Neutron Stars with Quark Matter

Bernd-Jochen Schaefer

JLU
NEUE WEGE. SEIT 1607.

HFHF
Helmholtz Forschungsakademie Hessen für FAIR

JUSTUS-LIEBIG-
UNIVERSITÄT
GIESSEN

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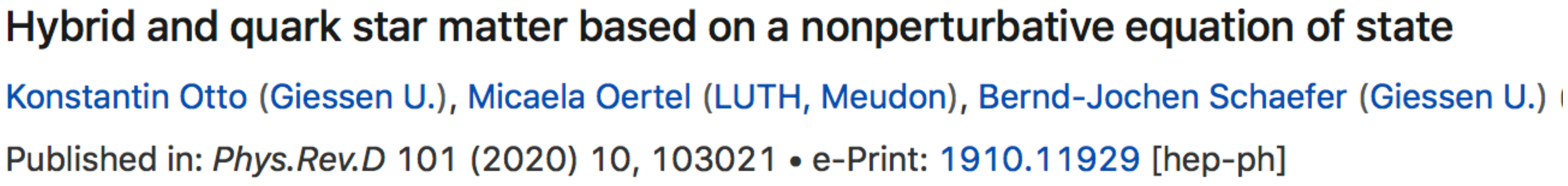
ACHT 2023: Non-Perturbative Aspects of Nuclear, Particle and Astroparticle Physics

27–29 Sept 2023

DER RETZHOF
Dorfstraße 17
A-8435 Wagna

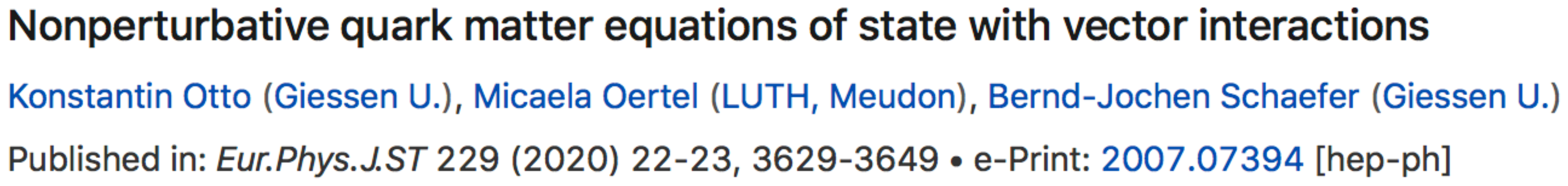
Agenda

- 

  Hybrid and quark star matter based on a nonperturbative equation of state

  Konstantin Otto (Giessen U.), Micaela Oertel (LUTH, Meudon), Bernd-Jochen Schaefer (Giessen U.)

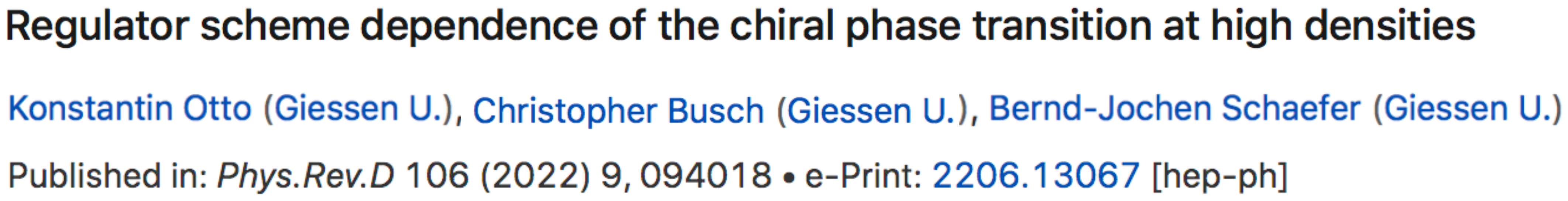
  Published in: *Phys.Rev.D* 101 (2020) 10, 103021 • e-Print: 1910.11929 [hep-ph]

- 

  Nonperturbative quark matter equations of state with vector interactions

  Konstantin Otto (Giessen U.), Micaela Oertel (LUTH, Meudon), Bernd-Jochen Schaefer (Giessen U.)

  Published in: *Eur.Phys.J.ST* 229 (2020) 22-23, 3629-3649 • e-Print: 2007.07394 [hep-ph]

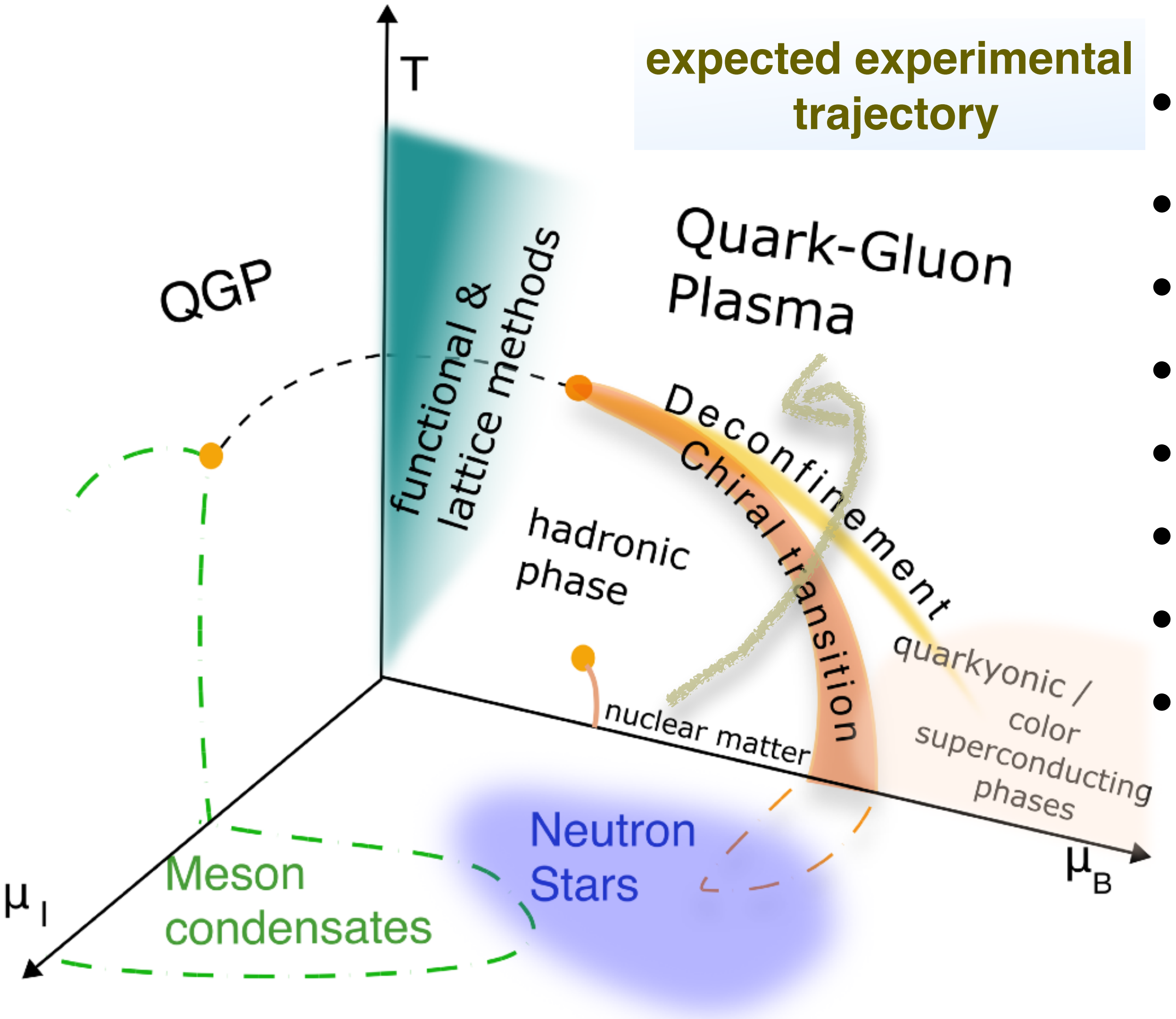
- 

  Regulator scheme dependence of the chiral phase transition at high densities

  Konstantin Otto (Giessen U.), Christopher Busch (Giessen U.), Bernd-Jochen Schaefer (Giessen U.)

  Published in: *Phys.Rev.D* 106 (2022) 9, 094018 • e-Print: 2206.13067 [hep-ph]

conjectured QCD phase structure

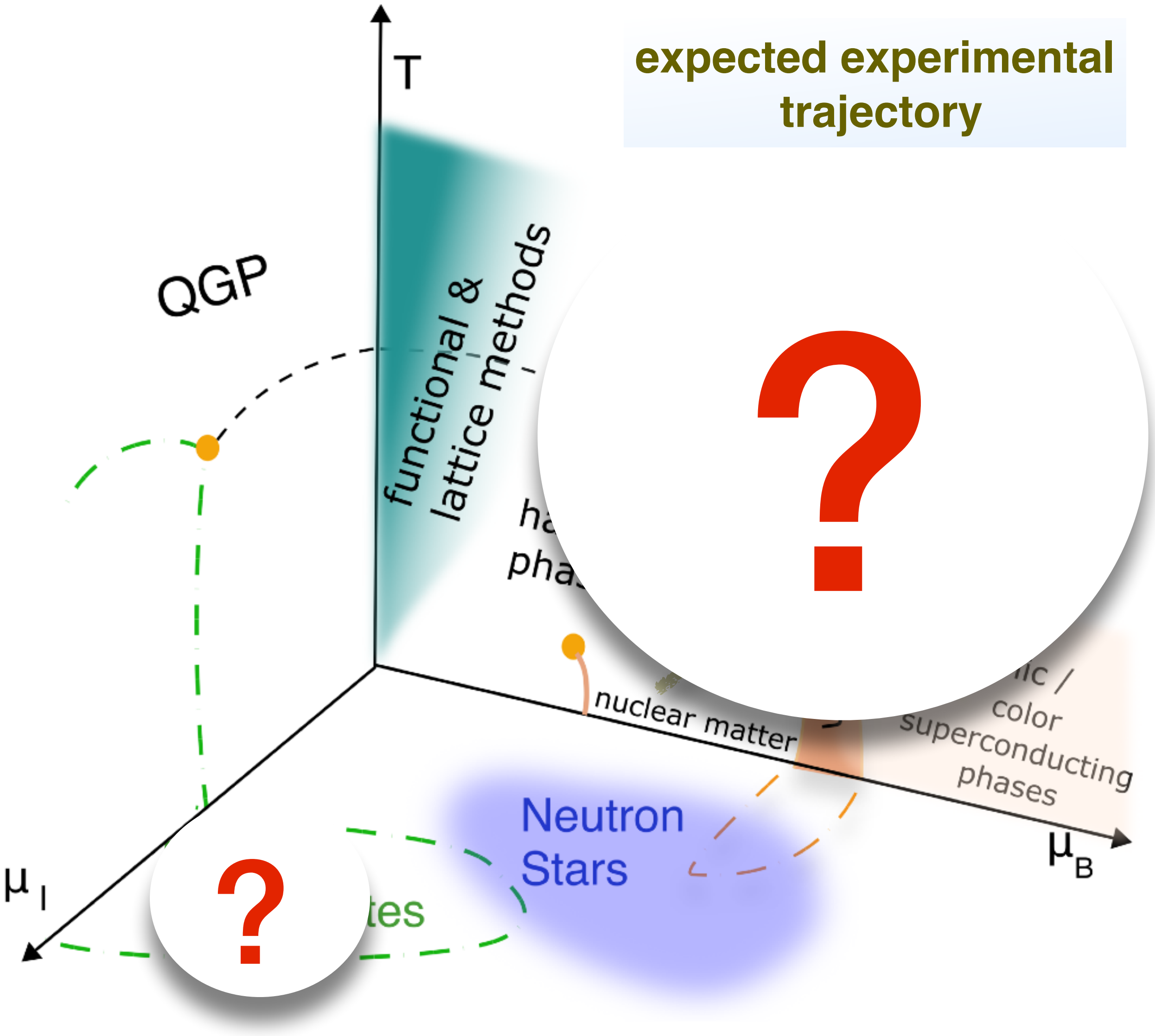


Open issues

- **Critical endpoint (CEP)?** chiral ⇔ deconfinement?
- **Chiral-Spin symmetry / Quarkyonic phase/s?**
- **inhomogeneous phase/s?**
- **axial anomaly restoration?**
- **finite volume effects?**
- **role of fluctuations?**
- **experimental signatures?**
- **….**

usual assumptions: equilibrium, homogeneous phases, infinite volume, ...

conjectured QCD phase structure

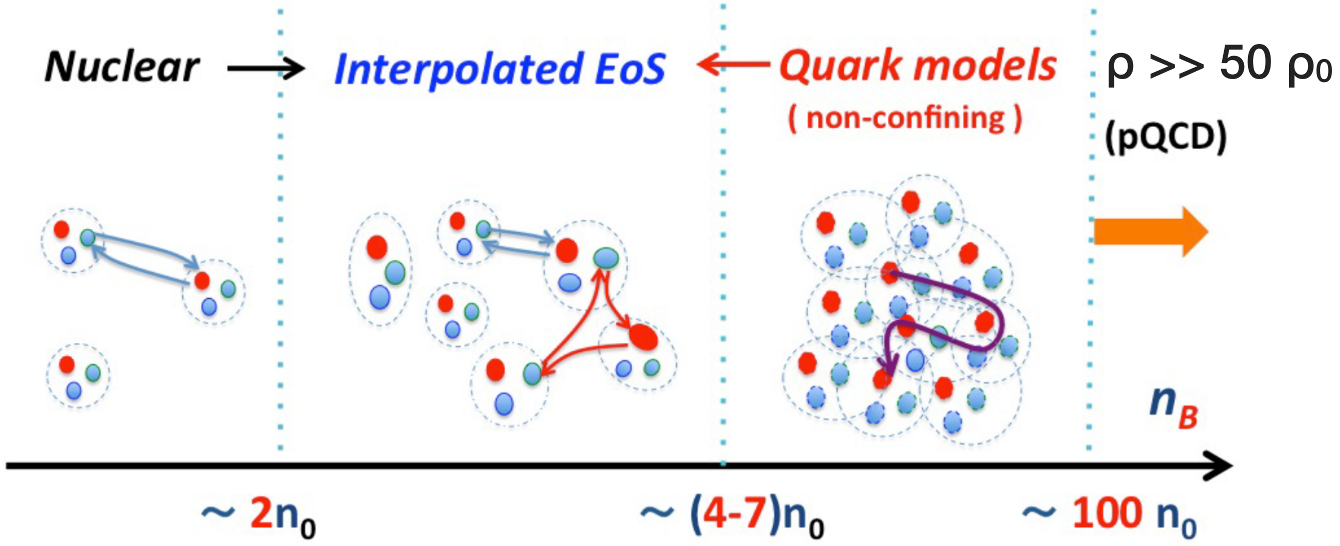


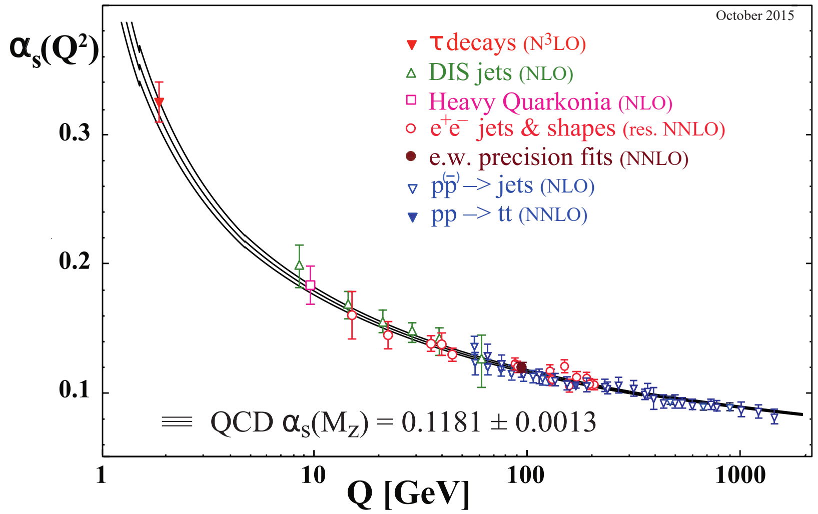
- **basically only corners known from first principle QCD**
- **alternative to HIC** to probe cold dense QCD matter
  ➜ **massive neutron star** (e.g. PSR J0348+0432)
- theory:
  only effective low-energy realisation of QCD: e.g. (P)QM models

ultimate goal: microscopic description of EoS guided by QCD first principle

EoS for dense matter

[Baym, Kojo et al 2018] three window model of dense matter





Nuclear phase:
1-2 meson/quark
exchanges

EoS from
nuclear physics
$\rho < 2\rho_0$ χEFT

interpolated EoS
many meson/quark
exchanges

system gradually changes
from hadronic to quark matter
- diquarks, colored quarks virtually …

- role of strangeness / hyperons

$2\rho_0 < \rho < 7\rho_0$ Neutron stars

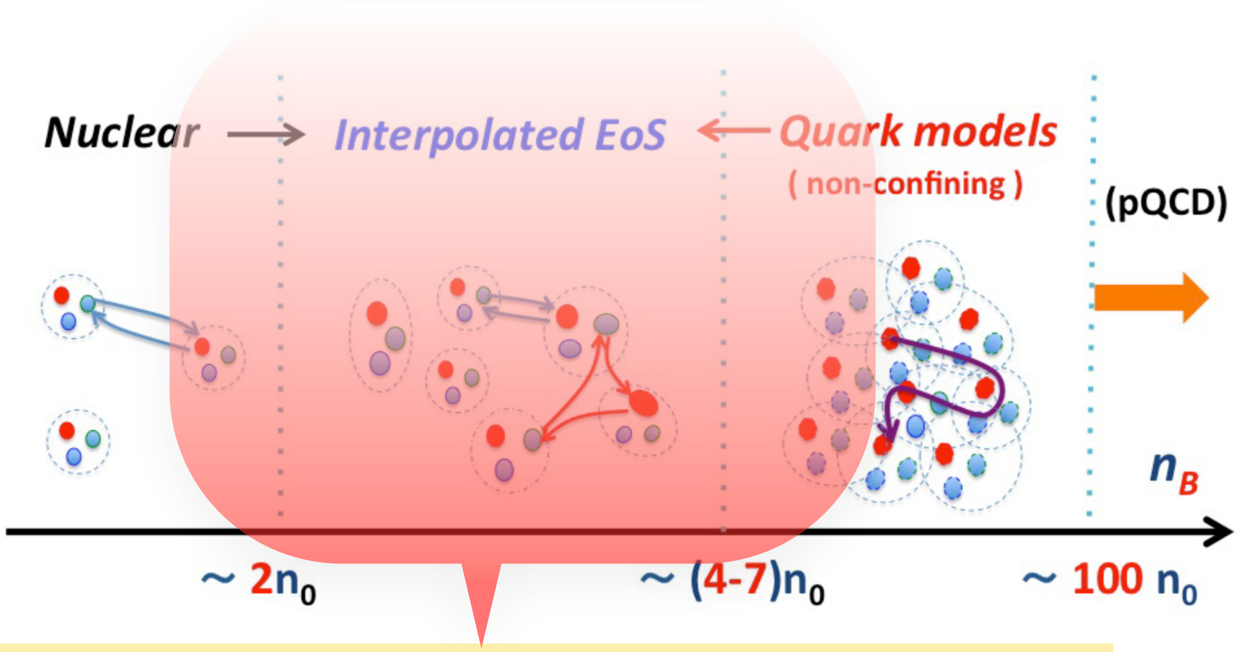
Quark phase:
quarks no longer
specific to baryons

mostly mean-field investigations
like NJL-type or phenomenological
models

➜ upgrade with FRG methods

[Hebeler, Lattimer, Pethick,Schwenk et al. 2010]
[Schaffner-Bielich et al. 2008]
[Blaschke, Fischer, Oertel et al. 2018]

Experimental facts

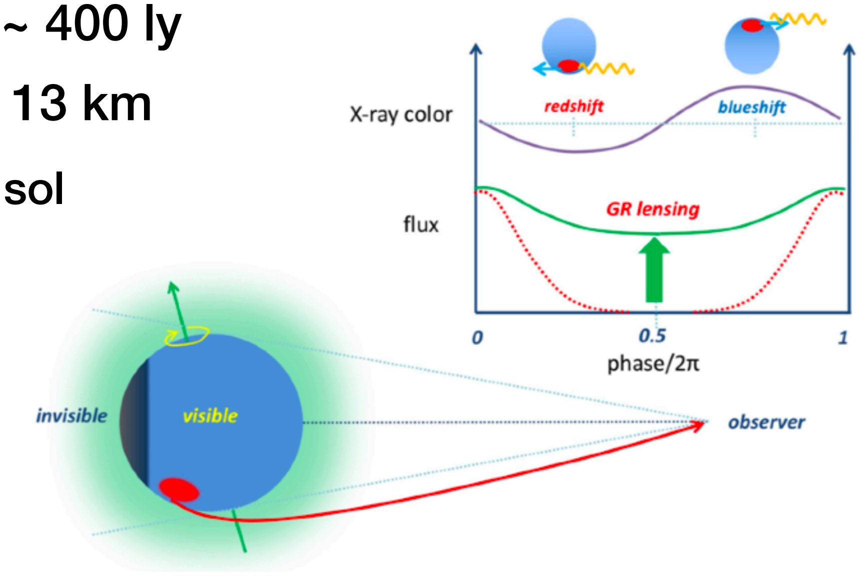


- $M_{max} = 2.05\ M_\odot$ & R ~ 12.8 km ; $\rho_{central} \sim 5\rho_0$
- M = 1.4 $M_\odot$ & R = 12,8 km ; ΔR = 0
- M = 2.4 $M_\odot$ & R = 12,8 km ; ~ as 1.4 $M_\odot$ stars

- PSR J0740+6620 & J0030+0451
  both M (> $M_\odot$) & R measured
  GW170817 constraints tidal deformability -> R

- detection phase transition possible:
  increase post-merger dominant oscillation frequency
  only a few events expected (even w/ 3rd generation detectors)

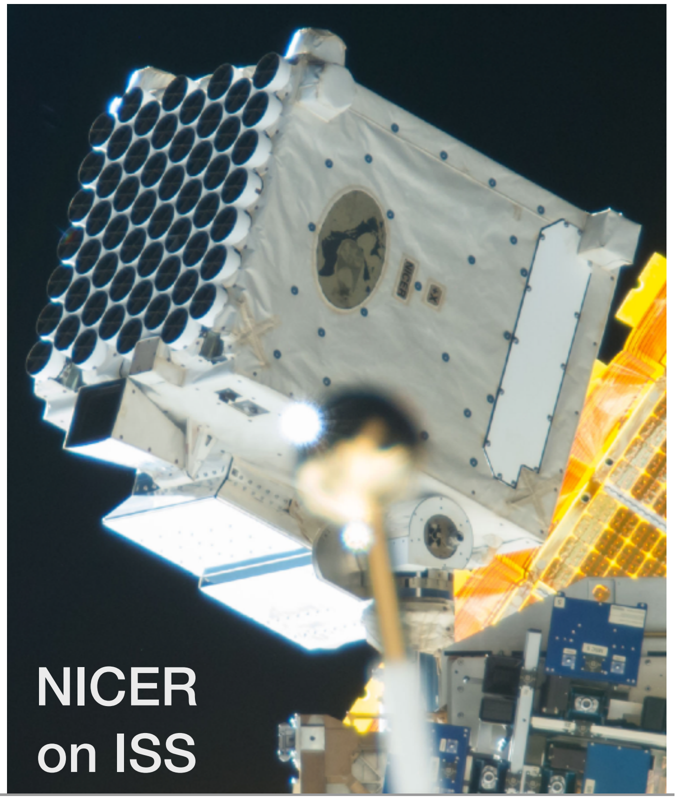
nearest ~ 400 ly
R ~ 10 - 13 km
M > 2 Msol



future: ~ 2035

- new 3rd generation detectors:
- (American) Cosmic Explorer
- (European) Einstein Telescope

running „accelerator“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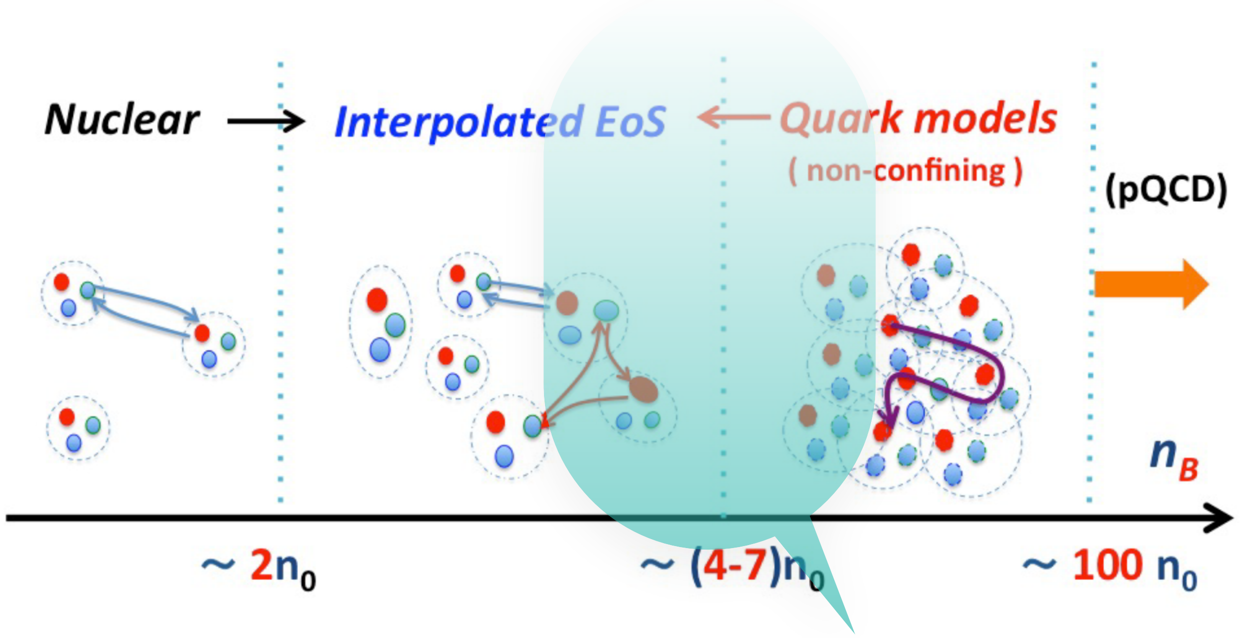
on ISS

Transition from hadronic to quark matter



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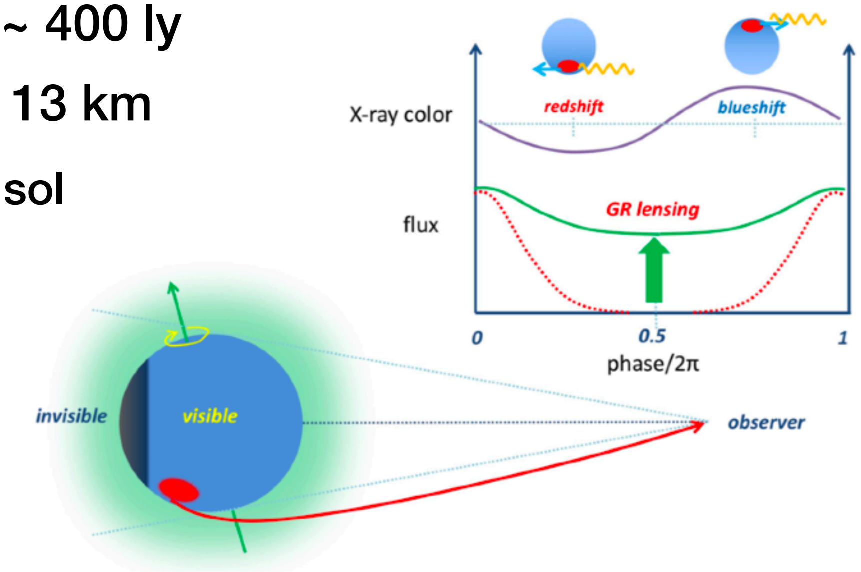
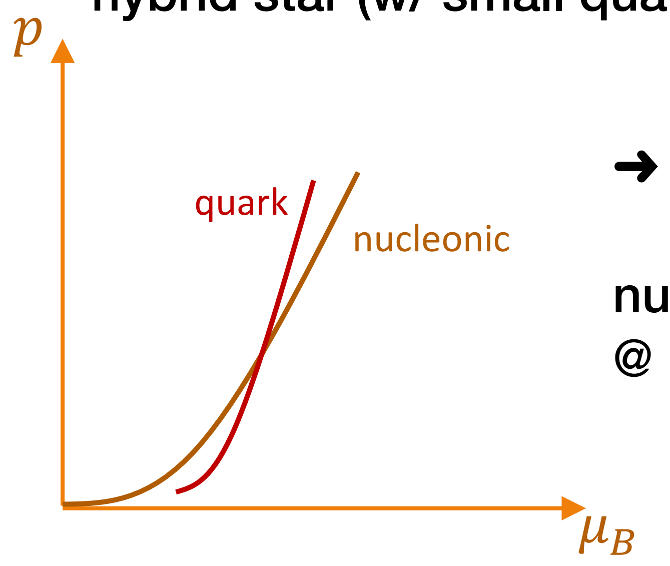


several possibilities (if transition): Maxwell construction or continuous interpolation

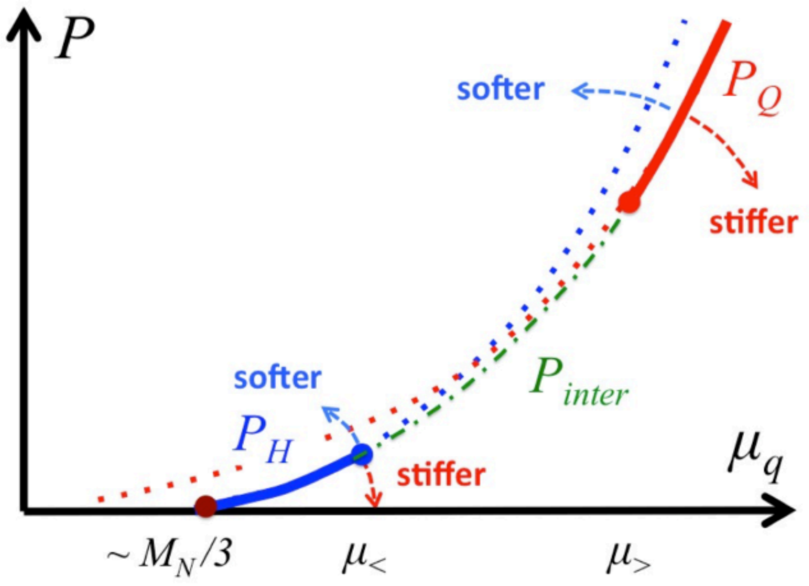
first-order transition

quark-hadron continuity

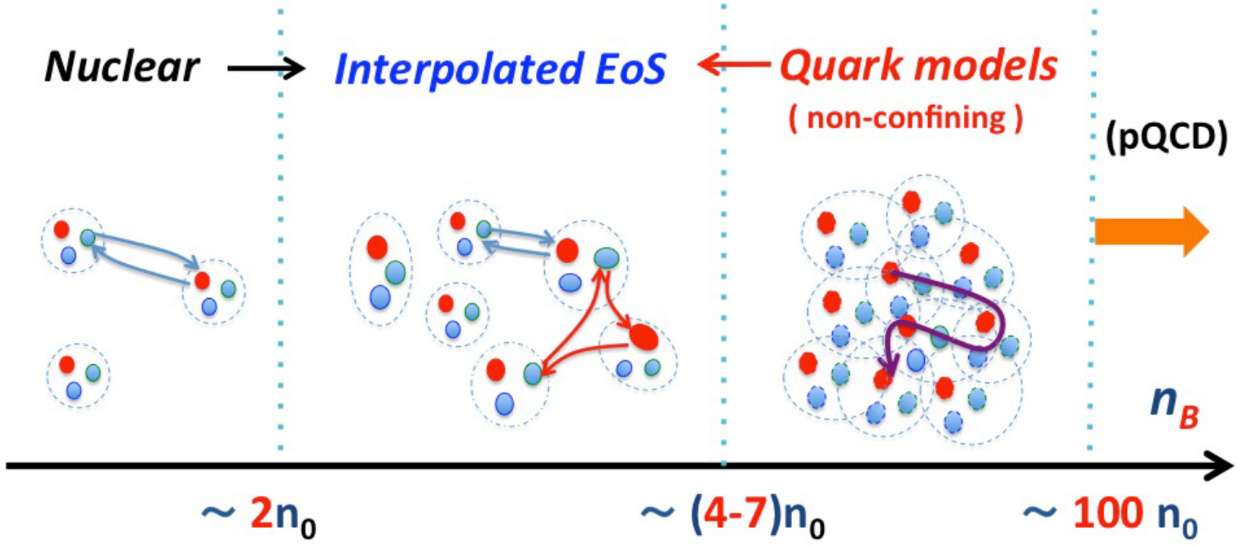
hybrid star (w/ small quark core)

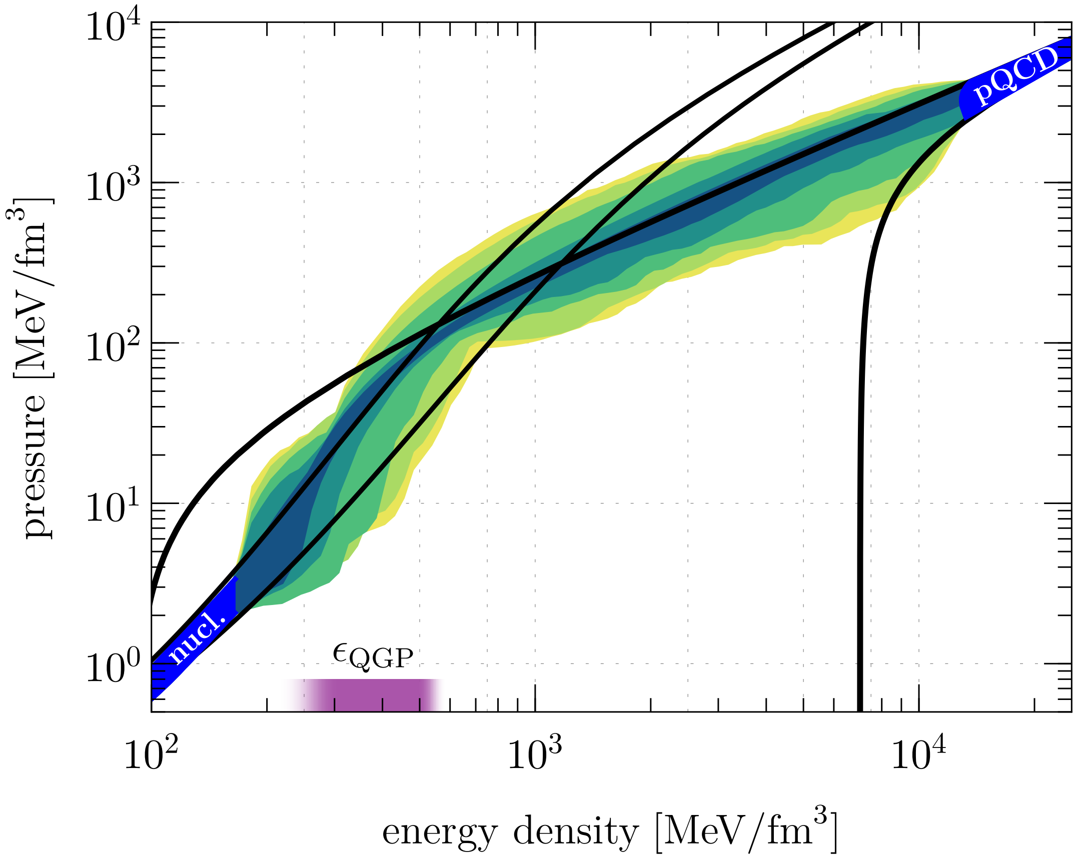


➜ quark EoS softer than nucleonic EoS @ higher densities



Conflicting constraints on EoS

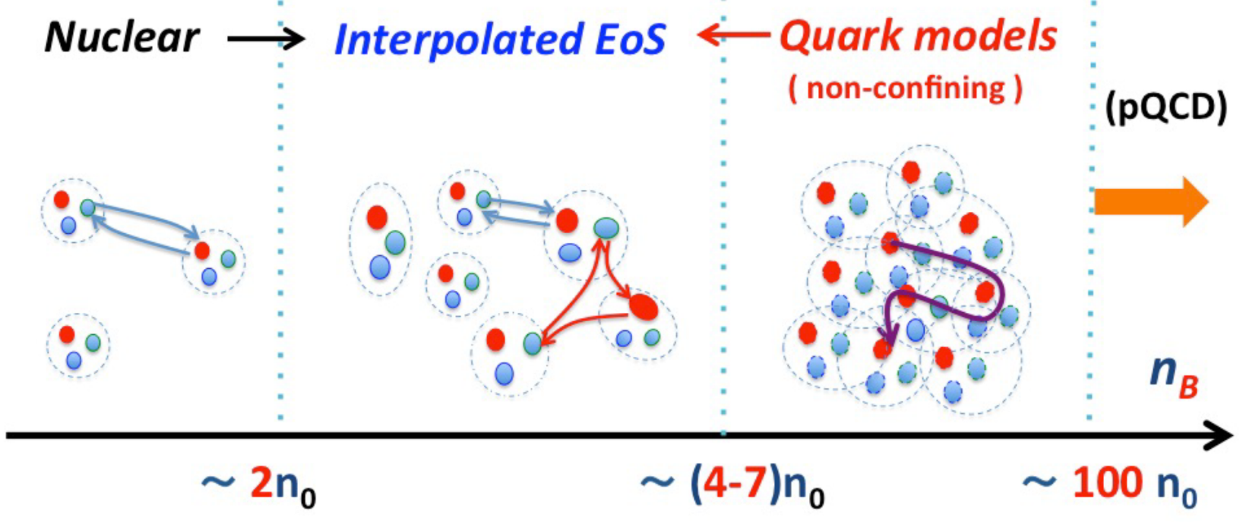


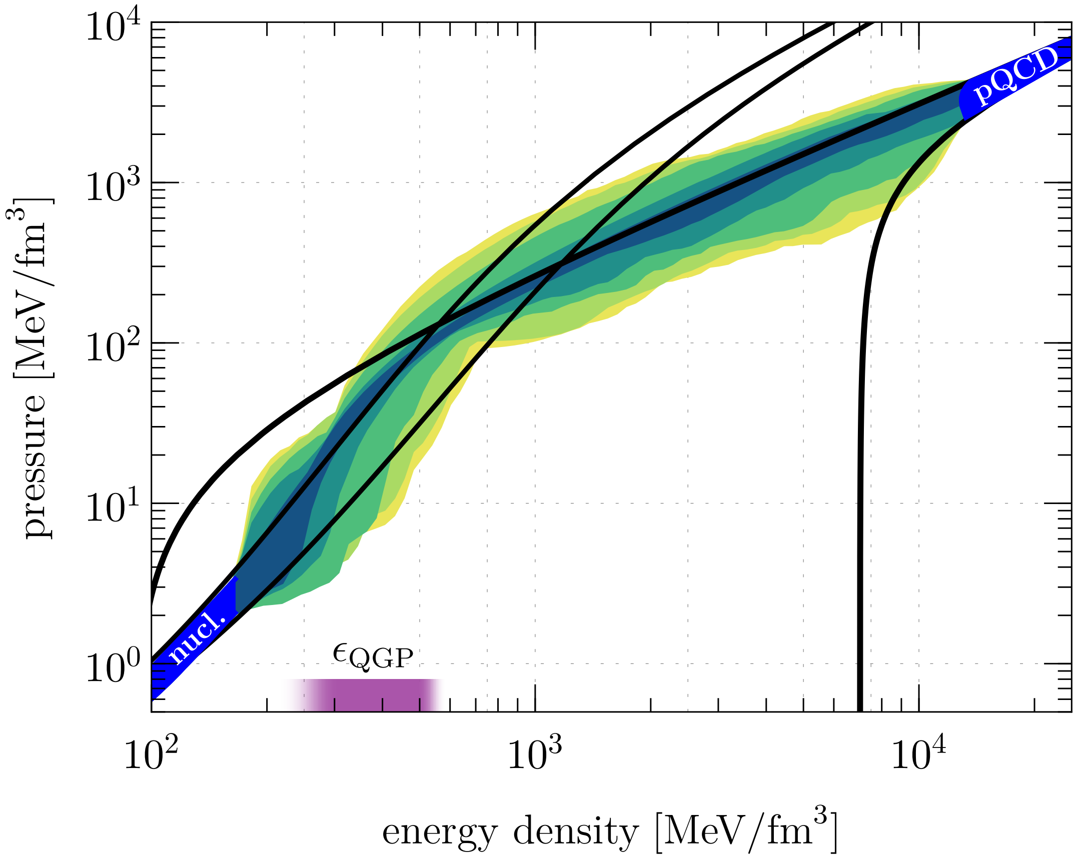


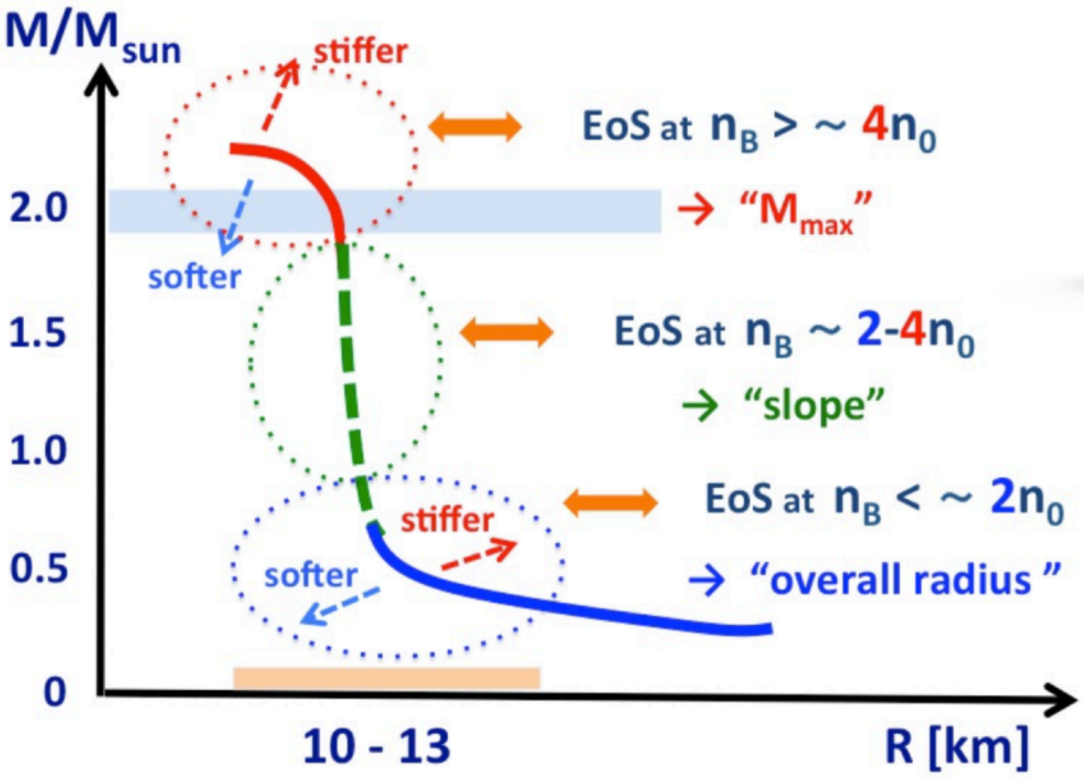
EoS ↔ TOV equation ↔ M-R relation (observables)

Conflicting constraints on EoS

[Baym 2018]







EoS ↔ TOV equation ↔ M-R relation (observables)

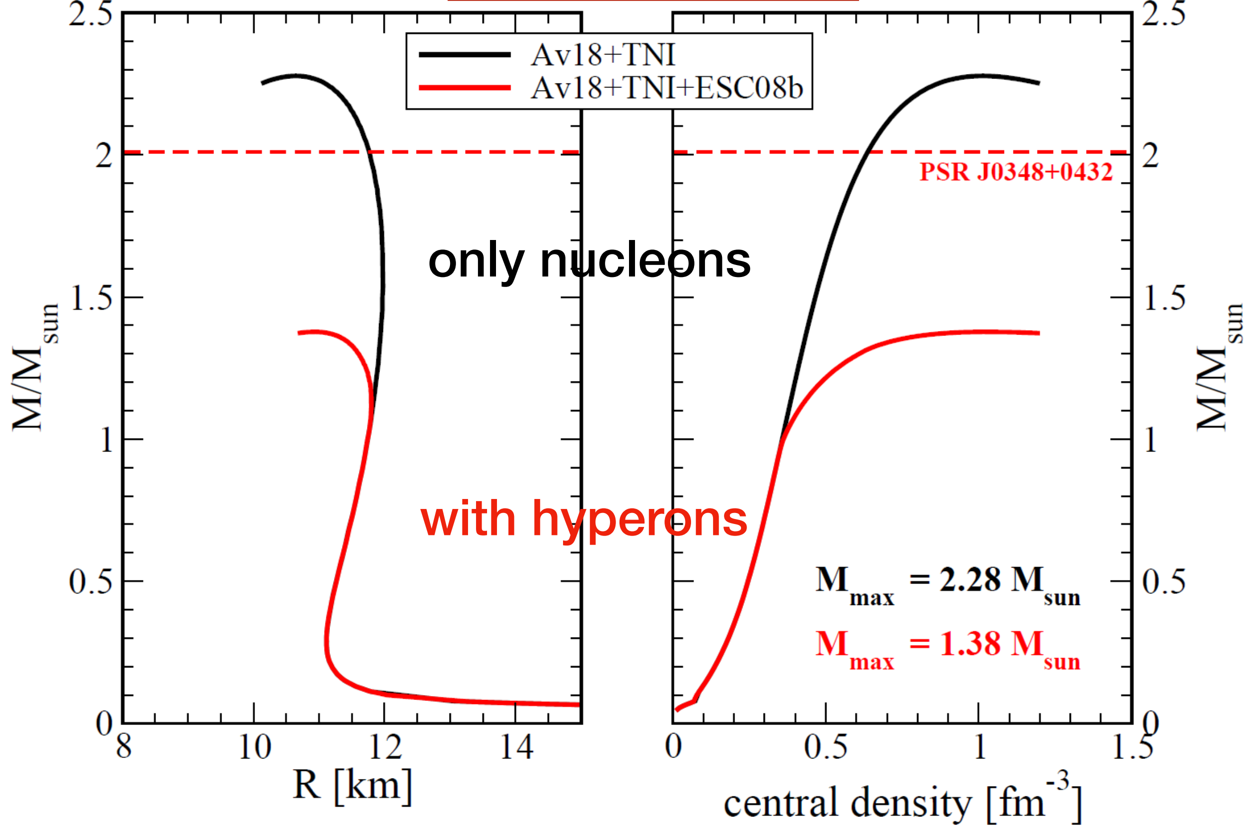
three constraints on the EoS:

1. stiff enough (@high density) ➜ $2M_\odot$
2. soft enough (@low density) ➜ Radius
3. speed of sound < 1

unsolved Puzzles / open Issues

[Bombaci 2016]

hyperons puzzle



only nucleons

with hyperons

Further constraints:

causality

charge neutrality: $n_p = n_e + n_\mu$

β-equilibrium: $\mu_n = \mu_p + \mu_e$

simplification:

➜ electrons and muons as free Fermi gas in EoS

General problems (physical theory input required):

➜ hyperon puzzle

onset of strangeness in hadronic phase or quark phase
➜ soften EoS

[Djapo, BJS, Wambach 2010]

➜ masquerade problem

many EoS look similar ➜ similar M-R relation
increasing #dof soften EoS,
repulsive interactions stiffen EoS

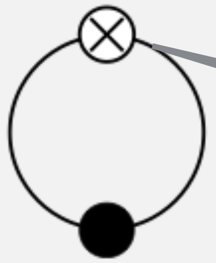
[Alvarez-Castillo, Blaschke 2014]

Functional Renormalization Group

- Wetterich Equation (average effective action)

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\,\mathrm{Tr}\;\partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k}\right)$$

$$k\partial_k \Gamma_k[\phi] \sim \frac{1}{2}$$



$$t = \ln(k/\Lambda)$$

$$\Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

R_k regulators

[Wetterich 1993]

- Ansatz effective action Quark-Meson truncation in LPA (LO derivative expansion)

$$\Gamma_k = \int d^4x \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + V_k(\phi^2)$$

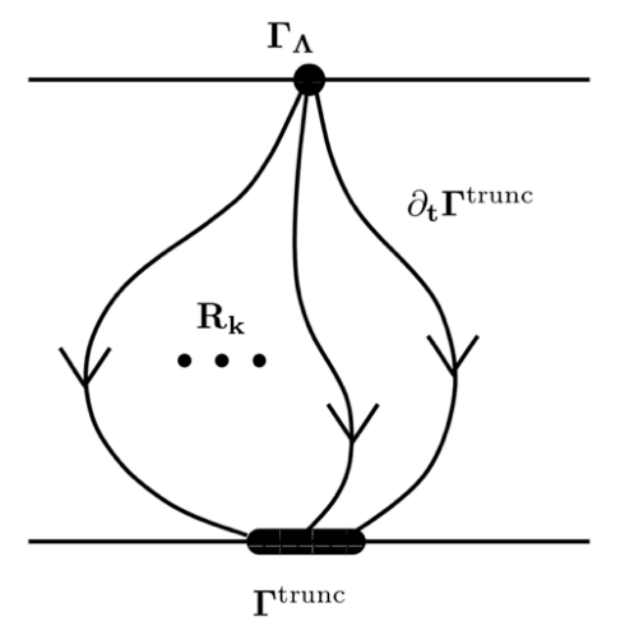
arbitrary potential

$$V_{k=\Lambda}(\phi^2) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

shape function conditions:

$$R_k(p^2) = p^2 r(p^2/k^2)$$

- $\lim_{p^2/k^2 \to \infty} R_k(p^2) = 0$
- $\lim_{p^2/k^2 \to 0} R_k(p^2) > 0 \; (= k^2)$
- $\lim_{k \to \infty} R_k(p^2) \to \infty$



Impact of fluctuations on EoS

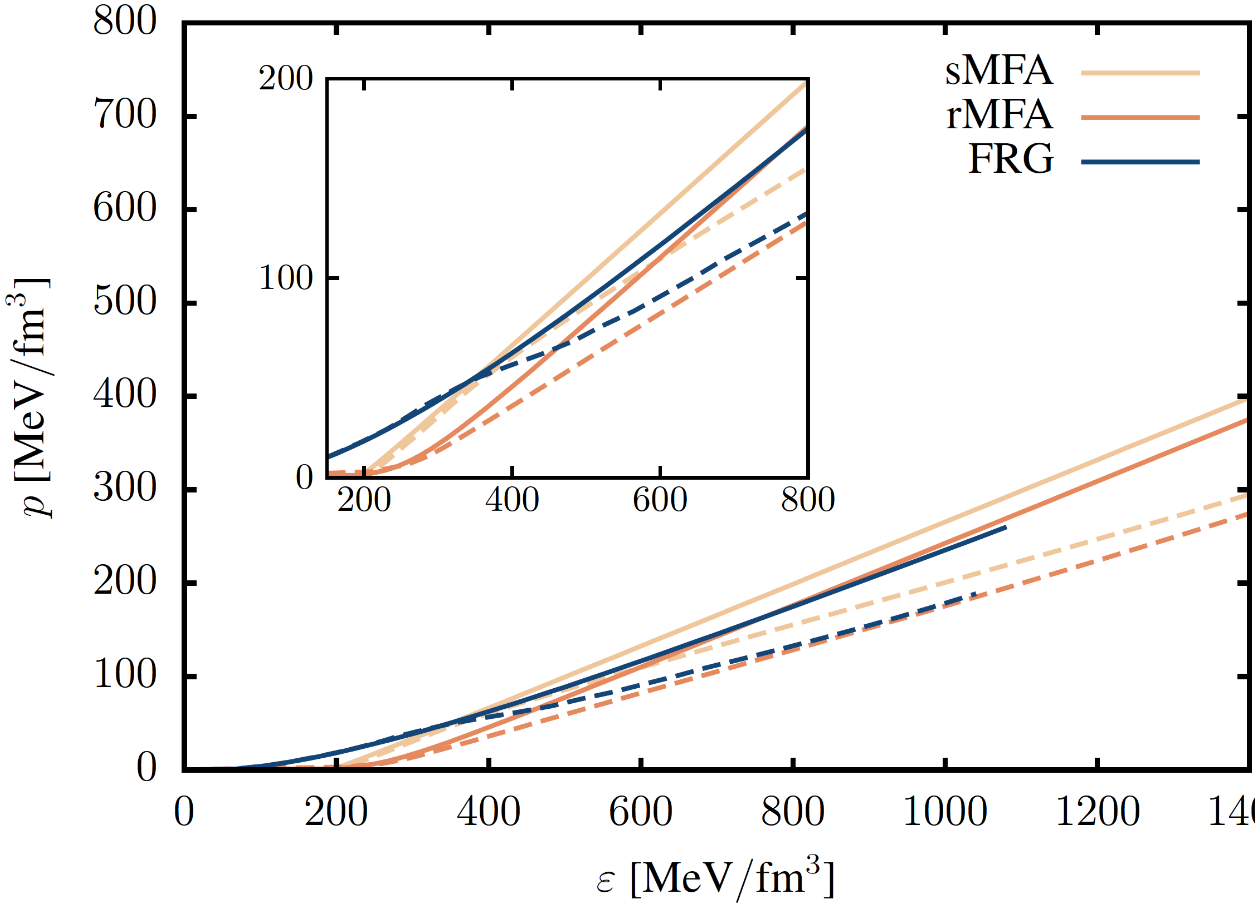
[Otto, Oertel, BJS 2020]

Impose β-equilibrium and charge neutrality conditions

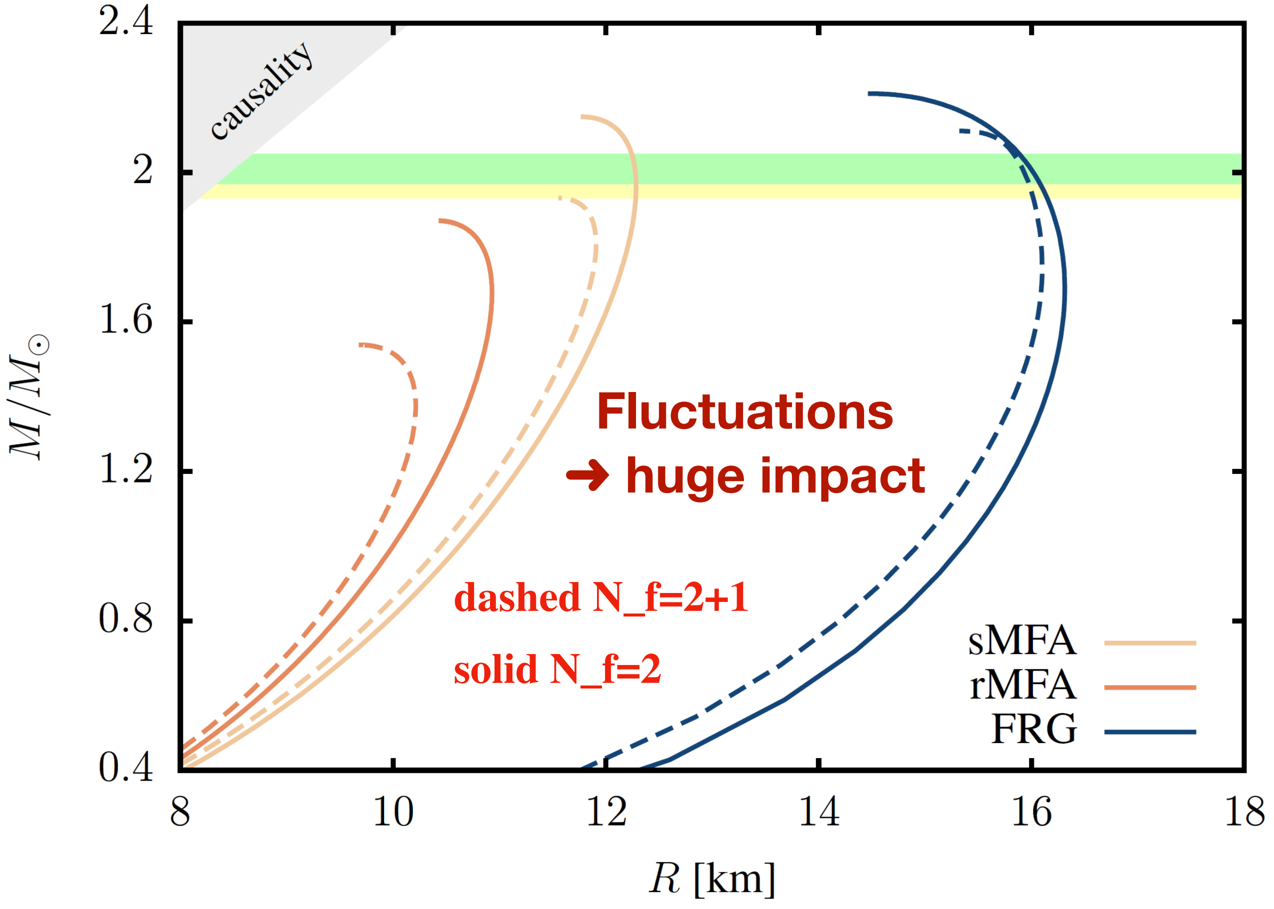
$$\mu_u = \mu_q - \frac{2}{3}\mu_e$$
$$\mu_d = \mu_q + \frac{1}{3}\mu_e$$
$$\mu_s = \mu_q + \frac{1}{3}\mu_e$$

Tolman-Oppenheimer-Volkoff equations

EoS



Mass-Radius relation



Fluctuations
➜ huge impact

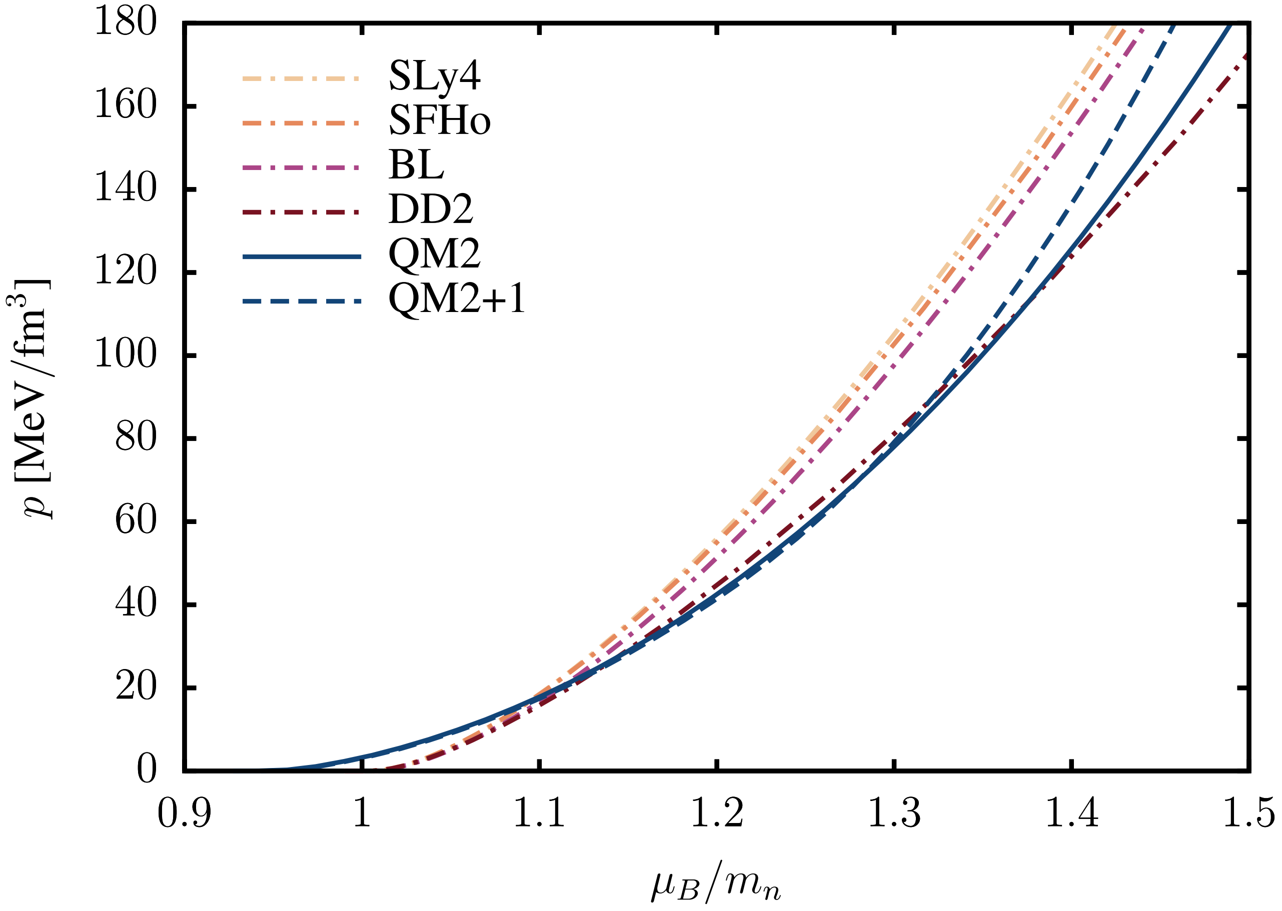
dashed N_f=2+1

solid N_f=2

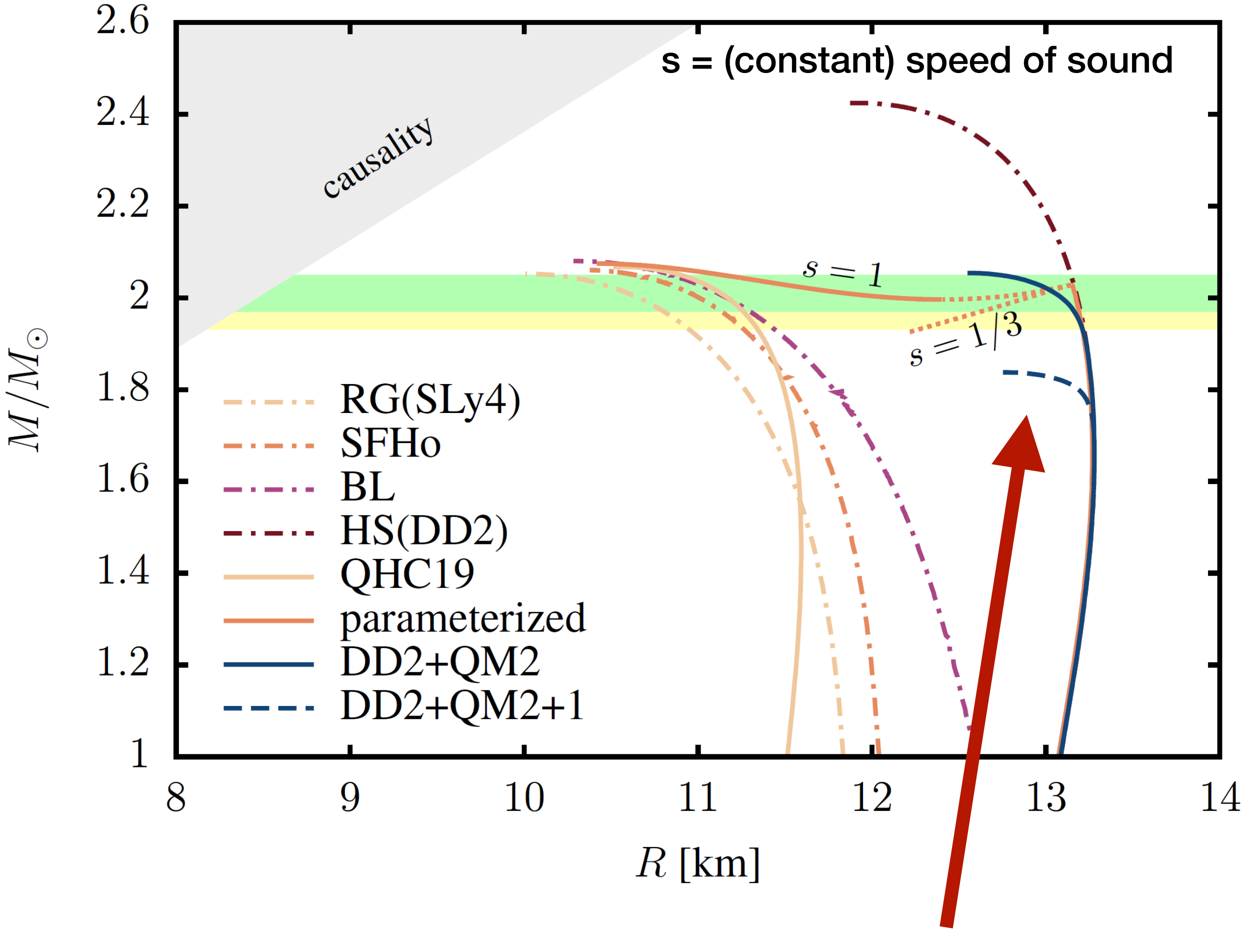
Hybrid star construction possible? - yes

[Otto, Oertel, BJS 2020]

combine nuclear EoS (DD2) with FRG QM truncation



➜ continuous nuclear-hybrid branch



2 $M_\odot$ limit violated for N_f= 2+1

can a repulsive vector interaction remedy this behavior?

Vector mesons & the FRG EoS

[Otto, Oertel, BJS 2020]

[Rennecke 2015]
[Pereira, Stiele, Costa 2020]

- Yukawa type interaction of temporal component and mean-field potential

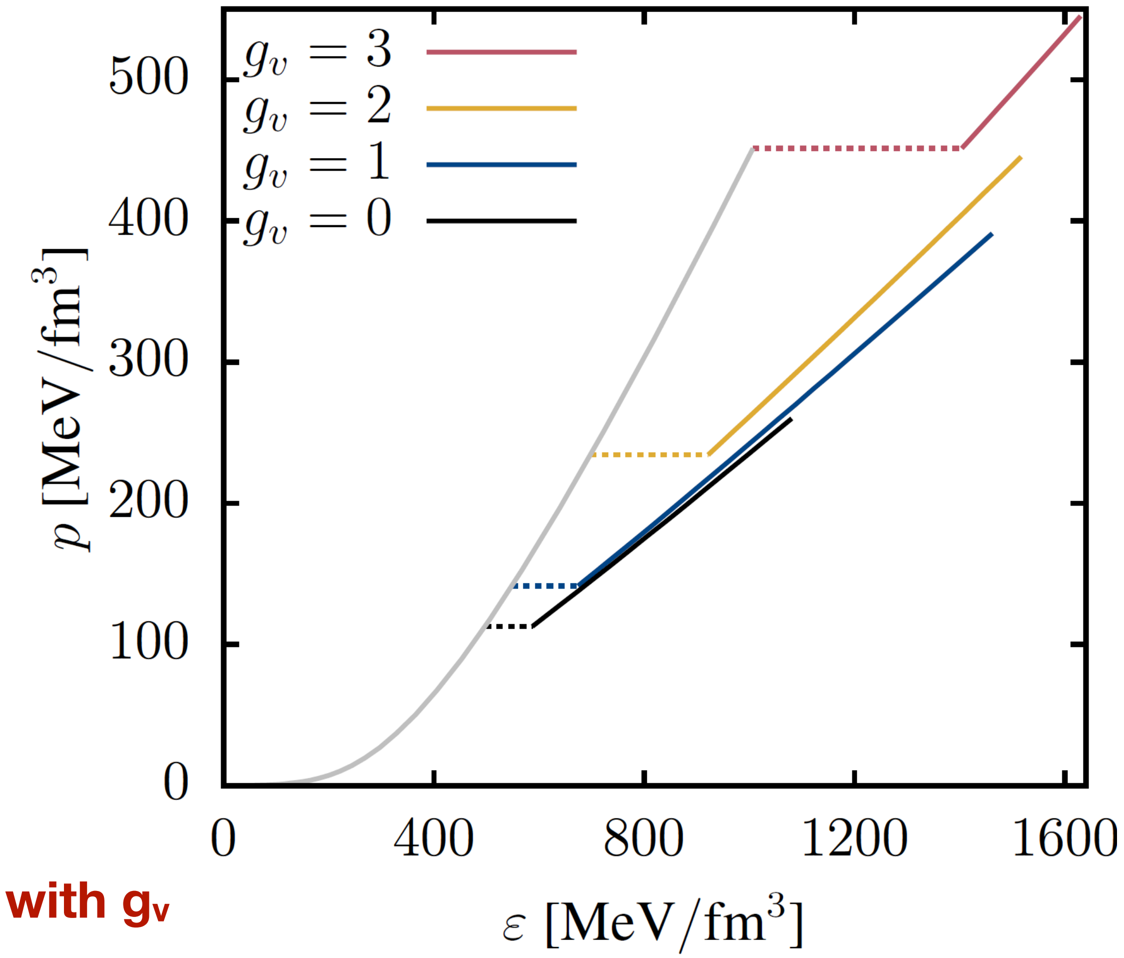
$$\Gamma_{\text{vec}} = \int_x \left[\frac{g_v}{2}\, \bar{q}\, \gamma_0\, \text{diag}_{\text{f}}(\omega, \omega, \sqrt{2}\phi)\, q - \frac{1}{2}\left(m_\omega^2 \omega^2 + m_\phi^2 \phi^2\right)\right]$$

- effectively shifts the chemical potentials:

$$\tilde{\mu}_u = \mu_q - \frac{2}{3}\mu_e - \frac{g_v}{2}\omega$$

$$\tilde{\mu}_d = \mu_q + \frac{1}{3}\mu_e - \frac{g_v}{2}\omega$$

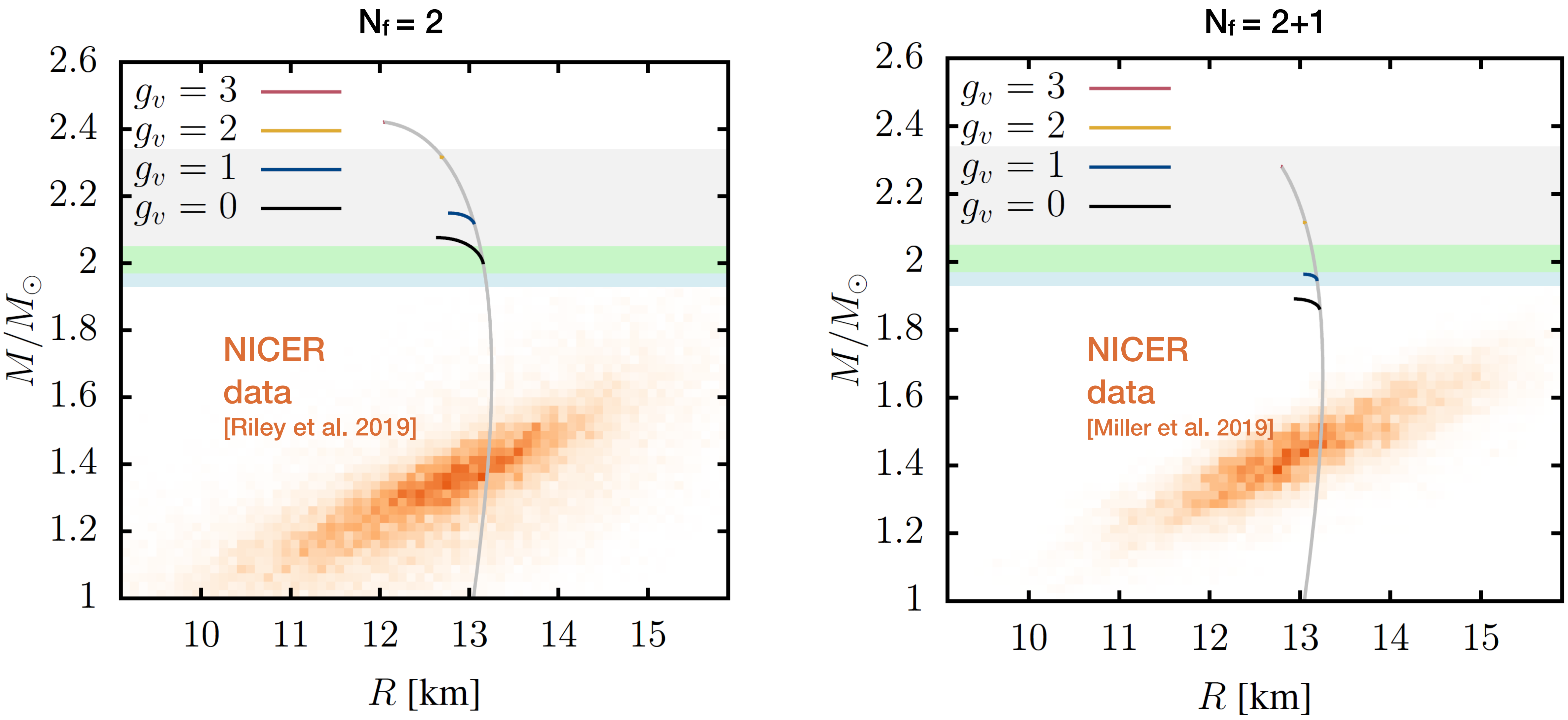
$$\tilde{\mu}_s = \mu_q + \frac{1}{3}\mu_e - \frac{g_v}{\sqrt{2}}\phi$$



➜ energy gap and transition pressure increases with g_v

Mass-Radius relations

[Otto, Oertel, BJS 2020]



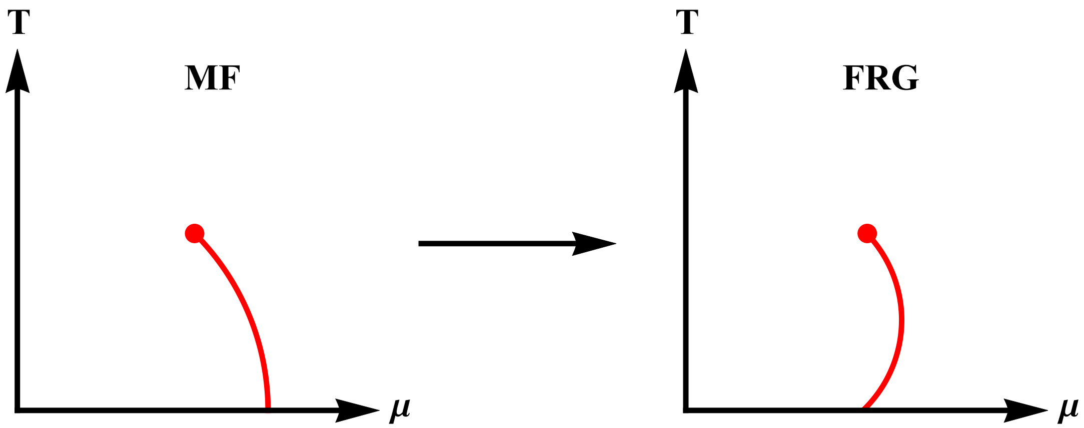
➜ including strange quarks: **finite vector coupling is needed to achieve $2M_\odot$ limit**

➜ at the same time: **larger vector coupling lead to smaller quark cores!**

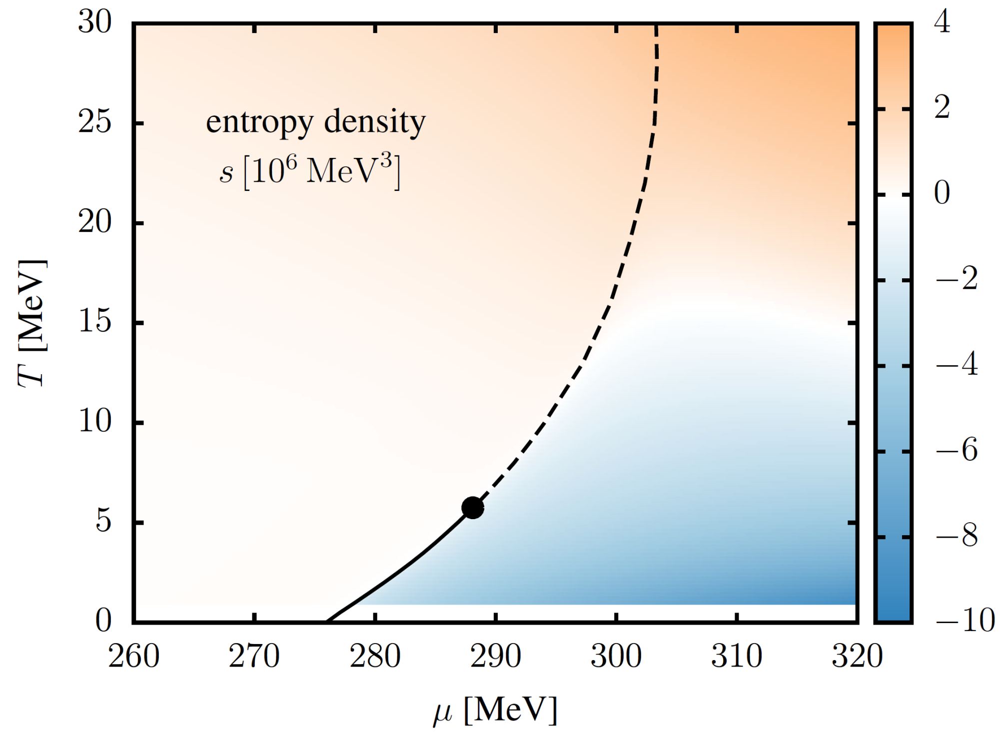
so far so good … BUT

Back-bending / negative entropy density

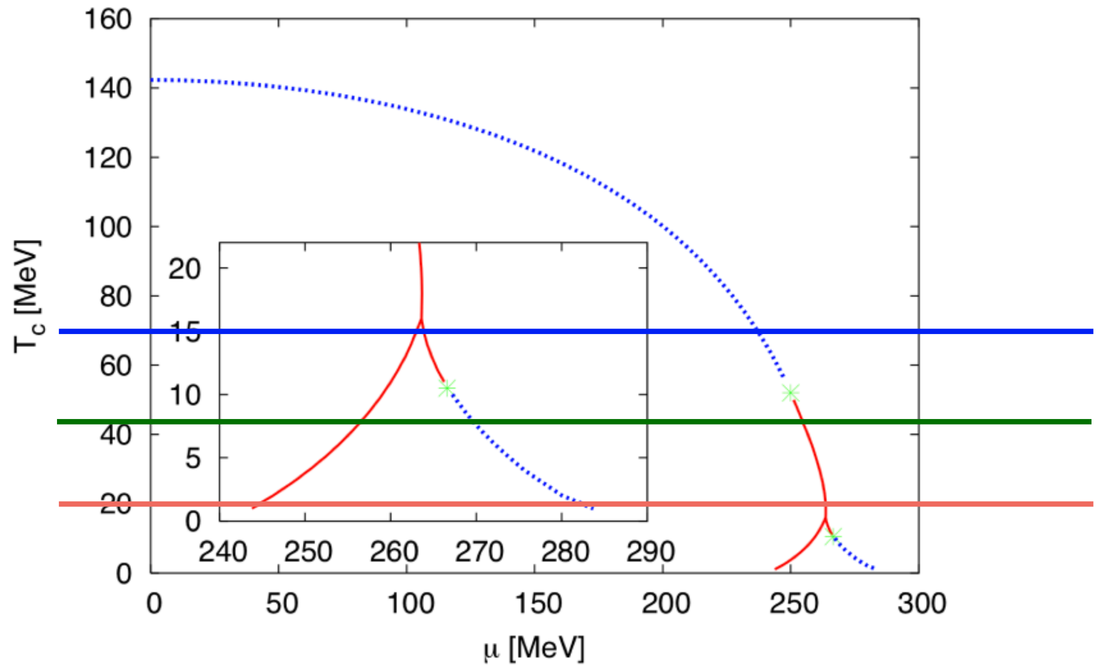
[R-A Tripolt, BJS, L von Smekal, J Wambach 2018]

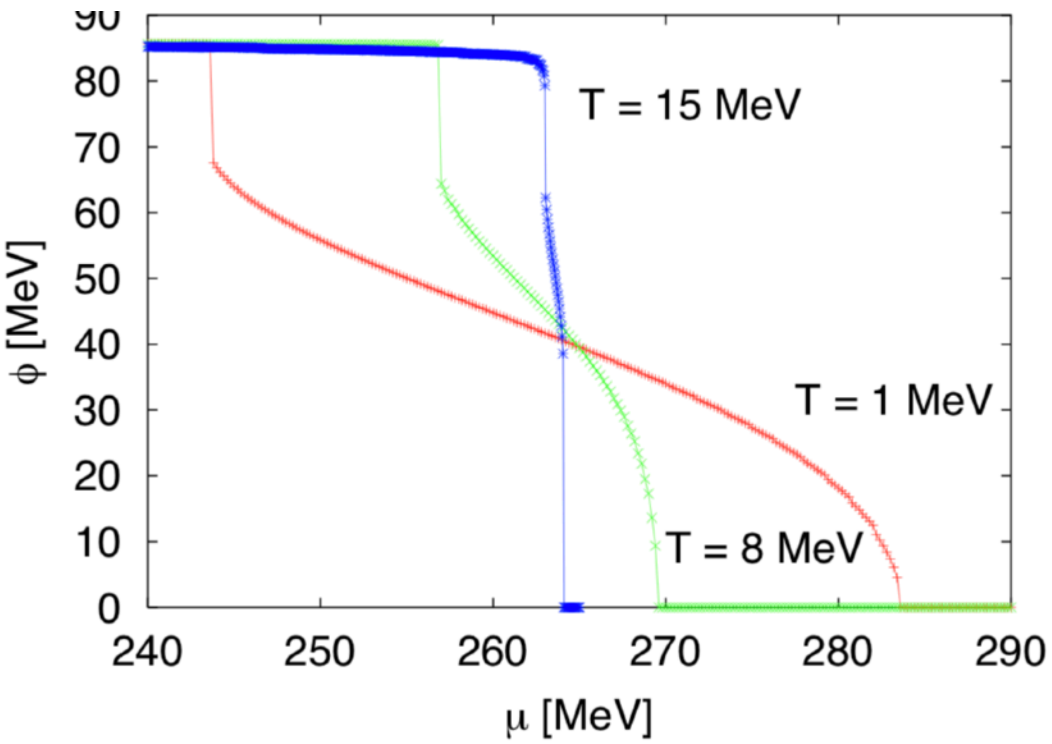


- Phase diagram quark-meson model
- Entropy density: s/T^3



[BJS, Wambach 2005]





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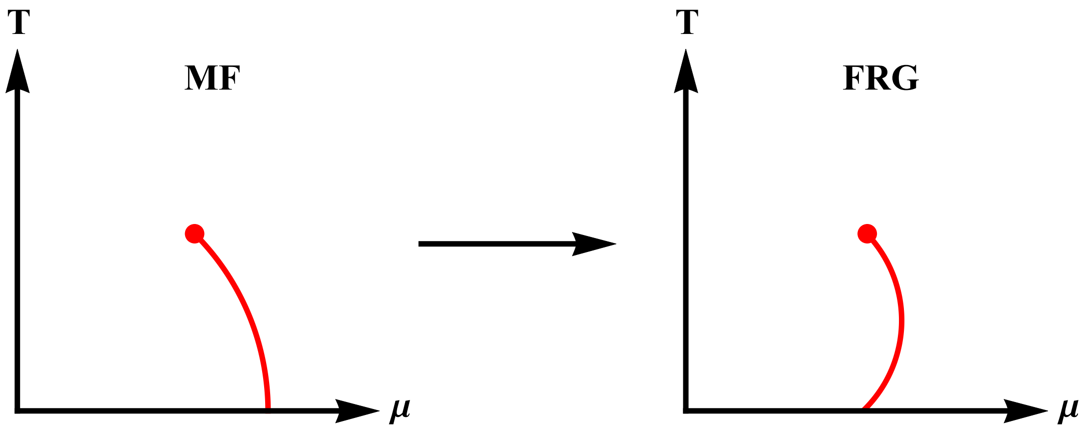

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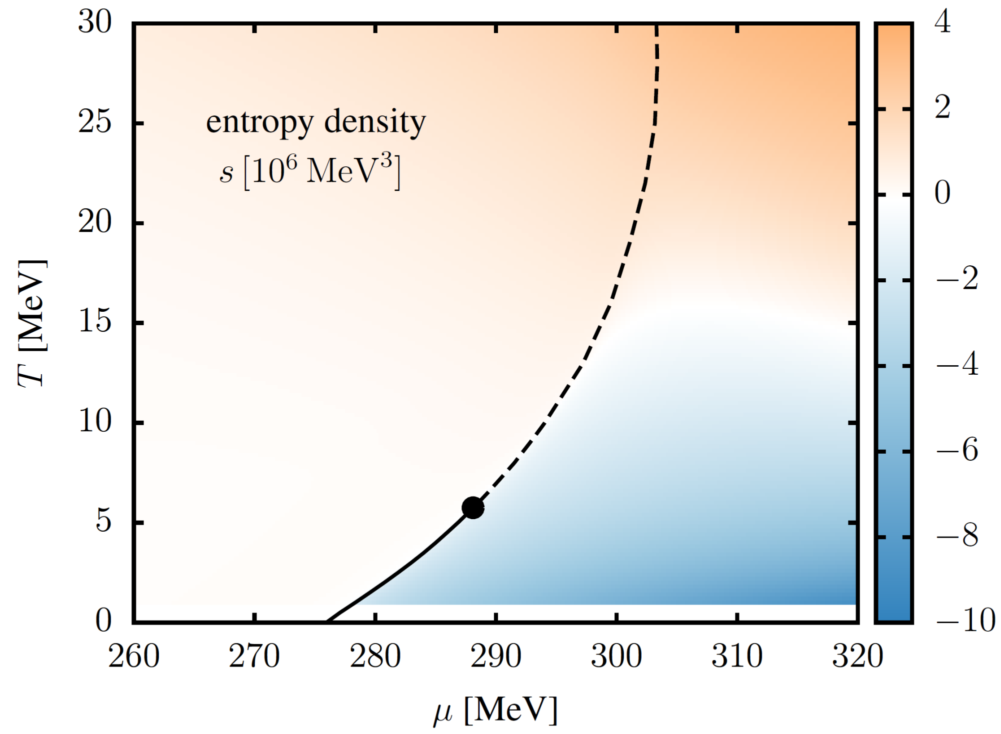

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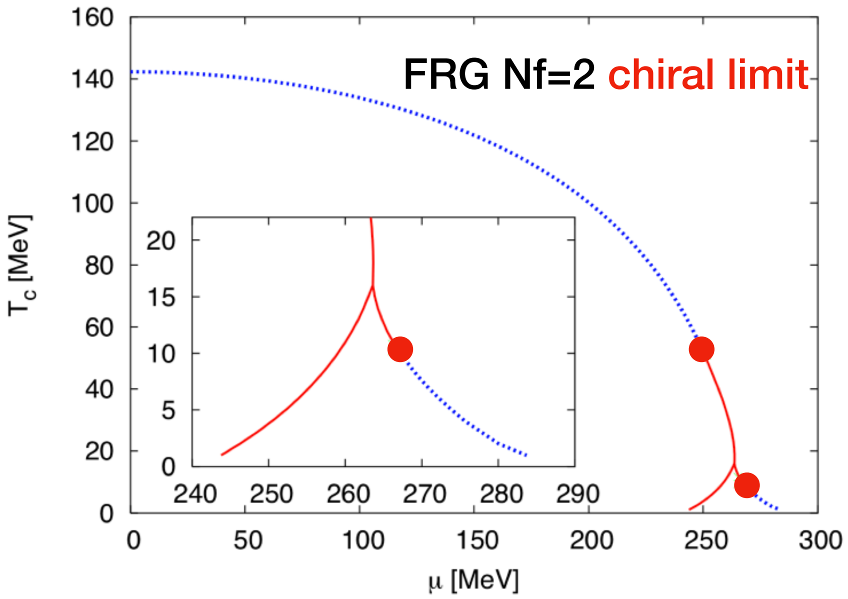

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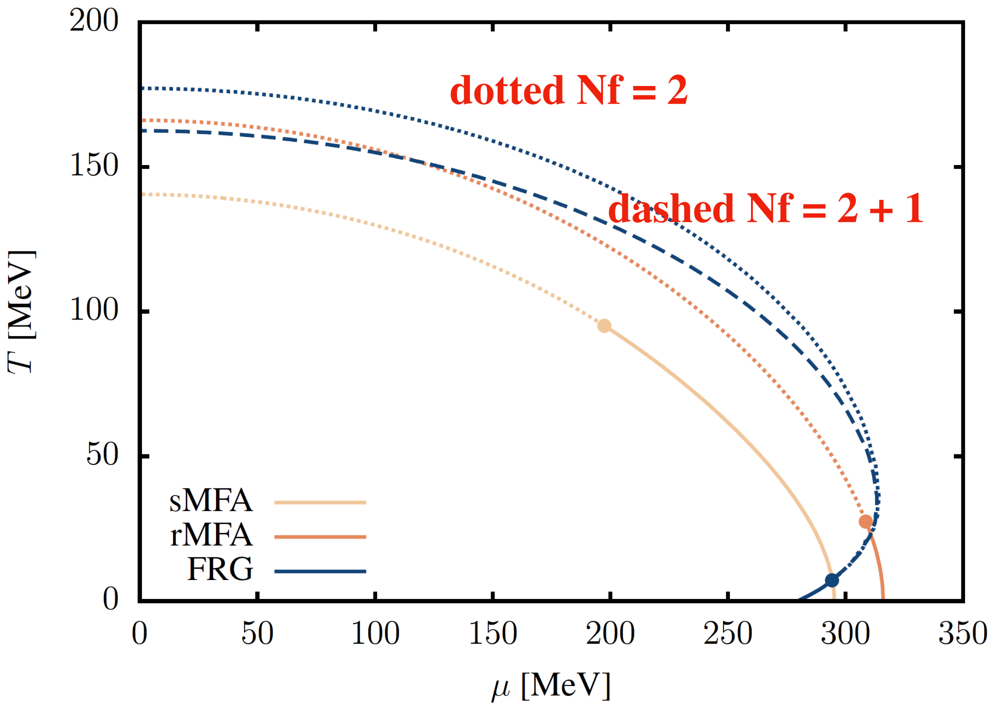

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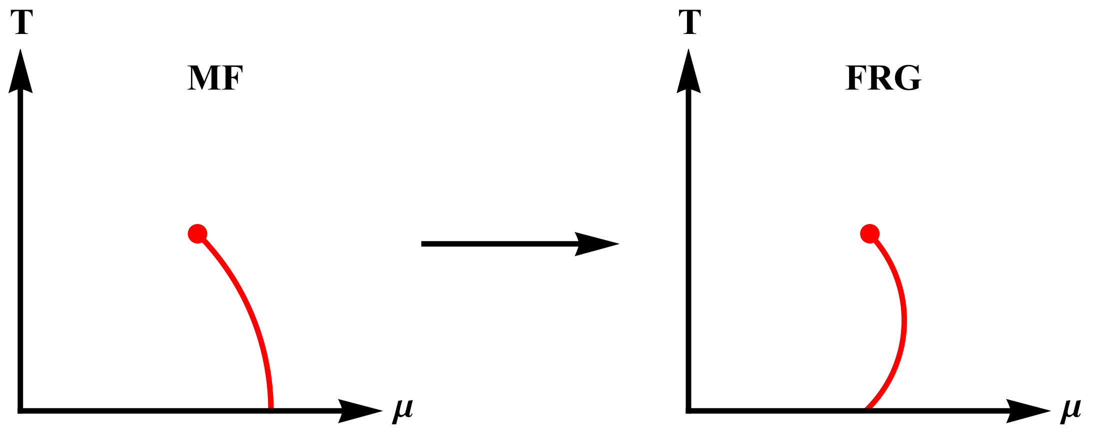

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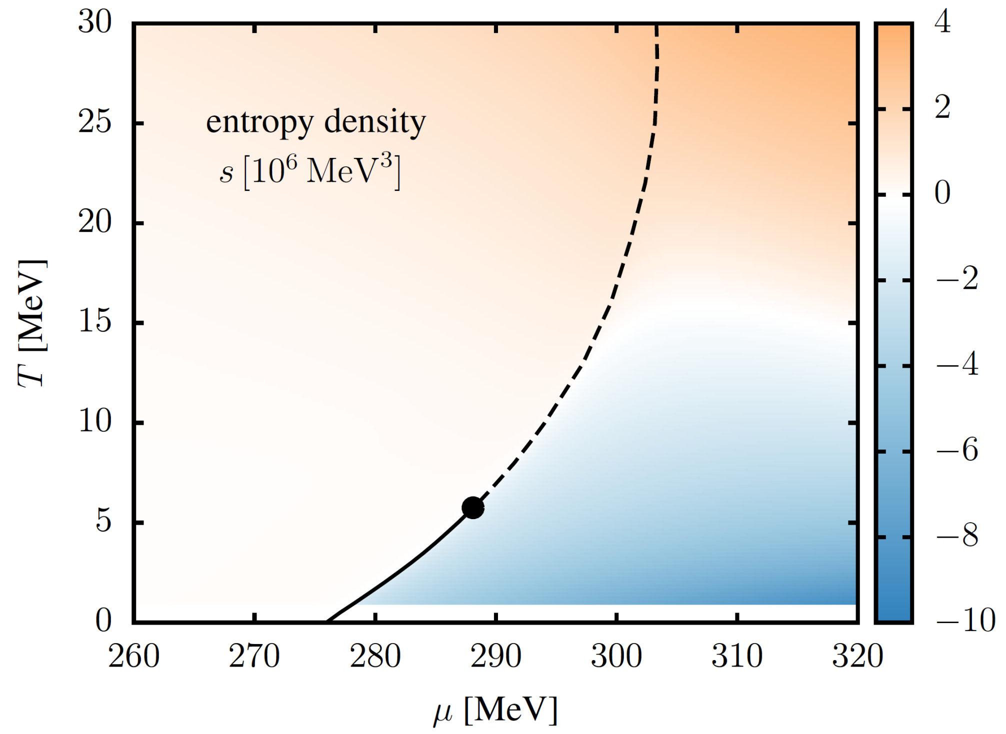
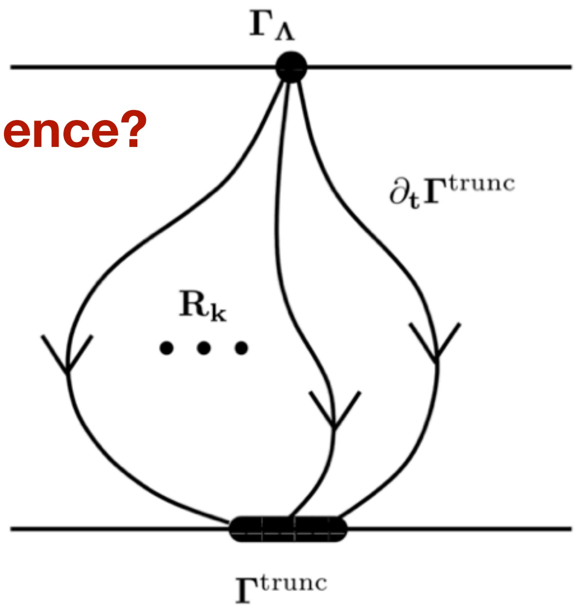


➜ **Regulator scheme dependence?**

less pronounced when more channels are included
e.g. pairing channel
s. next talk by Ugo Mire



- fermionic regulator

$$R_k^F(p,\mu) = R_k^F(\tilde{p},0) \qquad \tilde{p} = \begin{pmatrix} p_0 + \mathrm{i}\mu \\ \vec{p} \end{pmatrix}$$

shift required to preserve **Silver Blaze** property (T=0)

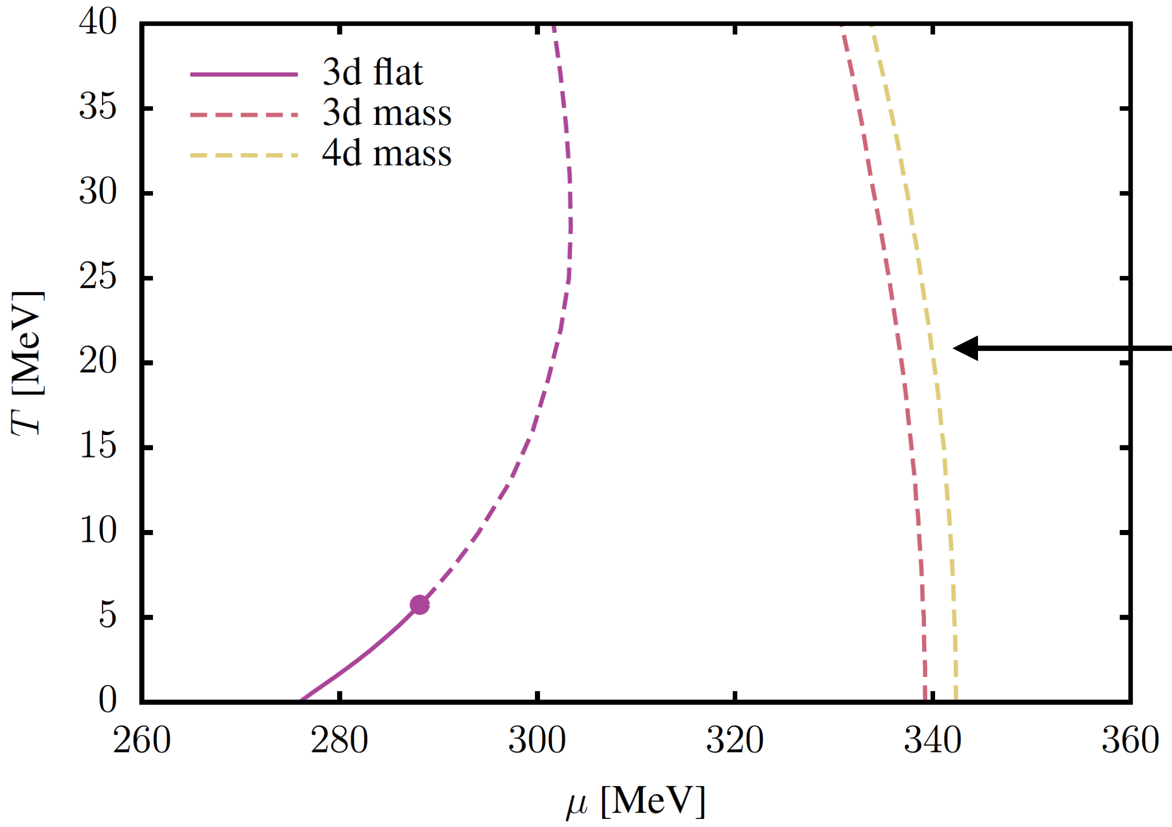
(necessary but not sufficient)

- relative shift in the cutoff-scales between bosonic & fermionic regulator
  ➜ is needed beyond LPA

e.g. field-dependent Yukawa-coupling

(multi quark-antiquark-meson scatterings)

Chiral transition at low temperature



no back-bending with

Callan-Symanzik mass-like regulator

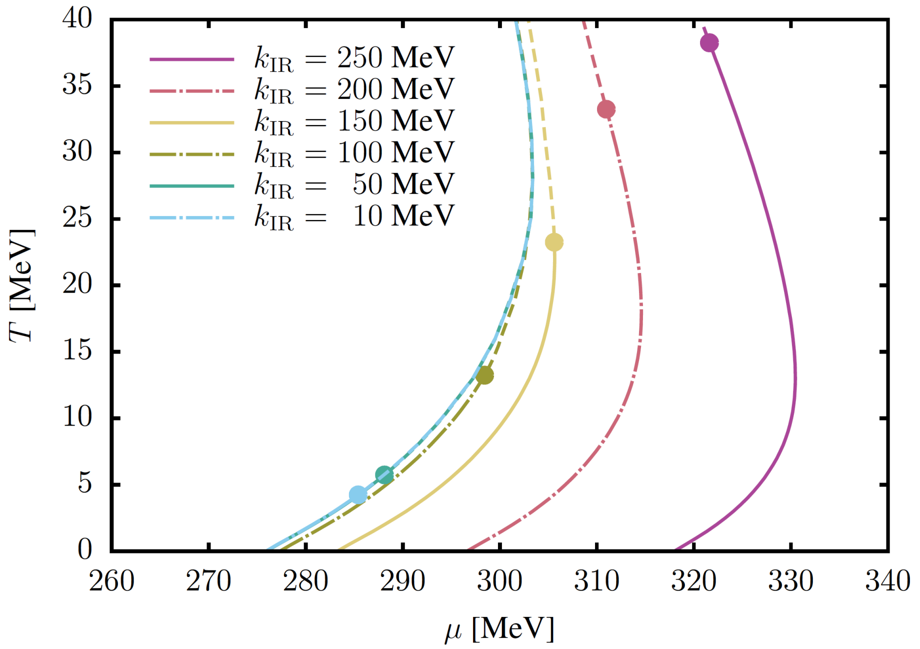
small differences between 3d and 4d regulators

purely crossover ➜ pseudocritical μ_c larger than m_q

could finite IR cutoff play a role? ➜ No

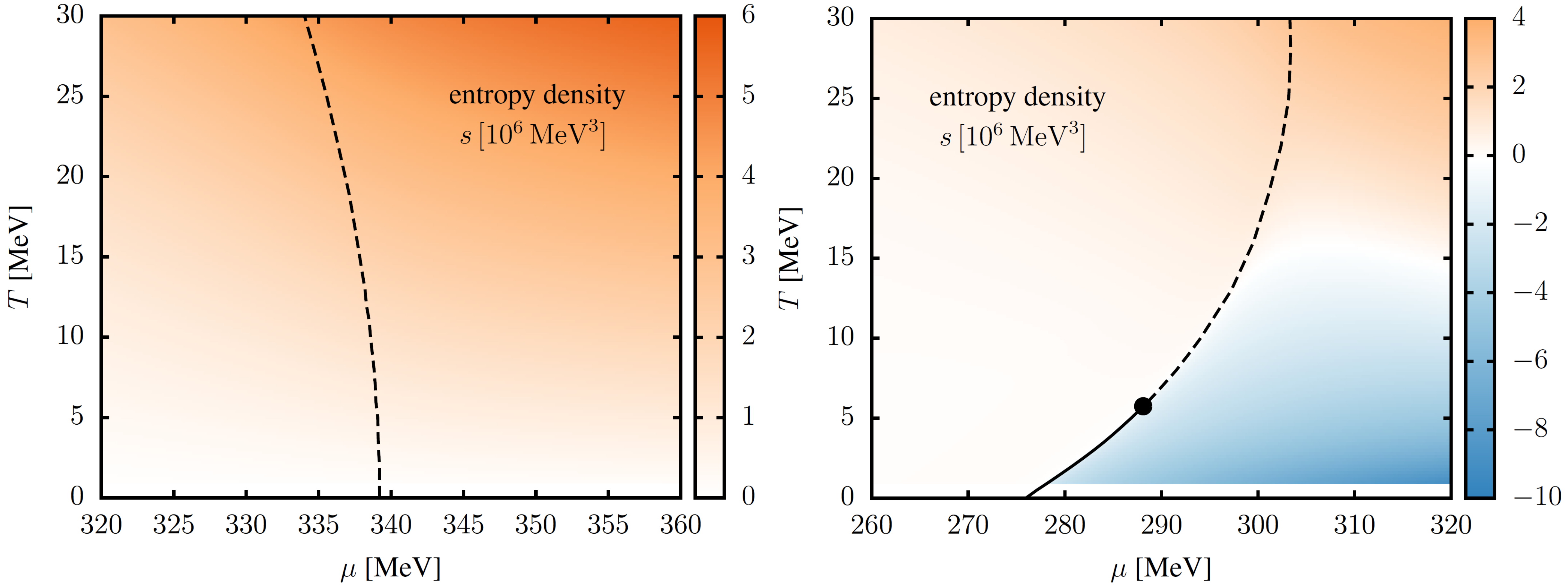
transition line shifts and CEP moves down

but back-bending over large k_{IR} range



Chiral transition at low temperature

➜ no negative entropy density anymore for Callan-Symanzik mass-like regulator



The ultimate Goal

… solving first principle QCD

EoS from QCD

- QCD procedure: start @O(100 GeV) (deep high-energy perturbative region)

[Braun et al. 2012++]

Λ ~ O(10GeV) quarks, gluons

- symmetric regime

quarks, gluons ➜ mesons

k_Φ

- symmetry-broken regime

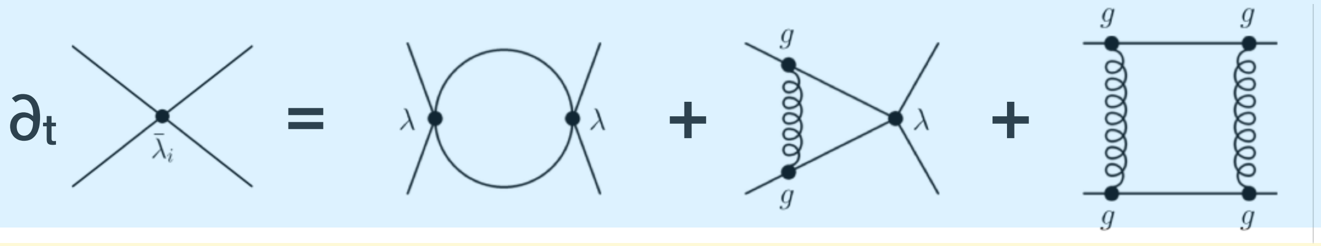
quarks, diquarks, mesons etc

k_{IR}

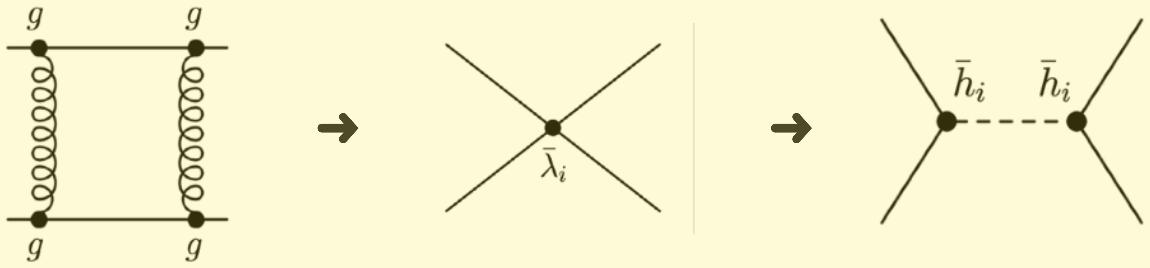
$$S = \int \mathrm{d}^4x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar\psi \left(\mathrm{i}\not{\partial} + \bar g \not{A} + \mathrm{i}\gamma_0 \mu \right) \psi \right\}$$

- quark-gluon vertex ➜ many quark self-interaction channels
- dynamical hadronisation:

  4-quark correlators ➜ bound states /resonances



- general picture:



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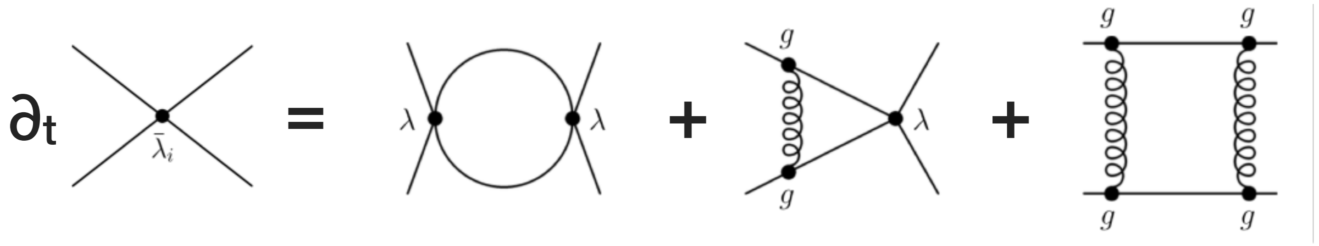
k_{IR}

$$S = \int \mathrm{d}^4x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi} \left(\mathrm{i}\partial\!\!\!/ + \bar{g} A\!\!\!/ + \mathrm{i}\gamma_0 \mu \right) \psi \right\}$$



- symmetry breaking ➜ condensates

➜ onset Landau-pole-type behavior
λ ~ 1/m

➜ Ginzburg-Landau effective potential

Quark-meson-diquark truncation

4-quark correlators

- QCD procedure: start @O(100 GeV) (deep high-energy perturbative region)

Λ ~ O(10GeV) quarks, gluons

- symmetric regime

quarks, gluons ➜ mesons

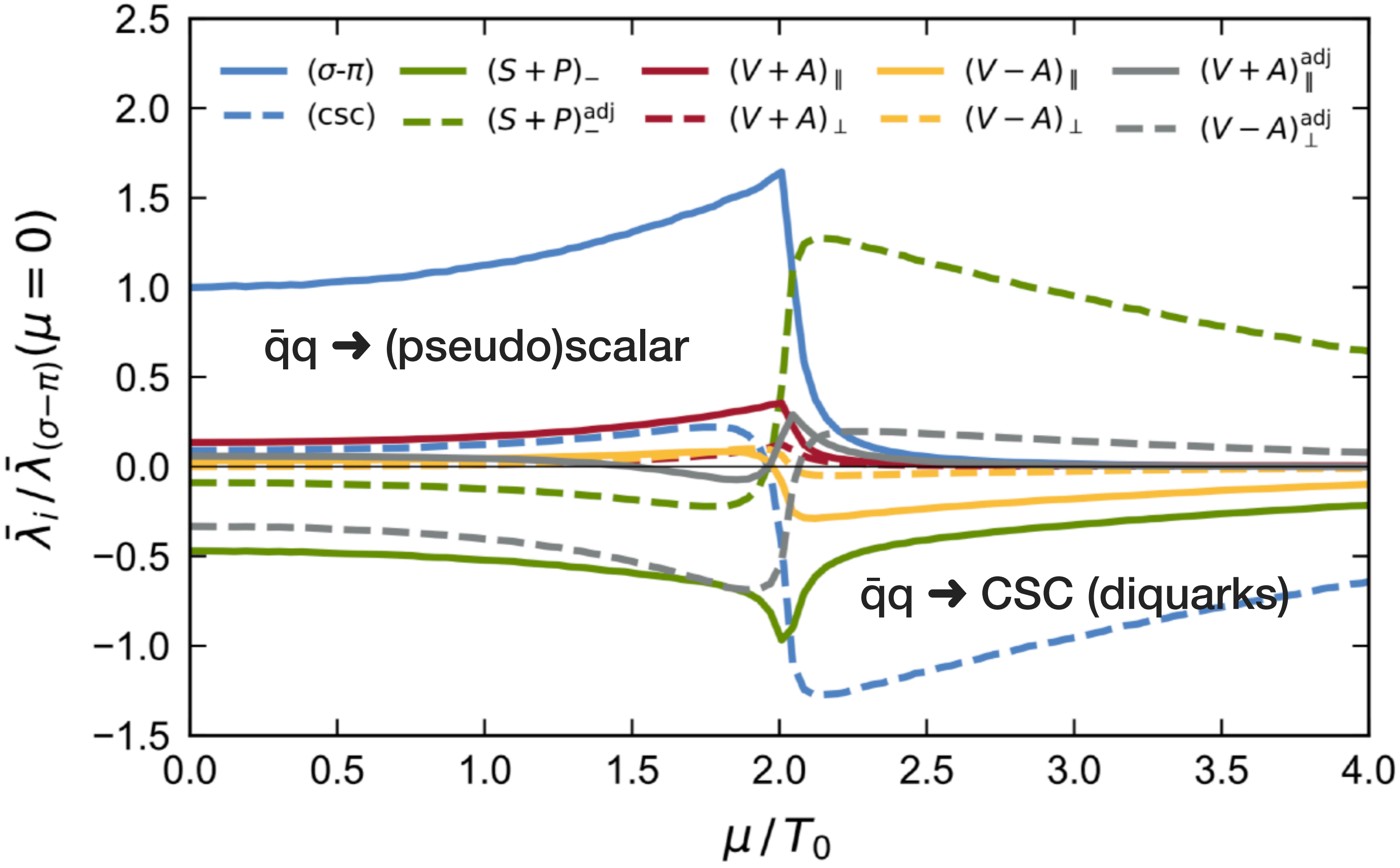
k_Φ

- symmetry-broken regime

quarks, diquarks, mesons etc

k_{IR}

[Braun et al. 2020]



Quark-meson-diquarks

- QCD procedure: start @O(100 GeV) (deep high-energy perturbative region)

[Mire, BJS to be published]

Λ ~ O(10GeV) quarks, gluons

- symmetric regime

quarks, gluons ➜ mesons

k_Φ

- symmetry-broken regime

quarks, diquarks, mesons etc

k_{IR}

Quark-meson-diquark truncation

$$
\begin{aligned}
\mathcal{L}_{\mathrm{QMD}} = {} & \bar{q}\left(\partial\!\!\!/ - \hat{\mu}\gamma_0 + g_\phi\left(\sigma + i\gamma_5\vec{\pi}\,\vec{\tau}\right)\right) q \\
& + \frac{g_\Delta}{2}\left(\Delta_{\mathrm{A}}\bar{q}_C\gamma_5\tau_2\lambda_{\mathrm{A}}q - \Delta_{\mathrm{A}}^{*}\bar{q}\gamma_5\tau_2\lambda_{\mathrm{A}}q_C\right) \\
& + \left(\left(\partial_\nu + \delta_{\nu 0}2\mu\right)\Delta_{\mathrm{A}}^{*}\right)\left(\partial_\nu - \delta_{\nu 0}2\mu\right)\Delta_{\mathrm{A}} \\
& + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + U(\rho, d) - c\sigma
\end{aligned}
$$

with $\rho = \frac{1}{2}(\sigma^2 + \vec{\pi}^2)$ and $d = \frac{1}{2}\Delta_{\mathrm{A}}^{*}\Delta_{\mathrm{A}}$

Quark-meson-diquarks

- QCD procedure: start @O(100 GeV) (deep high-energy perturbative region)

[Mire, BJS to be published]

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quarks, diquarks, mesons etc

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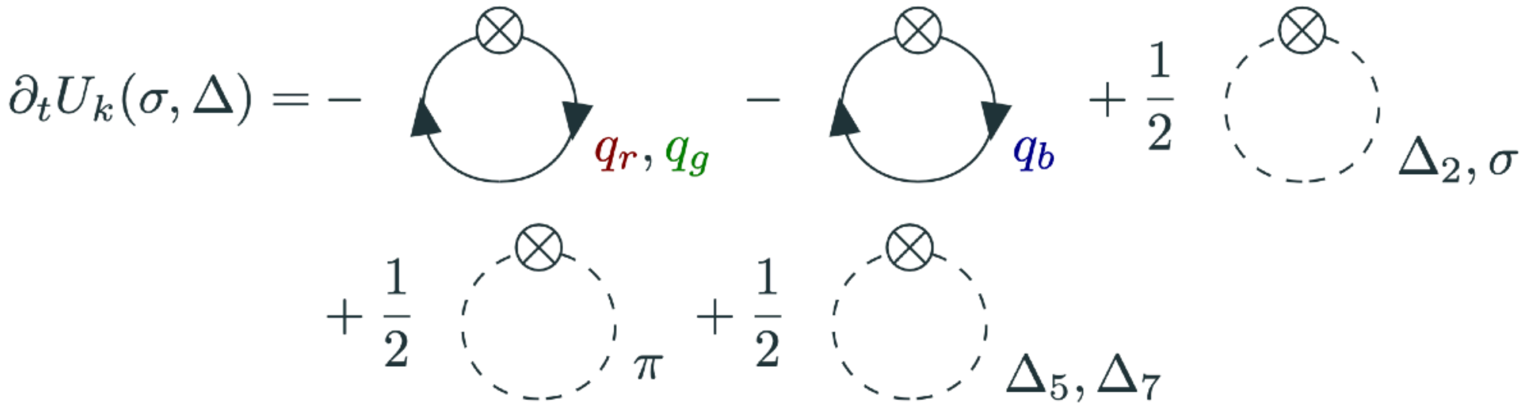
Quark-meson-diquark truncation

$$\mathcal{L}_{\text{QMD}} = \bar{q}\left(\not{\partial} - \hat{\mu}\gamma_0 + g_\phi\left(\sigma + i\gamma_5\vec{\pi}\,\vec{\tau}\right)\right) q$$

$$+\frac{g_\Delta}{2}\left(\Delta_{\text{A}}\bar{q}_C\gamma_5\tau_2\lambda_{\text{A}}q - \Delta_{\text{A}}^*\bar{q}\gamma_5\tau_2\lambda_{\text{A}}q_C\right)$$

$$+\left(\left(\partial_\nu + \delta_{\nu 0}2\mu\right)\Delta_{\text{A}}^*\right)\left(\partial_\nu - \delta_{\nu 0}2\mu\right)\Delta_{\text{A}}$$

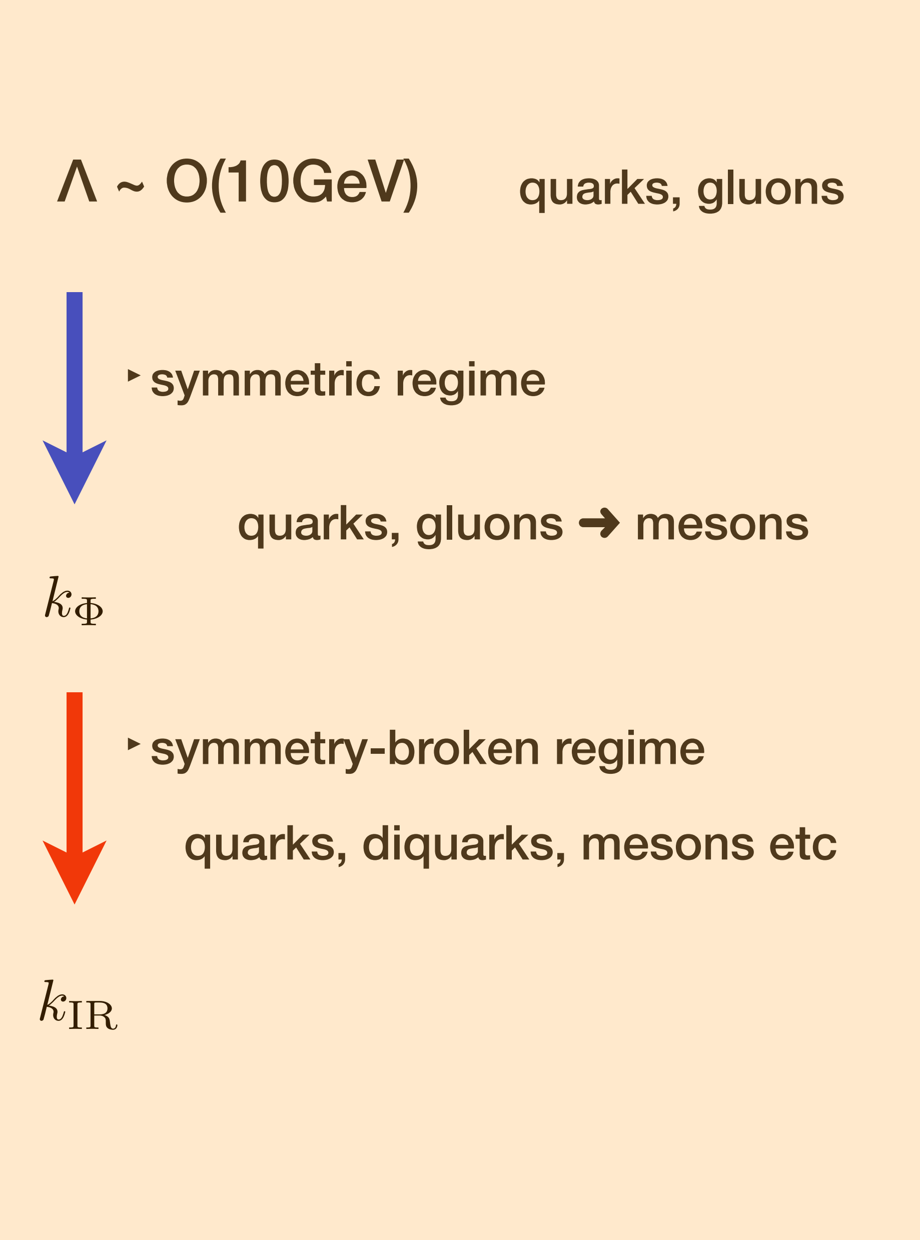
$$+\frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + U(\rho, d) - c\sigma$$



Fermi-surface

- QCD procedure: start @O(100 GeV) (deep high-energy perturbative region)

[Mire, BJS to be published]



Λ ~ O(10GeV) quarks, gluons

- symmetric regime

quarks, gluons ➜ mesons

k_Φ

- symmetry-broken regime

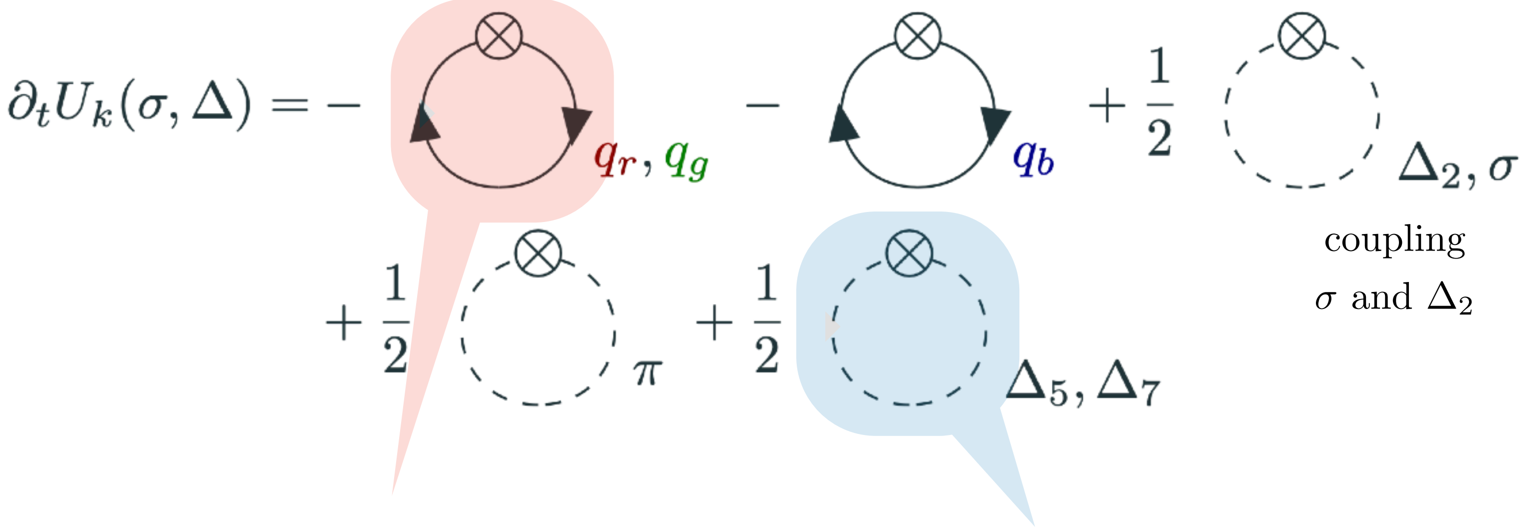
quarks, diquarks, mesons etc

k_{IR}

- dispersion relations

$$E_\Delta = \sqrt{(\epsilon_q \pm \mu)^2 + g_\Delta^2 d} \qquad \epsilon_{\{q,\pi,\sigma,\Delta\}} = \sqrt{k^2 + m^2_{\{q,\pi,\sigma,\Delta\}}}$$

- flow equation



$$\sim \frac{\epsilon_q - \mu}{\sqrt{(\epsilon_q - \mu)^2 + g_\Delta^2 d}} \qquad \sim \coth \frac{\epsilon_\Delta - 2\mu}{2T}$$

$$\longrightarrow \qquad \sim \frac{\epsilon_q - \mu}{((\epsilon_q - \mu)^2 + g_\Delta^2 d)^{3/2}}$$

diverges at Fermi-surface

flow around Fermi-surface?

no diquark loops approximation

- QCD procedure: start @O(100 GeV) (deep high-energy perturbative region)

[Mire, BJS to be published]

Λ ~ O(10GeV) quarks, gluons

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Quark-meson-diquark truncation

$$\mathcal{L}_{\mathrm{QMD}} = \bar{q}\left(\not{\partial} - \hat{\mu}\gamma_0 + g_\phi\left(\sigma + i\gamma_5\vec{\pi}\,\vec{\tau}\right)\right) q$$

$$+\frac{g_\Delta}{2}\left(\Delta_{\mathrm{A}}\bar{q}_C\gamma_5\tau_2\lambda_{\mathrm{A}}q - \Delta^*_{\mathrm{A}}\bar{q}\gamma_5\tau_2\lambda_{\mathrm{A}}q_C\right)$$

$$\xcancel{+\left((\partial_\nu + \delta_{\nu 0}2\mu)\Delta^*_{\mathrm{A}}\right)(\partial_\nu - \delta_{\nu 0}2\mu)\Delta_{\mathrm{A}}}$$

no diquark loops

$$+\frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + U(\rho, d) - c\sigma$$

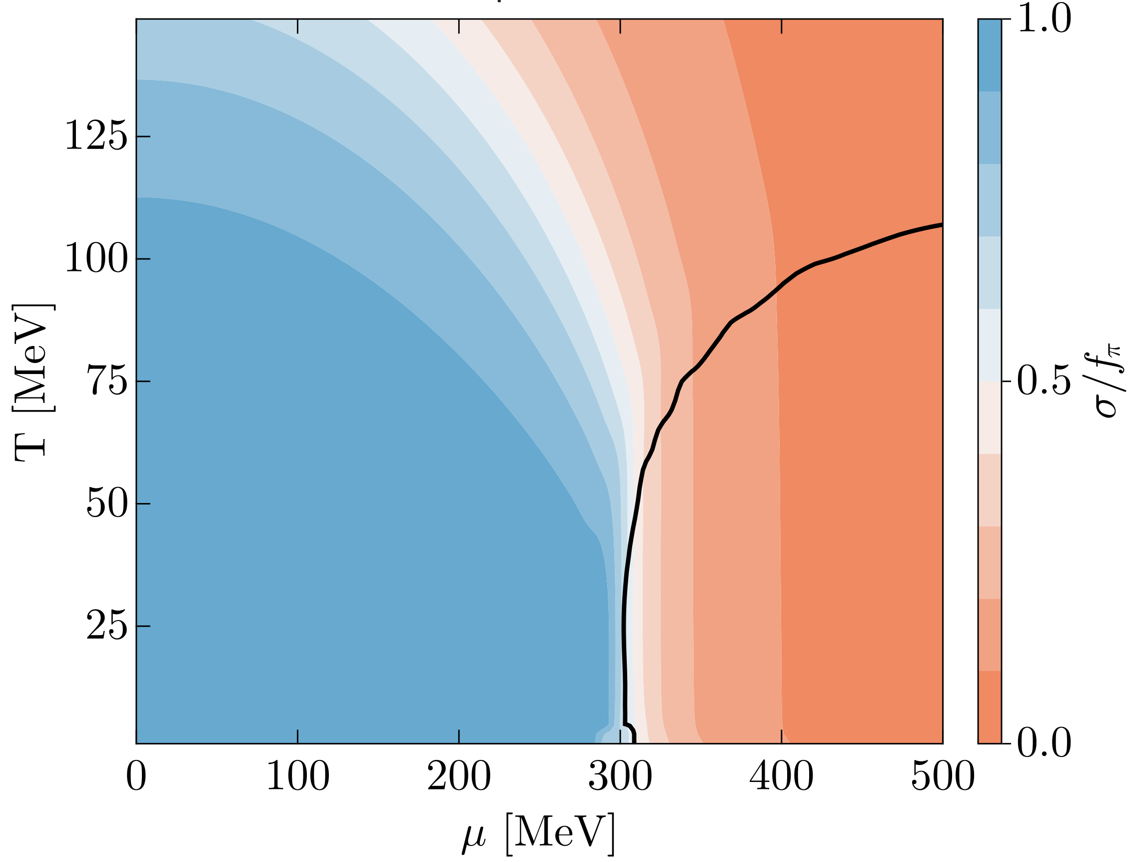
with $\rho = \frac{1}{2}(\sigma^2 + \vec{\pi}^2)$ and $d = \frac{1}{2}\Delta^*_{\mathrm{A}}\Delta_{\mathrm{A}}$

Phase diagram

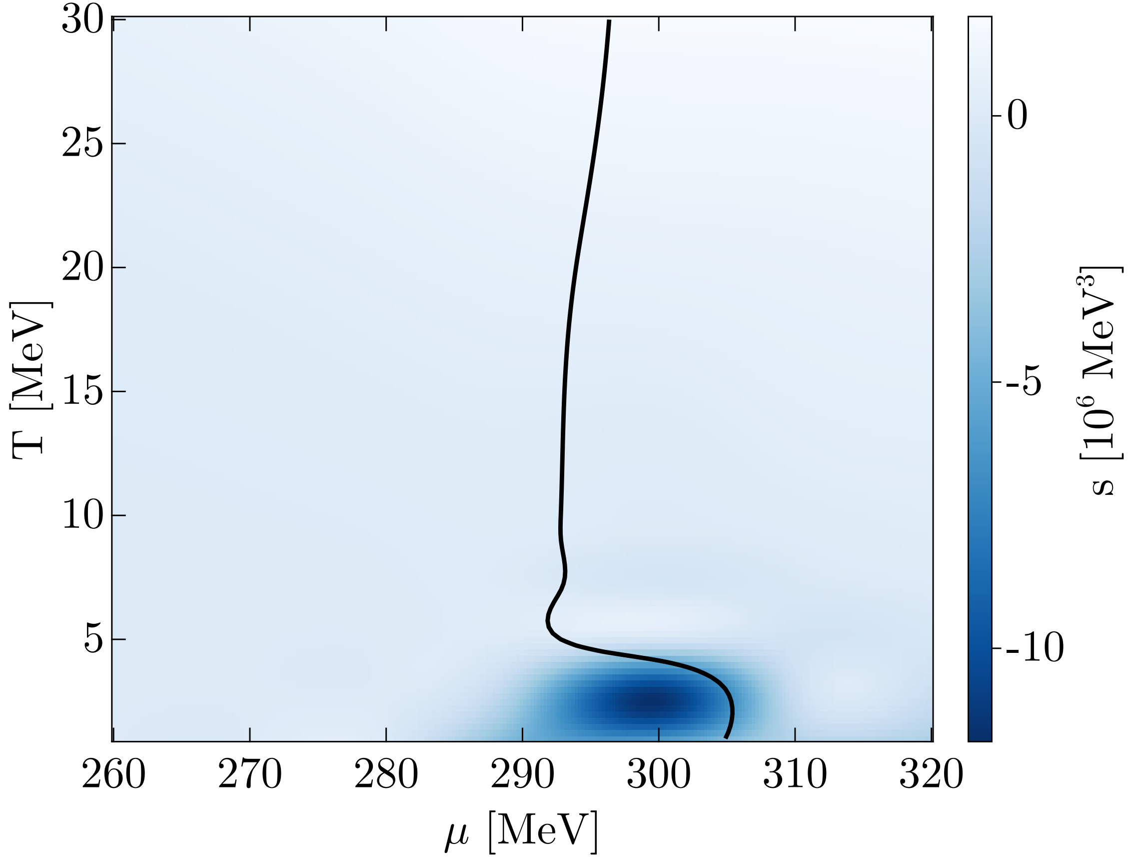
- **First QMD results**

[Mire, BJS to be published]

chiral and diquark condensates



entropy density for small temperatures



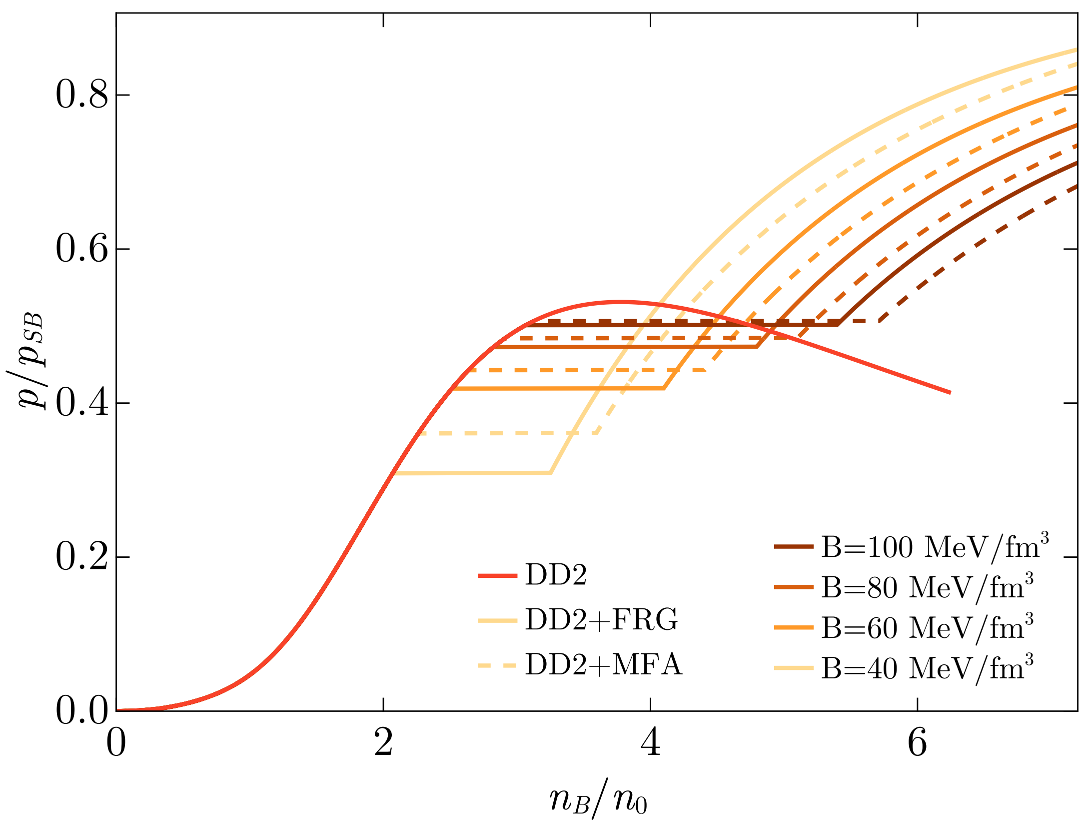
▸ flow in LPA and flat (Litim) 3d regulator:

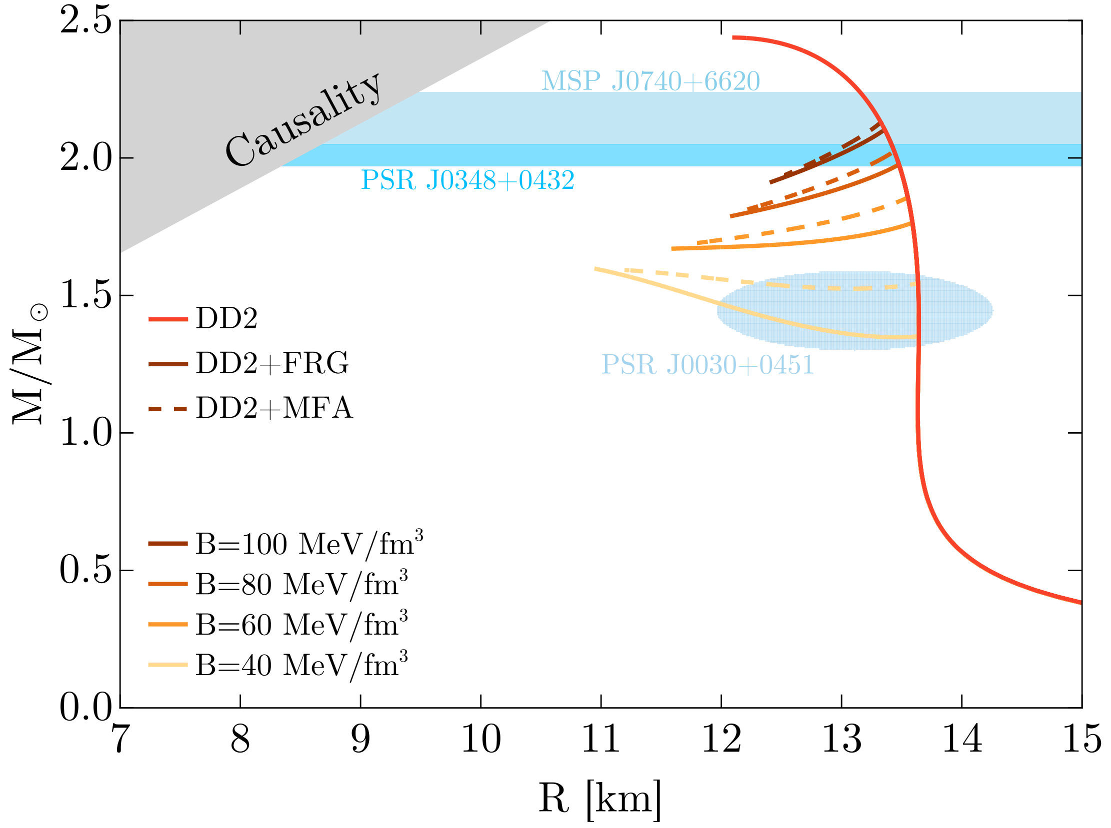
diquarks reduce negative entropy region & back-bending

EoS & Mass-Radius relation

- **First QMD results**

[Mire, BJS to be published]


Maxwell construction: hadronic EoS (DD2)


M-R relation

▸ **no crossing of quark EoS with hadronic EoS HS(DD2)**

introduce bag-constant

▸ **superconducting quark core**

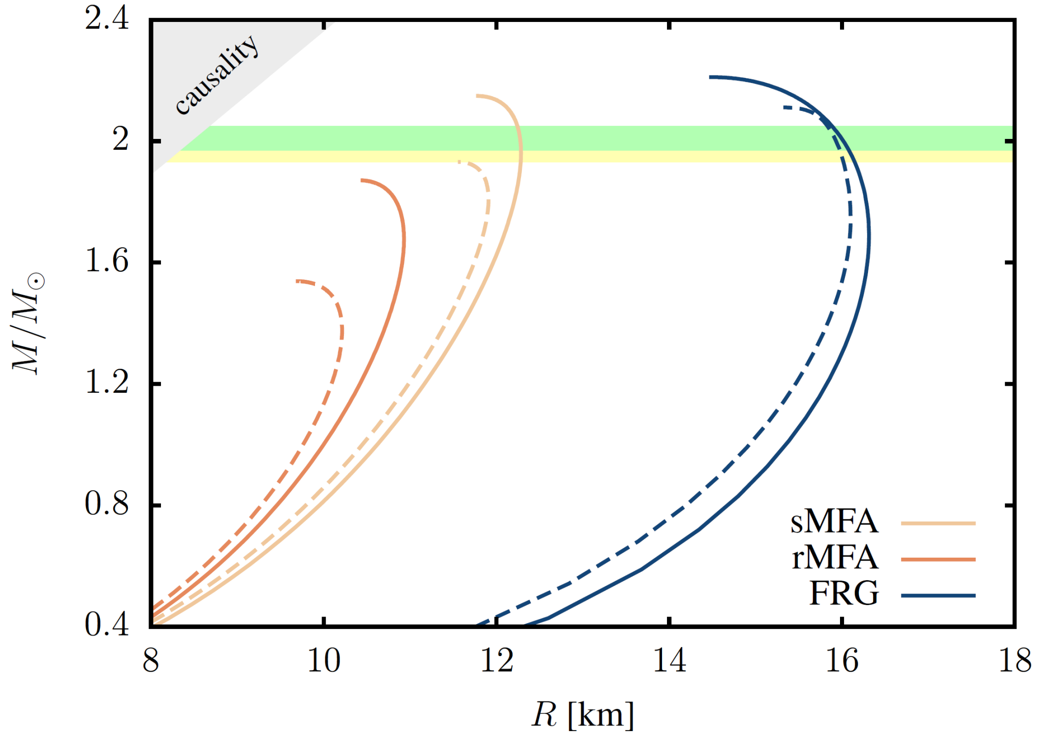
mostly unstable with present diquark parameters

Summary

Four home messages:

1. EoS with the FRG for two and three quark flavor:

   ➜ significant impact of fluctuations
   on M-R relation for NSs



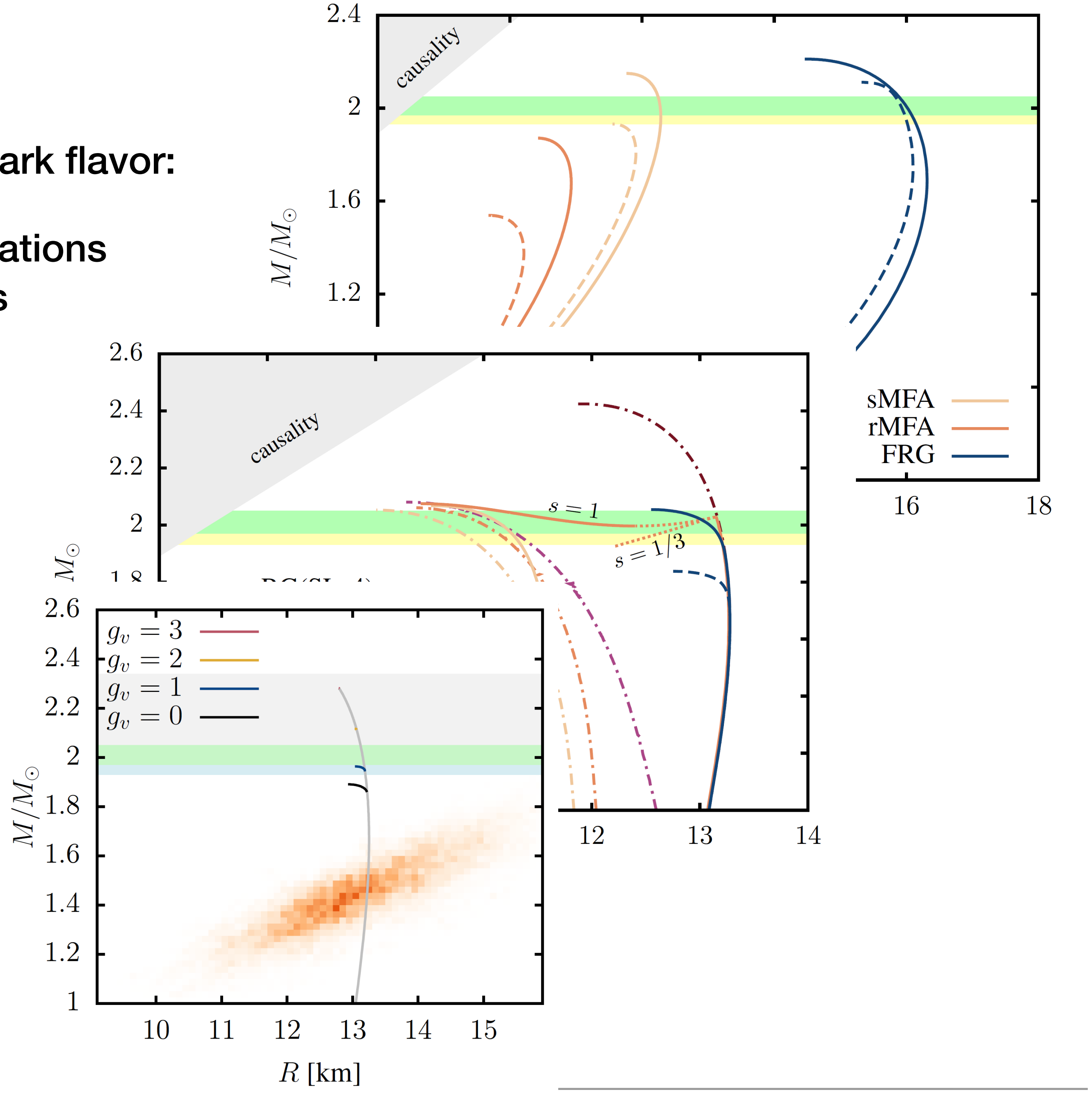
Summary

Four home messages:

1. EoS with the FRG for two and three quark flavor:

   ➜ significant impact of fluctuations
   on M-R relation for NSs

2. hybrid stars are possible

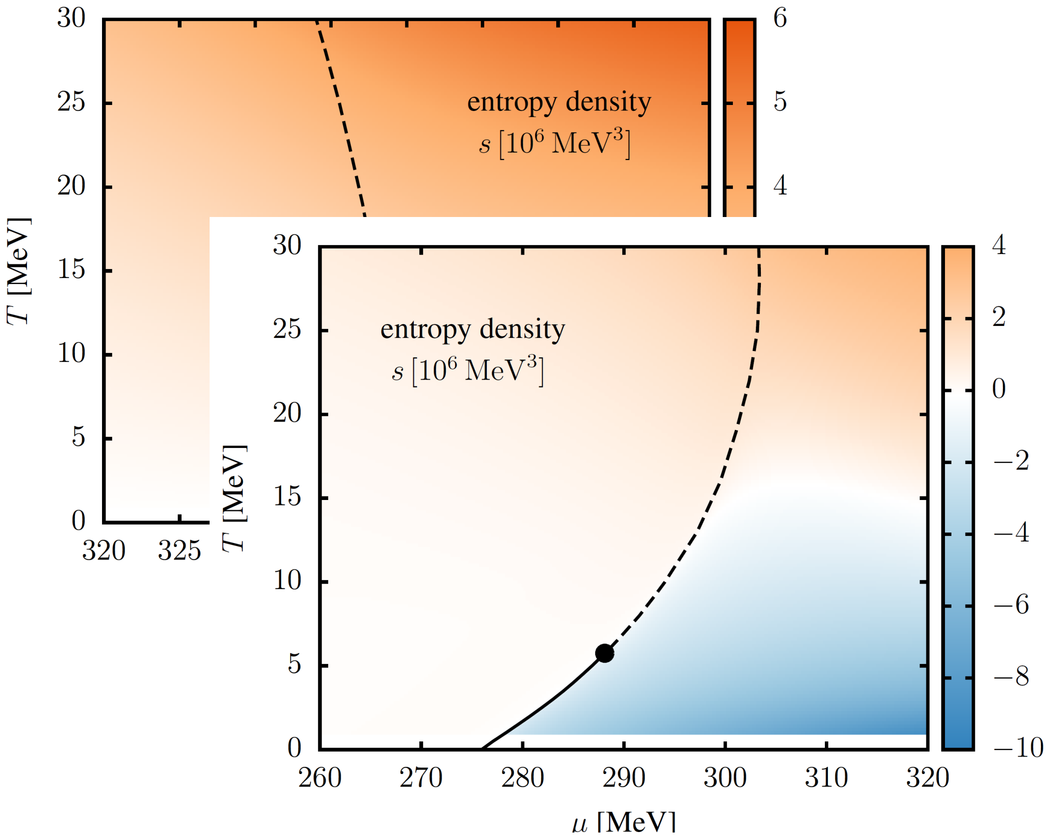


Non-zero vector coupling needed
➜ to reach 2 M⊙ with strangeness

similar findings with pairing channels

Summary

3. in LPA no back-bending / negative entropy density
for
CS mass-like regulators



CS type regulators closer to

poles compared to flat regulator

➜ (vacuum) flows numerically harder

4. no back-bending / reduction of negative entropy density
for
diquark (pairing) channel