New Horizons in the Holographic Conformal Phase Transition

@ 5th NPKI Workshop

Or…Cosmology of the Holographic Dilaton

Jay Hubisz @ Syracuse University

With: Cem Eröncel, Seung J. Lee, Gabriele Rigo, and Bharath Sambasivam

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Conformal Theories and BSM

- Many extensions of the SM: approximately conformal gauge theory that manifests above the few $\times$ TeV (or other scales)
- IR scales emergent through dimensional transmutation
- Dual, via AdS/CFT (large N limit), to stabilized Randall-Sundrum I models
  - 5D AdS: UV/IR branes at $y_0 = 0,\ y_1 = R$ cut off the extra dimension
  - Dual CFT spontaneously broken at scale $f = \sqrt{\frac{6}{\kappa^2 k}} e^{-kR}$
  - Dynamical brane = goldstone boson $\leftrightarrow$ ***dilaton***/radion
  - KK modes (composite resonances) around scale $M_{\text{KK}} \approx f/N$
  - Perturbative in $\frac{1}{N^2} \equiv \frac{\kappa^2 k^3}{8\pi^2}$ - expansion parameter in 5D gravity theory

The Holographic Conformal Phase Transition

- At high temperatures, $T \gg f$, expect restoration of conformal symmetry
    - In the hot early universe, a phase transition between deconfined/confined phases
- Understanding the thermodynamics/cosmology crucial to judge validity of these models of new physics

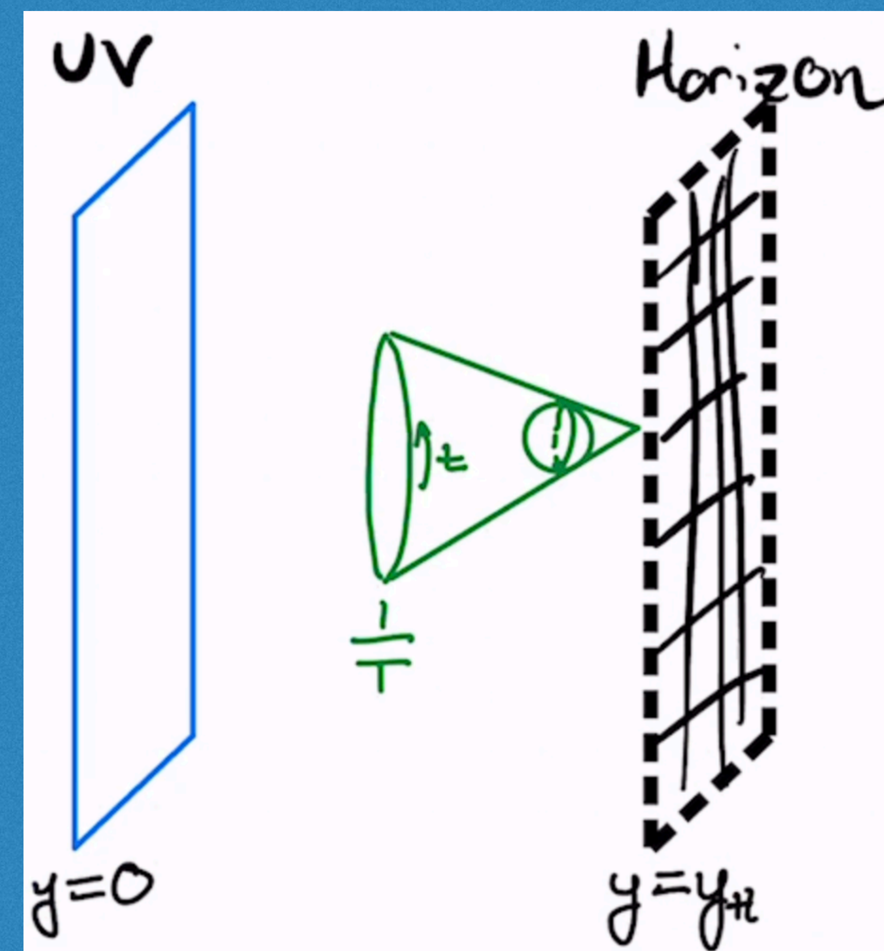
PT: The Standard Analysis

The dual to thermal 4D CFT state:
AdS-Schwarzchild solution to the 5D Einstein Eqs

$$ds^2 = e^{-2ky}\left(h(y)dt^2 + d\vec{x}^2\right) + \frac{1}{h(y)}\frac{dy^2}{G(y)}$$

$$h \approx 1 - e^{-4(y-y_H)}$$

$G(y)$ encodes backreaction
due to stabilization mechanism
e.g. Goldberger-Wise field ϕ(y)



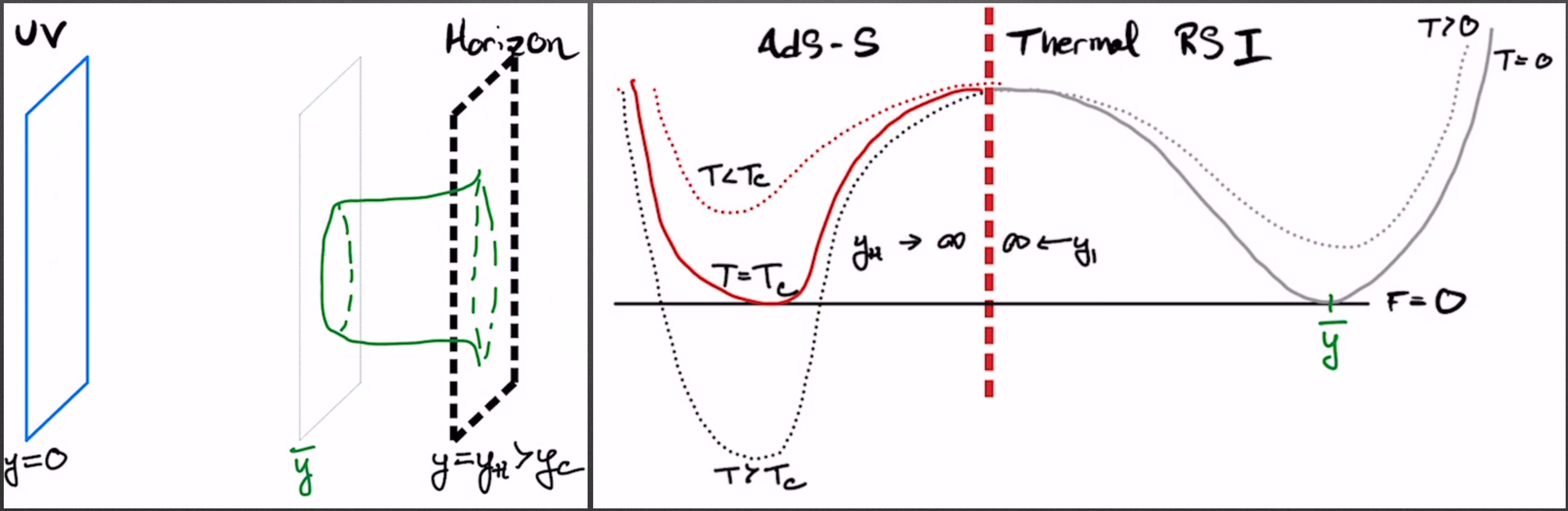
Euclidean path integral, compactified time $t \in (0, 1/T)$, compute free energy:

$$F = e^{-4y_0}\left[\sqrt{h(y_0)}V_0(\phi(y_0)) - \frac{6}{\kappa^2}h(y_0)\sqrt{G(y_0)}\right] - \frac{4\pi T}{\kappa^2}e^{-3y_h}$$

PT: The Standard Analysis

Below critical temperature, Hawking-Page like transition to thermal stabilized two-brane geometry

Patches of IR brane nucleate on the horizon



Nucleation on inflating background due to VE at $f=0$

If $\Gamma \propto e^{-S_{\rm bounce}}$ is not fast enough, PT never happens

A Slow Transition

S_{bounce} grows like N^2

(# of colors in dual conformal gauge theory)

Exponential suppression of the bubble action

A successful transition is at odds with perturbativity of 5D theory

Fixes?

- Focus primarily on changing the shape of the dilaton potential
  - Modifying the type of operator that deforms the geometry/explicitly breaks the CFT
  - Interplay between other dynamics (like QCD phase transition)

5D Cosmology

- Importantly, the dynamics are ***not*** simply static thermal physics - temperature controlled by FRW expansion
  - Conformal symmetry is part of sum total spacetime symmetries
  - 5D: Cosmological dynamics, thermodynamics, and the dilaton are all part of one picture
- What are the dynamics of a CFT in an evolving big bang cosmology? Is thermal equilibrium a good assumption?
  - 5D Cosmology can help us understand the answer

The 4D non-linear dilaton action

In 4D, can construct dilaton theory from ground up

$\tilde{g}_{\mu\nu} = e^{-2\tau} g_{\mu\nu}$ Write diff x Weyl invariant action

$$S = f^2 \int d^4x \sqrt{\tilde{g}} \left(\frac{1}{6} \tilde{R} + \Lambda + \text{higher derivative gravity terms} \right)$$

Expand on some background geometry (e.g. Mink.):

$$(g_{\mu\nu} = \eta_{\mu\nu}) \to f^2 \int d^4x \left[e^{-2\tau} \partial\tau^2 + \Lambda e^{-4\tau} + \text{higher derivatives in } \tau \right]$$

Aside: Non-Minkowski — get conformal coupling to curvature

$$\frac{1}{6} \sqrt{\tilde{g}} \tilde{R} \ni \frac{1}{6} e^{-2\tau} \sqrt{g} R$$

Higher order terms contain information about the parent CFT

e.g. a-anomaly associated with $(\partial\tau)^4$ term

5D AdS: all-orders dilaton action

5D AdS: Dual to large N strongly interacting CFT

$$S = -\int d^5x\sqrt{g}\left[\frac{1}{2\kappa^2}R + \Lambda + \text{higher derivative gravity terms}\right]$$

Suppressed by powers of $1/N$

$$-\int d^4\xi_0\sqrt{g_0}\lambda_0 - \int d^4\xi_1\sqrt{g_1}\lambda_1$$

$$\Lambda = -\frac{6k^2}{\kappa^2} \text{ and } N^2 = \frac{8\pi^2}{\kappa^2 k^3}$$

UV and IR branes

UV: turn on 4D gravity, break CFT explicitly

IR: spontaneous breaking of CFT, dynamical brane=dilaton

$$ds^2 = e^{-2A(x,y)}(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu - G(x,y)dy^2$$

Dilaton: $A(x, y_1) \equiv \tau(x)$

At leading order in $1/N$, EFT of dilaton has ∞ tower of higher derivative operators

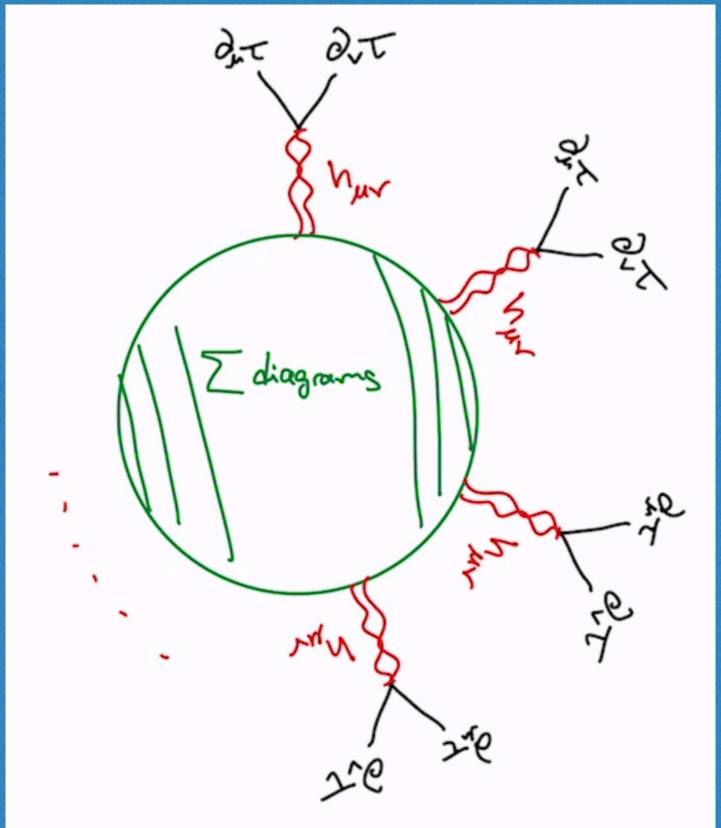
Early universe cosmology → Relativistic motion of brane through 5D space

5D geometry encodes complicated states/dynamics far from vacuum

All orders in derivatives

Integrating out 5D gravity trees gives ∂^{2n} terms in 4D dilaton EFT

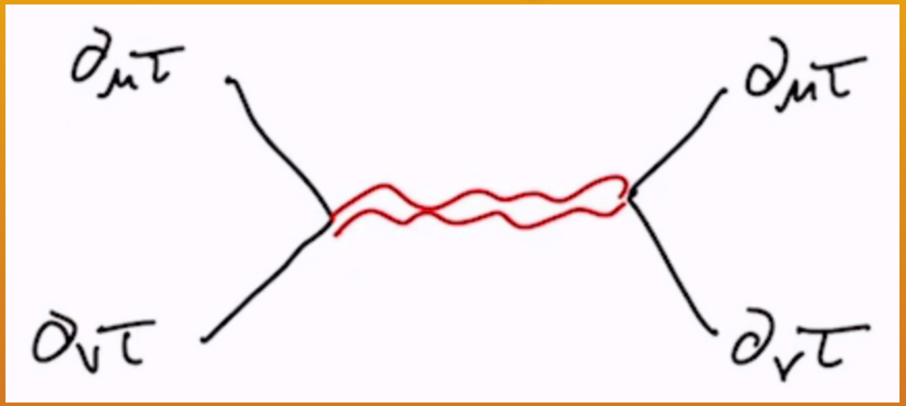
Cumbersome to work out explicitly:



$$\rightarrow c_n e^{(2n-4)\tau}(\partial\tau)^{2n}$$

Also novel couplings to stress energy tensor for light fields

"Simplest" diagram correctly reproduces the holographic a-anomaly



$$\rightarrow 2a_{\rm RS}(\partial\tau)^4$$

$$a_{\rm RS} = \frac{1}{8\kappa^2 k^3} = \frac{N^2}{4(16\pi^2)}$$

Csáki, JH, Ismail, Rigo, Sgarlata: 2205.15324

Moral:

If you can find solutions to full non-linear 5D gravity

<u>Solutions correctly sum ∞ tower of higher derivative operators at leading order in $1/N$</u>

Stabilization: A Tale of Two Mistunes

The original sin of RS I is stabilization - the UV and IR branes have tensions $\lambda_{0,1}$ that are free parameters

$$S = -\int d^5x\sqrt{g}\left[\frac{1}{2\kappa^2}R + \Lambda\right] - \int d^4\xi_0\sqrt{g_0}\lambda_0 - \int d^4\xi_1\sqrt{g_1}\lambda_1$$

Under the assumption of a static AdS background, integrating the action on eom yields an effective potential:

$$V_{\text{eff}} = \frac{6k}{\kappa^2}\left[\delta_0 + e^{-4kR}\delta_1\right]$$

The δ's are "mistunes" of the brane tensions against the bulk CC

$$\lambda_{0,1} = \pm\frac{6k}{\kappa^2}(1 \pm \delta_{0,1})$$

Naive: δ_0 causes inflation, δ_1 causes the dilaton to roll

Stabilization: Dynamical Mistunes

- Axion solution to SCP: promote the QCD vacuum energy to a field
- Goldberger-Wise stabilization promotes the δ's to dynamical quantities that depend on brane separation

$$V_{\text{eff}} = \underbrace{\left[V_0(\phi_0) - \frac{6}{\kappa^2}\sqrt{G_0}\right]}_{\tilde{\delta}_0(R)} + e^{-4kR}\underbrace{\left[V_1(\phi_R) + \frac{6}{\kappa^2}\sqrt{G_R}\right]}_{\tilde{\delta}_1(R)}$$

For a ***static Minkowski minimum*** these both vanish

Dilaton quartic and 4D CC relax to zero

Presume tuning of V_0 so min of V_{eff} is at zero

Away from the minimum, mistunes are no longer zero

In many models, vary ***slowly*** with R

Party like it's 1999!

Mistuned RS I Dilaton Cosmology

Time dependent metric, GN wrt UV brane @ y=0

$$ds^2 = n^2(y,t)dt^2 - a^2(y,t)dx_3^2 - dy^2$$

Fix $n(y=0,t) = 1$

Well known that cosmology for UV brane (fundamental) observer is "dark" radiation + CC:

$$H_{\mathrm{UV}}^2 = \frac{4\bar{\lambda}}{\bar{a}^4} + \delta_0(2+\delta_0)$$

Mistune of UV brane indeed *related* to CC

FRW geometry is actually closely related to AdS-S crucial difference of UV brane embedding

Gubser: arXiv:hep-th/9912001

Cosmology is FRW with initial radiation domination, followed by CC domination

Bulk geometry

Completely determined by UV brane mistune/radiation

Geometry contains a *Rindler* horizon at $y_H(t)$

$$ds^2 = n^2(y,t)dt^2 - a^2(y,t)dx_3^2 - dy^2$$

$$n(y_H(t),t) = 0$$

In radiation domination, $y_H(t) \approx \frac{1}{4}\log(1+4t)$

Thermal interpretation:

Cooling conformal plasma: $T = \frac{k}{\pi}e^{-ky_H(t)} \propto \frac{1}{\bar{a}}$

Late times → CC dominates if δ_0 positive: $y_H = \frac{1}{2k}\log\frac{2+\delta_0}{\delta_0}$

$$T = \frac{k}{\pi}e^{-ky_H} \approx \frac{H}{2\pi}$$

The usual dS temperature

Dynamics of the IR brane

Geometry is ended at $y = R(t) \equiv \langle \tau \rangle$

Junction conditions: *first order* equation for R

$$\frac{\frac{\dot{R}}{n}\frac{\dot{a}}{an} + \frac{a'}{a}}{\sqrt{1 - \left(\frac{\dot{R}}{n}\right)^2}} = -1 + \delta_1$$

$$\beta \equiv \frac{\dot{R}}{n} \qquad \tilde{H} \equiv \frac{\dot{a}}{an}$$

Bulk Hubble

$$\gamma\beta\tilde{H} + \gamma\frac{a'}{a} = -1 + \delta_1$$

A sort of relativistic energy-momentum relation for a DBI brane

Can square and solve for relativistic velocity:

$$\beta_\pm = \dot{R}/n\Big|_\pm = F_\pm(R(t), t)$$

not all solutions are valid

complex, or don't solve original eq.

Taxonomy of the Cosmological Dilaton

Signs of $\delta_{0,1}$:

$(++)$ $(+-)$

$(-+)$ $(--)$

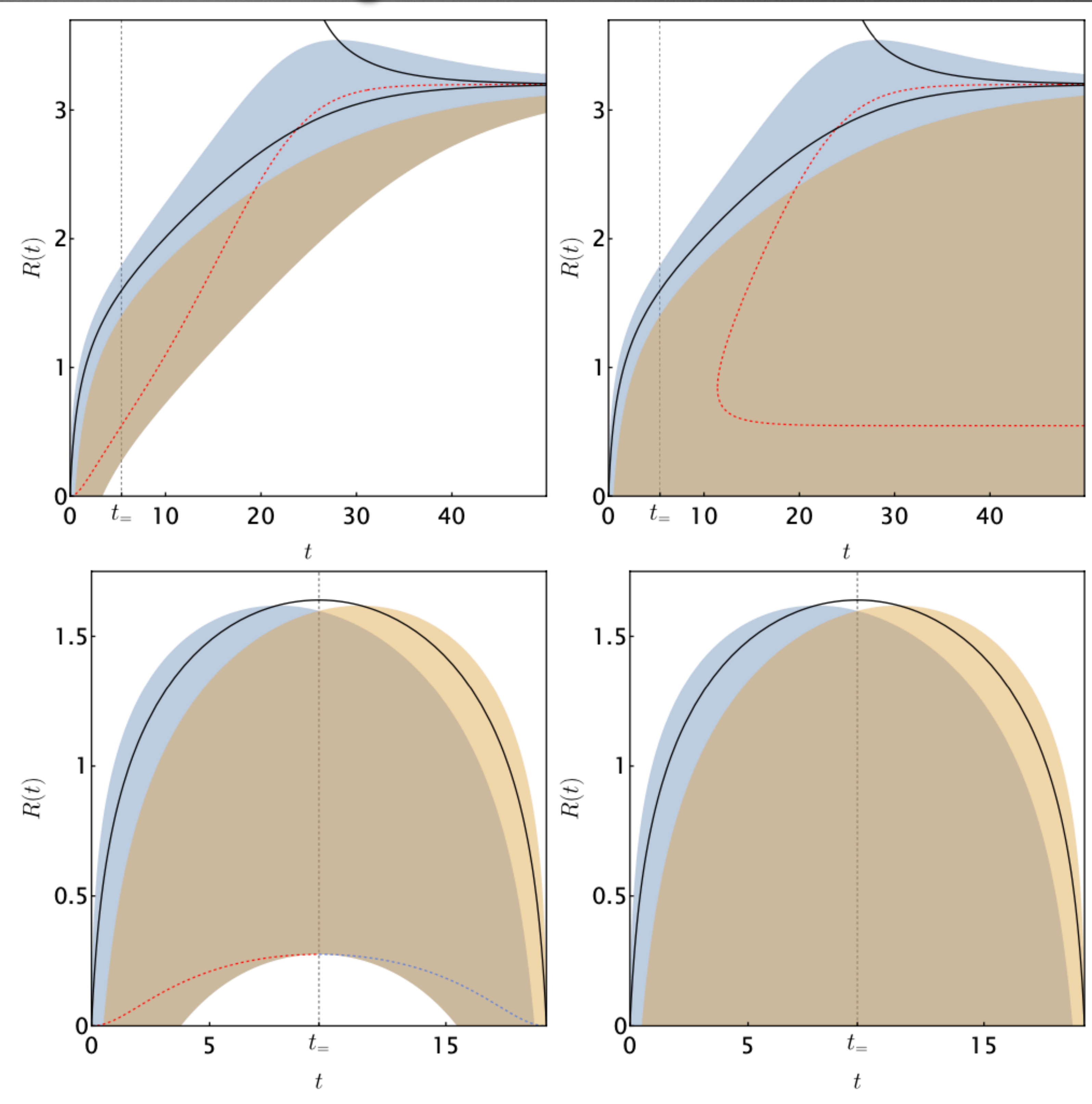
$$\gamma\beta\tilde{H} + \gamma\frac{a'}{a} = -1 + \delta_1$$

The brane can't just be anywhere! Not all initial conditions are allowed.

Only the blue branch admits existence of brane back to $t = 0$

Red lines:
$\dot{R} = 0$ turn-around points

Receding horizon: Thermal FRW



Second Order Equation

To help understand the physics, employ EE's to get second order equation for $R(t)$

Exact: $$\ddot{R} + \left[\left(3 - \frac{1}{n^2}\frac{\partial V}{\partial R} + \widetilde{f}(R,\beta)\right)\frac{\dot{a}}{a} - \frac{\dot{n}}{n}\right]\dot{R} + \frac{\partial V}{\partial R} = 0$$

Evolution of a scalar field with complicated friction term, and an effective potential

The Dilaton Effective Potential:

$$\frac{\partial V}{\partial R} = -n^2\left[4\delta_1 + \frac{1}{n}\dot{\widetilde{H}} + 2\widetilde{H}^2\right] \approx -4(\delta_1 e^{-2kR} + \delta_0) + \mathcal{O}((\mu_{\rm IR}/f)^8)$$

The first piece is just the expected dilaton quartic

Second piece: coupling of the dilaton to the 4D scalar curvature $\xi R_{(4)} e^{-2R}$

$\xi = 1/6$ - conformally coupled scalar

Explicit breaking propagates to the dilaton

Hidden corrections are T^8 contributions from couplings to $T_{\mu\nu}^2$

Hubble rate in inflating backgrounds gives positive mass^2 to the dilaton: late-time $f = 0$ solution is always stable!

The cosmological phase transition

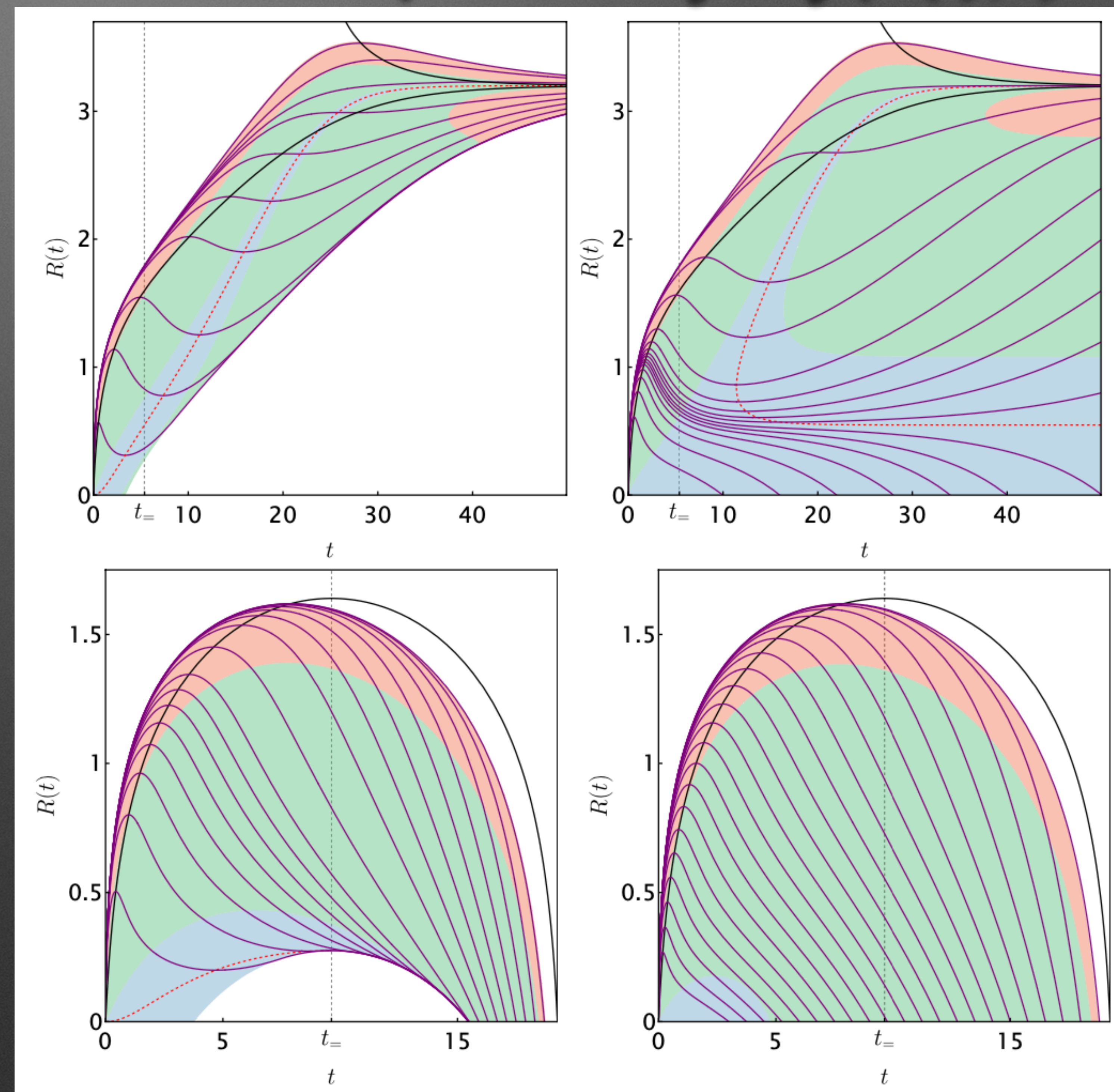
All trajectories for the IR brane begin *behind the horizon*, and *highly relativistic*

Out of equilibrium: $f = f(T(t), t)$

Signs of $\delta_{0,1}$:

$(++)$ $(+-)$

$(-+)$ $(--)$

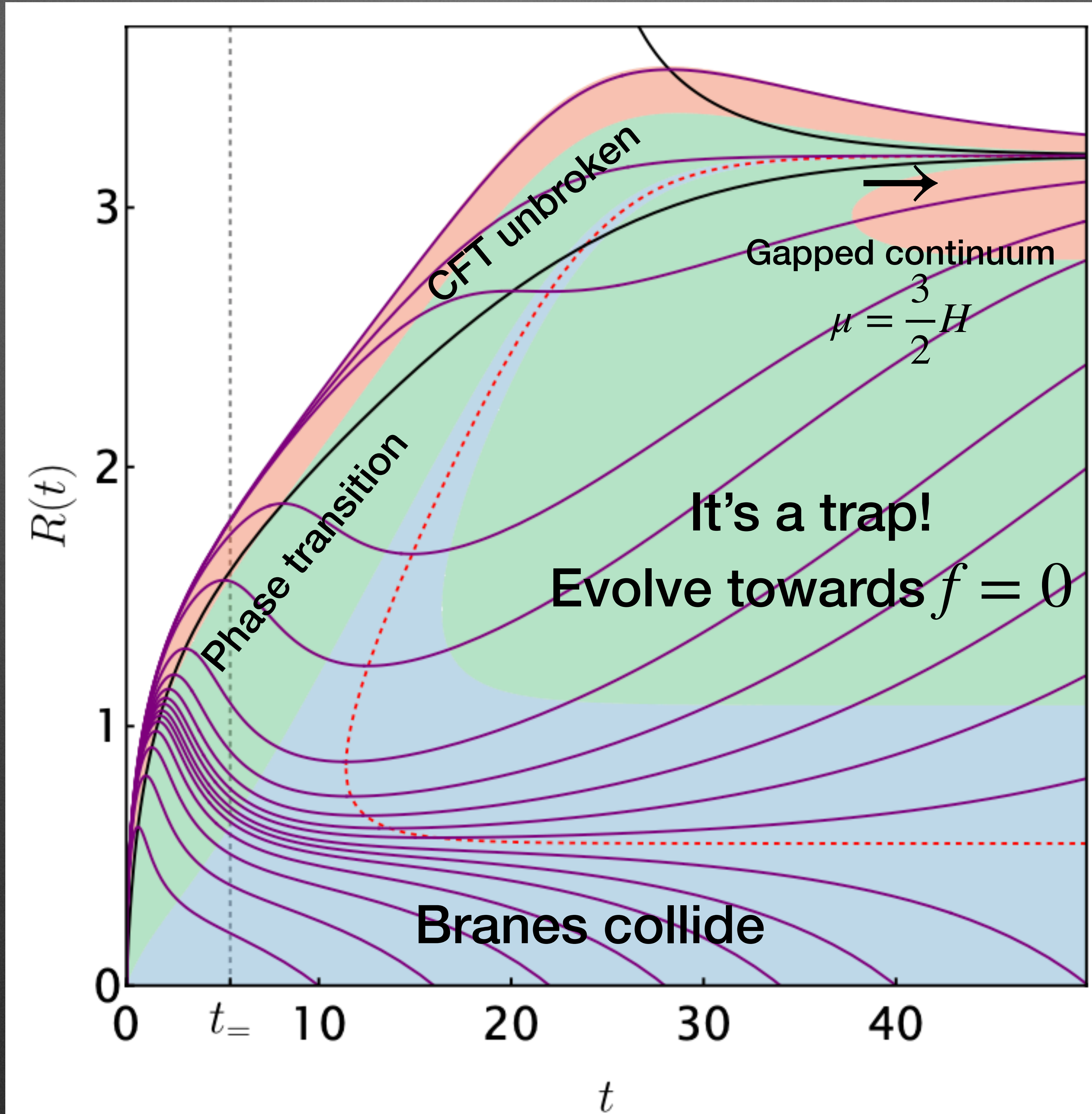


Each trajectory has a different temperature at which it passes through the horizon

For fundamental observer
this is the phase transition

Fate of the dilaton? Depends on initial conditions!

Example: Positive CC, Negative Quartic



Highly relativistic - PT happens later (or not at all)

(Preliminary)

Aside: What about trace anomalies?

$$\frac{\partial V}{\partial R} = -n^2\left[4\delta_1 + \frac{1}{n}\dot{\widetilde{H}} + 2\widetilde{H}^2\right] \approx -4(\delta_1 e^{-2kR} + \delta_0) + \mathcal{O}((\mu_{\rm IR}/f)^8)$$

The dilaton effective potential gets no contributions at order T^4

The CFT "dark" radiation is conformal - does not contribute to T^μ_μ

T^8 contributions overcome δ_0 effects at early times (destabilize $f = 0$)

Near-conformal (e.g. SM) radiation with $\beta(g) \neq 0$?
Does it re-stabilize the origin?

Expect it depends on the sign of the beta function

$$\frac{\partial V}{\partial R} \ni -T^\mu_\mu \propto \beta(g)\ T^4$$

Marginally relevant (asymptotically free) - destabilizes $f = 0$

AF radiation likely does not prevent phase transition

Stabilized Dilaton Cosmology

We had simple equations for the case of constant $\delta_{0,1}$

$$H_{\rm UV}^2 = \frac{4\bar{\lambda}}{\bar{a}^4} + \delta_0(2+\delta_0)$$

δ_0 determines bulk geometry

$$\frac{\frac{\dot{R}}{n}\frac{\dot{a}}{an} + \frac{a'}{a}}{\sqrt{1-\left(\frac{\dot{R}}{n}\right)^2}} = -1 + \delta_1$$

δ_1 determines motion of IR through that geometry

Remember the story of inflation:

In slow roll, friction dominates, dS with nearly constant H
dynamical equations are approximately first order

Approximate dynamics by simply promoting $\delta_{0,1}$ in these equations to be functions of R: $\delta_{0,1} \to \tilde{\delta}_{0,1}(R)$?

Stabilized Dilaton Cosmology

Exact equation for cosmology on the UV brane:

$$\frac{1}{2}\left[\dot{H}+2H^2\right]=\frac{\kappa^2}{12}\left[1-\frac{\kappa^2}{12}(\phi_0')^2\right]+\frac{\kappa^2}{6}V(\phi_0)=\frac{\kappa^2}{12}\left[\left(\bar{\phi}_0'\right)^2-\left(\phi_0'\right)^2\right]$$

$$\sim 2\tilde{\delta}_0(R)$$

Exact equation for IR brane evolution:

$$\ddot{R}+\left[\left(3-\frac{6}{\kappa^2 n^2(-T_1)}\frac{\partial V}{\partial R}+\widetilde{f}(R,\beta)\right)\frac{\dot{a}}{a}-\frac{\dot{n}}{n}\right]\dot{R}+\frac{\partial V}{\partial R}=0,$$

$$\sim -2\tilde{\delta}_1(R)$$

$$\frac{\partial V}{\partial R}=\frac{12}{\kappa^2(-T_1)}n^2\left[\frac{\kappa^2}{12}\left(\left(\bar{\phi}_1'\right)^2-\left(\phi_1'\right)^2\right)-\frac{1}{2}\left(\frac{1}{n}\dot{\widetilde{H}}+2\widetilde{H}^2\right)\right]$$

Can derive slow-roll conditions under which it is valid to approximate with simple substitution $\delta_{0,1}\rightarrow\tilde{\delta}_{0,1}(R(t))$

Slow-roll dilaton:

$$\epsilon_{\rm UV}\equiv\left|\frac{\dot{\tilde{\delta}}_0}{4H\widetilde{\delta}_0}\right|<1$$

$\frac{\dot{a}}{a}>\frac{a'}{a}\dot{R}$

$$\eta_{\rm IR}\equiv\left|\frac{n^2\widetilde{\delta}_1'}{3(\dot{a}/a)^2}\right|<1$$

$\frac{\dot{a}}{a}<\frac{a'}{a}\dot{R}$

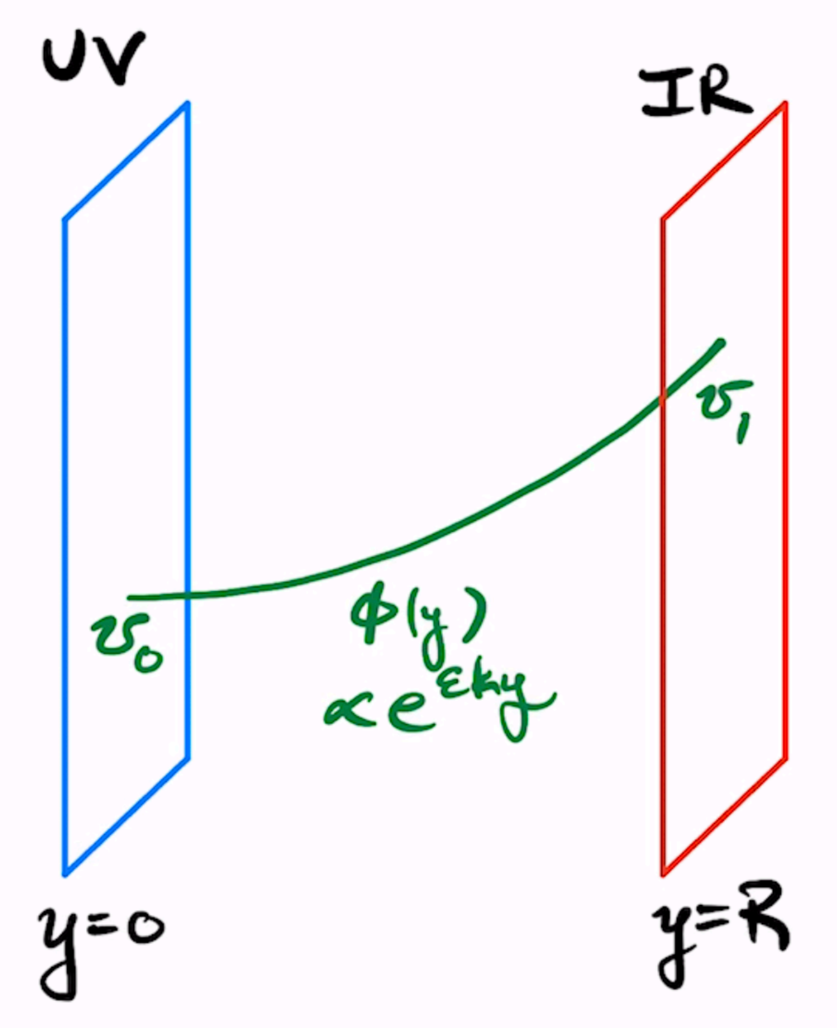
$$\epsilon_{\rm IR}\equiv\left|\frac{\widetilde{\delta}_1'}{4\left(\widetilde{\delta}_1+e^{2R}\widetilde{\delta}_0\right)}\right|<1$$

Implement simplest stabilization

5D scalar field ϕ with $m^2 = -\epsilon(2-\epsilon)k^2$

$\phi = \phi_\epsilon e^{\epsilon k y} + \phi_4 e^{(4-\epsilon)ky}$

Dual to sourcing near marginal operator, $\Delta \approx 4 - \epsilon$



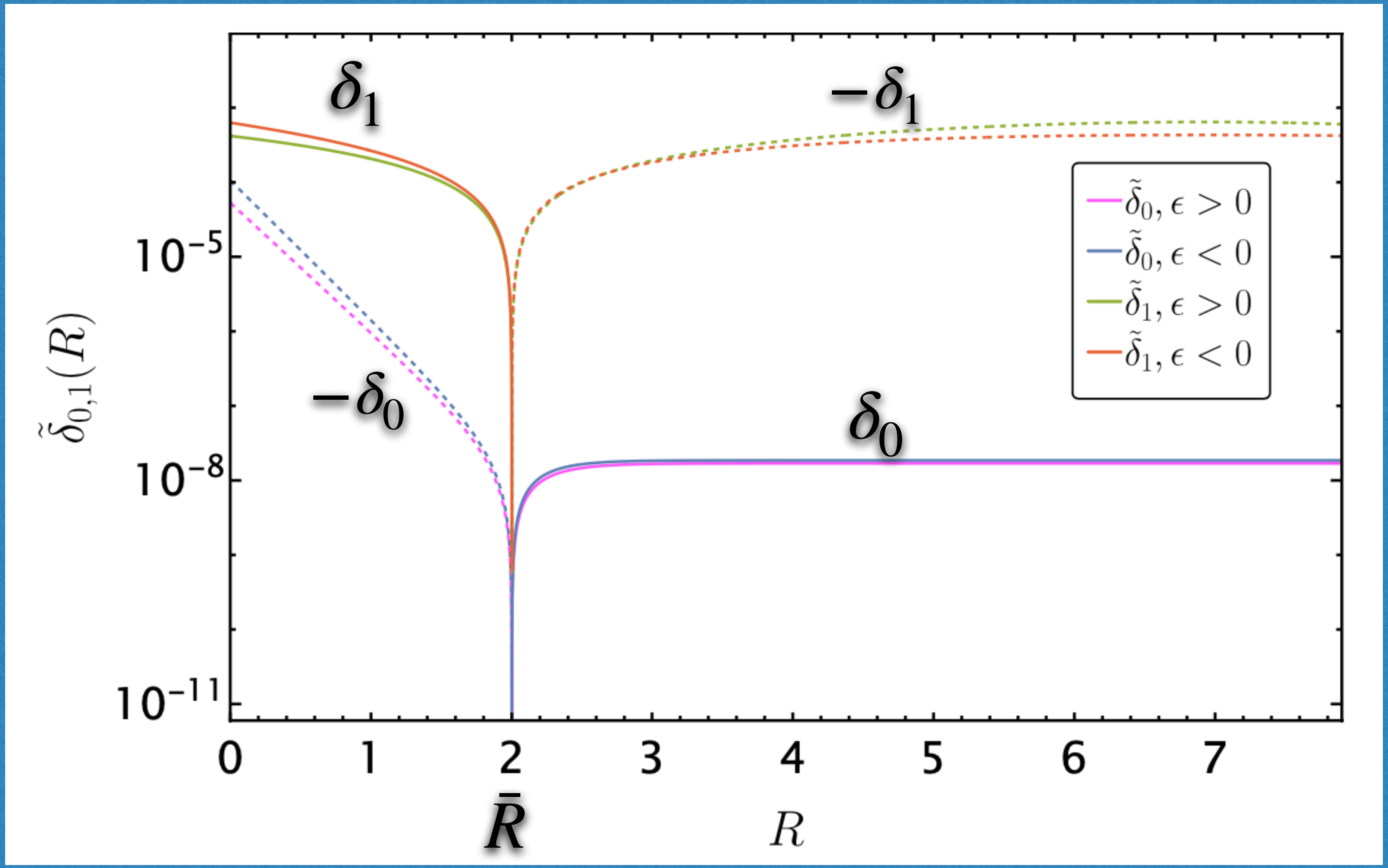
$$\tilde{\delta}_0(R) = \frac{\kappa^2}{24}\left[(\bar{\phi}_0')^2 - (\phi_0')^2\right]$$

$$\tilde{\delta}_1(R) = \frac{\kappa^2}{24}\left[(\phi_1')^2 - (\bar{\phi}_1')^2\right]$$

$$\tilde{\delta}_0(R) = \frac{\kappa^2}{3}\epsilon v_0 v_1 e^{-(4-\epsilon)\bar{R}}\left[\left(1 - e^{-(4-\epsilon)(R-\bar{R})}\right) - \frac{v_0}{v_1}e^{\epsilon\bar{R}}\left(1 - e^{-(4-2\epsilon)(R-\bar{R})}\right)\right]$$

$$\tilde{\delta}_1(R) = \frac{4\kappa^2}{3}v_0 v_1 e^{\epsilon\bar{R}}\left[\left(1 - e^{\epsilon(R-\bar{R})}\right) - \frac{1}{2}\frac{v_0}{v_1}e^{\epsilon\bar{R}}\left(1 - e^{2\epsilon(R-\bar{R})}\right)\right].$$

Mistunes of the stabilized dilaton



Minimum of potential at $\bar{R}$:
scale associated with dimensional transmutation

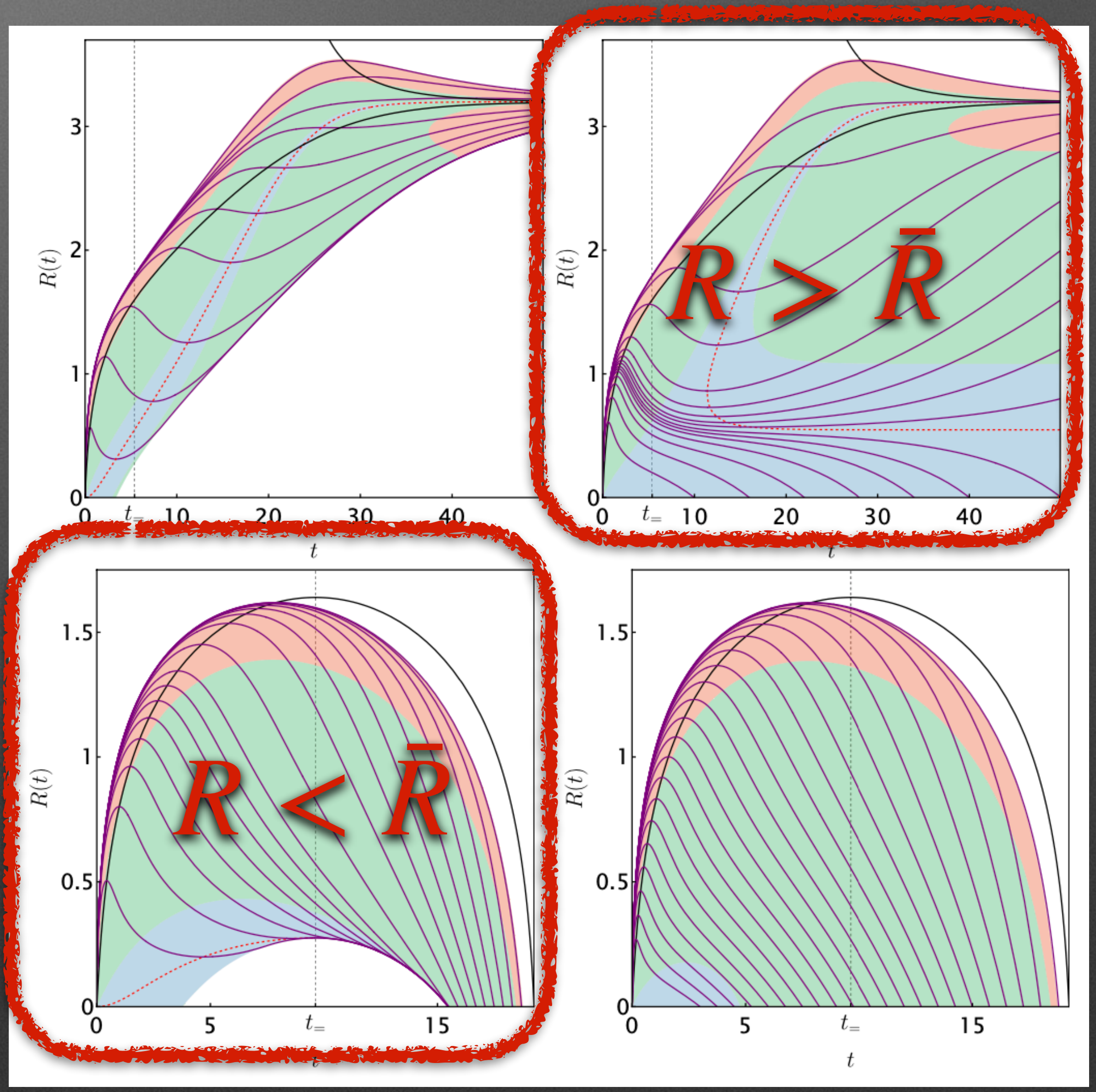
Note: with the exception of δ_0 below the minimum, mistunes vary only *mildly* with R

For marginally ir(relavant) deformations, have (-+) below the minimum, (+-) above it

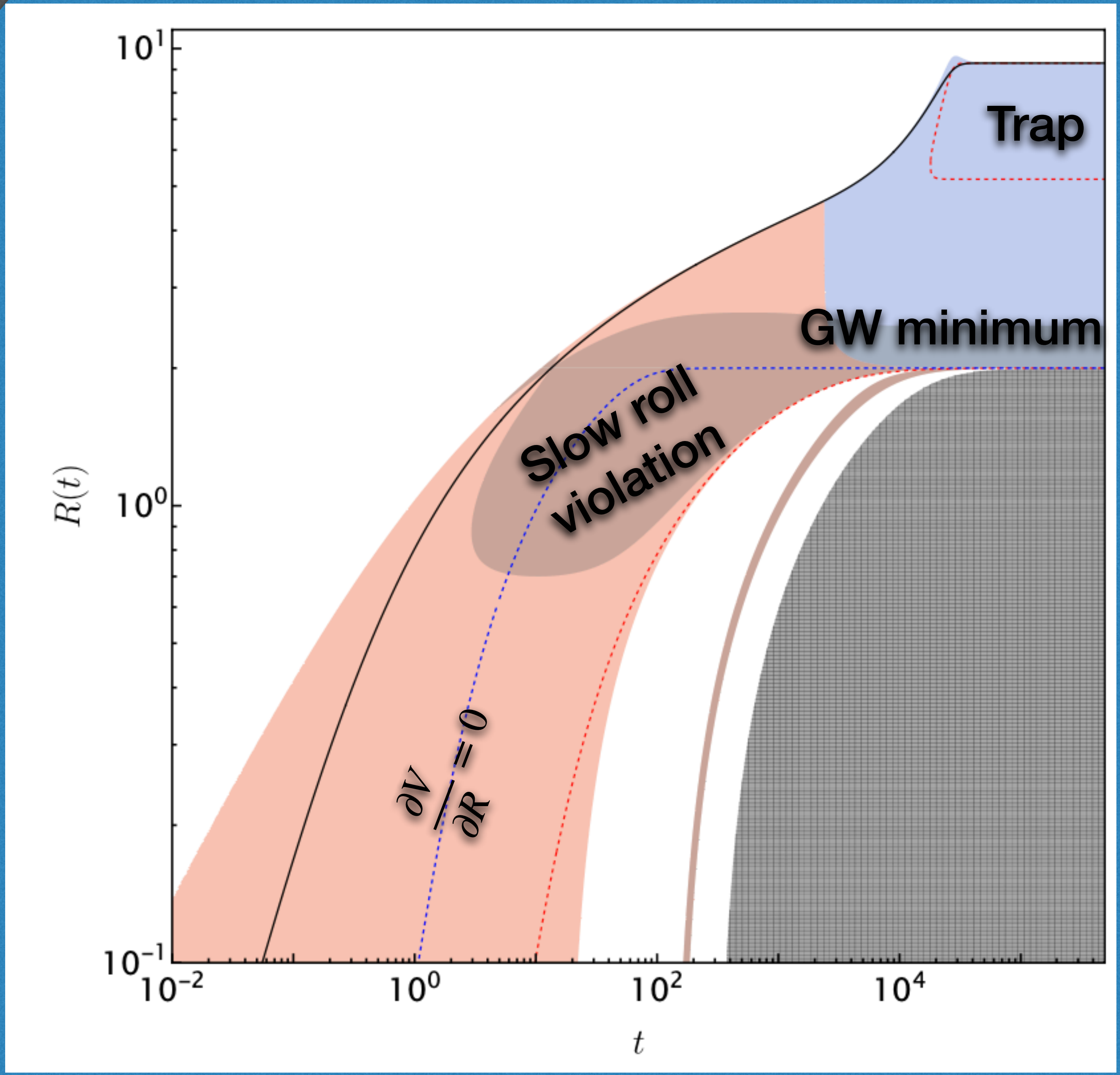
Comparison with 4D inflation

Inflation is a quasi-deSitter phase of the universe, where deSitter isometries are broken mildly by the shallow slope of the inflaton potential

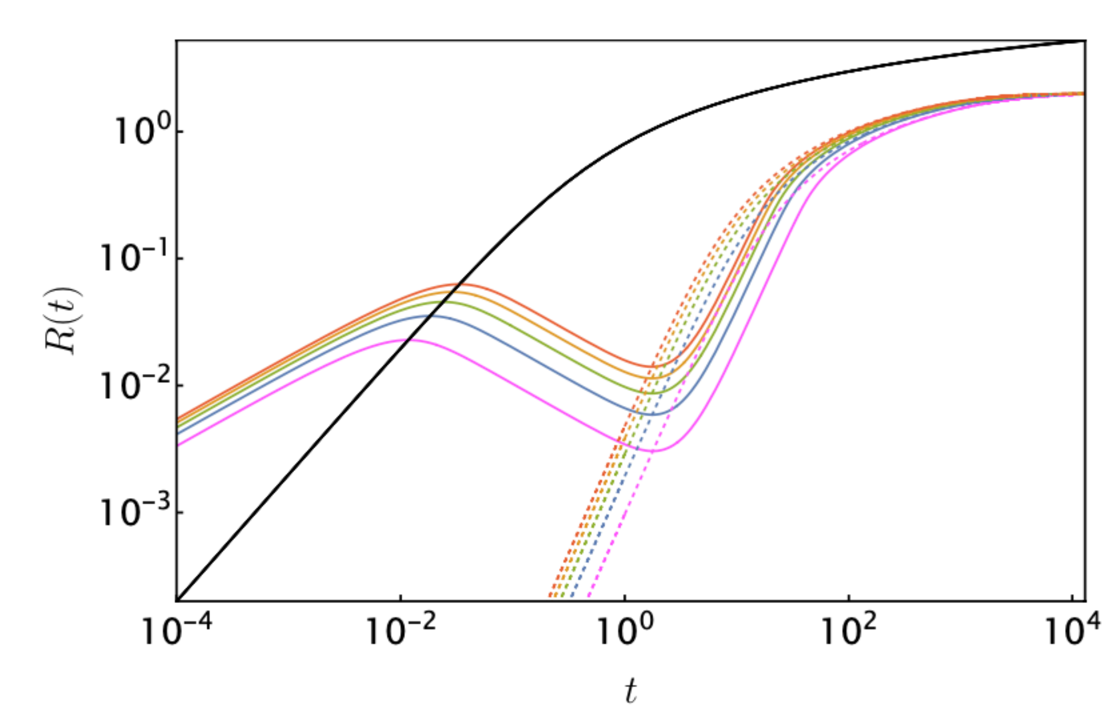
The trajectories of the stabilized dilaton (over much of its field evolution) are approximately described by either of 2 of the sign scenarios described:



Putting it all together



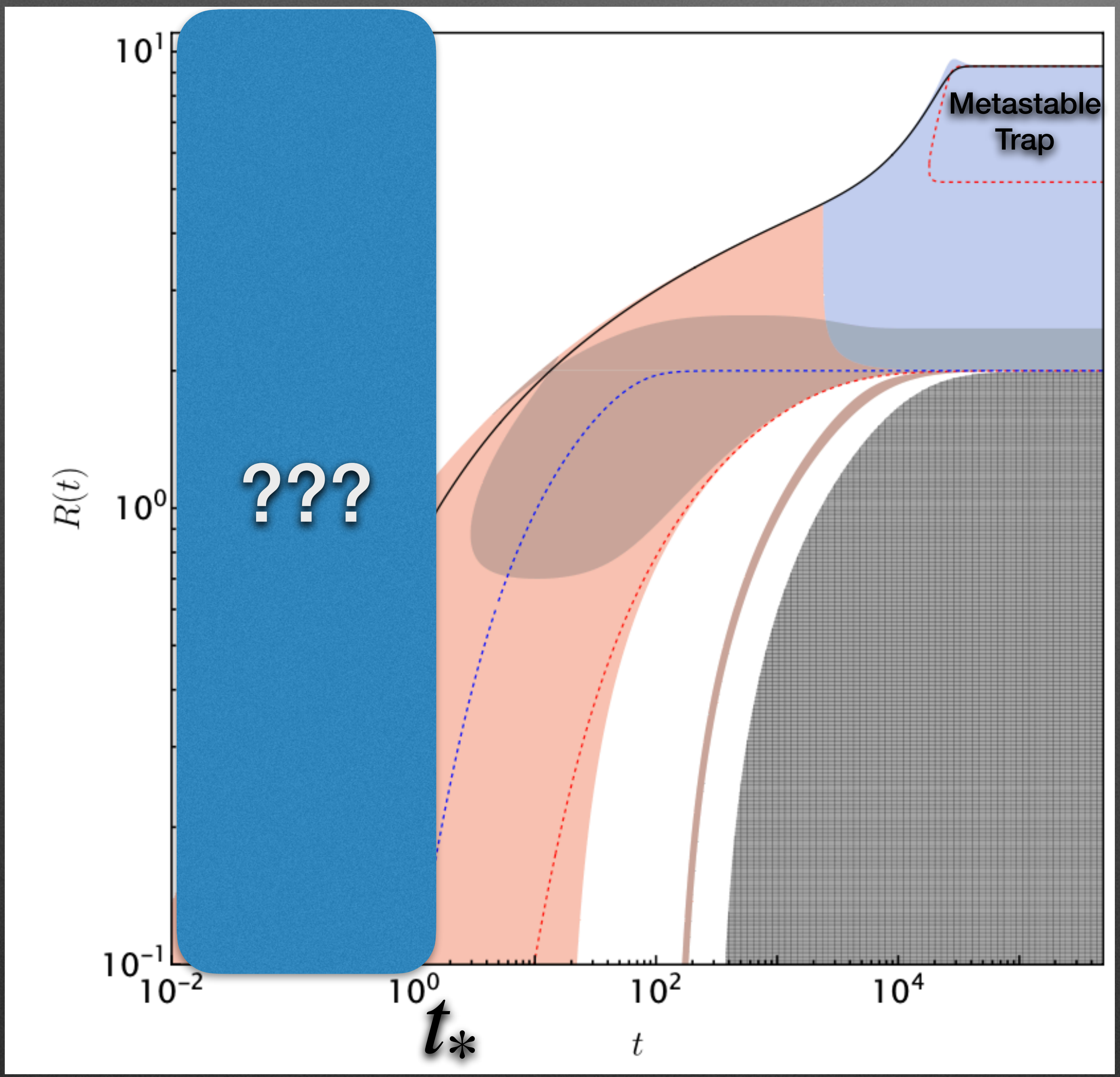
Some characteristic trajectories



Unless the brane is *extremely* relativistic at early times
Brane comes through horizon, settles to GW minimum

Initial conditions

To completely understand the question of the phase transition, need to specify boundary conditions at some early time



Some dynamics dumped energy into the CFT:

$$T = \frac{k}{\pi} e^{-k y_H(t_*)}$$

And into the dilaton:

$R(T(t_*), t_*)$ and velocity

If too much energy is dumped into the dilaton might get trapped

Equipartition?

As an example, might imagine early time dynamics didn't discriminate between dilaton and other CFT dof

At cutoff time t_* set by cutoff of 5D EFT:

$$\rho_{\text{dilaton}} \sim \frac{1}{N^2}\rho_R \sim \frac{\pi^2}{30}T^4$$

Energy of the dilaton? Examine the boundary condition:

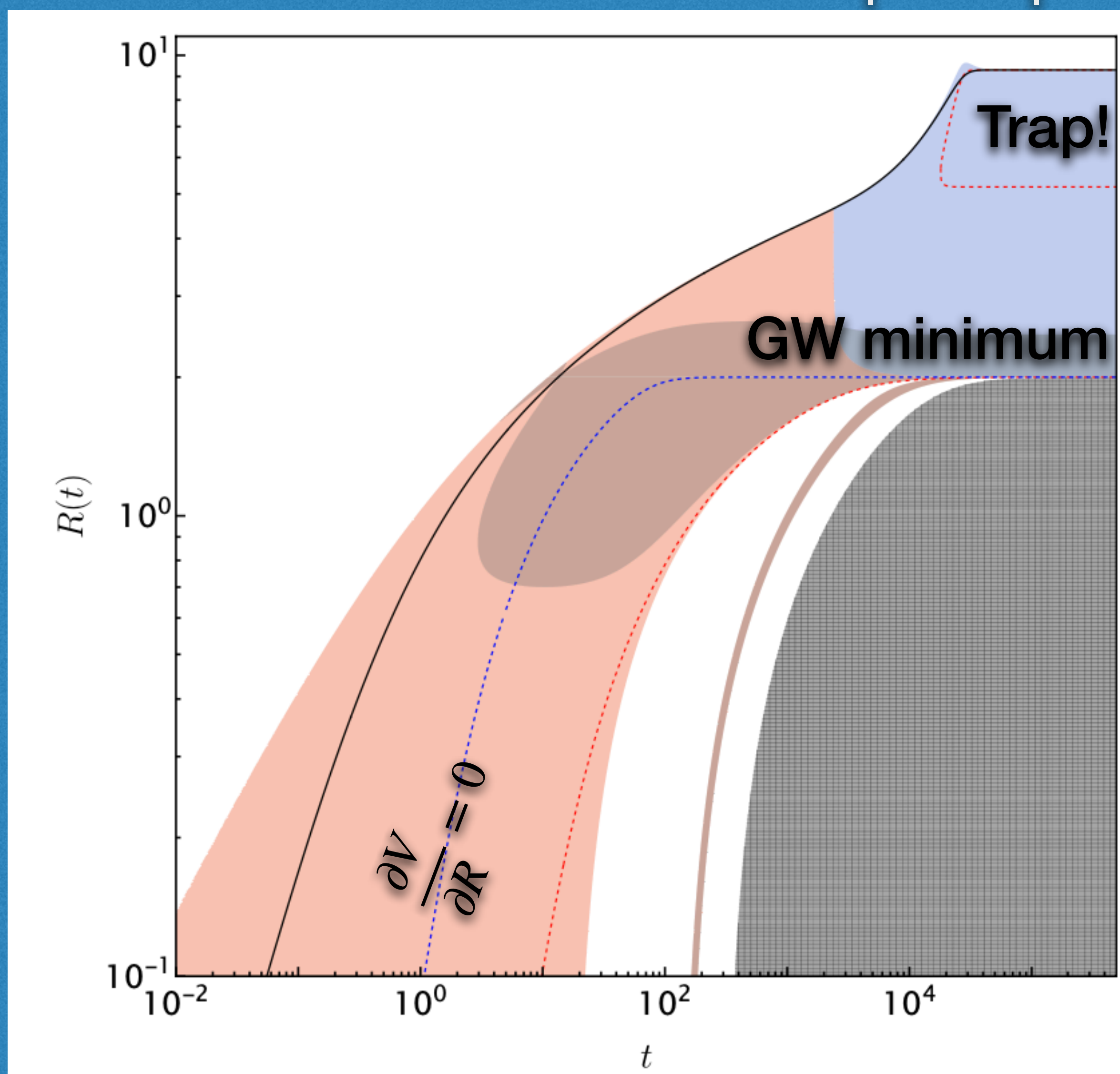
$$\frac{6}{\kappa^2}\frac{a^3 n}{\bar{a}^3}\left[\underbrace{\gamma\beta\frac{\dot{a}}{an}}_{\vec{P}} + \underbrace{\gamma\frac{a'}{a} + (1-\delta_1)}_{E}\right] = 0$$

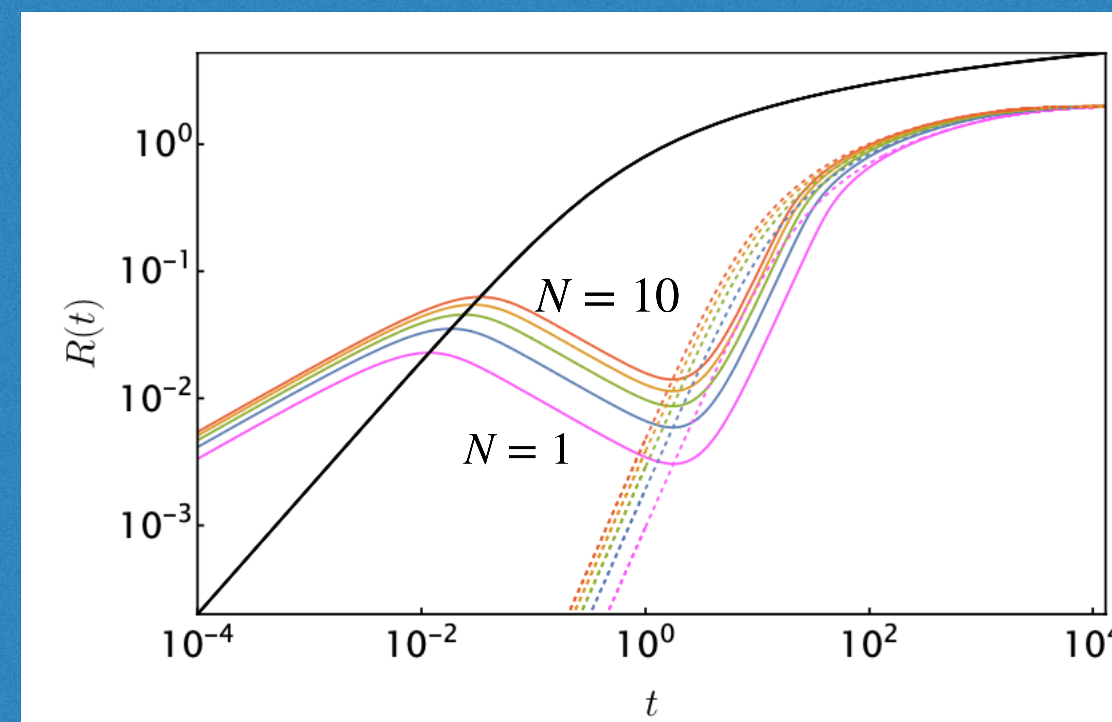
Energy and momentum arising from DBI-like dilaton EFT

$$S_{\text{dilaton}} = \int d^4x\sqrt{g}f(R,t)\left[\lambda(R,t)\sqrt{1-(\partial R)^2} - \lambda_1\right]$$

N scaling of trajectories

Arrive at estimate: $\left|\frac{a^3 n}{\bar{a}^3}\right| \gamma(t_*) = \frac{1}{720}\left(\frac{3\pi^2}{2}\right)^{1/3} N^{-4/3}$





For realistic models where $\bar{R} \approx 37k^{-1}$:

Equipartition: only begin to run into trouble around $N \sim \mathcal{O}(100)$

Or $N = 5$: $\gamma \sim 2 \cdot 10^4$ — dilaton singled out with 1000 times equipartition energy

Conclusions:

- A proper analysis of the early universe holographic phase transition necessarily involves finding 5D cosmological solutions
  - Highly relativistic brane motion at high temperatures: sensitivity to ∞ tower of operators in dilaton EFT
- Subtleties of conformal symmetry and its breaking:
  - Metastable inflating solution screened by T^8 effects until late times: $T < T_{\text{dS}}$
  - ***Initial conditions*** determine whether or not we land there
  - Must be extremely relativistic to get trapped!
- Future:
  - Early time "UV" completion — exit inflation, enter the cosmological dilaton?
  - Other dynamics: trace anomalies and how they change the story
  - Dynamics of the phase transition? Gravitational waves? Perturbations of the brane…
  - More….