The gravitational form factors of pseudoscalar meson

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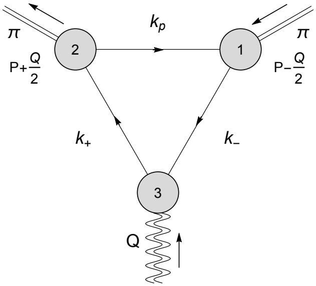
The pseudoscalar meson's gravitational form factors

The fascinating things about pseudoscalar meson's gravitational form factors

- It can help us better understand dynamical chiral symmetry breaking (DCSB).
- It provides a new entry point for us to explore the internal structure of mesons, for example, mass distribution and pressure distribution.

The gravitational form factors of pseudoscalar meson

In the impulse approximation, the Feynman diagram of meson's form factors are shown below



These couplings are given by

$$\Lambda^{\mu\nu}(P,Q) = iN_c \int_k \mathrm{Tr}\left[S(k_p)\Gamma_2(P_2;q_2)S(k_-)\Gamma_3^{\mu\nu}(P_3;q_3)S(k_+)\Gamma_1(P_1;q_1)\right], \quad (1)$$

The gravitational form factors of pseudoscalar meson

The symbols are shown below:

$$P_1 = P - \frac{Q}{2}; \quad P_2 = -(P + \frac{Q}{2}); \quad P_3 = Q,$$
$$q_1 = k - \frac{Q}{4}; \quad q_2 = k + \frac{Q}{4}; \quad q_1 = k + \frac{P}{2},$$
$$k_+ = k + \frac{P}{2} + \frac{Q}{2}; \quad k_- = k + \frac{P}{2} - \frac{Q}{2}; \quad k_p = k + P - \frac{Q}{2}.$$

the Γ_1 and Γ_2 means the Bethe-Salpeter amplitudes of pseudoscalar meson that can be obtained by solving the homogeneous Bethe-Salpeter equation and Γ_3 means the quark-graviton vertex

The gravitational form factors of pseudoscalar meson

For pseudoscalar meson, it has three gravitational form factors

$$\Lambda_{\mu\nu}^{uu\bar{d}}(P,Q)+\Lambda_{\mu\nu}^{\bar{u}\bar{u}d}(P,Q)=2P_\mu P_\nu A(t)+\frac{1}{2}\left(Q_\mu Q_\nu-g_{\mu\nu}Q^2\right)D(t)+2m^2\bar{c}(t)g_{\mu\nu} \quad (2)$$

$$=2P_\mu P_\nu\theta_2(t)-\frac{1}{2}\left(Q_\mu Q_\nu-g_{\mu\nu}Q^2\right)\theta_1(t)+2m^2\bar{c}(t)g_{\mu\nu} \quad (3)$$

θ_2 and θ_1 correspond to the mass and pressure distribution of pseudoscalar meson, respectively. According to conservation of energy and momentum, we have

$$\theta_2(0)=1,\bar{c}=0 \quad (4)$$

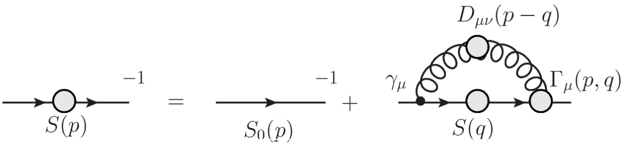
and in chiral limit

$$\lim_{m_\pi\to 0}\theta_1(0)=1 \quad (5)$$

Dyson-Schwinger equations

The quark propagator satisfies the Dyson-Schwinger equation:

$$S^{-1}(p) = Z_2(i\gamma\cdot p + m_{bm}) + Z_{1F}\int_q^{\Lambda} g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma^a_\nu(q,p). \tag{6}$$



$Z_{1F,2}$ are the quark-gluon vertex, quark wave-function renormalization constants, respectively. We employ a mass-independent momentum-subtraction renormalisation scheme. The self-energy is logarithmically divergent, $\int_q^\Lambda$ represents a translationally-invariant regularisation of the four-dimensional integral, with Λ the regularisation scale. In this work Pauli-Villars regularization was performed.

Gluon propagator

Gluon propagator: Qin-Chang Model[1]

$$Z_{1F} g^2 D_{\mu\nu}(k) = Z_2^2 \mathcal{G}(k^2) D_{\mu\nu}^{free}(k). \tag{7}$$

In this $D_{\mu\nu}^{free}(k)$ is the free gluon propagator and its form is as follows:

$$D_{\mu\nu}^{free}(k) = \frac{T_{\mu\nu}(k)}{k^2} = \frac{\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2}. \tag{8}$$

In this work we use a Landau gauge gluon propagator. In principle our result should not depend on the selection of the gauge, but here we use the bare vertex approximation for the quark-gluon vertex, and the Landau gauge, which is a fixed point of the renormalisation group, ensures that the result is not sensitive to the form of the quark-gluon vertex.

[1] Sixue Qin PRC2011

Gluon propagator

The effective interaction is

$$\frac{\mathcal{G}(k^2)}{k^2} = D\frac{8\pi^2}{\omega^4} e^{-k^2/\omega^2} + \mathcal{F}_{UV}(k^2), \tag{9}$$

$$\mathcal{F}_{UV}(k^2) \equiv \frac{8\pi^2\gamma_m \mathcal{F}(k^2)}{\ln[\tau + (1 + k^2/\Lambda^2_{QCD})^2]}, \; \mathcal{F}(k^2) = [1 - e^{-k^2/(4m_t^2)}]/k^2. \tag{10}$$

- This infrared term provides interaction strength at low momenta that is large enough to trigger dynamical chiral symmetry breaking (DCSB).
- The ultraviolet term comes form perturbative QCD, with $m_t = 0.5$ GeV, $\tau = e^2 - 1$, $N_f = 4$, $\Lambda^{N_f=4}_{QCD} = 0.234$ GeV, $\gamma_m = 12/(33 - 2N_f)$.

Quark-gluon vertex

Quark-gluon vertex: we choose the rainbow approximation so the quark-gluon vertex is replaced by bare vertex:

$$\Gamma_\nu^a \to \frac{\lambda^a}{2}\gamma_\nu . \tag{11}$$

The RL approximation is the leading term in a symmetry-preserving approximation scheme. The solution satisfy Ward–Takahashi identities[2].

[2] C. D. Roberts PPNP1994, P. Maris PRC1997

Quark propagator

Now the Dyson-Schwinger equation becomes this form:

$$S^{-1}(p) = Z_2 i\gamma \cdot p + Z_4 m\left(\mu^2\right) + Z_2^2 \int_q^\Lambda \frac{\mathcal{G}\left((p-q)^2\right)}{(p-q)^2} \frac{\lambda^a}{2} \gamma_\mu S(q) \frac{\lambda^a}{2} \gamma_\nu T_{\mu\nu}(p-q). \tag{12}$$

Consider the Lorentz structure of quark propagator, it should have two linear independent components and the general form is as follows:

$$S^{-1}(p) = i\gamma \cdot p A\left(p^2\right) + B\left(p^2\right), \tag{13}$$

Meson's Bethe-Salpeter Amplitudes

We can make R-L approximations to Bethe-Salpeter Equations and the truncated form is as follow:

$$\Gamma(P;k) = -Z_2^2 \int_q^\Lambda \frac{\mathcal{G}\left((q-k)^2\right)}{(q-k)^2} T_{\mu\nu}(q-k) \frac{\lambda^a}{2} \gamma_\mu S(q_+) \Gamma(P;k) S(q_-) \frac{\lambda^a}{2} \gamma_\nu, \tag{14}$$

with $P^2 = -M^2$. For pseudoscalar meson the basis are

$$i\gamma^5,\ \gamma^5 \not{P},\ \gamma^5 \not{k} P \cdot k,\ \frac{i}{2}\gamma^5 [\not{k}, \not{P}] \tag{15}$$

Then we can decouple Eq.(14), now it is a eigenvalue question. In this work we based on Arpack package to calculate it and canonical normalization is used.

Quark-graviton vertex

The Quark-graviton vertex $\Gamma^{\mu\nu}$ should satisfy tensor WGTI

$$iQ^{\mu}\Gamma_{\mu\nu}(Q,k) = S^{-1}(k_+)(k_-)_\nu - S^{-1}(k_-)(k_+)_\nu \tag{16}$$

According to Quark-graviton vertex DSE

$$\begin{aligned} i\Gamma^{\mu\nu}(P;k) &= Z_2(ik^\nu\gamma^\mu - g^{\mu\nu}(i\not{k}+m)) \\ &- Z_2^2\int_q^\Lambda \frac{\mathcal{G}\left((q-k)^2\right)}{(q-k)^2} T_{\alpha\beta}(q-k)\frac{\lambda^a}{2}\gamma_\alpha S(q_+)\, i\Gamma^{\mu\nu}(P;k)S(q_-)\,\frac{\lambda^a}{2}\gamma_\beta, \end{aligned} \tag{17}$$

The bare vertex satisfy the WGTI, but for the loop part, it still don't satisfy WGTI under Rainbow-Ladder truncation.

At first, based on the bare vertex, we construct an ansatz

$$\Gamma^{\mu\nu} = i\Gamma^{\mu}k^{\nu} - \frac{1}{2}g^{\mu\nu}\left(S^{-1}\left(k_{+}\right) + S^{-1}\left(k_{-}\right)\right) + \text{transverse terms} \tag{18}$$

with Γ^{μ} is quark-photon vertex. It satisfies tensor WGTI and tends to bare vertex at large relative momenta. For transverse term's basis we use

$$i\left(Q^2 g^{\mu\nu} - Q^{\mu}Q^{\nu}\right) \otimes \left(i\gamma^5,\ \gamma^5 \not{P},\ \gamma^5 \not{k} P\cdot k,\ \frac{i}{2}\gamma^5[\not{k}, \not{P}]\right) \tag{19}$$

And then we add a free parameter c to adjust the strength of the bare quark-gluon vertex

$$\begin{aligned} i\Gamma^{\mu\nu}(P;k) &= Z_2(ik^{\nu}\gamma^{\mu} - g^{\mu\nu}(i\not{k} + m)) \\ &- Z_2^2 \int_q^{\Lambda} \frac{\mathcal{G}\left(\left(q-k\right)^2\right)}{\left(q-k\right)^2} T_{\alpha\beta}(q-k)\frac{\lambda^a}{2}(c\gamma_{\alpha})S(q_{+})\, i\Gamma^{\mu\nu}(P;k)S(q_{-})\frac{\lambda^a}{2}(c\gamma_{\beta}), \end{aligned} \tag{20}$$

The value of c is determined in the chiral limit (~ 0.95). Now we can get the quark-graviton vertex's transverse part based on L-BFGS algorithm.

Numerical results

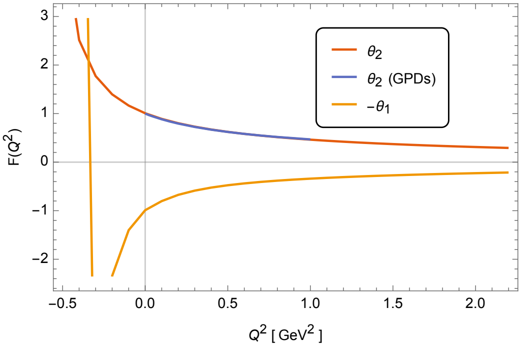


Figure: The gravitational form factors of pion. The GPD's result comes from Raya:2021zrz

Numerical result

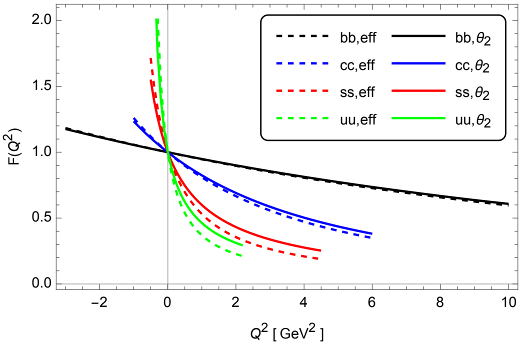


Figure: Compare θ_2 with electric form factor for pseudoscalar mesons.

	$u\bar{u}$	$s\bar{s}$	$c\bar{c}$	$b\bar{b}$
$\hat{m}$ [GeV]	0.0055	0.143	1.75	7.34
M [GeV]	0.135	0.688	2.977	9.262
f [GeV]	0.095	0.141	0.259	0.475
r^e [fm]	0.632	0.447	0.227	0.114
r^m [fm]	0.559	0.401	0.217	0.112

Summary

- The charge radius of pseudoscalar meson is larger than the mass radius.
- For pseudoscalar mesons, the mass distribution tends to coincide with the charge distribution as the mass of the meson increases
- θ_1 has a scalar meson's pole.

Thank you!