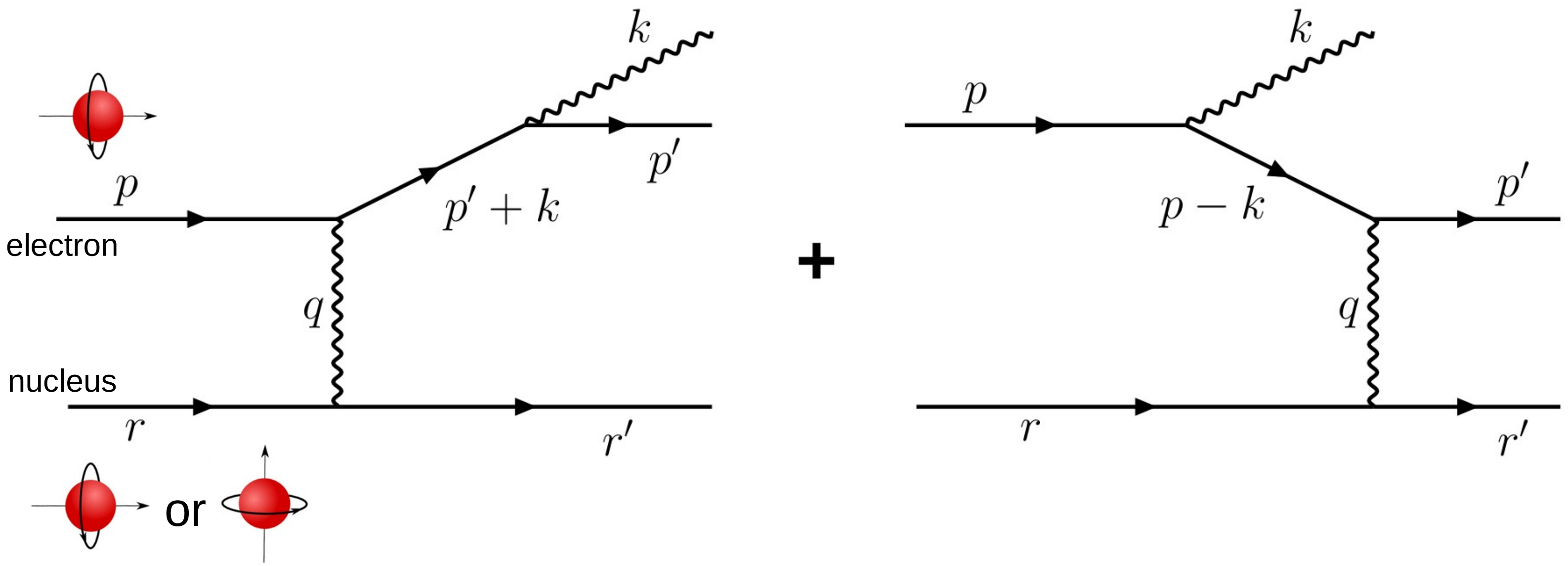
Polarized Bethe-Heitler Calculation at Leading Order



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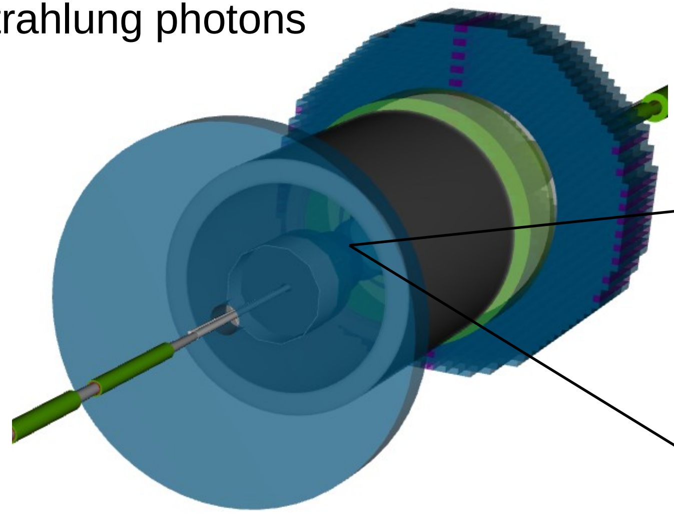
ePIC collaboration meeting
Warsaw July 28th 2023

Measuring Luminosity at the EIC

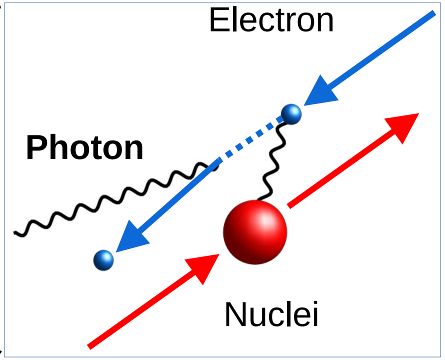
Goal of Luminosity program:

Measure the production rate of Bremsstrahlung photons
and use its calculable cross section
to extract the collider luminosity

$$\mathcal{L} = \frac{N_\gamma}{\sigma_{Brem}}$$



Dominant part of
total inelastic cross section
is QED Bremsstrahlung,
which is precisely calculable



Bethe & Heitler first calculated σ_{Brem} for *unpolarized* beams in 1934.

The EIC will accelerate *polarized* electrons *and* nuclei.

Goal of this work:

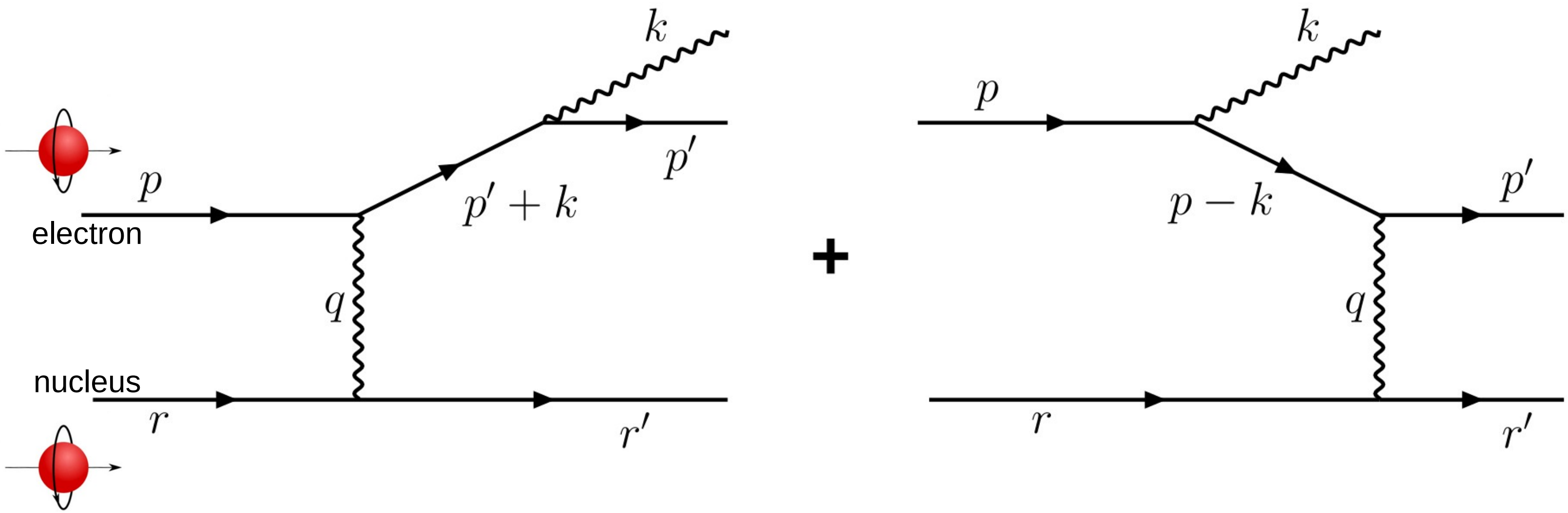
Calculate the polarized contribution to σ_{Brem}

Hasn't this already been calculated?? Not quite...

Highly related publications:

Akushevich et al. (1998) arXiv:hep-ph/9804361
Afanasev et al. (2001) arXiv:hep-ph/0102086

Two Feynman diagrams at Leading Order

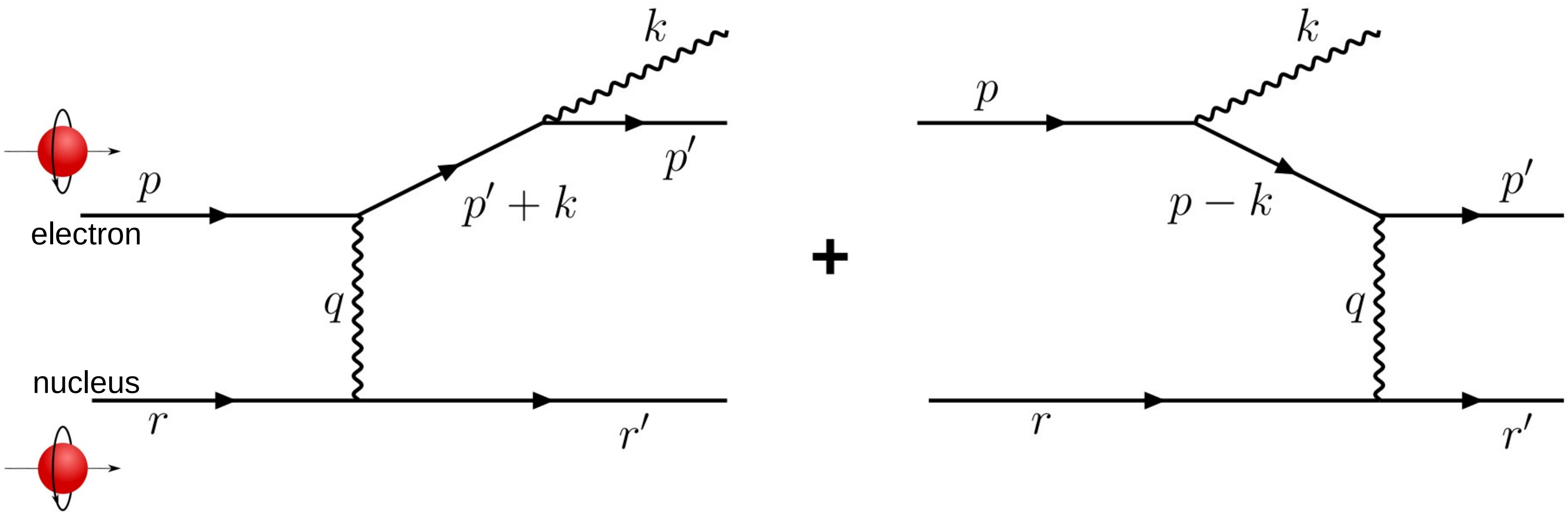


<u>3 references used:</u>

1) **Landau and Lifshitz, QED textbook (sections 93-97)**
2) Matthew Schwartz, QFT textbook
3) Gluckstern and Hull, Phys. Rev. 90 1953

- Before attempting the polarized case, the original Bethe-Heitler formula was re-derived.
- Main challenge is the tedious algebra of integrating over the angles in the final state.

Suitable approximations



Focus on dominant part of amplitude, for which: $q^2 \sim 0 \qquad (p' + k)^2 \sim 0 \qquad (p - k)^2 \sim 0$

This occurs at very small momentum transfer: $\mathbf{q} \sim m_e$.

<u>Suitable approximations (also used by Bethe & Heitler)</u>

- Neglect the structure of the hadron. Justified at low q^2.
- Ultrarelativistic particles: $p \approx \varepsilon - \dfrac{m_e^2}{2\varepsilon}$
- Neglect time component of $q = (0, \mathbf{q})$. "No-recoil" approximation. Resulting error is $\sim m_e / \varepsilon$

Lab Frame

Full 1D cross section

$\varepsilon,\ \varepsilon'$	energy electron, scat electron
ε_p	energy proton
ω	energy photon
$m_e,\ m_p$	electron and proton mass
$\mathbb{P}_e,\ \mathbb{P}_p$	electron and proton polarization
$\alpha,\ r_e$	fine struc const, classical electron radius

$$\frac{d\sigma}{d\omega} = \frac{4\alpha r_e^2}{\omega}\frac{\varepsilon'}{\varepsilon}\left[\mathcal{U} + \mathcal{P}\right]$$

Unpolarized
Bethe-Heitler term

$$\mathcal{U} = \left(\frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon} - \frac{2}{3}\right)\left(\ln\frac{4\varepsilon_p\,\varepsilon\,\varepsilon'}{m_p\,m_e\,\omega} - \frac{1}{2}\right)$$

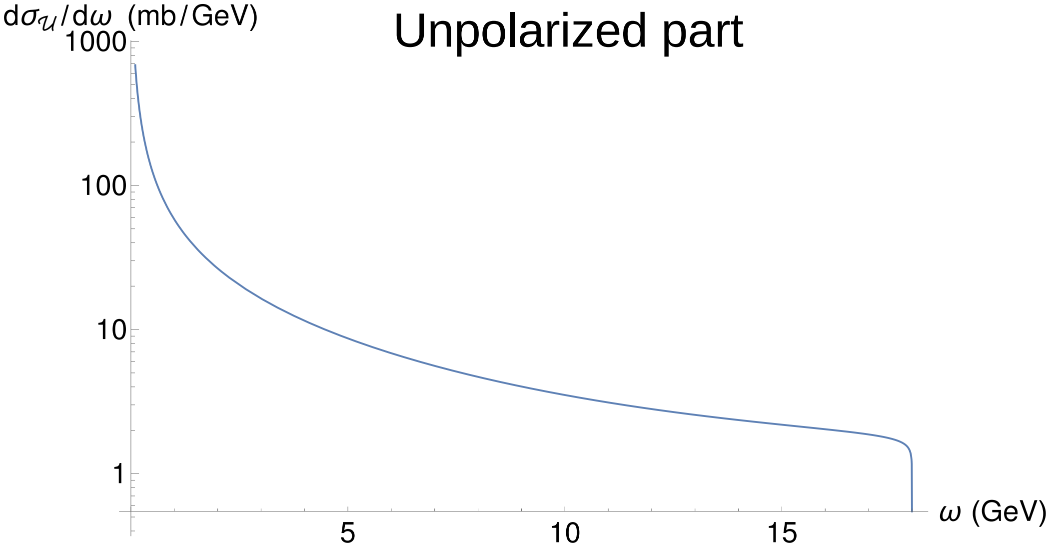
New Polarized term

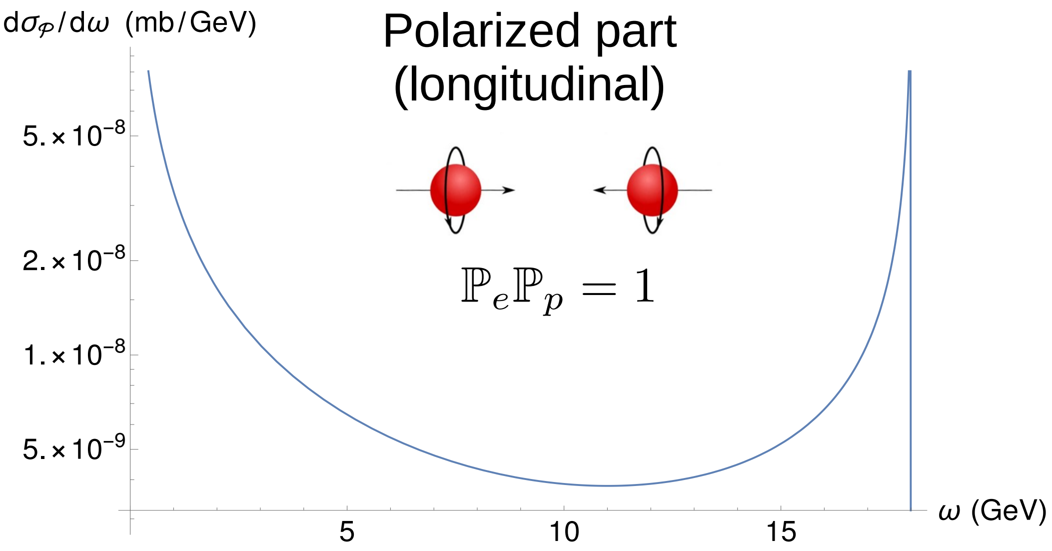
$$\mathcal{P} = \mathbb{P}_e\mathbb{P}_p\frac{m_e^2}{\varepsilon\,\varepsilon_p}\left(F_1 + \frac{\varepsilon}{4\varepsilon'}F_2 + \frac{\varepsilon'}{8\varepsilon}F_3 + \frac{\varepsilon^2}{2\varepsilon'^2}F_4\right)$$

F_1, F_2, F_3, F_4 are functions containing 3 types of logs (in backup).
One log is the same as that in Bethe-Heitler expression.

The polarized part is highly suppressed wrt unpolarized part.

Plots of cross section components
ep 18 x 275 GeV





- Polarized part of the cross section is completely negligible.
- For transverse ion polarizations at the EIC, the polarized part vanishes exactly when integrating over azimuthal angles.

Summary

- An analytic expression has been derived for polarized bremsstrahlung cross section relevant for the low q^2 regime of the EIC luminosity program.
- Polarized part is highly suppressed wrt unpolarized Bethe-Heitler component.
  - The origin of the suppression is the low q^2 scale that dominates the Bremsstrahlung process.
- The main feature of this calculation was shown to Andrei Afanasev, who agreed that polarization asymmetries would be proportional to q^2, and therefore suppressed.

Backup

Correction to Bethe-Heitler at high Z

Bethe-Maximon equation

Phys Rev 93, 768 & 788 (1954)

$$\frac{d\sigma_{\mathrm{BM}}}{d\omega} = \frac{4\alpha r_e^2}{\omega}\frac{\varepsilon'}{\varepsilon}\left(\frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon} - \frac{2}{3}\right)\left(\ln\frac{4\varepsilon\,\varepsilon'\,\varepsilon_p}{\omega\, m_p m_e} - \frac{1}{2} - \underline{f(\alpha Z)}\right)$$

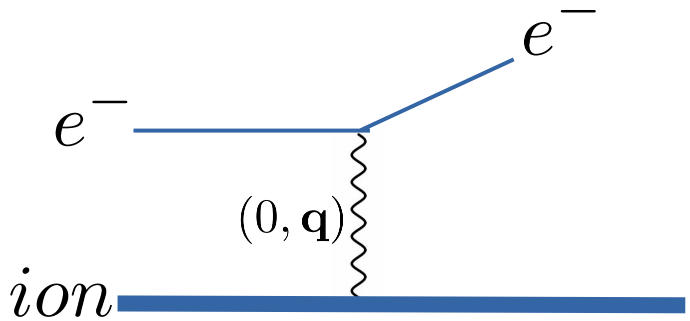
For large Z, the Bethe-Heitler expression is known to be inaccurate.

$$f(\alpha Z) = (\alpha Z)^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2 + (\alpha Z)^2)}$$

$$\text{for} \ \ \alpha Z \ll 1 \qquad f(\alpha Z) \approx 1.2(\alpha Z)^2$$

Relativistic Rutherford Scattering

- To gain some insight into the expected scale of the polarized part, compute the much simpler Rutherford scattering.



$$|M|^2 = \frac{e^4}{q^4}\left[\mathcal{U} + \mathcal{P}\right]$$

$$a^{(e)} \sim \frac{\varepsilon_I\, \varepsilon}{m_I\, m_e}$$

$$\mathcal{U} \approx 8m_I^2(2\varepsilon'\varepsilon + m_e^2 - p'p) \sim m_I^2\varepsilon'\varepsilon$$

$$a^{(I)} \sim 1$$

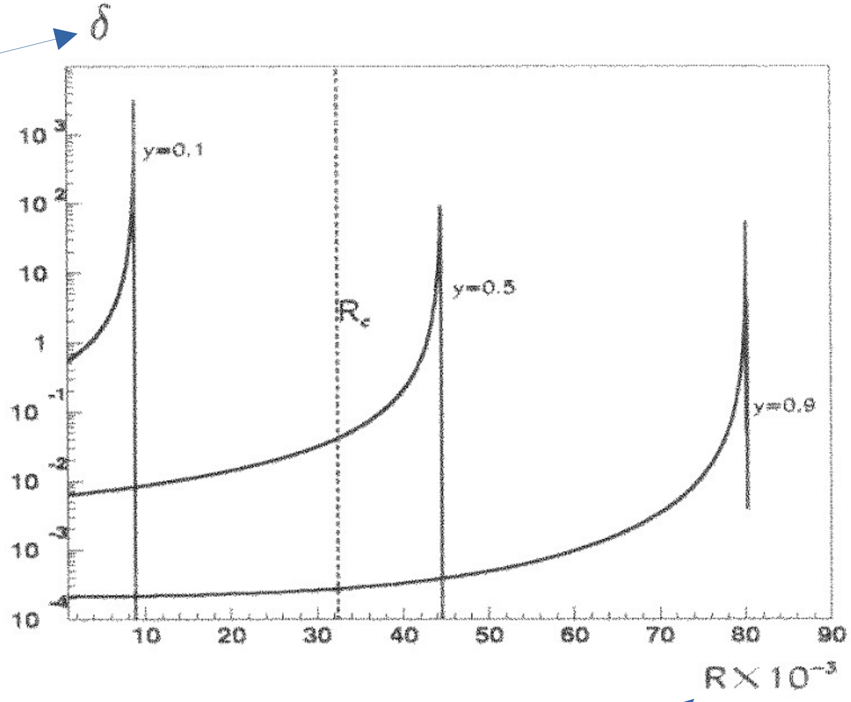
$$\mathcal{P} = -8m_e m_I(a^{(e)}a^{(I)}\, q^2 - qa^{(e)}\, qa^{(I)}) \sim \varepsilon_I \varepsilon\, m_e^2$$

$$q^2 \sim m_e^2$$

- **Polarized part scales with q^2, which is tiny in the Bremsstrahlung process.**

Sharp drop of cross section at upper energy limit observed before in numerical calculations

normalized cross section

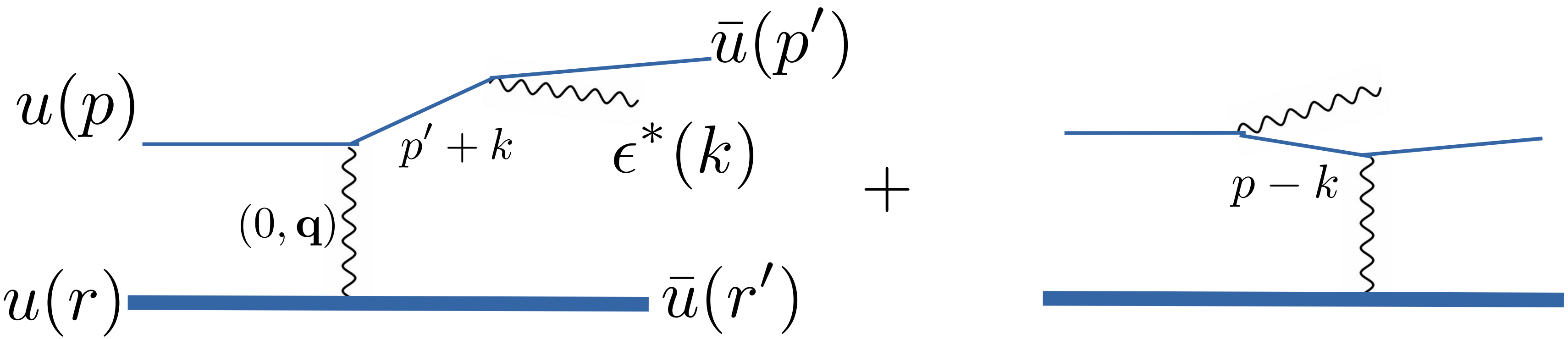


Akushevich et al. (1998)
arXiv:hep-ph/9804361

Figure 6. Integrand of radiative correction to the cross section (36) over R normalized to the Born one in the case of combination of the cross section (43). The broken line shows the used cut of photon energy ($E_\gamma < 10$ GeV).

R is proportional to the photon energy

QED LO Feynman Diagrams



<u>Beam electron</u>

$$p = (\varepsilon, \mathbf{p})$$

<u>Scattered electron</u>

$$p' = (\varepsilon', \mathbf{p}')$$

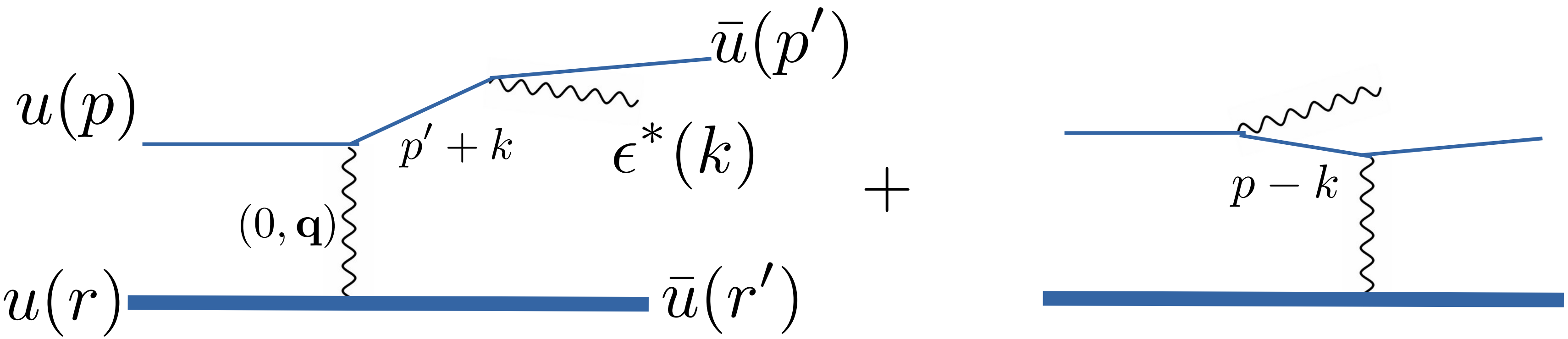
<u>Emitted photon</u>

$$k = (\omega, \boldsymbol{\omega})$$

<u>No-recoil approximation, as in ordinary Bethe-Heitler calculation</u>

$$q^0 = \varepsilon - (\varepsilon' + \omega) = 0$$

QED LO Feynman Diagrams



- This is a low q^2 process, so we neglect the structure of the ion. For heavy-ion beams at the EIC, the effect scales with Z^2.
- Feynman rules taken from Schwartz' book.

$$iM = ie^3 \frac{g_{\mu\nu}}{q^2} \epsilon^*_\sigma(k) \left[\bar{u}(r')\gamma^\mu u(r)\right] \left[\bar{u}(p') \left[\gamma^\sigma \frac{\not{p}' + \not{k} + m_e}{2p'k} \gamma^\nu + \gamma^\nu \frac{\not{p} - \not{k} + m_e}{-2pk} \gamma^\sigma\right] u(p)\right]$$

Amplitude Squared

Photon polarizations will be unmeasured:

$$\sum_{pol} \varepsilon_a^*(k)\varepsilon_b(k) \to -g_{ab}$$

$$|M|^2 = -\frac{e^6}{4q^4} Tr[u(r')\bar{u}(r')\gamma^\mu u(r)\bar{u}(r)\gamma^\alpha]\, Tr[u(p')\bar{u}(p')Q^\sigma_\mu u(p)\bar{u}(p)\bar{Q}_{\sigma\alpha}]$$

$$Q^\sigma_\mu = \gamma^\sigma \frac{\not{p}' + \not{k} + m_e}{p'k}\gamma_\mu - \gamma_\mu \frac{\not{p} - \not{k} + m_e}{pk}\gamma^\sigma$$

$$\bar{Q}_{\sigma\alpha} = \gamma_\alpha \frac{\not{p}' + \not{k} + m_e}{p'k}\gamma_\sigma - \gamma_\sigma \frac{\not{p} - \not{k} + m_e}{pk}\gamma_\alpha$$

$$|M|^2 \equiv -\frac{e^6}{4q^4} W^{\mu\alpha}\, w_{\mu\alpha}$$

Ion Tensor Electron Tensor

Unpolarized & Polarized Parts

Polarized part

$$W^{\mu\alpha} = \boxed{\mathcal{U}^{\mu\alpha}} + \boxed{\mathcal{P}^{\mu\alpha}}$$

$$w_{\mu\alpha} = \boxed{u_{\mu\alpha}} + \boxed{p_{\mu\alpha}}$$

Unpolarized part

- $U^{\mu\alpha}$ and $u_{\mu\alpha}$ are symmetric tensors
- $P^{\mu\alpha}$ and $p_{\mu\alpha}$ are antisymmetric.
- Thus, only $U^{\mu\alpha}u_{\mu\alpha}$ and $P^{\mu\alpha}p_{\mu\alpha}$ survive.
- $U^{\mu\alpha}u_{\mu\alpha}$ is the ordinary Bethe-Heitler
- **$P^{\mu\alpha}p_{\mu\alpha}$ is new and to be calculated.**

<u>Unmeasured spins in the final state</u>

$$\sum_{spin} u(p')\bar{u}(p') = \not{p}' + m_e \qquad \sum_{spin} u(r')\bar{u}(r') = \not{r}' + m_I$$

<u>Measured beam polarizations</u>

$$u(p)\bar{u}(p) = \frac{1}{2}(\not{p} + m_e)(1 - \gamma^5 \not{a}^{(e)})$$

$$u(r)\bar{u}(r) = \frac{1}{2}(\not{r} + m_I)(1 - \gamma^5 \not{a}^{(I)})$$

$a^{(e)}$ and $a^{(I)}$ are the electron and Ion spin 4-vectors

Spin 4-vectors

<u>Pauli-Lubanski pseudovector:</u> Purely vectorial in particle's respective Rest Frame

$$a^{(e)}_{RF} = \left(0, \vec{\zeta^{(e)}}\right) \qquad a^{(I)}_{RF} = \left(0, \vec{\zeta^{(I)}}\right)$$

- The Lab coordinate system is defined as for the ePIC detector where the Ion moves along +**Z**, and the electron moves along -**Z**.
- The above spin vectors are boosted to the Lab Frame where the beam polarizations are defined, which allows one to solve for ζ. Lifshitz QED section 29.
- Then, we further boost to Target Rest Frame (Ion), where we will remain until the very end.
- Focus on one of the EIC configurations: **longitudinally polarized beams**.

<u>In the Target Rest Frame (ultrarel limit: p->E)</u>

Beam energies as measured in Lab Frame

$$a^{(e)} = 2\mathbb{P}_e \frac{E_e E_I}{m_e m_I} \left(-1, 0, 0, +1\right)$$

electron beam polarization
= twice the mean spin vector along -**Z** in Lab Frame

$$a^{(I)} = \mathbb{P}_I \left(0, 0, 0, +1\right)$$

Ion beam polarization
= twice the mean spin vector along +**Z** in Lab Frame

Ion tensor

$$W^{\mu\alpha} = Tr[(\not{r}' + m_I)\gamma^\mu \frac{1}{2}(\not{r} + m_I)(1 - \gamma^5 \not{a}^{(I)})\gamma^\alpha]$$

$$\begin{aligned}
\mathcal{P}^{\mu\alpha} &= -\frac{1}{2} a_c^{(I)} Tr[(r'_a \gamma^a + m_I)\gamma^\mu (r_b \gamma^b + m_I)\gamma^5 \gamma^c \gamma^\alpha] \\
&= -\frac{m_I}{2} a_c^{(I)} \left[r'_a Tr[\gamma^a \gamma^\mu \gamma^5 \gamma^c \gamma^\alpha] + r_b Tr[\gamma^\mu \gamma^b \gamma^5 \gamma^c \gamma^\alpha] \right] \\
&= \frac{m_I a_c^{(I)}}{2} (r_a - r'_a) Tr[a\mu c\alpha 5] \\
&= 2i\, m_I\, q_a\, a_c^{(I)}\, \varepsilon^{a\mu c\alpha}
\end{aligned}$$

- *q* is momentum transfer in the exchange photon.
- This is a hint that the overall polarization effect will be small since $q^2 \sim m_e^2$. Lifshitz Eq 97.10
- Compare the scale to the unpolarized counter part: $U^{\mu\alpha} = 4m_I^2 \delta^{0\mu}\delta^{0\alpha} + \mathcal{O}(m_e^2)$

Electron tensor, polarized part

$$p_{\mu\alpha} \equiv p^{(1)}_{\mu\alpha} + p^{(2)}_{\mu\alpha} + p^{(3)}_{\mu\alpha} + p^{(4)}_{\mu\alpha}$$

- 4 terms arise from the product of Q and $\overline{Q}$ on slide 3.

$$p^{(1)}_{\mu\alpha} = 8i\frac{m_e a^{\lambda,(e)}}{(p'k)^2}\varepsilon_{a\mu\lambda\alpha}\left[m_e^2 q^a - p'k(p^a + k^a)\right]$$

$$p^{(2)}_{\mu\alpha} = 8i\frac{m_e a^{\lambda,(e)}}{(pk)^2}\left[(m_e^2 q^a - p'^a\, pk)\varepsilon_{a\mu\lambda\alpha} - p'^a k_\lambda(k^b - p^b)\varepsilon_{a\mu b\alpha}\right]$$

$$p^{(3)}_{\mu\alpha} = 8i\frac{m_e a^{\lambda,(e)}}{(pk)(p'k)}\left[\frac{q^2}{2}p'^a\varepsilon_{\lambda a\mu\alpha} + p'^a p^\mu(k^b - p^b)\varepsilon_{\lambda ab\alpha} + (p'^\alpha k^a(p^b - p'^b) + p^b(p^\alpha - k^\alpha)(p'^a + k^a))\varepsilon_{\lambda ab\mu}\right]$$

$$p^{(4)}_{\mu\alpha} = -p^{(3)}_{\alpha\mu}$$

Note: the simple relation between $p^{(3)}$ and $p^{(4)}$ doesn't exist between $p^{(1)}$ and $p^{(2)}$.

1st Result:
Fully differential polarized amplitude

$$
\begin{aligned}
\mathcal{P}^{\mu\nu}p_{\mu\nu} &= -32 m_e m_I \Bigg[\frac{1}{(pk)^2}\left(\frac{q^2}{2} ka^{(e)}(qa^{(I)} - 2p'a^{(I)})\right) \\
&+ \left(\frac{1}{(pk)^2} + \frac{1}{(p'k)^2}\right)\left(qa^{(e)}qa^{(I)}m_e^2 - a^{(e)}a^{(I)}q^2 m_e^2\right) \\
&+ \frac{1}{(pk)(p'k)}\left(-q^4 a^{(e)}a^{(I)} - 2qa^{(e)}qa^{(I)}m_e^2 + \frac{q^2}{2}(2p'a^{(e)}(p'a^{(I)} - pa^{(I)}) + ka^{(e)}(3qa^{(I)} + 2pa^{(I)}) + 4m_e^2 a^{(e)}a^{(I)})\right) \\
&+ \left(\frac{1}{p'k} - \frac{1}{pk}\right)\left((qa^{(e)} + ka^{(e)})qa^{(I)} - 2q^2 a^{(e)}a^{(I)}\right)\Bigg]
\end{aligned}
$$

- Beam polarizations appear in $a^{(e)}$ and $a^{(I)}$

Polarized cross section

$$|M|^2_{pol} = -\frac{e^6}{4q^4}\mathcal{P}^{\mu\alpha}p_{\mu\alpha}$$
$$= -(4\pi)^3\frac{\alpha\, r_e^2\, m_e^2}{4q^4}\mathcal{P}^{\mu\alpha}p_{\mu\alpha}$$

Fine-structure constant

$$\alpha = \frac{e^2}{4\pi}$$

Classical electron radius

$$r_e = \frac{e^2}{4\pi\, m_e}$$

$$d\sigma_{pol} = \frac{1}{2^5(2\pi)^5}|M|^2_{pol}\frac{|\mathbf{p}'|\omega d\omega}{|\mathbf{p}|m_I E_I}d\Omega' d\Omega_0$$

Ω’ = angular phase space of scattered electron

Ω_0 = angular phase space of emitted photon

- Polar axes strategically chosen → azimuthal integrations each yield a trivial (2π).

$$d\sigma_{pol} = -\frac{\alpha\, r_e^2\, m_e^2}{2^4 q^4}\mathcal{P}^{\mu\alpha}p_{\mu\alpha}\frac{|\mathbf{p}'|\omega d\omega}{|\mathbf{p}|m_I E_I}d\cos\theta' d\cos\theta_0$$

Integration over scattered electron angles

- The luminosity detectors at the EIC will typically not correlate scattered electron angles at very small q^2 with the emitted photon measurements. It's experimentally irrelevant, so integrate over scattered electron angles.
- This can be done analytically according to Ref 3), with integrals of type $I_{m,n}$
- All are elementary except 3, but they can be found in integral tables.
- The algebra is long and tedious. To help prevent mistakes, Mathematica is used to assemble all the terms, simplify, and apply the ultrarelativistic expansion to all terms where momenta appears, i.e. $p \sim \varepsilon - m_e^2/(2\varepsilon)$.
- **There are no terms of order m_e^0!**

Boost to Lab Frame for final result

- Now transform the expression from target rest frame to the lab frame in the ultrarelativistic limit.
- ε_I and m_I is the energy and mass of the Ion, respectively.

$$\omega \to \omega \frac{2\varepsilon_I}{m_I} \qquad \varepsilon \to \varepsilon \frac{2\varepsilon_I}{m_I} \qquad \varepsilon' \to \varepsilon' \frac{2\varepsilon_I}{m_I}$$

Lab Frame

2^{nd} result: double-differential cross section

- Here is the lowest surviving order in the small angle approximation: δ = θ ε/m_e

$$\frac{d\sigma_{\mathcal{P}}}{d\omega d\delta} = \mathbb{P}_e \mathbb{P}_p \frac{\alpha\, r_e^2\, m_e^2\, \delta}{\omega \varepsilon^3 \varepsilon' \varepsilon_p (1+\delta^2)^2} \Bigg[2\omega^3 + 2(-1+2L_2)\varepsilon'^3 + (8+6L_2)\varepsilon'^2\omega + 2(1+L_2)\varepsilon'\omega^2 \\ -2(L_1+L_2-L_\theta)(\varepsilon^2+\varepsilon'^2-\varepsilon\omega\delta^2)\varepsilon \\ -\left(4\varepsilon'^3 + 2(-1+L_2)\varepsilon'^2\omega + 5\varepsilon'\omega^2 + 2\omega^3\right)\delta^2 \\ + \frac{2\varepsilon\varepsilon'\omega}{1+\delta^2} + 2L_1\varepsilon\varepsilon'(\varepsilon+\varepsilon')(1+\delta^2) \Bigg]$$

$$L_1 = 2\ln\frac{4\varepsilon'\varepsilon_p}{m_e m_p} \qquad L_2 = \ln\frac{4\,\varepsilon\,\varepsilon'\,\varepsilon_p}{\omega\, m_e m_p} \qquad L_\theta = \ln(1+\delta^2)$$

Lab Frame

Integration over photon polar angle
Main result

- The integration over the photon polar angle is elementary but leads to a lot of algebra. Mathematica is used for the integration and simplification.

$$\frac{d\sigma_{pol}}{d\omega} = \frac{4\alpha r_e^2}{\omega}\frac{\varepsilon'}{\varepsilon}\mathbb{P}_e\mathbb{P}_p\frac{m_e^2}{\varepsilon\,\varepsilon_p}\left(F_1 + \frac{\varepsilon}{4\varepsilon'}F_2 + \frac{\varepsilon'}{8\varepsilon}F_3 + \frac{\varepsilon^2}{2\varepsilon'^2}F_4\right)$$

$$\begin{aligned}
F_1 &= \frac{1}{8}\left(5 + L_1(-2+4L_3) - 4L_3(-3+L_2) + 2L_2\right) \\
F_2 &= L_1 + (-1+L_3)(3+2L_3-2L_2) \\
F_3 &= -1 - 2L_2 + 2L_3(-9+2L_2) \\
F_4 &= (-1+L_3)(-2+L_1+L_2-L_3)
\end{aligned}$$

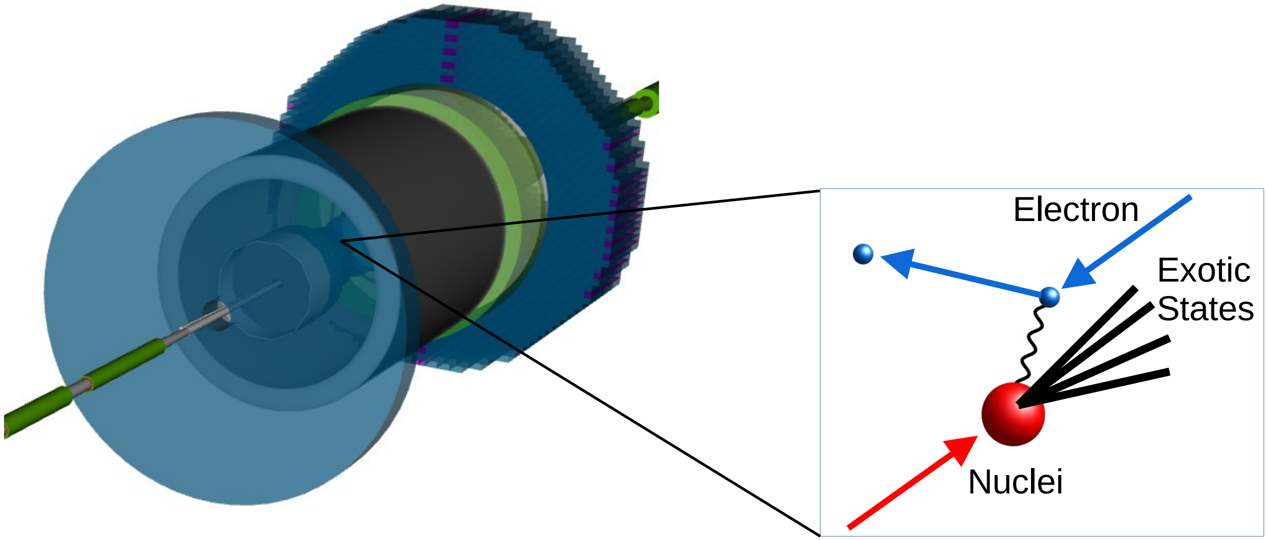
$$\begin{aligned}
L_1 &= \ln\frac{4\varepsilon'\,\varepsilon_p}{m_e m_p} \\
L_2 &= \ln\frac{4\varepsilon\,\varepsilon'\,\varepsilon_p}{\omega m_e m_p} \\
L_3 &= \ln\frac{\pi\varepsilon\,\varepsilon_p}{m_e m_p}
\end{aligned}$$

The Need for Luminosity Measurements

Main EIC Goal:

Measure the cross section of “exotic” states

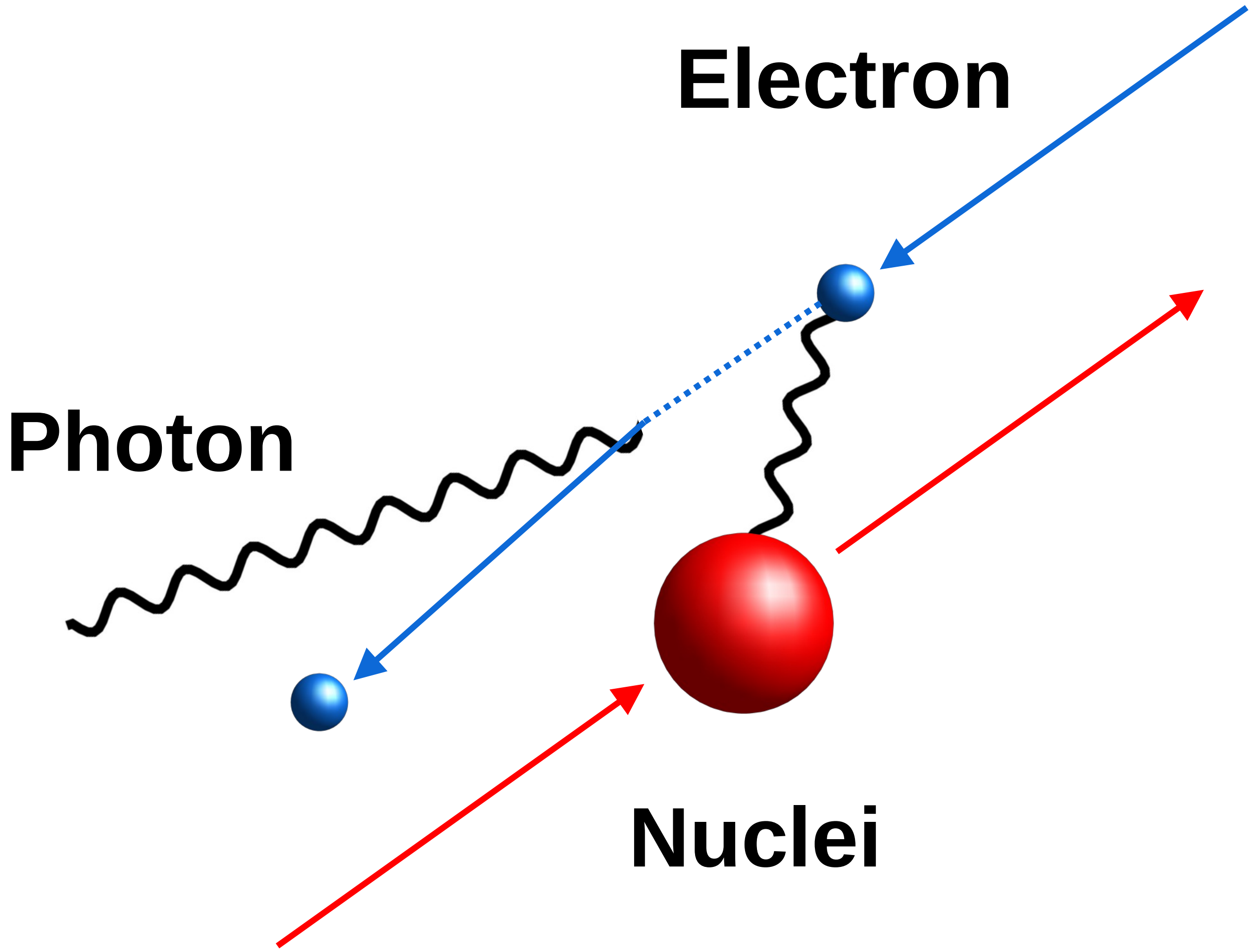
$$\sigma_{\text{exotic}} = \frac{N_{\text{exotic}}}{\mathcal{L}}$$



We need the a method to measure the collider luminosity

- Without luminosity, we won’t have our cross sections of interest

- Without cross sections, we won’t have PDFs, ...


Electron
Photon
Nuclei