Bridging Positivity and S-matrix Bootstrap bounds

or

how to bound dimension-six operators.

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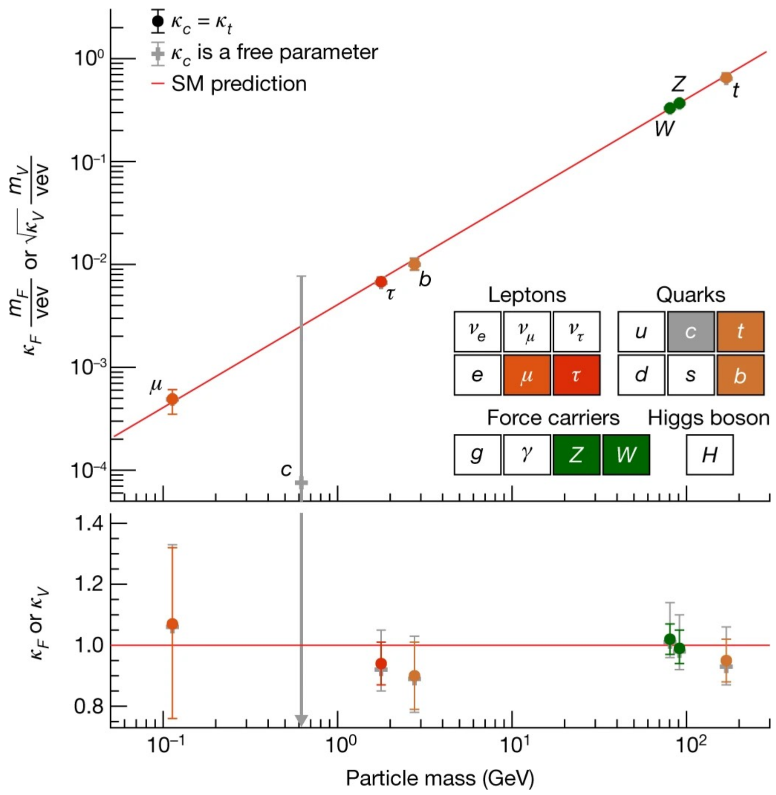
CERN Colloquium, February 22 2023

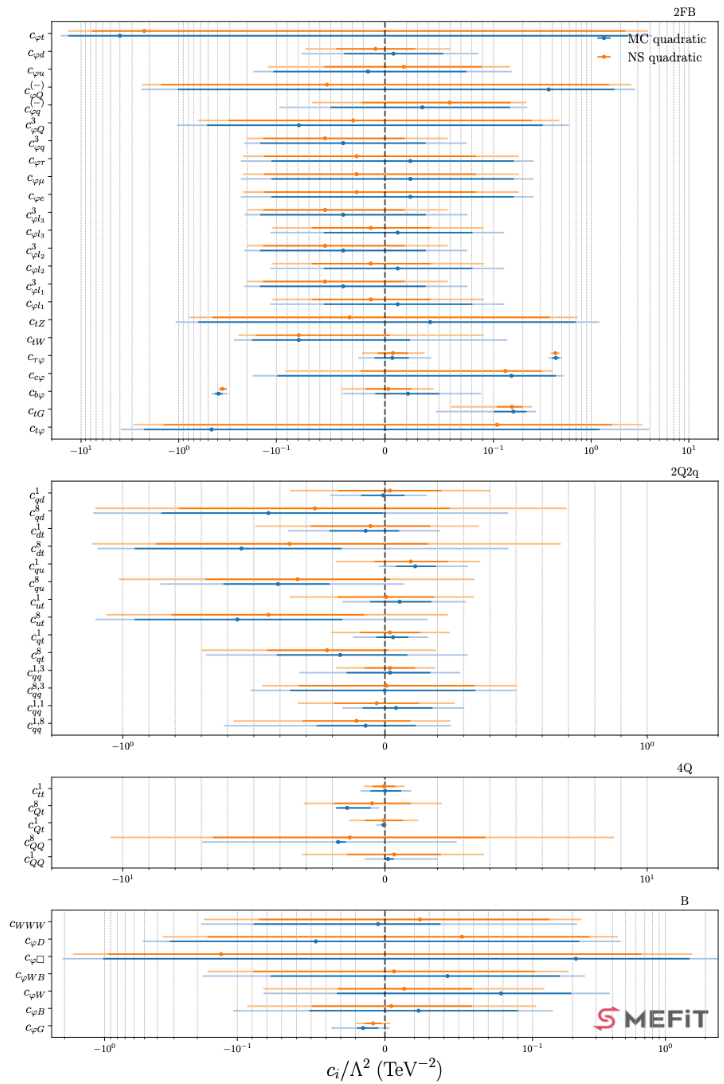
Based on hep-th/2210.01502 + ongoing work, with

Andrea Guerrieri

Mehmet Gümüş

The Standard Model …





… Effective Field Theory

The Standard Model Effective Field Theory

$$\mathscr{L}_{SMEFT} = \mathscr{L}_{\mathrm{dim}\leq 4} + HHLL/\Lambda + \mathscr{L}_6 + \cdots$$

The SM Formula T-Shirt

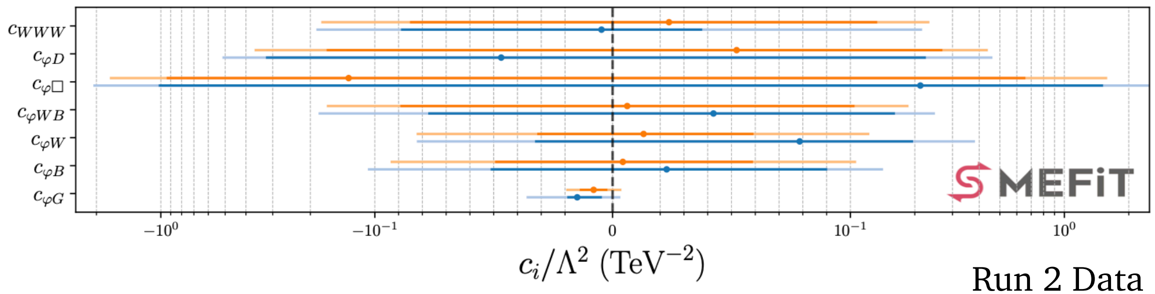
“Weinberg operator”,
high scale

Leading deviations at LHC:

$$M = M_{\mathrm{SM}} + O(E^2/\Lambda^2, Ev/\Lambda^2, v^2/\Lambda^2)$$

For instance:

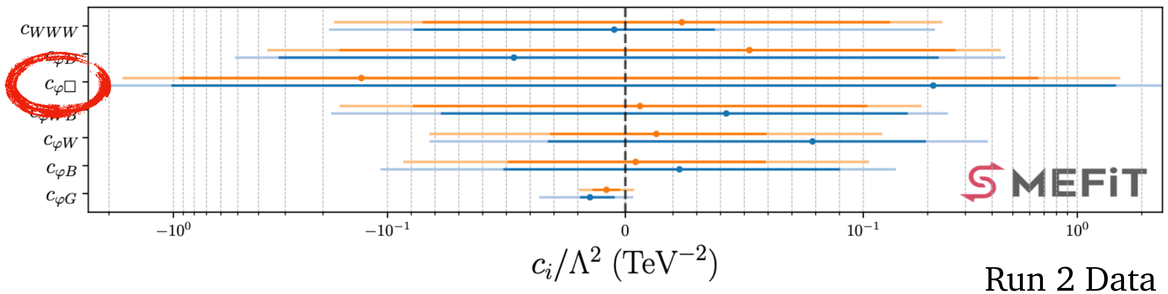
$$\mathscr{L}_6 = g_{4F}(\bar{\psi}\gamma^\mu\psi)^2 \; + \; g_D \bar{L}H\sigma_{\mu\nu}eF^{\mu\nu} \; + \; g_{FF}F_{\mu\nu}F^{\mu\nu}|H|^2 \; + \; g_H\partial_\mu|H|^2\partial^\mu|H|^2 \; + \; \cdots$$



Run 2 Data

See also ATLAS EFT fit,
HL-LHC should improve these bounds by $\times 2$
[SMEFit, HL-LHC & HE-LHC Working group]

Today I would like to explain how to analyse EFT amplitudes that are extremal, featuring the maximal/minimal feasible values of the Wilson coefficient.



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Today I would like to explain how to analyse EFT amplitudes that are extremal, featuring the maximal/minimal feasible values of the Wilson coefficient.

$$O_H = \partial_\mu |H|^2 \partial^\mu |H|^2$$

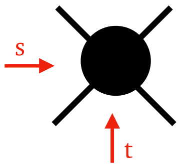
- Current bound $|g_H| \lesssim O(1) \times \Lambda^2/(1\ \text{TeV})^2$.
- Modifies all SM Higgs couplings $O_H = -O_{H\Box} = |H|^2 \Box |H|^2 = |D_\mu H|^2 |H|^2 + \text{e.o.m.}$
- Interesting operator in SILH (EFT parametrization of strongly interacting BSM Higgs),
- and in extensions of EWSB

Effective Field Theory Wilson coefficients take *a priori* unknown real values, but

not anything goes

Non-perturbative properties of the amplitude

- Unitarity of the S-matrix $S^\dagger S = \mathbb{1}$,
- Crossing-symmetry,



- Analytic properties that follow from causal evolution. Cartoon version $F(w) = \int_0^\infty e^{iwt} S(t) dt$

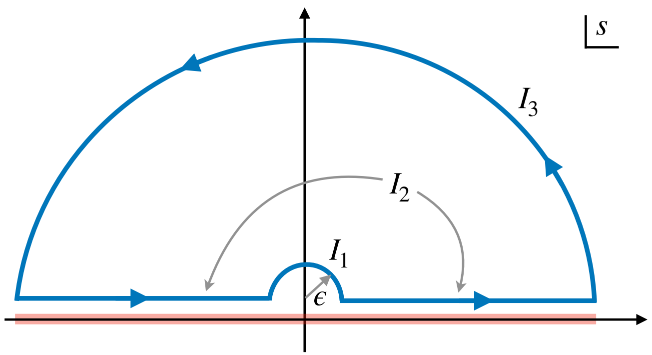
imply sharp bound on the max/min values that Wilson coefficients can attain.

A positivity bound

No dim-six operator:
By crossing it should contribute as $c_6/\Lambda^2(s+t+u)$
$s+t+u=4m^2 \to 0$

Classic example: $\mathscr{L} = \frac{1}{2}(\partial\phi)^2 + \frac{g_8}{\Lambda^4}(\partial\phi)^4 + \dots$ the forward amplitude is $A(s) = g_8/\Lambda^4\, s^2 + \cdots$

$$A''(\epsilon) = g_8 + O(\epsilon) = -I_1 = I_2 + I_3 = \frac{1}{\pi}\int_\epsilon^\infty \frac{\mathrm{Im}A(z)}{z^3}dz = \frac{1}{\pi}\int_\epsilon^\infty \frac{\sigma(s)}{z^3}dz > 0!$$



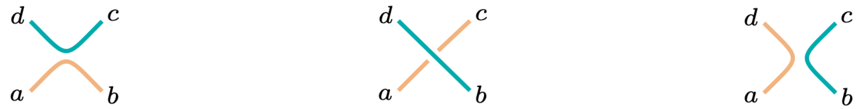
- Massive scalar, Froissart-Martin: $\lim_{s\to\infty} M(s,t)/|s|^2 = 0$
- Convergence of I_2 and $I_3 = 0$ holds very generally for massless flows.

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi ’06], …, [Komargodski, Schwimmer ’11], [Luty, Polchinski, Rattazzi ’12] , …

Dimension-six & Doubly subtracted dispersion relations

Consider the $O(n)$ massive scalar theory in 4D. In particular the $2 \to 2$ amplitude

$$M_{ab}^{cd}(s,t,u) = M(s\,|\,t,u)\delta_{ab}\delta^{cd} + M(t\,|\,s,u)\delta_a^c\delta_b^d + M(u\,|\,t,s)\delta_a^d\delta_b^c$$



The amplitude can be characterised by Taylor series below the cuts. Expanding around $\bar{x} = x/m^2 - 4/3$

$$M(s\,|\,t,u) = -\,c_0 + c_H\bar{s} + O(s^2)$$

Perturbatively $\mathscr{L} = \mathscr{L}_{\text{Kin}} - \frac{g_0}{4}(\overrightarrow{\phi}\cdot\overrightarrow{\phi})^2 + \frac{g_H}{4\Lambda^2}[\partial^\mu(\overrightarrow{\phi}\cdot\overrightarrow{\phi})]^2$

$c_0 = 2g_0 - 8/3g_Hm^2/\Lambda^2 + \cdots, \qquad c_H = 2g_Hm^2/\Lambda^2 + \cdots.$

Dimension-six & Doubly subtracted dispersion relations

Our low energy expansion is, $M(s\,|\,t,u) = -\,c_0 + c_H\bar{s} + O(s^2)$, thus we start with a subtracted dispersion relation:

$$M(s,t) = \frac{s^2}{2\pi i}\oint \frac{dz}{z-s}\frac{M(z,t)}{z^2}$$

After blowing up the contour and taking into account the three Isospin channels we get the sum rule

$$c_H = \overrightarrow{D}(s,t)\,.\,\overrightarrow{M}(s,t) + \frac{1}{\pi}\int_{4m^2}^{\infty} dz\,\mathrm{disc}_z\overrightarrow{M}(z,t)\,.\,\overrightarrow{K}(z,s,t;4/3)$$

where $\overrightarrow{M} = (M_{\mathrm{sing}}, M_{\mathrm{sym}}, M_{\mathrm{anti}})$, $\overrightarrow{D}$ and $\overrightarrow{K}$ are rational functions.

Dimension-six & Doubly subtracted dispersion relations

$$c_H = \overrightarrow{D}(s,t)\,.\,\overrightarrow{M}(s,t) + \frac{1}{\pi}\int_{4m^2}^{\infty} dz\,\mathrm{disc}_z\overrightarrow{M}(z,t)\,.\,\overrightarrow{K}(z,s,t;4/3)$$

c_H is not sign-definite — also known from un-subtracted sum rules [Low, Rattazzi, Vichi '09], [Falkowski, Rychkov, Urbano '12]

Project the last equation into the spin-0 partial waves

$$c_H \sim \mathbf{Re}f_0(s) + \frac{1}{\pi}\int_{4m^2}^{\infty} dz \sum_J \mathbf{Im}f_J(s)\,.\,\vec{k}'(z,s)$$

Compare with

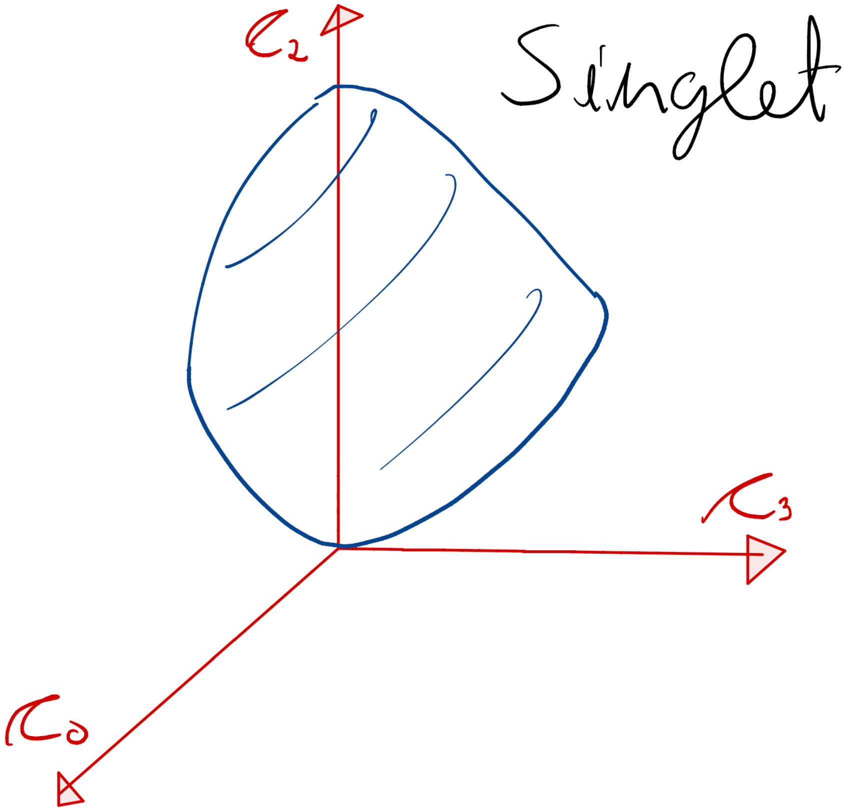
$$\left(g_8 = \frac{1}{\pi}\int_{\epsilon}^{\infty} \frac{\mathrm{Im}A(z)}{z^3}dz > 0 \right)$$

An opportunity for bootstrap methods.

e.g.1 the experimental bound is symmetric around $c_H = 0$, perhaps a highly asymmetric theory bound?

e.g.2 what are the theories achieving the maximally negative or maximally positive value?

A simpler example.



- singlet amplitude:

$$M(s,t,u) = -c_0 + c_2(\bar{s}^2 + \bar{t}^2 + \bar{u}^2) + c_3\bar{s}\bar{t}\bar{u} + O(s^4)$$

Around the origin

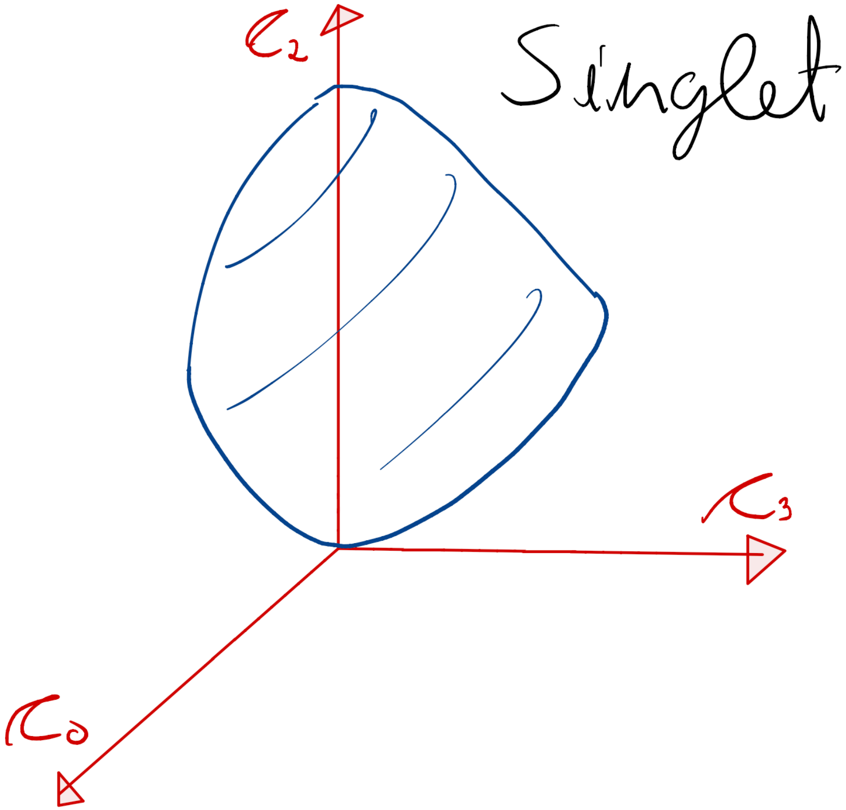
$$\mathcal{L} = \mathcal{L}_{\text{Kin}} - \frac{g_0}{4!}\phi^4 + \frac{g_2}{2\Lambda^4}(\partial\phi)^4 + \frac{g_3}{3\Lambda^6}(\partial^\mu\partial_\rho\phi)^3\phi + \cdots$$

$$c_0 = g_0 - 4/3 g_2\epsilon^2 + \cdots,$$

$$c_2 = g_2\epsilon^2 + \cdots,$$

$$c_3 = g_3\epsilon^3 + \cdots.$$

A simpler example.



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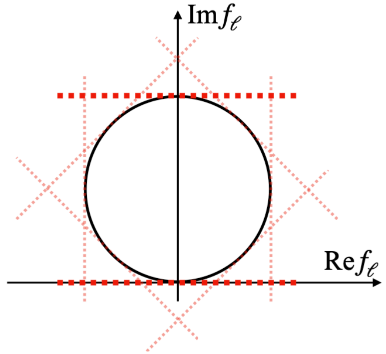
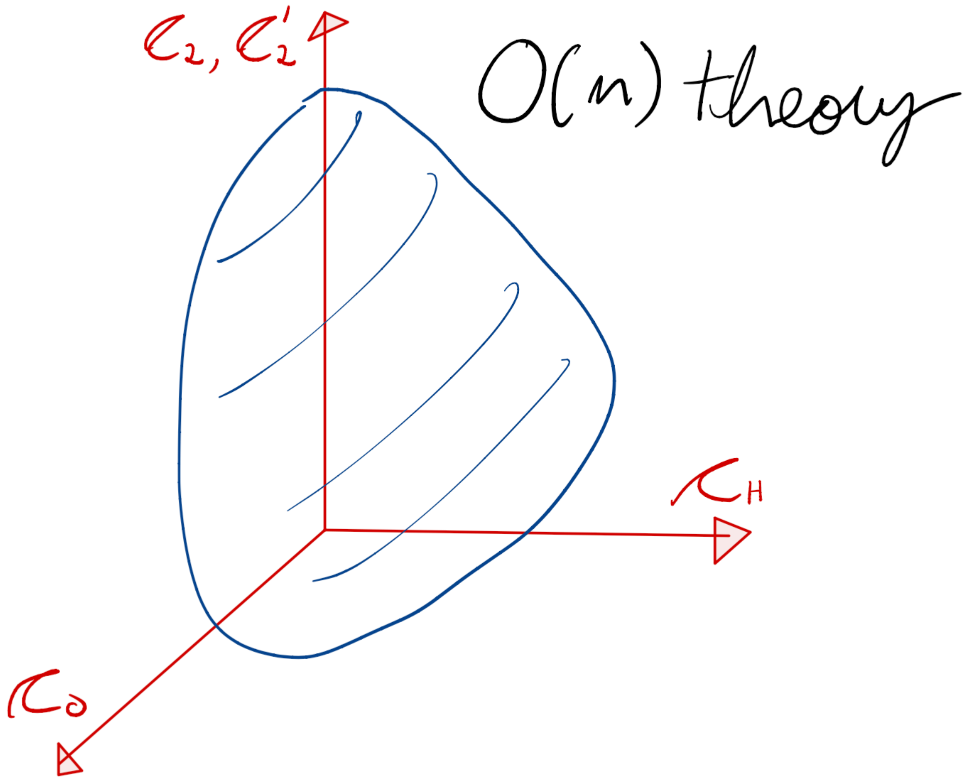
Analogous intuition for why this bound exists comes from fixed-t (doubly subtracted) sum rules, e.g.

$$-c_0 = M(s,t) - \frac{1}{\pi}\int_{4m^2}^{\infty} dz\, \mathrm{disc}_z M(z,t)\, K(z,s,t;4/3)$$





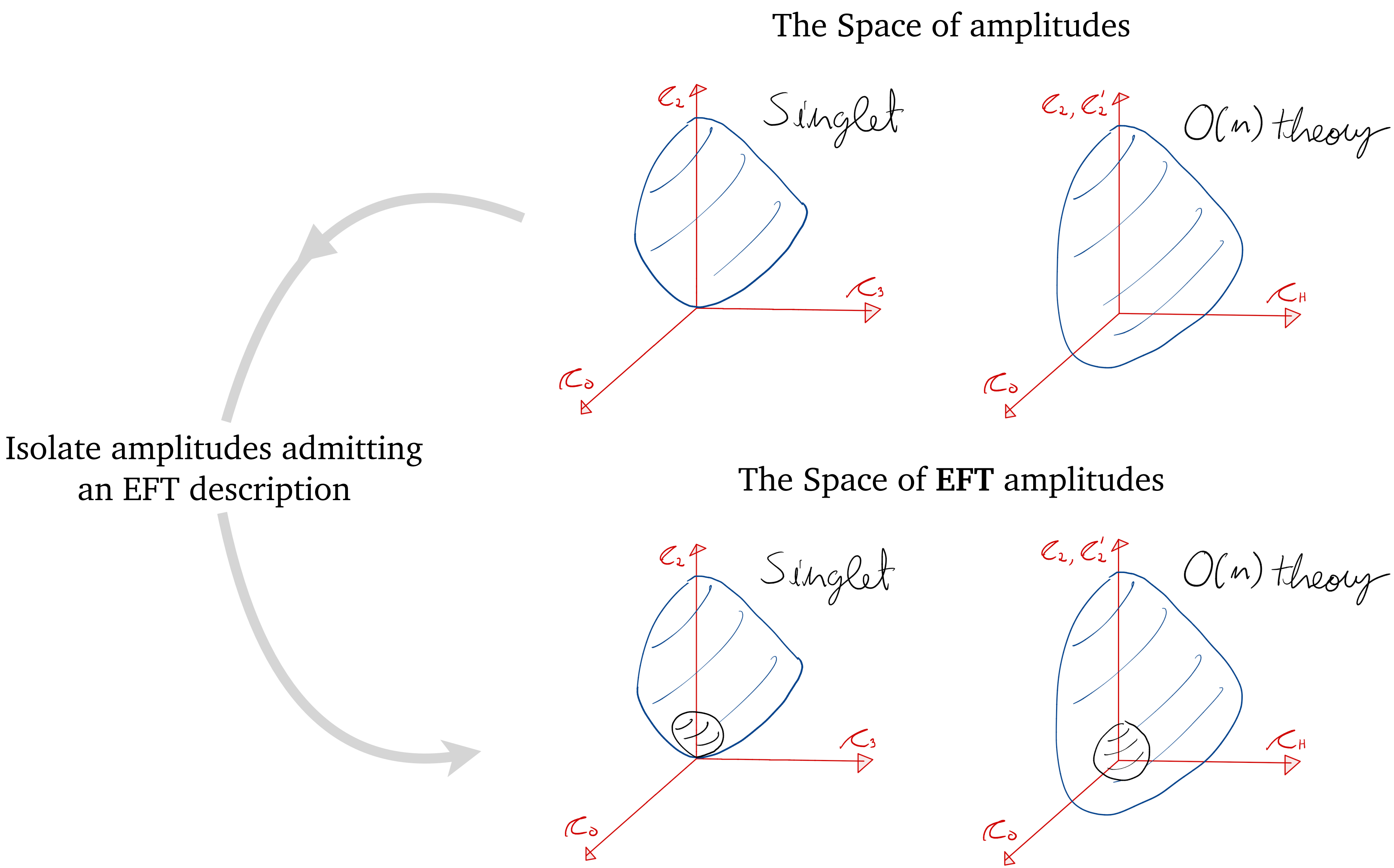
- Perturbatively

$$\mathscr{L} = \mathscr{L}_{\text{Kin}} - \frac{g_0}{4}(\vec{\phi}\cdot\vec{\phi})^2 + \frac{g_H}{4\Lambda^2}[\partial^\mu(\vec{\phi}\cdot\vec{\phi})]^2$$

$$c_0 = 2g_0 - 8/3 g_H m^2/\Lambda^2 + \cdots,$$

$$c_H = 2g_H m^2/\Lambda^2 + \cdots.$$

☞ **An opportunity for bootstrap methods, but …** $|c_H - g_H m^2/\Lambda^2| \gg O(1)$ **along the boundary.**


The Space of amplitudes
Singlet
O(n) theory
Isolate amplitudes admitting an EFT description
The Space of **EFT** amplitudes
Singlet
O(n) theory

Plan for the rest of the talk:

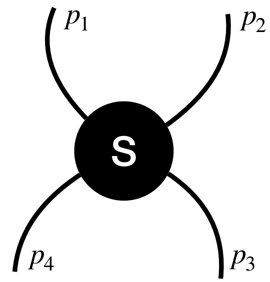
- ~~Motivation~~
- The space of two-to-two amplitudes
- The space of two-to-two **EFT** amplitudes
- Conclusions & outlook

Non-perturbative amplitudes

Min/max value of $\{M(s_0, t_0, u_0), \partial_{s_0} M(s_0, t_0, u_0), \partial^2_{s_0} M(s_0, t_0, u_0), \ \ldots\}$

subject to the constraints of

- Analyticity
- Crossing-symmetry $M(s, t, u) = M(t, s, u) = M(u, t, s)$
- Unitarity



Recall: $s = (p_1 + p_2)^2$
$t = (p_1 - p_3)^2$
$u = (p_1 - p_4)^2$
$s + t + u = 4m^2$

Optimization problems admit two complementary approaches

[Paulos, Penedones, Toledo, van Rees, Vieira, Homrich '16-'19] , …, [Correia, Zhivoedov, Sever '20-'21], …, [Guerrieri, Sever '20], …

Non-perturbative amplitudes

Min/max value of $\{M(s_0, t_0, u_0), \partial_{s_0} M(s_0, t_0, u_0), \partial_{s_0}^2 M(s_0, t_0, u_0), \dots\}$

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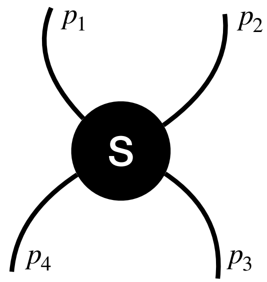

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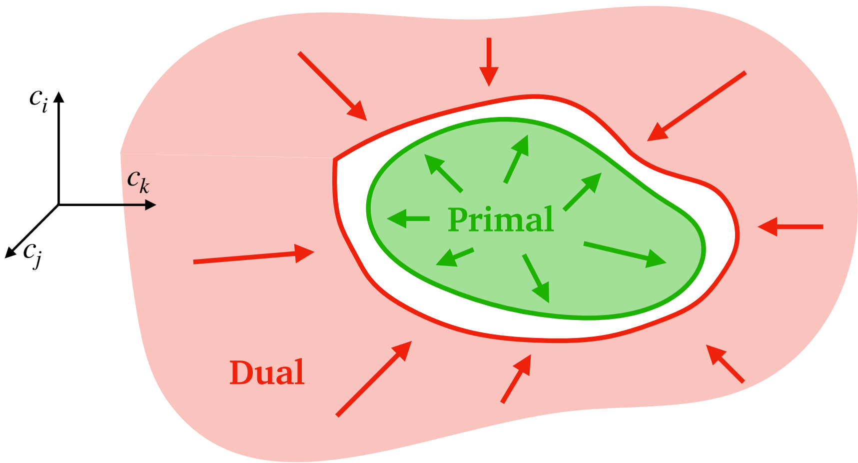


[Paulos, Penedones, Toledo, van Rees, Vieira, Homrich '16-'19] , …, [Correia, Zhivoedov, Sever '20-'21], …, [Guerrieri, Sever '20], …

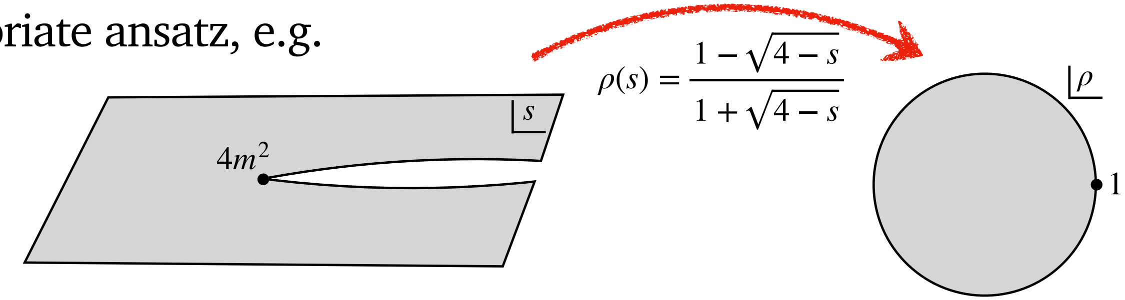
Primal approach

Primal bootstrap, in a nutshell

1.- Solve for analyticity and crossing with appropriate ansatz, e.g.

$$M^{\text{ans}}(s,t,u) = \sum_{p,q,r=0}^{N} c_{p,q,r}\,\rho^p(s)\rho^q(t)\rho^r(u)$$

with $s+t+u=4m^2$.



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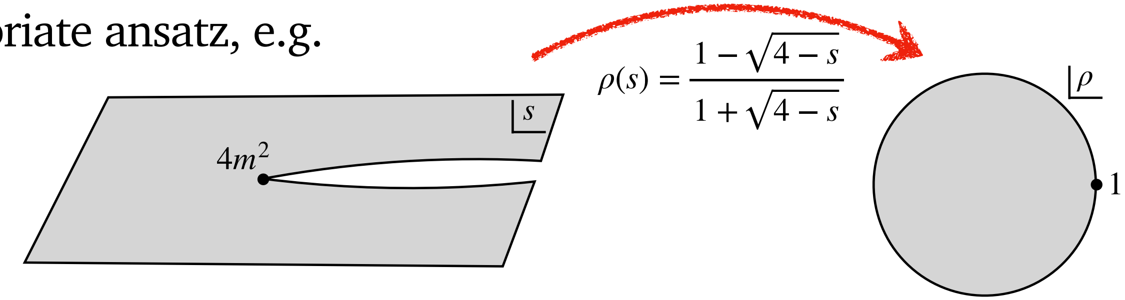

2.- Solve for unitarity numerically.

$$\left|1 + i\frac{\sqrt{s-4}}{s} f_\ell(s)\right| \le 1 \quad \text{for} \quad \ell = 1,\ldots,L \qquad \text{where } f_\ell^{\text{ans}}(s) = \frac{1}{32\pi}\int_{-1}^{1} P_\ell(x) M^{\text{ans}}(s, t(s,x), u(s,x))$$

Fill in the space of amplitudes by maximizing $M^{\text{ans}}(4/3, 4/3, 4/3)$ for large values of L, and then taking large N.

[Paulos, Penedones, Toledo, van Rees, Vieira, Homrich ’16-’19] , …

Dual approach

$$\mathcal{L}(\{P\},\{D\}) = R + \alpha_0(c_0 - R\sin\theta) + \alpha_2(c_2 - R\cos\theta - r_0)$$

$$+ \nu_{n,m}(\text{crossing})^{n,m} + (\text{unitarity})$$

Dual approach

$$\frac{1}{\pi}\int_4^\infty d\nu \sum_{\ell=0,2,\dots}' n_\ell \,\mathrm{Im} f_\ell(s)\, \frac{P_\ell(\dots)}{(\nu-4/3)^3}$$

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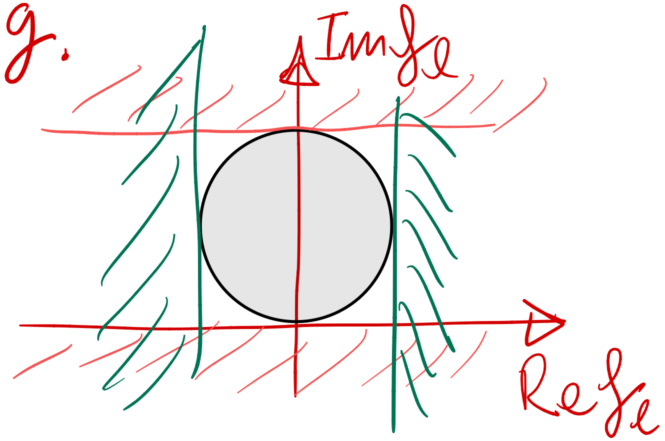
e.g.



$$0 \leq \operatorname{Im} f_\ell \leq \frac{2}{\rho^2(s)}$$

$$-\frac{1}{\rho^2} \leq \operatorname{Re} f_\ell \leq \frac{1}{\rho^2}$$

Dual approach

$$\Rightarrow M(s,t) = M(s_0,t_0) + \frac{1}{\pi}\int_4^\infty dz \left[M_z(z,t) K(z,s,t;t_0) + M_z(z,t_0) K(z,t,t_0;s_0) \right]$$

$$\text{w/ } K(z,s,t;t_0) = \frac{1}{z-s} + \frac{1}{z-4+s+t} - \frac{1}{z-t_0} - \frac{1}{z-4+t+t_0}$$

Explicit $s \leftrightarrow u$ symmetric

Impose $u \leftrightarrow t$ symmetry by expanding

$$M(s,t) - M(s, 4-s-t) = 0$$

into p.w.

$$\int_4^\infty dz \sum_{\ell=2}^{\infty}{}' (2\ell+1)\, \mathrm{Im} f_\ell(z)\, F_\ell(z,s,t;t_0,s_0) = 0$$

where

$$F_\ell = P_\ell\left(1+\frac{2}{t-4}\right) K(z,s,t;t_0) + P_\ell\left(1+\frac{2t_0}{z-4}\right) K(z,t,t_0;s_0) - (t \leftrightarrow u)$$

Finally act with derivatives

$$\mathcal{F}^{(\ell,k)} = \int_4^\infty dz \sum_{\ell=0}^{\infty}{}' (2\ell+1)\, \mathrm{Im} f_\ell(z)\, F_\ell^{(\ell,k)}(z) = 0$$

Equivalent to null constraints of Tolley, Wang, Zhou '20 Caron-Huot, Van Duong '20

with $F_\ell^{(\ell,k)}(z) \equiv \partial_{s^\ell} \partial_{t^k} F_\ell(z,s,t;\tfrac{4}{3},\tfrac{4}{3})\Big|_{t=s=\frac{4}{3}}$.

Dual approach

$$\mathcal{L}(\{P\},\{D\}) = R + \alpha_0\left(c_0 - R\sin\theta\right) + \alpha_2\left(c_2 - R\cos\theta - r_0\right)$$

$$+ \underbrace{\nu_{n,m}\,(\text{crossing})^{n,m}}_{} + (\text{unitarity})$$

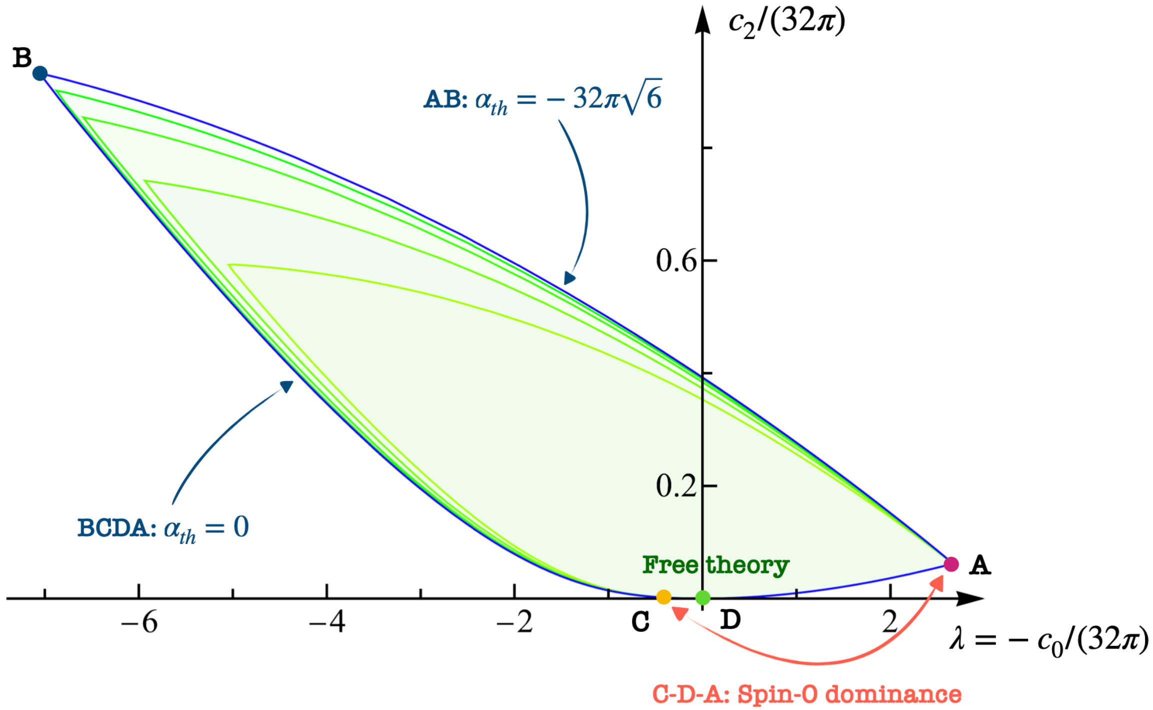
Involves only the imaginary part of the amplitude.

couples real and imaginary fields $\mathrm{Re}\, f_\ell(s)$, $\mathrm{Im}\, f_\ell(s)$

$$\mathcal{L}(\{P\},\{D\}) = R + \alpha_0 (c_0 - R\sin\theta) + \alpha_2 (c_2 - R\cos\theta - r_0)$$

$$+ \, \nu_{n,m} \underbrace{(\text{crossing})^{n,m}}_{\mathcal{L}^{(n,m)}} + (\text{unitarity})$$

Extremal values of (c_0, c_2), singlet theory



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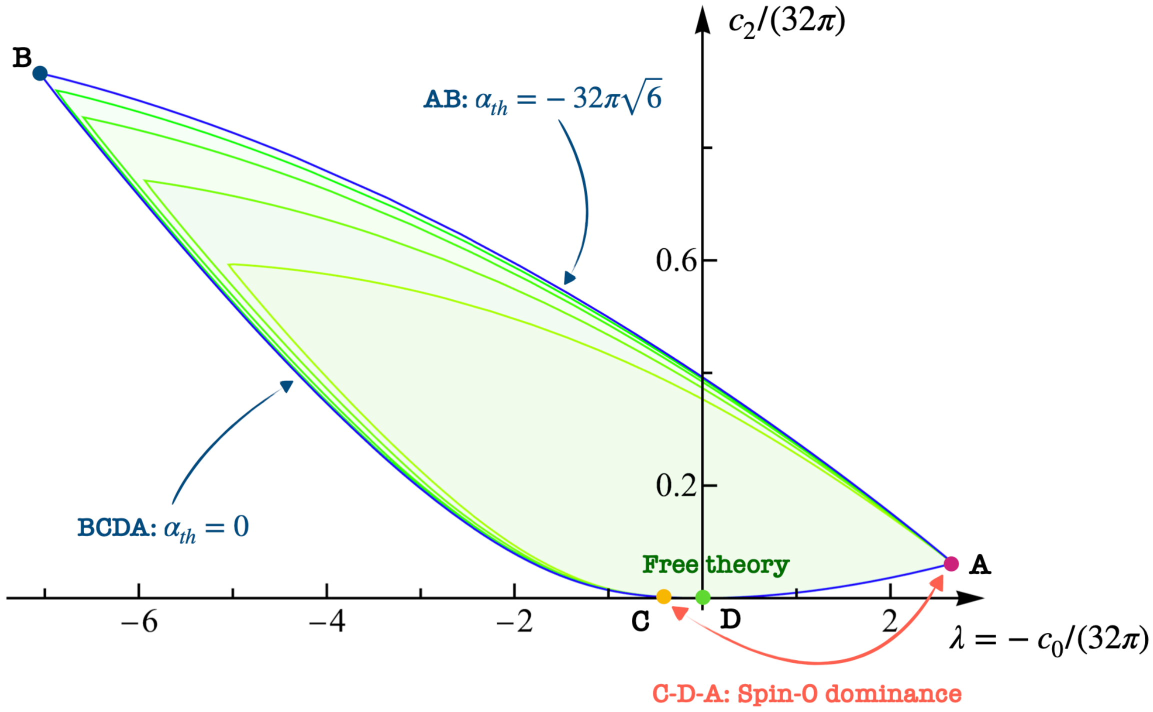
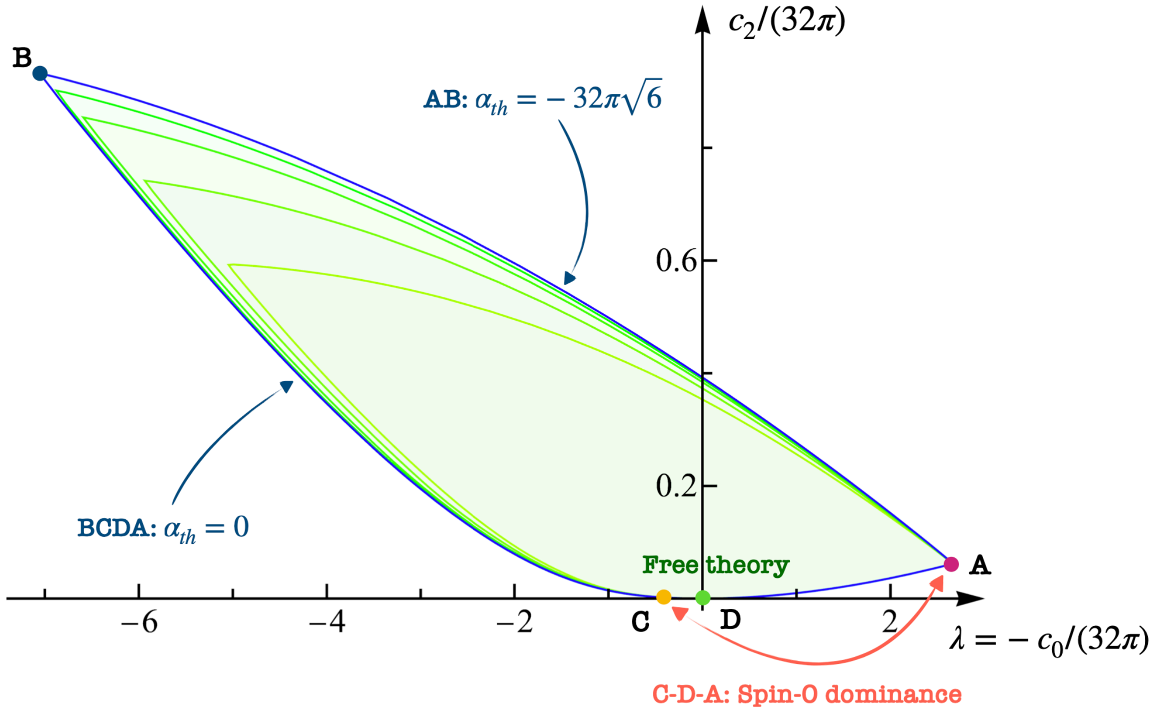


☞ Subtracted positivity constraints

☞ Wavelet-like expansion

Extremal values of (c_0, c_2), singlet theory

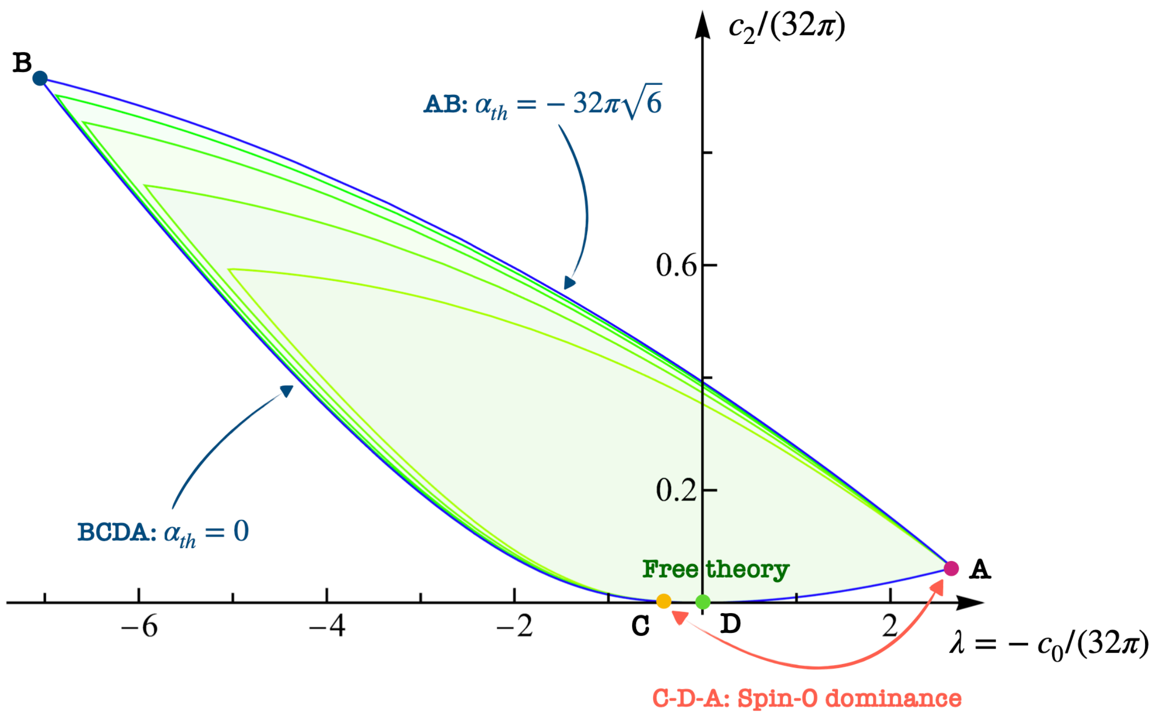


- ☞ Subtracted positivity constraints $\mathrm{Im}(s, 0 \leq t \leq 4m^2) \geq 0$; $\mathrm{Im}\,M(s,t) - 16\pi \sum'_{\ell=0} \mathrm{Im}\, f_\ell(s) P_\ell\left(1 + \frac{2t}{s-4}\right) \geq 0$

- ☞ Wavelet-like expansion $M(s,t) = \alpha_0 + \sum'_{\sigma \in \Sigma} \alpha_\sigma \left[\rho_s(\sigma) + \rho_t(\sigma)\right] + \sum'_{\sigma,\tau \in \Sigma^2} \alpha_{\sigma,\tau}\left(\rho_s(\sigma)\rho_t(\tau) + (s \leftrightarrow t)\right)$

$\Sigma = (4, \infty)$; $\rho_s(\sigma) = (\sqrt{\sigma - 4} - \sqrt{4 - s})/(\sqrt{} + \sqrt{})$

Extremal values of (c_0, c_2), singlet theory



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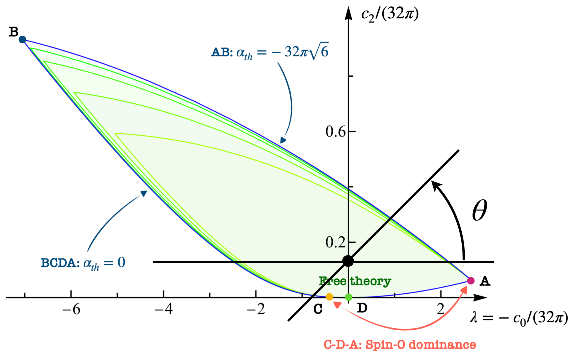
$\Sigma = (4, \infty)$; $\rho_s(\sigma) = (\sqrt{\sigma-4} - \sqrt{4-s})/(\sqrt{} + \sqrt{})$

Extremal values of (c_0, c_2), singlet theory

$$c_2 \sim \int_4^\infty \frac{\mathrm{Im}\, M(s, 4/3)}{(s - 4/3)^3}\, ds$$

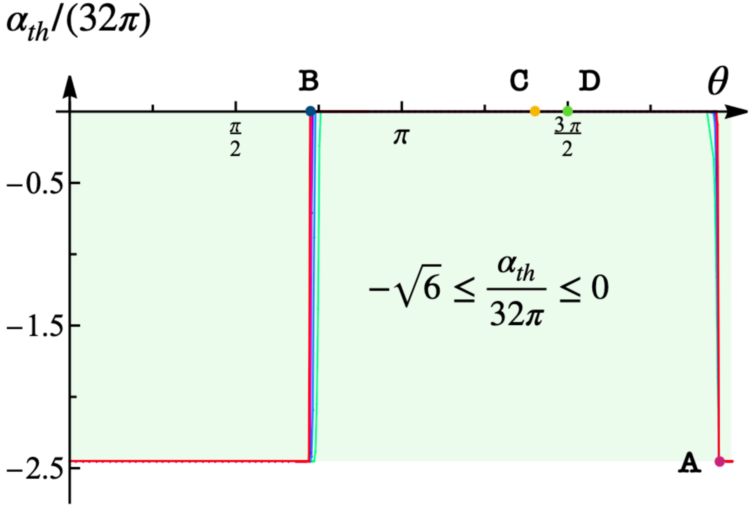
Pole at threshold

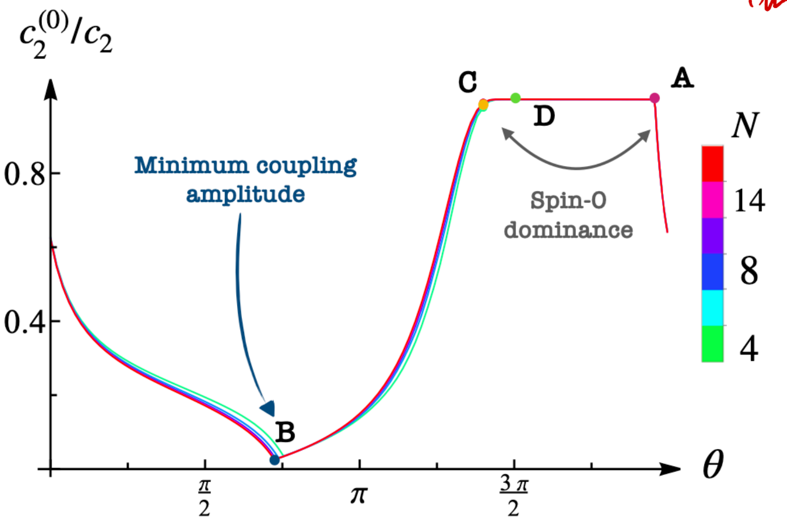


$$M(s,t,u) = \frac{\alpha_{th}}{\rho(s) - 1} + \dots$$

$$c_2 = \sum_{\ell=0}^{\infty} c_2^{(\ell)} \quad \text{w/}$$

$$c_2^{(\ell)} = \frac{m^4}{\pi}(2\ell+1)16\pi \int_{4m^2}^{\infty} \frac{\mathrm{Im} f_\ell(s) P_\ell(\dots)}{(z - \frac{4}{3}m^2)^3}$$





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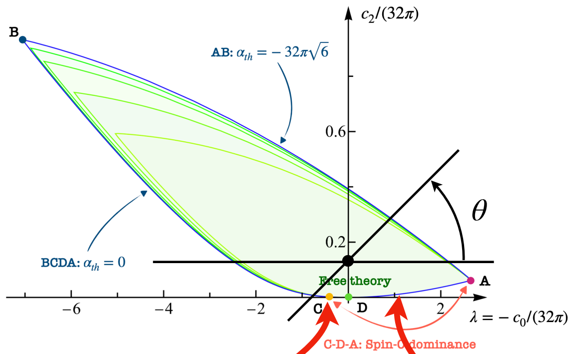
Pole at threshold



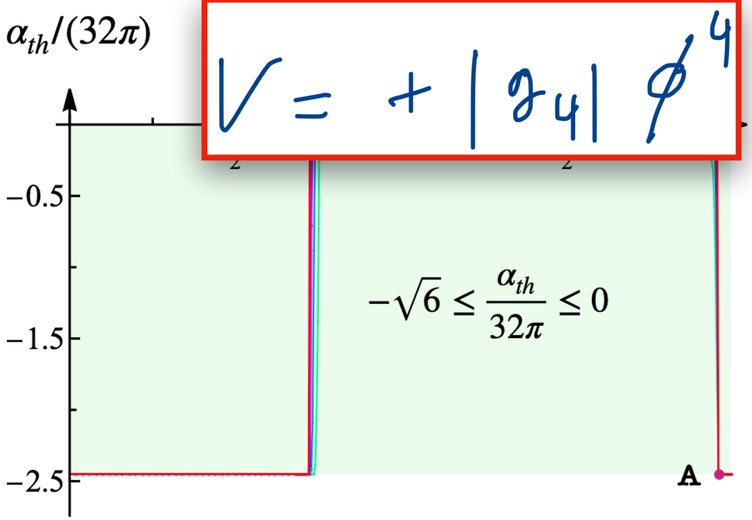
$$M(s,t,u) = \frac{\alpha_{th}}{\rho(s)-1} + \dots$$

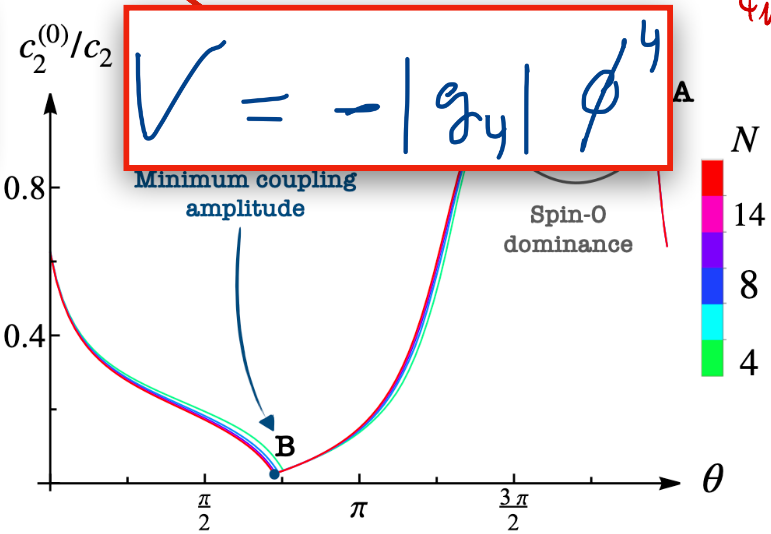
$$c_2 = \sum_{\ell=0}^{\infty} c_2^{(\ell)} \quad w/$$

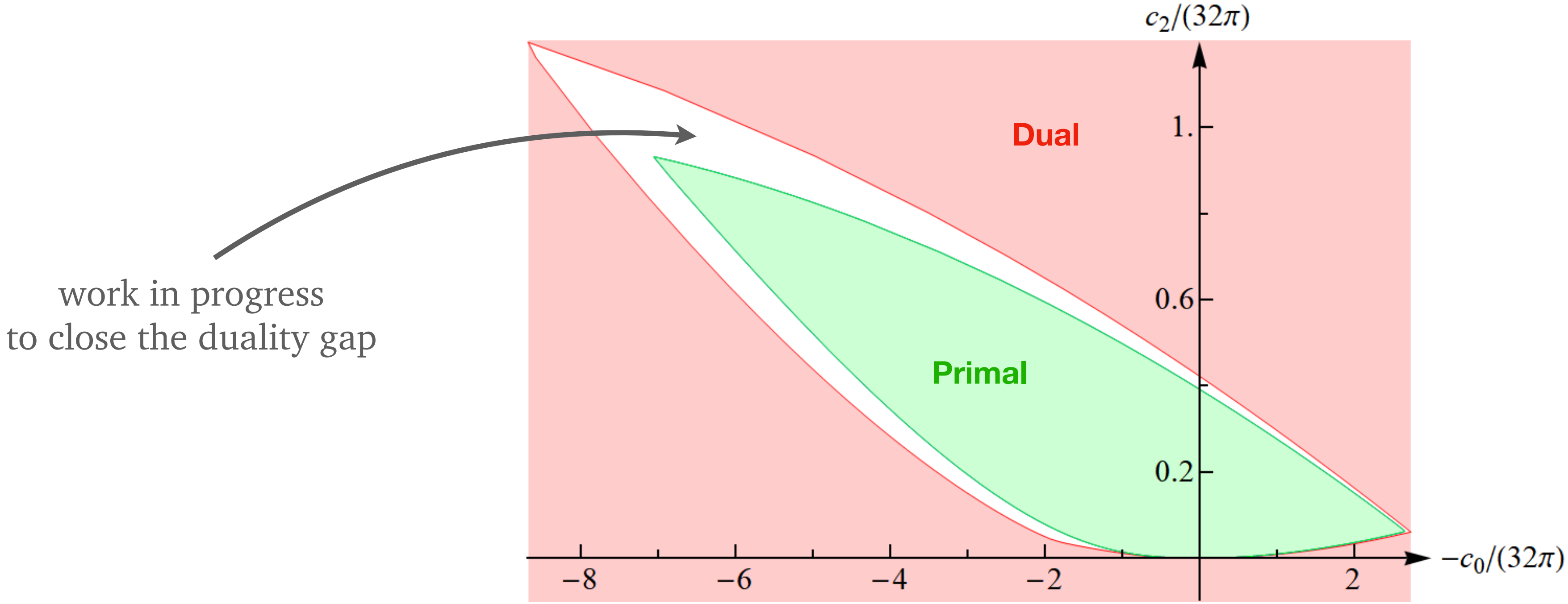
$$c_2^{(\ell)} = \frac{m^4}{\pi}(2\ell+1)16\pi \int_{4m^2} \frac{\mathrm{Im} f_\ell(s) P_\ell(\dots)}{(z-\frac{4}{3}m^2)^3}$$

$$V = +|g_4|\phi^4$$

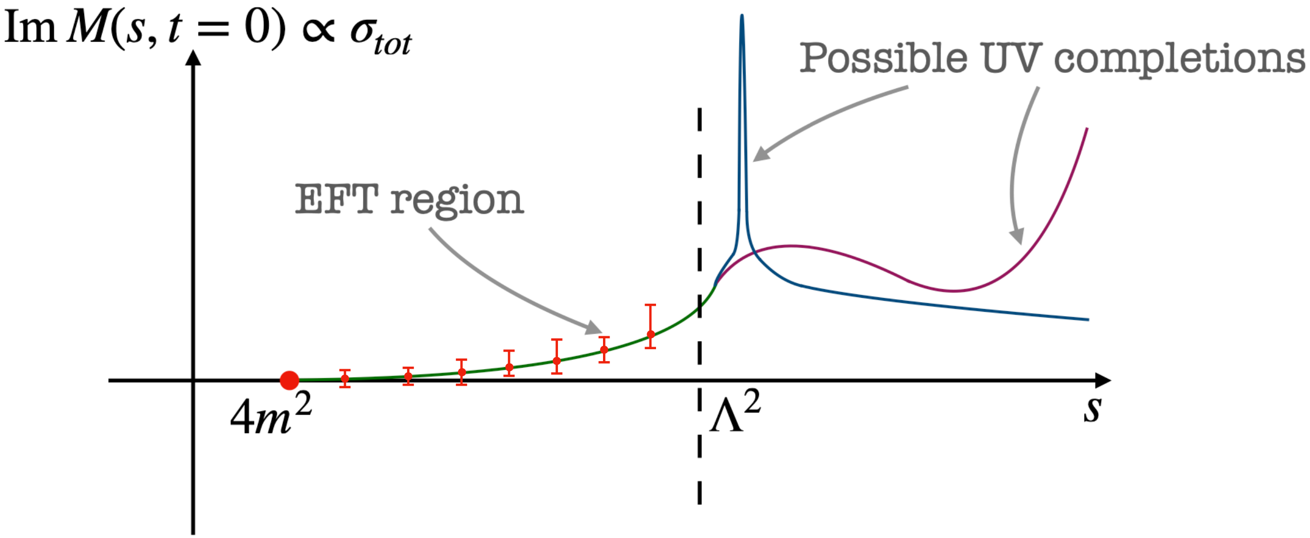
$$V = -|g_4|\phi^4$$



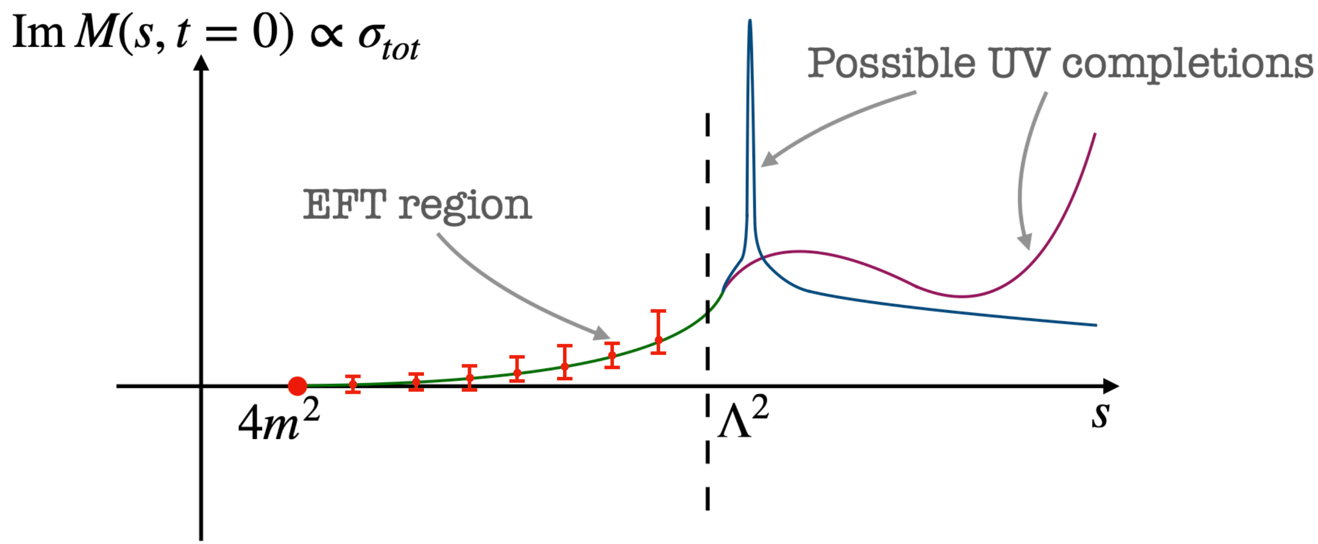



work in progress
to close the duality gap
Dual
Primal
$c_2/(32\pi)$
$-c_0/(32\pi)$
1.
0.6
0.2
−8
−6
−4
−2
2

The space of EFT amplitudes



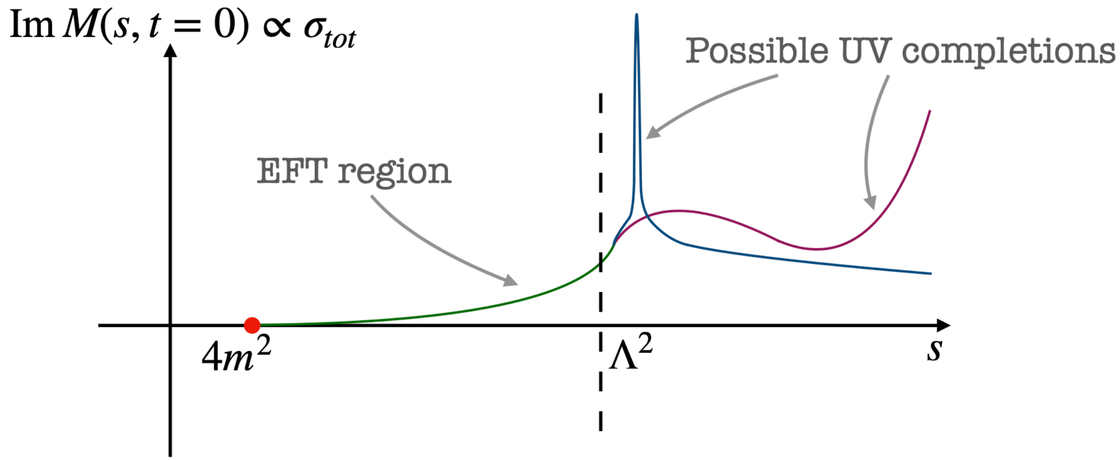
The space of EFT amplitudes



$$M(s,t) = M^{EFT}(s,t) \longrightarrow \operatorname{Im} M(s,t) = \operatorname{Im} M^{EFT}(s,t)$$

$$\int \operatorname{Im} M(s,t) \leq \int \operatorname{Im} M^{EFT}(s,t) \longleftarrow \operatorname{Im} M(s,t) \leq \operatorname{Im} M^{EFT}(s,t)$$

The space of EFT amplitudes



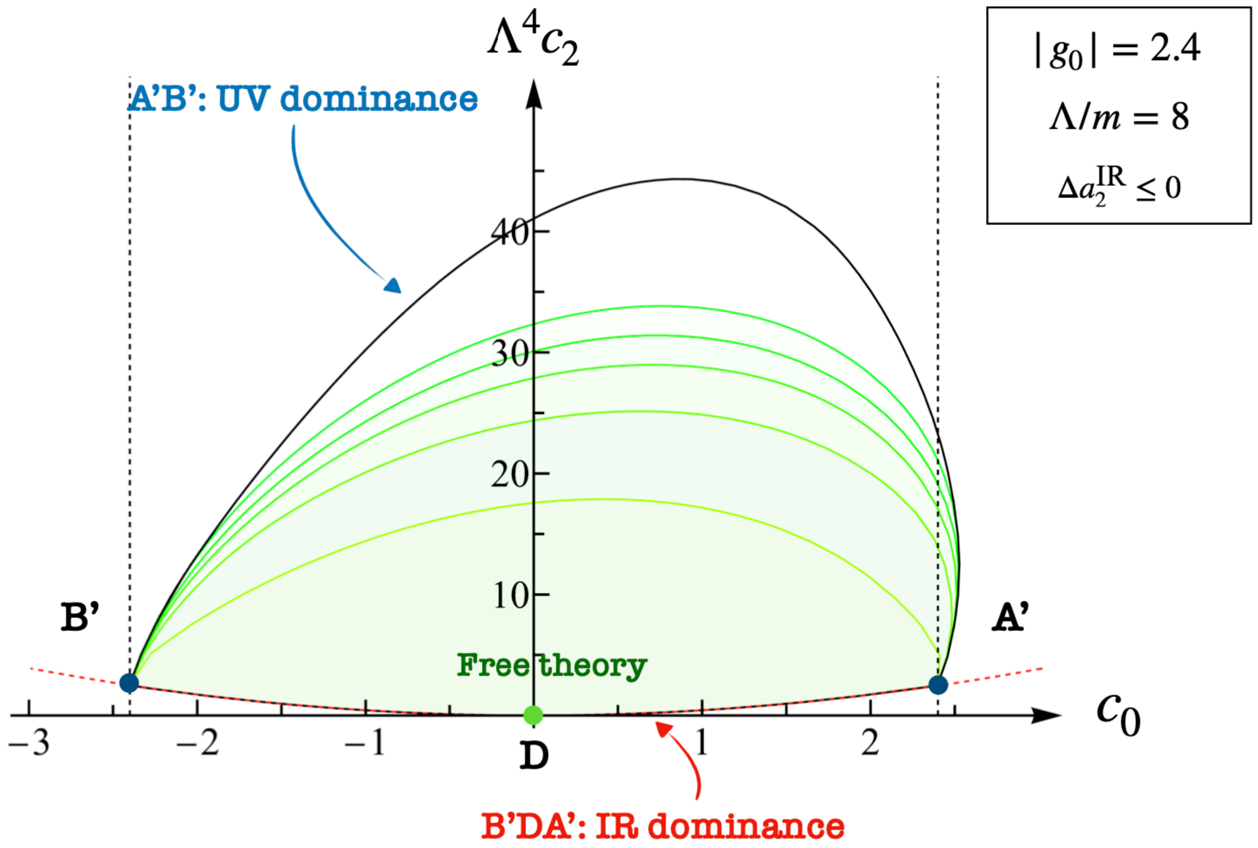
Two methods:

M1: Impose $$\Delta_n a^{\mathrm{IR}} \equiv \int_{4m^2}^{\Lambda^2} \frac{M_z^{\mathrm{ans}}(z,4/3) - M_z^{\mathrm{EFT}}(z,4/3)}{(z-4/3)^{n+1}} dz \leq 0$$

with $M_z^{\mathrm{EFT}}(z,t) = \frac{\sqrt{s-4}}{\sqrt{s}} \frac{g_0^2}{32\pi} + \ldots$

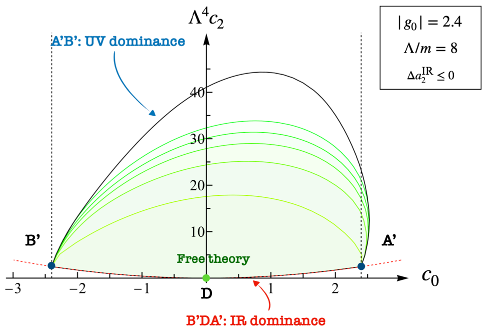
M2: Point wise constraints $\mathrm{Im} f_\ell(s) \leq \mathrm{Im} f_\ell^{\mathrm{EFT}}(s)$ for $s \leq \Lambda^2$.

Extremal values of (c_0, c_2), singlet **EFT** theory



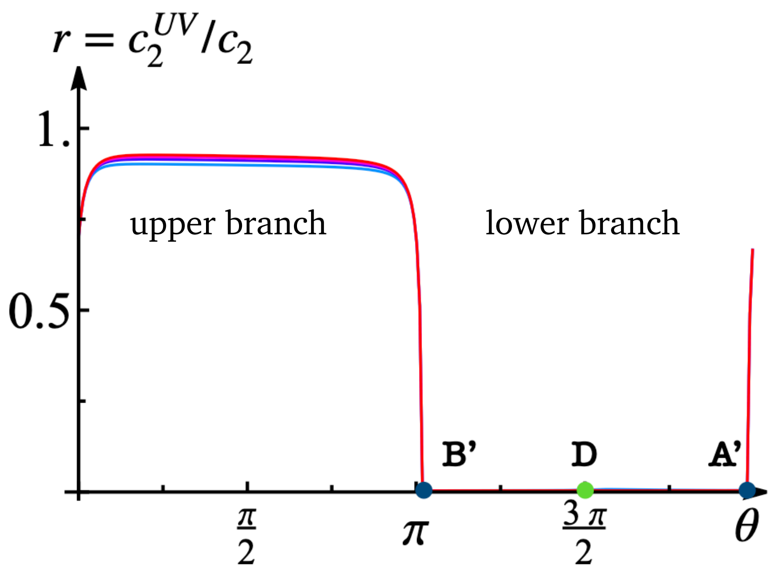
$$r \equiv \frac{c_2^{UV}}{c_2}$$

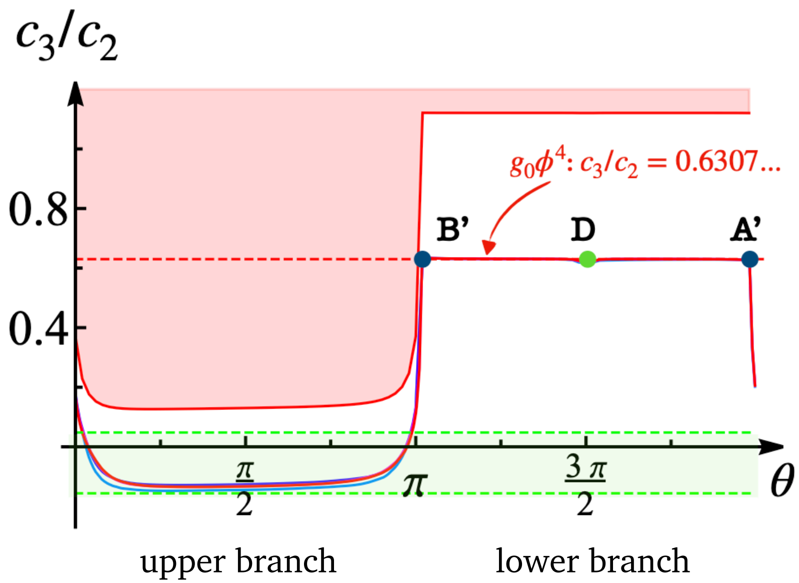
$$c_2^{UV} = \frac{1}{\pi}\int_{\Lambda^2}^{\infty} \frac{\operatorname{Im} M(s,t=4/3)}{(s-4/3)^3}\, ds$$



$$c_3 = \frac{1}{\pi}\int_4^{\infty} dz \left(3\,\frac{M_z(z,\frac{4}{3})}{(z-\frac{4}{3})^4} - 2\,\frac{\partial_t M_z(z,t)\big|_{t=4/3}}{(z-4/3)^3}\right)$$

$$< \frac{1}{\pi}\int_4^{\infty} dz\, \frac{3M_z(z,\frac{4}{3})}{(z-4/3)^4} \leq \left(\frac{9}{8}(1-r) + \frac{3}{\Lambda^2-4/3}\, r\right) c_2$$

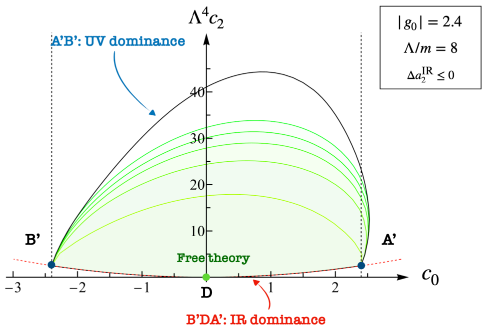


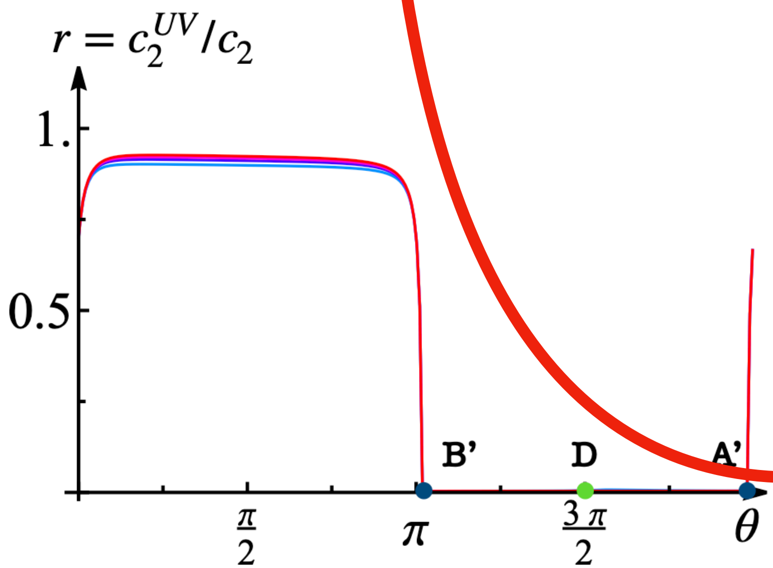


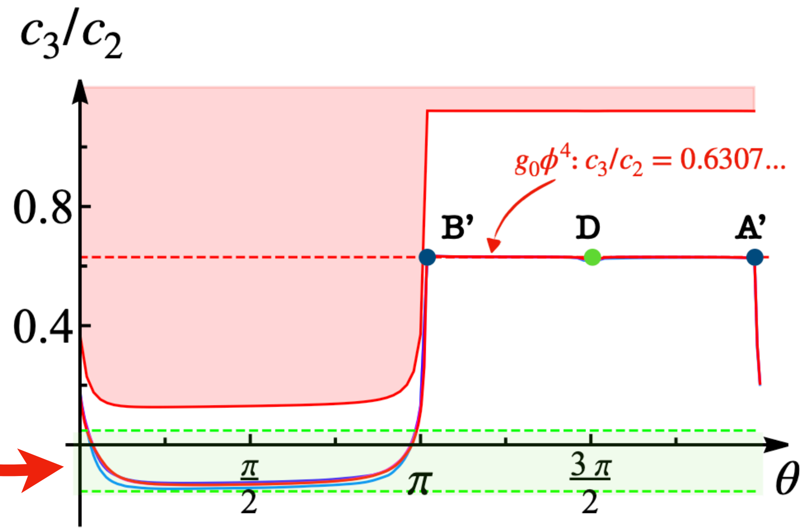
UV domination:

IR cuts can be neglected:

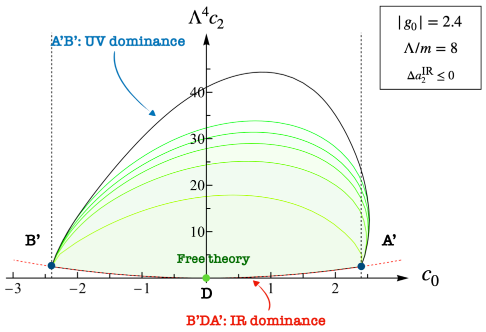
$$-0.1564 \leq c_3/c_2 \leq 9/188 \simeq 0.0479$$



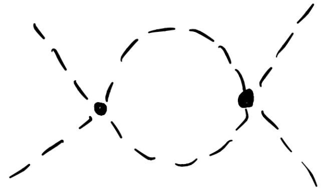




[Caron-Huot, Van Duong '21], …, [Haldar, Sinha, Zahed '21],…

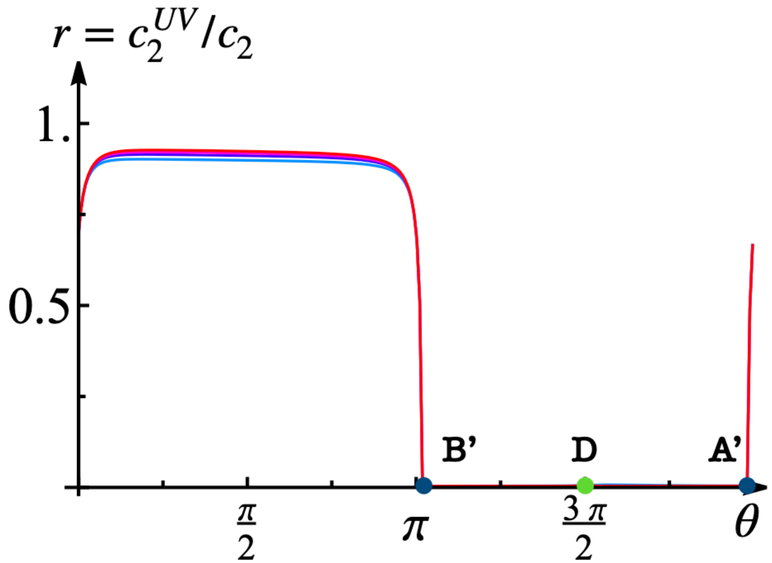


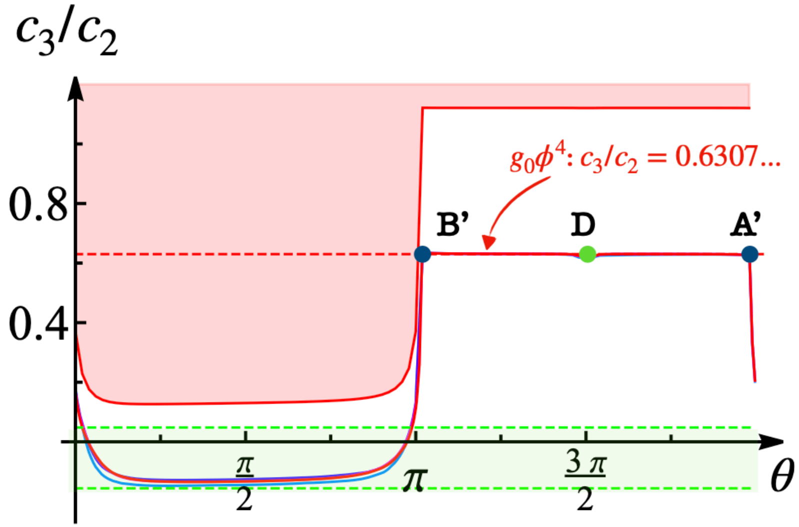
IR domination:

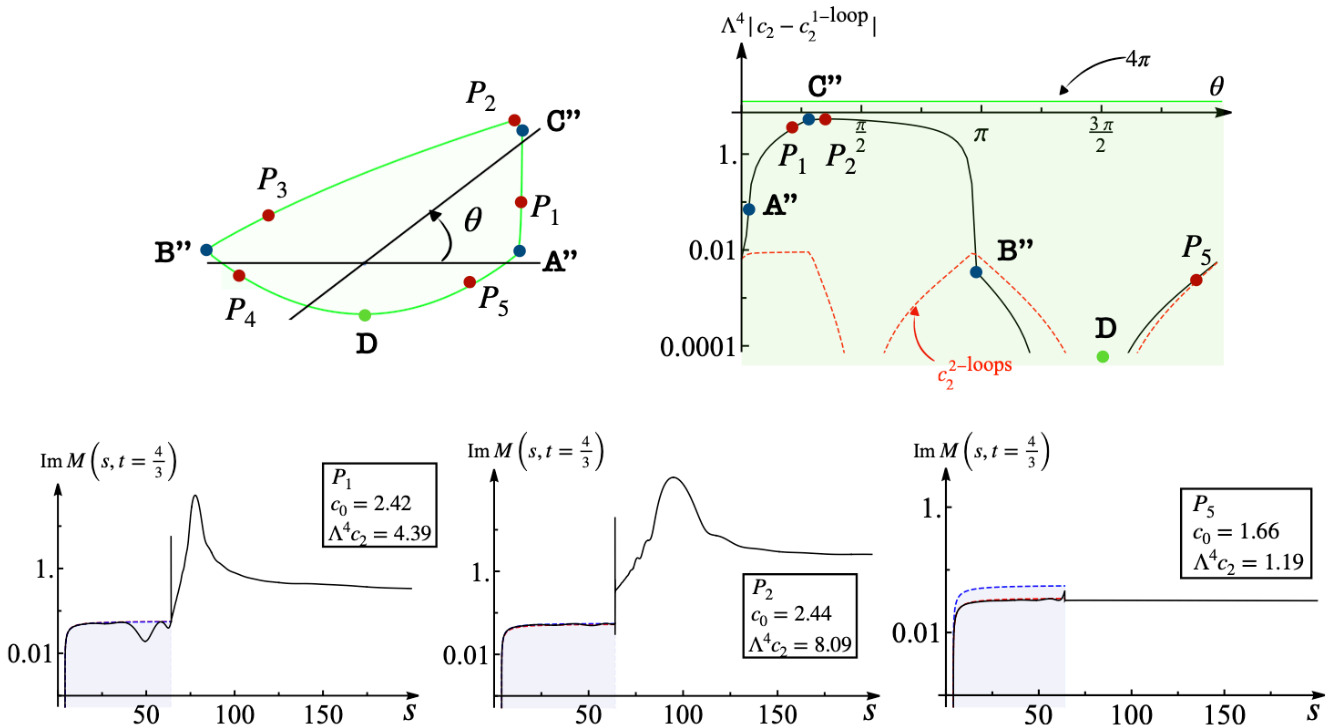


$$= -c_0^{(1)} + (\bar{s}^2+\bar{t}^2+\bar{u}^2)\,c_2^{(1)} + \bar{s}\,\bar{t}\,\bar{u}\,c_3^{(1)}$$

$$\frac{c_3^{(1)}}{c_2^{(1)}} = \frac{390 - 459\sqrt{2}\cot^{-1}\sqrt{2}}{-224 + 240\sqrt{2}\cot^{-1}\sqrt{2}} \simeq 0.6307$$



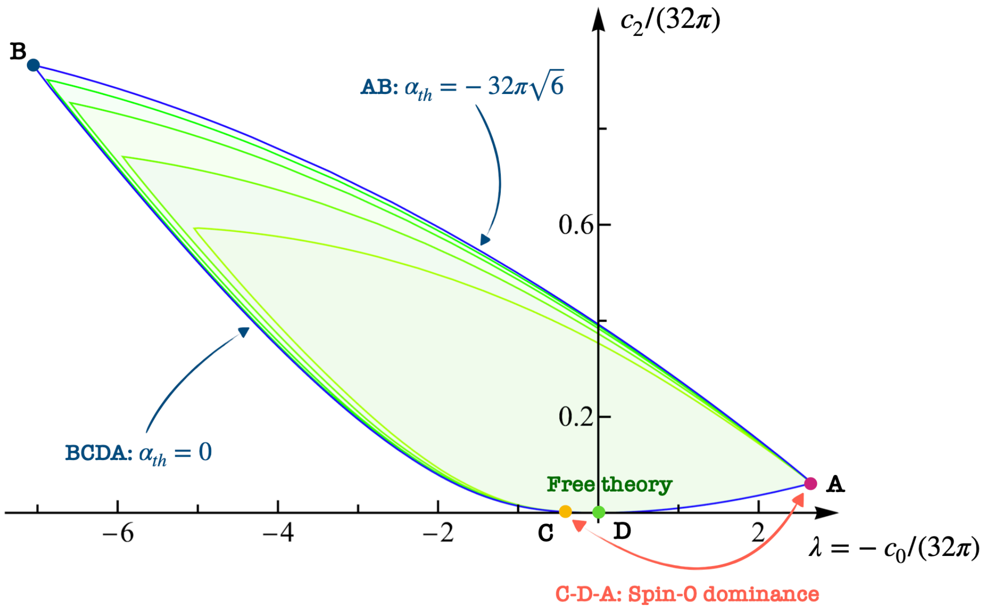




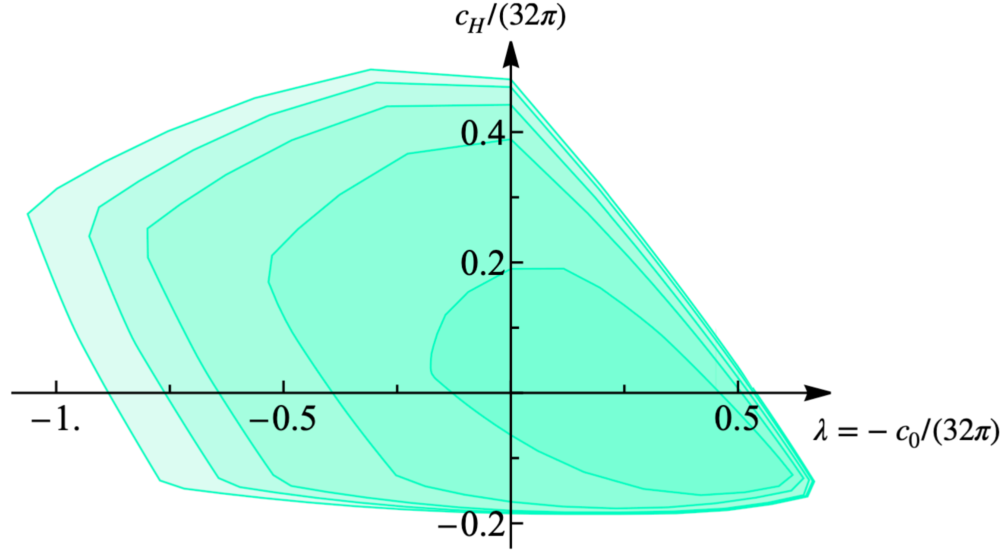
P_2
C''
P_3
θ
P_1
B''
A''
P_4
P_5
D
$\Lambda^4 |c_2 - c_2^{1\text{-loop}}|$
4π
θ
C''
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
1.
P_1 P_2
A''
0.01
B''
P_5
D
0.0001
$c_2^{2\text{-loops}}$
$\mathrm{Im}\, M\left(s, t = \frac{4}{3}\right)$
P_1
$c_0 = 2.42$
$\Lambda^4 c_2 = 4.39$
1.
0.01
50 100 150 s
$\mathrm{Im}\, M\left(s, t = \frac{4}{3}\right)$
1.
0.01
P_2
$c_0 = 2.44$
$\Lambda^4 c_2 = 8.09$
50 100 150 s
$\mathrm{Im}\, M\left(s, t = \frac{4}{3}\right)$
1.
P_5
$c_0 = 1.66$
$\Lambda^4 c_2 = 1.19$
0.01
50 100 150 s

The space of amplitudes

singlet theory



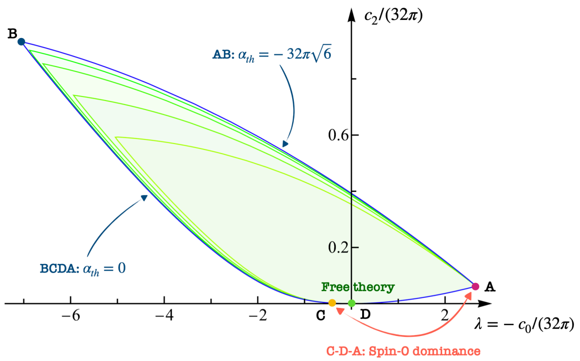
$O(n=4)$ theory

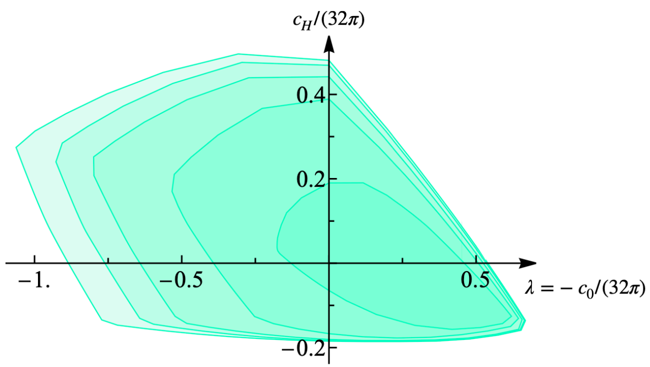


Cutoff dependence

$\Lambda^2 \to 4m^2$:

Strongly coupled all the way to the IR





$\Lambda^2/m^2 \to \infty$:

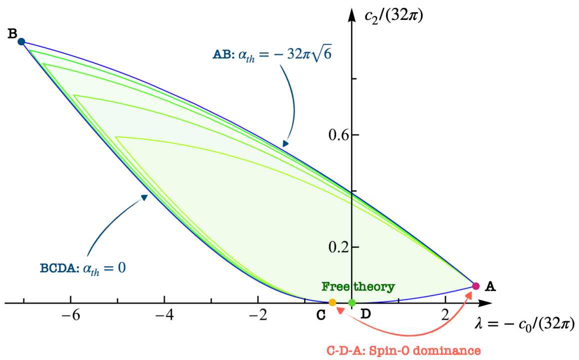
☞ If $\Lambda^2 \to \infty$ with m^2 held fixed and $g_0 \lesssim O(1)$, **single scale problem:** m^2
i.e. exact IR domination

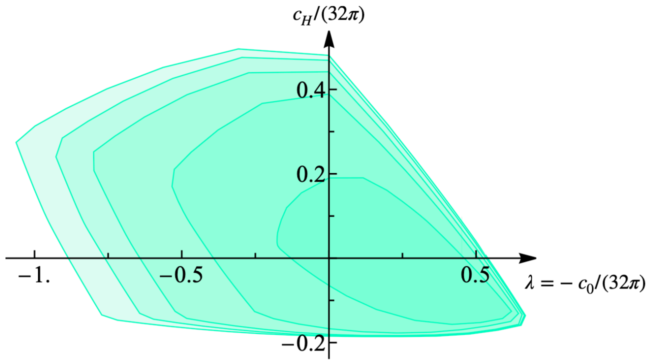
☞ If $g_0 \to 0$ first and then $m^2 \to 0$, **single scale problem:** Λ^2
i.e. exact UV domination

Cutoff dependence

$\underline{\Lambda^2 \to 4m^2:}$

Strongly coupled all the way to the IR





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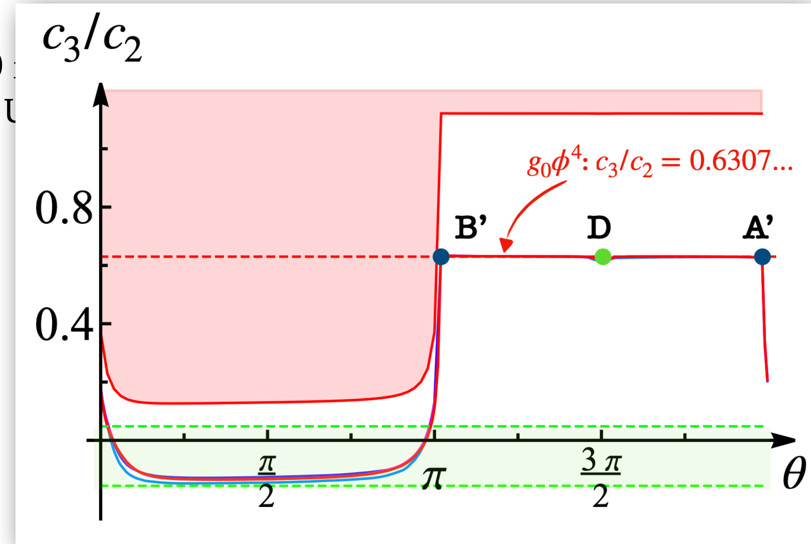
i.e. exact IR domination

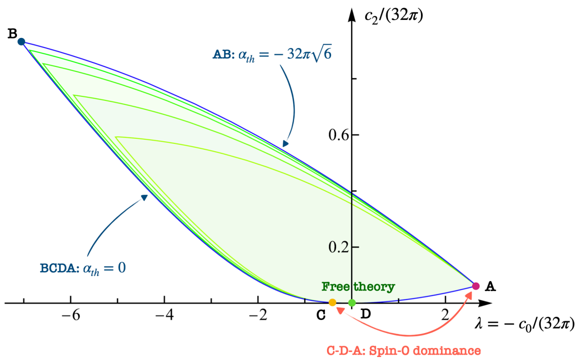
If $g_0 \to 0$
i.e. exact U

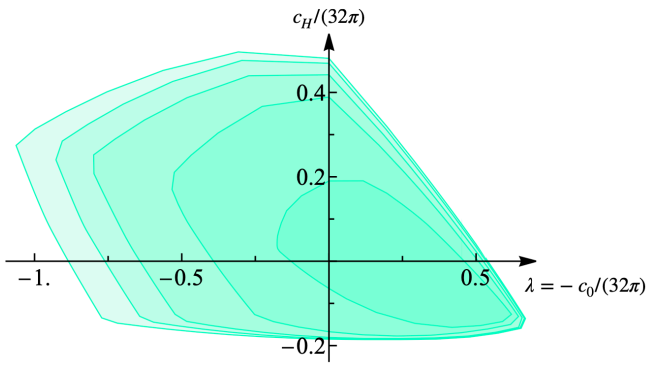


Cutoff dependence

$\Lambda^2 \to 4m^2$:

Strongly coupled all the way to the IR





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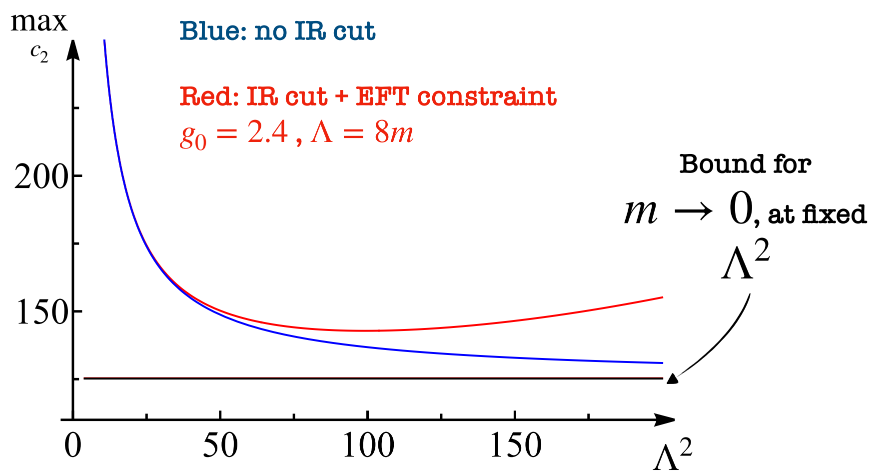
If $g_0 \to 0$ first (and then $m^2 \to 0$), **single scale problem:** Λ^2
i.e. exact UV domination



The construction that I presented allows to ask precise questions for EFTs that, on one hand feature non-negligible IR physics, but on the other hand are maximally strongly coupled at energies above a physical cutoff Λ.

Conclusions:

- I showed how to isolate EFT amplitudes in the space of non-perturbative amplitudes.

- Discussed the interplay with positivity (see paper for further analyses).

- Discussed interesting observables along the boundary, such as UV/IR domainance and the spin dominance.

- Construction allows to set bounds on operators of dimensions less than or equal to six.

- Presented novel bounds on 4D $O(n)$ amplitudes.

Outlook:

- Characterise further the physics of the extremal amplitudes, in particular for the $O(n)$ theory (more to appear soon, w/ A.Guerrieri and M.Gümüş)

- Close the gap between dual and primal.

- Make a more realistic model (turn on gauge couplings, etc.), and then

- take input data from experiment to constraint the low energy EFT.

- Here I asked what is perhaps the simplest pheno question, the answer to many more interesting pheno questions will become available as we develop further this strategy.