Green functions from lattice QCD

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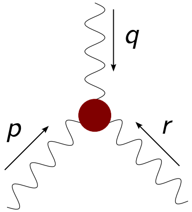
Quantum Chromodynamics

QCD Lagrangian

$$\mathcal{L}_{QCD} = \bar{\psi}\left(i\gamma^{\mu} D_{\mu} - m\right)\psi - \frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a}, \qquad D_{\mu} = \partial_{\mu} - i\ g\ A^{i}_{\mu} \frac{\sigma^{i}}{2}$$

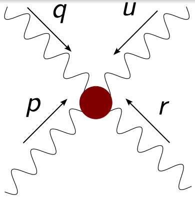
Quantum Electrodynamics

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$$

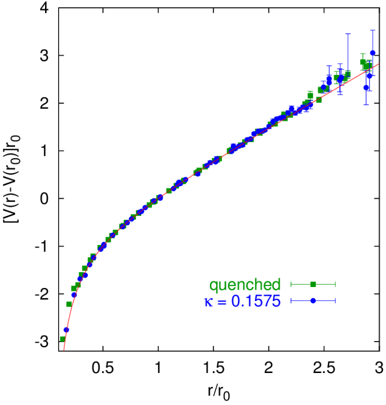


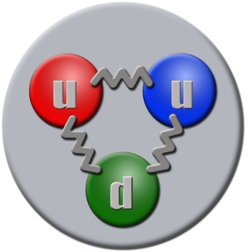
Quantum Chromodynamics

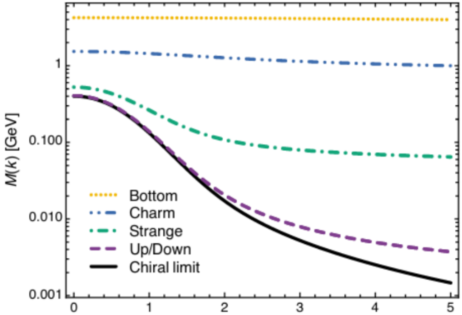
$$G^{a}_{\mu\nu} = \partial_{\mu} A^{a}_{\nu} - \partial_{\nu} A^{a}_{\mu} + g\ f_{abc} A^{b}_{\mu} A^{c}_{\nu}$$



Non-perturbative phenomena



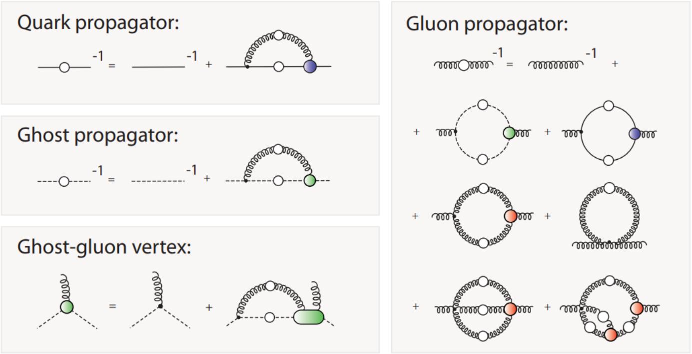




Non-perturbative methods

Dyson-Schwinger Equations:

- Allow nonperturbative calculations
- Coupled integral equations
- Truncated at some point
- Can be used in combination with other non-perturbative tools such as lattice QCD



arXiv:0909.0703 [hep-ph]

Lattice QCD

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int [dU][d\psi][d\bar{\psi}] e^{-S} \hat{\mathcal{O}} = \frac{1}{Z} \int [dU] e^{-S_G(U)} \det(D) \; \hat{\mathcal{O}}$$

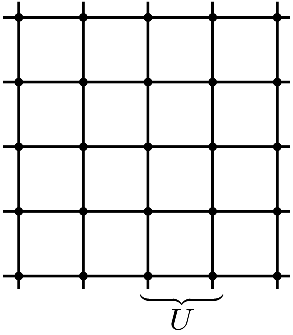
Pros

- Just QCD!
- Wick rotation $t \to t' = it$
- Regularized through $\Lambda \sim a^{-1}$

Cons

- Broken rotational symmetry
- Finite volume and discretization errors
- Expensive for chiral fermions

Discrete Space-time



Lattice setups

Exploited quenched gauge field configurations with:

β	L^4/a^4	a (fm)	confs
5.6	32^4	0.236	2000
5.8	32^4	0.144	2000
6.0	32^4	0.096	2000
6.2	32^4	0.070	2000

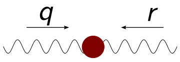
- Absolute calibration for $\beta = 5.8$ taken from [S. Necco and R. Sommer, Nucl. Phys. B622, 328 (2002)].
- Relative calibrations based in gluon propagator scaling [Phys. Rev. D 98, 114515 (2018)],

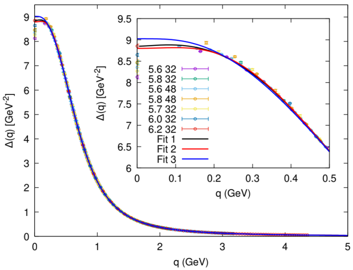
Gluon propagator

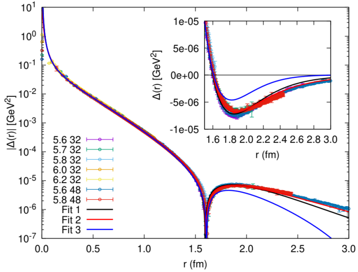
Landau gauge

Landau gauge $\partial_\mu A_\mu^a = 0$ fixed numerically, allowing to compute gauge dependent quantities.

$$\Delta_{\mu\nu}^{ab}(q^2) = \langle A_\mu^a(q) A_\nu^b(-q) \rangle = \delta^{ab} \Delta(q^2) P_{\mu\nu}(q)$$







F. de Soto, EPJ Web Conf. 274 (2022) 02013

Three-gluon vertex

$$f^{abc}\mathcal{G}_{\alpha\mu\nu}(q,r,p) = \langle A^a_\alpha(q) A^b_\mu(r) A^c_\nu(p)\rangle$$

Three-gluon vertex: $\mathcal{G}^{\alpha\mu\nu}(q,r,p) = g\bar{\Gamma}^{\alpha\mu\nu}(q,r,p)\ \Delta(q^2)\ \Delta(r^2)\ \Delta(p^2)$ which corresponds to the transverse projection of the 1PI vertex:

$$\bar{\Gamma}^{\alpha\mu\nu}(q,r,p) = \Gamma^{\alpha'\mu'\nu'}(q,r,p) P^{\alpha}_{\alpha'}(q) P^{\mu}_{\mu'}(r) P^{\nu}_{\nu'}(p)$$

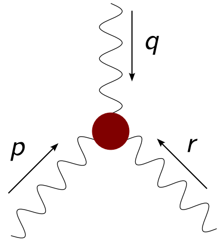
No access to longitudinal part

If the 1PI vertex, $\Gamma^{\alpha\mu\nu}(q,r,p)$ has a longitudinal and a transverse part:

$$\Gamma^{\alpha\mu\nu}(q,r,p) = {\Gamma_L}^{\alpha\mu\nu}(q,r,p) + {\Gamma_T}^{\alpha\mu\nu}(q,r,p)$$

we will only access the transverse part

Three-gluon vertex



Momentum conservation

$$q + r + p = 0$$

Ball-Chiu decomposition [Phys. Rev. D22 (1980) 2550]

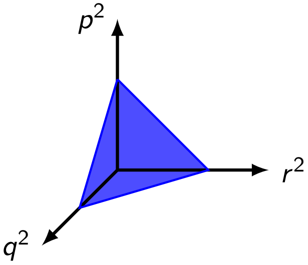
- 14 independent tensors $(\ell_1, \ell_2, \cdots, \ell_{10}, t_1, \cdots, t_4)$
- Transverse projection $\to$ 4 independent tensors

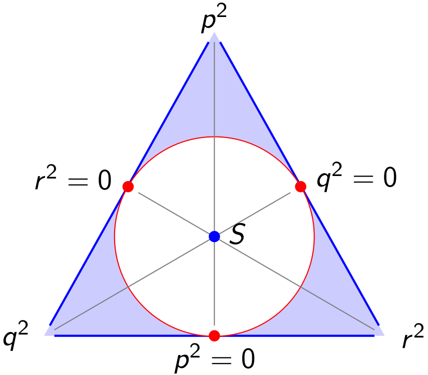
Transversely projected vertex

$$\bar{\Gamma}^{\alpha\mu\nu}(q, r, p) = \bar{\Gamma}_1 \lambda_1^{\alpha\mu\nu} + \bar{\Gamma}_2 \lambda_2^{\alpha\mu\nu} + \bar{\Gamma}_3 \lambda_3^{\alpha\mu\nu} + \bar{\Gamma}_4 \lambda_4^{\alpha\mu\nu}$$

Three-gluon vertex

$\bar{\Gamma}^{\alpha\mu\nu}(q, r, p)$ depends on three momenta, with $q + r + p = 0$. The scalar form factors can be cast in terms of q^2, r^2 and p^2.





Three-gluon vertex

Particular cases:

Case	Def.	$\hat{q}r$	Tensors
Sym.	$q^2 = r^2 = p^2$	$\frac{2\pi}{3}$	$\lambda_{1,2}^{sym}$
Soft gluon	$p = 0$	π	λ_3^{sg}
Collinear	$q = r = -p/2$	0	(none)
Bisectoral	$q^2 = r^2$	$(0, \pi)$	3
General		–	4

Symmetric and soft-gluon cases already studied in [Phys.Lett.B 818 (2021) 136352]

Three-gluon vertex

$$\bar{\Gamma}^{\alpha\mu\nu} = \underbrace{\bar{\Gamma}_1\lambda_1^{\alpha\mu\nu}}_{Tree-level} + \bar{\Gamma}_2\lambda_2^{\alpha\mu\nu} + \bar{\Gamma}_3\lambda_3^{\alpha\mu\nu} + \bar{\Gamma}_4\lambda_4^{\alpha\mu\nu}$$

Basis for the transversely projected vertex

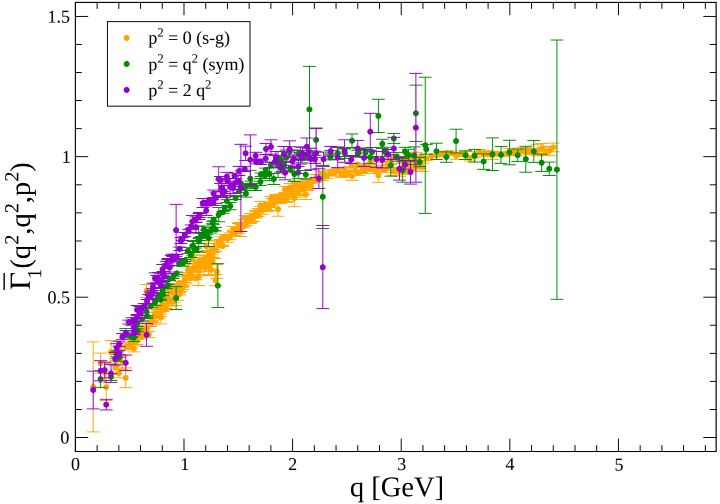
$$\lambda_1^{\alpha\mu\nu}(q,r,p) = \left(\ell_1^{\alpha'\mu'\nu'} + \ell_4^{\alpha'\mu'\nu'} + \ell_7^{\alpha'\mu'\nu'}\right) P_{\alpha'}^{\alpha}(q) P_{\mu'}^{\mu}(r) P_{\nu'}^{\nu}(p)$$

$$\lambda_2^{\alpha\mu\nu}(q,r,p) = 3\frac{(r-p)^{\alpha'}(p-q)^{\mu'}(q-r)^{\nu'}}{q^2+r^2+p^2} P_{\alpha'}^{\alpha}(q) P_{\mu'}^{\mu}(r) P_{\nu'}^{\nu}(p)$$

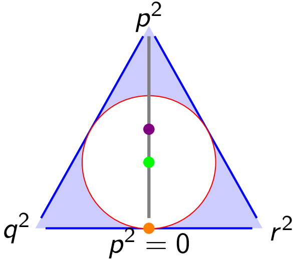
$$\lambda_3^{\alpha\mu\nu}(q,r,p) = \frac{3(\ell_3^{\alpha'\mu'\nu'} + \ell_6^{\alpha'\mu'\nu'} + \ell_9^{\alpha'\mu'\nu'})}{q^2+r^2+p^2} P_{\alpha'}^{\alpha}(q) P_{\mu'}^{\mu}(r) P_{\nu'}^{\nu}(p)$$

$$\lambda_4^{\alpha\mu\nu}(q,r,p) = \left(\frac{3}{q^2+r^2+p^2}\right)^2 (t_1^{\alpha\mu\nu} + t_2^{\alpha\mu\nu} + t_3^{\alpha\mu\nu})$$

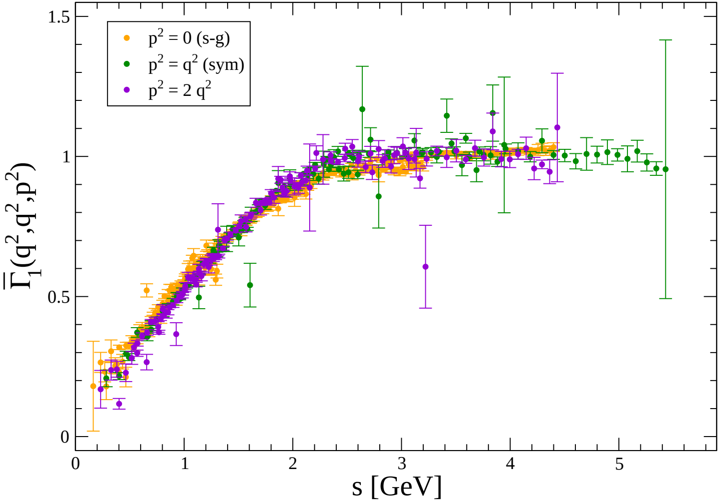
Results for the bisectoral case $q^2 = r^2$.



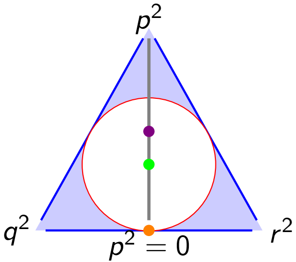
- Different running for each angle $\hat{qr}$ ($\pi/2$, $2\pi/3$, and π. in the plot)



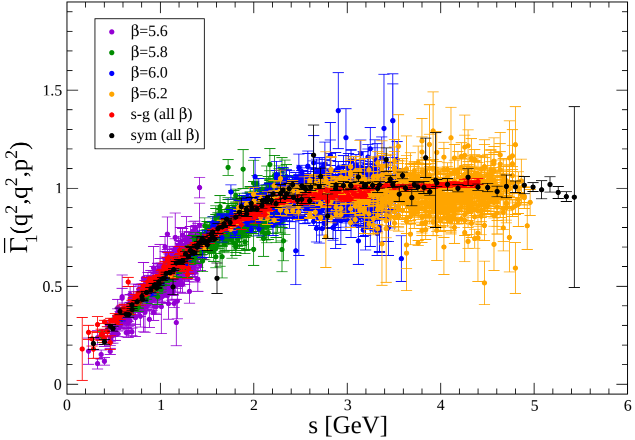
Results for the bisectoral case $q^2 = r^2$.



- The different angles scale in terms of $\sqrt{(q^2+r^2+p^2)/2}$.

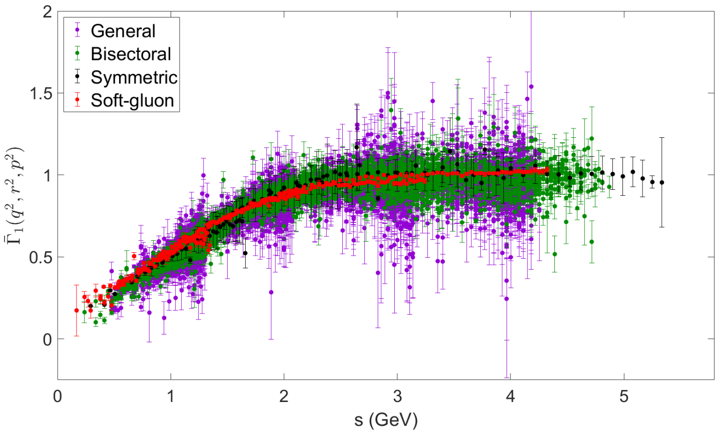


Results for the bisectoral case $q^2 = r^2$.



- Scales in terms of $(q^2 + r^2 + p^2)/2$.
- Renormalized $\bar{\Gamma}_3^{sg}(q^2)\Big|_{q^2=\mu^2}$ at $\mu = 4.3GeV$.

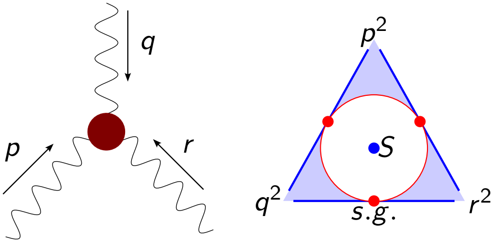
Results for the general case.



- Scales in terms of $(q^2 + r^2 + p^2)/2$.
- Renormalized $\bar{\Gamma}_3^{sg}(q^2)\Big|_{q^2=\mu^2}$ at $\mu = 4.3 GeV$.

Three-gluon vertex

- Three-gluon vertex studied for all kinematics.
- Scale with $s^2 = (q^2 + r^2 + p^2)/2$.
- $\bar{\Gamma}_1$ dominates.
- $\bar{\Gamma}_2 \ll \bar{\Gamma}_1$ (and $\bar{\Gamma}_3, \bar{\Gamma}_4 \approx 0$).



Planar degeneracy[a]: the full vertex seems to be well described by

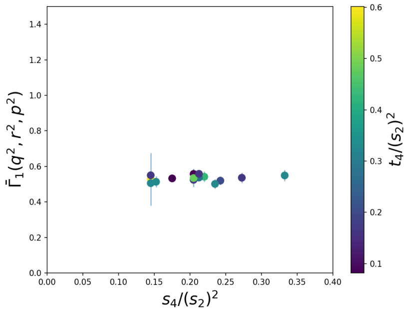
$$\bar{\Gamma}^{\alpha\mu\nu}(q,r,p) \approx \bar{\Gamma}_3^{sg}(s^2)\Big|_{s^2} \bar{\Gamma}_0^{\alpha\mu\nu}(q,r,p)$$

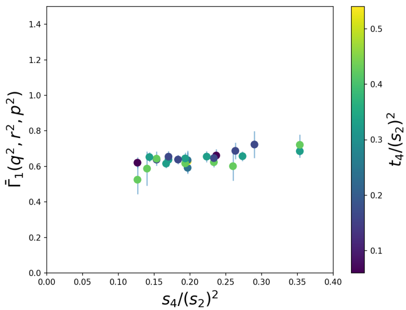
[a] F. Pinto-Gómez, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, arXiv:2208.01020

Three-gluon vertex: Hypercubic errors (Ongoing)

H4 invariants:

$$q^{[4]} = q_1^4 + q_2^4 + q_3^4 + q_4^4, s_2 = (q^2 + r^2 + p^2)/2,$$
$$s_4 = (q^{[4]} + r^{[4]} + p^{[4]})/4, t_4 = (q^2 - r^2)^2 + (r^2 - p^2)^2 + (p^2 - q^2)^2/3.$$





Three-gluon vertex: Schwinger Mechanism

Schwinger mechanism

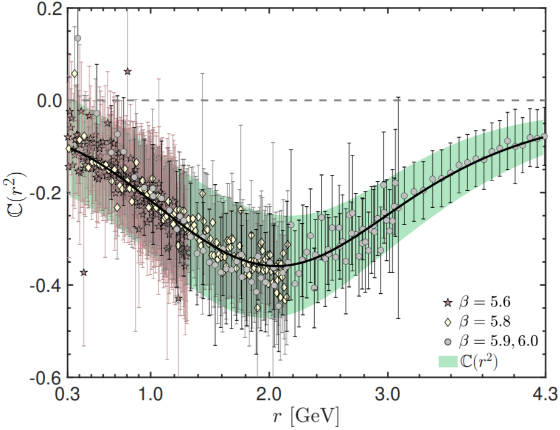
Explains how a gluon mass can be dynamically generated in QCD

For this mechanism to be active it is necessary and sufficient that a special type of longitudinally-coupled, negative-residue, simple pole structure is dynamically generated in the three-gluon vertex.

This condition shows as a non-null displacement in the Ward identity:

$$\mathbb{C}(r^2) = L_{sg}(r^2) - F(0)\left\{\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \widetilde{Z}_1\,\frac{d\Delta^{-1}(r^2)}{dr^2}\right\}$$

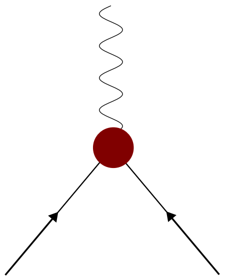
Three-gluon vertex: Schwinger Mechanism



Schwinger mechanism for gluons from lattice QCD. A.C. Aguilar, F. De Soto, et al. (2211.12594)

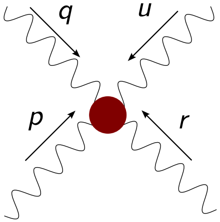
Three-gluon vertex: phenomenology

Schwinger-Dyson equation for quark-gluon vertex:



Using lattice results as input for the SD equations.

Four-gluon vertex



Momentum conservation

$$q + r + p + u = 0$$

Vertex definition

$$\mathcal{G}^{abcd}_{\alpha\beta\gamma\delta}(q, r, p, u) = \langle A^a_\alpha(q) A^b_\beta(r) A^c_\gamma(p) A^d_\delta(u) \rangle$$

Tensors for the general four-gluon vertex

$$\text{Color:} \quad \rightarrow \quad f^{abe} f^{cde}(3) \; ; \quad f^{abe} d^{cde}(6) \; ; \quad d^{abe} d^{cde}(3) \; ; \quad \delta^{ab}\delta^{cd}(3)$$

$$\text{Lorentz:} \quad \rightarrow \quad g_{\alpha\beta} g_{\gamma\delta}(3) \; ; \quad g_{\alpha\beta} q_\gamma r_\delta(54) \; ; \quad q_\gamma r_\delta p_\alpha u_\beta(81)$$

Four-gluon vertex: colinear momenta

In the lattice we only have access to the transversely projected vertex. In the case all the momenta are parallel, there is no transverse tensor with any momenta, simplifying the Lorentz structure to $g_{\alpha\beta}g_{\gamma\delta}$-like terms.

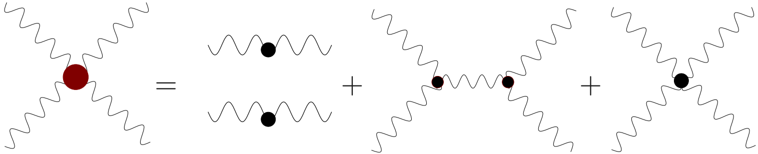
We have performed a preliminary study on the following kinematical cases:

- A: $(p, p, -p, -p)$
- B: $(p, -p, q, -q)$
- C: $(p, p, p, -3p)$
- D: $(p, q, -p-q, 0)$
- F: other collinear cases

Tree-level four-gluon vertex

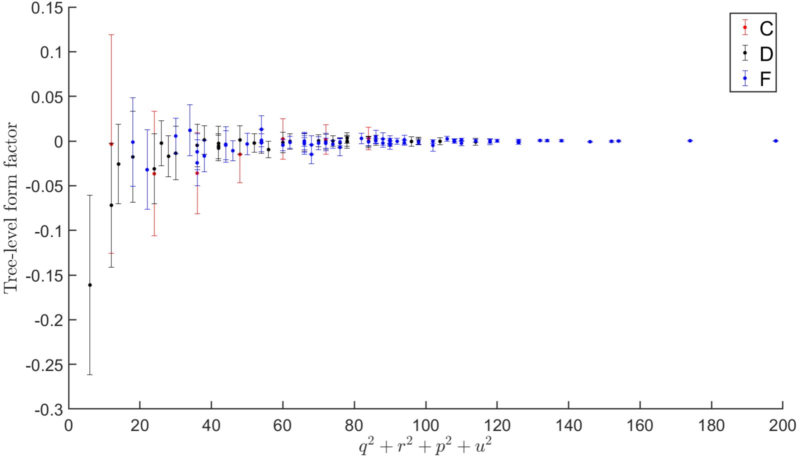
$$\Gamma^{(0)abcd}_{\alpha\beta\gamma\delta} = -f^{ade}f^{bce}(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\beta}g_{\gamma\delta}) - f^{abe}f^{cde}(g_{\alpha\delta}g_{\beta\gamma} - g_{\alpha\gamma}g_{\beta\delta}) - f^{ade}f^{bce}(g_{\alpha\delta}g_{\beta\gamma} - g_{\alpha\beta}g_{\gamma\delta})$$

Four-gluon vertex: disconnected terms

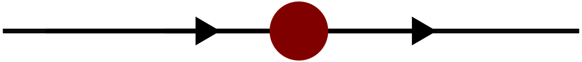


- A: $(p, p, -p, -p)$
- B: $(p, -p, q, -q)$
- C: $(p, p, p, -3p)$
- D: $(p, q, -p-q, 0)$
- F: other collinear cases

Four-gluon vertex: preliminary results



Quark propagator



In collaboration with S. Zafeiropoulos and J. Karpie

- Domain-Wall discretization for fermions
- Application of the H4 method to the quark propagator
- Spectral analysis of quark propagator.
- Coupling strength calculation
- Quark-gluon vertex

Perspectives

- H4 method application for the three-gluon vertex
- Study the SD equation for the quark-gluon vertex using the lattice results as input
- Keep studying the four-gluon vertex for restricted kinematics
- Determination of α_s from general kinematics three gluon vertex
- Study the quark propagator, applying the H4 method, and its possible applications.