The Standard Model

Classical to QFT

Classical

- Newtonian mechanics and Maxwell's equations
- Deterministic
- Introducing field theories
- Valid at everyday scales

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

Quantum

- Schrödinger's equation and matrix mechanics
- Non-deterministic
- Calculations using probabilities
- Not relativistic

$$\frac{-\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} = i\hbar\frac{\partial \Psi}{\partial t}$$

$$\boldsymbol{\sigma_Q \sigma_R} \geq \left|\frac{\mathbf{1}}{\mathbf{2}\boldsymbol{i}}\langle[\hat{\boldsymbol{Q}}, \hat{\boldsymbol{R}}]\rangle\right|$$

Quantum Field Theory (QFT)

- Relativistic quantum mechanics
- Dirac equation, etc.
- Fields as fundamental basis

$$(i\not{\partial} - m)\psi = 0$$

$$\mathcal{L} = g^{\alpha\beta}\partial_\alpha\phi\,\partial_\beta\phi + m_0\phi^2$$

Gauge theories

- A gauge transformation is a modification to an underlying field that does not change the observable physics
- In electromagnetism, the observables E and B are defined in terms of the scalar and vector potentials φ and A

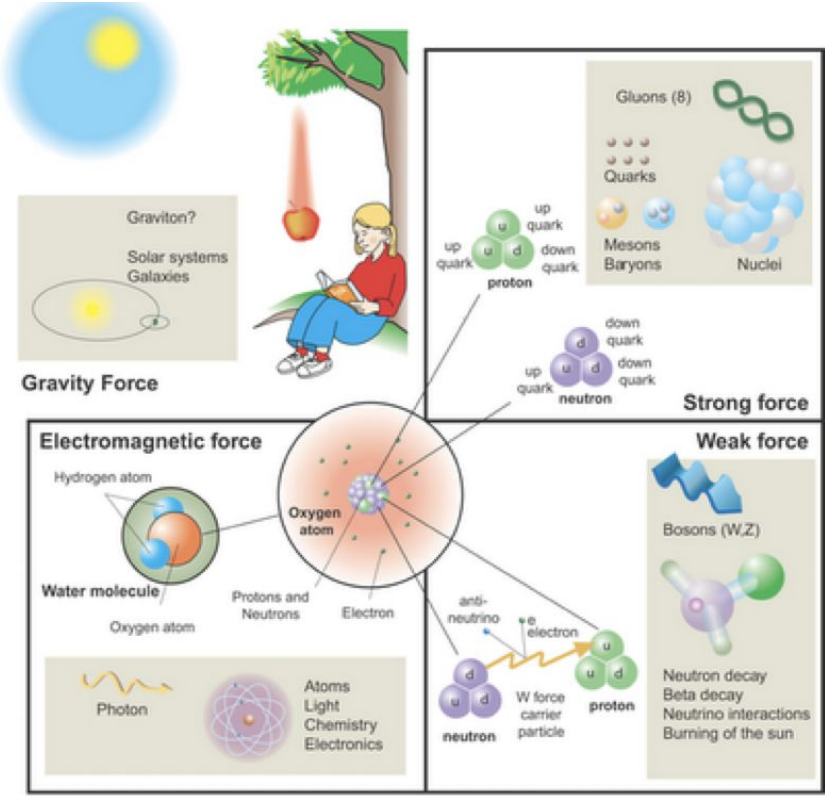
$$\mathbf{E} = -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t} \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

- Maxwell’s equations are invariant under the following gauge transformations

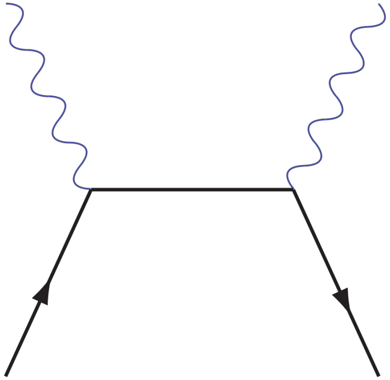
$$\varphi \to \varphi - \frac{\partial \psi}{\partial t} \qquad \mathbf{A} \to \mathbf{A} + \nabla\psi$$

- The Standard Model must be gauge invariant

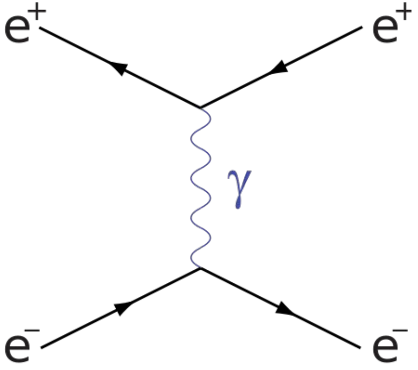
Fundamental forces



QED



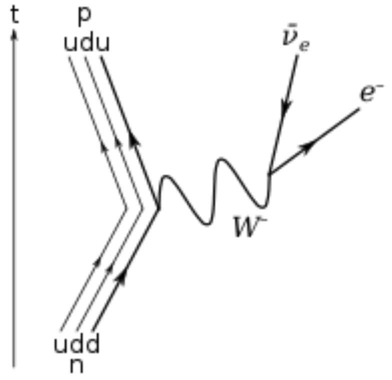
- Quantum ElectroDynamics - quantum field theory of E&M
- Abelian gauge theory with U(1) symmetry group
- Mediates interactions between charged fields
    - Electric charge defines interactions
- Photon (γ) is the mediating particle and couples to charge



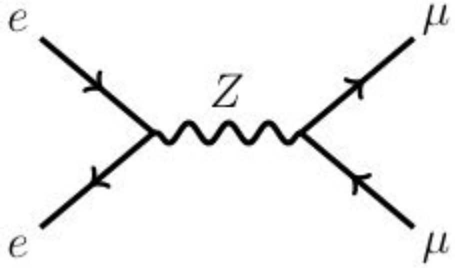
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - e j^\mu A_\mu$$

Weak interactions



- Sometimes referred to as Quantum FlavorDynamics (QFD)
- Quantum field theory of flavor physics
  - Flavor refers to particle identity
  - Governs radioactive decay
- Weak isospin (T_3) governs interactions
- Charged current ($W^\pm$ mediator) allows flavor mixing
- Neutral current (Z^0 mediator) does not permit flavor mixing
- Massive mediating particles limit range to sub-atomic scales
- Breaks parity (P) and charge-parity (CP) symmetries

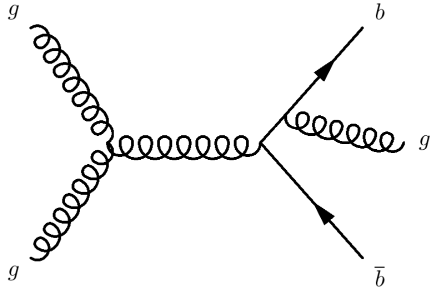


Electroweak unification

- QED and weak interactions can be unified into a single theory
- Resulting Yang-Mills field has an SU(2)xU(1) symmetry group
    - Weak hypercharge (Y_W) is defined from T_3 and electric charge
- At high energies, mediators are unified to be 4 massless gauge bosons
    - Spontaneous symmetry breaking gives rise to massive $W^\pm$ and Z^0 bosons
- More details next week…

QCD

- Quantum ChromoDynamics - quantum field theory of strong interactions
- Non-abelian gauge theory with SU(3) symmetry group
- Mediates interactions between particles carrying color charge
  - Color has 3 possible values
  - Combination of all three or color/anti-color is known as a color singlet
- Mediated by gluons, which also carry color charge
- Color confinement - bare quarks or gluons are not allowed
- Asymptotic freedom - interactions weaken at high energy scales
  - Stronger interactions at low energy ⇒ non-perturbative
  - High energy interactions can be treated as perturbative



Particle content of the SM

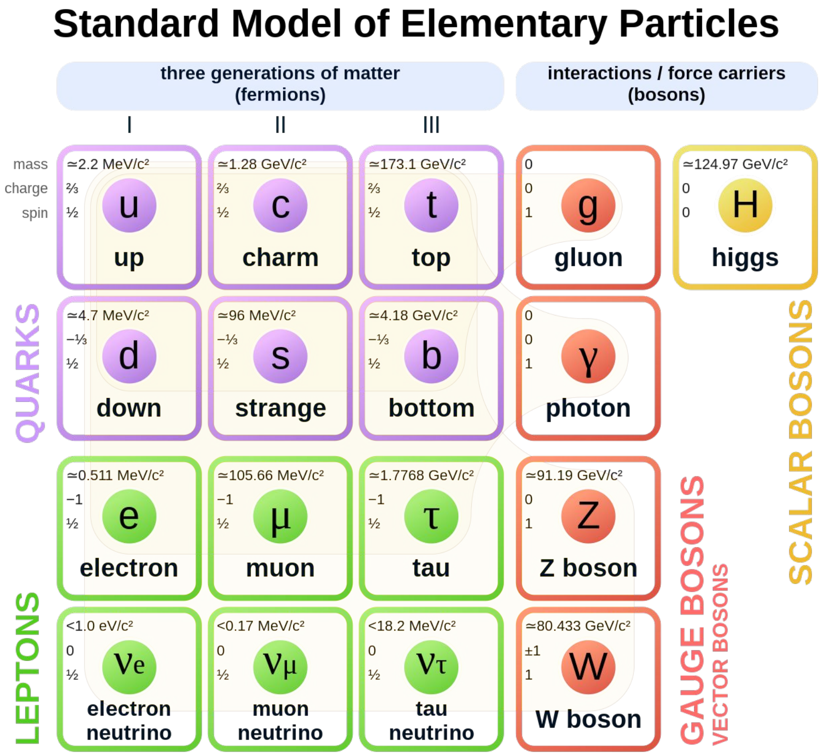
Fermions:

- Spin-½
- The “stuff” of matter
- Quarks and leptons
- Subject to Pauli exclusion principle
    - Fermi-Dirac statistics

Bosons

- Integer spin
- Force carrying particles
- Gauge and scalar bosons
- Not subject to Pauli exclusion
    - Bose-Einstein statistics

Particle content of the SM



Standard Model of Elementary Particles

	three generations of matter (fermions)			interactions / force carriers (bosons)	
	I	II	III		
mass charge spin	≃2.2 MeV/c^2 ⅔ ½ u up	≃1.28 GeV/c^2 ⅔ ½ c charm	≃173.1 GeV/c^2 ⅔ ½ t top	0 0 1 g gluon	≃124.97 GeV/c^2 0 0 H higgs
QUARKS	≃4.7 MeV/c^2 −⅓ ½ d down	≃96 MeV/c^2 −⅓ ½ s strange	≃4.18 GeV/c^2 −⅓ ½ b bottom	0 0 1 γ photon	SCALAR BOSONS
	≃0.511 MeV/c^2 −1 ½ e electron	≃105.66 MeV/c^2 −1 ½ μ muon	≃1.7768 GeV/c^2 −1 ½ τ tau	≃91.19 GeV/c^2 0 1 Z Z boson	GAUGE BOSONS VECTOR BOSONS
LEPTONS	<1.0 eV/c^2 0 ½ ν_e electron neutrino	<0.17 MeV/c^2 0 ½ ν_μ muon neutrino	<18.2 MeV/c^2 0 ½ ν_τ tau neutrino	≃80.433 GeV/c^2 ±1 1 W W boson	

Particle Data Group

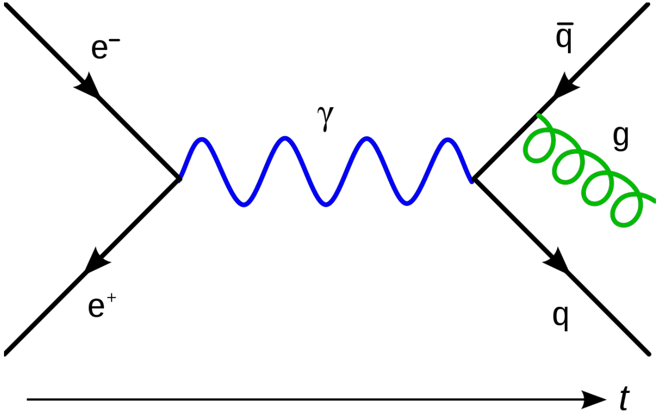
- The Particle Data Group (PDG) summarizes all available HEP information

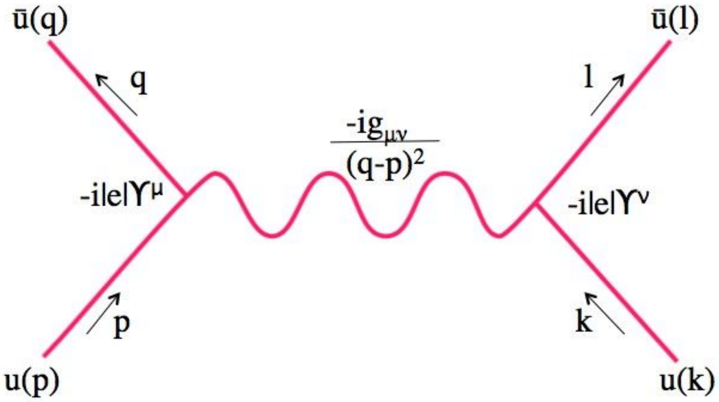
  https://pdg.lbl.gov/

- Summary tables as well as review articles
- Available online and in print
  - Comprehensive book and reference handbook can be ordered online for free
- PDG defines standard ID numbers for all known and hypothetical particles
  - Useful for Monte Carlo generators - to be discussed in more detail later

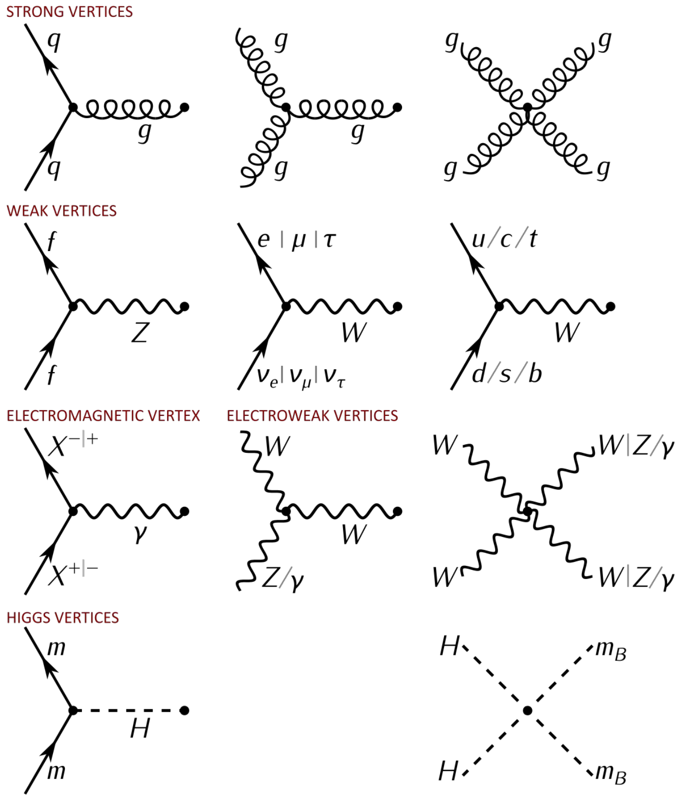
Feynman diagrams

- Graphical representation of physical particle interactions
  - Basic Feynman rules replace complex path integral formulation
- Typically time axis runs left to right
- Each mediator (line) and vertex contribute to calculations





Feynman diagrams - SM propagators and vertices



Perturbative field theories

- A perturbative field theory treats interactions as small deviations from a system with no interactions (free field theory)
  - Each additional interaction has a diminishing effect
- Very useful for calculating infinite sums of contributing effects
  - Can be depicted using Feynman diagrams of each degree of contribution
- A complete mathematical description can be found for perturbative QFT
- Non-perturbative field theories cannot be fully calculated
  - Low-energy QCD is the prime example
  - Various models can be used to approximate these interactions

Feynman diagrams - LO, NLO, etc.

- All diagrams with same set of incoming and outgoing particles contribute
    - Calculations are performed at a given order - defined by number of vertices
- Leading order (LO) diagrams have the minimum number of vertices
- Next-to-leading order (NLO) have the next smallest number of vertices
- For perturbative interactions, higher order corrections generally decrease