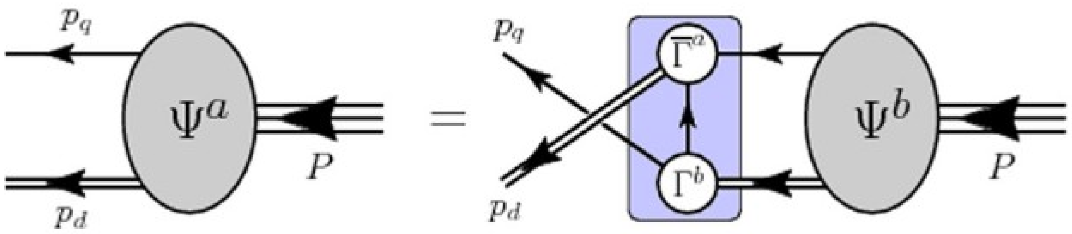
Diquarks, baryons, and transition form factors

Khépani Raya Montaño



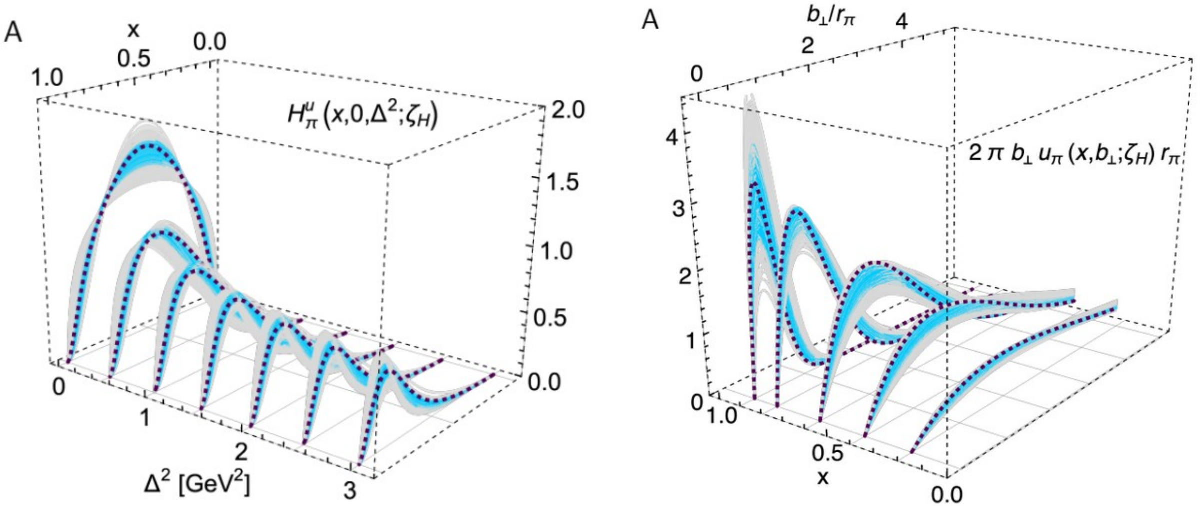
Universidad de Huelva

MeV2TeV 2023

Feb 16-17, 2022. Córdoba (Spain)

A data-driven construction of the pion GPD

Khépani Raya Montaño



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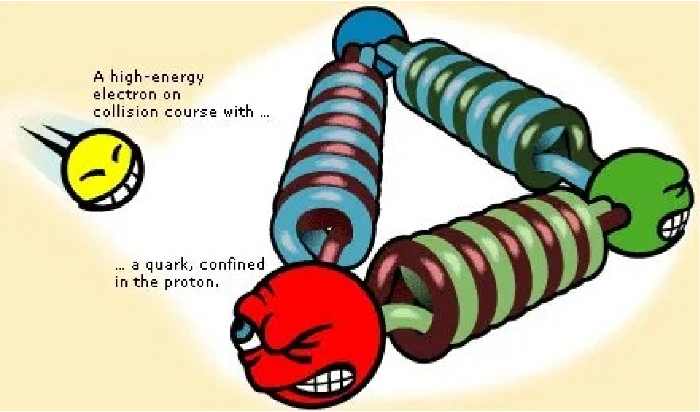
QCD: Basic Facts

- QCD is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

$$\mathcal{L}_{\rm QCD} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$
$$D_\mu = \partial_\mu + ig\tfrac{1}{2}\lambda^a A^a_\mu\,,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu},$$

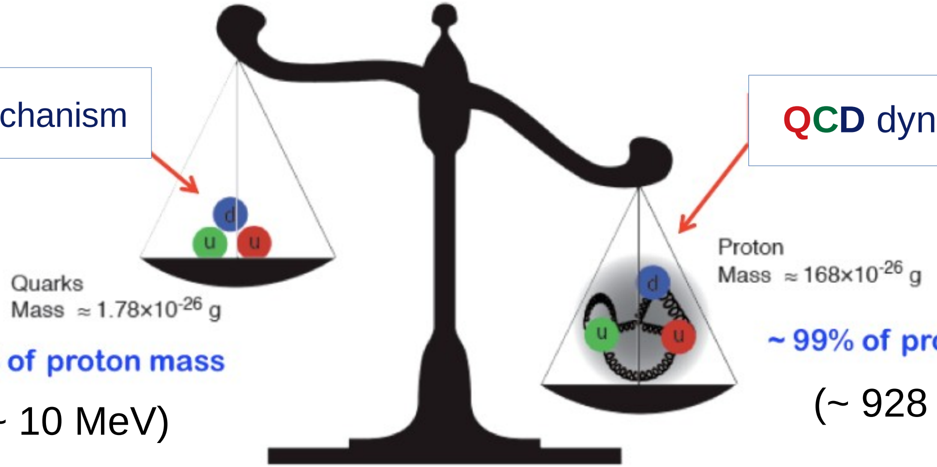
- Quarks and gluons not *isolated* in nature.
  - Formation of colorless bound states: "**Hadrons**"
  - **1-fm scale** size of hadrons?

- Emergence of hadron masses (EHM) from QCD **dynamics**



Higgs mechanism

QCD dynamics



~ 1% of proton mass

(~ 10 MeV)

~ 99% of proton mass

(~ 928 MeV)

QCD: Basic Facts

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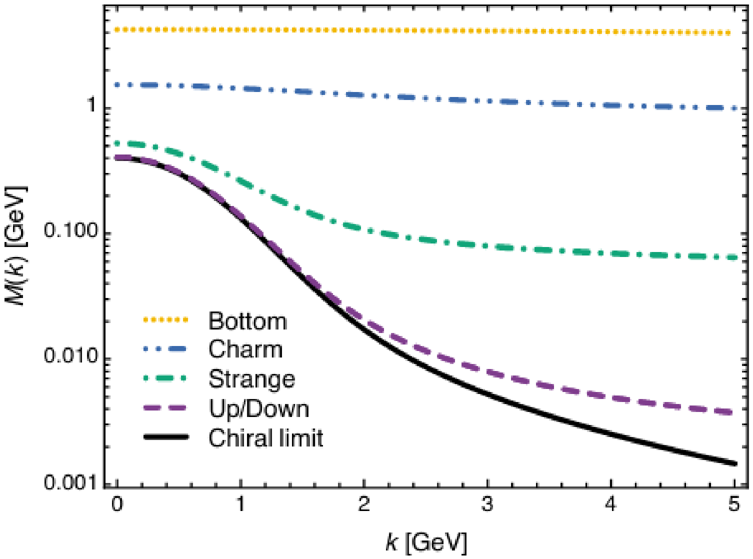
Can we trace them down to fundamental d.o.f?

Dynamical masses

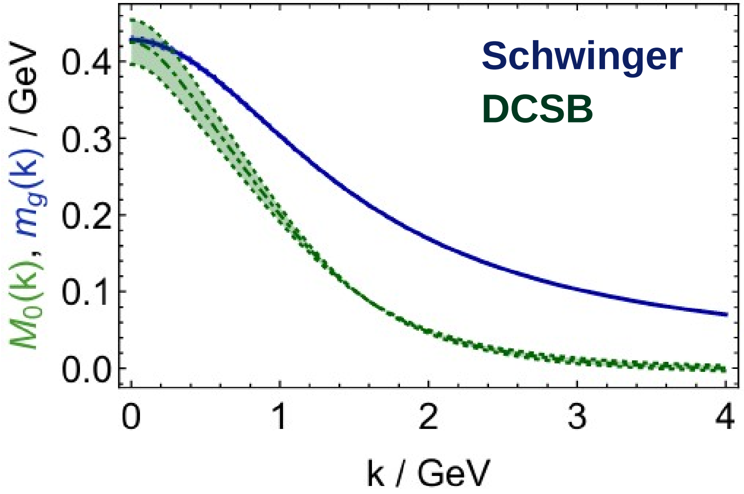
(**D**ynamical **C**hiral **S**ymmetry **B**reaking)



"Higgs" masses

$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma \cdot p + \mathbf{M_f(p^2)})$$

- Emergence of hadron masses (EHM) from QCD **dynamics**



Gluon and quark *running masses*

QCD: Basic Facts

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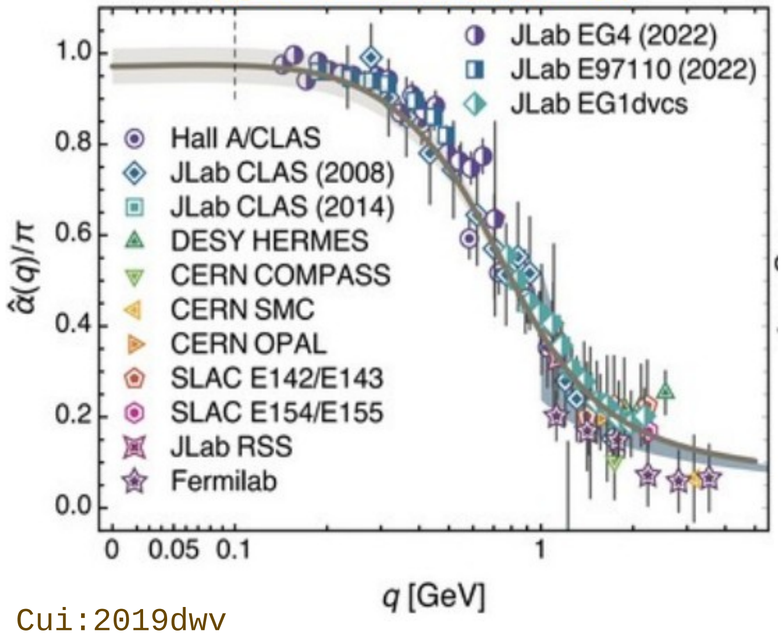
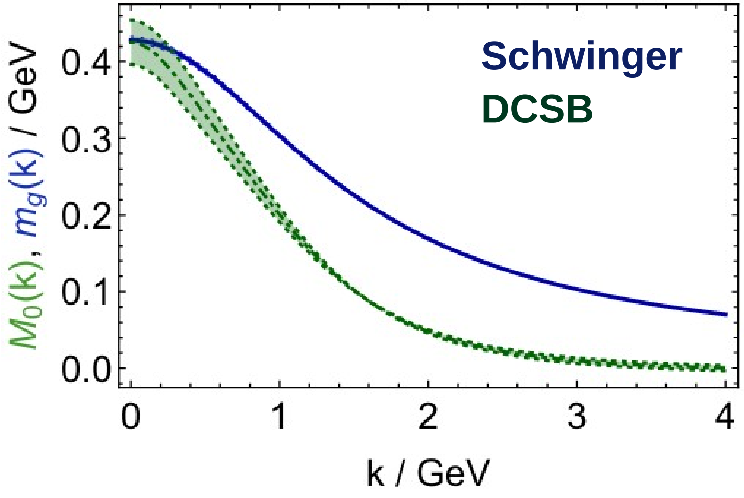
Can we trace them down to fundamental d.o.f?

(figure: D. Binosi's courtesy!)



Cui:2019dwv

- Emergence of hadron masses (EHM) from QCD **dynamics**

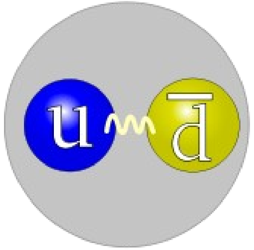


Gluon and quark *running masses*

Pion **Structure**

Pion

- **“Two”** quark **bound-state**

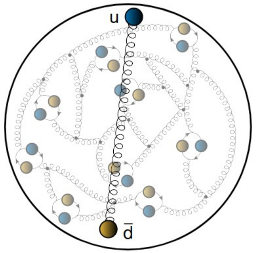


- **Archetype** of nuclear exchange **forces**
  - *Our favorite mediator*
- The **lightest** hadron in nature

Pion Structure

Pion

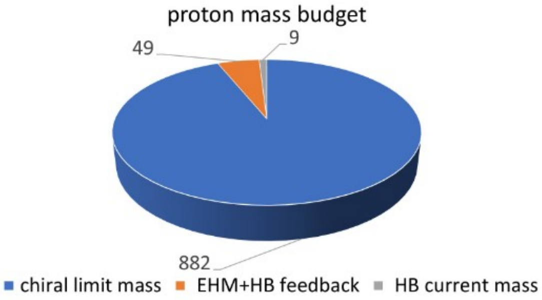
- **“Two”** quark **bound-state**

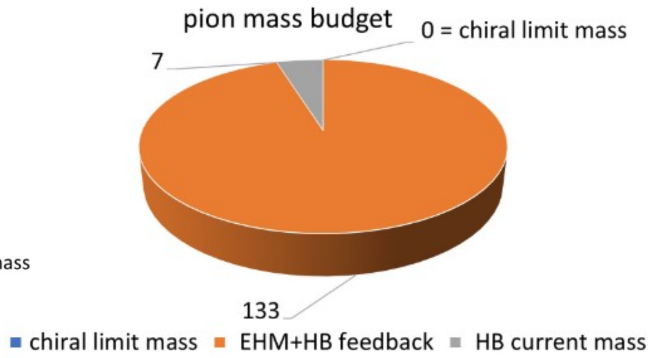


$M_{u,d} \sim 310 \text{ MeV}$

$$M_u + M_d \neq 140 \text{ MeV}$$

- Unlike the proton, pion is **massless** in the absence of **Higgs** mass generation





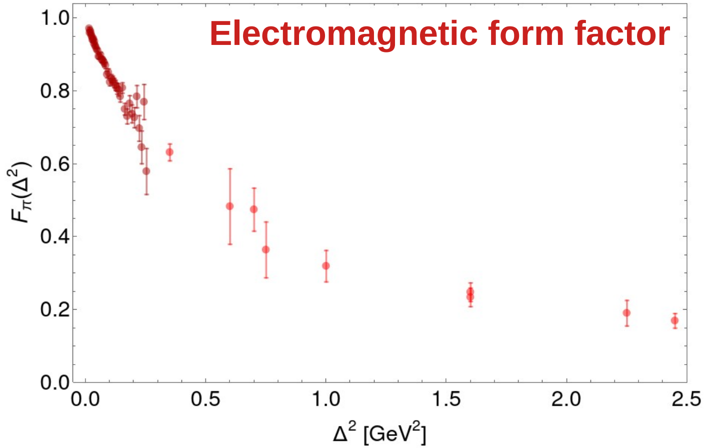
- **Archetype** of nuclear exchange **forces**
  - *Our favorite mediator*
- The **lightest** hadron in nature

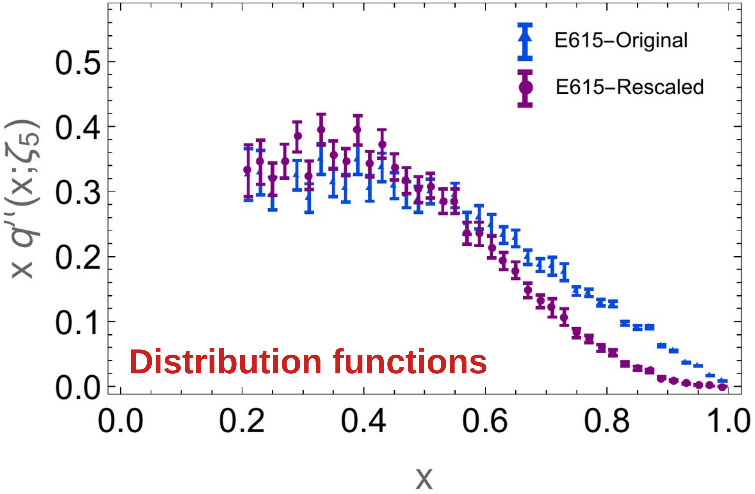
→ Both a quark-antiquark **bound-state** and a **Golstone Boson**

→ Its mere **existence** is connected with **mass** generation in the **SM**

Pion **Structure**

- The experimental access to the pion structure is via electromagnetic probes, yielding e.g.:





- Generalized parton distributions (**GPDs**) encode them **both** (and many more):

$$F_\pi(\Delta^2) = \int H_\pi^u(x,\xi,-\Delta^2;\zeta)\;,\;\; u^\pi(x;\zeta) = H_\pi^u(x,0,0;\zeta)$$

  - But the experimental access and theoretical derivation is **far more complicated**.

Pion **GPD**

- **Question**:
  From 1-dimensional distributions (**EFF** and **PDF**), can we obtain the 3-dimensional **GPD**?

Pion GPD

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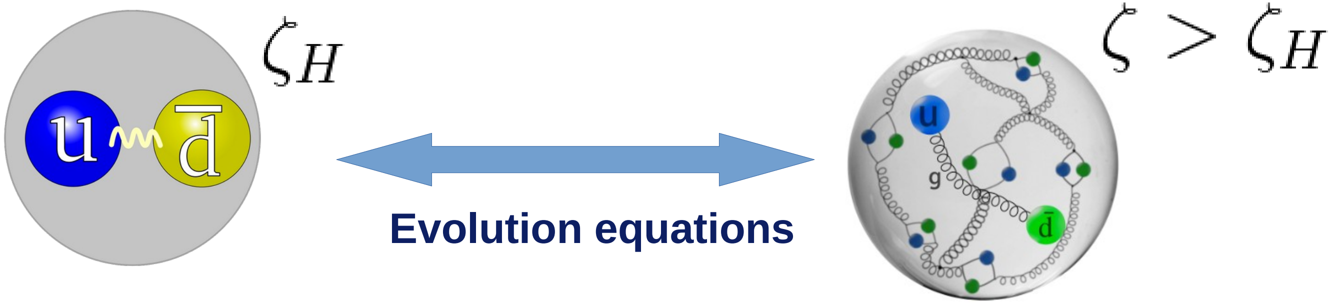
  From 1-dimensional distributions (**EFF** and **PDF**), can we obtain the 3-dimensional **GPD**?

- ~~Partial~~ **Answer:**
  Yes, we can. Under two premises:

  - *There exists at least one effective charge such that, when used to integrate the leading-order perturbative DGLAP equations, an evolution scheme for parton DFs is defined that is all-order exact.*



Thus, we can plainly connect the **parton** and **quasiparticle** picture in a well determined manner.

Pp's talk

Pion GPD

- **Question**:
  From 1-dimensional distributions (**EFF** and **PDF**), can we obtain the 3-dimensional **GPD**?

- ~~Partial~~ **Answer**:
  Yes, we can. Under two premises:

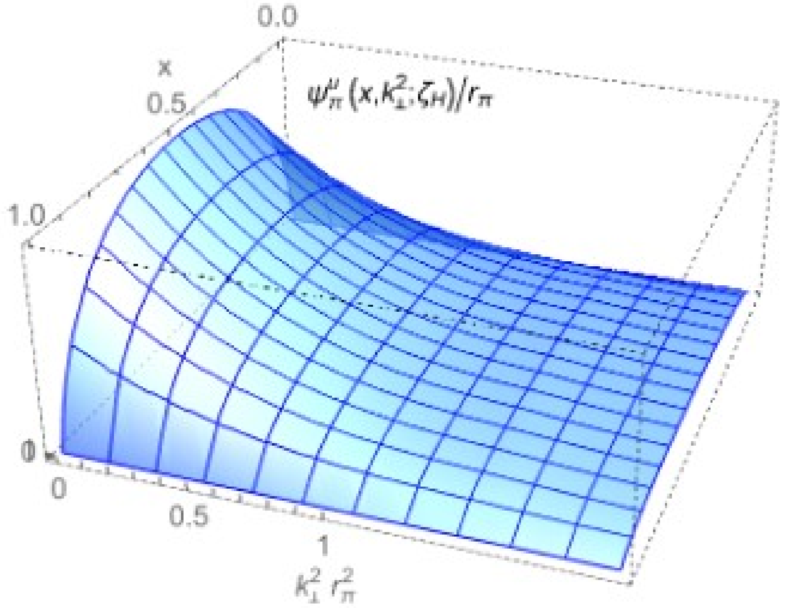
  - *There exists at least one effective charge such that, when used to integrate the leading-order perturbative DGLAP equations, an evolution scheme for parton DFs is defined that is **all-order** exact.*

  - *A **factorised** representation of the pion light-front wave function (**LFWF**), from which the (DGLAP) **GPD** is derived, at the hadronic scale, is a sensible approximation.*

Overlap: $$H_\pi^u(x,\xi,-\Delta^2;\zeta_H) = \int \frac{d^2k_\perp}{16\pi^3}\psi_\pi^{u*}(x_-,k_{\perp-}^2;\zeta_H)\psi_\pi^u(x_+,k_{\perp+}^2;\zeta_H)$$

Factorization: $$\psi_\pi^u(x,k_\perp^2;\zeta_H) = \tilde{\psi}_\pi^u(k_\perp^2)\left[u^\pi(x;\zeta_H)\right]^{1/2}$$

Light-front wave functions



$$\psi_P^u(x, k_\perp^2; \zeta)$$

“One ring to rule them all”

Raya:2021zrz
Raya:2022eqa

Light-front **wave functions**

- Many **distributions** are related via the leading-twist light-front wave function (**LFWF**), e.g.:

Distribution amplitudes

$$f_{\mathsf{P}}\varphi^u_{\mathsf{P}}(x,\zeta_{\mathcal{H}}) = \int \frac{dk^2_\perp}{16\pi^3}\psi^u_{\mathsf{P}}\left(x,k^2_\perp;\zeta_{\mathcal{H}}\right)$$

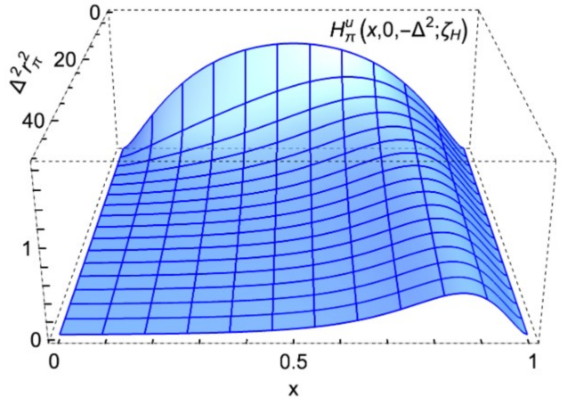
Distribution functions

$$u^{\mathsf{P}}(x;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3}\left|\psi^u_{\mathsf{P}}\left(x,k^2_\perp;\zeta_{\mathcal{H}}\right)\right|^2$$

- In the **DGLAP** kinematic domain, this is also the case of the valence-quark **GPD**:

$$H^u_{\mathsf{P}}(x,\xi,t;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3}\psi^{u*}_{\mathsf{P}}\left(x_-,k^2_{\perp -};\zeta_{\mathcal{H}}\right)\psi^u_{\mathsf{P}}\left(x_+,k^2_{\perp +};\zeta_{\mathcal{H}}\right)$$

$$x_\mp = (x\mp\xi)/(1\mp\xi)\,,\; t=-\Delta^2$$
$$k_{\perp\mp} = k_\perp \pm (\Delta_\perp/2)(1-x)/(1\mp\xi)$$



- ✔ The overlap approach guarantees the **positivity** of the **GPD**.
- It is, in principle, limited to the **DGLAP** kinematic region. $|x|\leq\xi$
- ✔ Nonetheless, it can be exteded to the **ERBL** domain.

JM and Pietro's talks Chavez:2021koz

Light-front **wave functions**

Albino:2022gzs
Raya:2021zrz
Raya:2022eqa

- Many **distributions** are related via the leading-twist light-front wave function (**LFWF**), e.g.:

Distribution amplitudes

$$f_{\mathsf{P}}\varphi_{\mathsf{P}}^{u}(x,\zeta_{\mathcal{H}}) = \int \frac{dk_{\perp}^{2}}{16\pi^{3}}\psi_{\mathsf{P}}^{u}\left(x,k_{\perp}^{2};\zeta_{\mathcal{H}}\right)$$

Distribution functions

$$u^{\mathsf{P}}(x;\zeta_{\mathcal{H}}) = \int \frac{d^{2}k_{\perp}}{16\pi^{3}}\left|\psi_{\mathsf{P}}^{u}\left(x,k_{\perp}^{2};\zeta_{\mathcal{H}}\right)\right|^{2}$$

- This connection already **suggests** that:

$$u^{\mathbf{P}}(x;\zeta_{H}) \sim [\varphi_{\mathbf{P}}^{u}(x;\zeta_{H})]^{2}$$

is a fair approximation, implying:

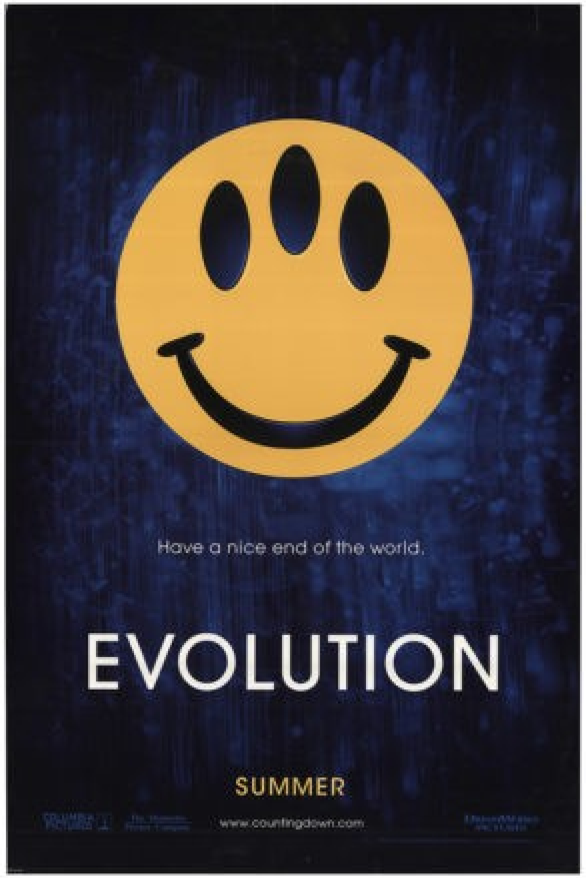
$$\psi_{\mathbf{P}_u}^{\uparrow\downarrow}(x,k_{\perp}^{2};\zeta_{H}) = \tilde{\psi}_{\mathbf{P}_u}^{\mathbf{u}}(k_{\perp}^{2})\,[u^{\mathbf{P}}(x;\zeta_{H})]^{1/2}$$

- In fact, we have learned that *x*-*k* crossed terms are weighted by: $M_{\mathbf{P}}^{2},\ M_{h}^{2}-M_{q}^{2}$ (factorised LFWF)

→ So a **factorised Ansatz** should be sensible for the **pion,** implying**:**

$$H_{\mathsf{P}}^{u}(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_{-})\sqrt{u^{\mathbf{P}}(x_{-};\zeta_{H})u^{\mathbf{P}}(x_{+};\zeta_{H})}\,\Phi_{\mathbf{P}}(z;\zeta_{H})$$

$$z = \frac{(1-x)^{2}}{(1-\xi^{2})^{2}}\Delta_{\perp}^{2}$$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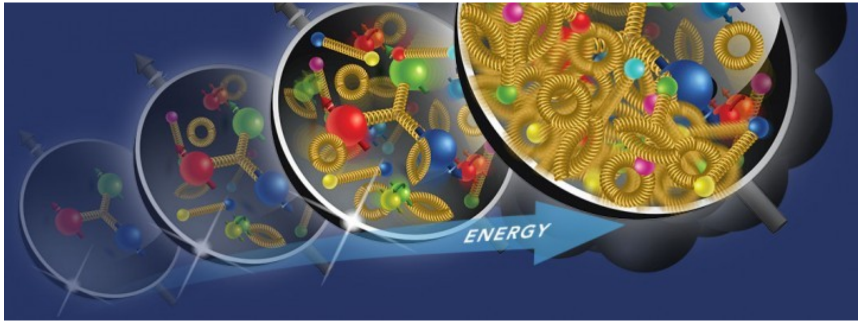
DGLAP: All orders evolution

Idea. Define an **effective** coupling such that:

"All orders evolution"

Starting from fully-dressed **quasiparticles**, at ζ_H → **Sea** and **Gluon** content unveils, as prescribed by **QCD**

$$\left\{\zeta^2\frac{d}{d\zeta^2}\int_0^1 dy\,\delta(y-x) - \frac{\alpha(\zeta^2)}{4\pi}\int_x^1 \frac{dy}{y}\begin{pmatrix} P_{qq}^{\rm NS}\left(\frac{x}{y}\right) & 0 \\ 0 & \mathbf{P}^{\rm S}\left(\frac{\mathbf{x}}{\mathbf{y}}\right)\end{pmatrix}\right\}\begin{pmatrix} H_\pi^{{\rm NS},+}(y,t;\zeta) \\ \mathbf{H}_\pi^{\rm S}(y,t;\zeta)\end{pmatrix} = 0$$



- Not the **LO** QCD coupling but an **effective** one.
- Making this equation **exact**.
- And connecting with the **hadron scale.**

Raya:2021zrz Cui:2020tdf

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi} S(\zeta_H, \zeta_f)\right) \langle x^n(\zeta_H)\rangle_q$$

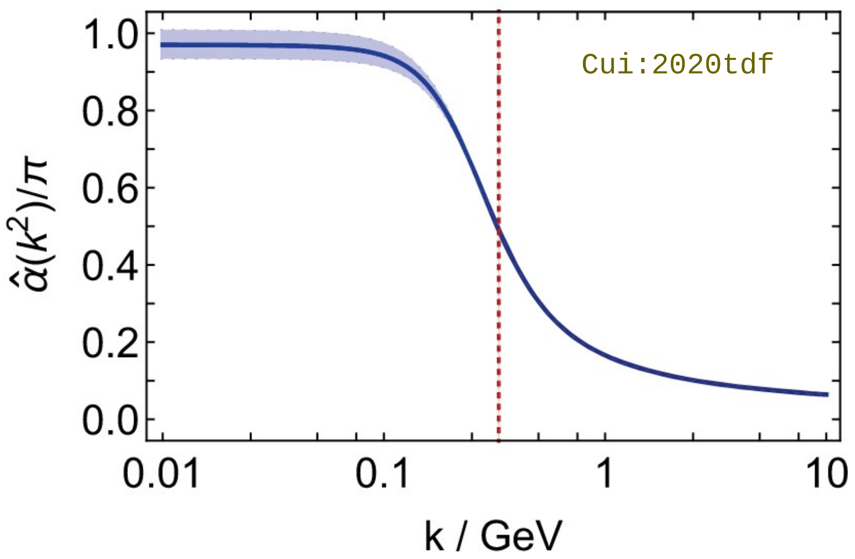
$$S(\zeta_0, \zeta_f) = \int_{2\ln(\zeta_0/\Lambda_{QCD})}^{2\ln(\zeta_f/\Lambda_{QCD})} dt\, \alpha(t)$$

Explicitly depending on the **effective charge**

$$\langle x^n(t;\zeta)\rangle_F = \int_0^1 dx\, x^n\, F(x,t;\zeta)$$

$$\gamma_{AB}^{(n)} = -\int_0^1 dx\, x^n P_{AB}^C(x)$$

- The **QCD PI effective charge** is our best candidate to accommodate our **all orders scheme**.

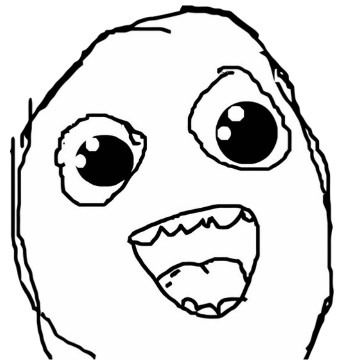


$$\hat{\alpha}(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{M}^2(k^2)}{\Lambda_{QCD}^2}\right]} \Rightarrow \zeta_H = 0.331\ \text{GeV}$$

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q \left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$



$$S(\zeta_0,\zeta_f) = \int_{2\ln(\zeta_0/\Lambda_{\rm QCD})}^{2\ln(\zeta_f/\Lambda_{\rm QCD})} dt\,\alpha(t)$$

This contains, ***implicitly***, the information of the **effective charge**

- No actual ***need*** to know it. Assuming its existence is **sufficient**.
- ***Unambiguous*** definition of the **hadron scale**:

$$\langle x(\zeta_H)\rangle_q = 0.5 \;\Rightarrow\; \langle x^n(\zeta_f)\rangle_q = \langle x^n(\zeta_H)\rangle_q \left(\langle 2x(\zeta_f)\rangle_q\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

(flavor symmetric case)

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q \underbrace{\left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)}^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

Information on the charge is here

- Details of the effective charge are **encoded** in the ratio of first moments.
- Natural connection with the **hadron scale**.

Implication 2:

$$\begin{aligned}
\langle 2x(\zeta_f)\rangle_q &= \exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_f)\right), \qquad q=u,\bar{d}\,;\\
\langle x(\zeta_f)\rangle_{\text{sea}} &= \langle x(\zeta_f)\rangle_{\sum_q q+\bar{q}} - \left(\langle x(\zeta_f)\rangle_u + \langle x(\zeta_f)\rangle_{\bar{d}}\right),\\
&= \frac{3}{7} + \frac{4}{7}\langle 2x(\zeta_f)\rangle_u^{7/4} - \langle 2x(\zeta_f)\rangle_u\\
\langle x(\zeta_f)\rangle_g &= \frac{4}{7}\left(1-\langle 2x(\zeta_f)\rangle_u^{7/4}\right)\,;
\end{aligned}$$

- **Sea** and **gluon** determined from valence-quark moments

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q \left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

Information on the charge is here

- Can **jump** from one scale to another (both ways)
- Natural connection with the **hadron scale**.

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\end{aligned}$$

- **Sea** and **gluon** determined from valence-quark moments
- **Asymptotic** (massless) limits are evident.

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q \left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

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- Can **jump** from one scale to another (both ways)
- Natural connection with the **hadron scale**.

Implication 2:

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&= \frac{3}{7} + \frac{4}{7}\langle 2x(\zeta_f)\rangle_u^{7/4} - \langle 2x(\zeta_f)\rangle_u \\
\langle x(\zeta_f)\rangle_g &= \frac{4}{7}\left(1 - \langle 2x(\zeta_f)\rangle_u^{7/4}\right)\,;
\end{aligned}$$

- **Sea** and **gluon** determined from valence-quark moments
- **Asymptotic** (massless) limits are evident.
- And, of course, the momentum **sum rule**:

$$\langle 2x(\zeta_f)\rangle_q + \langle x(\zeta_f)\rangle_{\text{sea}} + \langle x(\zeta_f)\rangle_g = 1$$

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q\left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

Information on the charge is here

- Can **jump** from one scale to the another (even downwards)
- Natural connection with the **hadron scale**.

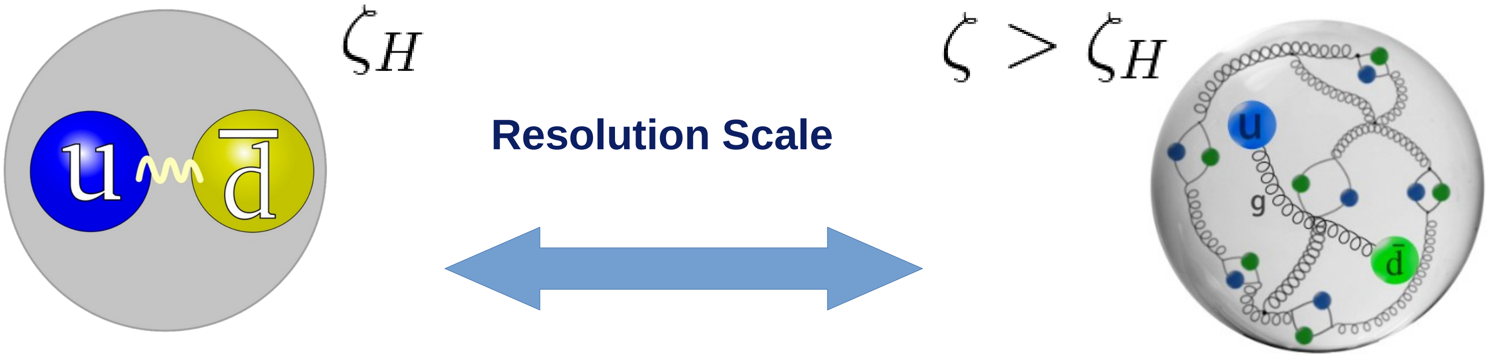
Implication 3: Recurrence relation

$$\langle x^{2n+1}\rangle_{u_\pi}^{\zeta} = \frac{(\langle 2x\rangle_{u_\pi}^{\zeta})^{\gamma_0^{2n+1}/\gamma_0^1}}{2(n+1)} \times \sum_{j=0,1,\ldots}^{2n}(-)^j\binom{2(n+1)}{j}\langle x^j\rangle_{u_\pi}^{\zeta}(\langle 2x\rangle_{u_\pi}^{\zeta})^{-\gamma_0^j/\gamma_0^1}\,.$$

- Since **isospin symmetry** limit implies:

$$q(x;\zeta_H) = q(1-x;\zeta_H)$$

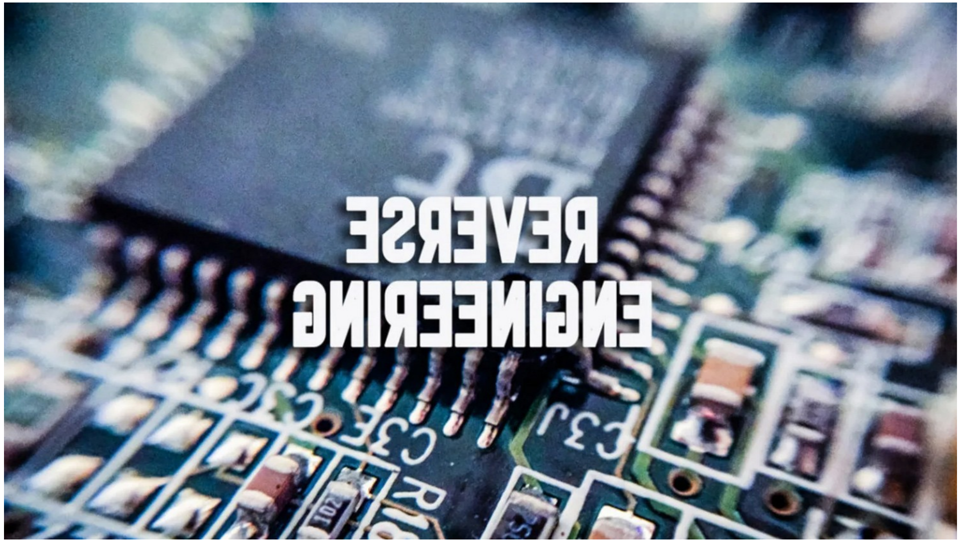
- **Odd** moments can be expressed in terms of previous **even** moments.
- Thus arriving at the recurrence **relation** on the left.



- **Fully-dressed valence quarks**
  (quasiparticles)

- **Unveiling of glue and sea d.o.f**
  (partons)

Reverse engineering: PDF data



Pion PDF

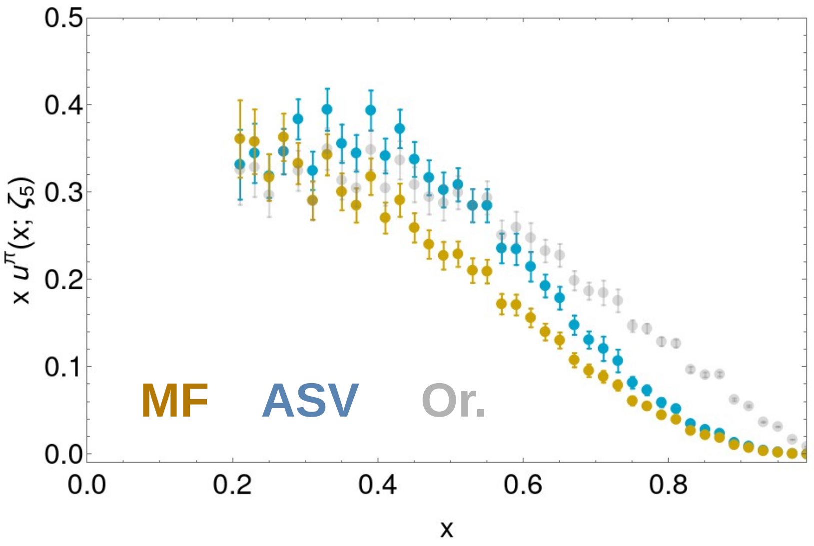
- Let us assume the data can be parameterized with a certain functional form, i.e.:

$$u^\pi(x;[\alpha_i];\zeta) = n_u^\zeta x^{\alpha_1^\zeta}(1-x)^{\alpha_2^\zeta}(1+\alpha_3^\zeta x^2)$$

$$\{\alpha_i^\zeta | i = 1,2,3\}$$

Normalization

Free parameters



- Then, we proceed as follows:

1) Determine the **best values α$_i$** via least-squares fit to the data.

2) Generate new **values α$_i$,** distributed randomly around the best fit.

3) Using the latter set, evaluate:

$$\chi^2 = \sum_{l=1}^{N} \frac{(u^\pi(x_l;[\alpha_i];\zeta_5) - u_j)^2}{\delta_l^2}$$

Data point with error

4) Accept a replica with probability:

$$\mathcal{P} = \frac{P(\chi^2;d)}{P(\chi_0^2;d)}\,,\quad P(y;d) = \frac{(1/2)^{d/2}}{\Gamma(d/2)} y^{d/2-1} e^{-y/2}$$

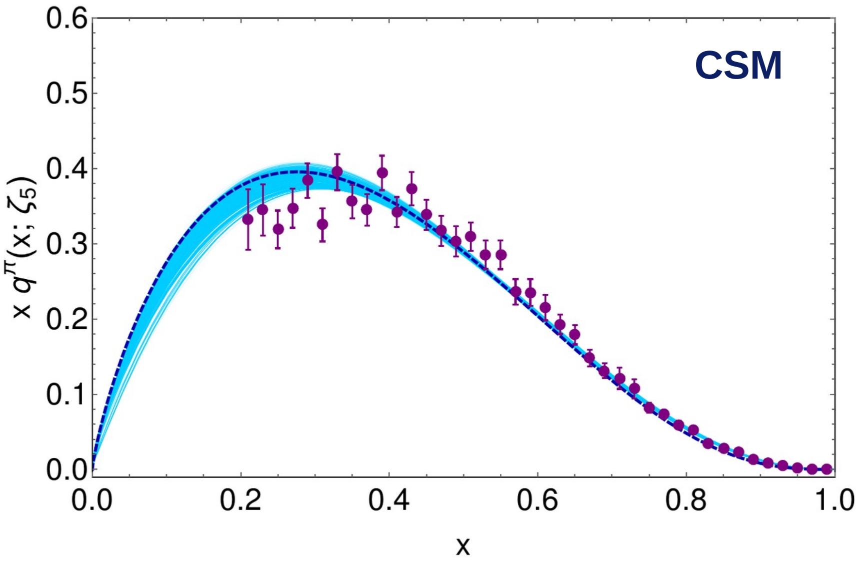
5) Evolve back to ζ_H

Repeat (2-5).

Pion PDF: ASV Data

➢ Applying this algorithm to the **ASV data** yields:

(average)



Mean values (of moments) and errors

{{0.5, 2.75144 × 10^-17}, {0.299833, 0.00647045}, {0.199907, 0.00735448}, {0.142895, 0.0068623}, {0.107274, 0.00608759}, {0.0835168, 0.00532834}, {0.0668711, 0.0046596}, {0.0547511, 0.00409028}, {0.0456496, 0.00361041}, {0.0386394, 0.00320609}}

Moments from SCI, ζ_H

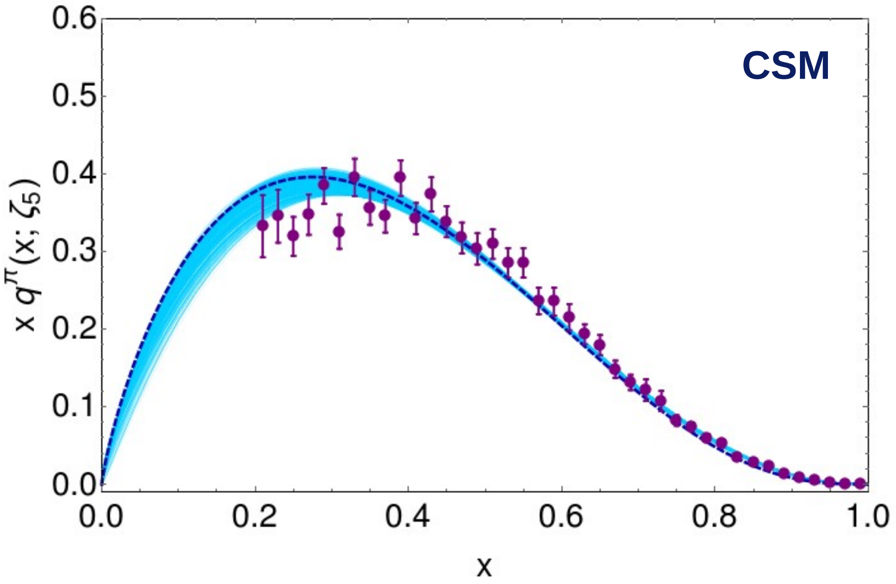
{0.5, 0.332885, 0.249327, 0.199231, 0.165865, 0.142056, 0.124215, 0.11035, 0.0992657, 0.090203, 0.0826552, 0.0762721, 0.0708035, 0.0660661, 0.0619225}

✔ The produced moments are compatible with a **symmetric PDF** at the **hadronic scale**.

✔ Not at all similar to those from SCI

Pion PDF: ASV Data

- Applying this algorithm to the **ASV data** yields:

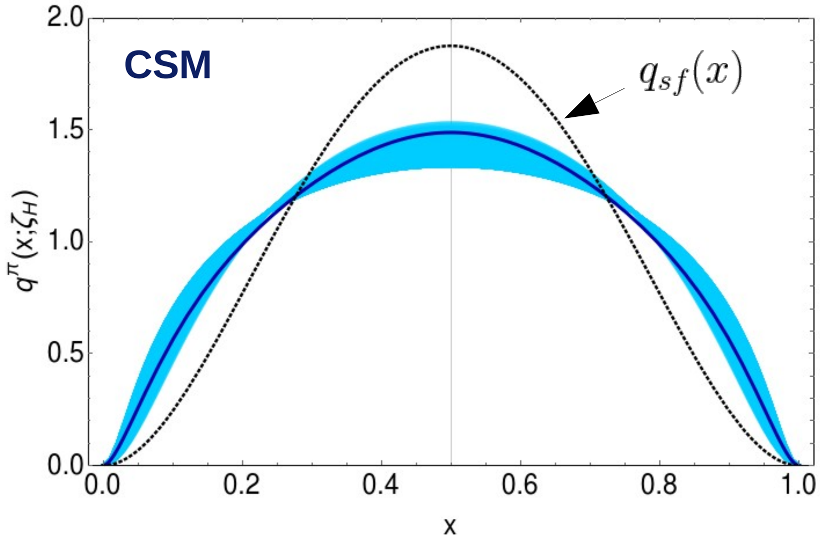


Mean values (of moments) and errors

{{0.5, 2.75144 × 10^-17}, {0.299833, 0.00647045}, {0.199907, 0.00735448}, {0.142895, 0.0068623}, {0.107274, 0.00608759}, {0.0835168, 0.00532834}, {0.0668711, 0.0046596}, {0.0547511, 0.00409028}, {0.0456496, 0.00361041}, {0.0386394, 0.00320609}}

✔ Then, we can **reconstruct** the moments produced by each replica, using the single-parameter **Ansatz**:

$$u^{\pi}(x;\zeta_{\mathcal{H}}) = n_0 \ln(1 + x^2(1-x)^2/\rho^2)$$



✔ The produced moments are compatible with a **symmetric PDF** at the **hadronic scale**.

✔ It exhibits a **soft end-point** behavior…

Pion PDF: Lattice Data

- We can follow an analogous procedure to infer, based upon **lattice data**, how the **hadronic scale PDF** should look like.

- Let us consider the list of **lattice QCD** moments:

n	[61]	[62]	[63]
1	0.254(03)	0.18(3)	0.23(3)(7)
2	0.094(12)	0.064(10)	0.087(05)(08)
3	0.057(04)	0.030(05)	0.041(05)(09)
4			0.023(05)(06)
5			0.014(04)(05)
6			0.009(03)(03)

Joo:2019bzr
Sufian:2019bol
Alexandrou:2021mmi

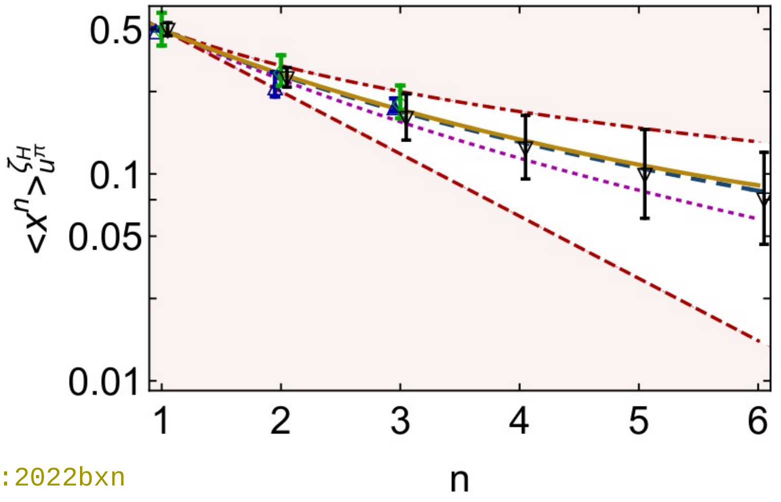
- Those verify the recurrence relation, thus being compatible with a **symmetric PDF** at ζ_H

- While also falling within the **physical bounds**.

$$\frac{1}{2^n} \leq \langle x^n \rangle^{\zeta}_{u_\pi} \left(\langle 2x \rangle^{\zeta}_{u_\pi} \right)^{-\gamma_0^n/\gamma_0^1} \leq \frac{1}{1+n}$$

Produced by $q(x;\zeta_H) = \delta(x - 1/2)$ (infinitely heavy valence quarks)

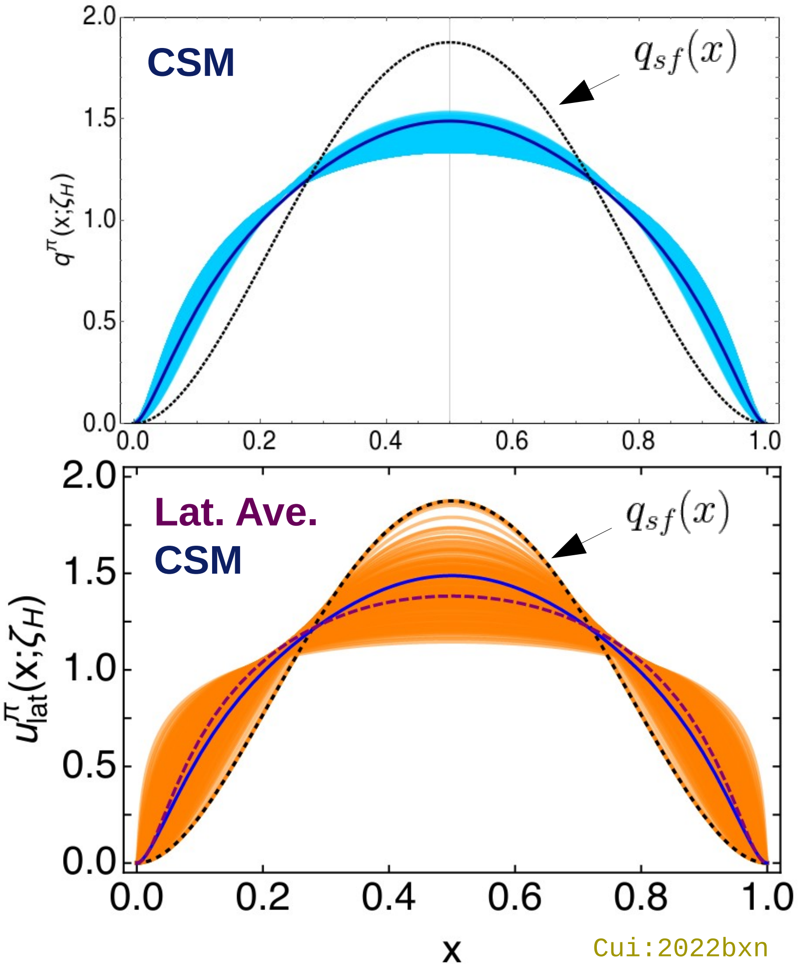
Produced by $q(x;\zeta_H) = 1$ (massless SCI case)



Cui:2022bxn

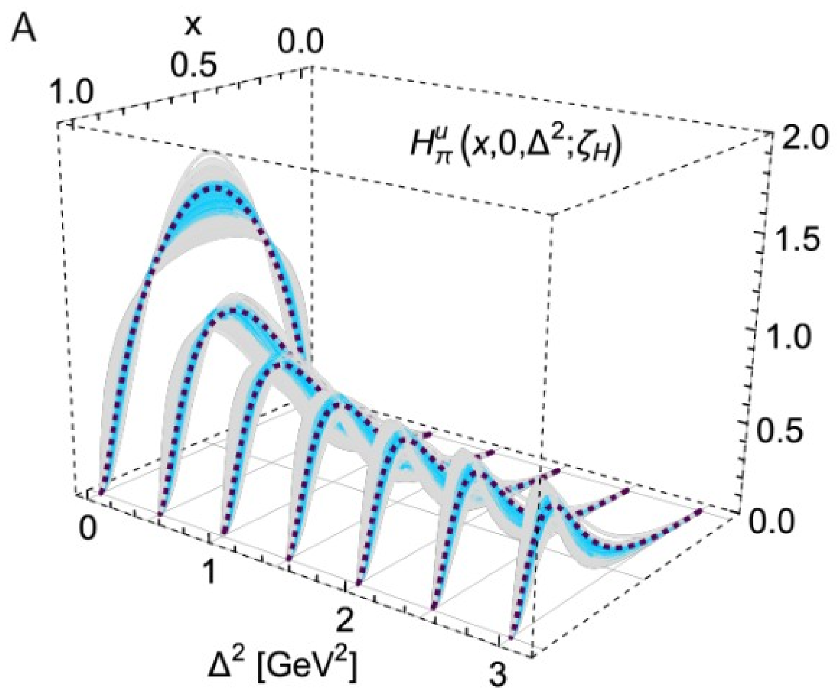
Pion PDF: Recap.

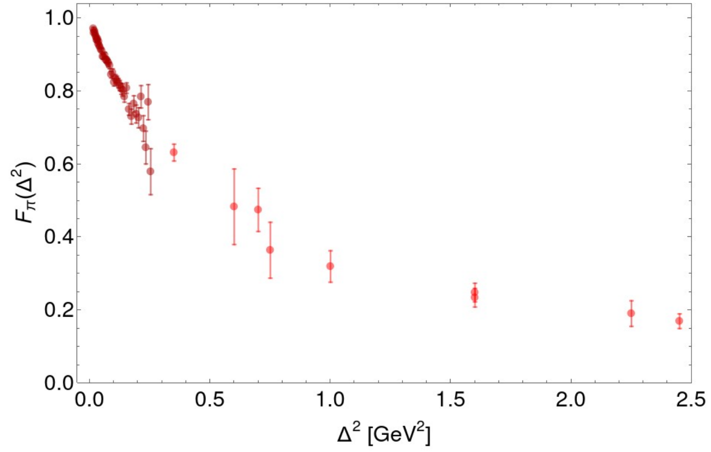
- The (**original**) **experimental** data yield a hadronic scale **PDF** compatible with **SCI results**.
  - ➔Thus should be disfavored since it does not produce the expected large-x behavior.
- Both (**ASV**) **experimental** and **lattice** data yield hadronic scale **PDFs** exhibiting soft end-point behavior and **EHM-induced broadening**.
- The results are **compatible**, although current precision of the lattice moments still leaves us with a somewhat **wide band** of **uncertainty**.
- Thus we focus on the **ASV** data for the rest of the discussion.

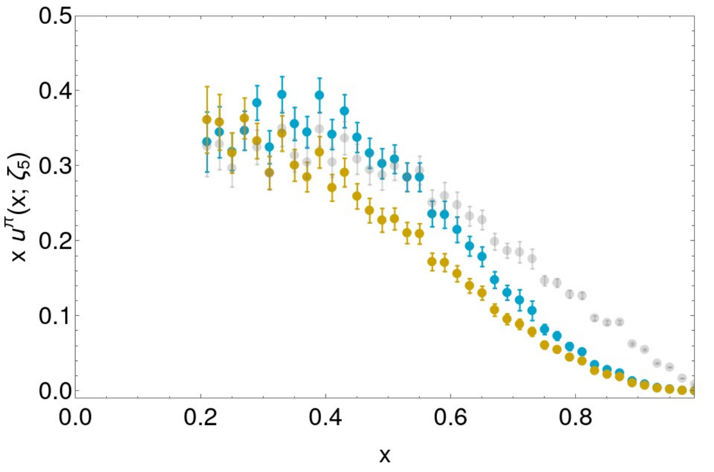


Cui:2022bxn

GPD from PDF and EFF







LFWF: Factorized models

Raya:2021zrz

- Starting with a **factorized** **LFWF**, $\psi^{\uparrow\downarrow}_{\mathbf{P}u}(x,k_\perp^2;\zeta_H) = \tilde{\psi}^{\mathbf{u}}_{\mathbf{P}u}(k_\perp^2)\,[u^{\mathbf{P}}(x;\zeta_H)]^{1/2}$

This one shall be obtained as described previously

- The overlap representation for the **GPD** entails:

$$H^u_{\mathrm{P}}(x,\xi,t;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3}\psi^{u*}_{\mathrm{P}}\left(x_-,k^2_{\perp -};\zeta_{\mathcal{H}}\right)\psi^{u}_{\mathrm{P}}\left(x_+,k^2_{\perp +};\zeta_{\mathcal{H}}\right)$$

$$= \Theta(x_-)\sqrt{u^{\mathbf{P}}(x_-;\zeta_H)u^{\mathbf{P}}(x_+;\zeta_H)}\Phi_{\mathbf{P}}(z;\zeta_H)$$

Heaviside Theta

This dictates the off-forward behavior of the GPD

- Where $z = s_\perp^2 = -t(1-x)^2/(1-\xi^2)^2$ and:

… will be driven by the electromagnetic form factor

$$\Phi^u_{\mathbf{P}}(z;\zeta_H) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\tilde{\psi}^{u*}_{\mathbf{P}}\left(\mathbf{k}_\perp^2;\zeta_H\right)\tilde{\psi}^{u}_{\mathbf{P}}\left((\mathbf{k}_\perp-\mathbf{s}_\perp)^2;\zeta_H\right)$$

Setting the Stage

➢ Recall a **GPD** arising from a factorised **LFWF** adopts the form:

$$H_\pi^u(x,\xi,-\Delta^2;\zeta_H) = \theta(x_-)\sqrt{u^\pi(x_-;\zeta_H)u^\pi(x_+;\zeta_H)}\,\Phi^\pi(z^2;\zeta_H)$$

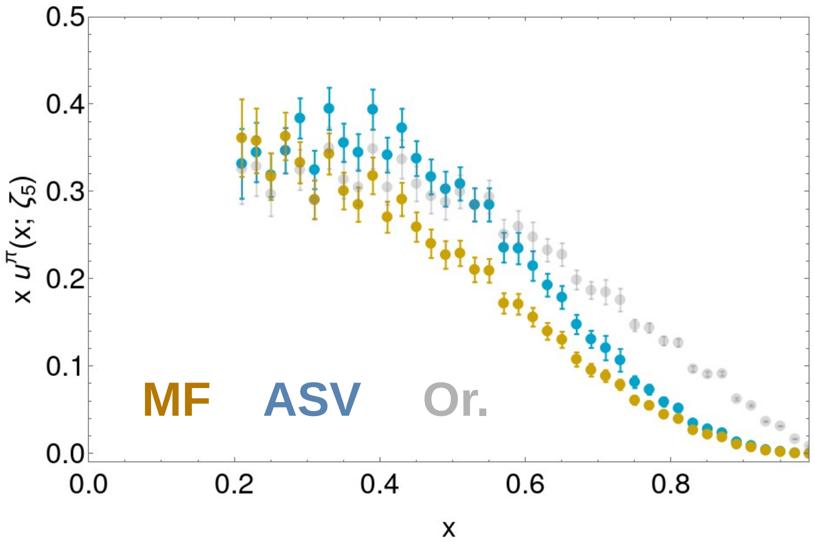
$$u^\pi(x;\zeta_H) = n_0 \ln(1 + x^2(1-x)^2/\rho^2)$$

➢ The empirical data on **PDF** to contrast with:

- **ASV** analysis.
- **MF** resummation.
- **Lattice** QCD moments.

For references, see:

Cui:2022bxn
Cui:2021mom



Setting **the Stage**

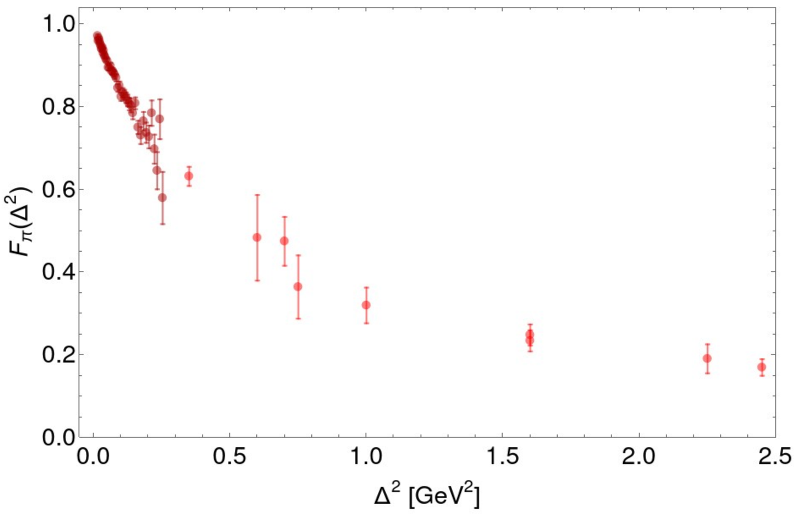
$$F_\pi(t) = \int_0^1 dx\, u^\pi(x;\zeta_H)\Phi_\pi(z;\zeta_H)$$

- We thus employ a **3-parameter** model for the **GPD:**: $\{\rho, \beta, \gamma\}$

$$H_\pi^u(x,\xi,-\Delta^2;\zeta_H) = \theta(x_-)\sqrt{u^\pi(x_-;\zeta_H)u^\pi(x_+;\zeta_H)}\,\Phi^\pi(z^2;\zeta_H)$$

$$\lambda = \beta - \frac{r_\pi^2}{6\langle x^2\rangle_{u_\pi}^{\zeta_H}}$$

$$\Phi^\pi(y;\zeta_H) = \frac{1+\lambda y}{1+\beta y+\gamma^2 y^2}$$



- The empirical data on **EFF**:
  - **JLab** data.
  - **Charge** radius: $r_\pi = 0.64(2)\,\text{fm}$

SPM extraction

Cui:2021aee

Conservative "Gaussian" error

  - Given r_π, low-Q^2 data is redundant.

The **Algorithm**

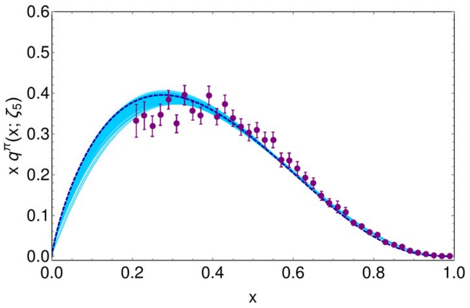
$$H_\pi^u(x,\xi,-\Delta^2;\zeta_H) = \theta(x_-)\sqrt{u^\pi(x_-;\zeta_H)u^\pi(x_+;\zeta_H)}\,\Phi^\pi(z^2;\zeta_H)$$

1. For the chosen **PDF data** set, generate a **replica**. The replica would be accepted following the aforementioned *chi-2* criteria.

2. After acceptance, **evolve** it to the **hadronic scale** using many Mellin moments. The *de-evolved* PDF shall be reconstructed using the functional form:

$$u^\pi(x;\zeta_H) = n_0 \ln(1 + x^2(1-x)^2/\rho^2)$$

3. Store both the value $\boldsymbol{\rho_i}$ and the probability of acceptance $\mathbf{P(\rho_i)}$.



The Algorithm

$$H_\pi^u(x,\xi,-\Delta^2;\zeta_H) = \theta(x_-)\sqrt{u^\pi(x_-;\zeta_H)u^\pi(x_+;\zeta_H)}\,\Phi^\pi(z^2;\zeta_H)$$

4. Keeping the selected **PDF**, we now constrast **Φ** with the **EFF** data, via:

$$F_\pi(t) = \int_0^1 dx\, u^\pi(x;\zeta_H)\Phi_\pi(z;\zeta_H)$$

$$\Phi^\pi(y;\zeta_H) = \frac{1+\lambda y}{1+\beta y+\gamma^2 y^2}$$

$$\lambda = \beta - \frac{r_\pi^2}{6\langle x^2\rangle_{u_\pi}^{\zeta_\mathcal{H}}}$$

5. Employing a *chi-2* criteria, we compute the probability of acceptance **P($\Phi_i|\rho_i$)**.

6. The GPD is accepted with probability **P($\Phi_i|\rho_i$)P(ρ)**.

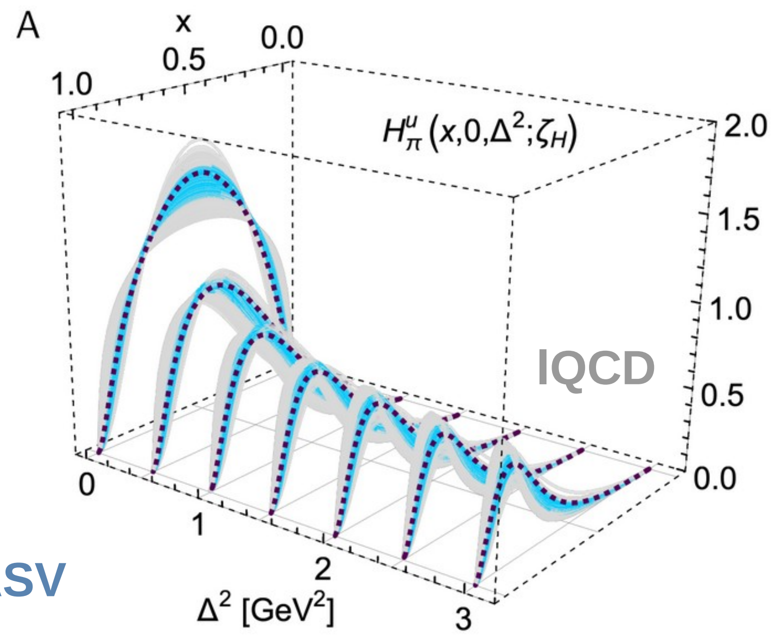
REPEAT

Numerical Results

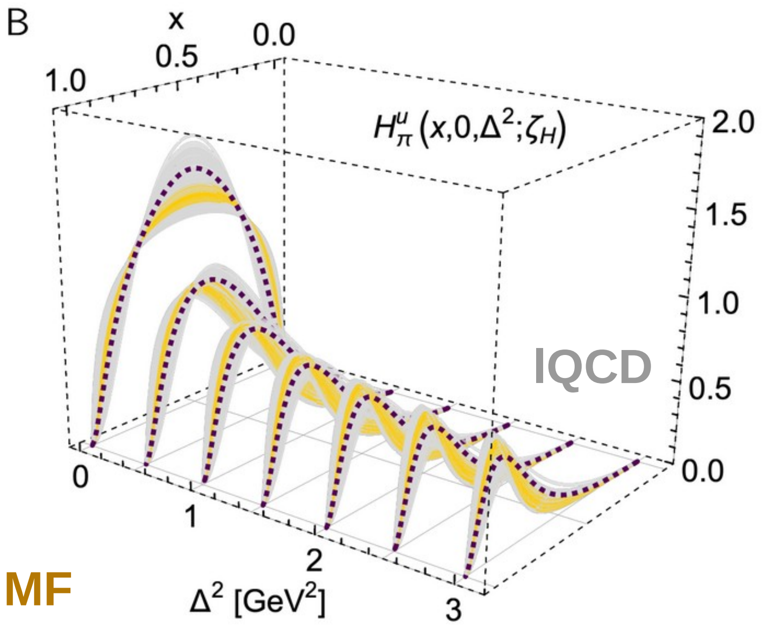
Pion GPD

$$H^u_\pi(x,\xi,-\Delta^2;\zeta_H) = \theta(x_-)\sqrt{u^\pi(x_-;\zeta_H)u^\pi(x_+;\zeta_H)}\,\Phi^\pi(z^2;\zeta_H)$$

- Applying this procedure, from the pion **PDF** and **EFF** empirical data, one gets the **GPDs**:



ASV



MF

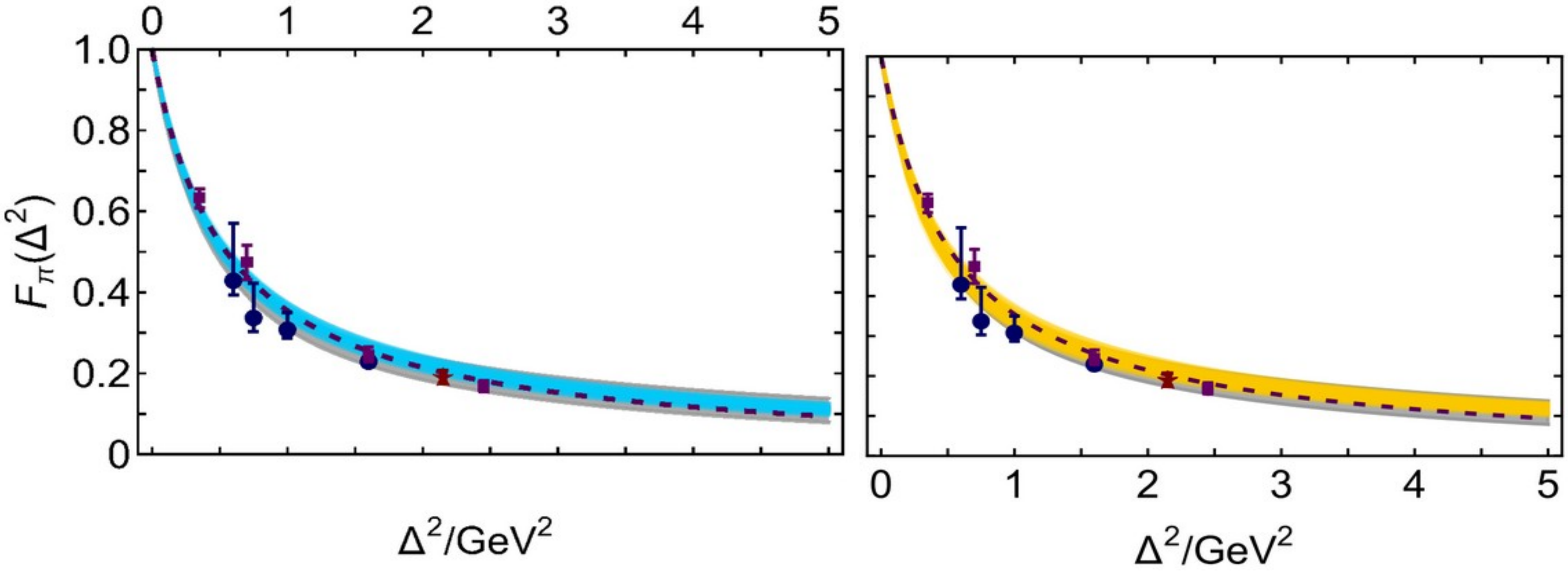
CSM:

Raya:2021zrz
Raya:2022eqa

Pion **EFF**

$$H_\pi^u(x,\xi,-\Delta^2;\zeta_H) = \theta(x_-)\sqrt{u^\pi(x_-;\zeta_H)u^\pi(x_+;\zeta_H)}\,\Phi^\pi(z^2;\zeta_H)$$

- For the **EFF**, we essentially arrive at the same results.

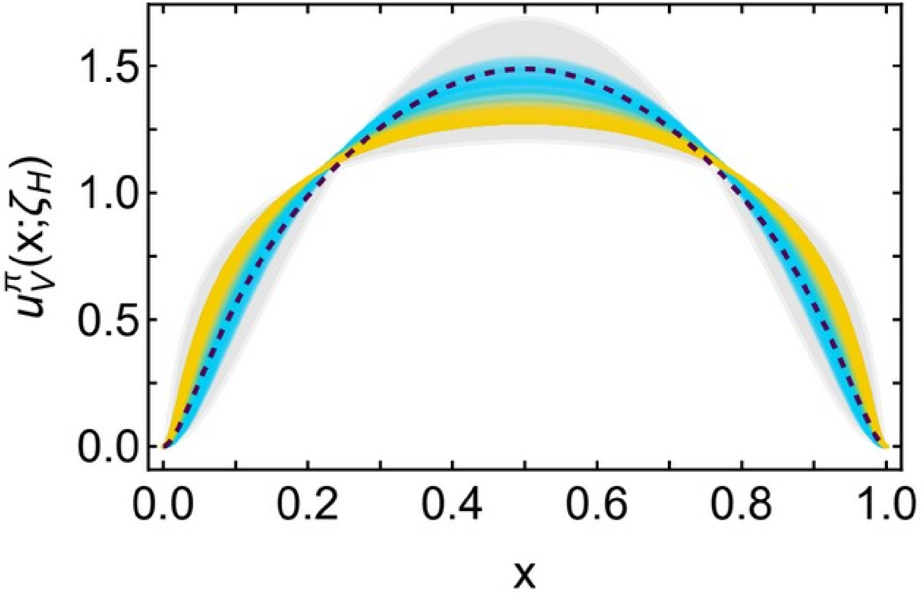


	r_π/fm
ASV	$0.640(20)$
MF	$0.638(18)$
lQCD	$0.639(19)$

Pion **PDF**

$$H_\pi^u(x,\xi,-\Delta^2;\zeta_H) = \theta(x_-)\sqrt{u^\pi(x_-;\zeta_H)u^\pi(x_+;\zeta_H)}\,\Phi^\pi(z^2;\zeta_H)$$

- The **PDFs** agree within errors, but...



- **Lattice** QCD **cannot distinguish** between **ASV**, **MF** or the *scale-free* profiles.

n	[61]	[62]	[63]
1	0.254(03)	0.18(3)	0.23(3)(7)
2	0.094(12)	0.064(10)	0.087(05)(08)
3	0.057(04)	0.030(05)	0.041(05)(09)
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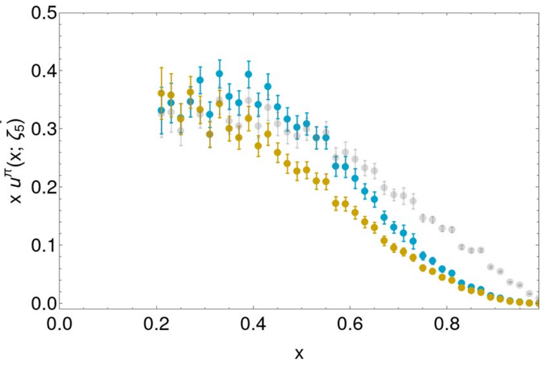
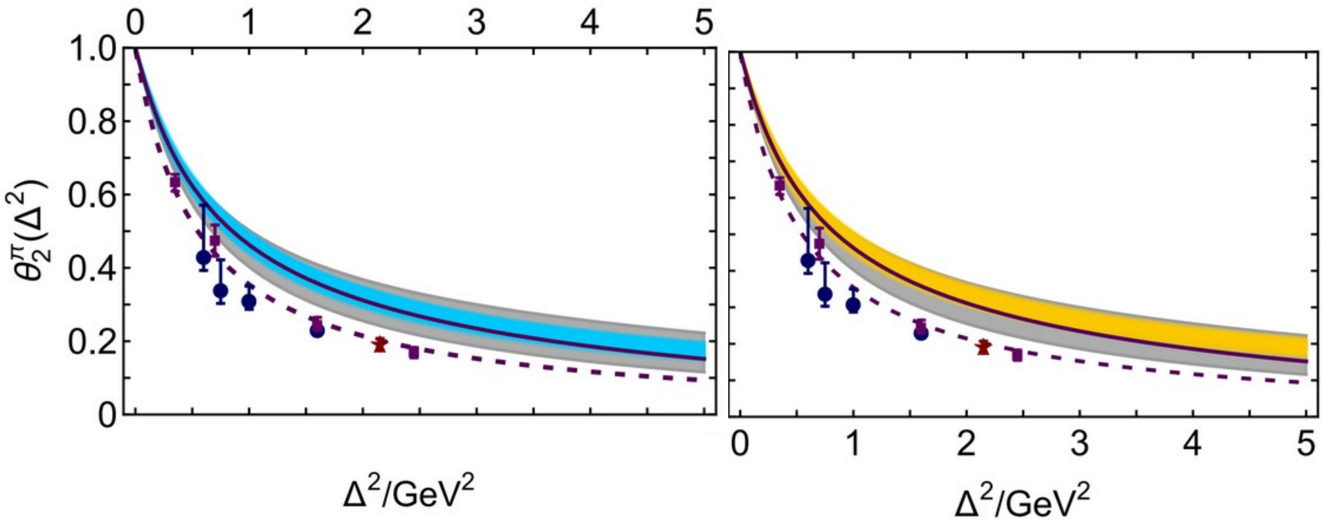


Mass **Distribution**

- The first Mellin moment of the GPD yields the **gravitational form factor**s:

$$\int_{-1}^{1} dx\, 2x H_\pi^u(x,\xi,-\Delta^2\ \zeta) = \theta_2^\pi(\Delta^2) - \xi^2\theta_1^\pi(\Delta^2)$$

- θ_1 currently escapes our approach, but θ_2 is within reach: $\theta_2^\pi(\Delta^2) = \int_0^1 dx\, 2x H_\pi(x,\xi=0,-\Delta^2)$



θ_2 is associated with the **mass distribution.**

- We found the **mass** radii:

	ASV	MF	lQCD
$r_\pi^{\theta_2}$	0.518(16)	0.498(14)	0.512(21)

Recall: $r_\pi = 0.64(2)\,\text{fm}$

About Radii

$$H_{\mathrm{P}}^{u}(x,\xi,t;\zeta_H) = \theta(x_-)\left[u^{\mathrm{P}}(x_-;\zeta_H)u^{\mathrm{P}}(x_+;\zeta_H)\right]^{1/2}\Phi_{\mathrm{P}}(z;\zeta_H)$$

- In the **factorized** models:

$$\frac{\partial^n}{\partial^n z}\Phi_{\mathrm{P}}^{u}(z;\zeta_{\mathcal{H}})\Big|_{z=0} = \frac{1}{\langle x^{2n}\rangle_{\bar{h}}^{\zeta_{\mathcal{H}}}}\left.\frac{d^n F_{\mathrm{P}}^{u}(\Delta^2)}{d(\Delta^2)^n}\right|_{\Delta^2=0}$$

PDF moments — Derivatives of **EFF**

$$\frac{\partial}{\partial z}\Phi_{\mathrm{P}}^{u}(z;\zeta_{\mathcal{H}})\Big|_{z=0} = -\frac{r_{\mathrm{P}}^2}{4\chi_{\mathrm{P}}^2(\zeta_{\mathcal{H}})},$$

$$\frac{\partial}{\partial z}\Phi_{\mathrm{P}}^{\bar{h}}(z;\zeta_{\mathcal{H}})\Big|_{z=0} = (1-d_{\mathrm{P}})\frac{\partial}{\partial z}\Phi_{\mathrm{P}}^{u}(z;\zeta_{\mathcal{H}})\Big|_{z=0}$$

Asymmetry term = **0** for **pion**

- Therefore, the **mass *radius***:

$$r_{\mathrm{P}_u}^{\theta_2 2} = \frac{3r_{\mathrm{P}}^2}{2\chi_{\mathrm{P}}^2}\langle x^2(1-x)\rangle_{\mathrm{P}_{\bar{h}}},$$

$$r_{\mathrm{P}_{\bar{h}}}^{\theta_2 2} = \frac{3r_{\mathrm{P}}^2}{2\chi_{\mathrm{P}}^2}(1-d_{\mathrm{P}})\langle x^2(1-x)\rangle_{\mathrm{P}_u}$$

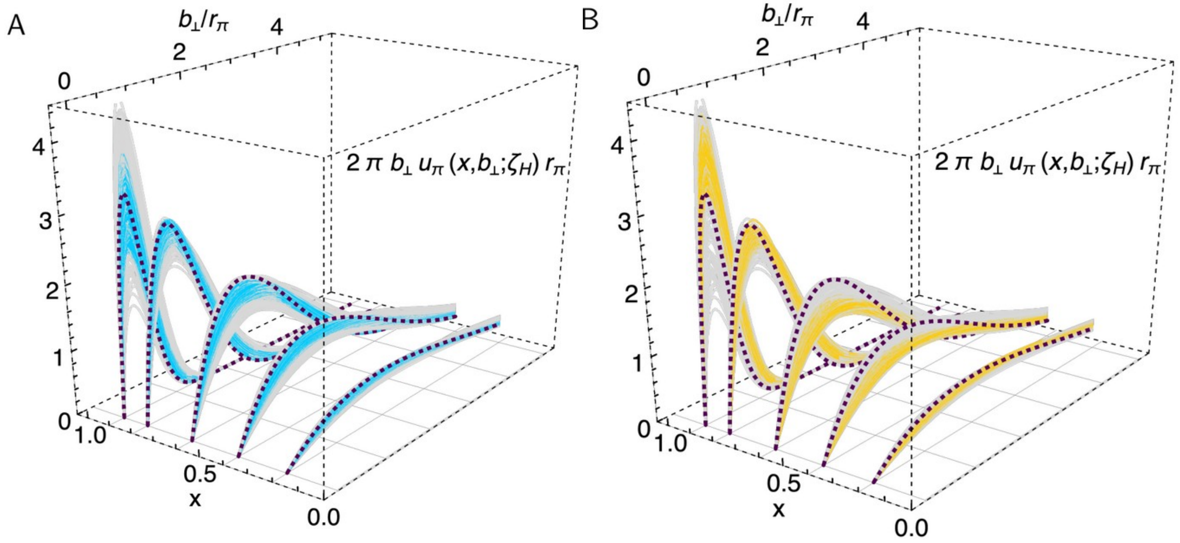
$$\left(\frac{r_\pi^{\theta_2}}{r_\pi^{E}}\right)^2 = \frac{<x^2(1-x)>_{\zeta_H}^{q}}{<x^2>_{\zeta_H}^{q}} \approx \left(\frac{4}{5}\right)^2$$

Determined from **PDF moments!**

IPS **GPDs**

- **Impact parameter** space **GPDs** are defined as:

$$u^\pi(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(|b_\perp|\Delta)\, H_\pi^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$



- Such that, in **factorized** models:

$$u^\pi(x, b_\perp^2; \zeta_H) = \frac{u^\pi(x;\zeta_H)}{(1-x)^2} \Psi^\pi\left(\frac{|b_\perp|}{1-x}; \zeta_H\right)$$

- The location and values of the **maxima**:

	x	$b_\perp/r_\pi$	i_π
CSM [57]	0.88	0.13	3.29
ASV	0.89(2)	0.10(2)	3.21(30)
MF	0.95(1)	0.05(1)	4.58(50)
lQCD	0.91(6)	0.08(5)	4.04(1.67)

- Furthermore:

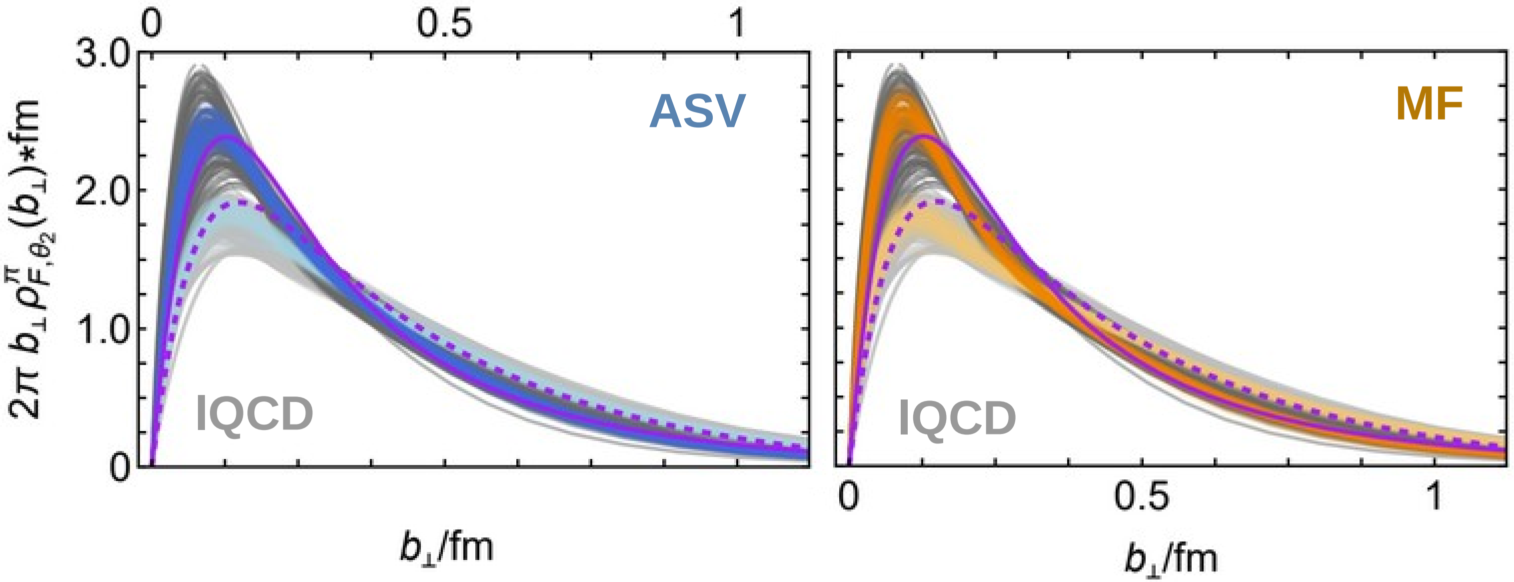
$$\langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_u^\pi = \frac{2}{3} r_\pi^2 = \langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_d^\pi$$

Algebraic result !

Distributions: Mass & Charge

- **Density** distributions are obtained by integrating the **IPS-GPD.**

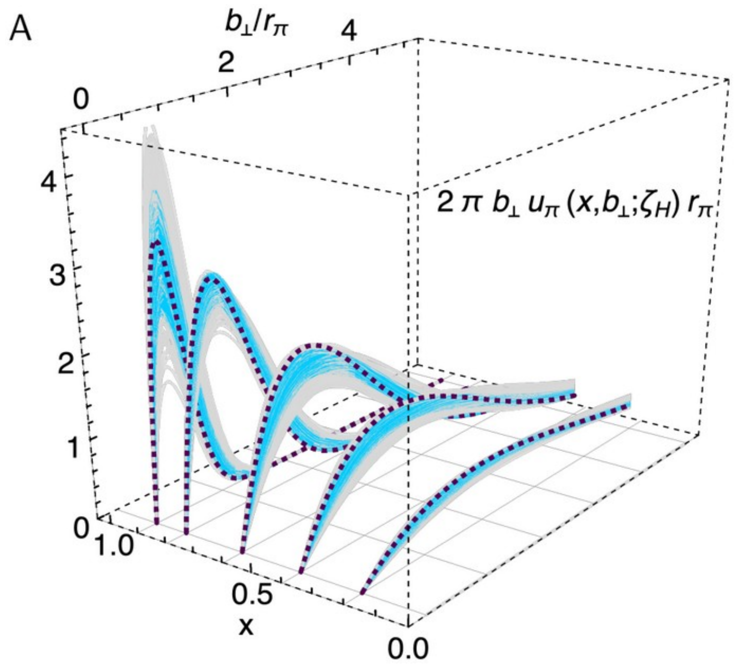
$$\rho^{\pi}_{\{F,\theta_2\}}(|b_\perp|) = \int_{-1}^{1} dx\,\{1,2x\}\,u^{\pi}(x,b_\perp^2;\zeta_{\mathcal{H}}) = \int_0^{\infty}\frac{d\Delta}{2\pi}\Delta J_0(|b_\perp|\Delta)\{F_\pi(\Delta^2),\theta_2(\Delta^2)\}$$

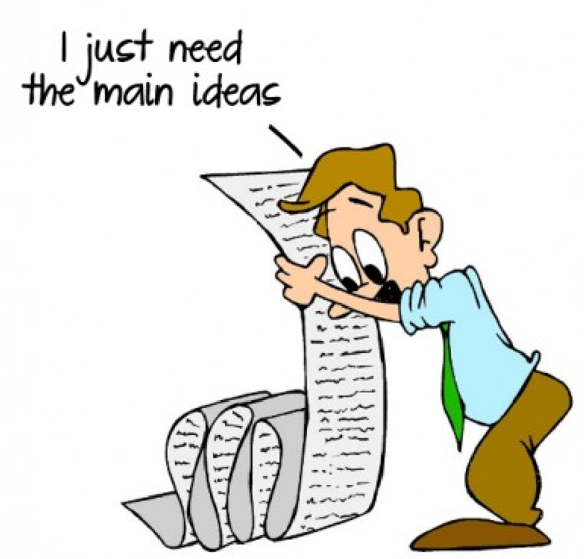


$$r_\pi^{\theta_2} < r_\pi^{E}$$

- The narrower curves correspond to the mass distribution, demonstrating that: **Charge** effects span over a **larger** domain than **mass** effects.

Conclusions and Scope

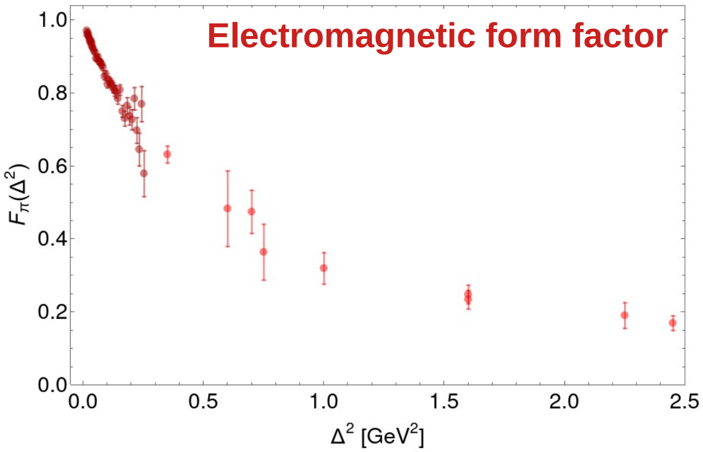


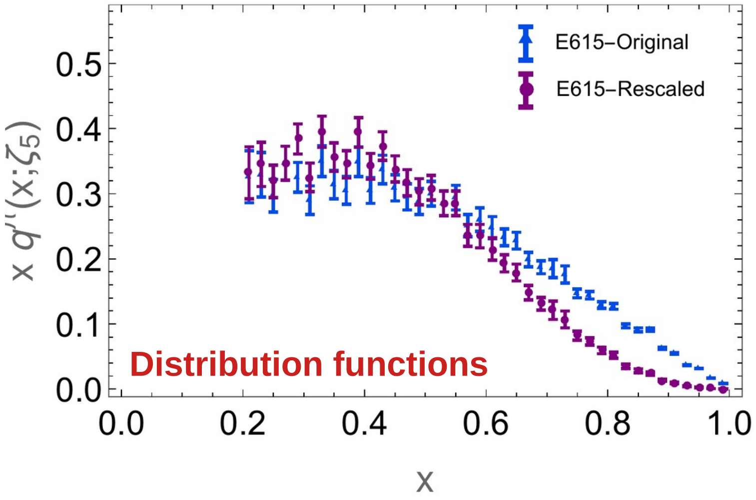


Conclusions and Scope

- **Question**:
  From the empirical knowledge of 1-dimensional distributions (**EFF** and **PDF**), can we obtain the 3-dimensional **GPD**?

$$u^\pi(x;\zeta_{e/l})\ ,\ F_\pi(\Delta^2) \longrightarrow H_\pi(x,\xi,-\Delta^2;\zeta)$$ **???**





Conclusions and Scope

- **Question:**
  From the empirical knowledge of 1-dimensional distributions (**EFF** and **PDF**), can we obtain the 3-dimensional **GPD**?

$$u^\pi(x;\zeta_{e/l})\ ,\ F_\pi(\Delta^2) \longrightarrow H_\pi(x,\xi,-\Delta^2;\zeta)$$

???

- ~~Partial~~ **Answer:**

DGLAP GPD

All orders evolution

$$u^\pi(x;\zeta_{e/l})$$

$$H_\pi^u(x,\xi,-\Delta^2;\zeta_H) = \theta(x_-)\sqrt{u^\pi(x_-;\zeta_H)u^\pi(x_+;\zeta_H)}\,\Phi^\pi(z^2;\zeta_H)$$

Factorised LFWF

$$H_\pi^u(x,\xi,-\Delta^2;\zeta_H) \sim \int_{k_\perp} \psi^*\psi$$

Sum rule

$$F_\pi(\Delta^2) = \int_0^1 dx\, H_\pi^u(x,0,-\Delta^2)$$

Conclusions and Scope

- **Question:**
  From the empirical knowledge of 1-dimensional distributions (**EFF** and **PDF**), can we obtain the 3-dimensional **GPD**?

- **Answer:**
  Yes, but so far we are limited to the **DGLAP** region.

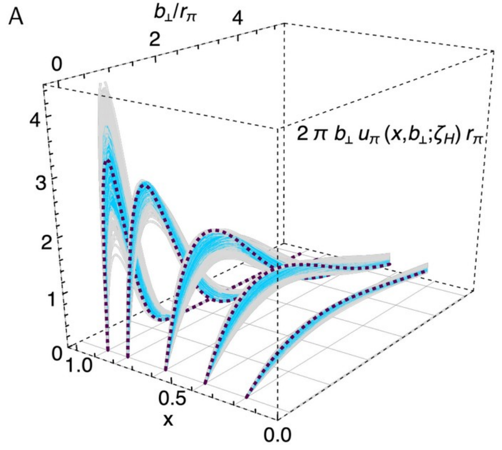
  - **Nevertheles...**

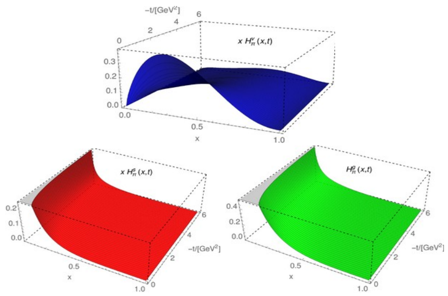
    - **Charge**, **Mass** and **spatial** distributions are already within the reach of DGLAP GPDs.

    - In this domain, we can also evolve the **GPDs** to disentangle **valence**, **glue** and **sea** content.

    - Sophisticated covariant extensions to the **ERBL** domain are known.

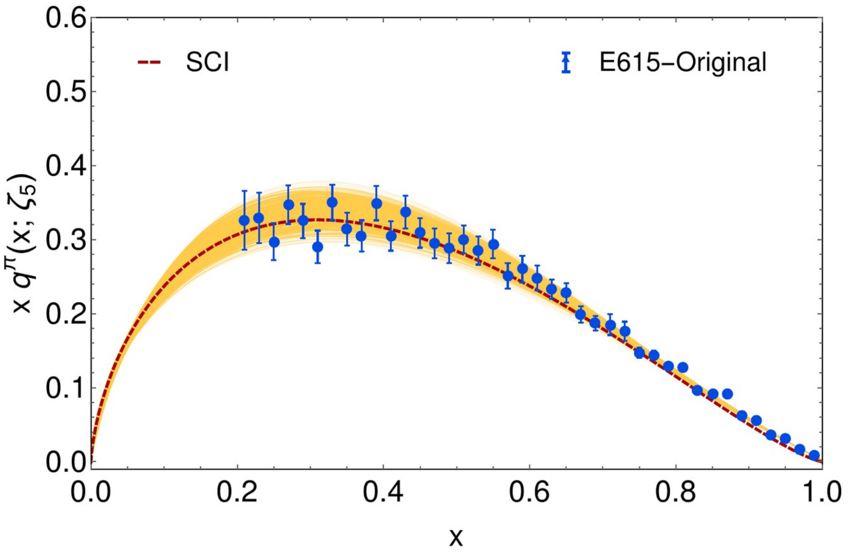
JM and Pietro’s talks





Pion PDF: Original Data

➢ Applying this algorithm to the original data yields:



(average)

```
Mean values (of moments) and errors, ζH
{{0.5, 2.52187 × 10^-17}, {0.331527, 0.00803273}, {0.247615, 0.0110893},
 {0.19784, 0.0121977}, {0.165066, 0.0124911}, {0.141928, 0.0124198},
 {0.124755, 0.0121811}, {0.111521, 0.0118683}, {0.101021, 0.0115275},
 {0.0924926, 0.0111824}, {0.085431, 0.010845}, {0.0794897, 0.0105214},
 {0.0744232, 0.0102142}, {0.0700521, 0.00992435}, {0.0662432, 0.00965182}}
```

(SCI)

```
Moments from SCI, ζH
{0.5, 0.332885, 0.249327, 0.199231, 0.165865, 0.142056, 0.124215, 0.11035,
 0.0992657, 0.090203, 0.0826552, 0.0762721, 0.0708035, 0.0660661, 0.0619225}
```

✔ The produced moments are compatible with a **symmetric PDF** at the **hadronic scale**.

✗ But also exhibit agreement with the **SCI results**.

$$q_{\rm SCI}(x;\zeta_H) \approx 1$$

Thus, given the **QCD prescription**,

$$u^{\pi}(x;\zeta) \overset{x\simeq 1}{\sim} (1-x)^{\beta = 2+\gamma(\zeta)}$$

We shall **discard** this for the upcoming construction of the valence quark GPD