Semi Inclusive Deep Inelastic Scattering at NEik Order

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Ongoing Work

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EICUG 2023

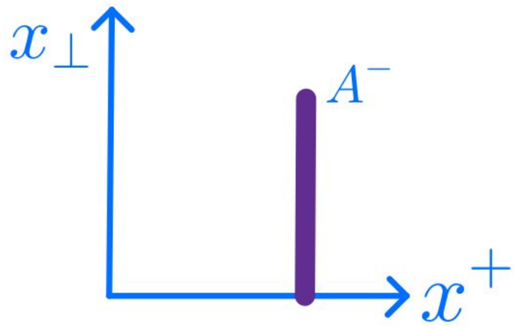
Introduction

- **Generally in saturation physics in color glass condensate (CGC) framework two approximations are considered:**
  - Semi-classical approximation
    - Dense target represented by Strong semi-classical gluon field $A^{\mu}(x)$
  - Eikonal approximation
    - Limit of infinite boost of $A^{\mu}(x)$
    - There is hierarchy between components of $A^{\mu}(x)$ with respect to Lorentz boost factor γ^{t} of the target:

$$A^{-} = \mathcal{O}(\gamma_t) >> A^{j} = \mathcal{O}(1) >> A^{+} = \mathcal{O}(1/\gamma_t)$$

    - Only leading order energy term (leading term in γ^{t}) considered.

Eikonal order and Beyond it



Zero Width
1.Highly boosted background field (target) is localised in the longitudinal direction $x^+ = 0$ (zero width).

Leading Component
2.Only leading component of target (-component) is considered and subleading components are neglected (suppressed by Lorentz boost factor).

x^- independence
3.Dynamics of the target are neglected due to time dilation (x^- dependence of target neglected).

Background field of target is : $A^\mu(x^-, x^+, \boldsymbol{x}) \approx \delta^{\mu -}\delta(x^+)A^-(\boldsymbol{x})$

Beyond Eikonal:

- In very high energy accelerators (γ^t~1000 order), next-to-eikonal (NEik) order terms (power suppressed terms in high energies) are negligible while calculating observables.

- But to analyze the data from RHIC and future electron ion collider (**EIC**) (γ^t~ 10-100 order), NEik order terms might be sizable!

Eikonal order and Beyond it

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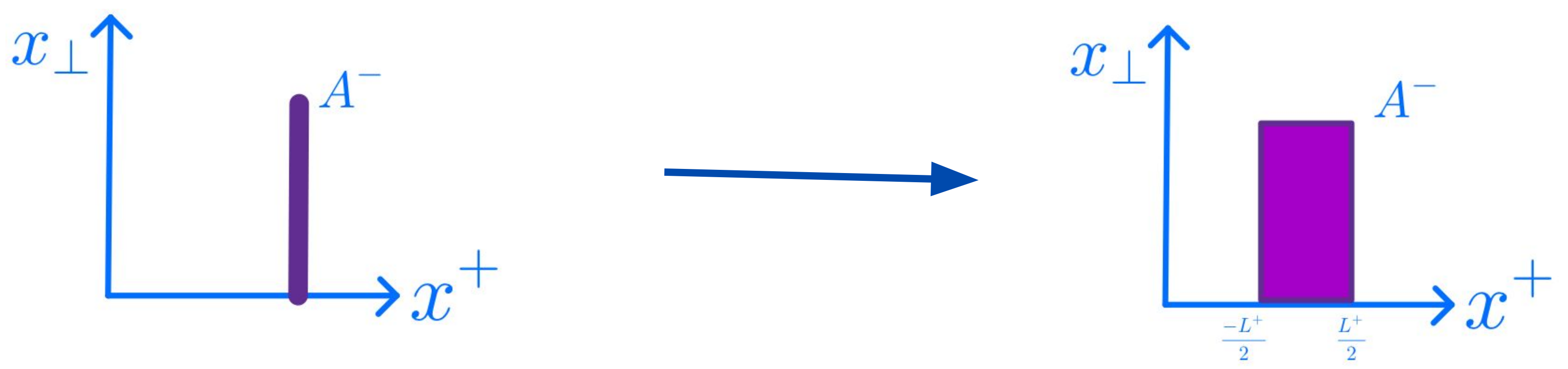
To go Beyond Eikonal:

Finite Width
1.Instead of infinitely thin shockwave as a target, we consider finite width of a target.

Transverse Component
2.Instead of neglecting sub-leading components, we include transverse component of background field.

x^- dependence
3.We take into account corrections coming due to the x^- dependence of a target (consider background field is x^- dependent).



Quark Background Field

- At NEik order, we also have to take into account interaction with quark background field $\Psi(z)$.
- Due to large boost of the target along x^-: its localized in longitudinal x^+ direction around small support.
- If we consider projections on quark background field then,

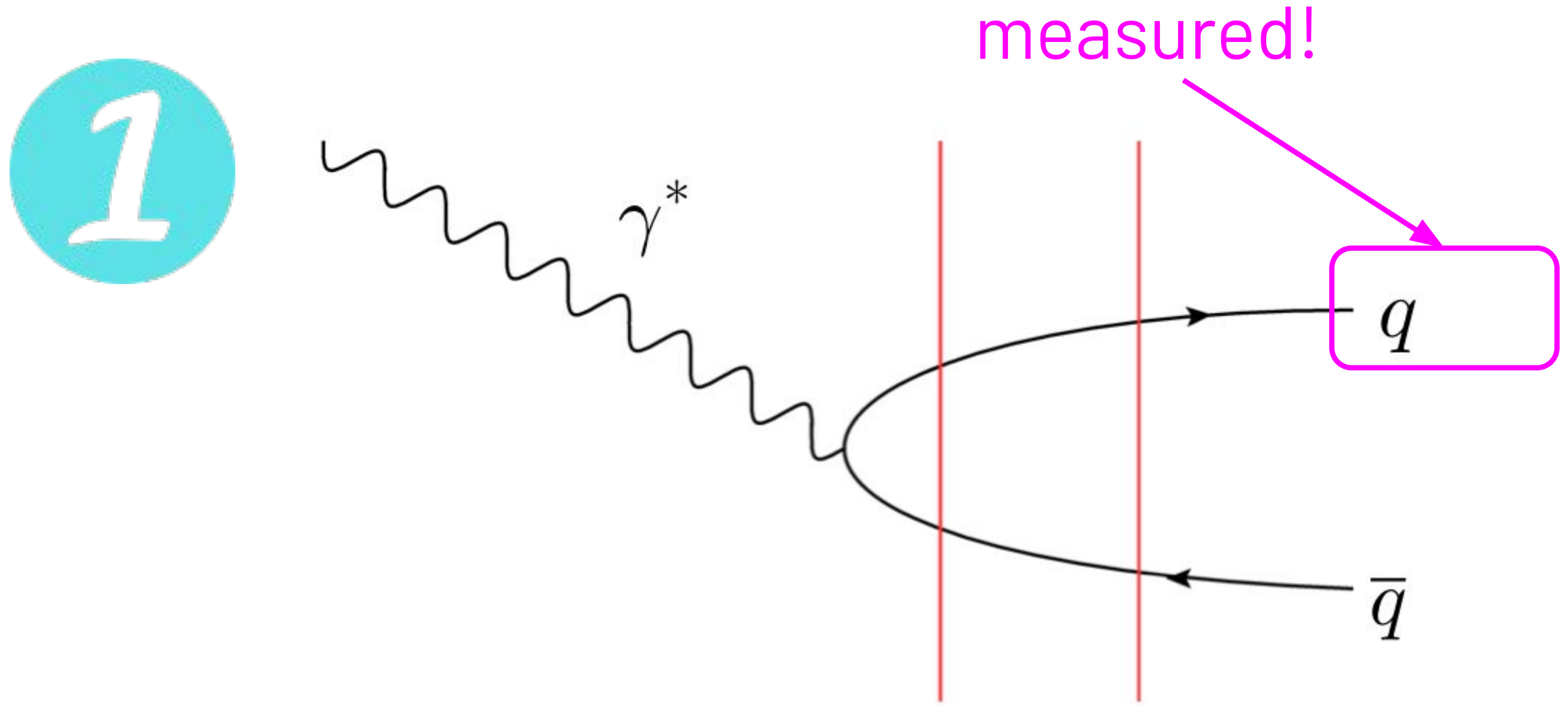
$$\Psi(z) = \frac{\gamma^+\gamma^-}{2}\Psi(z) + \frac{\gamma^-\gamma^+}{2}\Psi(z) = \Psi^-(z) + \Psi^+(z)$$

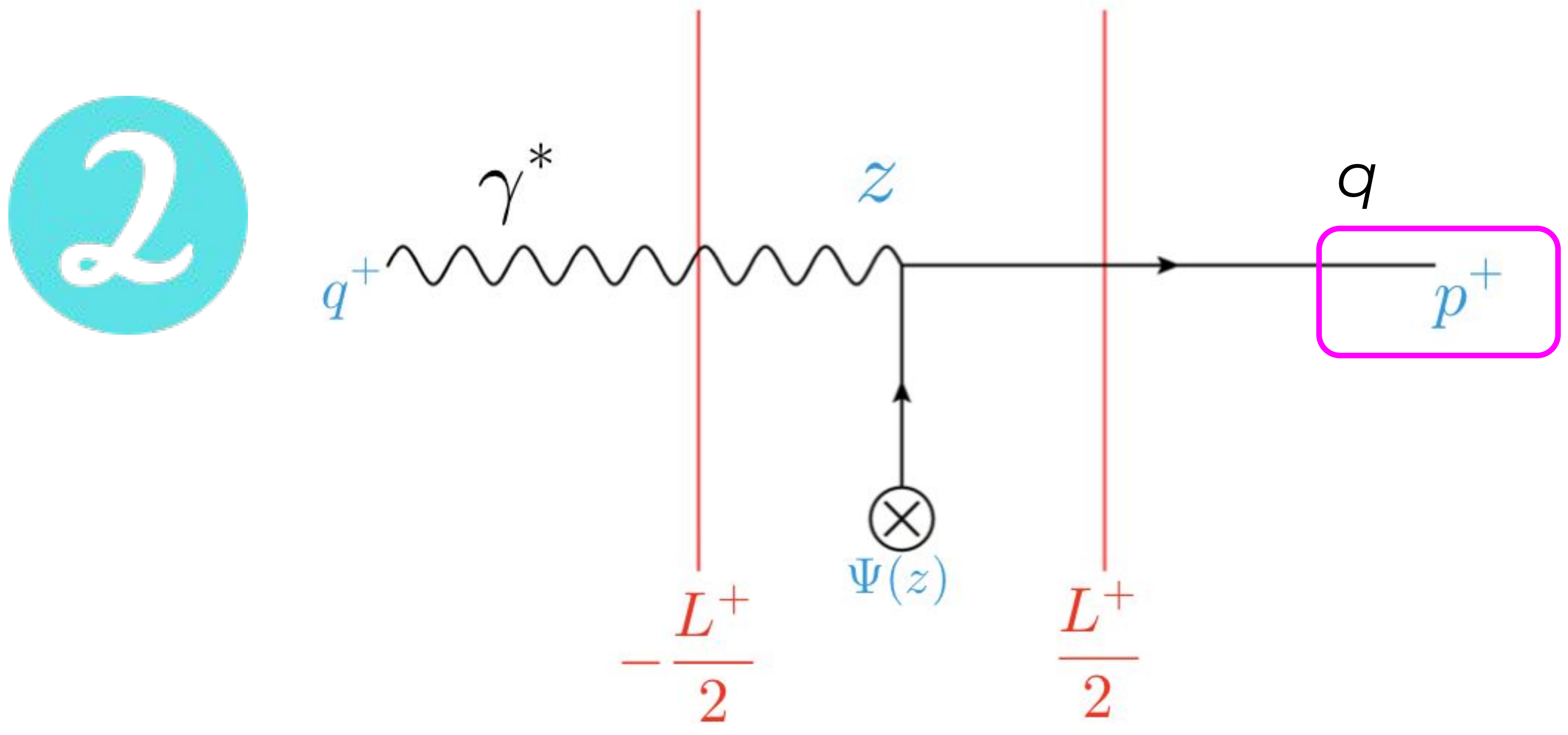
$\mathcal{O}(\sqrt{\gamma_t})$ $\mathcal{O}(1/\sqrt{\gamma_t})$

- For NEik corrections, only - component considered and + component is neglected (contribute at NNEik only).

Semi Inclusive Deep Inelastic Scattering (SIDIS):

- In CGC, for this process: two kinds of contributions!
- Each of them are expected to be dominant in different kinematic regions.





- Contribution (1) is studied by Marquet, Xiao, Yuan [arXiv:0906.1454]. There is contribution at eikonal order.
- In this talk contribution coming due to (2) is discussed. No contribution at eikonal order.

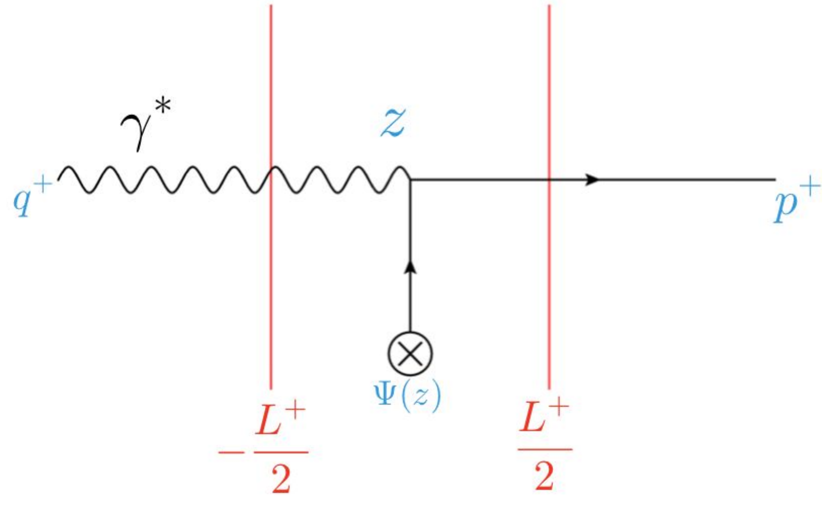
SIDIS: S-matrix computation

- S-matrix at NEik order calculated : only $\Psi^-(z)$ of component considered

$$S_{\gamma*\to q} = \lim_{x^+\to\infty} \int d^2x_\perp \int dx^- \, e^{i\check{p}\cdot x} \int d^4z \, \epsilon^\lambda_\mu(q) \, e^{-iq\cdot z} \, \overline{u}(p,h) \, \gamma^+ \, S_F(x,z)|^{IA}_{Eik}(-iee_f\gamma^\mu)\Psi^-(z)$$

- Two polarizations of photons are considered: No contribution at Eikonal
  - Longitudinal Polarization: no contribution at NEik order
  - Transverse Polarization: Contribution at NEik order

Finally, S-matrix for SIDIS process:



$\mathcal{O}(1/\gamma_t)$

$$S_{\gamma^*_T\to q} = 2\pi\delta(q^+ - p^+) \int dz^+ \int d^2z_\perp \, e^{i(q_\perp - p_\perp)z_\perp} \, \overline{u}(p,h)$$
$$\times \, \epsilon^j_\lambda(iee_f)U_F(\infty, z^+, z_\perp) \, \left(\frac{\gamma^j\gamma^+\gamma^-}{2}\right)\Psi(z)$$

$\mathcal{O}(\sqrt{\gamma_t})$

Similar calculations in case of q-g dijets are done in Altinoluk et al. (arXiv:2303.12691)

SIDIS: Cross-Section

Squaring amplitudes, we get cross-section for SIDIS process, in terms of Wilson lines:

$$\frac{d^2\sigma^{\gamma_T^*\to q}}{d^2p_\perp} = \frac{e^2e_f^2}{(2\pi)^2}\frac{1}{2}\frac{1}{2q^+}\int d^2z'_\perp \int d^2z_\perp \, e^{i(q_\perp - p_\perp)(z_\perp - z'_\perp)} \int dz'^+ \int dz^+ \times \left\langle \overline{\Psi}(z')\gamma^- \, \mathcal{U}_F^\dagger(\infty, z'^+, z'_\perp) \;\; \mathcal{U}_F(\infty, z^+, z_\perp) \; \Psi(z) \right\rangle$$

Over all suppression of $\mathcal{O}(1/\gamma_t)$: NEik order

SIDIS: Relation at small-x between CGC and TMD calculations

- Any color operator $\mathcal{O}$, the CGC-like target average $\langle \mathcal{O} \rangle$ is proportional to the quantum expectation value in the momentum state of target.

$$\langle \mathcal{O} \rangle = \lim_{P'_{tar} \to P_{tar}} \frac{\langle P'_{tar} | \mathcal{O} | P_{tar} \rangle}{\langle P'_{tar} | P_{tar} \rangle}$$

- Using this relation, we can relate obtained cross-section with unpolarized transverse momentum dependent (TMD) quark distribution.
- By comparing with quark TMD function, we get cross section:

$$\frac{d^2\sigma^{\gamma_T^* \to q}}{d^2 p_\perp} = \frac{\pi e^2 e_f^2}{W^2} f_1^q(x = 0, p_\perp - q_\perp)$$

Suppression by centre of mass energy $1/W^2$ characterizes NEik contribution in terms of exchange t channel quark!

Summary

- SIDIS cross-section is calculated at NEik order by including quark background field.
  - expressed in terms of TMD quark distribution function.

Thank you!