Using Lattice QCD to test the Standard Model

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Flavour Phenomenology

Why is it important?

- Flavour physics is the study of the different types of quark flavours, their spectrum and their interactions (transitions among them).

u	c	t
d	s	b

- Flavour-violating and CP-violating processes allow us to test high energy physics since they are strongly suppressed and governed by quantum fluctuations.
  - Unveiling New Physics effects
  - Constraining NP models.

- Tests not limited by available energy by available precision (both in experiment and theory)

$$\textbf{Experiment} = (\text{Known factors}) \times (V_{CKM}) \times \underbrace{(\text{matrix elements})}_{\text{Lattice QCD}}$$

- Hadronic matrix elements: They encode the non-perturbative QCD interactions responsible for the hadronization of quarks/antiquarks into hadrons. We can parameterize those in terms of decay constants, form factors, bag parameters...

CKM Matrix

Interaction eigenvectors are not the same as mass eigenvector but they can be related by a change of basis.

$$d_L = S_d d'_L \quad u_L = S_u u'_L \,,$$

Thus, the Flavour Changing Charged Currents (FCCC):

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}} \left\{ W_\mu^\dagger \bar{u} \gamma^\mu (1-\gamma^5) V d + \text{h.c.} \right\} \,,$$

where $V = S_u S_d^\dagger$ is the so-called Cabibbo-Kobayashi-Maskawa (CKM) matrix.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Since the CKM Matrix comes from the multiplication of two change of basis matrices, it needs to be **unitary**.
- It is important to determine each of them **independently** to test unitarity.
- The first row:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0021(2)_{V_{us}}(6)_{V_{ud}} \ . \qquad (1)$$

- This is a 3σ deviation that could be a signal of New Physics effects.

How to determine $|V_{cb}|$

- One interesting element is $|V_{cb}|$, some loop observables are very sensitive to $|V_{cb}|$. For example, for the SM prediction of ε_K, the parameter of indirect CP violation for kaons, |Vcb| is the dominant error.
- Another reason to study this specific parameter is because there is an on-going tension in its determination.

Inclusive determinations

$$\boxed{B \to X_c \ell \nu_\ell}$$

$$|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3}$$

Exclusive determination

$$\boxed{B \to D^* \ell \nu_\ell}$$

$$|V_{cb}| = (39.36 \pm 0.68) \times 10^{-3}$$

There exists a 3 σ **tension** between these two values.

We will focus on the exclusive determination using the semileptonic decay:

$$\boxed{B \to D^* \ell \nu_\ell}$$

The differential decay width can be expressed in terms of some form factors as:

$$\begin{aligned}\frac{d\Gamma}{d\omega} =& |V_{cb}|^2 |\eta_{EW}|^2 \frac{G_F^2 M_B^5}{16\pi^3} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 r^3 (\omega^2 - 1)^{1/2} \times \\ & \times \left\{ \frac{1}{3y^2} \left(1 + \frac{m_\ell^2}{2q^2}\right) \left[|H_+|^2 + |H_-|^2 + |H_0|^2\right] + \frac{m_\ell^2}{2M_B^2} |H_S|^2 \right\} .\end{aligned} \tag{2}$$

This expression needs to be compared with experimental data in order to extract the $|V_{cb}|$ value.

BGL parametrization

- We are going to use the BGL parametrization for the form factors:

$$
\begin{aligned}
H_{\pm}(\omega) &= \frac{1}{M_B\sqrt{r}} f(\omega) \mp \sqrt{\omega^2 - 1} M_B \sqrt{r} g(\omega)\,, \\
H_0(\omega) &= \frac{y}{M_B^2\sqrt{r}} \mathcal{F}_1(\omega)\,, \\
H_S(\omega) &= y\sqrt{(w^2 - 1)r} \mathcal{F}_2(\omega)\,,
\end{aligned}
$$

- These can be computed using Lattice QCD

In order to compute the form factors, we are going to do first a z-expansion, where we do a conformal mapping from ω to a new variable z:

$$z = \frac{\sqrt{\omega+1} - \sqrt{2}}{\sqrt{\omega+1} + \sqrt{2}}$$

The maximum value for z is $z_{max} = 0.056$ so we can do a power expansion on this parameter. Thus:

$$g = \frac{1}{P_{1-}(z)\phi_g(z)} \sum_{j=0}^{\infty} a_j z^j \,, \qquad \mathcal{F}_1 = \frac{1}{P_{1+}(z)\phi_{\mathcal{F}_1}(z)} \sum_{j=0}^{\infty} c_j z^j \,,$$

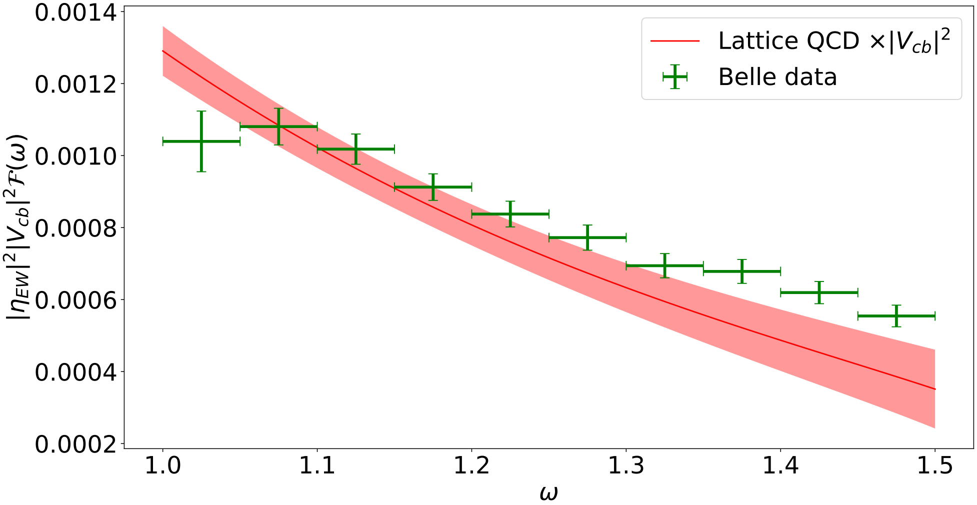
$$f = \frac{1}{P_{1+}(z)\phi_f(z)} \sum_{j=0}^{\infty} b_j z^j \,, \qquad \mathcal{F}_2 = \frac{1}{P_{0-}(z)\phi_{\mathcal{F}_2}(z)} \sum_{j=0}^{\infty} d_j z^j \,,$$

Now we can use Lattice QCD in order to find the value of the expansion's coefficients. The FNAL-MILC collaboration obtained:

a_0	$0.0330(12)$	b_0	$0.01229(23)$
a_1	$-0.155(55)$	b_1	$-0.003(12)$
a_2	$-0.12(98)$	b_2	$0.07(53)$
c_0	$0.002059(38)$	d_0	$0.0509(15)$
c_1	$-0.0058(25)$	d_1	$-0.327(67)$
c_2	$-0.013(91)$	d_2	$-0.03(96)$

Using these values we can reconstruct the theoretical expression for the decay width and, comparing them with the experimental data, we can obtain the $|V_{cb}|$ value.

arXiv: 2105.14019



Datos Belle; arXiv: 1809 . 03290

Two different determinations

FNAL-MILC

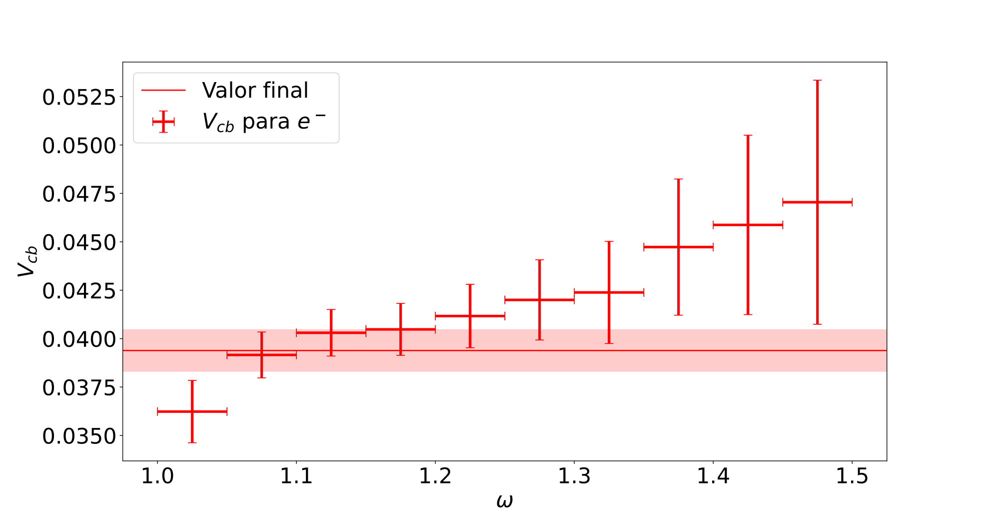
- Global fit in the entire energy range.
- $|V_{cb}| = (38.76 \pm 0.75) \times 10^{-3}$
- Compatible with past exclusive determinations.

  arXiv: 2105.14019

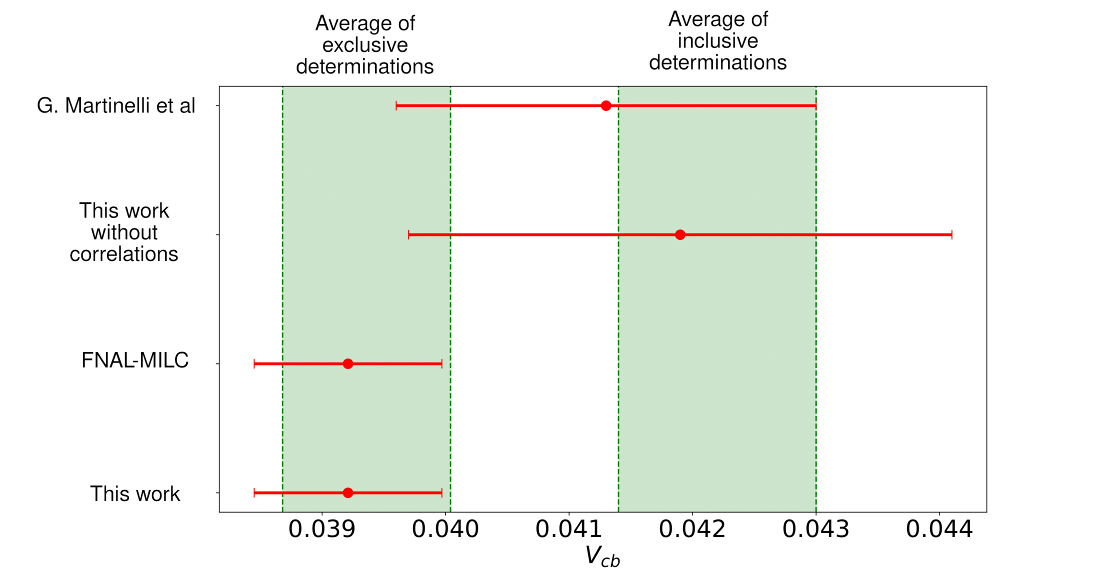
G. Martinelli et al

- Bin by bin determination.
- Unitarity imposed by hand
- $|V_{cb}| = (41.3 \pm 1.7) \times 10^{-3}$
- Seem to resolve the tension with inclusive determinations.

  arXiv: 2109.15248



$$|V_{cb}| = (38.76 \pm 0.44_{\text{Th}} \pm 0.54_{\text{Exp}}) \times 10^{-3} = (38.76 \pm 0.75) \times 10^{-3}\,.$$



Average of exclusive determinations
Average of inclusive determinations
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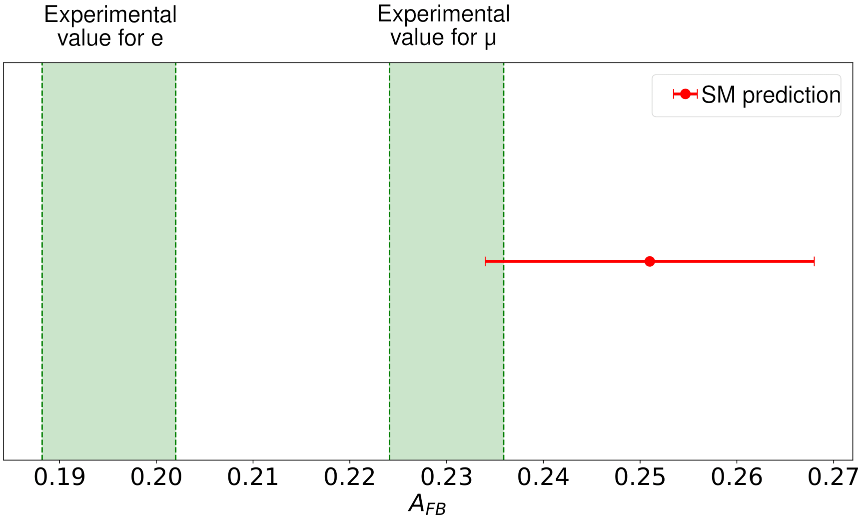
This work without correlations
FNAL-MILC
This work
0.039
0.040
0.041
0.042
0.043
0.044
V_{cb}

- Apart from determining some parameters of the SM we can also use the theoretical expression for the decay width to study some observables that can be compared with experimental data.
- One of them is the so-called Forward Backward Asymmetry which is defined as:

$$A_{FB} = \frac{\int_{\omega_{\min}}^{\omega_{\max}} \int_0^1 \frac{d^2\Gamma}{d\omega d\cos\theta_\ell} - \int_{\omega_{\min}}^{\omega_{\max}} \int_{-1}^0 \frac{d^2\Gamma}{d\omega d\cos\theta_\ell}}{\int_{\omega_{\min}}^{\omega_{\max}} \int_{-1}^1 \frac{d^2\Gamma}{d\omega d\cos\theta_\ell}} .$$

- The SM prediction is:

$$A_{FB(\mathbf{teo})} = 0.251(17)$$


Experimental value for e
Experimental value for μ
SM prediction
0.19
0.20
0.21
0.22
0.23
0.24
0.25
0.26
0.27
A_{FB}

Another observable is $R(D^{(*)})$ which is related to lepton universality. It is defined as:

$$R(D^{(*)}) \equiv \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu_\ell)}, \qquad \ell = e,\, \mu\,,$$

where

$$\mathcal{B}(B \to D^{(*)}\ell\nu_\ell) = \tau_B \int_1^{\omega_{\mathrm{Max},\ell}} d\omega \frac{d\Gamma}{d\omega}\,,$$

$$R(D^*)^{(e)} = 0.265(13) \qquad R(D^*)^{(\mu)} = 0.266(13)$$

The experimental value $R(D^*)_{\text{exp}} = 0.295 \pm 0.010 \pm 0.010 = 0.295 \pm 0.014$. There used to be a 3σ tension when this observable was computed along $R(D)$, but the computations where revisited recently and the tension was resolved.

QCD at low energies

Lattice Setup

- FNAL/MILC collaboration has generated data to calculate decay constants f_{xy} and hadron masses m_{xy} for:
  - 6 different values of a.
  - Several values of the valence quark masses m_x, $m_y \in [\sim m_l, \sim m_s]$
  - Physical and unphysical values of the light $(m_l \equiv m_u = m_d)$ and strange quark masses.

- Perform a NNLO (2-loops) ChPT analysis of the data to:
  - Extrapolate (or interpolate) the data to the physical point (physical masses, continuum limit, infinite volume limit ...) to calculate the decay constants f_π, f_K, and f_K/f_π.

$$\frac{\Gamma(K^+ \to \ell\nu_\ell(\gamma))}{\Gamma(\pi^+ \to \ell\nu_\ell(\gamma))} \propto \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{f_K^2}{f_\pi^2} \tag{3}$$

$$f_K, f_\pi + \Gamma(K \to \ell\nu), \Gamma(\pi \to \ell\nu) \Rightarrow |V_{us}|, |V_{ud}| \tag{4}$$

    Test first-row CKM unitarity and tensions in the extraction of V_{us}.
  - Extract the ChPT LECs (low energy constants) that enter in the ChPT calculation of decay constants and meson masses.
  - Test the dependence on m_s^{sea} of M_π^2.

Strong CP problem

- When studying the QCD vacuum, we need to introduce the θ term in order to take into account its non-trivial structure:

$$\mathcal{L}_\theta = -\frac{1}{16}\theta G_{\mu\nu}\tilde{G}^{\mu\nu} \tag{5}$$

- This term violates CP in QCD.
- The instanton contribution, $G\tilde{G}$, is giving mass to the η', that is why it is much heavier than the η.
- This term contributes to the neutron dipole moment. Experimental results constrict θ to be smaller than 10^{-10}.
- This is introducing a fine-tuning problem into the theory of QCD. If we have something really small, we would think that there is a symmetry that is producing such a small number (making that to be 0).

- Several solutions have been proposed.
- One of the most popular ones, is to introduce a new hypothetical particle called the Axion.
- However there is one solution within the SM.

$$\boxed{\text{Massless Solution}} \tag{6}$$

- If $m_u = 0$, the θ term can be removed by doing a $U(1)$ rotation $u \to e^{i\alpha\gamma^5}u$, which makes $\theta \to \theta + \alpha$.

- We know that the experimental quark mass is not zero, so this has to be ruled out from the very begining. However, note that there are two contributions to the experimental mass:

$$m_{u,exp} = m_u + \frac{m_d m_s}{\Lambda_{\text{instaton}}} \,. \tag{7}$$

- If the topological contribution is big enough, we can set $m_u = 0$ solving the strong CP problem. This mass contribution is the one related to the Higgs condensate, proportional to the Yukawa couplings.

- An estimation to the topological contribution can be estimated by studying how the pion mass varies with m_s^{sea}.

$$M_\pi^2 = \beta_1(m_u + m_d) + \beta_2 m_s^{sea}(m_u + m_d) + \text{higher orders} \ , \tag{8}$$

where m_u is the mass from the coupling to the Higgs (the one we need to be zero to solve the strong CP problem).

- The goal is to check whether the topological contribution $m_d m_s/\Lambda_{\text{instanton}}$ can mimic the effect of m_u to reproduce $m_{u,exp}$.

$$m_d m_s/\Lambda_{\text{instanton}} < \beta_2 m_d m_s \sim \beta_1 m_{u,exp} \quad \rightarrow \quad \beta_2/\beta_1 \approx 5\text{GeV}^{-1} \ . \tag{9}$$

- Funckle et al. computed an estimation to the topological contribution by assuming equal and fixed masses of the light quarks, and varying the m_s in the chiral limit:

$$\frac{\beta_2}{\beta_1} = \left. \frac{M^2_{\pi,1} - M^2_{\pi,2}}{m_{s,1}M^2_{\pi,2} - m_{s,2}M^2_{\pi,1}} \right|_{M_\pi \to 0} . \tag{10}$$

- They found:

$$\frac{\beta_2}{\beta_1} = 0.63(39)\mathrm{GeV}^{-1} \tag{11}$$

- Low enough to rule out massless solution but with high error.

arXiv: 2111.00288

- We want to improve this measurement.
- We can do that by using FNAL/MILC data to perform a fit to the NNLO chiral expressions, for different values of both the m_s^{sea} and the valence quark masses.
- This fit would give us a value for the expansion coefficients β_1 and β_2.
- Then we can relate them with the Low Energy Constants of ChPT.

$$\beta_1 = B_0 \; , \qquad \beta_2 = \frac{16 B_0}{F^2}(2L_6 - L_4) \tag{12}$$

Thank you for your attention!