

Muon $g-2$ and lepton flavor violation in SUSY-GUT theories

Mario E. Gómez

Departamento de Ciencias Integradas

Universidad de Huelva Spain



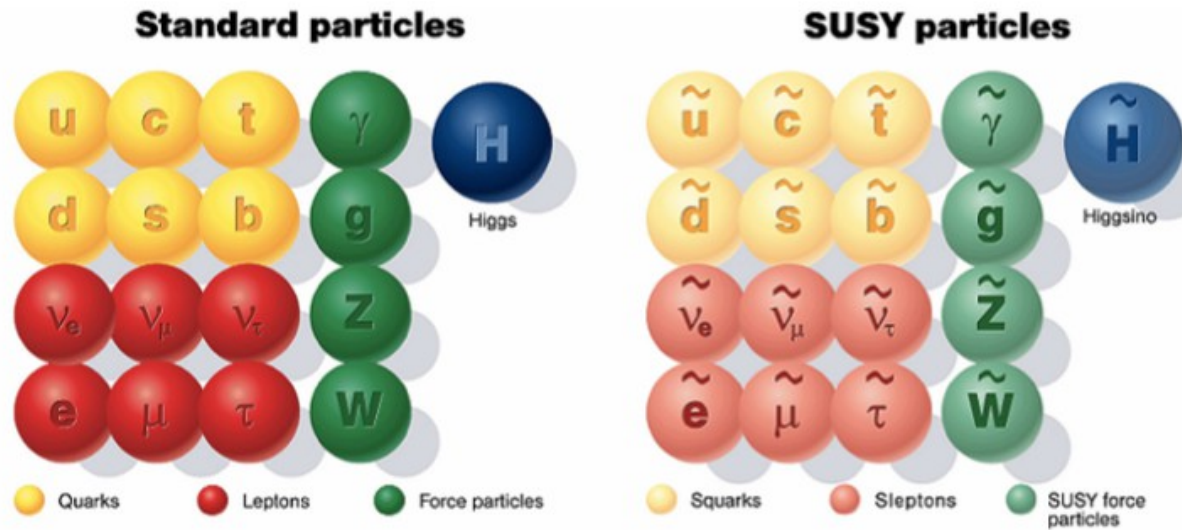
Universidad de Huelva

Colaboration with A. Tiwari, Q. Shafi, C. Ün, J. Ellis, S. Lola, Ruiz de Austri, JHEP 07 (2020) 07, 096 JHEP 09 (2020), Eur. Phys. J. C (2022) 82:561 + work in progress.

OUTLINE

- GUT's and SUSY.
- PS: $SU(4) \times SU(2) \times SU(2)$
- Fitting muon $(g-2)$ in $SU(4) \times SU(2) \times SU(2)$ models
- *Neutralino relic density and DM detection.*
- *LFV with a See-Saw mechanism.*

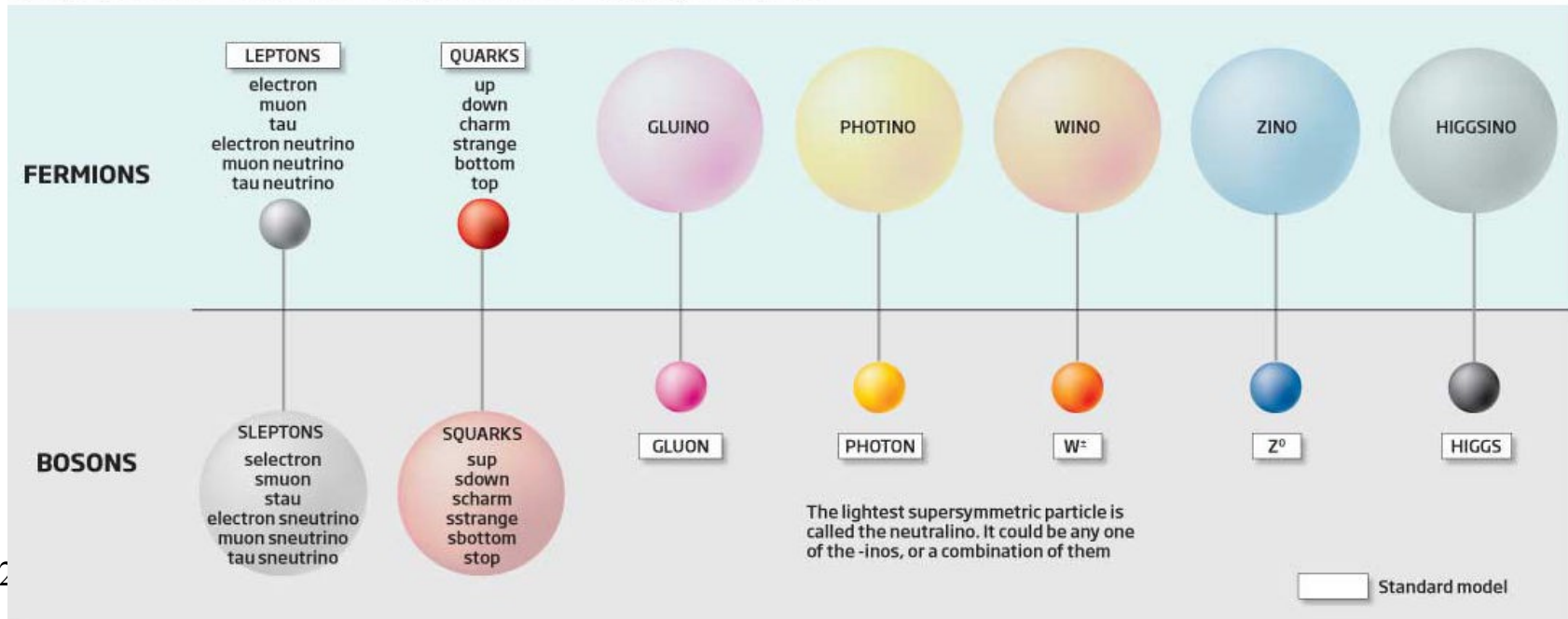
Conclusions.



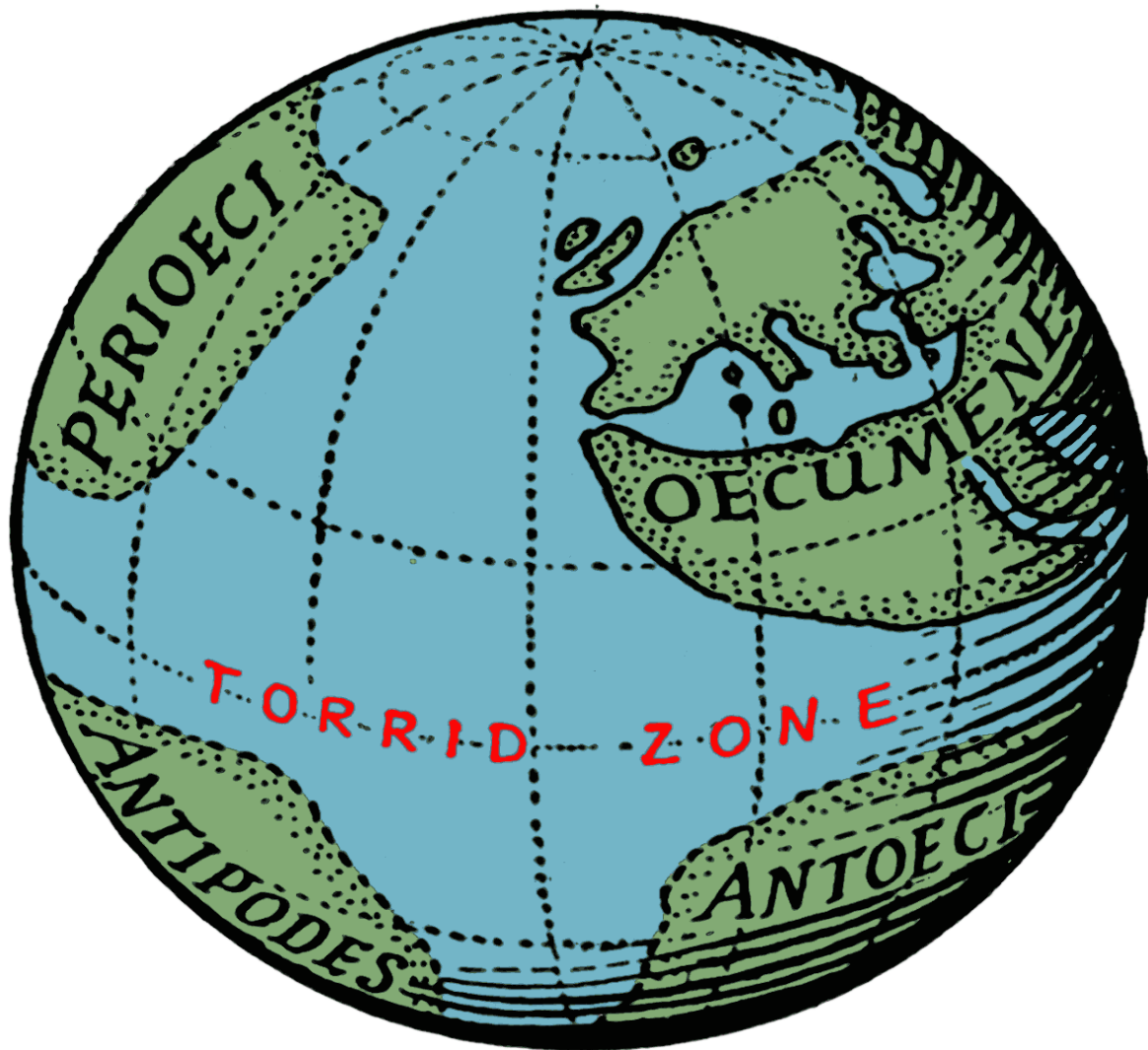
Particle zoo

©NewScientist

Particles are divided into two families called bosons and fermions. Among them are groups known as leptons, quarks and force-carrying particles like the photon. Supersymmetry doubles the number of particles, giving each fermion a massive boson as a super-partner and vice versa. The LHC is expected to find the first supersymmetric particle



Crates of Mallus 150 B.C.



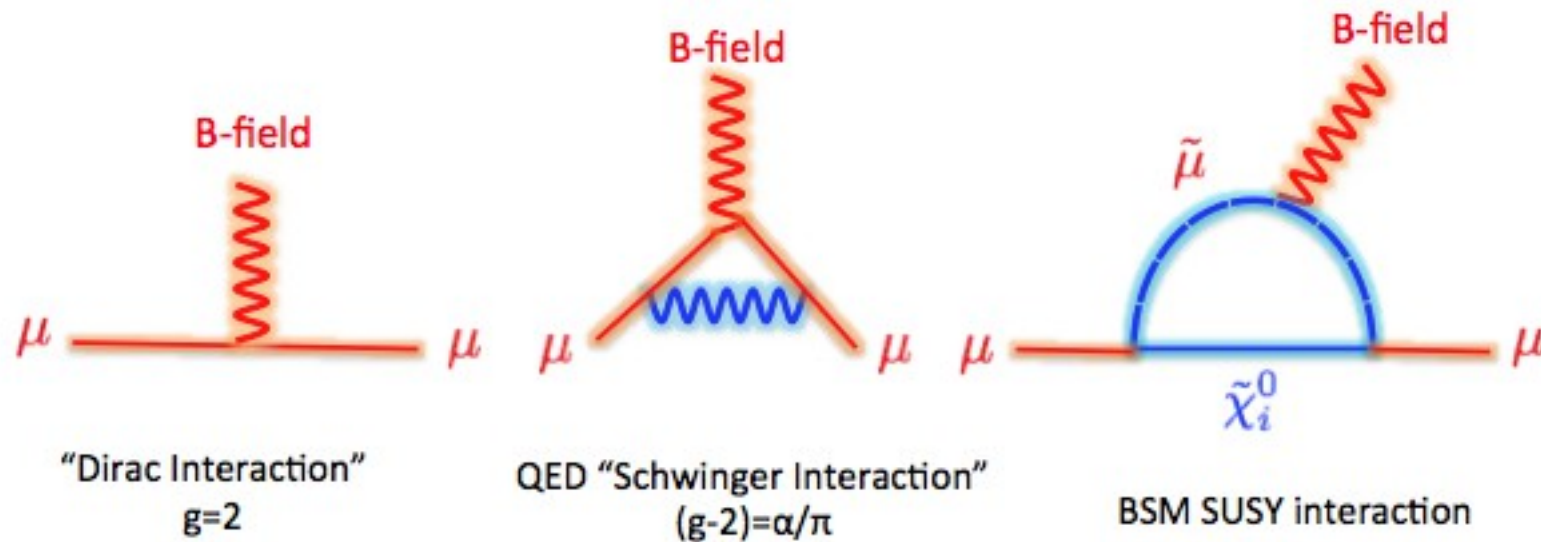
Antipodes theory:

Popular debate in the Middle age.

Probably referred in all explation trip proposals until it was eliminated by direct observation.

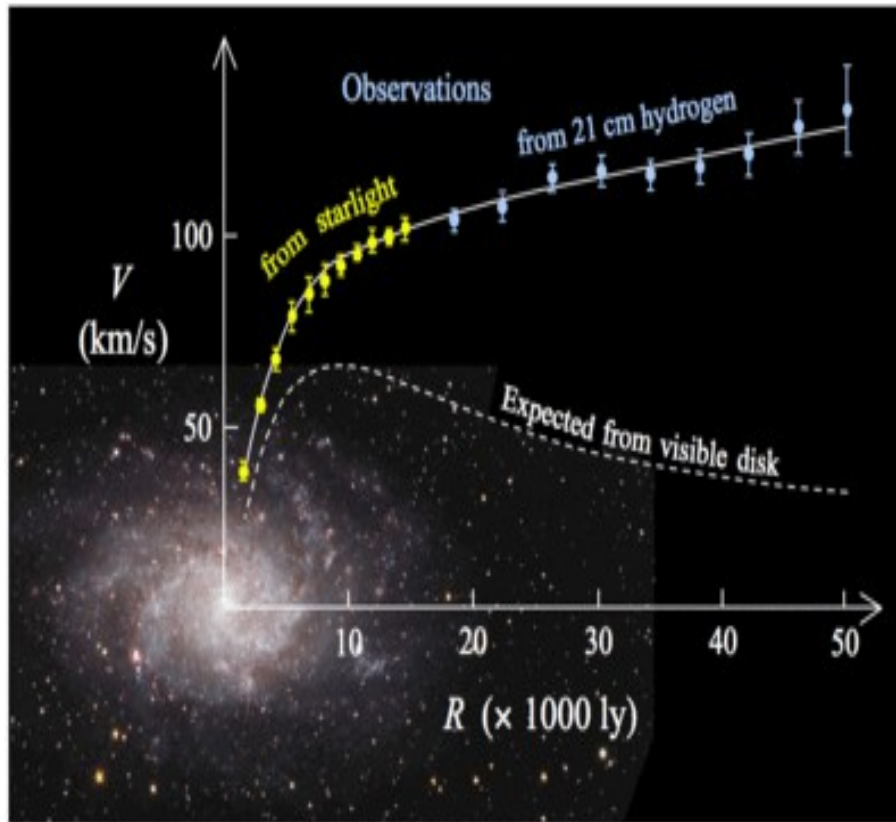


One loop contribution to SM proc.

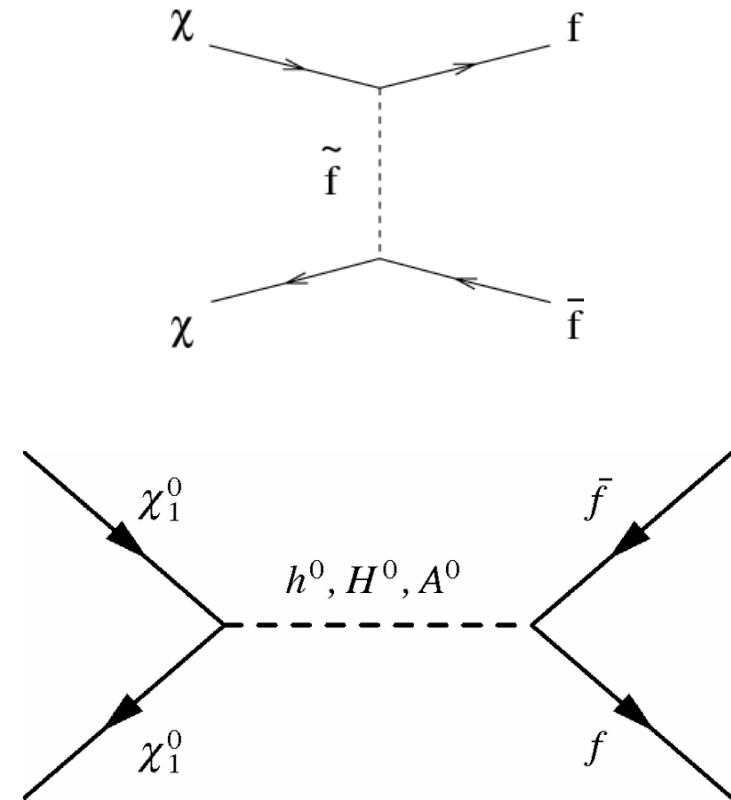


SM vs Experiment Discrepancies
anomalous magnetic dipole moment ($g_\mu - 2$)

Dark Matter problem

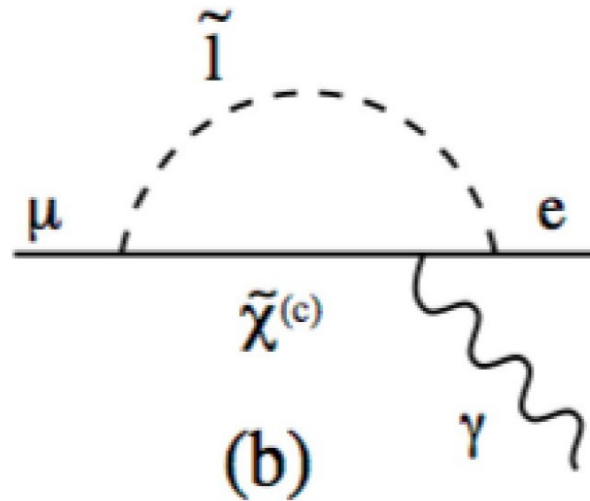
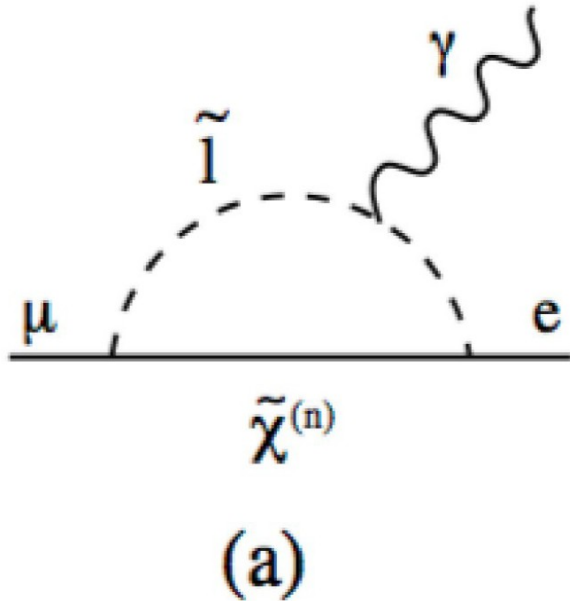


Rotation curve of spiral galaxy M 33 (yellow and blue points with error bars), and a predicted one from distribution of the visible matter (white line). The discrepancy between the two curves can be accounted for by adding a dark matter halo surrounding the galaxy



$$\Omega h^2 \sim \frac{3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}}{\langle \sigma_{\text{ann}} v_{\text{Mol}} \rangle}$$

In SUSY flavor mixing lepton-slepton vertices can induce LFV diagrams:



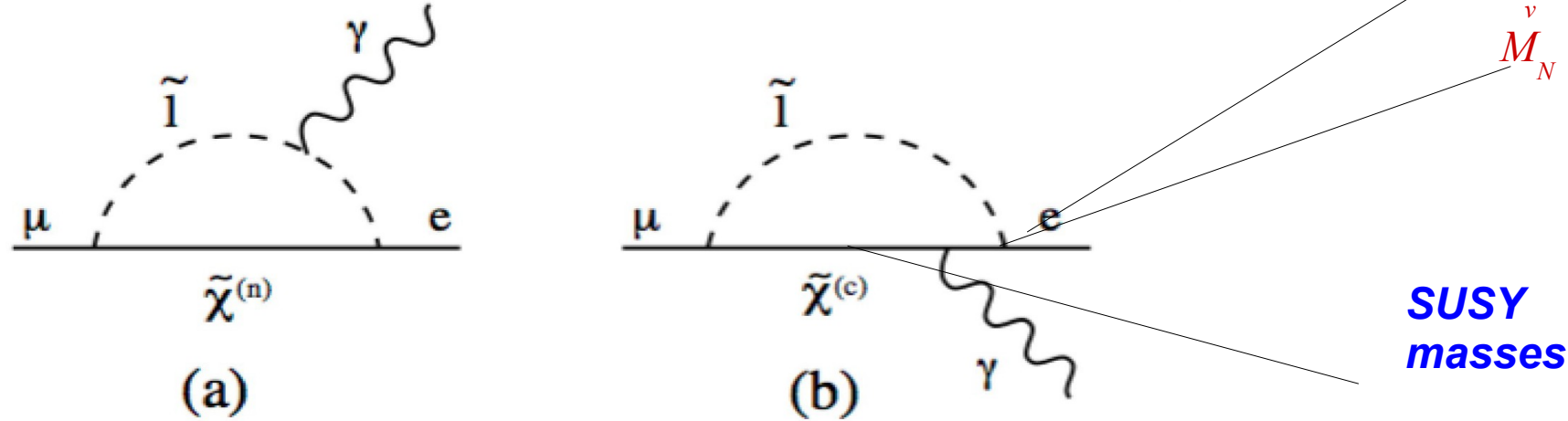
Lepton-slepton flavor mixing is very constrained by the experimental limits:

$$BR(\mu \rightarrow e \gamma) < 5.7 \times 10^{-13}$$

$$BR(\tau \rightarrow \mu \gamma) < 4.4 \times 10^{-8}$$

$$BR(\tau \rightarrow e \gamma) < 3.3 \times 10^{-8}$$

In SUSY flavor mixing lepton-slepton vertices can induce LFV diagrams:



Lepton-slepton flavor mixing is very constrained by the experimental limits:

$$BR(\mu \rightarrow e \gamma) < 4.2 \times 10^{-13}$$

$$BR(\tau \rightarrow \mu \gamma) < 4.4 \times 10^{-8}$$

$$BR(\tau \rightarrow e \gamma) < 3.3 \times 10^{-8}$$

Soft SUSY Breaking Terms

The soft SUSY breaking masses

$$\begin{aligned}
 -\mathcal{L}_{\text{soft}} = & -\frac{1}{2} \left(M_3 \lambda_g^a \lambda_g^a + M_2 \lambda_{\tilde{W}}^i \lambda_{\tilde{W}}^i + M_1 \lambda_{\tilde{B}} \lambda_{\tilde{B}} + \text{h.c.} \right) \\
 & + M_L^2 \tilde{L}^\dagger \tilde{L} + M_Q^2 \tilde{Q}^\dagger \tilde{Q} + M_U^2 \tilde{U}^* \tilde{U} + M_D^2 \tilde{D}^* \tilde{D} + M_E^2 \tilde{E}^* \tilde{E} + \\
 & m_{H_d}^2 \tilde{H}_d^\dagger \tilde{H}_d + m_{H_u}^2 H_u^\dagger H_u - \left(B\mu \tilde{H}_d^T H_u + \text{h.c.} \right) \\
 & + \left(y_\ell A_\ell H_d^\dagger \tilde{L} \tilde{E} + y_d A_d H_d^\dagger \tilde{Q} \tilde{D} - y_u A_u H_u^T \tilde{Q} \tilde{U} + \text{h.c.} \right),
 \end{aligned}$$

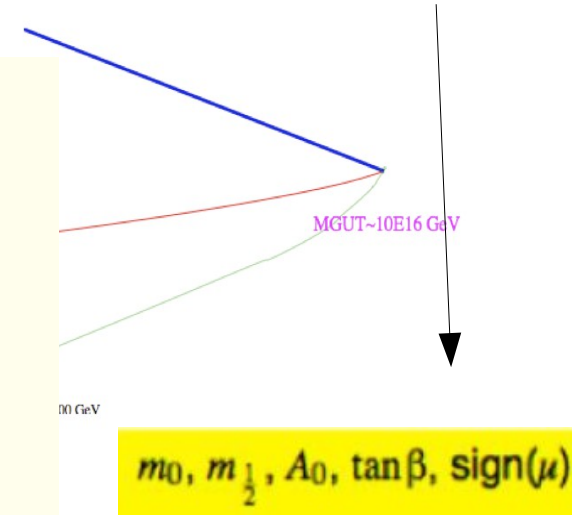
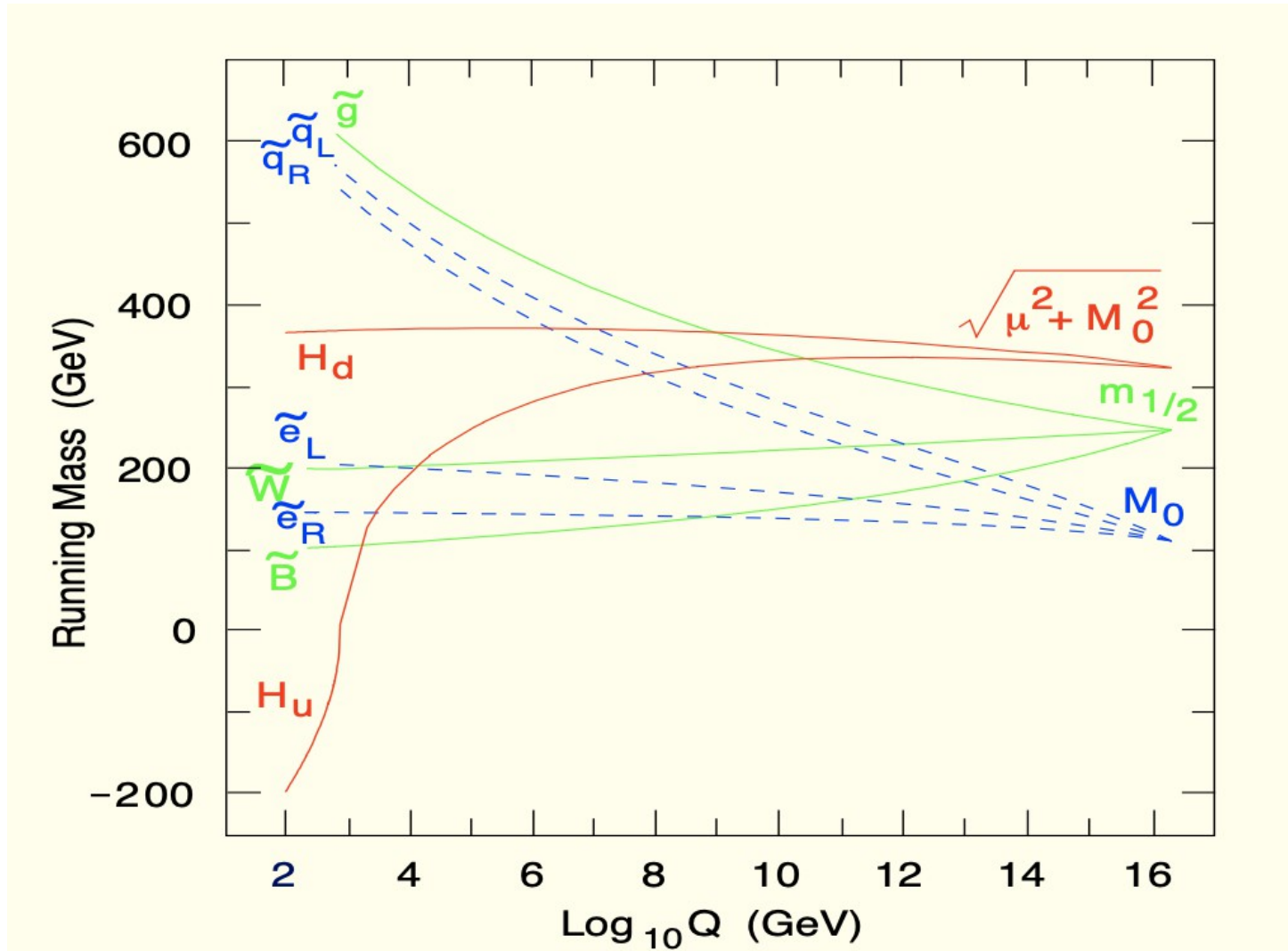
Inspired from supergravity assume universal soft breaking, $\mathcal{L}_{\text{soft}}$:

$$\sum_{f,H} m_0^2 \tilde{f} \tilde{f} + \sum_{\lambda} m_{\frac{1}{2}} \lambda \lambda + \sum_f A_0 Y_f \tilde{f} \tilde{F} H_f + B\mu H_u H_d$$

$$m_0, m_{\frac{1}{2}}, A_0, \tan\beta, \text{sign}(\mu)$$

μ and A_0 can be complex, however their phases constraint to be $< 0,2$ rad by the bounds c the fermion EDM.

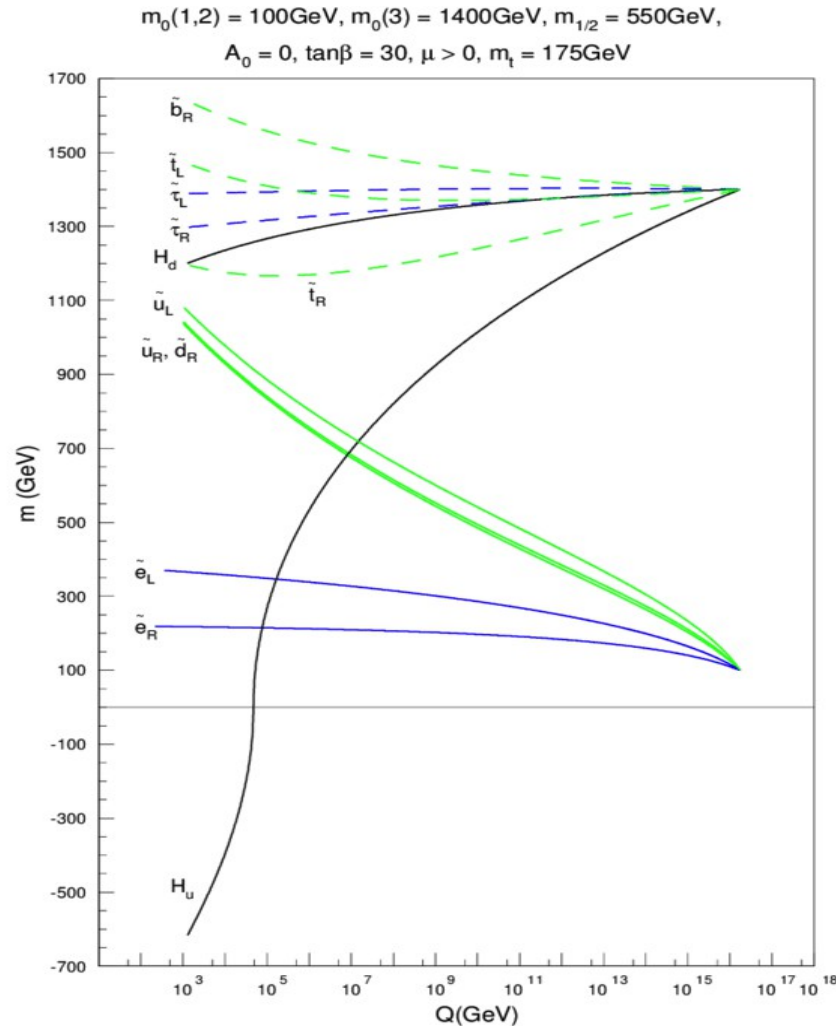
GUT initial conditions



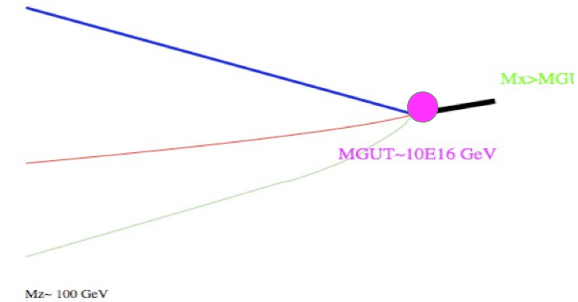
$m_0, m_{1/2}, A_0, \tan\beta, \text{sign}(\mu)$

Gunion,
Int. J. Mod. Phys. A 2010

Non Universal scenarios



Baer et al., HEP 06 (2004) 044



CMSSM choice:

- m_0 Universal soft masses.
- $m_{1/2}$ Universal gaugino masses.
- A_0 Universal Trilinear terms.

Representation-dependent choice

$$m_r = x_r m_0$$

$$A_r = Y_r A_0, \quad A_0 = a_0 m_0$$

PATI-SALAM Unification

$$G_{PS} \equiv SU(4) \times SU(2)_L \times SU(2)_R$$

$4_c 2_L 2_R$

MATTER FIELDS

$$F_r \quad \begin{pmatrix} d_r & -u_r \\ e_r & -\nu_r \end{pmatrix} \quad (4, 2, 1)$$

$$F_r^c \quad \begin{pmatrix} u_r^c \\ d_r^c \end{pmatrix}, \begin{pmatrix} \nu_r^c \\ e_r^c \end{pmatrix} \quad (\bar{4}, 1, 2)$$

$$\langle \tilde{\nu}_H^c \rangle = \langle \tilde{\nu}_H^c \rangle \sim M$$

HIGGS FIELDS

$$H^c \quad \begin{pmatrix} u_H^c \\ d_H^c \end{pmatrix}, \begin{pmatrix} \nu_H^c \\ e_H^c \end{pmatrix} \quad (\bar{4}, 1, 2)$$

$$\bar{H}^c \quad \begin{pmatrix} \bar{u}_H^c & \bar{d}_H^c \\ \bar{\nu}_H^c & \bar{e}_H^c \end{pmatrix} \quad (4, 1, 2)$$

$$h \quad \begin{pmatrix} h_2^+ & h_1^0 \\ h_2^0 & h_1^- \end{pmatrix} \quad (1, 2, 2)$$

$$G_{PS} \rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

$$M_4 = M_3; M_2 = y_{LR} M_{2R}$$

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Condition for gaugino masses.

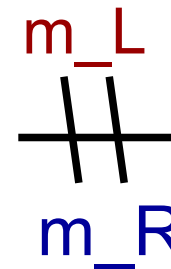
$$m_{H_{u,d}}^2 = m_{10}^2 \mp 2M_D^2$$

PS(4-2-1) *LR Asymmetry*

MATTER FIELDS

$$F_r \quad \begin{pmatrix} d_r & -u_r \\ e_r & -\nu_r \end{pmatrix} \quad (4, 2, 1)$$

$$F_r^c \quad \begin{pmatrix} u_r^c \\ d_r^c \end{pmatrix}, \begin{pmatrix} \nu_r^c \\ e_r^c \end{pmatrix} \quad (\bar{4}, 1, 2)$$



Gaugino Masses

$$M_1 = \frac{3}{5}M_{2R} + \frac{2}{5}M_4 ,$$

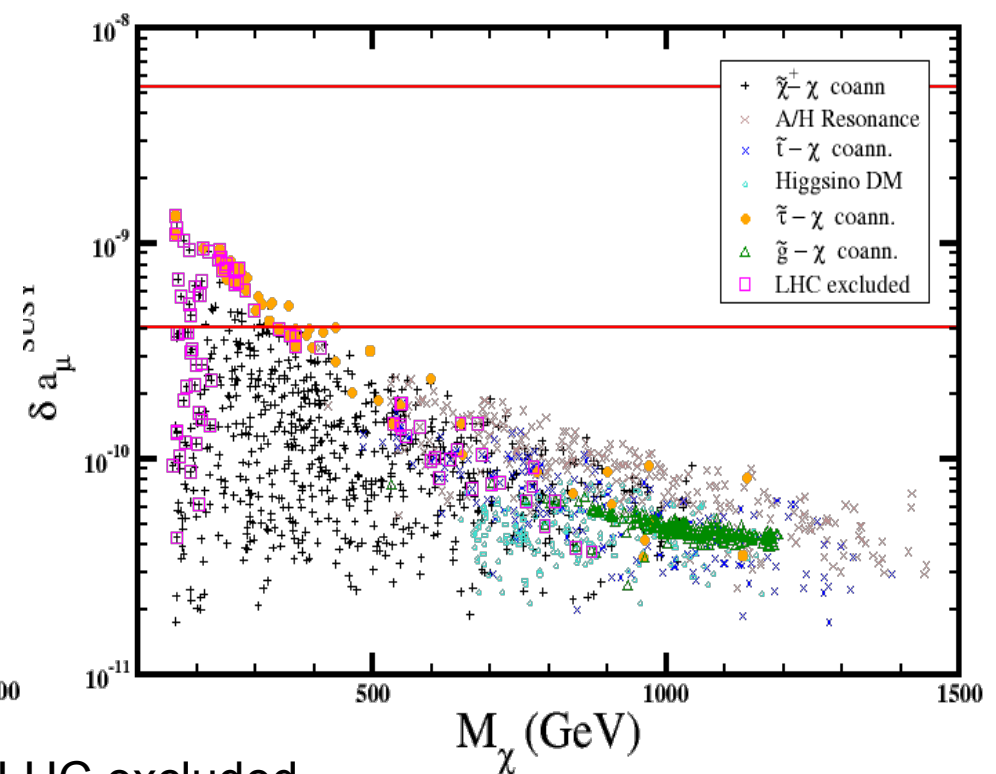
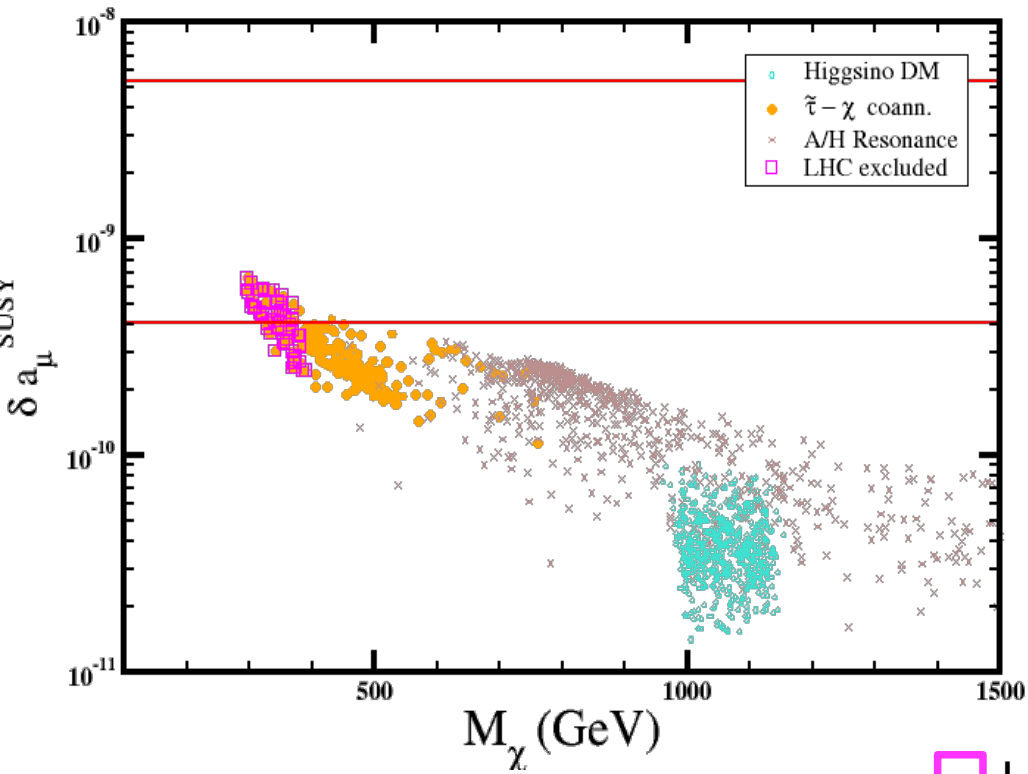
$$M_4 = M_3 ; M_2 = y_{LR} M_{2R}$$

New Parameter

$$X_{LR} = \frac{m_R}{m_L}$$

SO(10)


PS(4-2-2)



 LHC excluded.

Higgsino DM

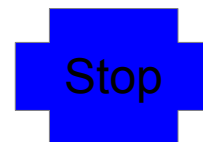
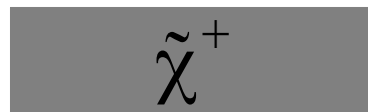
A/H Resonances

$h_f > 0.1, |m_A - 2m_\chi| > 0.1 m_\chi.$ 

$|m_A - 2m_\chi| \leq 0.1 m_\chi$ 

$h_f \equiv |N_{13}|^2 + |N_{14}|^2,$

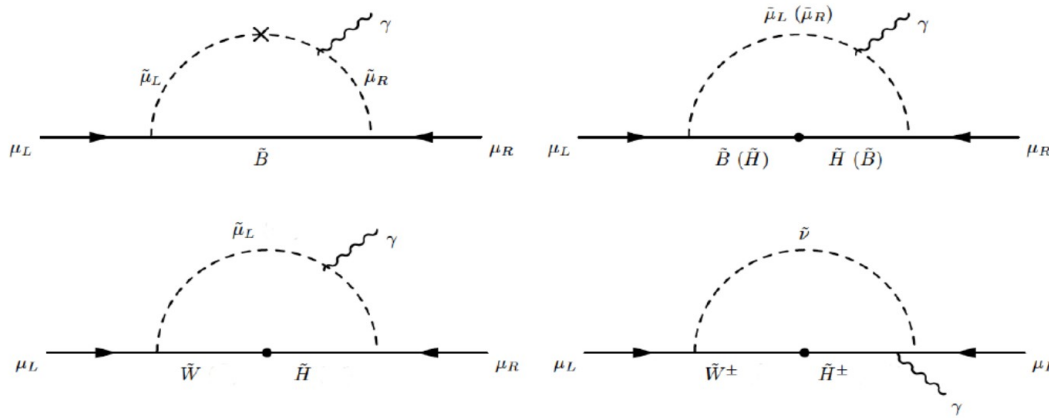
Coannihilations: $(m_z - m_{LSP}) < 0.1 m_{LSP}$



Muon g-2 combining Fermilab + BNL data

$$\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (25.1 \pm 5.9) \times 10^{-10} .$$

SUSY Contribution to Muon g-2



| Low Scale | GUT Scale |
|--------------------------------------|--------------------|
| $m_{\tilde{\mu}_L}, m_{\tilde{\nu}}$ | m_L |
| $m_{\tilde{\mu}_R}$ | m_R |
| $M_{\tilde{B}}$ | M_1 |
| $M_{\tilde{W}}$ | M_2 |
| μ | m_{H_u}, m_{H_d} |
| A_μ | A_0 |
| $\tan \beta$ | $\tan \beta$ |

$$m_h = 123 - 127 \text{ GeV}$$

$$m_{\tilde{g}} \geq 2.1 \text{ TeV (800 GeV if it is NLSP)}$$

$$0.8 \times 10^{-9} \leq \text{BR}(B_s \rightarrow \mu^+ \mu^-) \leq 6.2 \times 10^{-9} (2\sigma)$$

$$2.99 \times 10^{-4} \leq \text{BR}(B \rightarrow X_s \gamma) \leq 3.87 \times 10^{-4} (2\sigma)$$

$$0.114 \leq \Omega_{\text{CDM}} h^2 \leq 0.126 .$$

$$0 \leq m_L \leq 5 \text{ TeV}$$

$$0 \leq M_{2L} \leq 5 \text{ TeV}$$

$$-3 \leq M_3 \leq 5 \text{ TeV}$$

$$-3 \leq A_0/m_L \leq 3$$

$$1.2 \leq \tan \beta \leq 60$$

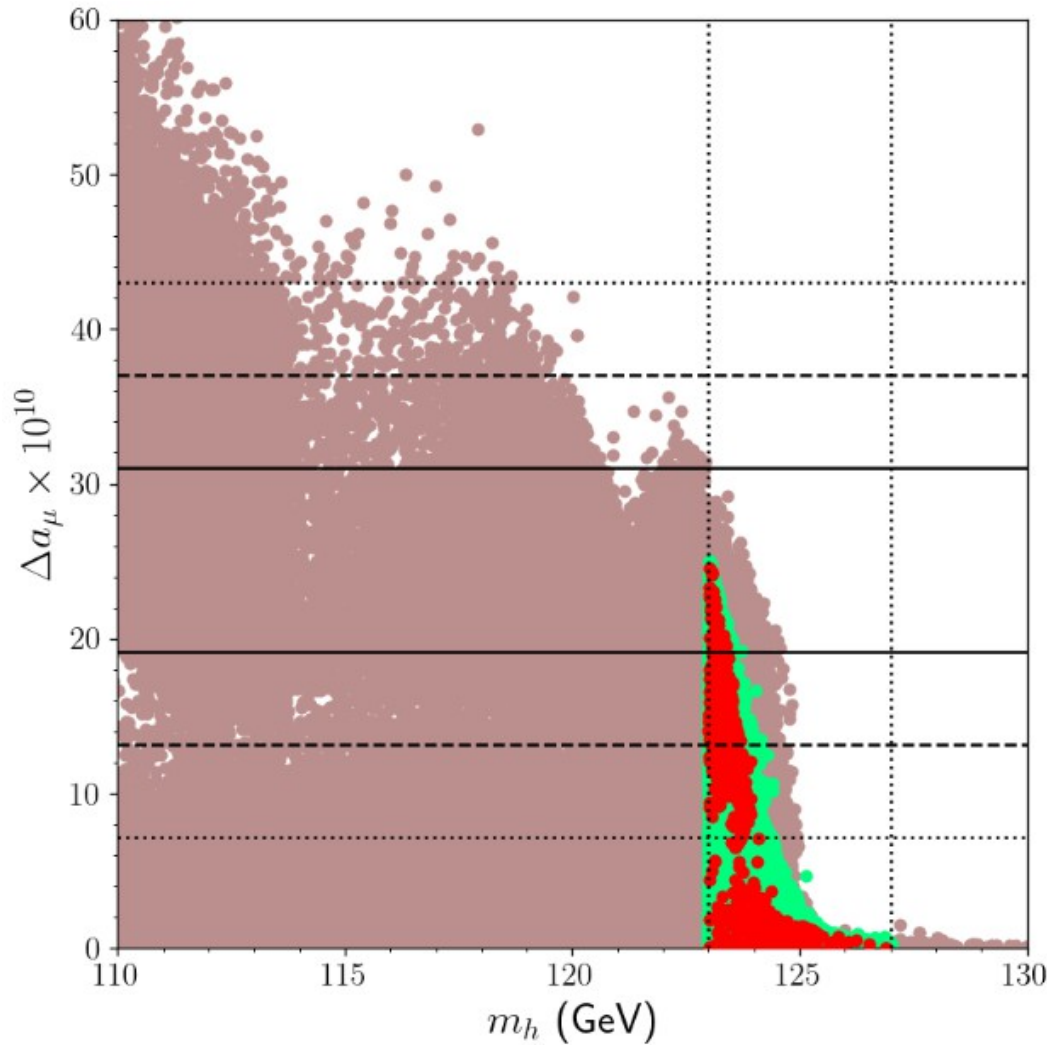
$$0 \leq x_{\text{LR}} \leq 3$$

$$-3 \leq y_{\text{LR}} \leq 3$$

$$0 \leq x_{\text{d}} \leq 3$$

$$-1 \leq x_{\text{u}} \leq 2 .$$

Important contribution is in tension with the Higgs mass



$$\Delta a_{\mu}^{\tilde{B}\tilde{\mu}_L\tilde{\mu}_R} \simeq \frac{g_1^2}{16\pi^2} \frac{m_{\mu}^2 M_{\tilde{B}} (\mu \tan \beta - A_{\mu})}{m_{\tilde{\mu}_L}^2 m_{\tilde{\mu}_R}^2} F_N \left(\frac{m_{\tilde{\mu}_L}^2}{M_{\tilde{B}}^2}, \frac{m_{\tilde{\mu}_R}^2}{M_{\tilde{B}}^2} \right)$$

$$\Delta m_h^2 \simeq \frac{m_t^4}{16\pi^2 v^2 \sin^2 \beta} \frac{\mu A_t}{M_{\text{SUSY}}^2} \left[\frac{A_t^2}{M_{\text{SUSY}}^2} - 6 \right]$$

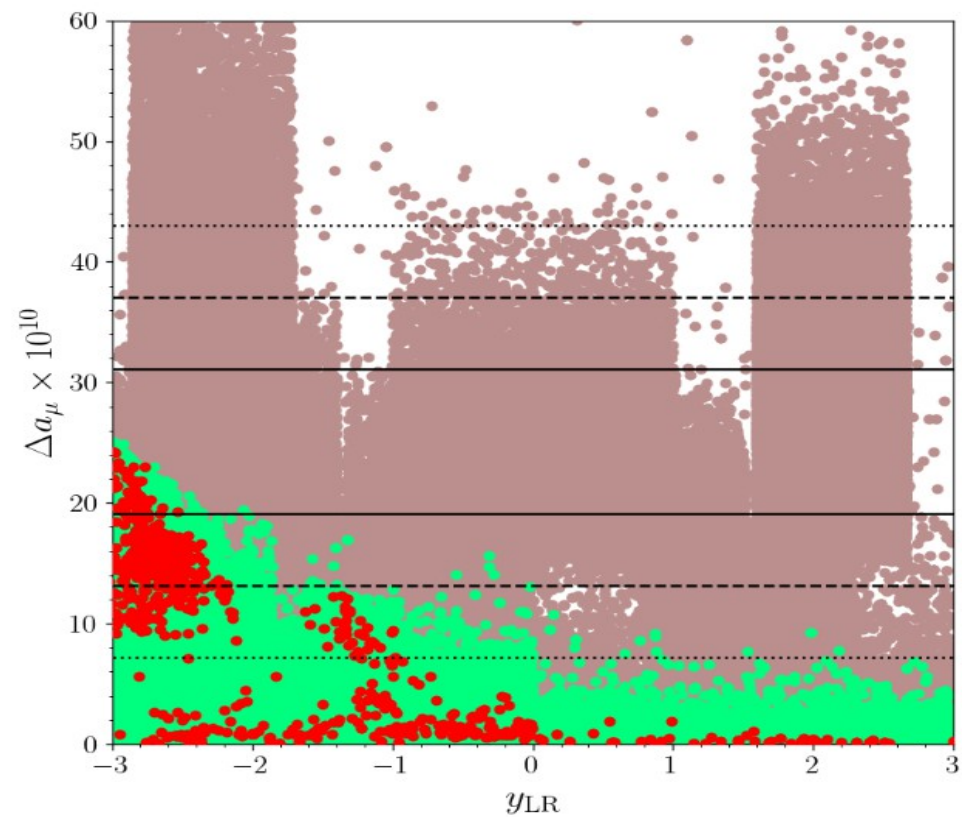
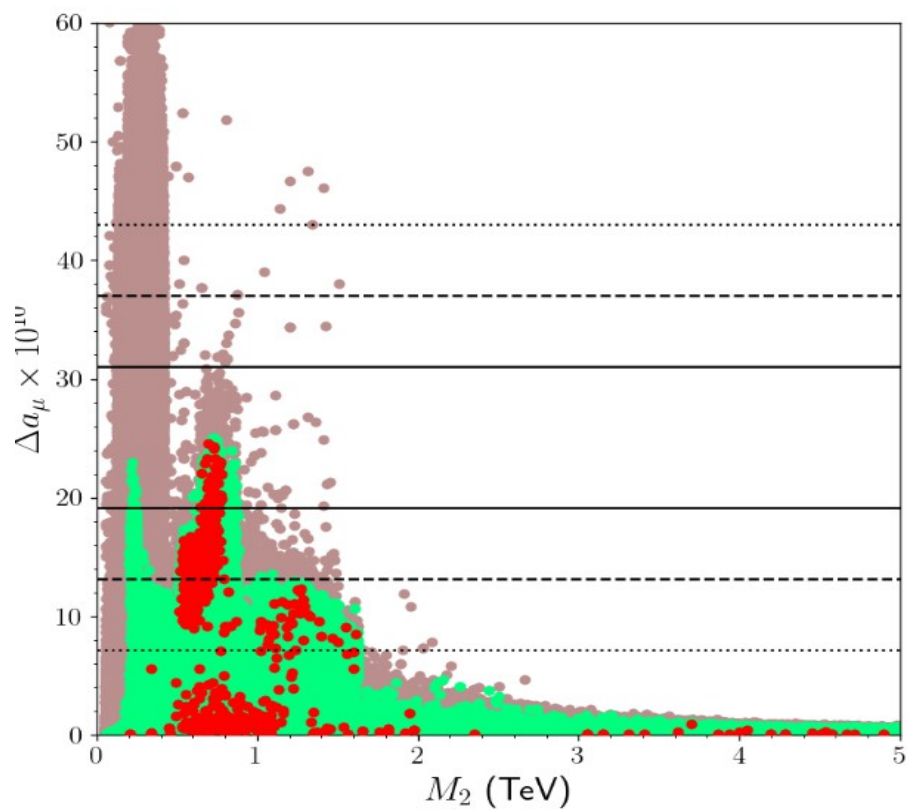
$$\begin{aligned}
\frac{dm_{\tilde{l}_{R_i}}^2}{dt} &= -\frac{1}{16\pi^2} \left[2(Y_l^i)^2 P_{\tilde{l}}^i + g_1^2 \text{Tr}(Y m^2) - 4g_1^2 M_1^2 \right] \\
\frac{dm_{\tilde{L}_i}^2}{dt} &= -\frac{1}{16\pi^2} \left[(Y_l^i)^2 P_{\tilde{l}}^i - \frac{1}{2} g_1^2 \text{Tr}(Y m^2) - (g_1^2 M_1^2 + 3g_2^2 M_2^2) \right] \\
\frac{dm_{\tilde{d}_{R_i}}^2}{dt} &= -\frac{1}{16\pi^2} \left[2(Y_d^i)^2 P_{\tilde{d}}^i + \frac{1}{3} g_1^2 \text{Tr}(Y m^2) - \left(\frac{4}{9} g_1^2 M_1^2 + \frac{16}{3} g_3^2 M_3^2 \right) \right] \\
\frac{dm_{\tilde{u}_{R_i}}^2}{dt} &= -\frac{1}{16\pi^2} \left[2(Y_u^i)^2 P_{\tilde{u}}^i - \frac{2}{3} g_1^2 \text{Tr}(Y m^2) - \left(\frac{16}{9} g_1^2 M_1^2 + \frac{16}{3} g_3^2 M_3^2 \right) \right] \\
\frac{dm_{\tilde{Q}_i}^2}{dt} &= -\frac{1}{16\pi^2} \left[(Y_u^i)^2 P_{\tilde{u}}^i + (Y_d^i)^2 P_{\tilde{d}}^i + \frac{1}{6} g_1^2 \text{Tr}(Y m^2) - \left(\frac{1}{9} g_1^2 M_1^2 + 3g_2^2 M_2^2 + \frac{16}{3} g_3^2 M_3^2 \right) \right]
\end{aligned}$$

$$m_{\tilde{u}_{iL}}^2 = m_{\tilde{q}_{i0}}^2 + m_{u_i}^2 + \tilde{\alpha}_G \left(\frac{8}{3} f_3 m_3^2 + \frac{3}{2} f_2 m_2^2 + \frac{1}{30} f_1 m_1^2 \right)$$

$$m_{\tilde{u}_{iR}}^2 = m_{\tilde{u}_{i0}}^2 + m_{u_i}^2 + \tilde{\alpha}_G \left(\frac{8}{3} f_3 m_3^2 + \frac{8}{15} f_1 m_1^2 \right)$$

$$m_{\tilde{e}_{iL}}^2 = m_{\tilde{\ell}_{i0}}^2 + m_{e_i}^2 + \tilde{\alpha}_G \left(\frac{3}{2} f_2 m_2^2 + \frac{3}{10} f_1 m_1^2 \right)$$

$$m_{\tilde{e}_{iR}}^2 = m_{\tilde{e}_{i0}}^2 + m_{e_i}^2 + \tilde{\alpha}_G \frac{6}{5} f_1 m_1^2.$$



- Gaugino masses not unified at GUT
- $M_3 \gg M_2, M_1$

$$M_1 = \frac{3}{5}M_{2R} + \frac{2}{5}M_4 ,$$

$$M_4 = M_3 ; M_2 = y_{LR} M_{2R}$$

MSSM extended by seesaw mechanism

- The superpotential for MSSM-Seesaw I can be written as

$$W = W_{\text{MSSM}} + Y_{\nu}^{ij} \epsilon_{\alpha\beta} H_2^{\alpha} N_i^c L_j^{\beta} + \frac{1}{2} M_N^{ij} N_i^c N_j^c, \quad (5)$$

- The full set of soft SUSY-breaking terms is given by,

$$\begin{aligned} -\mathcal{L}_{\text{soft,SI}} = & -\mathcal{L}_{\text{soft}} + (m_{\tilde{\nu}}^2)_j^i \tilde{\nu}_{Ri}^* \tilde{\nu}_R^j + \left(\frac{1}{2} B_{\nu}^{ij} M_N^{ij} \tilde{\nu}_{Ri}^* \tilde{\nu}_{Rj}^* \right. \\ & \left. + A_{\nu}^{ij} h_2 \tilde{\nu}_{Ri}^* \tilde{l}_{Lj} + \text{h.c.} \right), \end{aligned} \quad (6)$$

$$\mathcal{M} = \begin{pmatrix} 0 & m_\nu^D \\ m_\nu^{D^T} & M_R \end{pmatrix}$$

“See-Saw” explanation for tiny masses.

• The physical masses are:

1. $m_1 \equiv m_{light} \simeq \frac{(m_\nu^D)^2}{M_R}$

2. $m_2 \simeq M_R$

• For $(m_\nu^D)_{33} \approx (200 \text{ GeV})$ ($\lambda_\nu \approx \lambda_t$) and $M_{N_3} \approx O(10^{14} \text{ GeV})$, $m_{eff} \approx 0.05 \text{ eV}$

$$W = W_{\text{MSSM}} + \frac{1}{2} (Y_\nu L H_2)^T M_N^{-1} (Y_\nu L H_2).$$

$$m_{\text{eff}} = -\frac{1}{2} v_u^2 Y_\nu \cdot M_N^{-1} \cdot Y_\nu^T, \quad m_\nu^\delta = U^T m_{\text{eff}} U$$

Slepton flavor mixings

$$(m_{\tilde{L}}^2)_{ij} \sim \frac{1}{16\pi^2} (6m_0^2 + 2A_0^2) (Y_\nu^\dagger Y_\nu)_{ij} \log \left(\frac{M_{\text{GUT}}}{M_N} \right)$$

$$(m_{\tilde{e}}^2)_{ij} \sim 0$$

$$(A_l)_{ij} \sim \frac{3}{8\pi^2} A_0 Y_{li} (Y_\nu^\dagger Y_\nu)_{ij} \log \left(\frac{M_{\text{GUT}}}{M_N} \right)$$

Orthogonal
matrix

$$Y_\nu = \frac{\sqrt{2}}{v_u} \sqrt{M_R^\delta} R \sqrt{m_\nu^\delta} U^\dagger$$

Casas + Ibarra

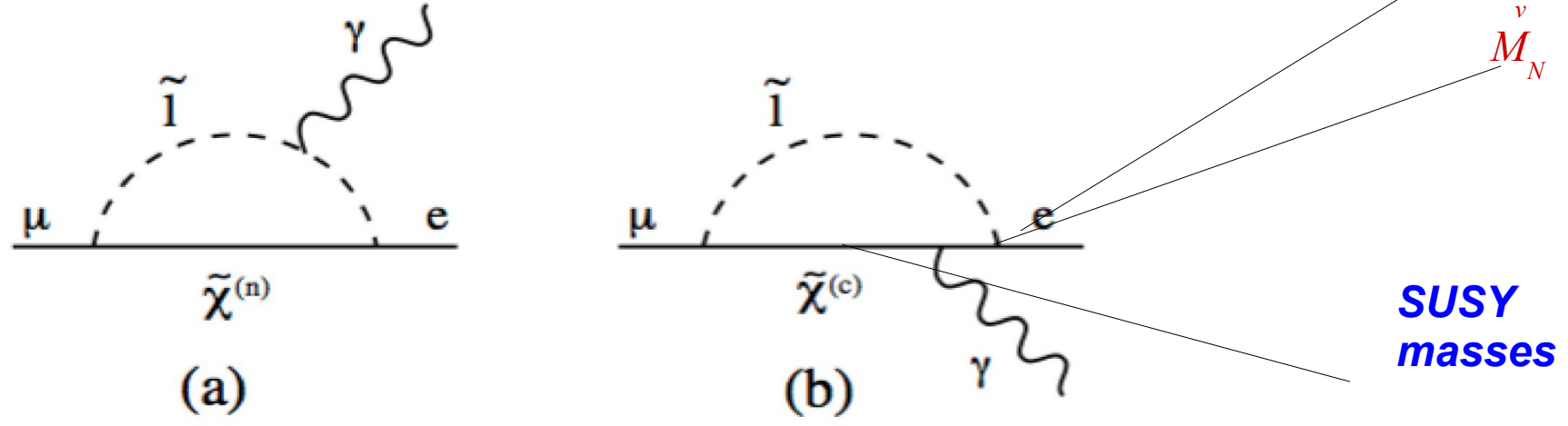
Diagonal Universal
1E14 GeV

Order 1

$$Y_\nu^\dagger Y_\nu = \frac{2}{v_u^2} M_R U m_\nu^\delta U^\dagger$$

Limit case of
degenerate MR

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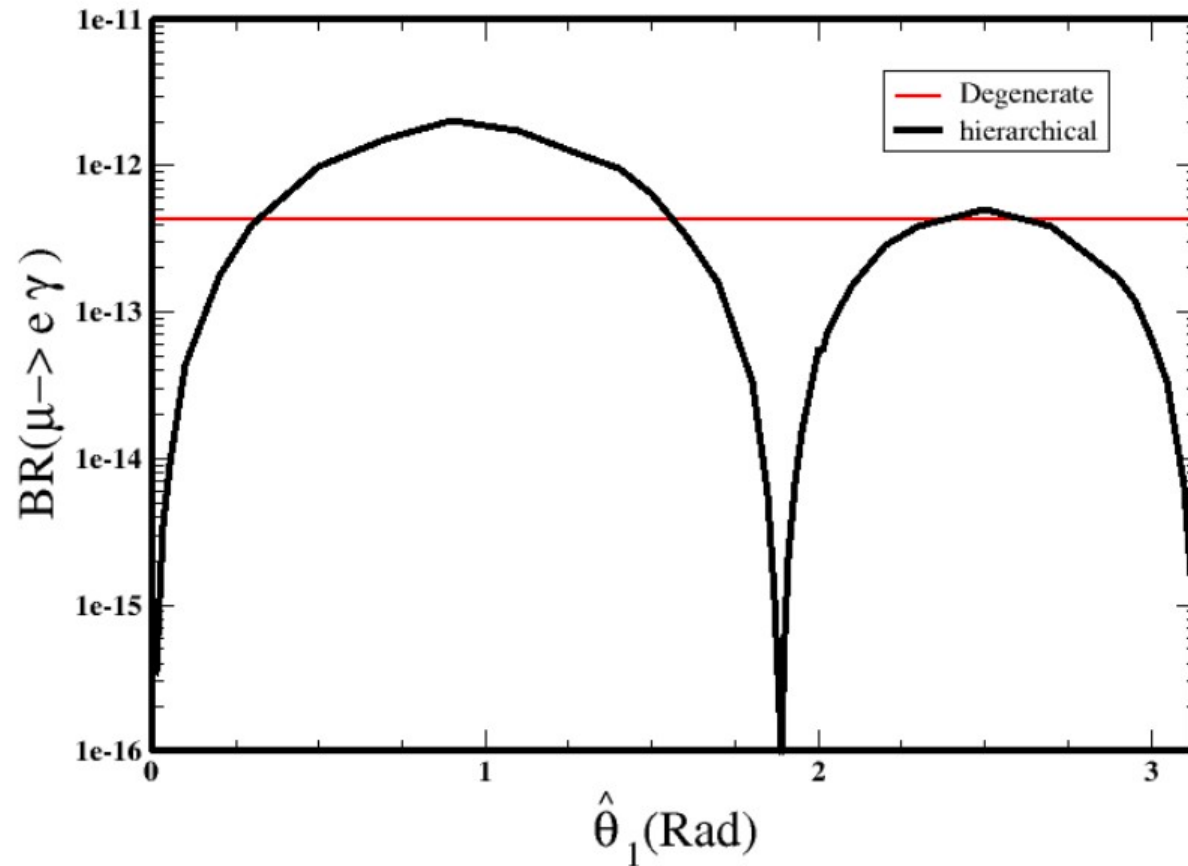
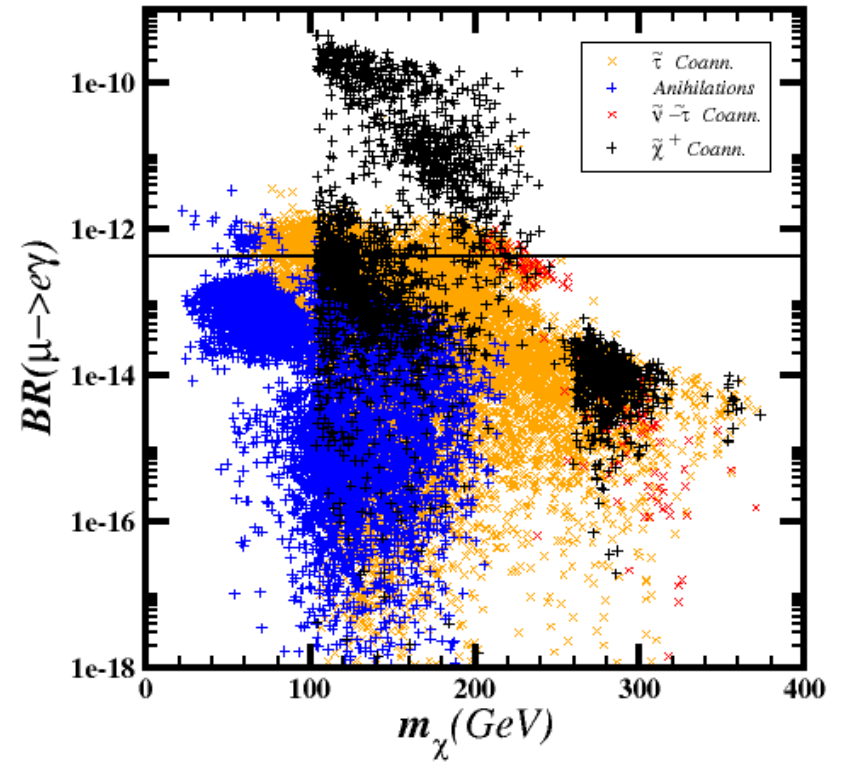
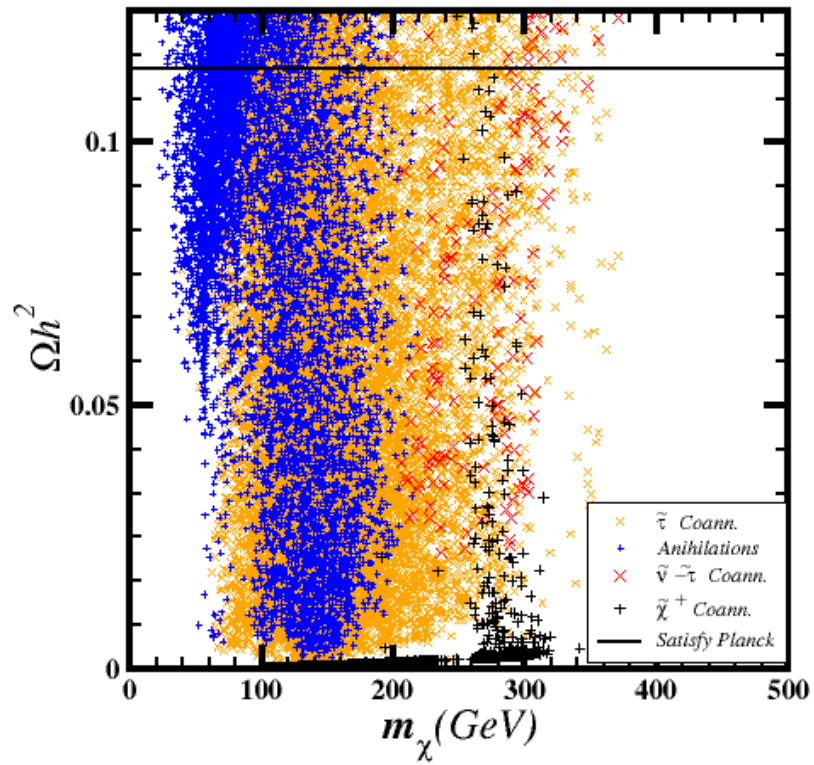
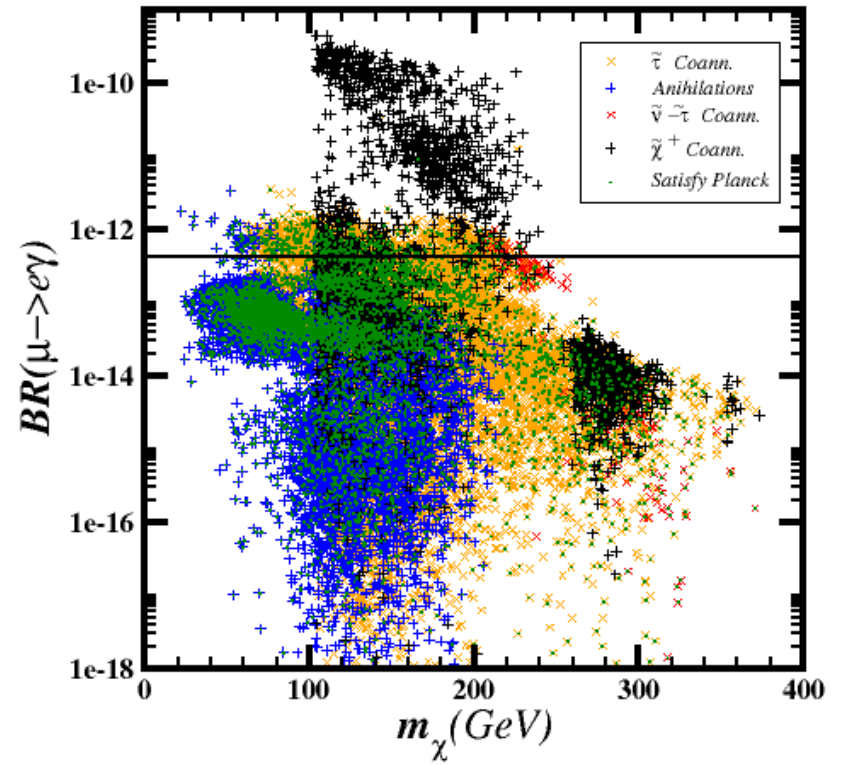
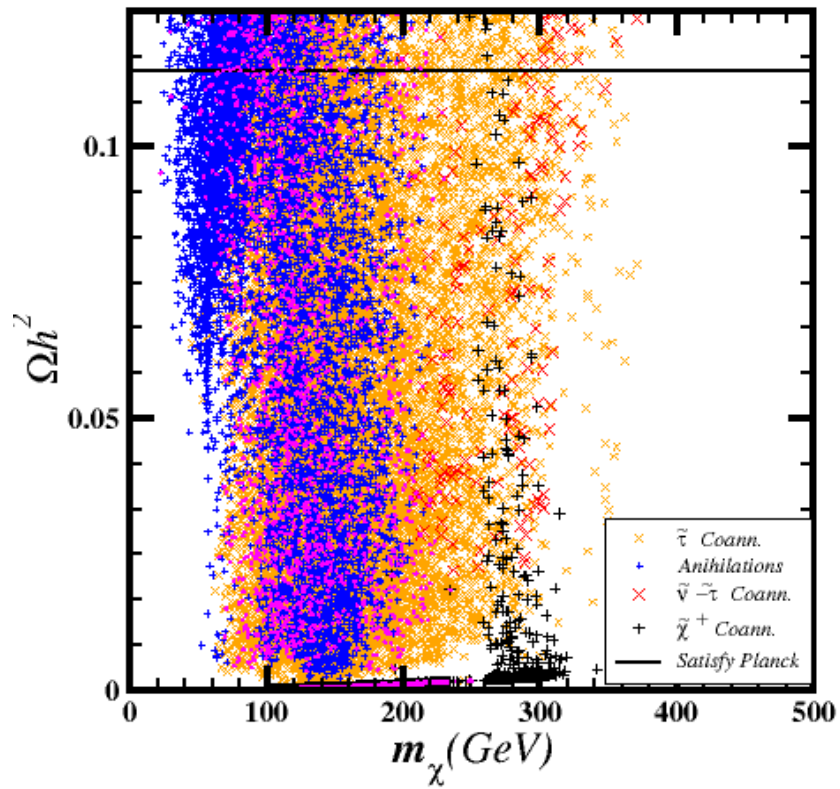
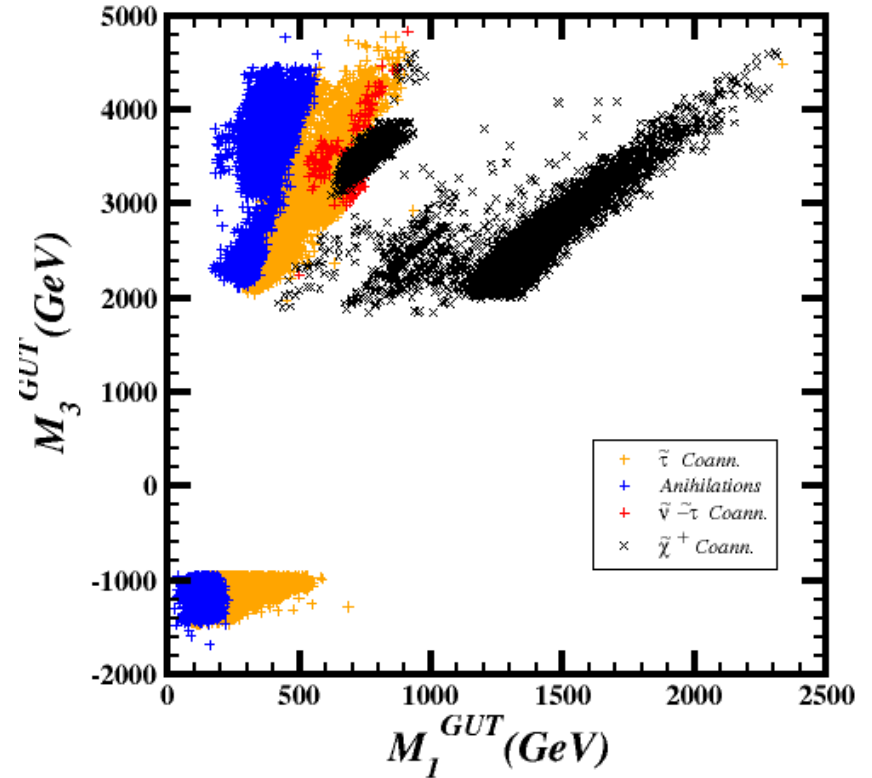
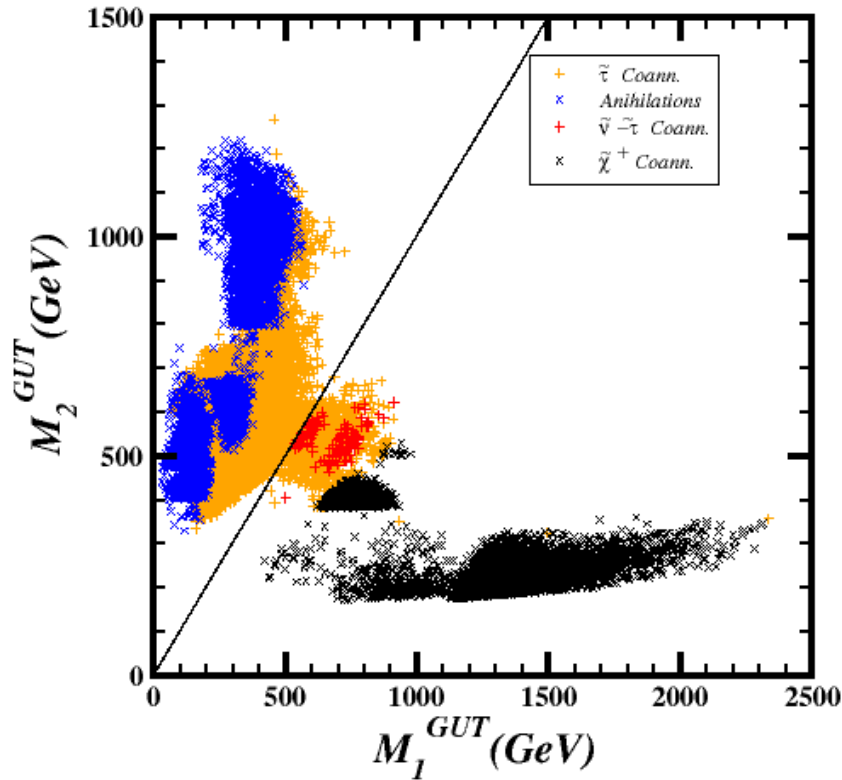


Figure 1: $BR(\mu \rightarrow e\gamma)$ vs $\hat{\theta}_1$ under two different assumptions for the right-handed neutrinos using the CMSSM with $m_0 = 650 \text{ GeV}$, $m_{1/2} = 700 \text{ GeV}$, $A_0 = -1400 \text{ GeV}$ and $\tan \beta = 40$, $\mu > 0$ and $M_3 = 2.5 \cdot 10^{12} \text{ GeV}$.



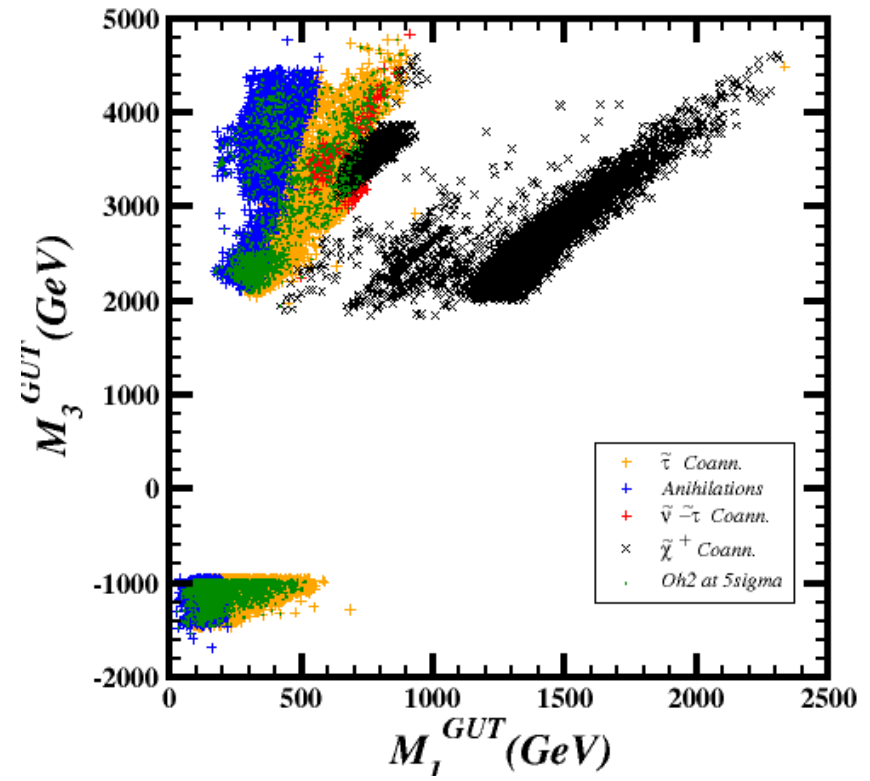
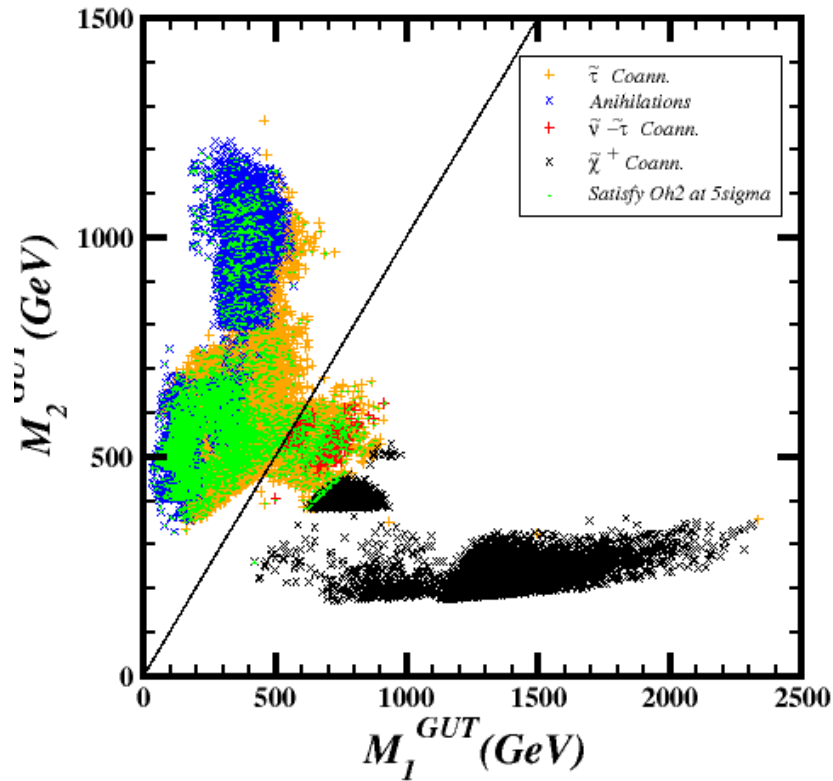




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$$M_1 = \frac{3}{5}M_{2R} + \frac{2}{5}M_4,$$

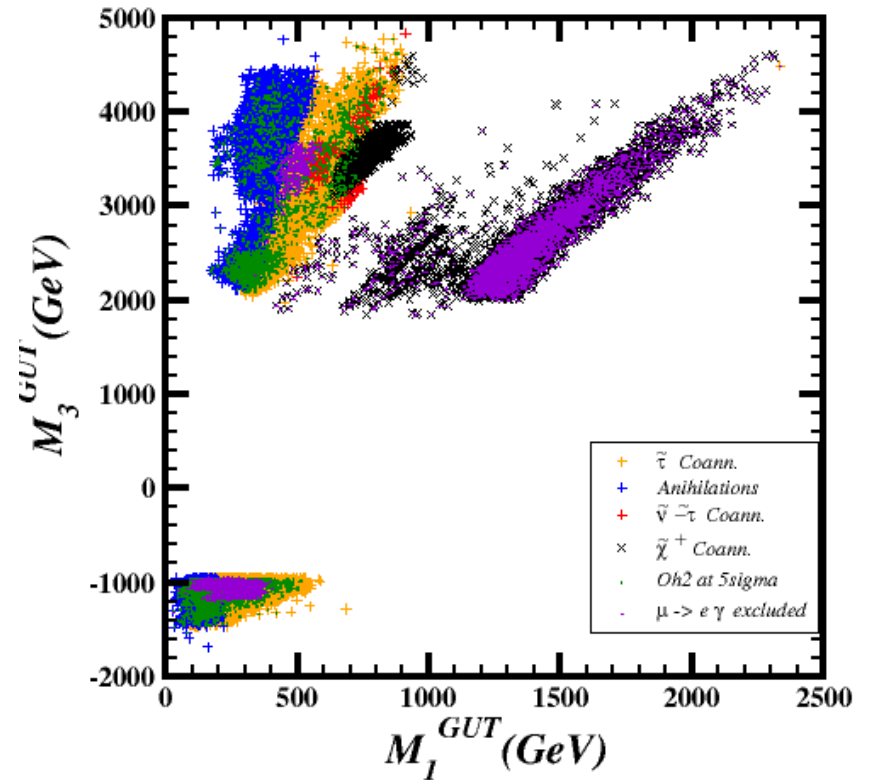
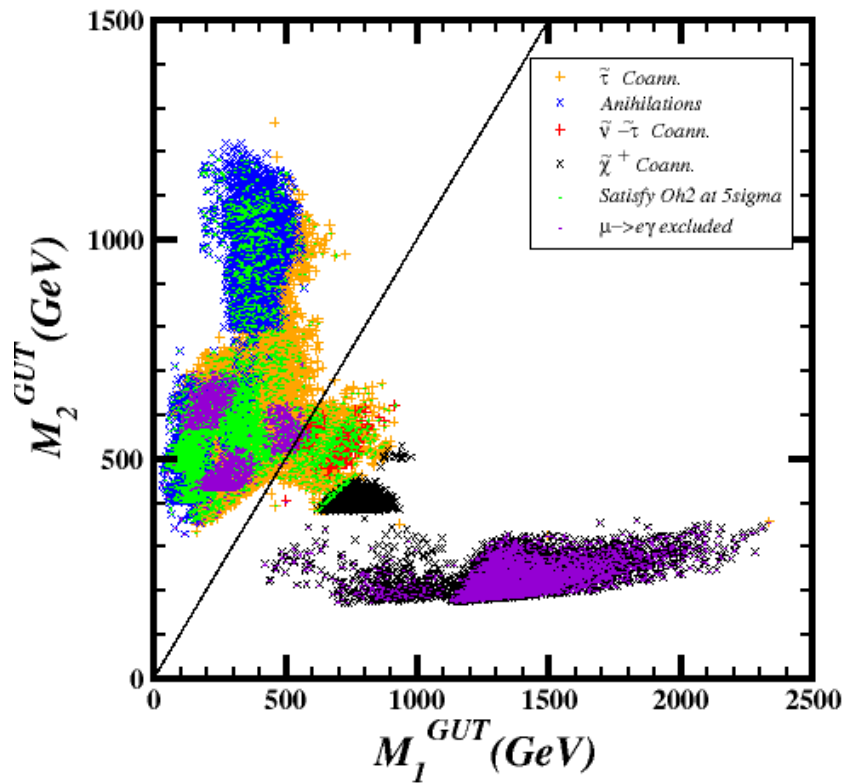
$$M_4 = M_3; M_2 = y_{LR} M_{2R}$$



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$$M_1 = \frac{3}{5}M_{2R} + \frac{2}{5}M_4,$$

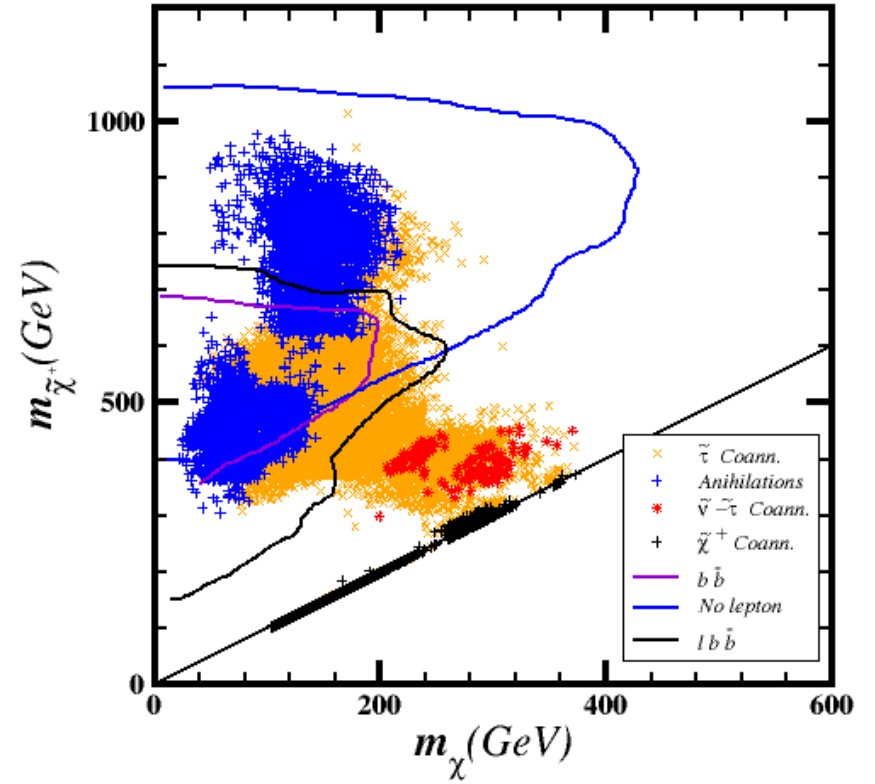
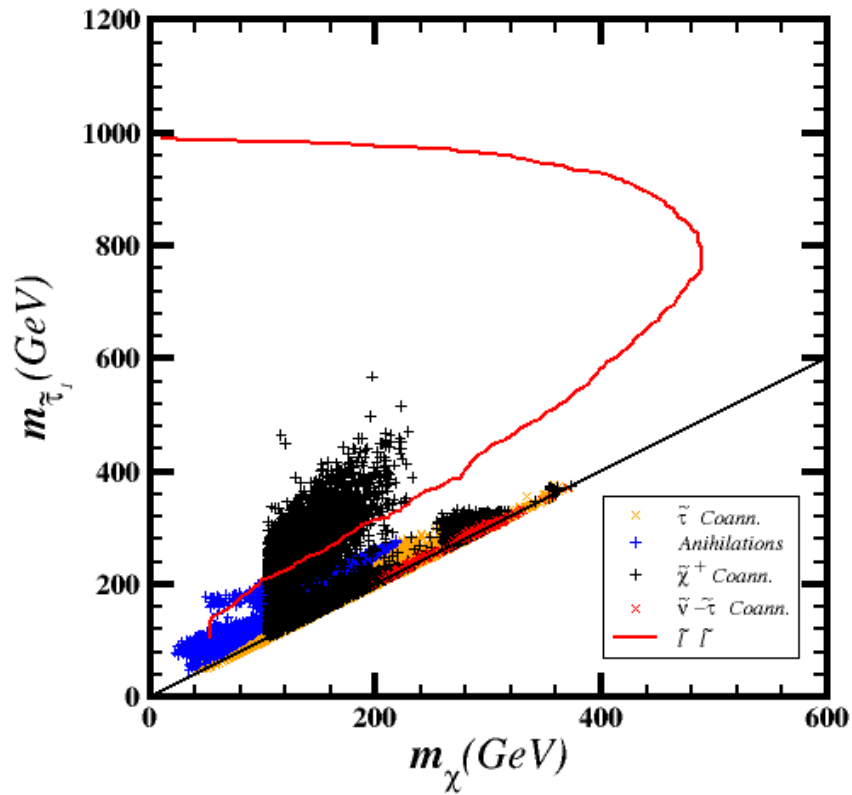
$$M_4 = M_3; M_2 = y_{LR}M_{2R}$$

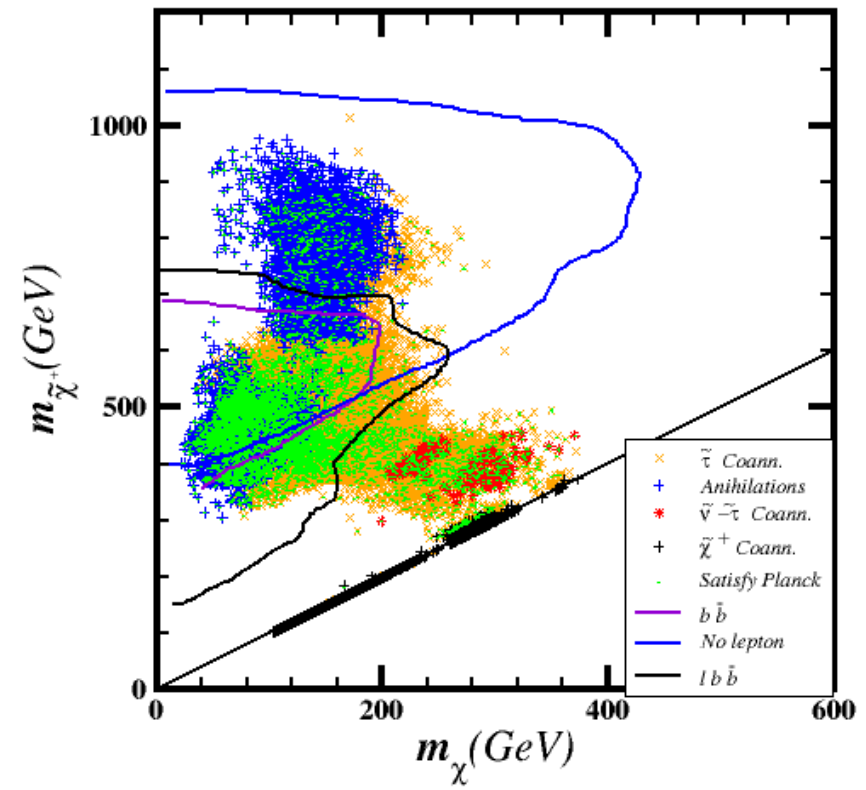
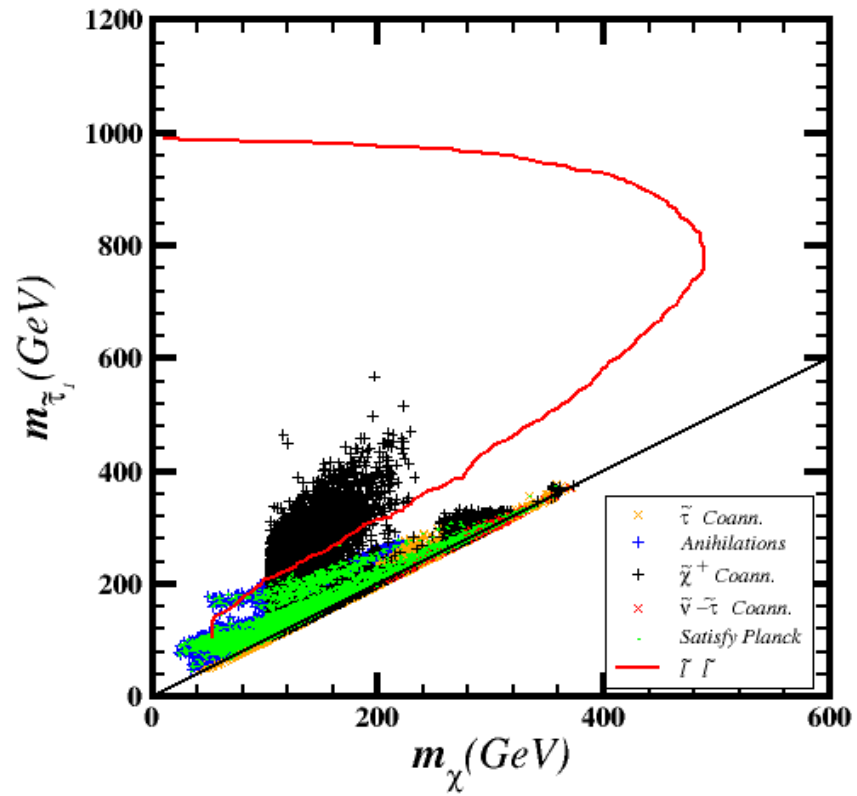


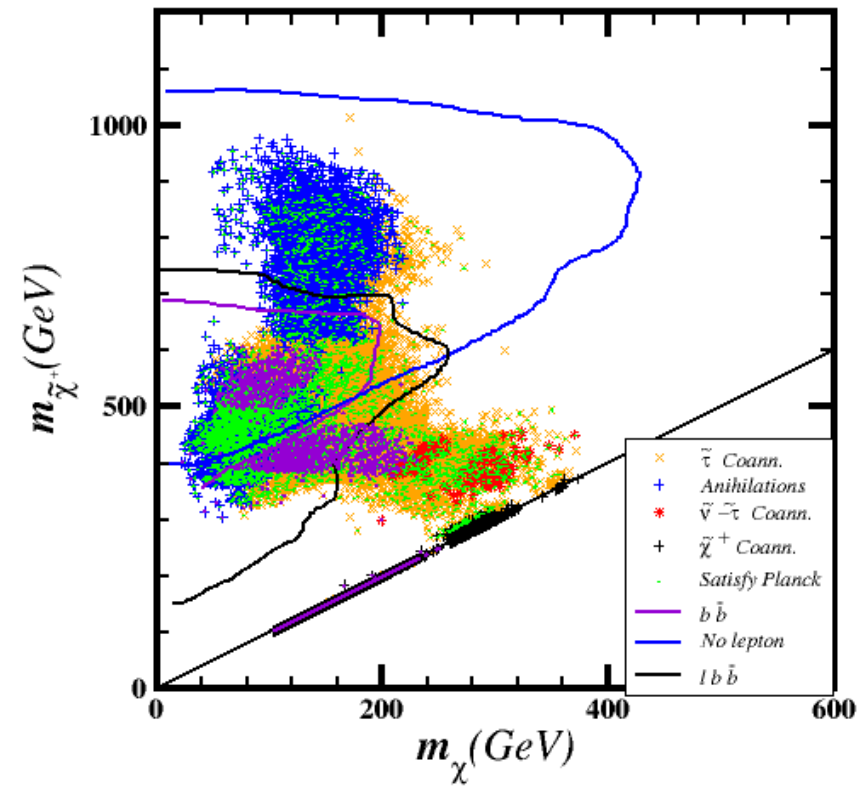
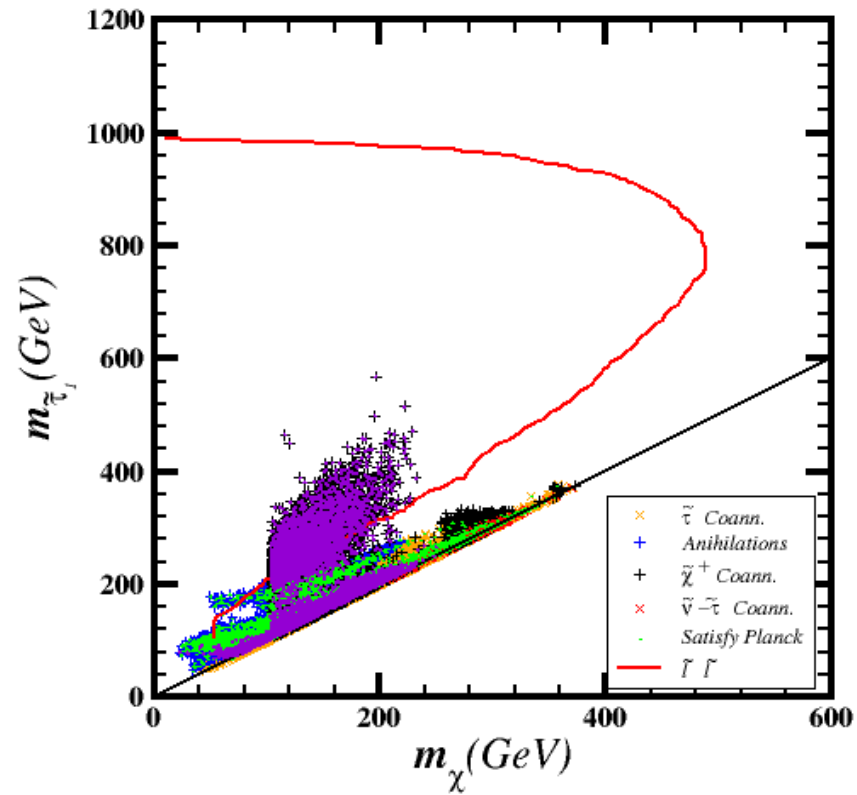
- Gaugino masses not unified at GUT
- $M_3 \gg M_2, M_1$

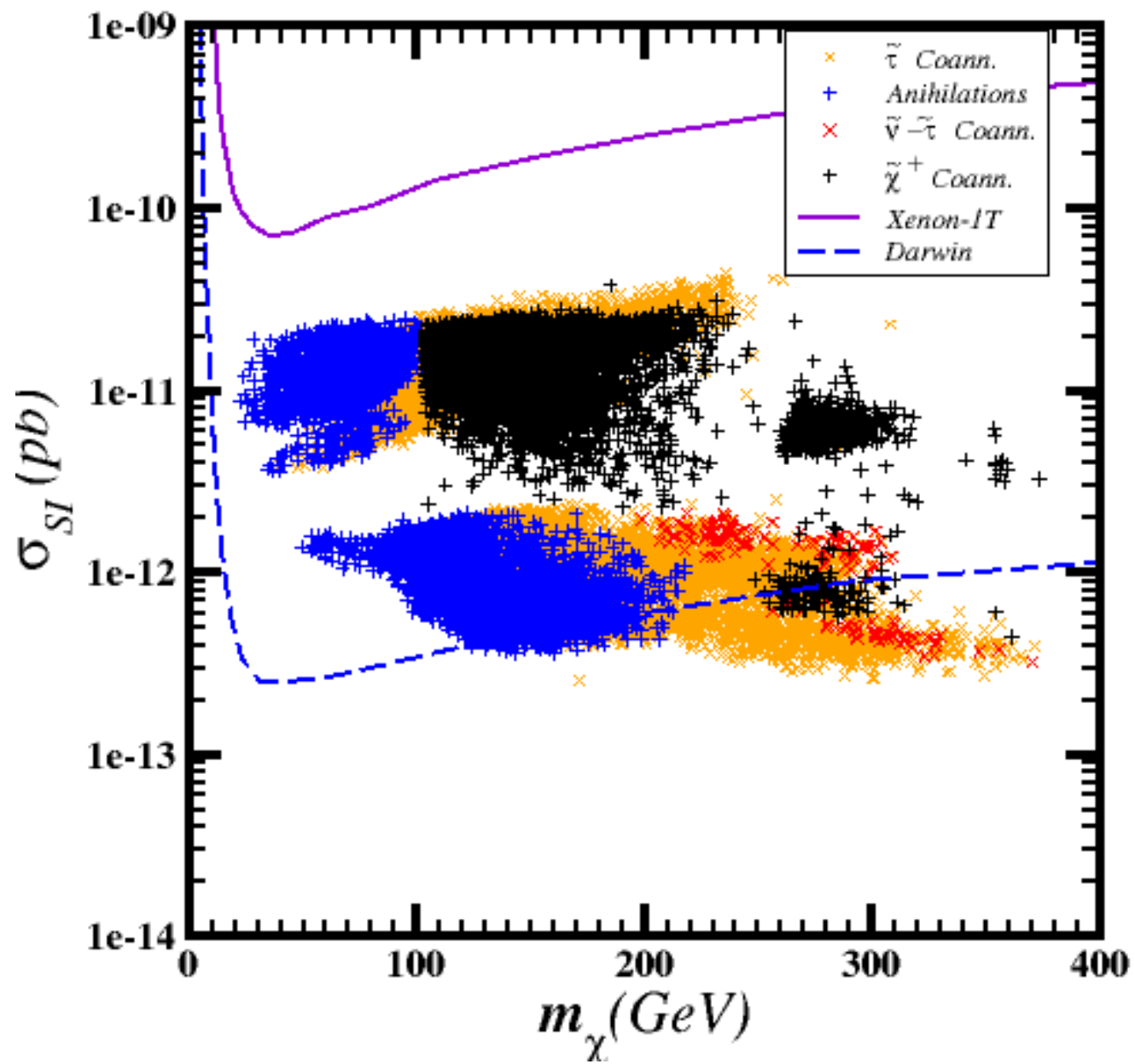
$$M_1 = \frac{3}{5}M_{2R} + \frac{2}{5}M_4,$$

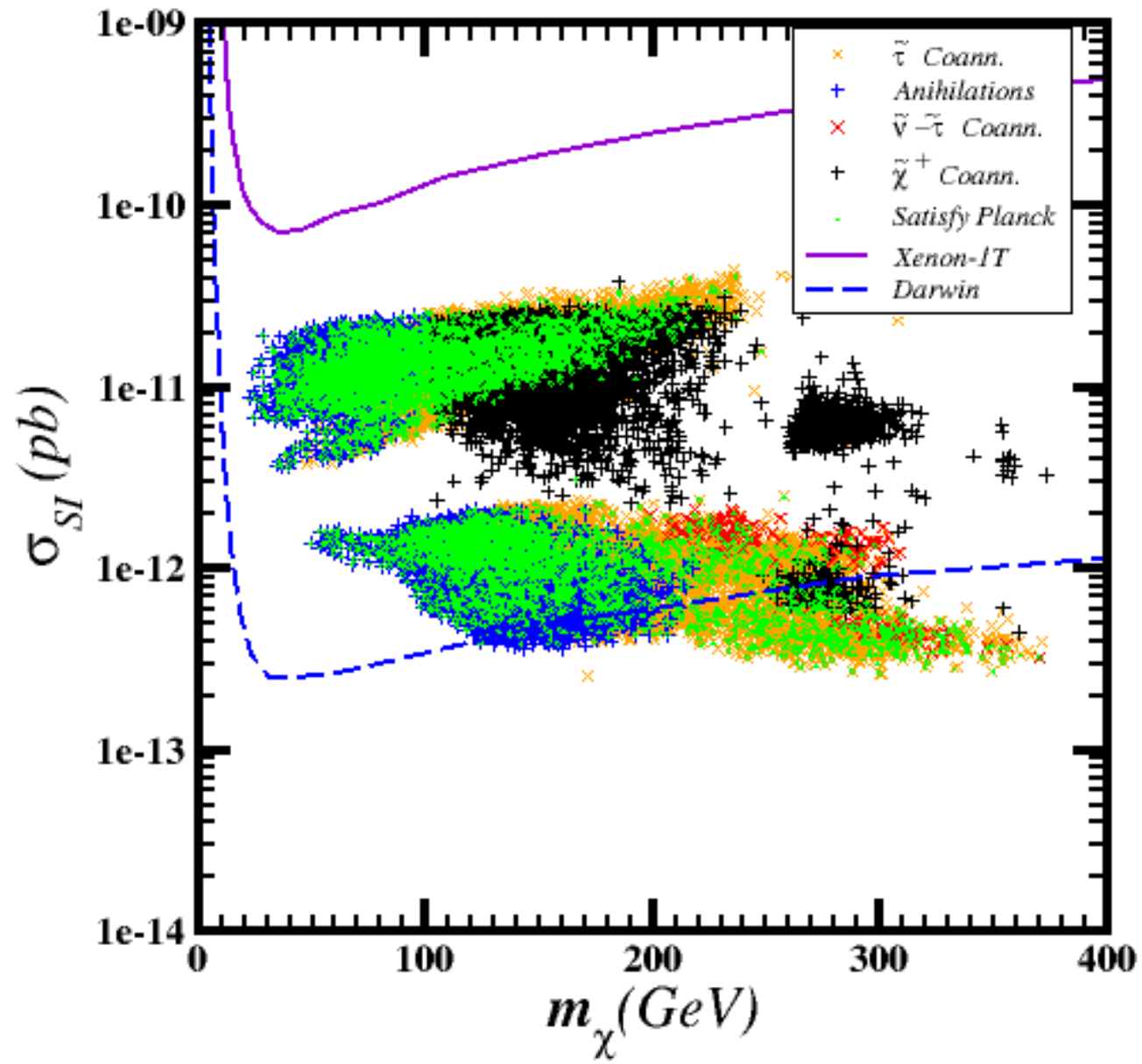
$$M_4 = M_3; M_2 = y_{LR} M_{2R}$$

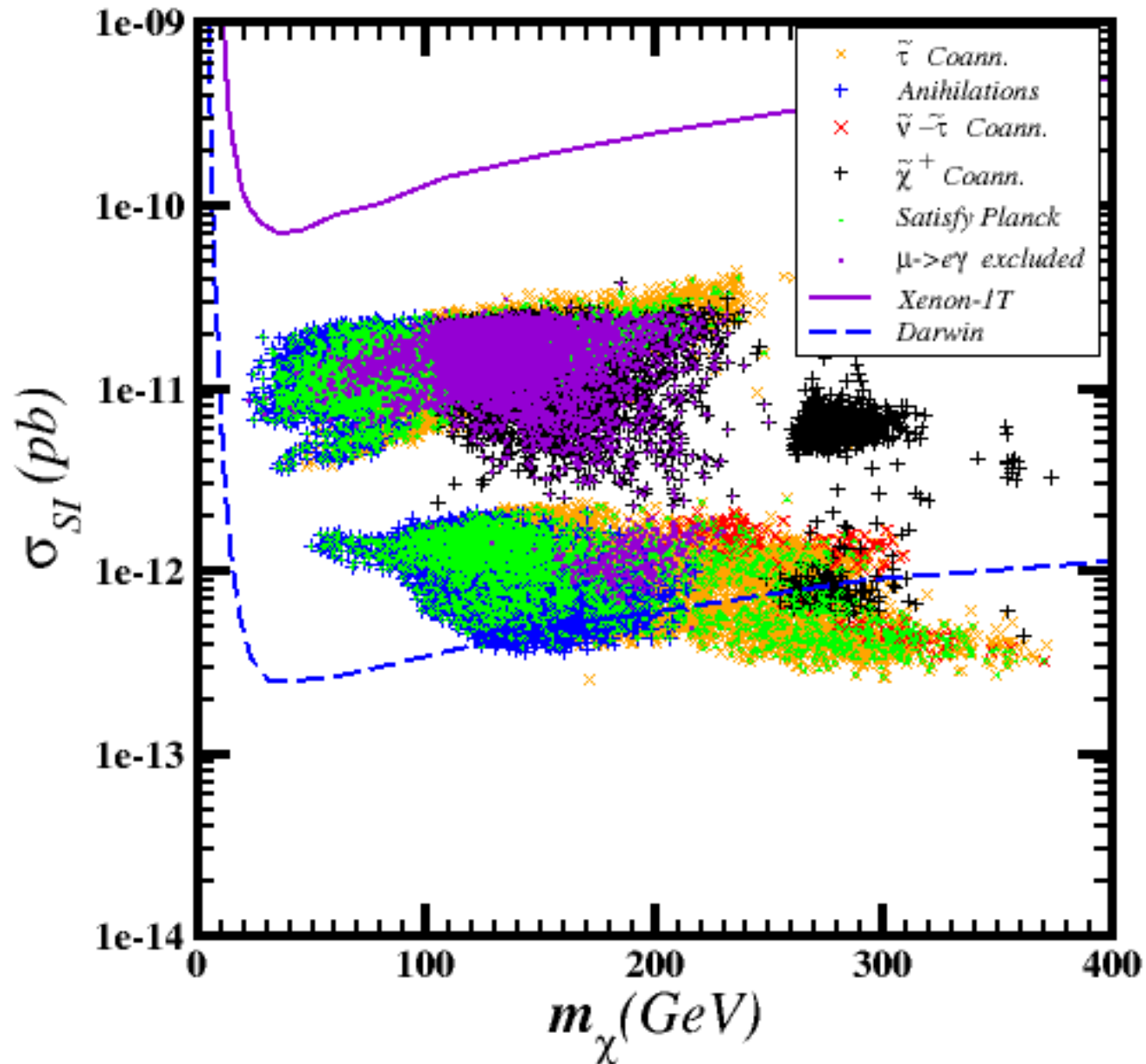












CONCLUSIONS

Susy Models with large gluino masses can explain the muon $(g-2)$ discrepancy via a SUSY contribution maintaining the prediction for the observed Higgs masses.

Models with Pati-Salam Unification, $SU(4) \times SU(2) \times SU(2)$ can motivate gaugino non-universality ($M_3 \gg M_2, M_1$) and a L/R asymmetry on the scalars such that muon $(g-2)$ can be explained while LSP that satisfies the relic abundance condition.

$(g-2) + \text{DM}$ satisfaction implies chargino and slepton masses at the LHC range. However, the small mass difference neutralino-stau makes difficult the identification of the signal. But that may be at the reach of planned linear colliders.

When the model is complemented with a simple see-saw with RH-neutrinos with mass $\sim 2.5 \times 10^{13}$ GeV, $\text{BR}(\mu \rightarrow e \gamma)$ falls in the experimental range.

Many of the points with relic density predict interesting SD cross sections above the neutrino floor.

Thanks for attention !!!!