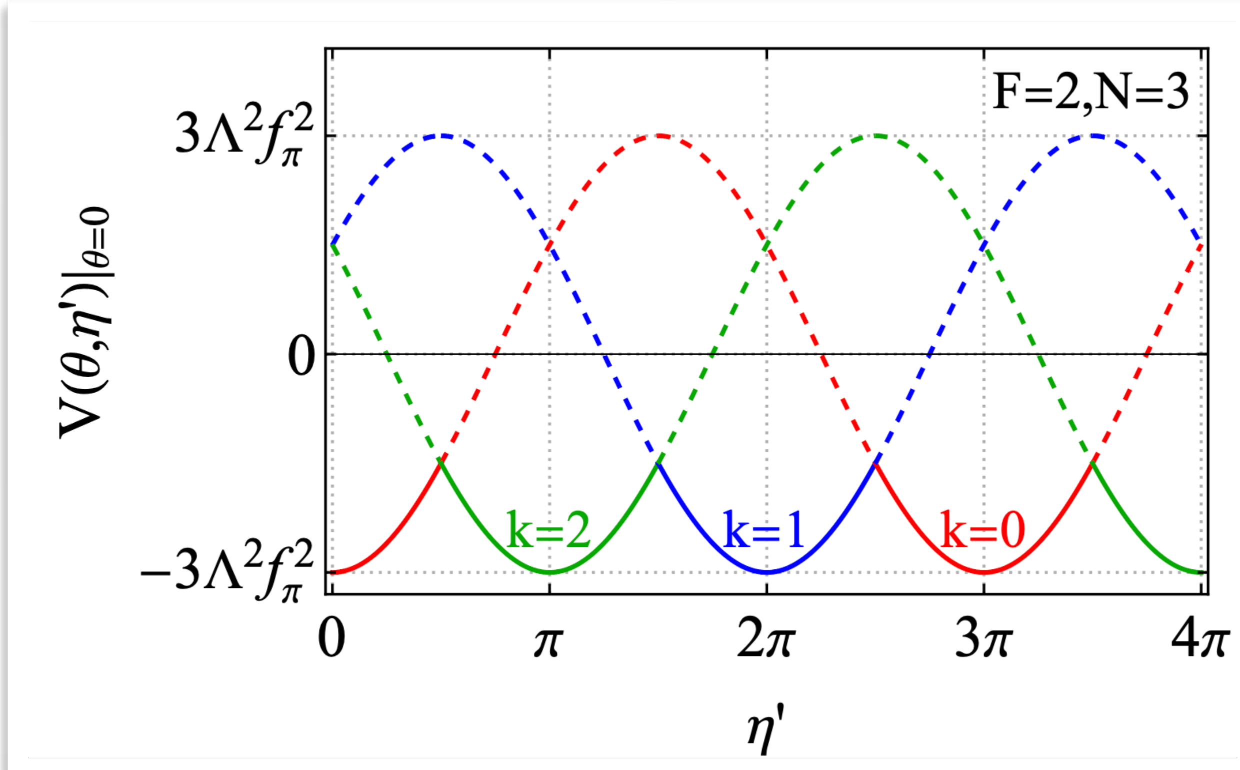

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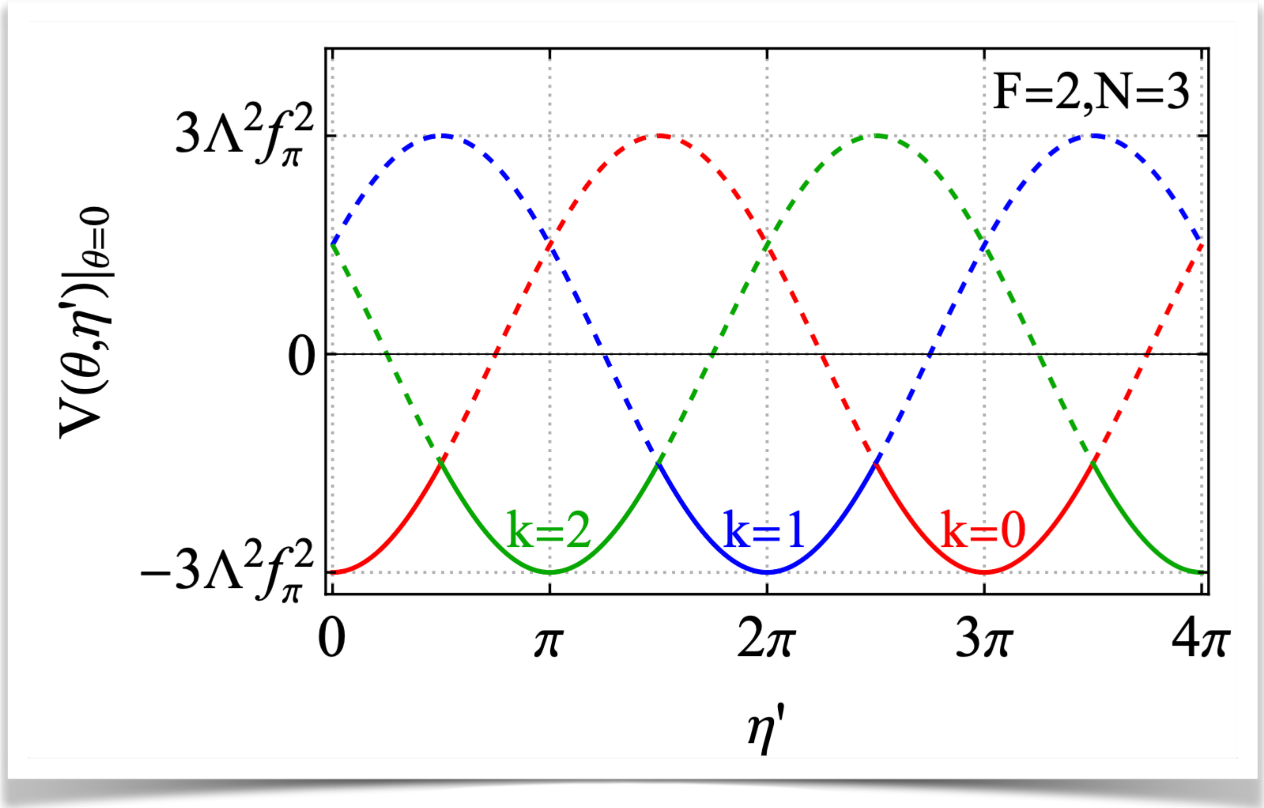

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But: branch structure is indication that it is **not** an instanton effect

Take Home Message Part 1

Large N QCD:

- disfavors instanton origin of η' potential
- generically leads to branched potential

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Let us check this in QCD-like theory!

Part 2: Lessons from SUSY QCD with AMSB

SUSY QCD

- $SU(N)$ SYM with F flavors, i.e. $Q_f, \bar{Q}_f, f = 1,\ldots, F$ in $N, \bar{N}$ representation

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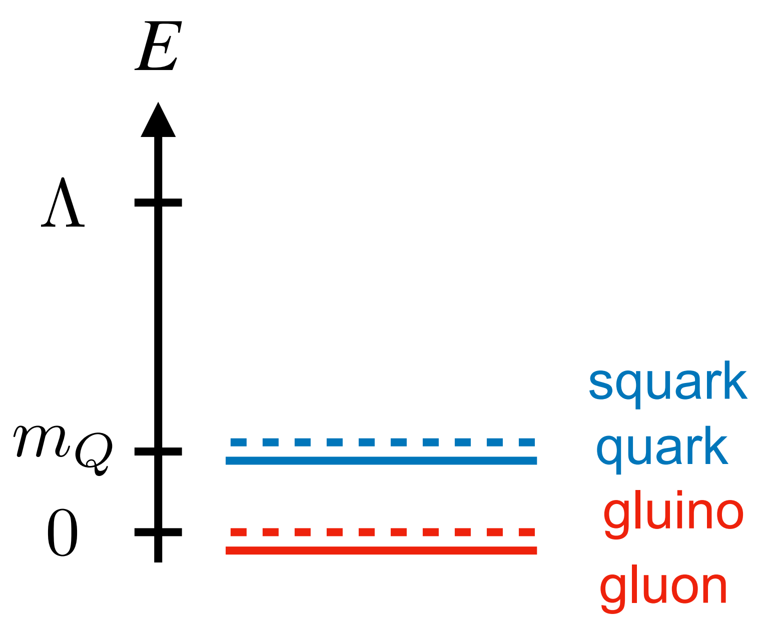

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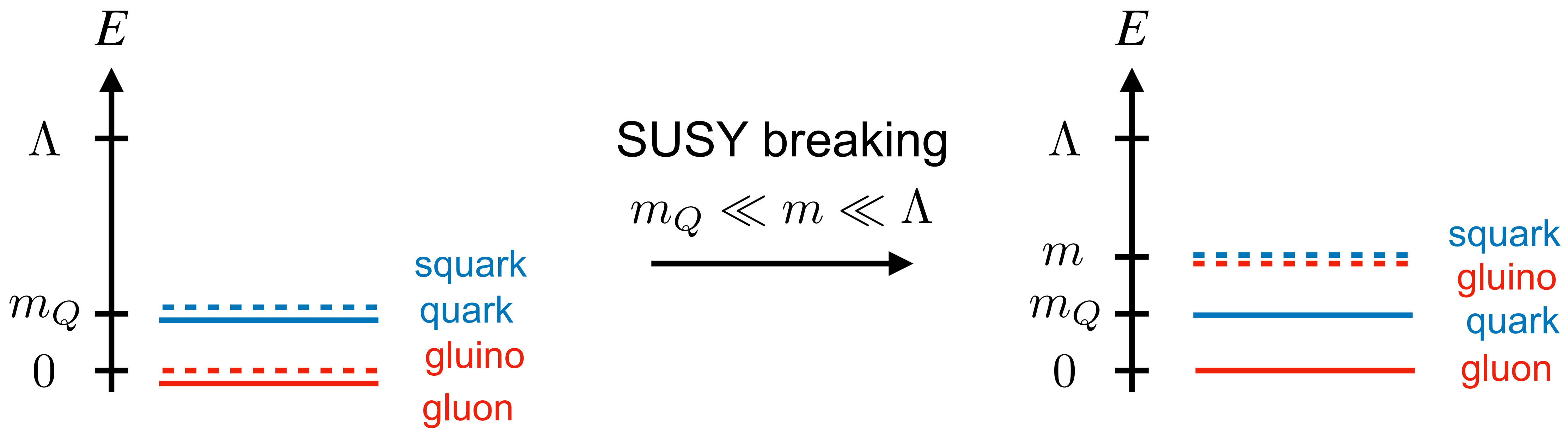

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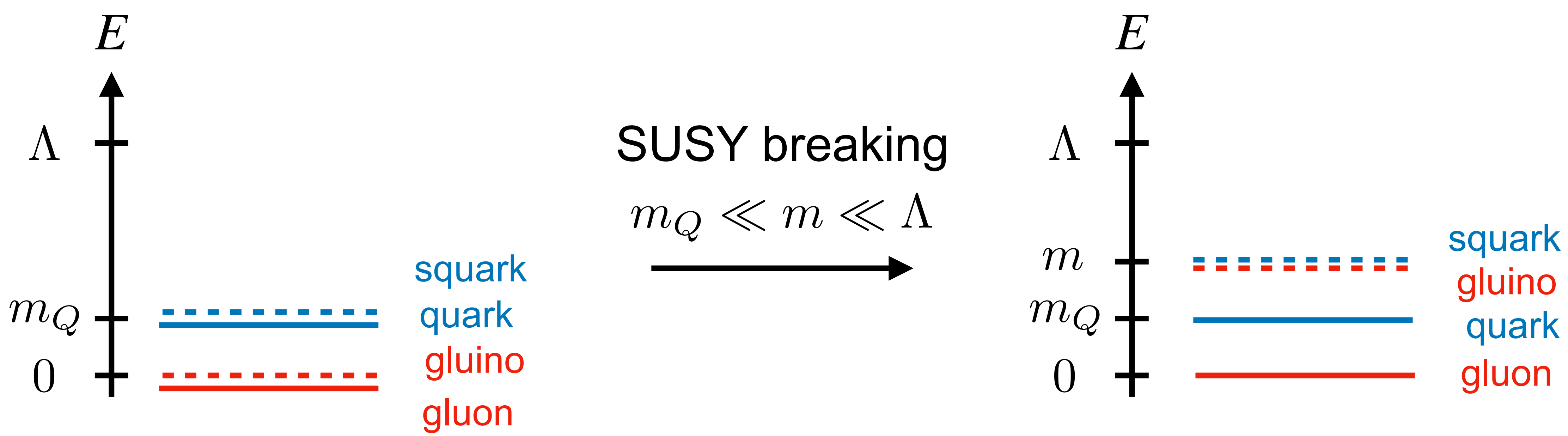

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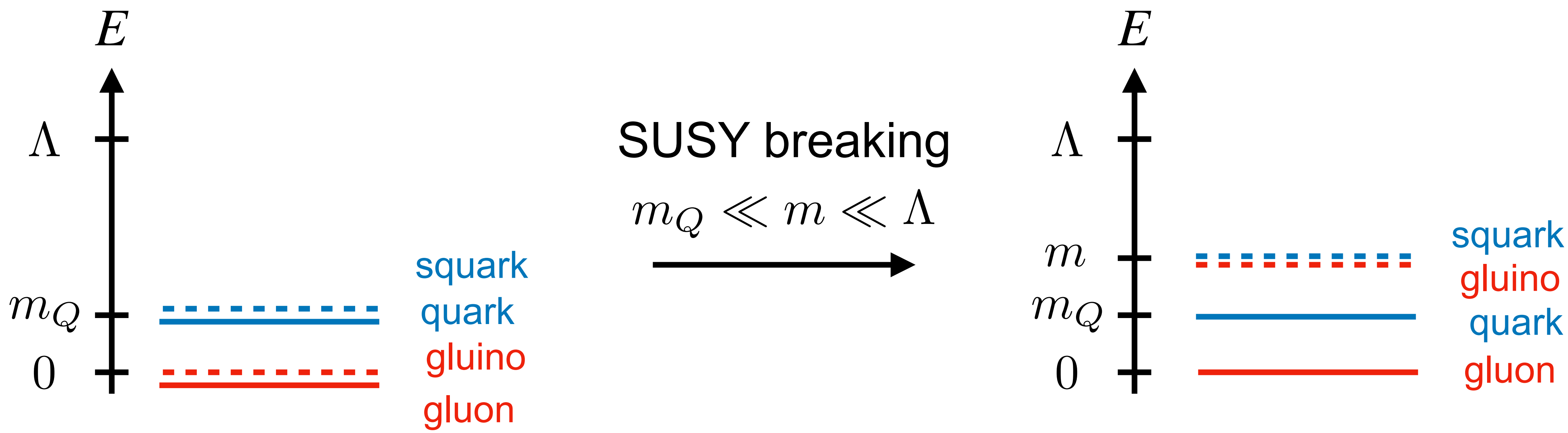

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➡ only QCD-like because gluino & squarks involved in confining dynamics

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  → superpotential is uniquely fixed

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- SUSY breaking via AMSB stabilizes chiral symmetry breaking vacuum

Murayama '21
Csaki et al '22

→ can construct chiral Lagrangian from top down

→ SUSY breaking effects where scale invariance is broken

$$V_{\text{tree}} = \partial_i W g^{ij^*} \partial_j^* W^* + m^* m \left(\partial_i K g^{ij^*} \partial_j^* K - K\right) + m \left(\partial_i W g^{ij^*} \partial_j^* K - 3W\right) + h.c.$$

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Csaki et al '22

- Similar results by Dine, Draper, Stephenson-Haskins and Xu (2016) using soft-breaking via explicit gaugino and squark masses

  → only reliable for $F < N$

F<N SUSY QCD

- D-flat directions parameterized by meson matrix

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holomorphic scale $\Lambda^{b_0} = \mu^{b_0} e^{2\pi i \tau}$ with $\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$

add mass term to mimick QCD

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- D-flat directions parameterized by meson matrix

$M_{ff'} = \bar{Q}^a_f Q^a_{f'}$

$\langle M \rangle \propto \mathbf{1}$ breaks chiral symmetry

- Non-perturbative ADS superpotential is generated

$$W = (N-F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)} + \mathrm{Tr}(m_Q M)$$

holomorphic scale $\Lambda^{b_0} = \mu^{b_0} e^{2\pi i \tau}$ with $\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$

add mass term to mimick QCD

- Scalar potential with AMSB has chiral symmetry breaking minimum

$$\langle M \rangle = f^2 \mathbb{1} \qquad \text{with} \qquad f = \Lambda \left(\frac{N+F}{3N-F}\frac{\Lambda}{m}\right)^{(N-F)/(2N)} + \mathcal{O}(m_Q/m)$$

F<N SUSY QCD: Chiral Lagrangian

- Parameterize GBs as

$$Q^a_f = |f|\delta^a_f\,, \quad \bar{Q}^a_f = Q^a_{f'}U_{f'f}\,, \quad M = |f|^2 U$$

is also the η' and pion decay constant $\frac{\eta'}{f}$ and $\frac{\pi^A}{f}$

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- Chiral Lagrangian

$$V = -\,m\left[(3N-F)\left(\frac{\Lambda^{3N-F}}{|f|^{2F}}\right)^{1/(N-F)}\det(U)^{-1/(N-F)} + |f|^2\mathrm{Tr}(m_Q U)\right] + h.c.$$
$$-\,2\left(\frac{\Lambda^{3N-F}}{|f|^{2F}}\right)^{1/(N-F)}\det(U)^{-1/(N-F)}\mathrm{Tr}(m_Q^\dagger U^\dagger) + h.c\,.$$

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complex root of $\Lambda^{3N-F} = |\Lambda|^{3N-F} e^{i\theta}$ and $\det U = e^{iF\eta'}$

→ get branch-like structure

F<N SUSY QCD: Chiral Lagrangian

- Full chiral Lagrangian

$$V_k = -2(3N-F)\left(\frac{N+F}{3N-F}\right)^{-F/N}\left(\frac{m}{|\Lambda|}\right)^{F/N} m|\Lambda|^3 \cos\left(\frac{F}{N-F}\eta' - \frac{\theta+2\pi k}{N-F}\right)$$

$$-2\left(\frac{N+F}{3N-F}\right)^{1-F/N}\left(\frac{m}{|\Lambda|}\right)^{F/N}|\Lambda|^3\sum_{i=1}^{F} m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{F-1} t_i^j \pi_j\right)$$

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Csaki, d'Agnolo, Gupta, Kuflik, Roy, MR

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- Has branch-like structure but with $1/(N-F)$ instead of $1/N$

→ **not** an instanton effect but gluino condensation in unbroken $SU(N-F)$

F<N SUSY QCD: Special Cases

- $F = 0$: vacuum energy as conjectured in large N pure QCD

$$V_k \xrightarrow{F=0} -6N^2 m|\Lambda_{\text{phys}}|^3 \cos\left(\frac{\theta + 2\pi k}{N}\right)$$

→ **But:** this also holds for finite N

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- $N \gg F$: branched η' potential

$$V_k \overset{N\gg F}{\rightarrow} -6N^2 m|\Lambda_{\text{phys}}|^3 \cos\left(\frac{F}{N}\eta' - \frac{\theta+2\pi k}{N}\right) - \frac{14}{3}N|\Lambda_{\text{phys}}|^3 \sum_{i=1}^{F} m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{F-1} t_i^j \pi^j\right)$$

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What happens when both F and N are large?

F<N SUSY QCD: fixed F/N

- For $F/N =$ fixed the η' mass is constant

$$m_{\eta'}^2 = \frac{(x-3)^2 x}{(x+1)(x-1)^2} m^2 \,, \quad \text{with} \quad x = \frac{F}{N}$$

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- $F = N - 1$: **no** branches and trivially 2π-periodic

$$V_k \overset{N \gg 1}{\to} -4N^{3/2} m^2 |\Lambda_{\text{phys}}|^2 \cos\left((N-1)\eta' - \theta\right) - 2N^{1/2} m |\Lambda_{\text{phys}}|^2 \sum_{i=1}^{F} m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{F-1} t_i^j \pi^j\right)$$
$$- 4N^{1/2} m |\Lambda_{\text{phys}}|^2 \sum_{i=1}^{F} m_i \cos\left(N\eta' + \theta_Q - \theta + \sum_{j=1}^{F-1} t_i^j \pi^j\right) .$$

→ **instanton** effect! Not exponentially suppressed for $N \gg 1$

F≥N SUSY QCD

- Larger moduli space, also baryon fields and different description for $F = N, F = N + 1, F > N + 1$

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F$\geq$N SUSY QCD

- Larger moduli space, also baryon fields and different description for $F = N, F = N + 1, F > N + 1$

- $F = N$ and $F = N + 1$ do **not** have branches

  → consistent with an **instanton** effect

- $F > N + 1$ has $(F - N)$ branches

  → from gluino condensation in the dual $SU(F - N)$ theory

  → similar to $F < N$ case

Spontaneous CP breaking at $\bar{\theta} = \pi$

- Gaiotto, Komargodski and Seiberg argued using large N and anomalies that at $\bar{\theta} = \pi$ CP might be spontaneously broken, depending on F

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- $F = 1$: there is a critical value for the quark masses $m_{Q,0} \sim \Lambda/N$ with
  - $m_Q < m_{Q,0}$ there is a unique CP conserving vacuum
  - $m_Q = m_{Q,0}$ η' is exactly massless
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We can see this explicitly in our results!

Csaki, d'Agnolo, Gupta, Kuflik, Roy, MR

Spontaneous CP breaking at $\bar{\theta} = \pi$

- $F = 1$:

$$V_k(\eta', \bar{\theta}) \propto -a\, m|\Lambda|^3 \cos\left(\frac{\eta' - (\bar{\theta} + 2\pi k)}{N-1}\right) - b\, m_Q|\Lambda|^3 \cos(\eta') - 2m_Q|\Lambda|^3 \cos\left(\frac{N\eta' - (\bar{\theta} + 2\pi k)}{N-1}\right)$$

Csaki, d'Agnolo, Gupta, Kuflik, Roy, MR

Spontaneous CP breaking at $\bar{\theta} = \pi$

- $F = 1:$ dominates for $m_Q \ll m$

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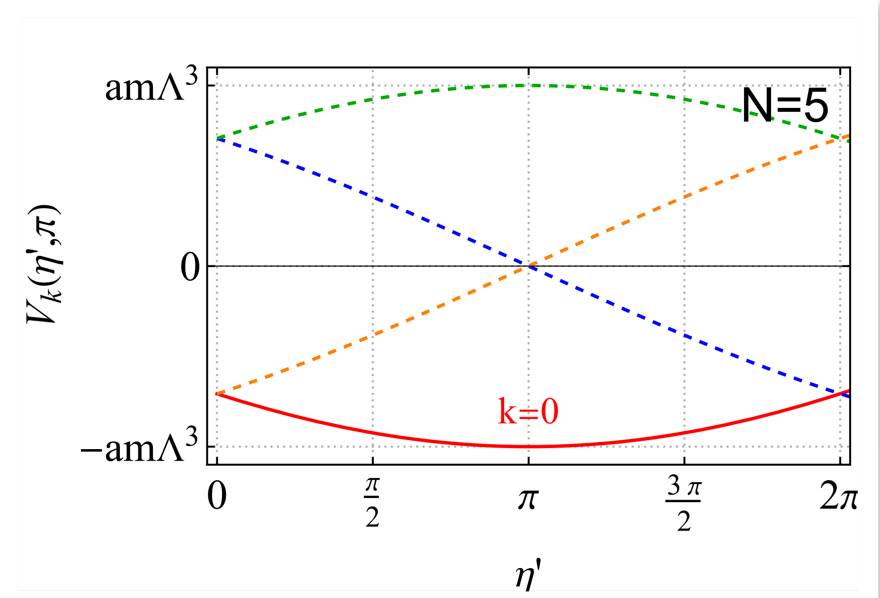
$m_Q < m_{Q,0}$



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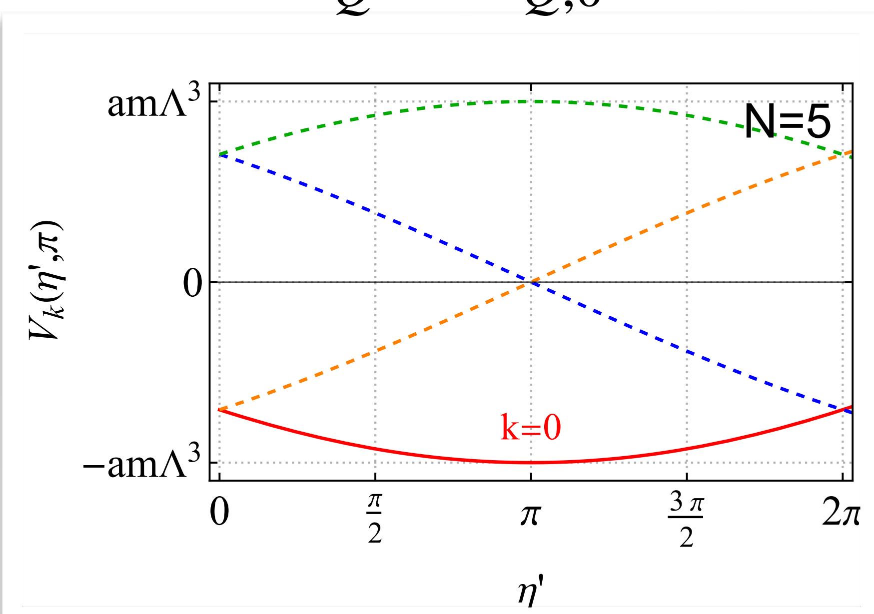

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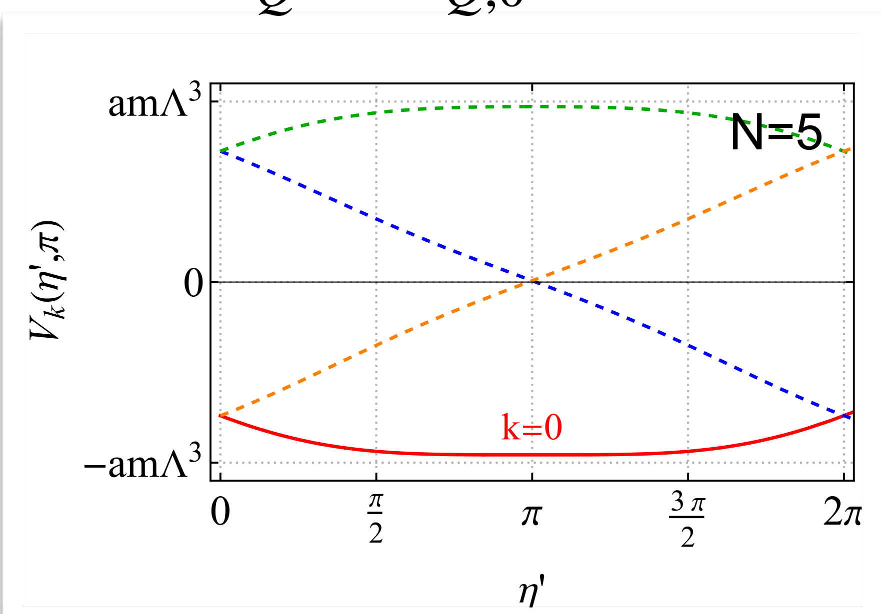

η' massless

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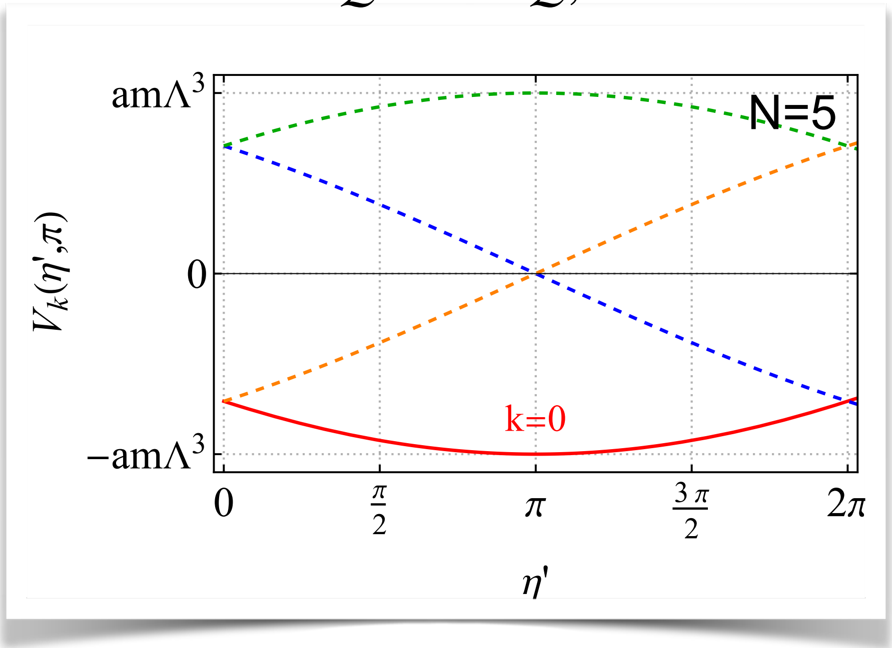

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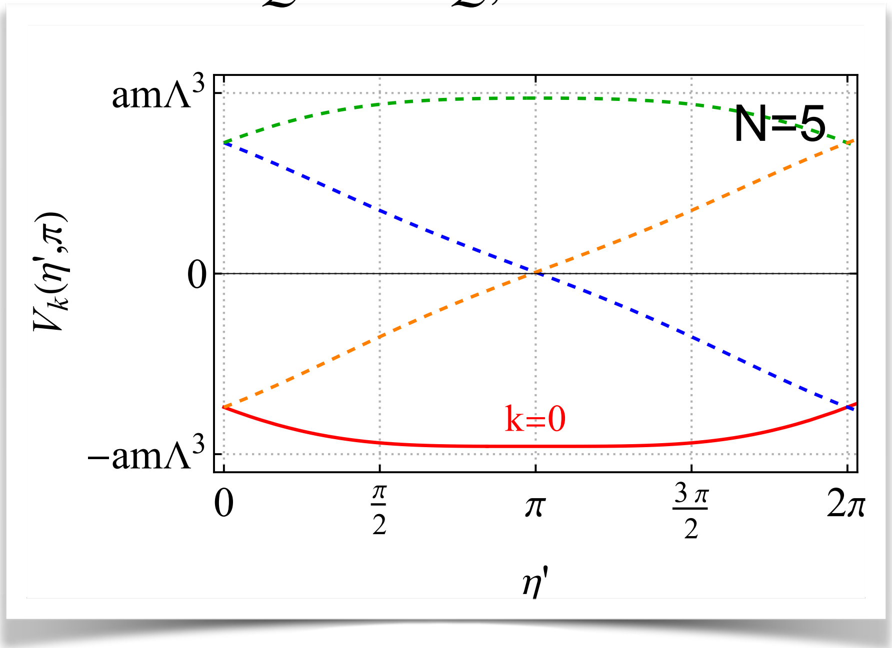

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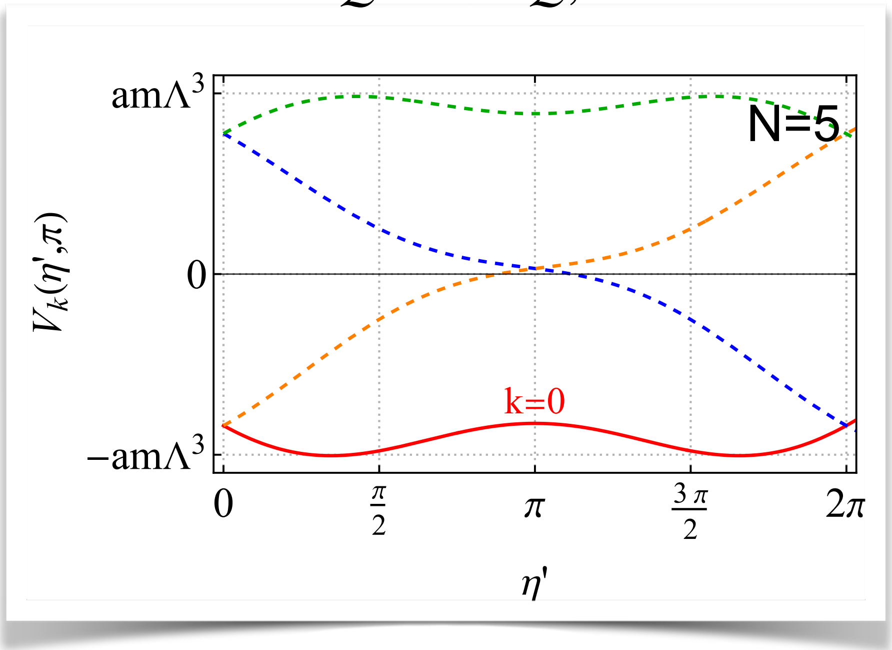

2 degenerate vacua
CP spont. broken

Csaki, d'Agnolo, Gupta, Kuflik, Roy, MR

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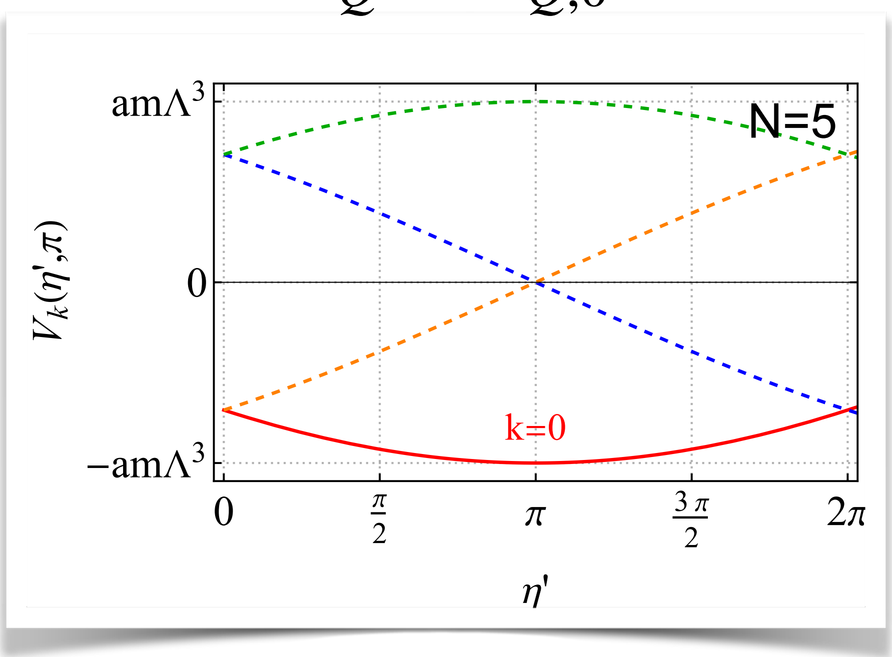

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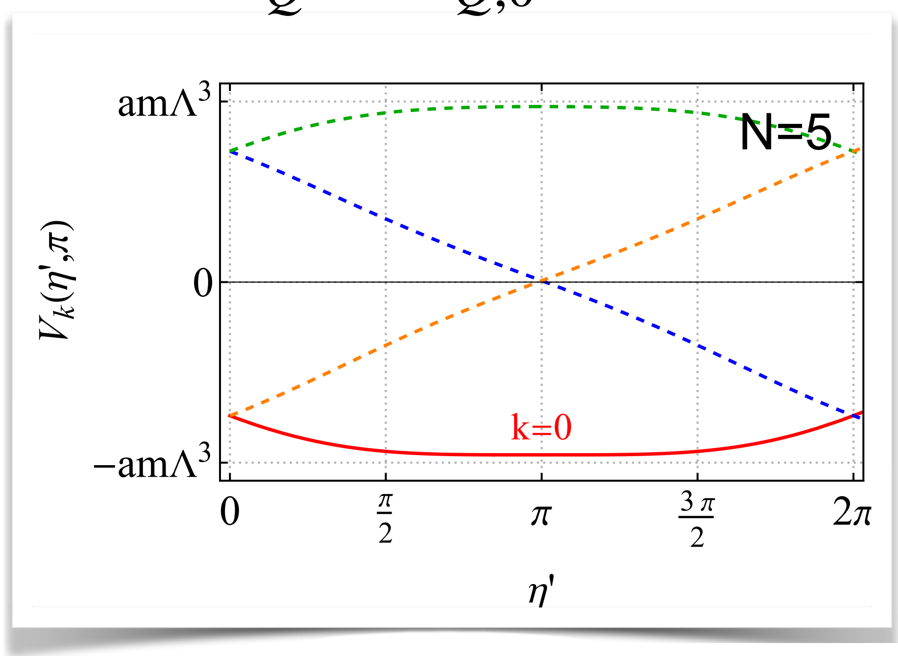

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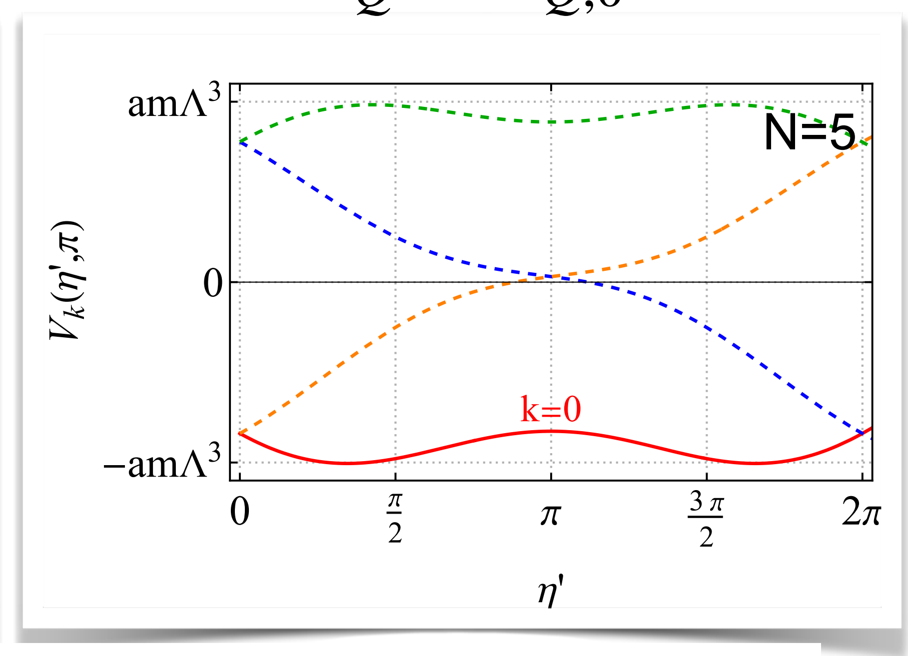

unique CP-conserving minimum

our assumption $m_Q \ll m$

→ only reliable for large N

Spontaneous CP breaking at $\bar{\theta} = \pi$

- $F > 1$: for equal quark masses

→ assume vacuum does not break residual $SU(F)$

$$U = e^{i\eta'} e^{2\pi i m/F} \mathbb{1}$$

VEV in center of $SU(F)$

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$$\langle \eta' \rangle = \frac{\bar{\theta} + 2\pi k}{F}\,, \quad k = 0, \ldots, F-1$$

Csaki, d'Agnolo, Gupta, Kuflik, Roy, MR

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→ $\langle U \rangle = e^{\pm i\pi/F} \mathbb{1}$

→ CP: $U \to U^\dagger$ is spontaneously broken

Lessons from SUSY

- η' potential has in general a branched structure with $|N - F|$ branches

  ➡ originates from confinement dynamics and **not** instantons

- For $F = N - 1, N, N + 1$ we find a single branch

  ➡ consistent with being an **instanton** effect

- η' mass vanishes for $F \ll N$ and is constant for $F/N =$ fixed

- SUSY QCD predicts spontaneous CP breaking at $\bar{\theta} = \pi$ in accordance with QCD

  Gaiotto, Komargodski and Seiberg '17

Which scenario is realized in QCD?

Backup

F=N SUSY QCD

completely antisymmetric color singlet combination of Q_f^a and $\bar{Q}_f^a$

- Larger moduli space: mesons and baryons

$$M_{ff'} = \bar{Q}_f^a Q_{f'}^a \qquad B = \epsilon^{f_1 \cdots f_N} B_{f_1 \cdots f_N}\,, \quad \bar{B} = \epsilon^{f_1 \cdots f_N} \bar{B}_{f_1 \cdots f_N}$$

➡ quantum modified constraint on moduli space $\det(M) - \bar{B}B = \Lambda^{2N}$

➡ implemented in superpotential through Lagrange multiplier X

$$W = X\left(\frac{\det(M) - \bar{B}B}{\Lambda^{2N}} - 1\right) + \mathrm{Tr}(m_Q M)$$

$$K = \frac{\mathrm{Tr}(M^\dagger M)}{\alpha|\Lambda|^2} + \frac{X^\dagger X}{\beta|\Lambda|^4} + \frac{\bar{B}^\dagger \bar{B}}{\gamma|\Lambda|^{2N-2}} + \frac{B^\dagger B}{\delta|\Lambda|^{2N-2}}$$

- Assume chiral symmetry breaking mesonic VEV

$$V = -2|\Lambda|^2(|\Lambda|^2 + (N-2)m^2)\cos\left(N\eta' - \theta\right) - 2m|\Lambda|^2 \sum_{i=1}^{N} m_i \cos\left((N-1)\eta' - \theta_Q - \theta - \sum_{j=1}^{N-1} t_i^j \pi^j\right)$$
$$- 4m|\Lambda|^2 \sum_{i=1}^{N} m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{N-1} t_i^j \pi^j\right).$$

No branches!
Consistent with instanton effect

F=N+1 SUSY QCD

- Baryons in (anti-)fundamental of $SU(F)$

$$B^f = \epsilon^{f_1 \cdots f_N f} B_{f_1 \cdots f_N}$$

$$\bar{B}_f = \epsilon_{f_1 \cdots f_N f} \bar{B}^{f_1 \cdots f_N}$$

- Superpotential implements constraint on moduli space

$$W = \frac{BM\bar{B} - \det(M)}{\Lambda^{2N-1}} + \mathrm{Tr}(m_Q M)$$

Kähler:

$$K = \frac{\mathrm{Tr}(M^\dagger M)}{\alpha|\Lambda|^2} + \sum_f \frac{\bar{B}_f^\dagger \bar{B}_f}{\beta|\Lambda|^{2N-2}} + \sum_f \frac{B_f^\dagger B_f}{\gamma|\Lambda|^{2N-2}}$$

- Chiral Lagrangian similar to $F = N$

$$\begin{aligned} V = &-2(N-2)\left(\frac{N-2}{N}\frac{m}{|\Lambda|}\right)^{(N+1)/(N-1)} m|\Lambda|^3 \cos\left((N+1)\eta' - \theta\right) \\ &-2\left(\frac{N-2}{N}\frac{m}{|\Lambda|}\right)^{N/(N-1)} |\Lambda|^3 \sum_{i=1}^{N+1} m_i \cos\left(N\eta' - \theta_Q - \theta - \sum_{j=1}^{N} t_i^j \pi^j\right) \\ &-4\left(\frac{N-2}{N}\frac{m}{|\Lambda|}\right)^{1/(N-1)} m|\Lambda|^2 \sum_{i=1}^{N+1} m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{N} t_i^j \pi^j\right) . \end{aligned}$$

Again no branches!

Consistent with being an instanton effect

F>N+1 SUSY QCD

- Study weakly-coupled dual $SU(F-N)$

→ DOF: F (anti-)fundamentals $q, \bar{q}$ under $SU(F-N)$ and meson matrix M

- Superpotential

$$W_d = \frac{1}{\mu} q_i M_{ij} \bar{q}_j + \mathrm{Tr}(m_Q M) \qquad \text{with} \qquad \Lambda^{3N-F} \tilde{\Lambda}^{3\tilde{N}-F} = (-1)^{F-N} \mu^F$$

mass term when M gets a VEV → integrate out dual quarks

- Pure $SU(F-N)$ SYM in IR → superpotential from gluino condensation

$$W_d^{\mathrm{eff}} = \tilde{N} \tilde{\Lambda}_{\mathrm{eff}}^3 + \mathrm{Tr}(m_Q M) = (N-F) \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{1/(N-F)} + \mathrm{Tr}(m_Q M)$$

scale matching $\left(\frac{\tilde{\Lambda}_{\mathrm{eff}}}{\det(M/\mu)^{1/F}} \right)^{3\tilde{N}} = \left(\frac{\tilde{\Lambda}}{\det(M/\mu)^{1/F}} \right)^{3\tilde{N}-F}$

F>N+1 SUSY QCD

$$W_d^{\text{eff}} = \tilde{N}\tilde{\Lambda}_{\text{eff}}^3 + \text{Tr}(m_Q M) = (N-F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)} + \text{Tr}(m_Q M)$$

- Same as $F < N$ with $F \leftrightarrow N$ with chiral symmetry breaking minimum

$$M_{ff'} = f^2 \delta_{ij}\,, \quad \text{with} \quad f^2 = |\Lambda|^2 \left(\frac{2F-3N}{N}\frac{m}{|\Lambda|}\right)^{(F-N)/(2N-F)}$$

- Chiral Lagrangian

$$\begin{aligned} V_k = &-4(3N-2F)\left(\frac{2F-3N}{N}\frac{m}{|\Lambda|}\right)^{F/(2N-F)} m|\Lambda|^3 \cos\left(\frac{F}{F-N}\eta' - \frac{\theta+2\pi k}{F-N}\right) \\ &-2F\left(\frac{2F-3N}{N}\frac{m}{|\Lambda|}\right)^{N/(2N-F)} |\Lambda|^3 \sum_{i=1}^{F} m_i \cos\left(\frac{N}{F-N}\eta' - \theta_Q - \frac{\theta+2\pi k}{F-N} - \sum_{j=1}^{F-1} t_i^j \pi_j\right) \\ &-\frac{4FN}{2F-3N}\left(\frac{2F-3N}{N}\frac{m}{|\Lambda|}\right)^{N/(2N-F)} |\Lambda|^3 \sum_{i=1}^{F} m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{F-1} t_i^j \pi_j\right)\,, \end{aligned}$$

$F-N$ branches!

Axion Mass

$$V_a = -2\alpha\Lambda^2 f_\pi^2 \sum_{i=1}^{F} \frac{m_i}{\Lambda} \cos\left(\frac{\bar\theta + an}{F} + \sum_{j=1}^{F-1} t_i^j \pi^j\right)$$

- Easy to obtain axion mass for general F
- Integrate out pions to leading order in $\frac{f_\pi}{f_a}$ instead of diagonalizing mass matrix

→ only need to solve linear equations

- Axion mass $m_a^2 = \alpha\Lambda n^2 \frac{f_\pi^2}{f_a^2}\left(\sum_{i=1}^{F} m_i^{-1}\right)^{-1}$

→ in terms of pion mass $m_a^2 = \frac{n^2 F}{2(F-1)} \frac{f_\pi^2}{f_a^2} \frac{\mathrm{Tr}\, m_\pi^2}{\mathrm{Tr}\, m_q \mathrm{Tr}\, m_q^{-1}}$

→ $F = 2:$ $m_a^2 = 2\alpha\Lambda n^2 \frac{f_\pi^2}{f_a^2} \frac{m_u m_d}{m_u + m_d} = n^2 m_\pi^2 \frac{f_\pi^2}{f_a^2} \frac{m_u m_d}{(m_u + m_d)^2}$

AMSB

- Anomaly mediation: SUSY breaking effects where scale invariance is broken

→ effects described with chiral compensator with F term

Pomarol, Rattazzi '99

$$\mathcal{L} = \int d^4\theta \Phi^* \Phi K + \int d^2\theta \Phi^3 W + c.c. \qquad \text{with} \quad \Phi = 1 + \theta^2 m$$

- Generates tree-level scalar potential

$$V_{\text{tree}} = \partial_i W g^{ij^*} \partial_j^* W^* + m^* m \left(\partial_i K g^{ij^*} \partial_j^* K - K \right) + m \left(\partial_i W g^{ij^*} \partial_j^* K - 3W \right) + h.c.$$

Csaki et al '22

- Gluino and squark masses are loop generated

$$m_\lambda = \frac{g^2}{16\pi^2}(3N - F)m \,, \quad m_{\widetilde{Q}}^2 = \frac{g^4}{(8\pi^2)^2} 2C_i (3N - F) m^2$$

Holomorphic vs Physical Scale

- RGE invariant scale at two-loops

$$\Lambda_c = \mu \left(\frac{b_0 g_c^2(M_c)}{8\pi^2} \right)^{-b_1/(2b_0^2)} \exp\left(-\frac{8\pi^2}{b_0 g_c^2(\mu)} \right)$$

finite in large N limit since $b_0 g_c^2 \propto N g_c^2 =$ const

$b_0 = 3N - F$

$b_1 = 6N^2 - 2NF - 4F(N^2-1)/(2N)$

- Holomorphic scale in superpotential (1-loop exact)

$$\Lambda = \mu e^{\frac{2\pi i \tau(\mu)}{b_0}} \qquad \text{with} \quad \tau = \frac{4\pi i}{g_h^2} + \frac{\theta}{2\pi}$$

- Relation between holomorphic and canonical coupling

wave-function renormalization

$$\text{Re}(\tau) = \frac{8\pi^2}{g_c^2} + 2T(Ad)\log g_c + \sum_i T(i) \log Z_i$$

Dynkin index for adjoint and matter representations

- Relation between holomorphic and canonical scale

$$|\Lambda| = g_c(\mu)^{-b_1/b_0^2} \mu \exp\left(-\frac{8\pi^2}{b_0 g_c^2(\mu)} \right) = \left(\frac{b_0}{8\pi^2} \right)^{b_1/(2b_0^2)} \Lambda_c$$

for $N \gg F$:
$|\Lambda| \propto N^{1/3} \Lambda_c$