# Gluon saturation/CGC theory towards precision at EIC


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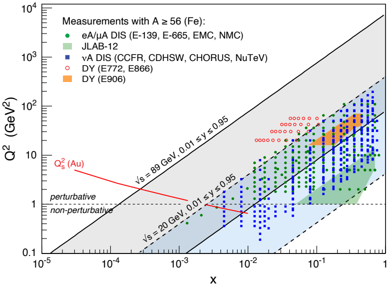
# Gluon saturation in DIS at low $x$




Study of the non-linear gluon saturation regime of QCD at low $x$ and moderate $Q^2$ :
$\rightarrow$ one of the majors goals of the EIC

- Breakdown of standard pQCD approaches (collinear/TMD/GPD) in that regime due to multiple parton scattering effects
- For $Q_s^2 \gtrsim Q^2 > 1$ GeV$^2$: regime of strong color fields but weak coupling $\alpha_s$
  $\rightarrow$ semi-classical QCD
- Nonlinear saturation effects enhanced for large nuclei: $Q_s^2(x_{Bj}, A) \propto A^{1/3}$

# CGC at LO and beyond

At leading power in high energy limit (or $x_{Bj} \to 0$) at fixed $Q^2$:
Color Glass Condensate effective theory (CGC) for scattering processes (includes gluon saturation)

In CGC, cross sections are typically convolutions of:

- Perturbatively calculable impact factors (processes dependent)
- Non-perturbative correlators of Wilson lines: dipoles and higher multipoles

These multipoles have a calculable dependence on $x_{Bj}$ (or energy or rapidity):
BK equation for dipole and JIMWLK equation for general multipoles: Nonlinear equations!

CGC at LO in $\alpha_s$ in qualitative agreement with HERA and LHC data for many observables but insufficient precision for unambiguous observation of gluon saturation in the data.

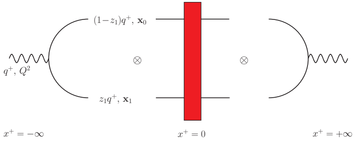
$\rightarrow$ Need for theory predictions at NLO in $\alpha_s$ in order to perform precise quantitative studies, in particular at the EIC.

$\Rightarrow$ Required theoretical input :

- NLO corrections to the rapidity evolution equations, in order to resum low $x$ NLLs
  – NLO BK [Balitsky, Chirilli - arXiv:0710.4330 / 1309.7644]
  – NLO JIMWLK [Kovner, Lublinsky, Mulian - arXiv:1310.0378]
  [Lublinsky, Mulian - arXiv:1610.03453]
- NLO corrections to process-dependent impact factors

# Inclusive DIS at low $x$ from LO to NLO (massless quarks)

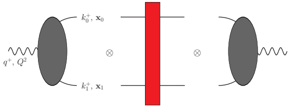
Dipole factorization at LO for inclusive DIS at low $x_{Bj}$:

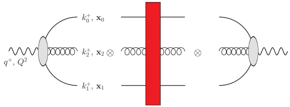



$$\sigma^{\gamma^* A \to X}_{T,L}(x_{Bj}, Q^2) \propto \int_{\mathbf{x}_0, \mathbf{x}_1} \int_0^1 dz_1 \Phi^{q\bar{q},LO}_{T,L}(\mathbf{x}_{01}, z_1, Q^2)\Big[1 - \langle s_{01} \rangle\Big]$$

$s_{01}$ is the color dipole operator

NLO impact factor for inclusive DIS structure functions with massless quarks:
[Balitsky, Chirilli - arXiv:1009.4729 /1207.3844]
[G.B. - arXiv:1112.4501 /1606.00777 / 1708.06557]
[Hanninen, Lappi, Paatelainen - arXiv:1711.08207]

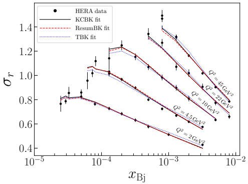


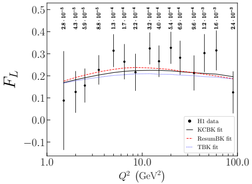




$$\sigma_{T,L}(x_{Bj}, Q^2) = \sum_{q\bar{q}\ st.} \Big|\Psi^{\gamma^*_{T,L}}_{q\bar{q}}\Big|^2 \Big[1 - \langle s_{01} \rangle_0\Big] + \sum_{q\bar{q}g\ st.} \Big|\Psi^{\gamma^*_{T,L}}_{q\bar{q}g}\Big|^2 \Big[1 - \langle s_{012} \rangle_0\Big]$$

- Perturbatively calculable $\Psi^{\gamma^*_{T,L}}_{q\bar{q}}$ LFWF at one lone loop, $\Psi^{\gamma^*_{T,L}}_{q\bar{q}g}$ at tree level
- UV divergences cancelled between $q\bar{q}$ and $q\bar{q}g$ contributions
- Extraction and resummation of low $x_{Bj}$ logs performed at the end, with BK equation

# Fits to HERA ep data at NLO: massless quarks




Fits of $\langle \mathbf{S}_{01} \rangle_{x_{Bj}}$ on HERA ep data for $\sigma_r$ ($\simeq F_2$) using :

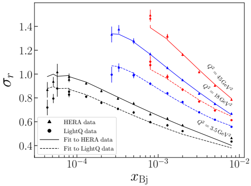
- The NLO corrections to dipole factorization in massless case
- Low $x$ LL resummation with an improved BK equation with running coupling and improved kinematics (collinear resummation) $\rightarrow$ Approx. low $x$ NLL resummation

$\rightarrow$ Overall successful fits, with weak dependence on details of resummation schemes ($\rightarrow$ 12 fits)

$\rightarrow$ $F_L$ obtained from the fit is consistent with HERA data

Moreover: Fits also performed on light quark $\sigma_r$ interpolated data extracted from HERA data thanks to a phenomenological dipole model

$\rightarrow$ Another 12 successful fits, but very different parameters obtained for light quark interpolated data vs full data

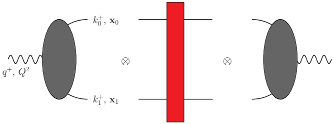


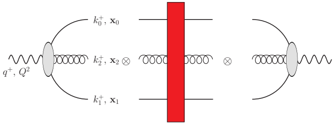

[G.B., Hanninen, Lappi, Mantysaari - arXiv:2007.01645]

# NLO Inclusive DIS: massive quarks

Charm and bottom quarks known to give a sizable contribution to DIS structure function at HERA
$\Rightarrow$ Precision cannot be achieved without including heavy quark mass effects!

Calculation of inclusive DIS at NLO in the dipole factorization with massive quarks:
[G.B., Lappi, Paatelainen - arXiv:2103.14549 / 2112.03158 / 2204.02486]

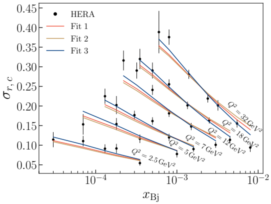


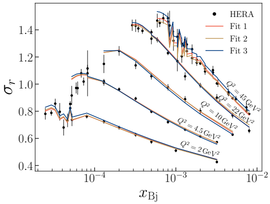




$$\sigma_{T,L}(x_{Bj}, Q^2) = \sum_{q\bar{q}\ st.} \left|\Psi_{q\bar{q}}^{\gamma^*_{T,L}}\right|^2 \left[1 - \langle s_{01}\rangle_0\right] + \sum_{q\bar{q}g\ st.} \left|\Psi_{q\bar{q}g}^{\gamma^*_{T,L}}\right|^2 \left[1 - \langle s_{012}\rangle_0\right]$$

- Same general method as in the massless quark case
- But new complications, like quark mass renormalization in Light-Front PT

# NLO results with massive quarks vs HERA ep data




Convolution of :

- the 12 fits of $\langle \mathbf{S}_{01} \rangle_{x_{Bj}}$ on light-quark interpolated data
- the NLO impact factors for inclusive DIS with massive quarks

$\Rightarrow$ 3 out of the 12 fits are consistent with charm $\sigma_r$ ep data from HERA
These 3 fits are also consistent with bottom $\sigma_r$ and with full $\sigma_r$ data from HERA

$\Rightarrow$ Total and heavy quark DIS structure functions HERA data both described for the first time in the dipole factorization with perturbative (BK) evolution in $x_{Bj}$!

[Hanninen, Mantysaari, Paatelainen, Penttala - arXiv:2211.03504]

In the future, including EIC data for charm and bottom structure functions in addition to fully inclusive structure functions is expected to provide very strong constraints on NLO fits in dipole factorization

$\Rightarrow$ Test of our understanding of gluon saturation physics

# Diffractive and Exclusive DIS observables at NLO

*Many DIS processes have been calculated recently at NLO in the dipole picture at low x :*

- Diffractive structure functions:

[G.B, Hanninen, Lappi, Mulian, Mantysaari - arXiv:2206.13161] : Partial NLO ($q\bar{q}g$ contr.)
[G.B., Lappi, Mantysaari, Paatelainen, Penttala - *to appear*] : Full NLO result

- Diffractive dijet production:

[Boussarie, Grabovsky, Szymanowski, Wallon - arXiv:1606.00419] : Full NLO result

Note: Diffractive dijets +(softer) jet argued to drive coherent diffractive dijet production in [Iancu, Mueller, Triantafyllopoulos - arXiv:2112.06353], [Iancu, Mueller, Triantafyllopoulos, Wei - arXiv:2207.06268]

- Diffractive dihadron production:

[Fucilla, Grabovsky, Li, Szymanowski, Wallon - arXiv:2211.05774]: Full NLO result

- Exclusive light vector meson production at NLO:

[Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon - arXiv:1612.08026]: Full NLO result, in momentum space
[Mantysaari, Penttala - arXiv:2203.16911]: Full NLO result, in mixed space

- Exclusive heavy vector meson production:

[Mantysaari, Penttala - arXiv:2104.02349]: Full NLO for $\gamma_L$ contribution
[Mantysaari, Penttala - arXiv:2204.14031]: Full NLO for $\gamma_T$ contribution

# Semi-inclusive observables in DIS at NLO

- Single inclusive hadron production (SIDIS):
[Bergabo, Jalilian-Marian - arXiv:2210.03208]: Full NLO for $\gamma_L$ contribution

- Double inclusive hadron production:
[Bergabo, Jalilian-Marian - arXiv:2207.03606]: Full NLO for $\gamma_L$ contribution
[Iancu, Mulian - arXiv:2211.04837]: Real NLO corrections for $\gamma_T$ contribution
[Bergabo, Jalilian-Marian - arXiv:2301.03117]: Full NLO for $\gamma_T$ contribution

- Photon+dijet production:
[Roy, Venugopalan - arXiv:1911.04530]: Full NLO result

- DIS dijet production:
[Caucal, Salazar, Venugopalan - arXiv:2108.06347]: Full NLO result

- Photoproduction of dijets ($Q^2 \to 0$):
[Taels, Altinoluk, G.B., Marquet - arXiv:2204.11650]: Full NLO result ($\gamma_T$ only)

# Back-to-back dijet at NLO : from CGC to TMD

[Dominguez, Marquet, Xiao, Yuan - arXiv:1101.0715]

Dijet production at LO in dipole (and quadrupole) factorization : Two typical transverse scales:

$k_t = p_1 + p_2$ : total momentum $\quad$ $P_T = z_2 p_1 - z_1 p_2$: relative momentum

In the back-to-back dijet limit ($k_t \ll P_T$):
Dipole and quadrupole operators $\rightarrow$ gluon TMDs with future staple gauge link

• What happens at NLO? What about large logs of $P_T/k_t$ (Sudakov logs)?

[Taels, Altinoluk, G.B., Marquet - arXiv:2204.11650]
[Caucal, Salazar, Schenke, Venugopalan - arXiv:2208.13872]

- Study of the back-to-back limit ($P_T \gg k_t$) of NLO dijet production in CGC
- Correct Sudakov double (and single) logs obtained only if rapidity evolution is performed using a JIMWLK evolution with improved kinematics.

[Caucal, Salazar, Schenke, Stebel, Venugopalan - arXiv:2304.03304]

At leading power in $P_T \gg k_t, Q_s$, NLO CGC result reduces to NLO TMD factorized expression:

NLO Coeff. function $\otimes$ Sudakov factor $\otimes$ gluon TMD $+O(k_t^2/P_T^2) + O(Q_s^2/P_T^2)$

- Valid to all orders in $Q_s/k_t$
- Sudakov factor contains both double and single large logs of $P_T^2/k_t^2$
- gluon TMDs obeys nonlinear rapidity evolution equation (derived from JIMWLK) !!!

# Conclusion

- Ongoing effort to push gluon saturation/CGC theory to NLO accuracy:
  - Numerous DIS observables calculated at NLO with gluon saturation
  - Feasibility of dipole fits at NLO demonstrated: require data on both structure function and their charm (and bottom) component.

    $\Rightarrow$ The saturation/CGC theory community should be ready with precise NLO predictions for the start of the EIC

- Not covered here: exploration by various groups of power-suppressed corrections to the CGC formalism in the high-energy limit
  - Might be necessary for precision at EIC c.o.m. energy
  - Can help to understand the transition between gluon saturation regime and other ones (collinear/TMD/GPD factorizations, ...)
  - Necessary for spin physics at low $x$