# Towards a Higher-power Cyclotron

**Koay Hui Wen**
TRIUMF

2023-03-09

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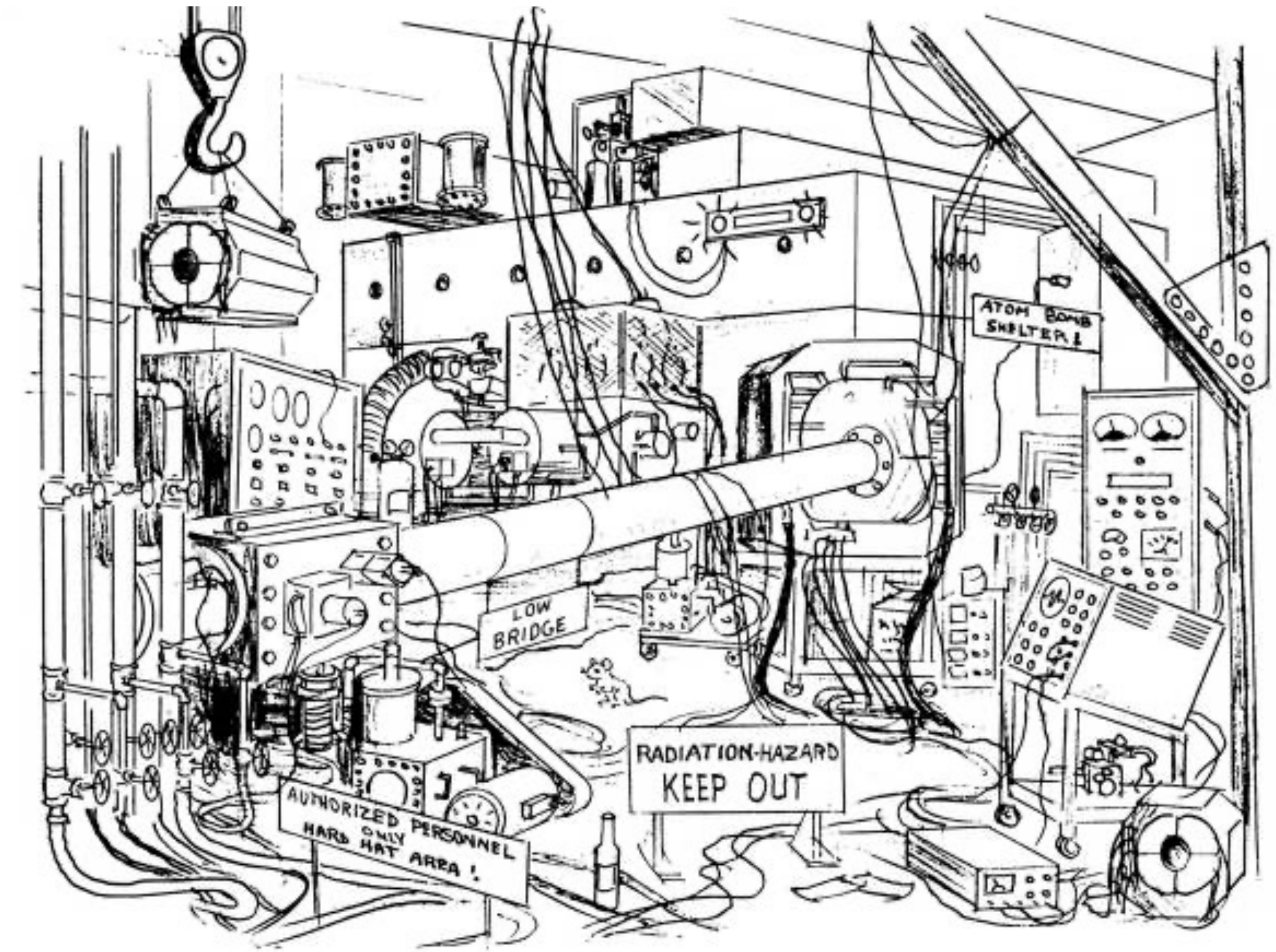
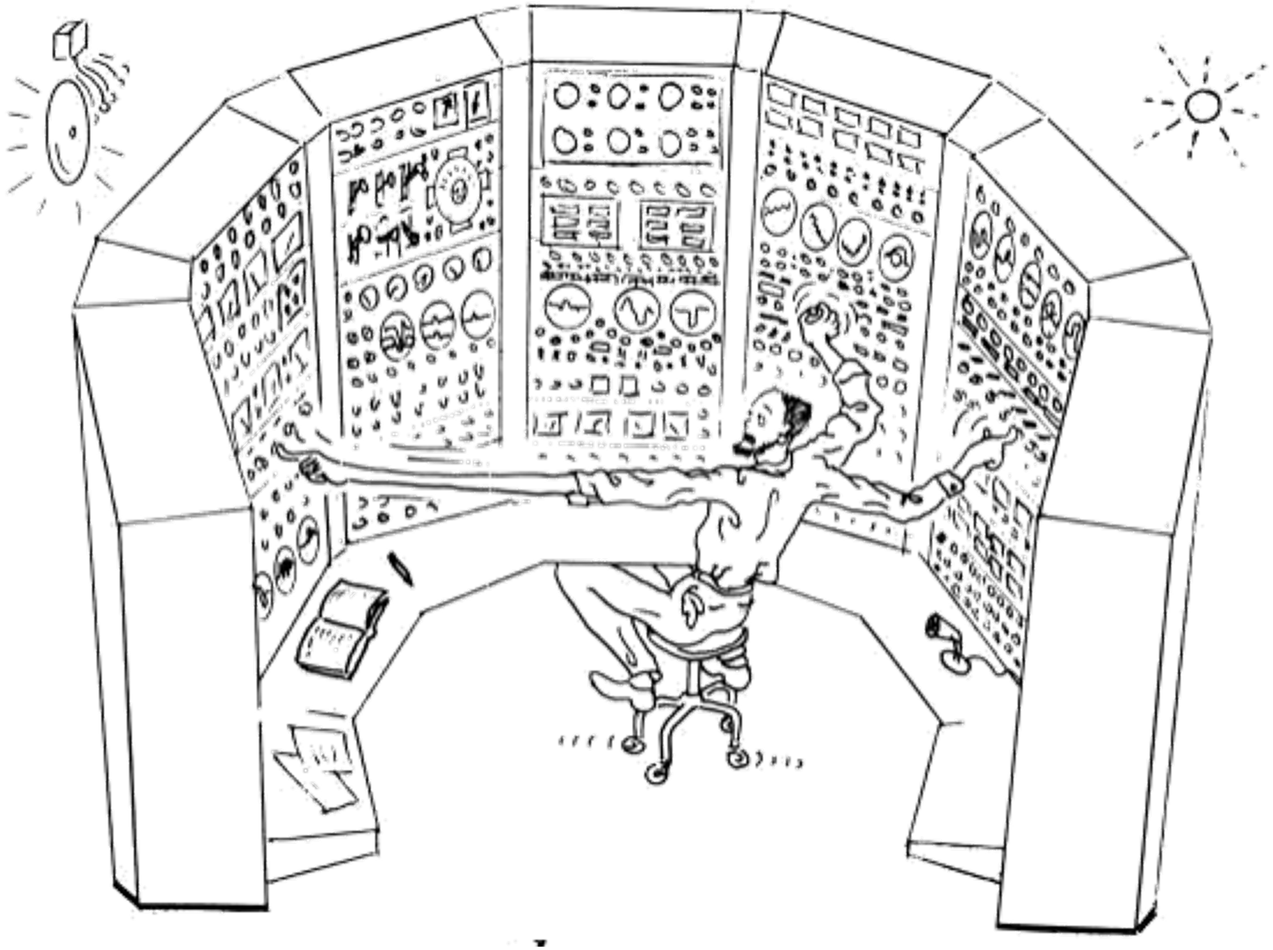
# What's a cyclotron?




*Judd, D. L. (1966). The Fluttering, Spiralling Flight of Cyclotron Evolution.*
*In Proc. of the International Conference on Isochronous Cyclotrons, Gatlinburg, Tennessee*

# What's a cyclotron?



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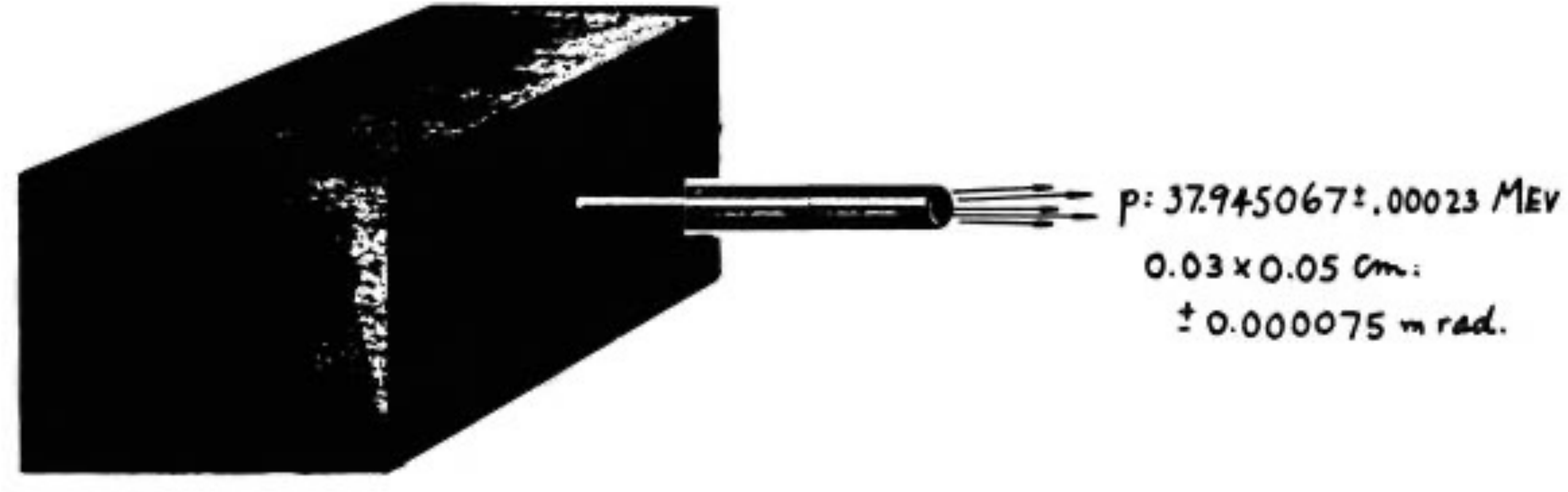
# What’s a cyclotron?




*Judd, D. L. (1966). The Fluttering, Spiralling Flight of Cyclotron Evolution.*
*In Proc. of the International Conference on Isochronous Cyclotrons, Gatlinburg, Tennessee*

# The cyclotron is.....

→ Lorentz force of a charged particle

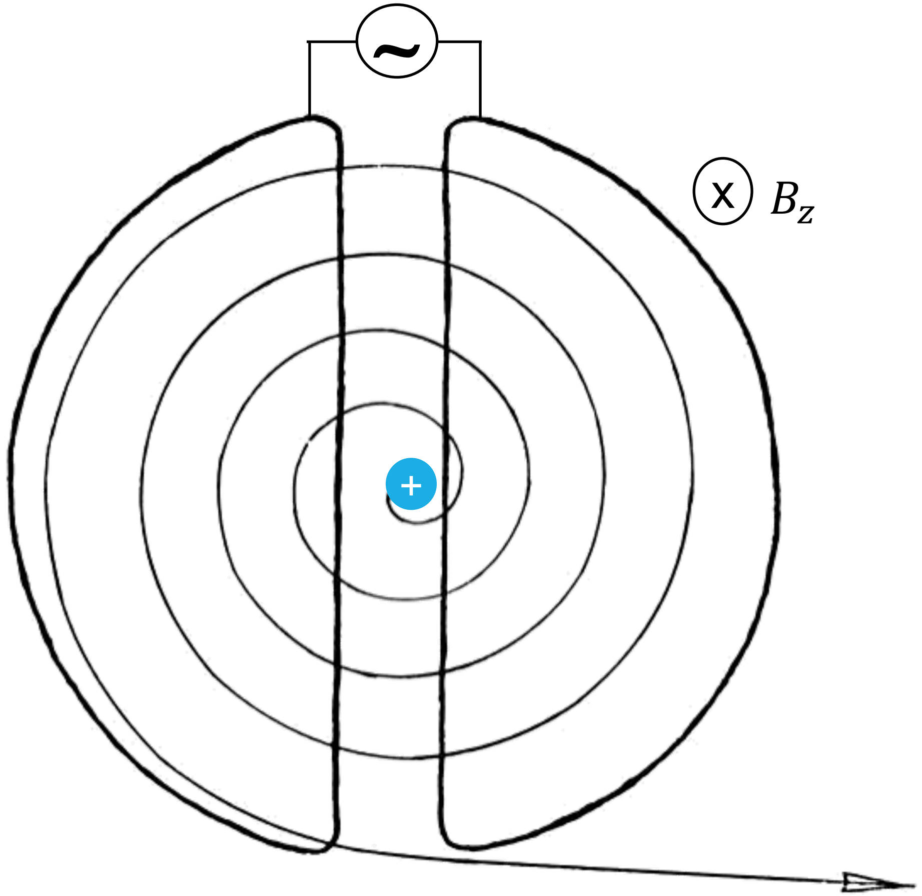
$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

→ As for a **classical cyclotron**, the magnetic field, $B_z$ is a constant

$$\frac{mv^2}{r} = qvB_z$$

→ Constant $B_z$ results in a fixed revolution frequency for non-relativistic events

$$\omega_{rev} = \frac{qB_z}{m} = \frac{v}{r}$$




*Judd, D. L. (1966). The Fluttering, Spiralling Flight of Cyclotron Evolution.*
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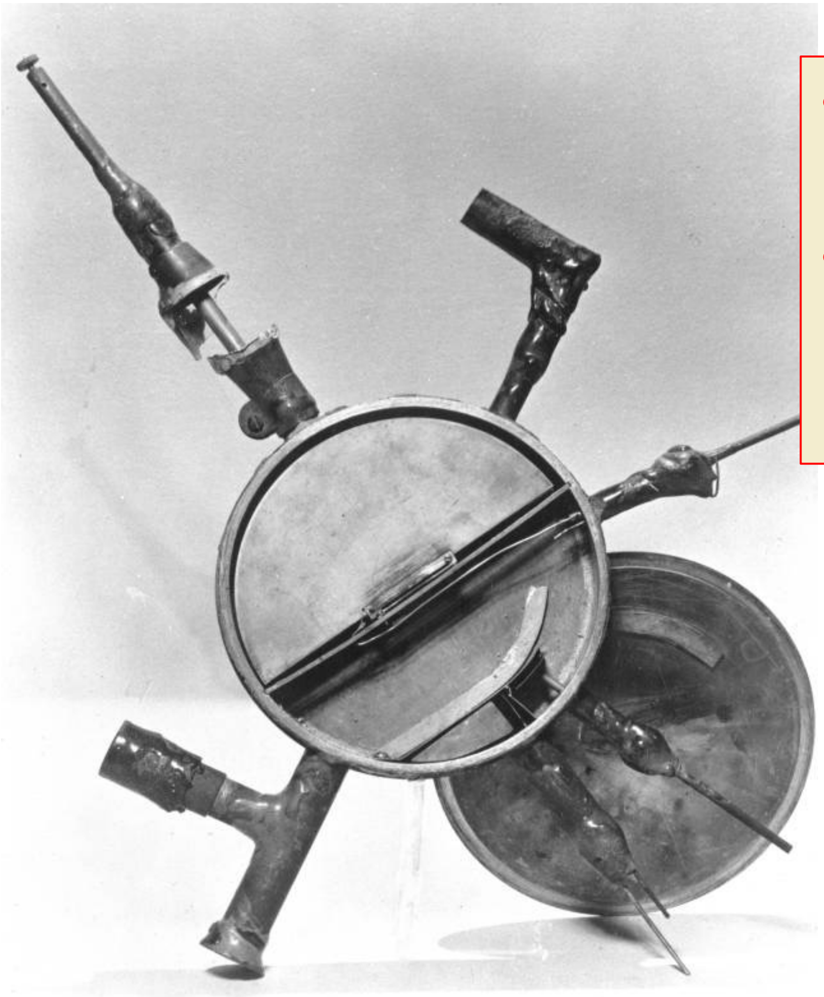
# The cyclotron is…..



- Simple, compact, wide usage…
- This invention won him the 1939 Nobel prize in Physics.

*…as seen by the inventor, Ernest Lawrence*



*LBL NEWS Magazine, Vol. 6, No. 3, Fall 1981, p. 11 The first successful cyclotron*

# Isochronous Cyclotrons

→ When energy increases, the relativistic effect becomes significant.

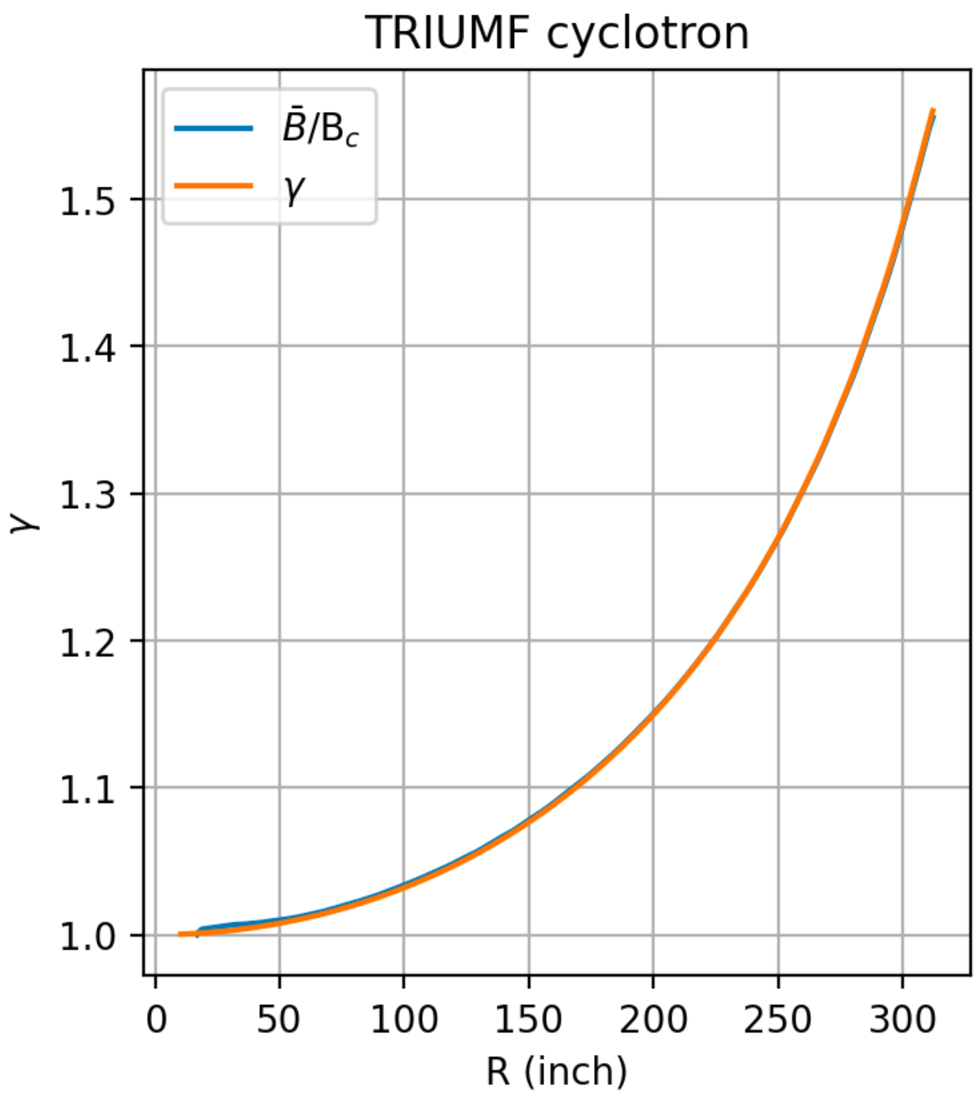
→ To keep $\omega_{rev}$ constant, $\vec{B}$ field to counter the relativistic effect

$$\omega_{rev} = \frac{q\bar{B}(r)}{\gamma(r)m_o} = \frac{qB_c}{m_o}$$

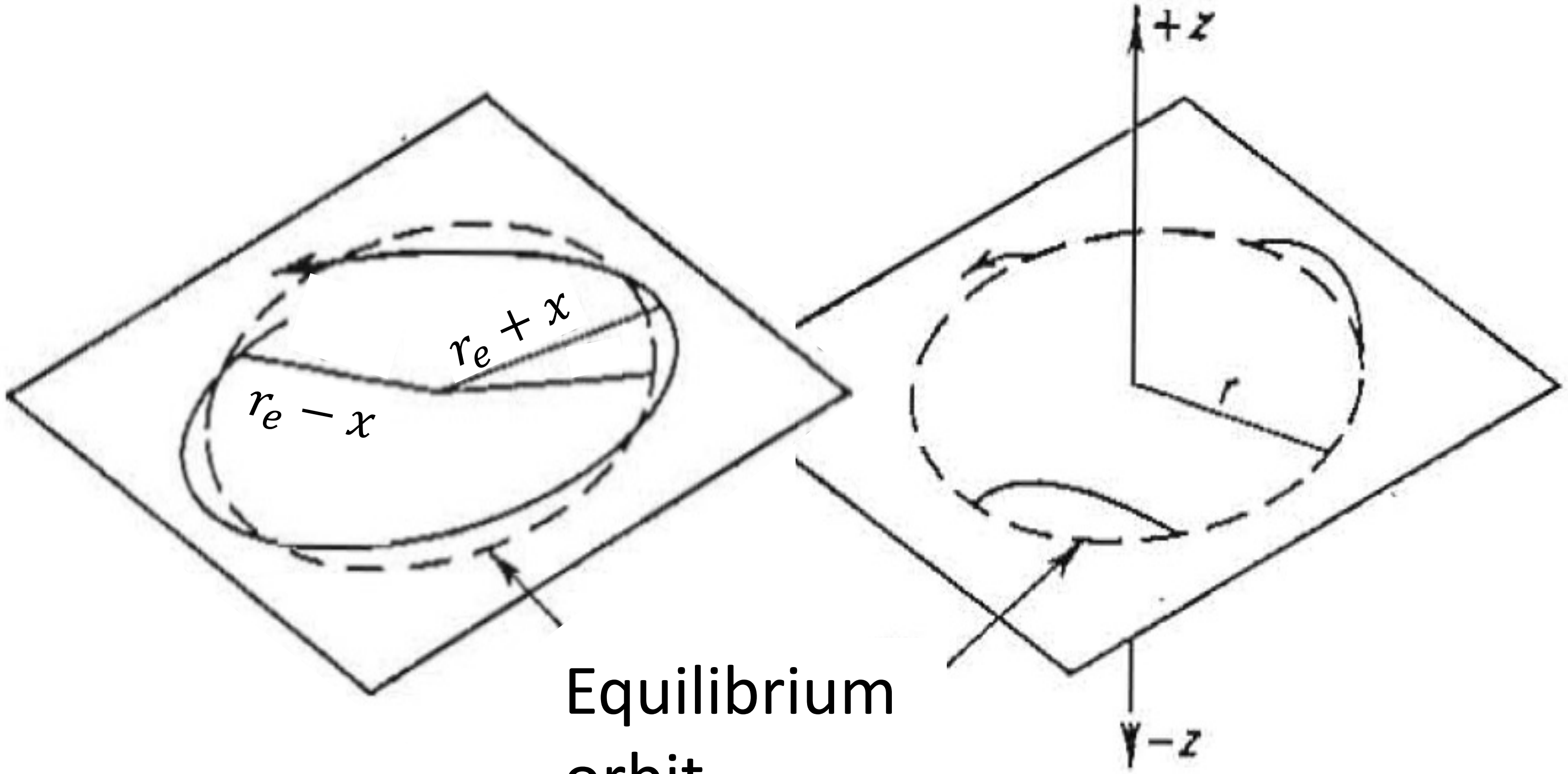
→ An isochronous cyclotron has

$$\bar{B}(r) = \gamma(r)B_c = \frac{B_c}{\sqrt{1-\left(\frac{R}{R_\infty}\right)^2}}$$

where $R_\infty = c/\omega_{rev}$



# Beam Focusing




Equilibrium orbit

- **Betatron oscillation** : Transverse oscillation about the equilibrium orbit
- **Betatron frequency (tune), $\boldsymbol{\nu}$** : The number of betatron wavelengths in one revolution

# Limiting Factor: Axial Focusing

→ Axial equation of motion

$$F_z = m\ddot{z} = qvB_r$$

where $B_r = z\left(\frac{\partial B_r}{\partial z}\right)_{z=0} + \frac{z^2}{2}\left(\frac{\partial^2 B_r}{\partial z^2}\right)_{z=0} + \cdots$

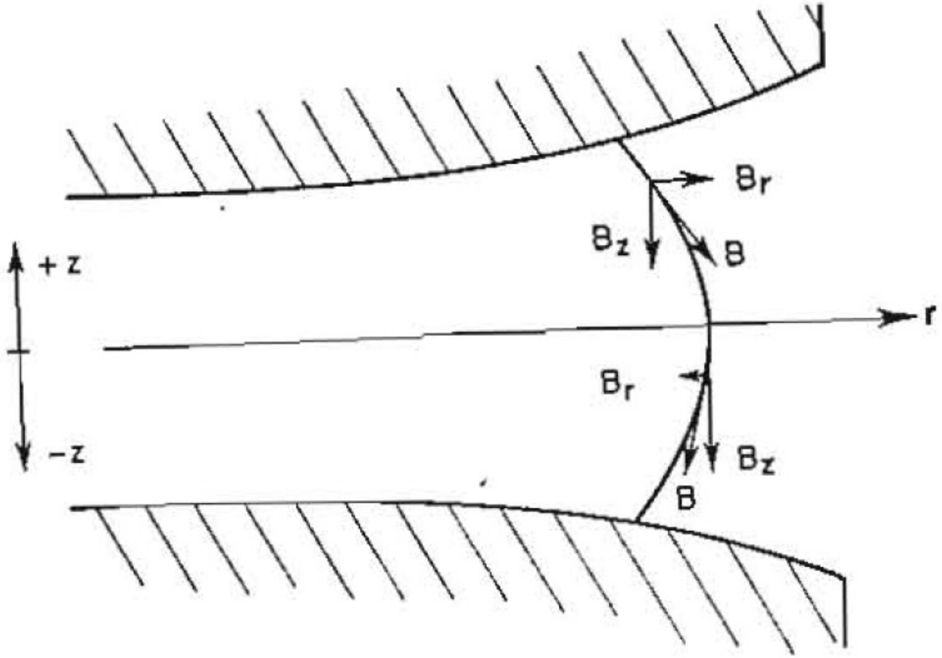
→ Since $\nabla \times \vec{B} = 0$ at region with no current,

$$\left(\frac{\partial B_r}{\partial z}\right)_{z=0} = \left(\frac{\partial B_z}{\partial r}\right)_{z=0}$$

$$\therefore \ddot{z} - \frac{r}{B}\frac{dB}{dr}\omega^2 z = 0$$

→ As, $k = \frac{r}{B}\frac{dB}{dr}$,

$$v_z^2 = -k$$



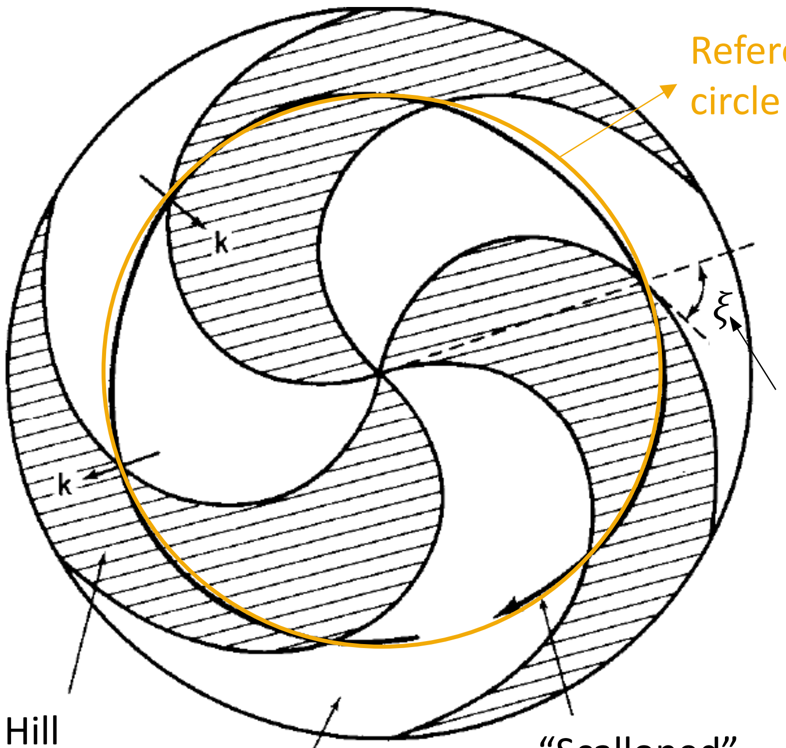

**$v_z > 0$ (or $k < 0$ ) for bounded solution!**

**Contradiction: Isochronous cyclotron, $\frac{dB}{dr} > 0$ !**

*Photo: Livingood, John J. et al. "Principles of Cyclic Particle Accelerators." (1961)*

# Beam Focusing of Isochronous Cyclotrons

- → "Thomas cyclotron" with azimuthally varying field was introduced in 1938 [1].
- → Spiral-edge focusing was introduced by Kerst in 1954 [2].
- → Alternating gradient to overcome axial defocusing from isochronous field.
- → Also known as an AVF/FFAG cyclotron
- → The resultant trajectory is a "scalloped" equilibrium orbit with an effective radius $r_{eff}$




Schematic of a three-sector isochronous cyclotron with a spiral-ridge magnetic field and its "scalloped" equilibrium orbit.


*[1] Thomas, L. H. (1938). The Paths of Ions in the Cyclotron. Physical Review, 54(8), 580–588. doi:10.1103/PhysRev.54.580*
*[2] Kerst, D. W. et. Al.(1954). A fixed field alternating gradient accelerator with spirally ridged poles. Technical Report MURA-042*

# TRIUMF Cyclotron

Injection energy: 300 keV

Max Energy: 520 MeV ($\gamma$~1.56)

Voltage per turn: ~ 0.4 MV

RF harmonic: 5th harmonic

RF frequency:~23 MHz

Sector number: 6

Peak intensity achieved: 0.42 mA

Accelerating ions: $H^-$

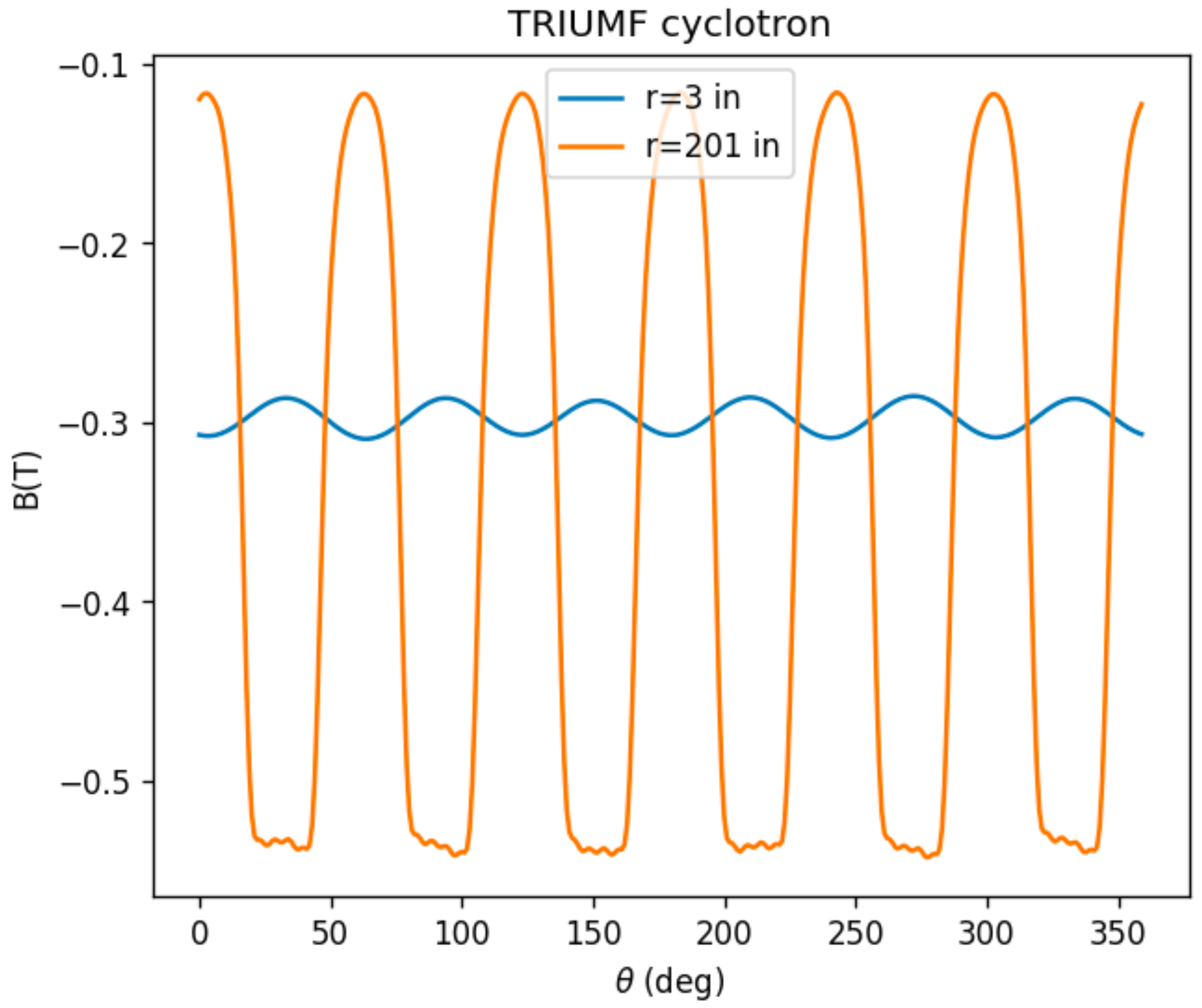
# Axial Focusing

For an isochronous cyclotron, axial focusing is mainly provided by:

a) Azimuthally varying field (AVF)

$$B(r,\theta) = \bar{B}(r)\left[1 + \sum f_n(r)\ \cos n\big(\theta - \phi_n(r)\big)\right]$$

where $\mathrm{F} = \frac{1}{2}\sum_{\mathrm{n}} \mathrm{f}_{\mathrm{n}}^{2}$ represents the flutter of the magnetic field variation



# Axial Focusing

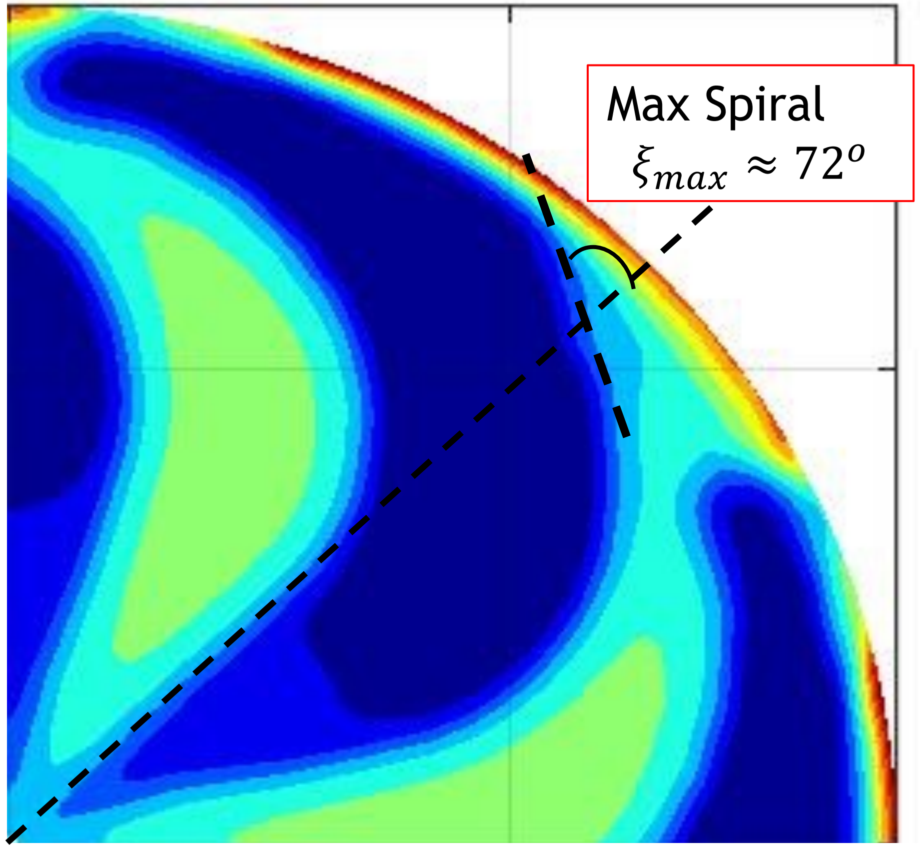
For an isochronous cyclotron, axial focusing is mainly provided by:

a) Azimuthally varying field (AVF)

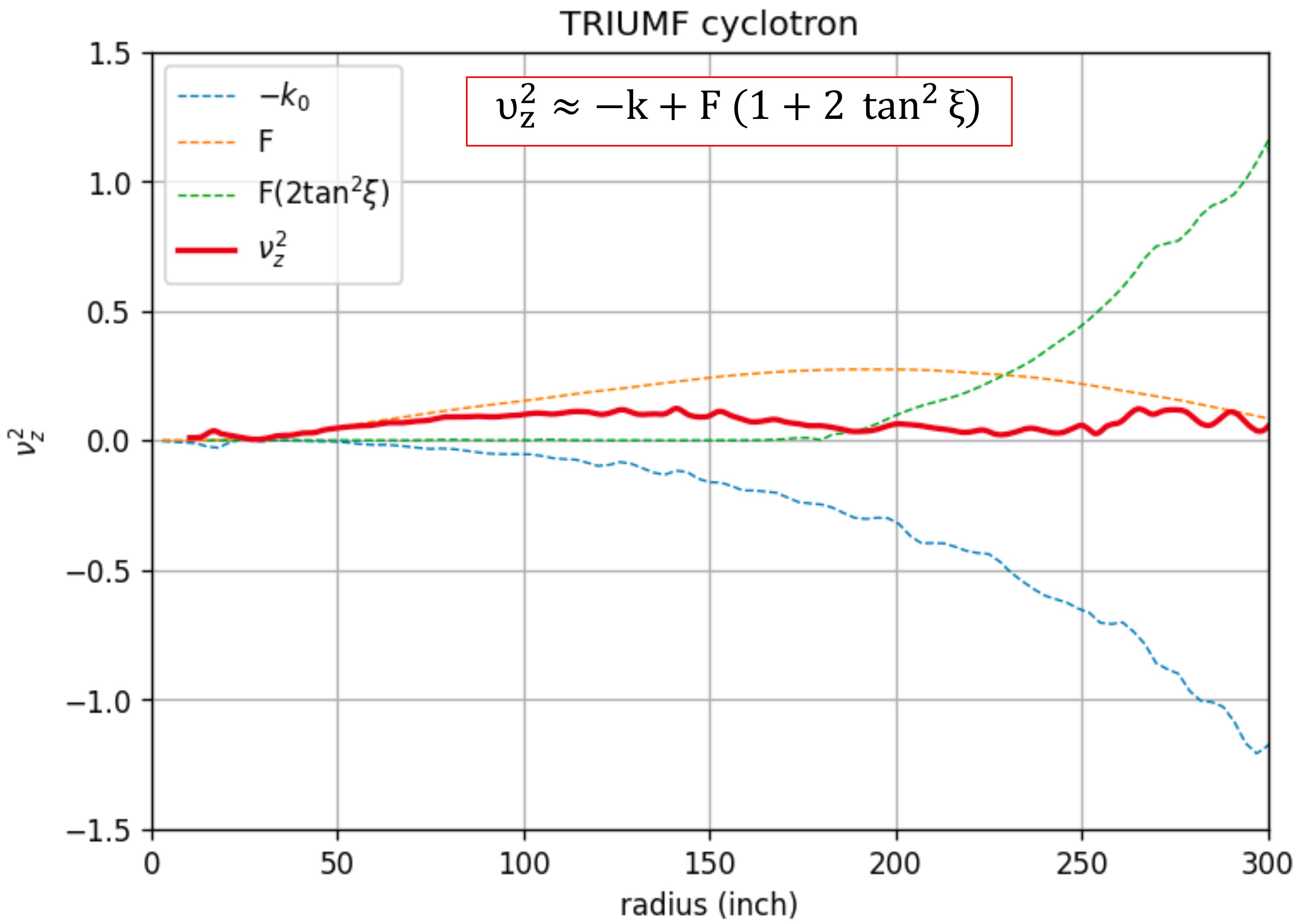
b) Spiral edges (ξ) from the N$^{th}$ sectors

$$\upsilon_z^2 \approx -k + F \cdot \boxed{(1 + 2\ \tan^2 \xi)} \cdot \frac{N^2}{N^2 - 1}$$

TRIUMF cyclotron



# Axial Focusing



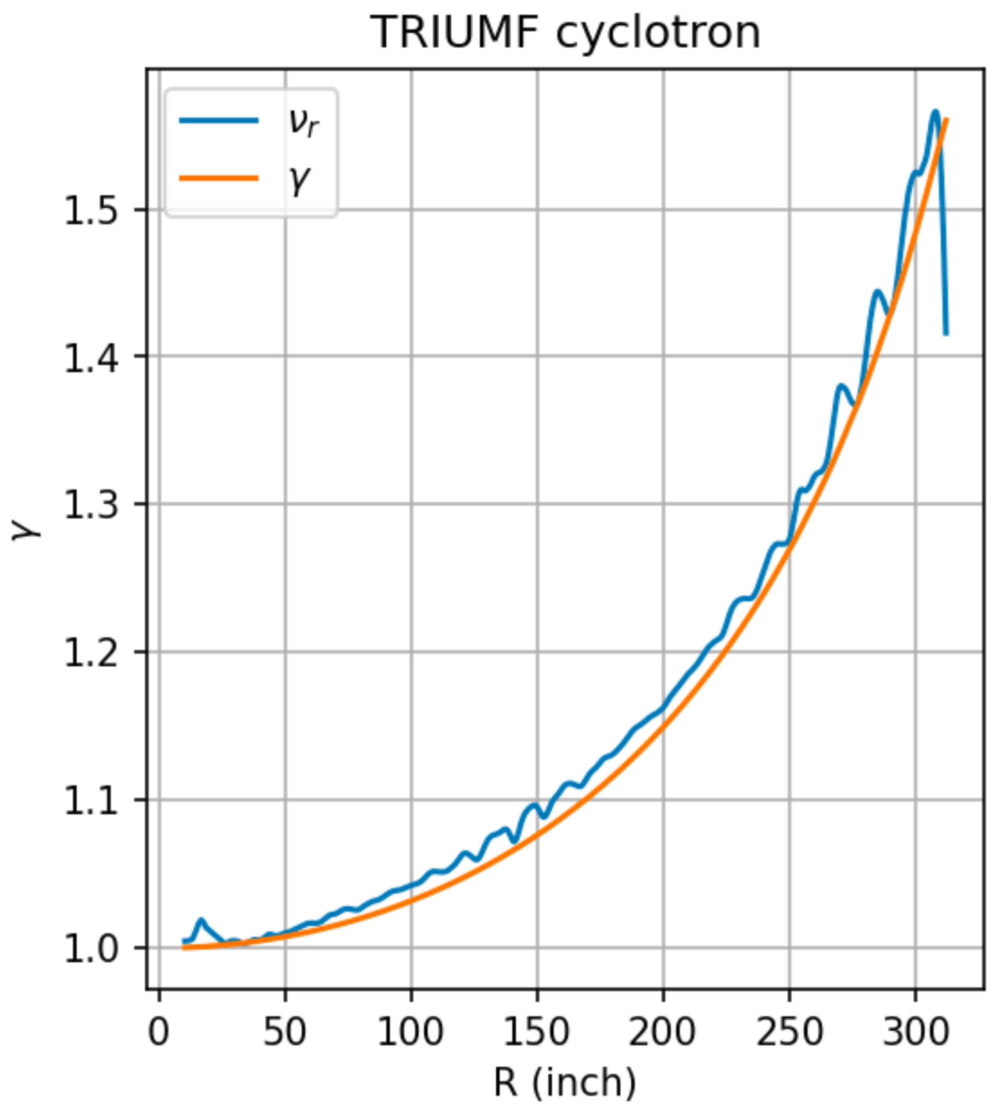
# Radial Focusing

→ Radial focusing provided by the increasing field gradient, $k = \frac{r}{B}\frac{dB}{dr}$:

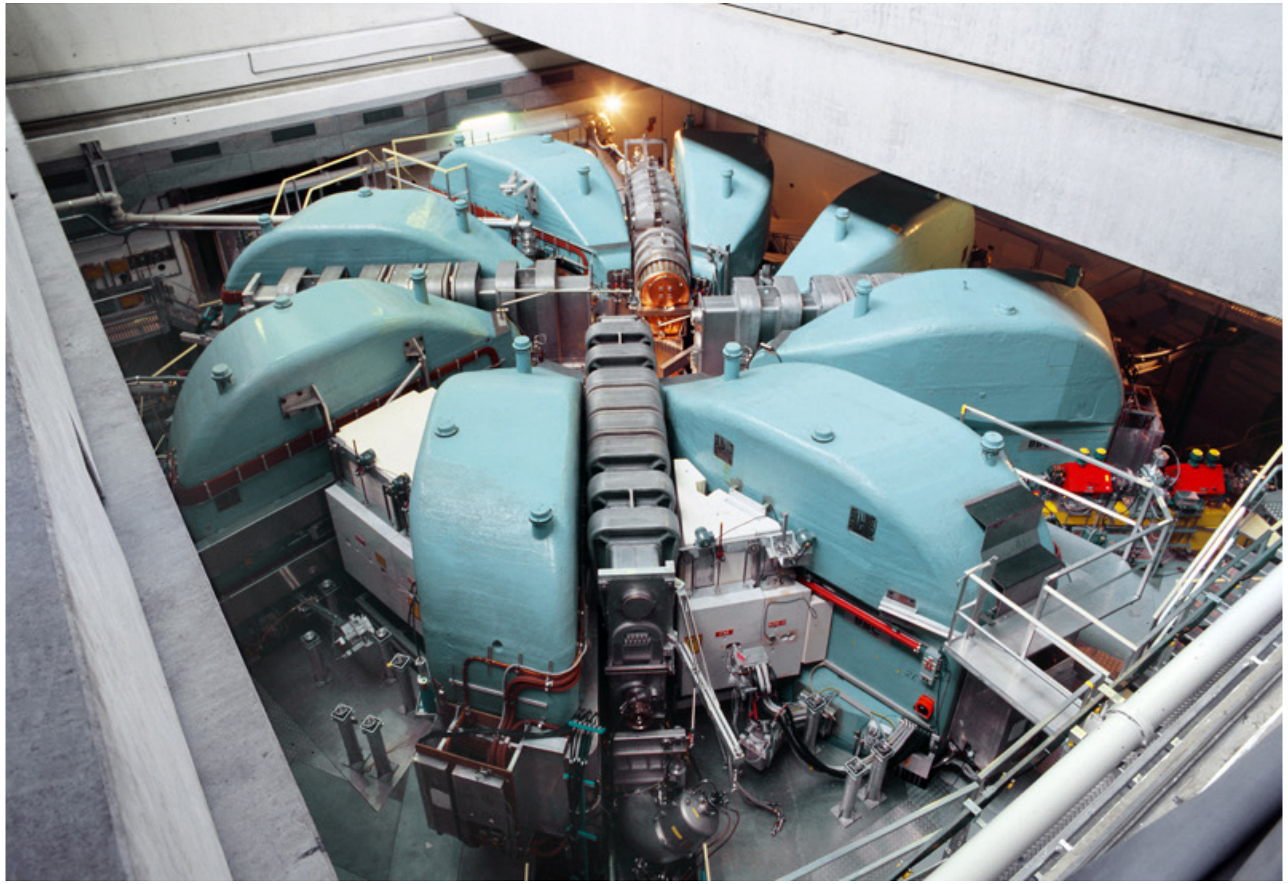
$$\upsilon_r^2 = 1 + k + \cdots$$

→ For an isochronous cyclotron,

$$\upsilon_r \approx \gamma$$



# PSI Ring Cyclotron



Injection energy: 72 MeV

Max Energy: 590 MeV ($\gamma$~1.64)

Voltage per turn: ~ 3 MV

RF harmonic: 3rd (flat top), 6th

RF frequency: 150 MHz (flat top), 300 MHz

Sector number: 8

Peak intensity achieved: 2.4 mA

Accelerating ions: $H^+$

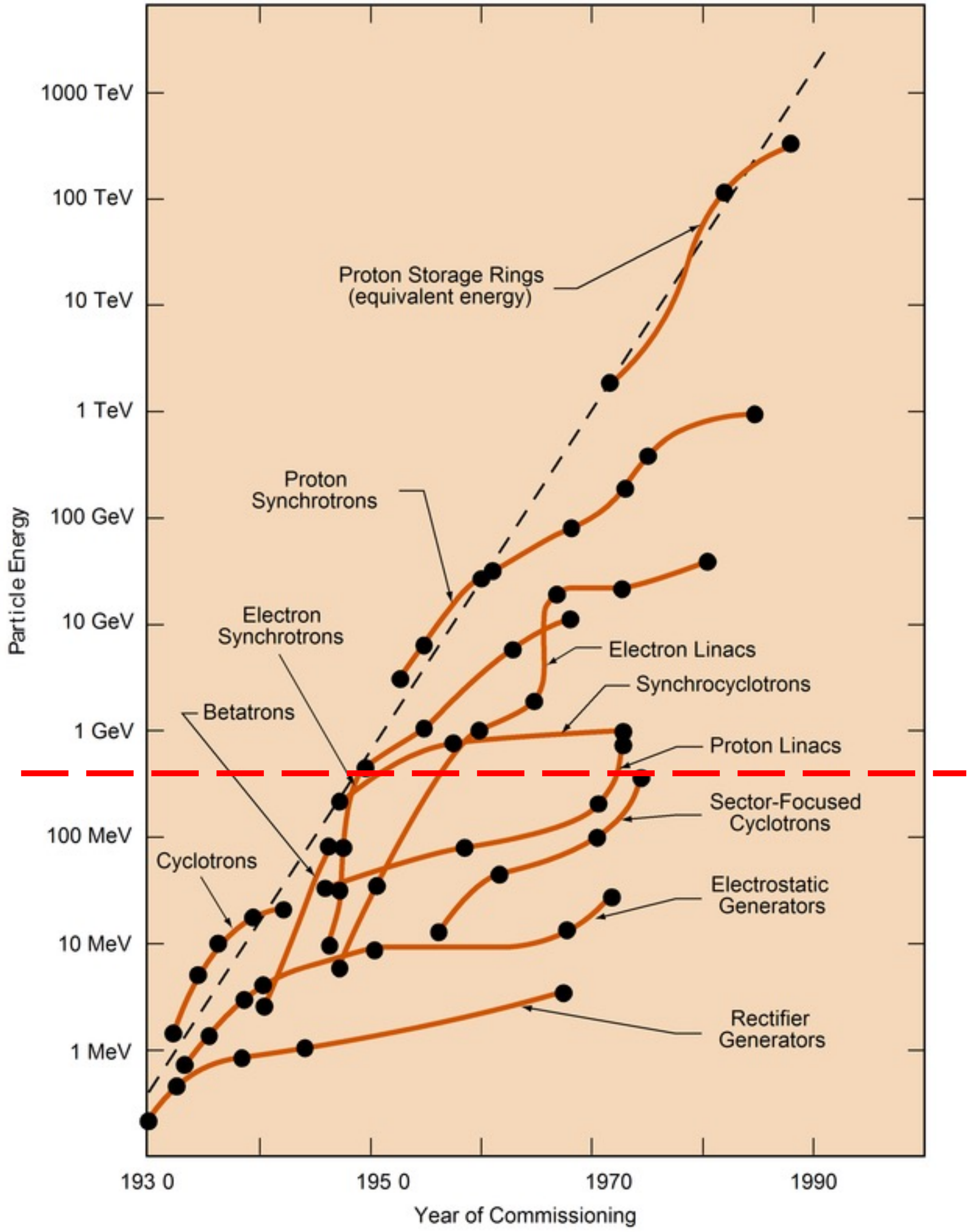
*Source: https://indico.mit.edu/event/150/contributions/380/attachments/140/272/snowmass2021_baumgarten.pdf*

# Comparison of Energy Efficiency of Different Accelerators

| | PSI cyclotron | SNS linac | J-PARC linac and RCS |
|---|---|---|---|
| Beam energy | 0.59 GeV | 1 GeV | 3 GeV |
| Beam Power | 1.4 MW | 1.4 MW | 1 MW |
| Power consumption | 4.5 (RF) in total 10 MW | 16.3 MW | 32.6 MW |
| Fraction of grid power converted to beam power | ~18-19% | ~9% | ~3% |

*V.P. Yakovlev, et. Al. "The Energy Efficiency of High Intensity Proton Driver Concepts", in Proc. 8th Int. Particle Accelerator Conf. (IPAC'17)*

# Livingston Chart: Evolution of Accelerators




**Despite the high efficiency, max. energy achieved by cyclotrons is still < 600 MeV**

*http://www.ischebeck.net/media/Accelerator%20Physics/Advanced%20Accelerator%20Concepts/Livingston%20Plot/slides/Livingston%20Plot%201.html*

# Towards higher power…

Higher energy × Larger current = High power

P=VI

**Higher energy**

1. Magnetic field
2. Resonances
3. Extraction

**Larger current**

1. Space charge
2. Erosion

**High power**

Other issues

# Higher Energy?

# Magnetic Field Limitation

- Larger field increment as $\bar{B}(r) \propto \gamma(r)$

$\rightarrow \gamma \approx 1.64$ for PSI ring cyclotron of 590 MeV (so far the highest $\gamma$ ever built)

$\rightarrow \gamma \approx 2$ for for energy of 1 GeV

$\rightarrow \gamma \approx 12$ for for energy of 10 GeV

- Axial focusing requirement increases as $k = (\beta\gamma)^2$ and $\upsilon_z^2 \approx -k + F \cdot \frac{N^2}{N^2-1} \cdot (1 + 2\ \tan^2 \xi)$

# Resonances

- In real application, perturbation due to field error/misalignment exists.
- Equation of motion due to perturbations (smooth approximation):

$$\frac{d^2x}{d\theta^2} + \nu_r^2 x = \boxed{a_0 + a_1 x + a_2 x^2 + b_1 xy + \cdots}$$

driving terms from perturbations

- When the perturbation terms involves a particular Fourier harmonics, resonances might occur in a circular accelerator.
- Like the periodic perturbations of a harmonic oscillator can cause a resonance when the perturbation frequency is equal to an eigenfrequency of the oscillator.

# Resonances

General form of resonance condition:

$$M_x \nu_x + M_z \nu_z = N$$

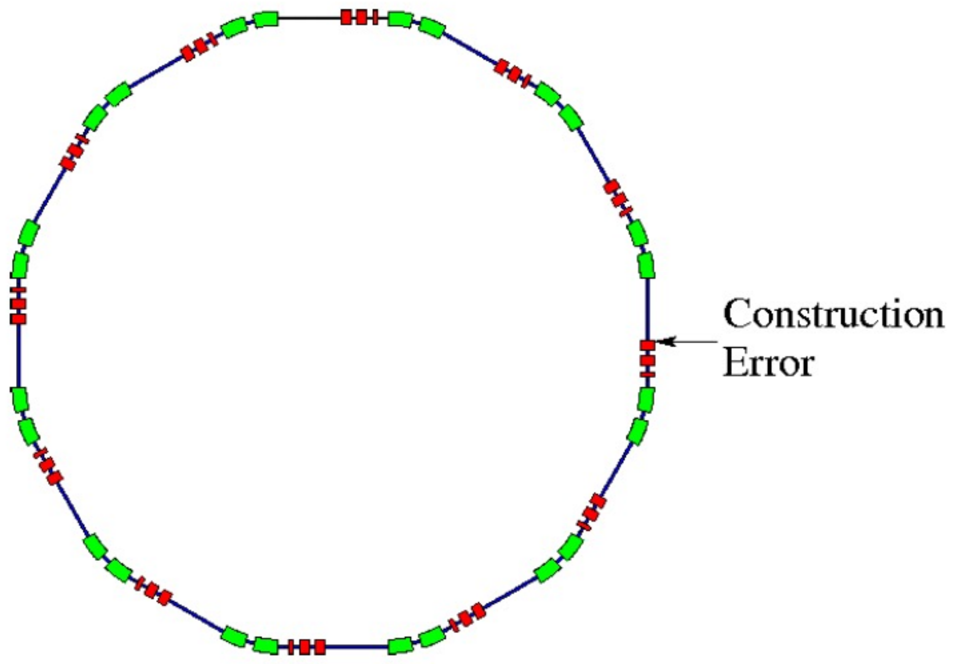
$\nu_x$ = radial betatron tune

$\nu_z$ = vertical betatron tune

$M_x, M_z, N$ = Integers

$|M_x| + |M_z|$ = Order of resonance

**Crossing of resonances may induce serious beam loss!!**




*Wiedemann, H. (2007). Particle Accelerator Physics*

# Possible solutions

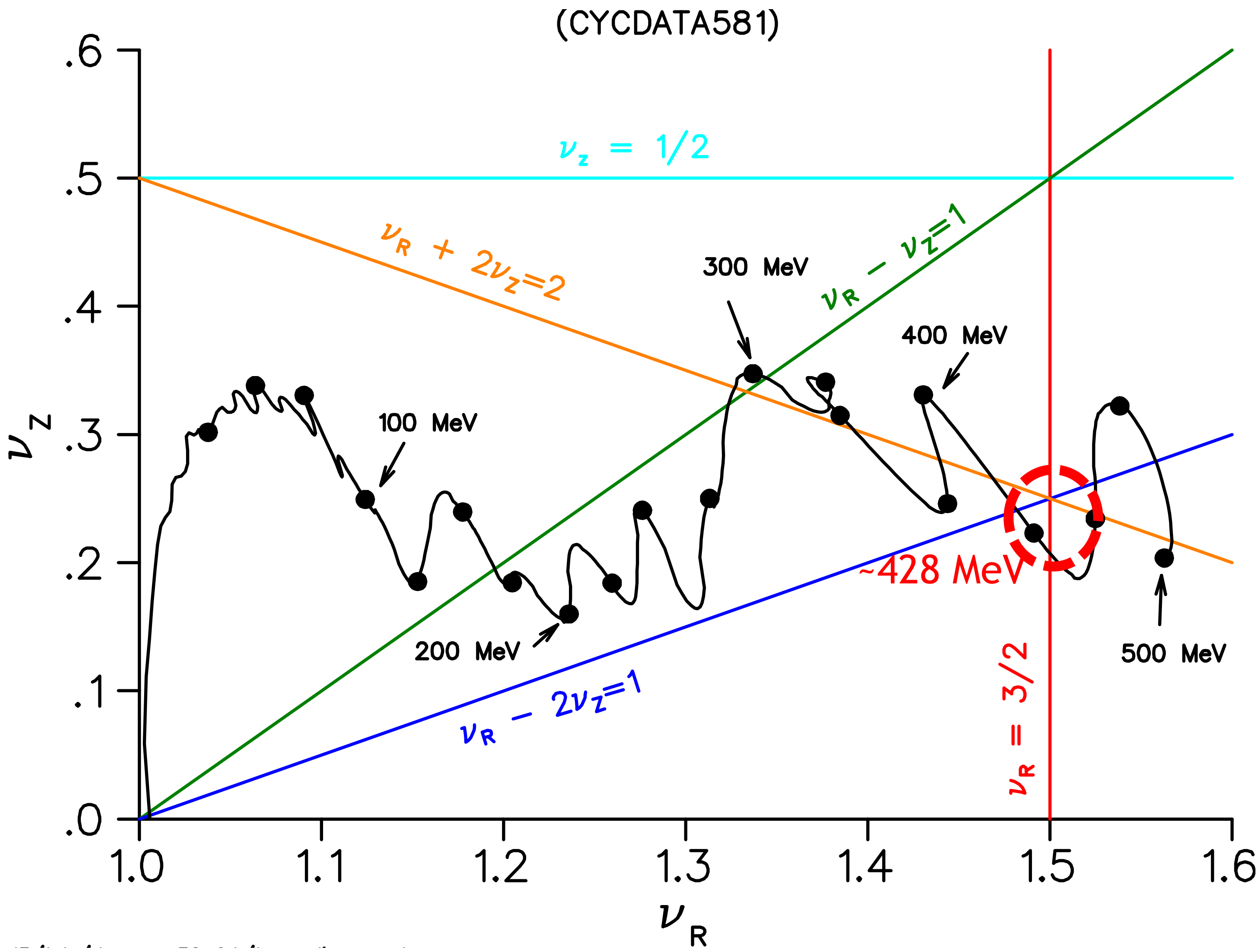
- Reduce the source of perturbation

    → Correction of misalignment or field error by harmonic coils or trim coils

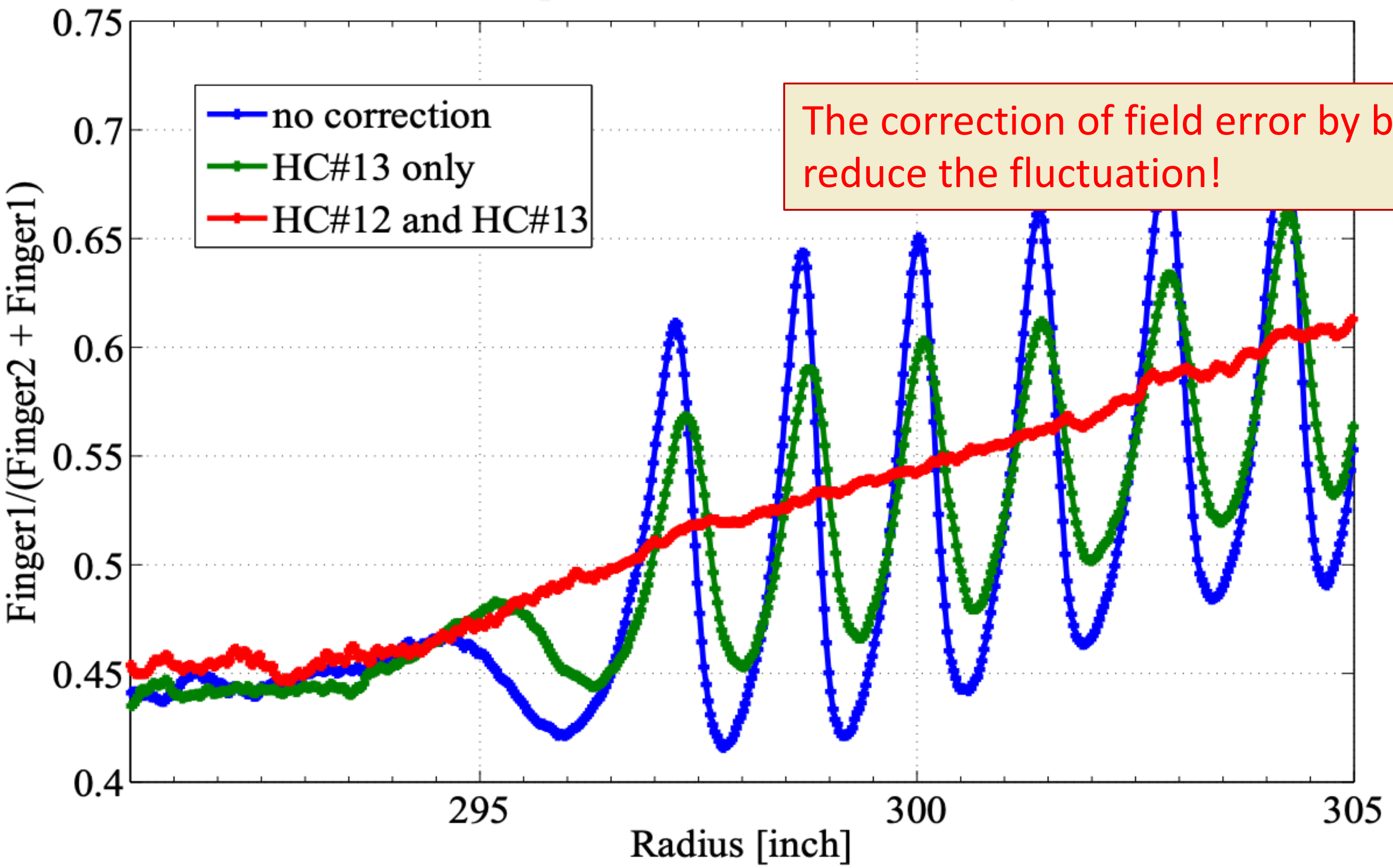
- Reduce the tune change

    → Reduce the frequency of resonance crossings

# Example: TRIUMF Tune Diagram



# Example: Correction of $\nu_r = \frac{3}{2}$ Resonance in TRIUMF Cyclotron

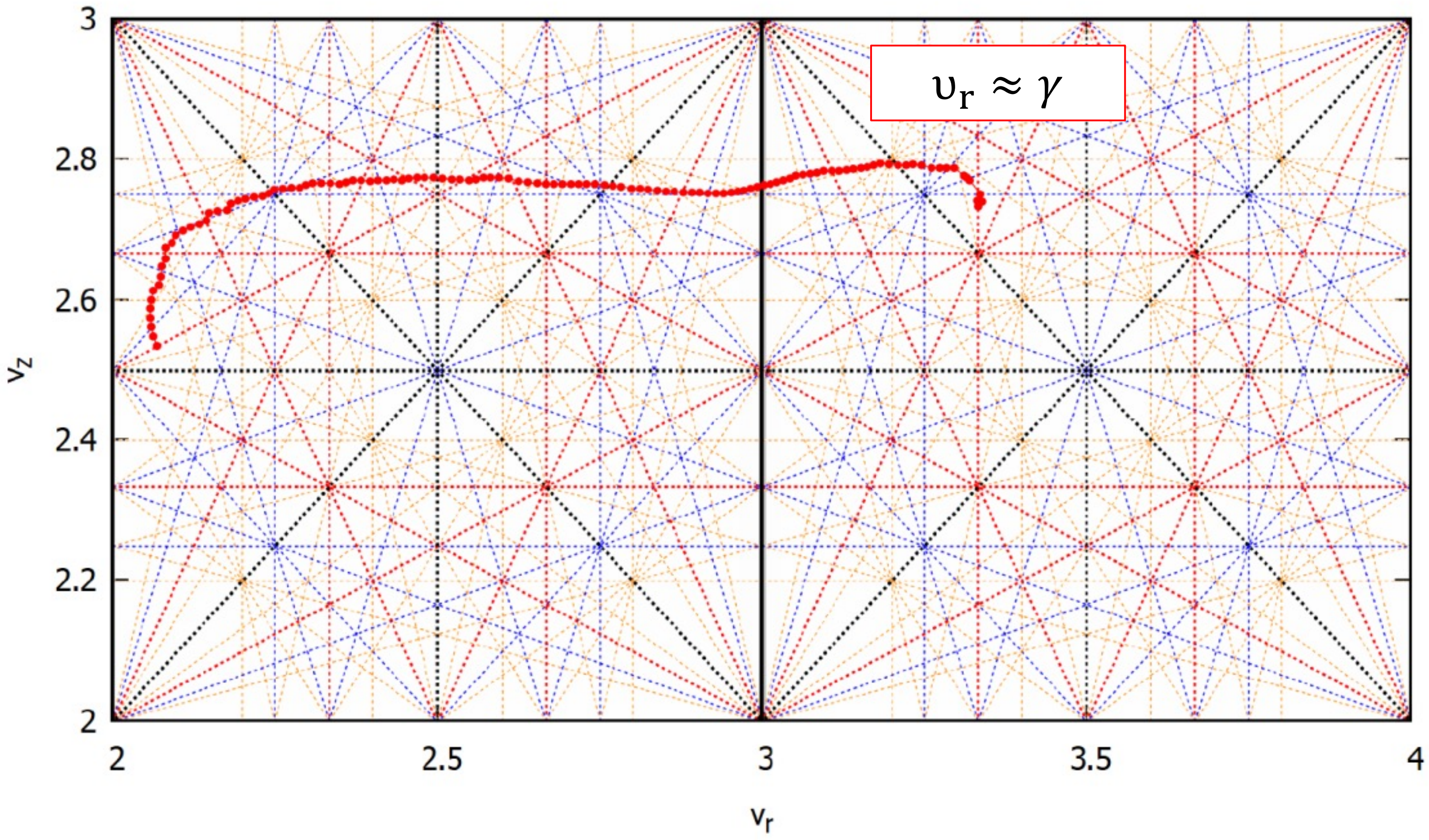



The correction of field error by both HC greatly reduce the fluctuation!

R. Baartman, T. Planche, and Y.-N. Rao, Correction of the vr = 3/2 Resonance in TRIUMF Cyclotron, Conf. Proc. C, 1205201 (2012), pp. 415–417

# Example: Constant Tune Cyclotron

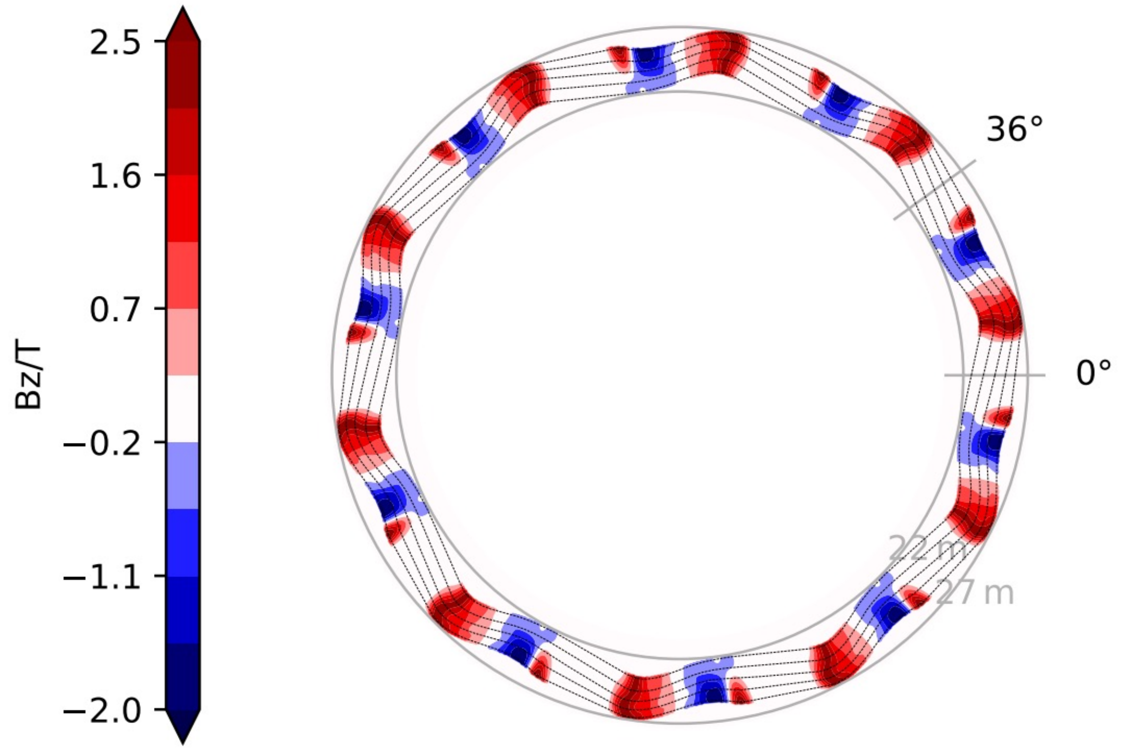
Cyclotron Proposed by China Institute of Atomic Energy (CIAE) to accelerate 800 MeV → 2 GeV

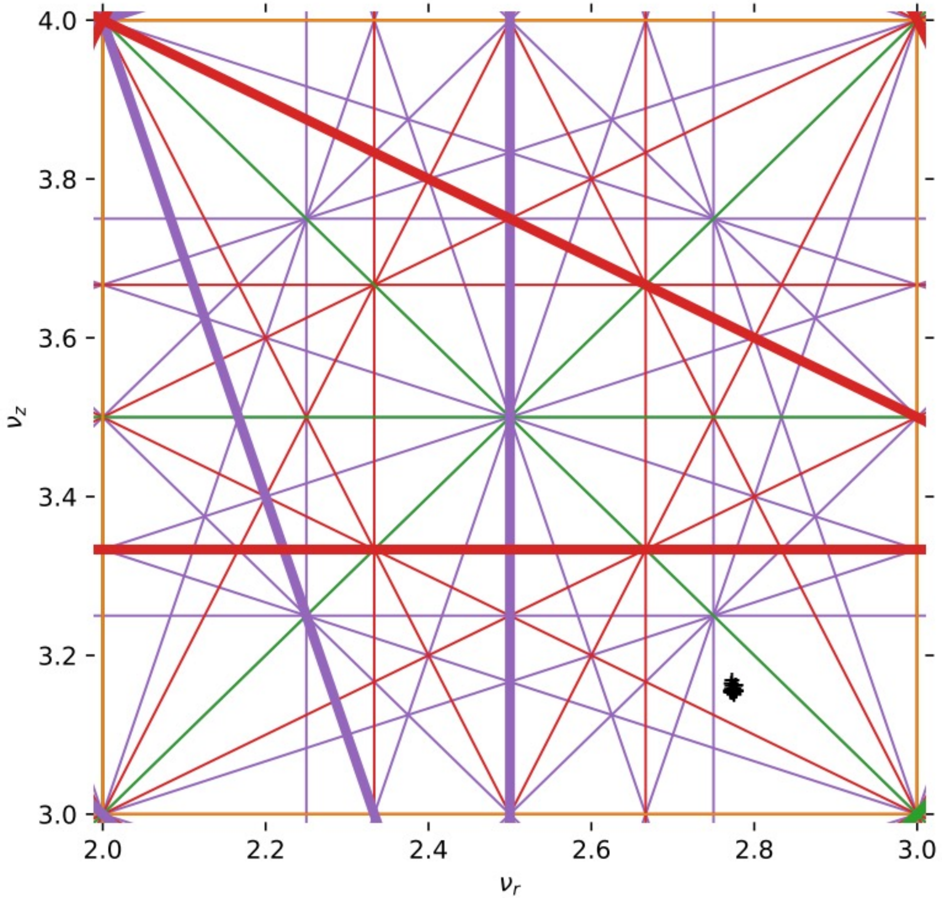



*T. Zhang, et al., A new solution for cost effective, high average power (2 GeV, 6 MW) proton accelerator and its R&D activities, in: Proc. Cyclotrons, 2019*

# Example: Constant Tune Cyclotron

Fix the tune from 800 MeV to 2 GeV so that no resonance is crossed → suppress beam loss! [Planche, 2022]






*Planche, T. (2022). Constant-tune cyclotrons. arXiv preprint arXiv:2211.02988.*

# Extraction of Cyclotrons

| Electrostatic extraction | Stripping extraction |
|---|---|
| $H^+$ | $H^-, H_2^+, H_3^+$ etc. |
| 1. PSI, iThemba, RIKEN etc.<br>2. Commercial cyclotrons: IBA-C235 | 1. TRIUMF, Dea$\delta$alus(plan)<br>2. Commercial cyclotrons: TR30, IBA-Cyclone 30 etc. |

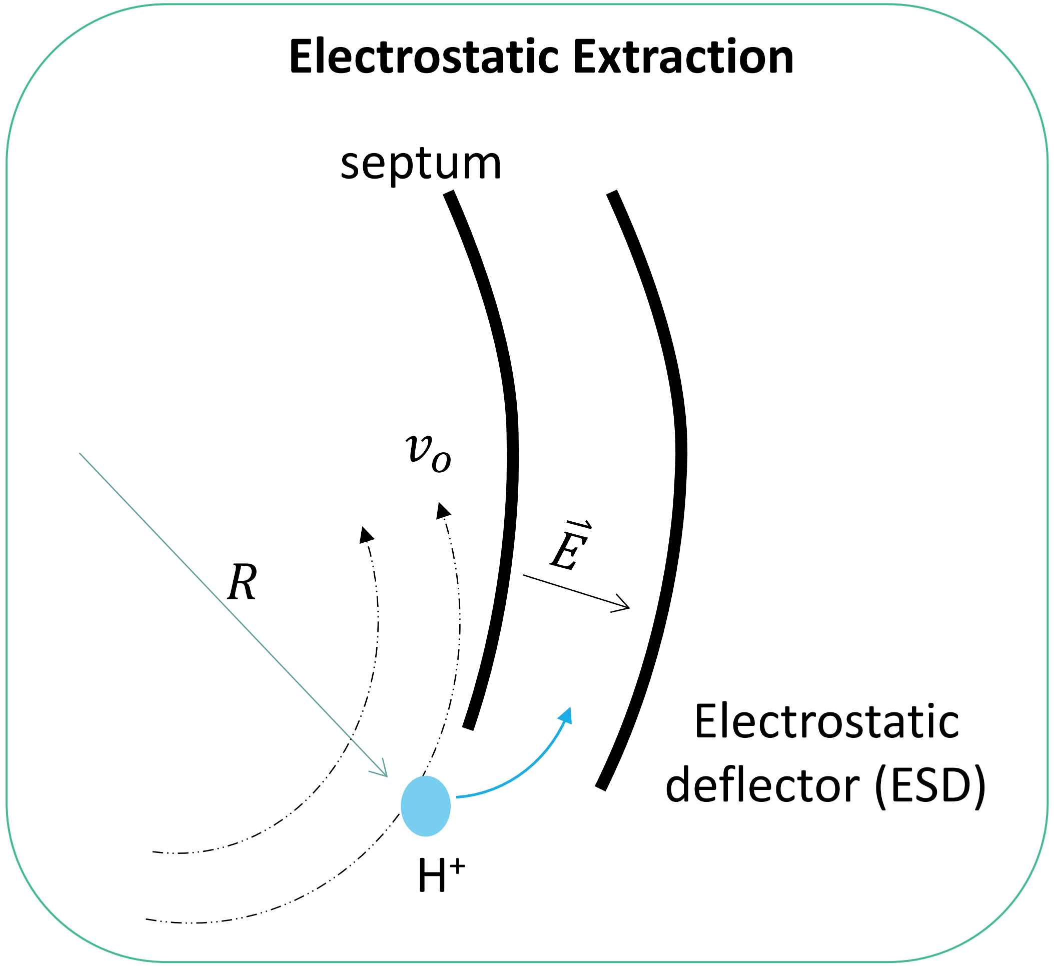
# Electrostatic Extraction

- Accelerate $H^+$ and extract them by electrostatic deflector.
- A large final turn separation is needed to clear the septum

Turn separation = $\Delta R_0 + x$

$\Delta R_0$ is the natural separation from energy gain;

$x$ is the separation from orbital oscillation



# Natural turn separation

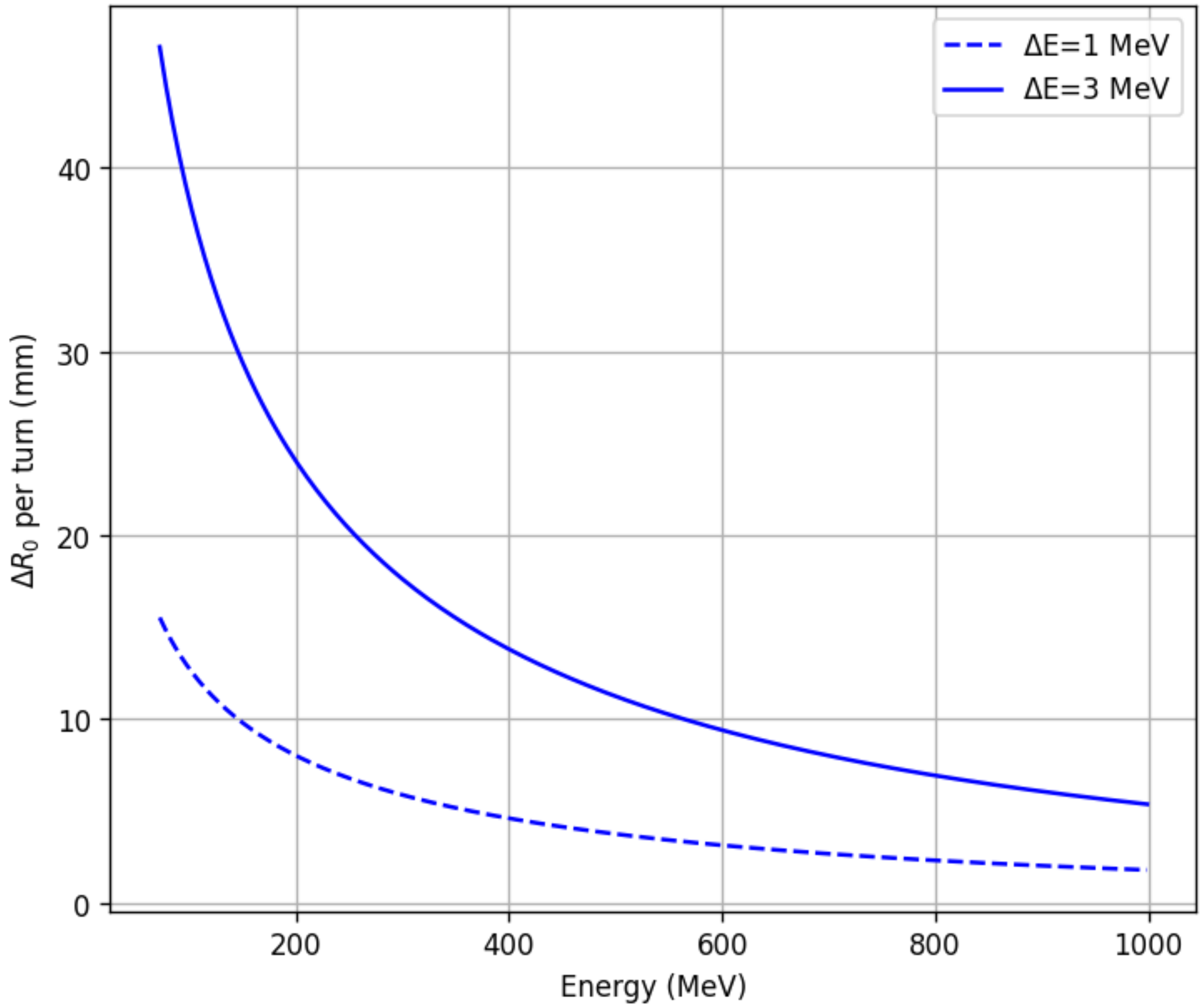
Turn separation = $\Delta R_0 + x$

Natural radial gain per turn:

$$\Delta R_0 = \frac{\Delta E}{E_0} \frac{R}{\gamma(\gamma^2 - 2)}$$

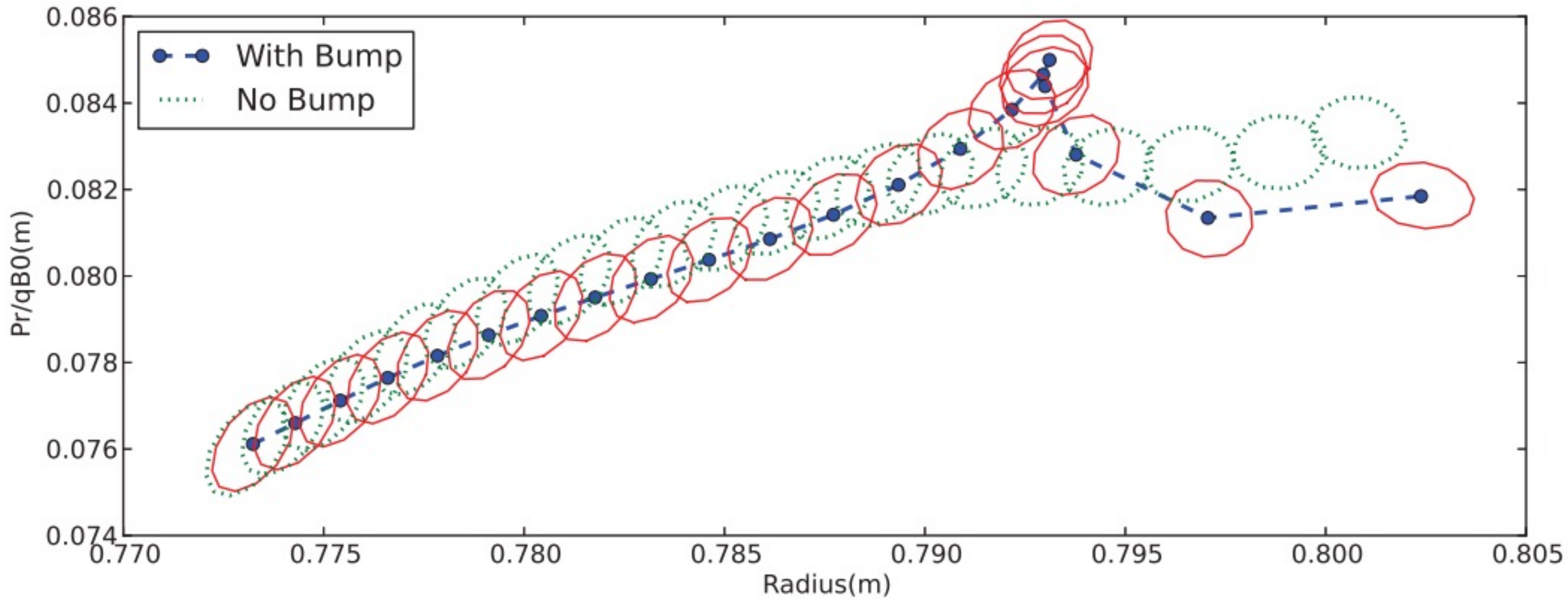
$\Delta E$ is the energy gain per turn;

$E_0$ is the rest mass energy



# Example: Orbital oscillation

Use a small first harmonic bump to induce coherent oscillation that increases the final turn separation [Qin, 2014]

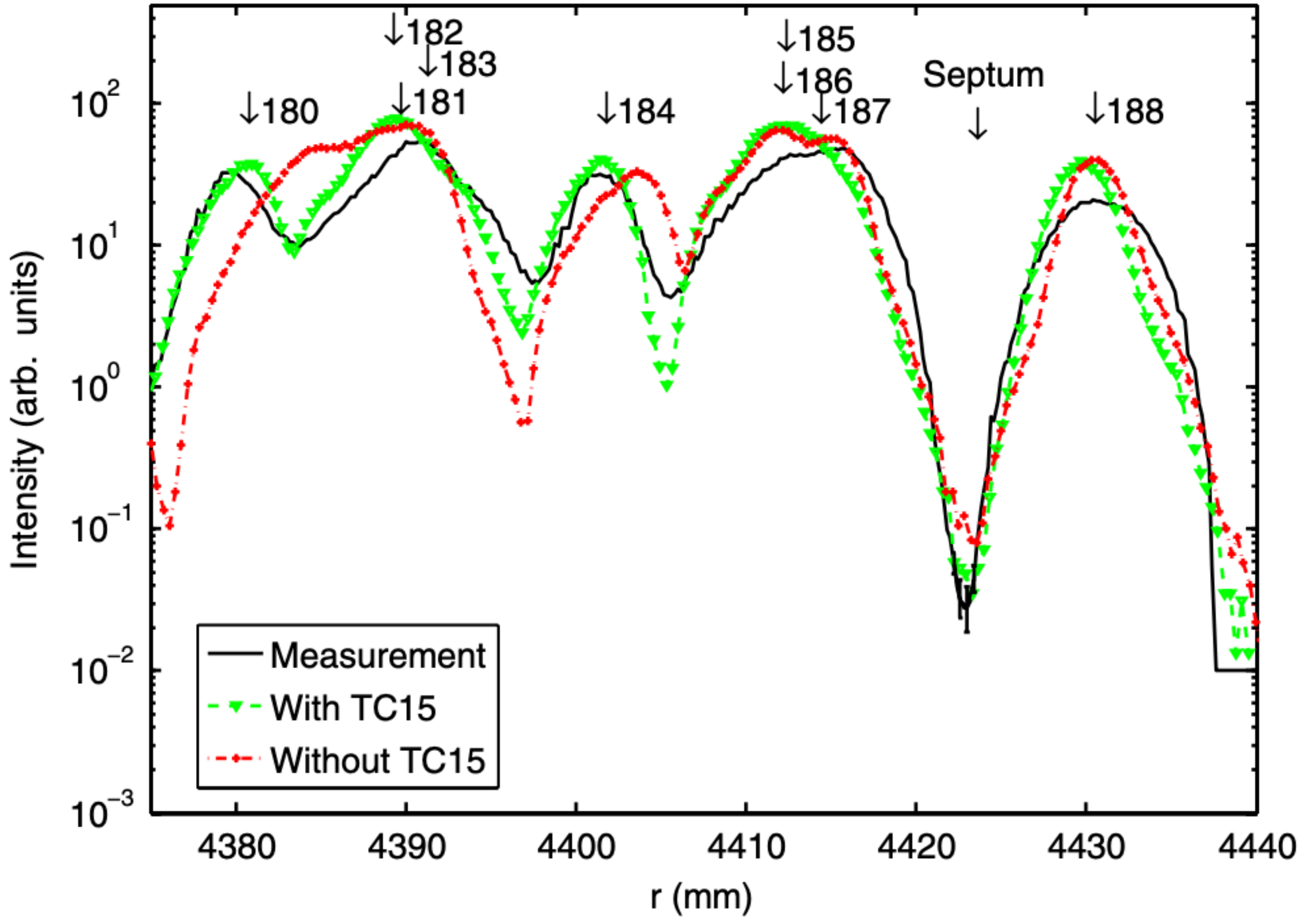





*Qin, Bin et. Al. (2014). Design Study Of a 250 MeV Superconducting Isochronous Cyclotron for Proton Therapy.*

# Example: Electrostatic Extraction at PSI

Radial beam profile with indicated turn numbers for a 2 mA beam extraction at PSI ring cyclotron [Bi, 2011]

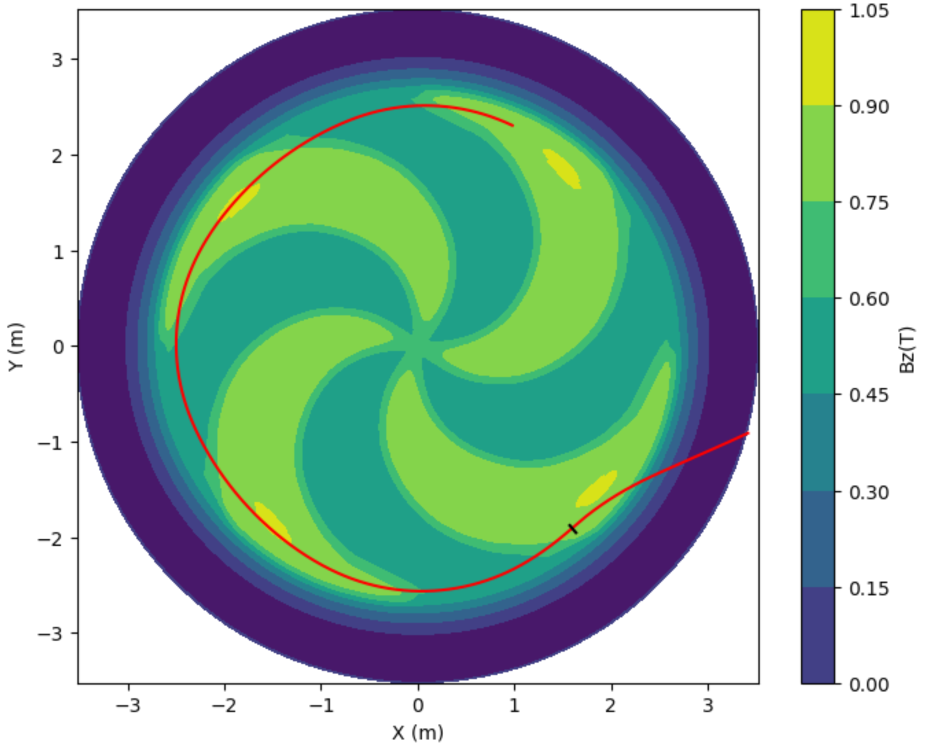



- Without orbit oscillations: turn separation is only 6 mm
- After introducing a first harmonic bump, the final turn separation is increased by almost 3 times

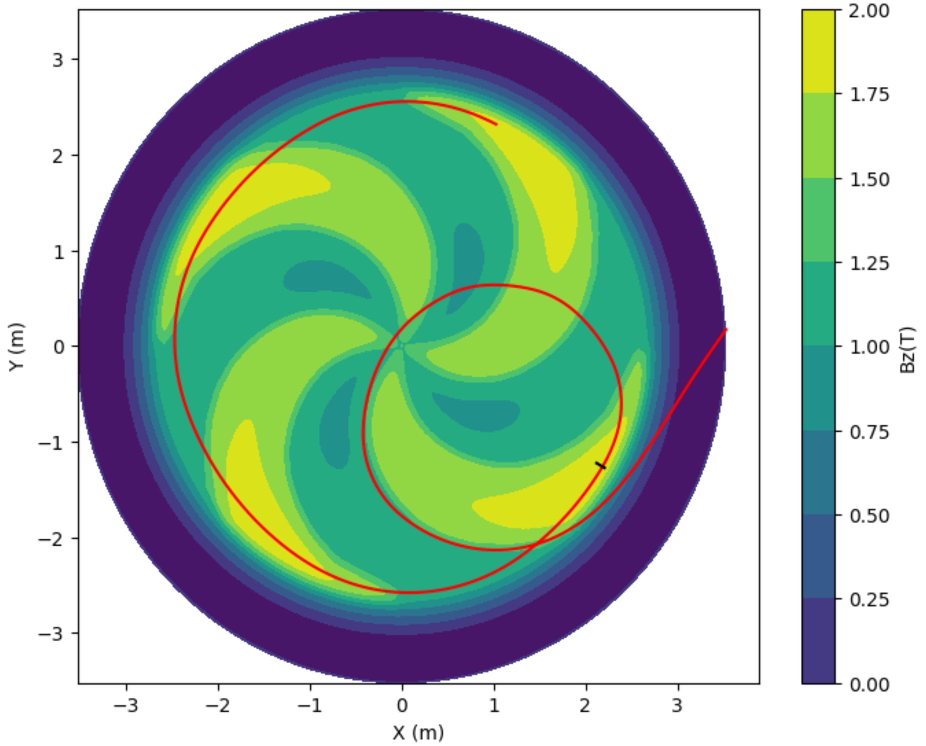
Bi, Y. J., et. Al. (2011). Towards quantitative simulations of high power proton cyclotrons. *Phys. Rev. ST Accel. Beams*, *14*, 054402. doi:10.1103/PhysRevSTAB.14.054402

# Stripping Extraction

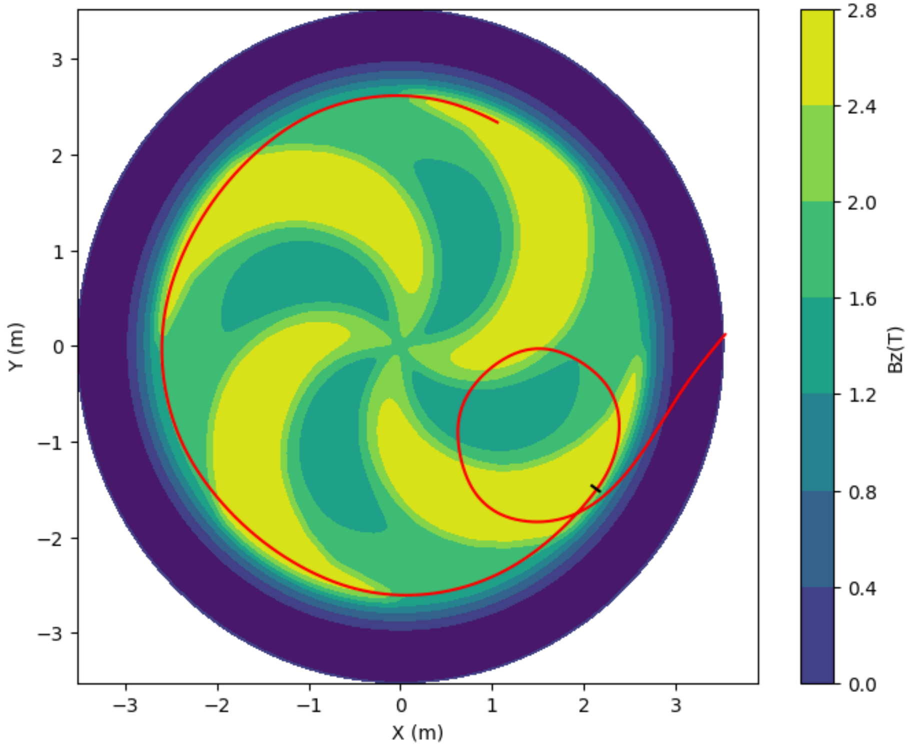
Accelerate $H^-$ or $H_n^+$ and extract them by stripping foils.




H- ($H^- \rightarrow H^+ + 2e^-$)




$H_2^+$ ($H_2^+ \rightarrow 2H^+ + e^-$)



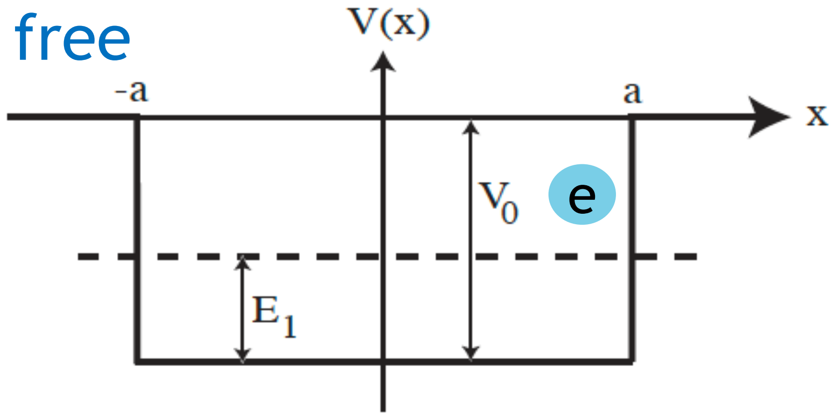

$H_3^+$ ($H_2^+ \rightarrow 3H^+ + 2e^-$)

Turn separation is not required!

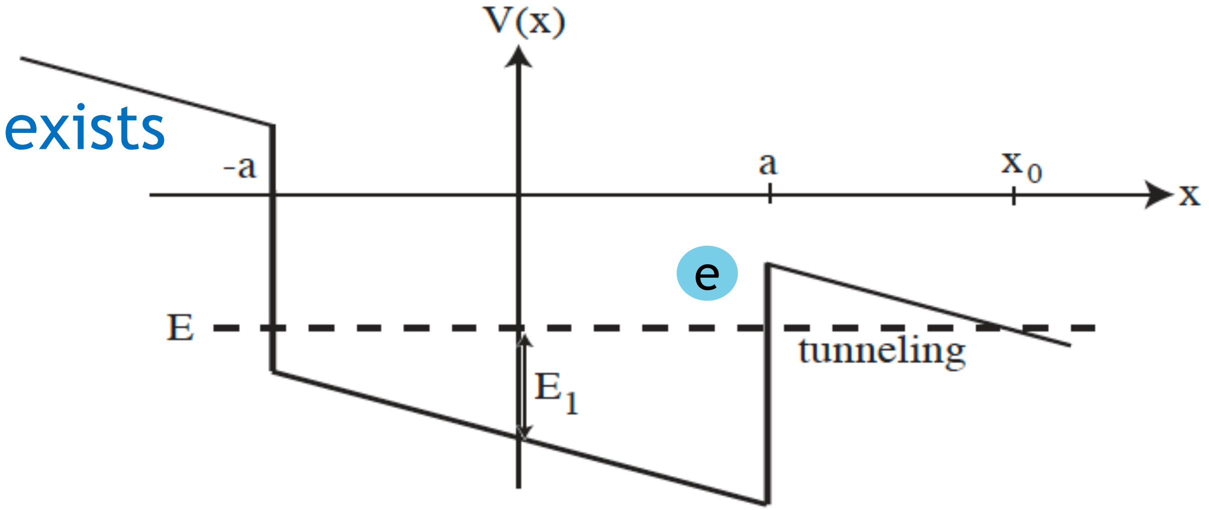
Koay H. W. (2022) *Stripping Extraction and Lorentz Dissociation*, CYC2022

# Lorentz Dissociation

Field free




Field exists



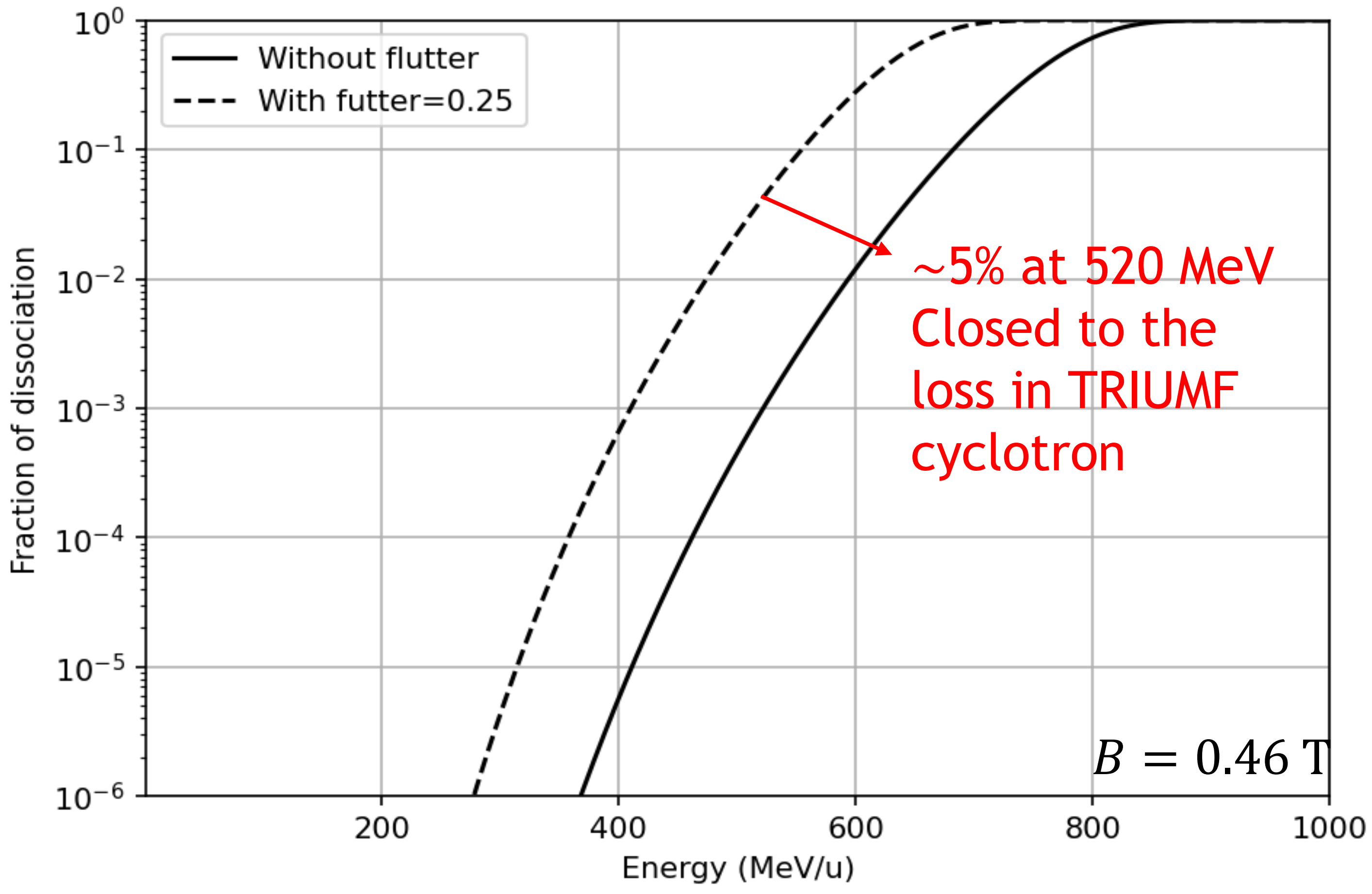

Lorentz dissociation is the dissociation of hydrogen ions due to the existence of equivalent electric field during acceleration

Equivalent electric field,

$$\mathcal{E} = \gamma\beta c B_z \cong (3\mathrm{MV/cm})\gamma\beta(B_z/\mathrm{T})$$

# Example: Lorentz Dissociation of $H^-$

Equivalent electric field,
$\mathcal{E} = \gamma\beta c B_z \cong (3\text{MV/cm})\gamma\beta(B_z/\text{T})$



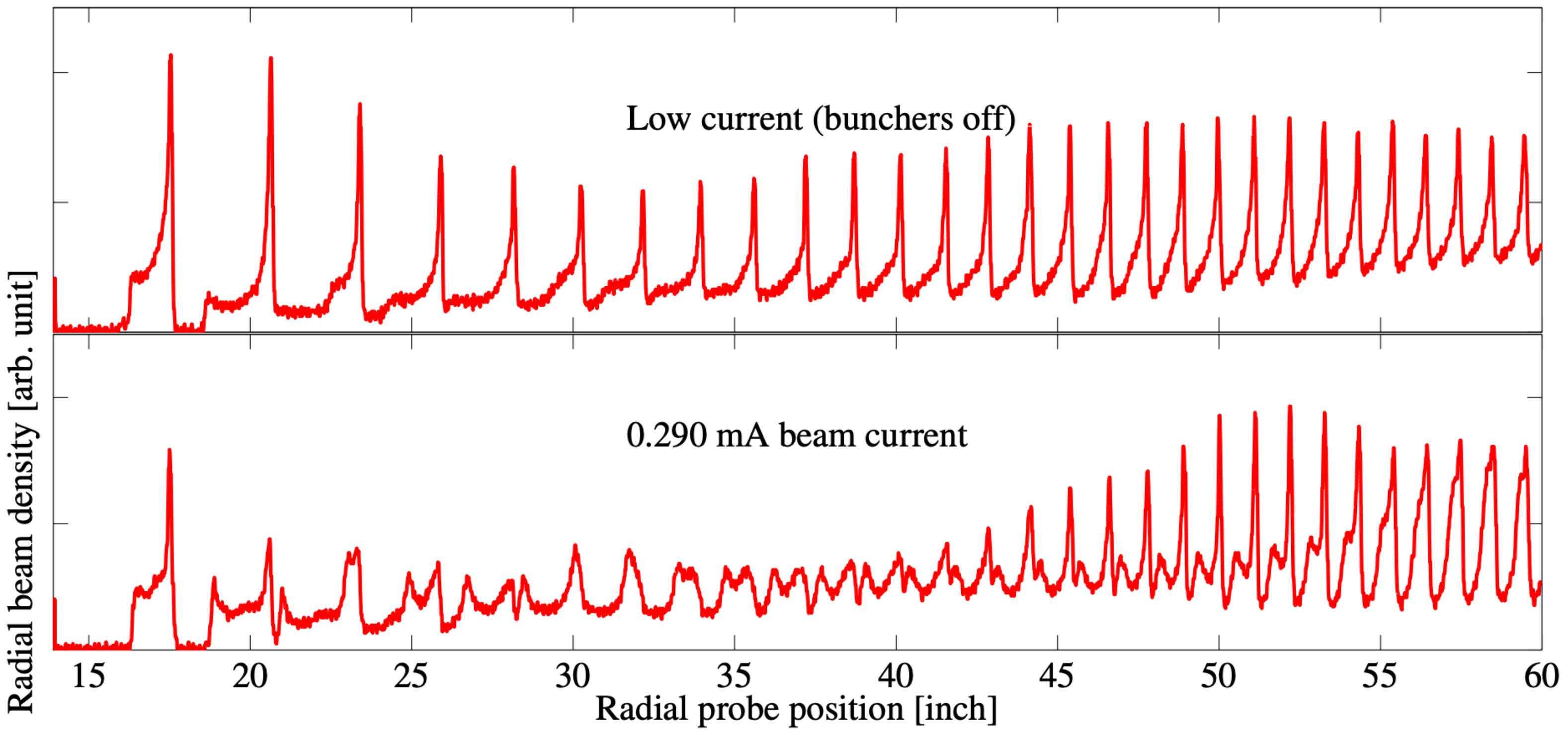

Similar problem also exists for $H_n^+$ ions…

*Koay H. W. (2022) Stripping Extraction and Lorentz Dissociation, CYC2022*

# Larger Current?

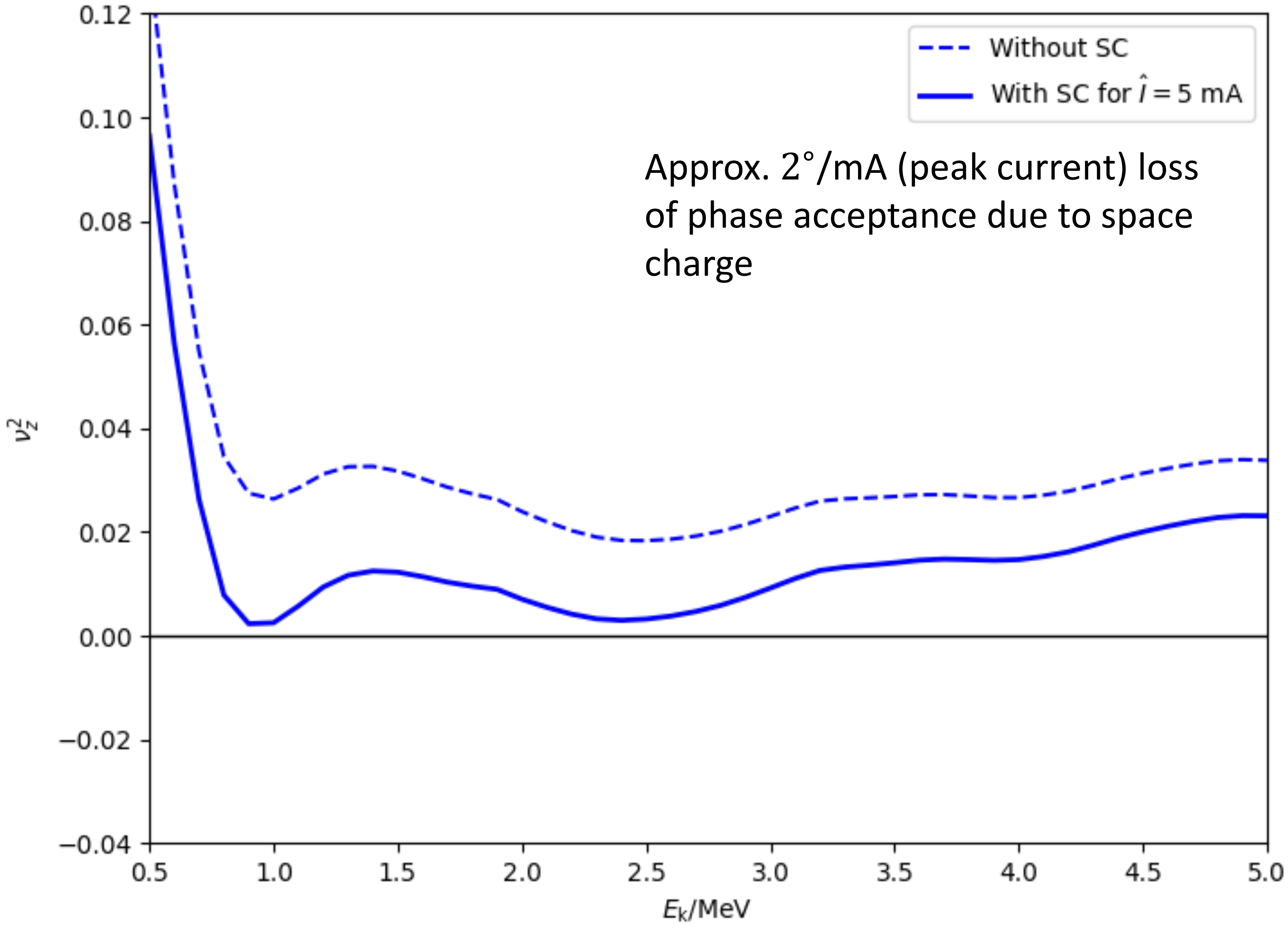
# Space Charge Effect

## TRIUMF cyclotron's first 30 turns




*Rick Baartman (2013)*

# Example: Tune shift due to space charge




Approx. 2°/mA (peak current) loss of phase acceptance due to space charge

*Thomas P. et. Al. (2022) arXiv:2211.05872*

# Example: Space Charge Limit in TRIUMF

- The current limit for the TRIUMF cyclotron was estimated to be 1.1 mA (Planche, 2022).

- However, the maximum ever reported is about 0.42 mA (measured at a reduced duty cycle).

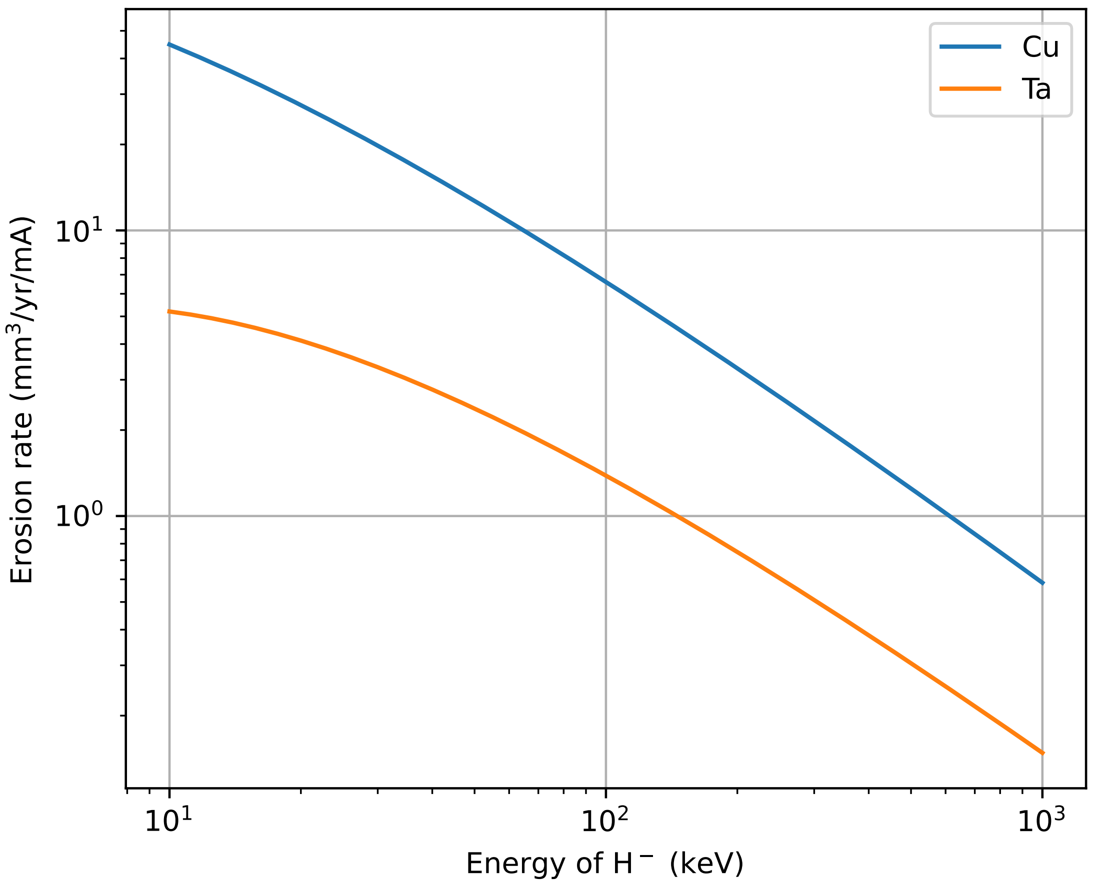
- This discrepancy is due to the insufficient brightness of the injected ion beam that has $I_{source} \leq 2.5$ mA (far below the $I_{source} = 12$ mA required to reach this limit.

*Thomas P. et. Al. (2022) arXiv:2211.05872*

# Other Issues

1. Activation: Ensure sufficient cooldown before annual maintenance (typical power loss limit for hands-on maintenance at 1 W/m)
2. Replacement of foils: At TRIUMF, 2 $mg/cm^2$ of pyrolytic graphite foils last for almost 500 mA-hr (roughly a year for 5000 hrs and 100 μA)
3. Erosion

# Sputtering erosion

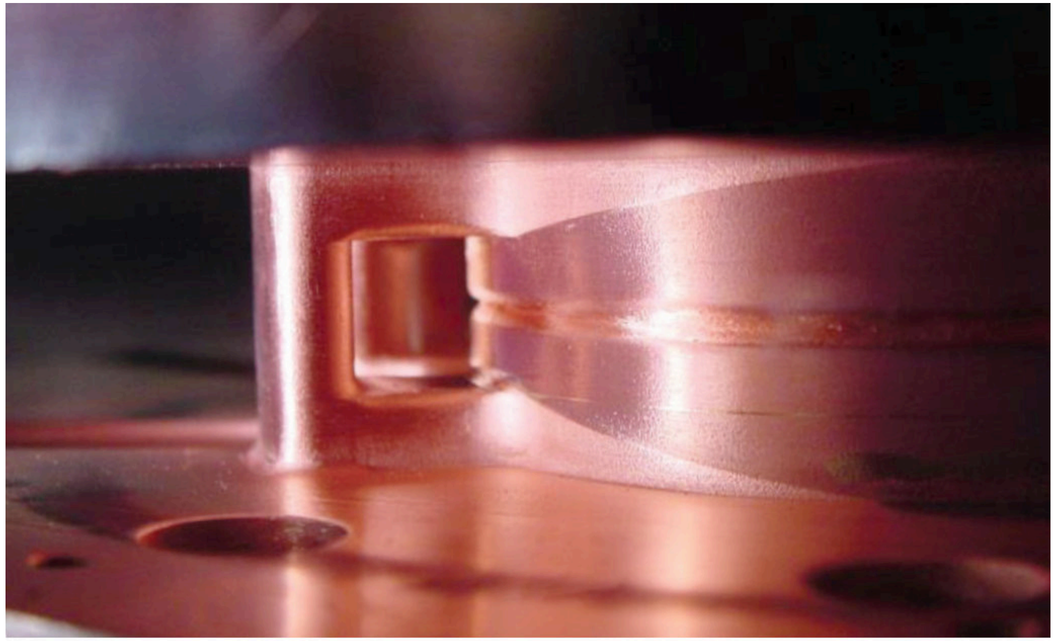



Erosion rate calculated using Yamamura's surface sputtering equation [Yamamura, 1996].
Assuming the cyclotron is operating continuously for a year.

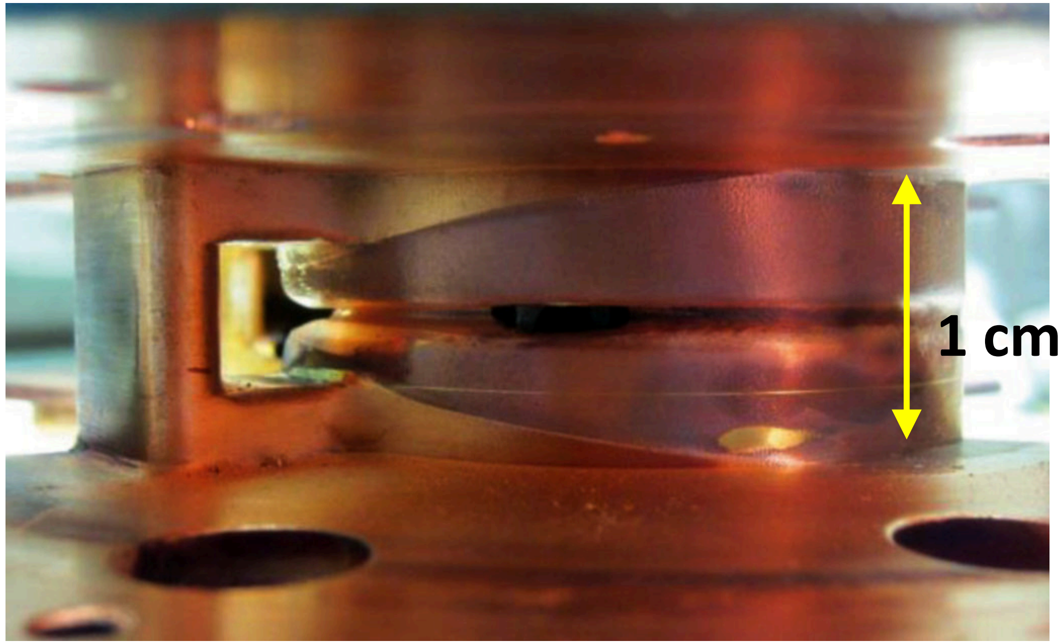
*Y Yamamura et. Al. Energy dependence of ion-induced sputtering yields from monatomic solids at normal incidence. Atomic Data and Nuclear Data Tables, 62(2):149–253, 1996.*

# Example: Erosion of TR-30

After 2 years



After 10 years




Erosion of the TR-30 copper centre post
(erosion ~70 $mm^3$/year for 25 keV 7mA of H+ beam loss)



*Thomas P. et. Al. (2022) arXiv:2211.05872*

# Summary

- As the cyclotron is the most efficient accelerator, it is worth further research to achieve a higher power.

- TRIUMF has provided a good understanding of the fundamental of a cyclotron by operating reliably for the past 50 years.

- Stand on this basis, we are optimistic of the future potential of a cyclotron to achieve beyond the max power we have now.

# Thank you!