Walking in the Hidden Valley

Modelling near-conformal dark sector theories

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The dark sector

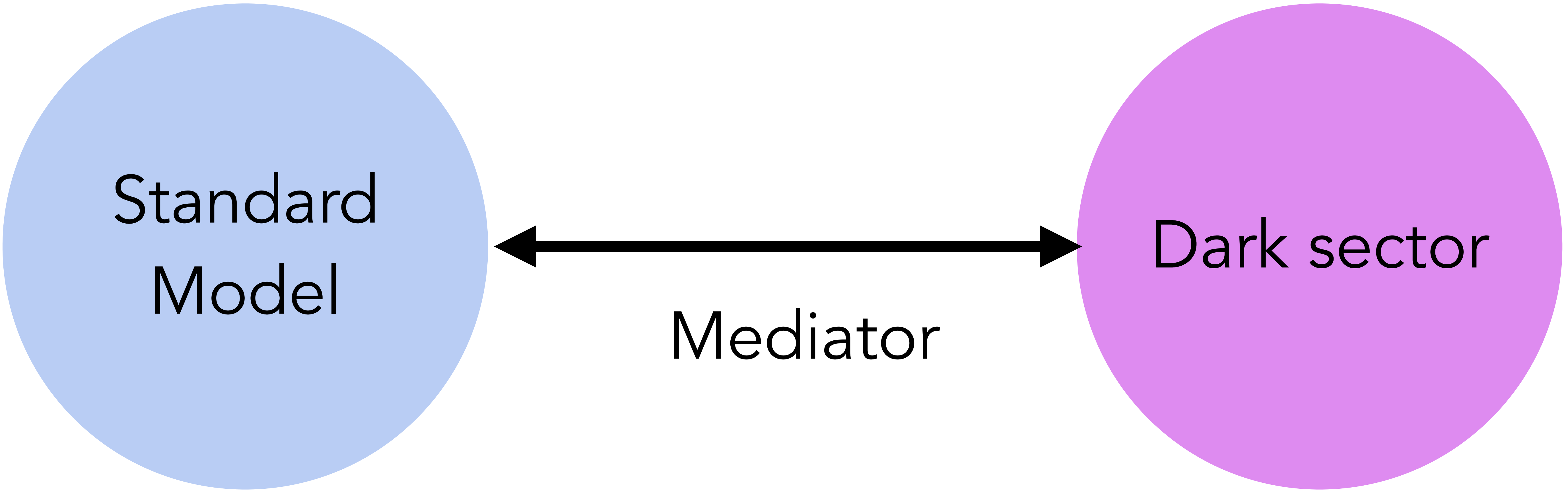
- Dark sectors extend the Standard Model (SM) with a new sector uncharged under the SM gauge group but instead connected to the SM through a mediator.

arxiv:0604261

arxiv:0712.2041

arxiv:0806.2385

- M.J. Strassler et al.

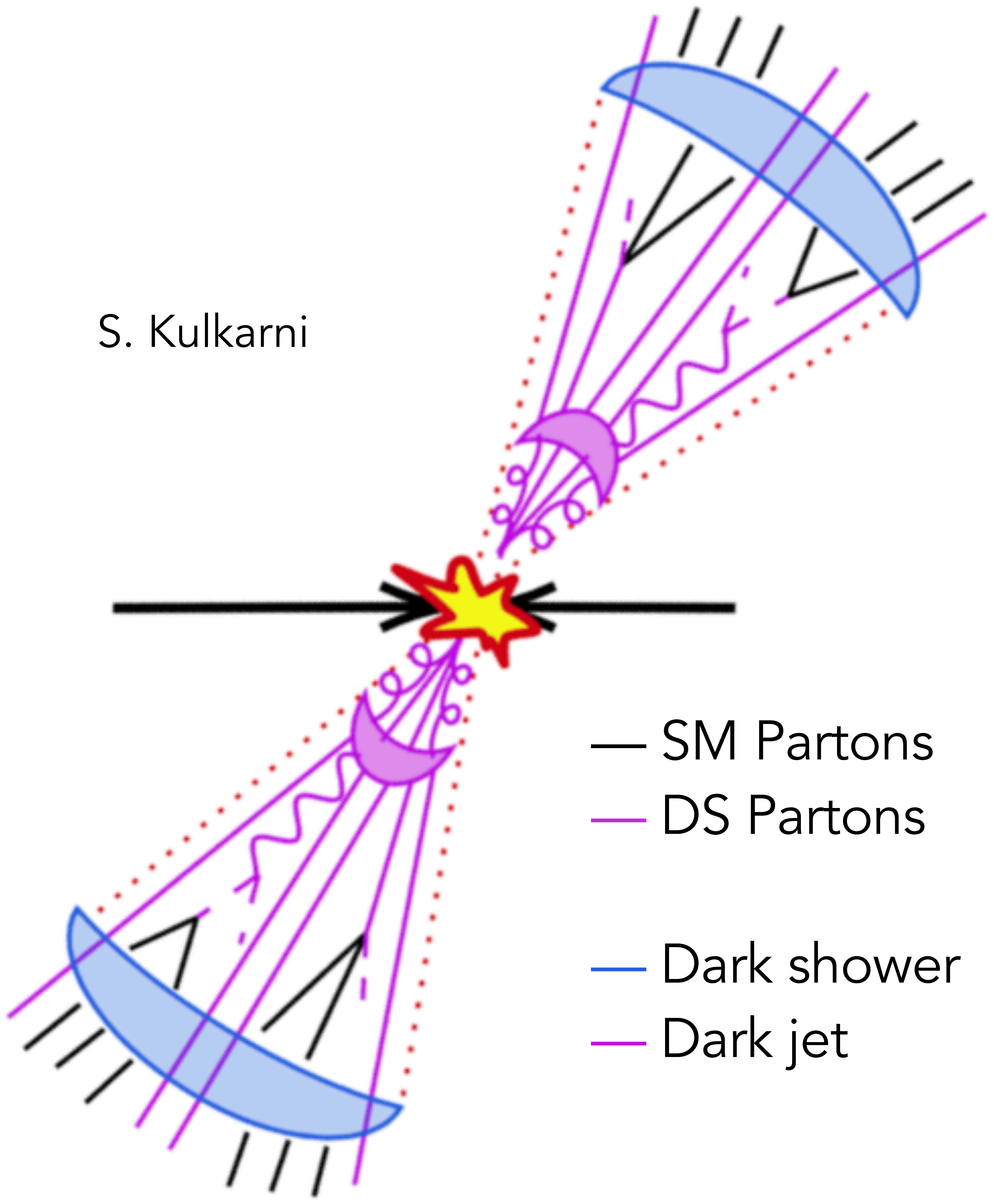


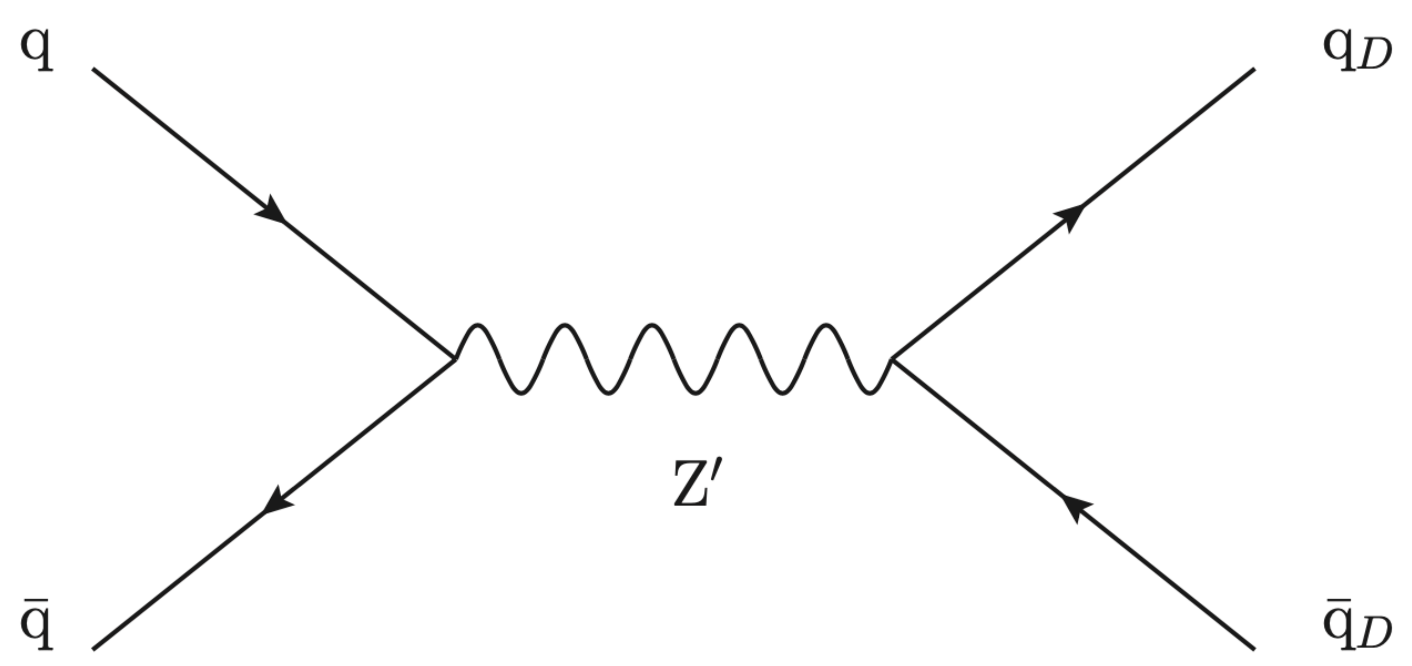
- We focus on confining“Hidden Valleys"; particularly interacting QCD-like dark sectors with a non-Abelian $SU\left(N_c\right)$ gauge group with N_f flavors of fundamental Dirac fermions (dark quarks).

- Analogous to QCD, confinement ensures the formation of bound states such as dark pions.

Dark sector signatures

S. Kulkarni

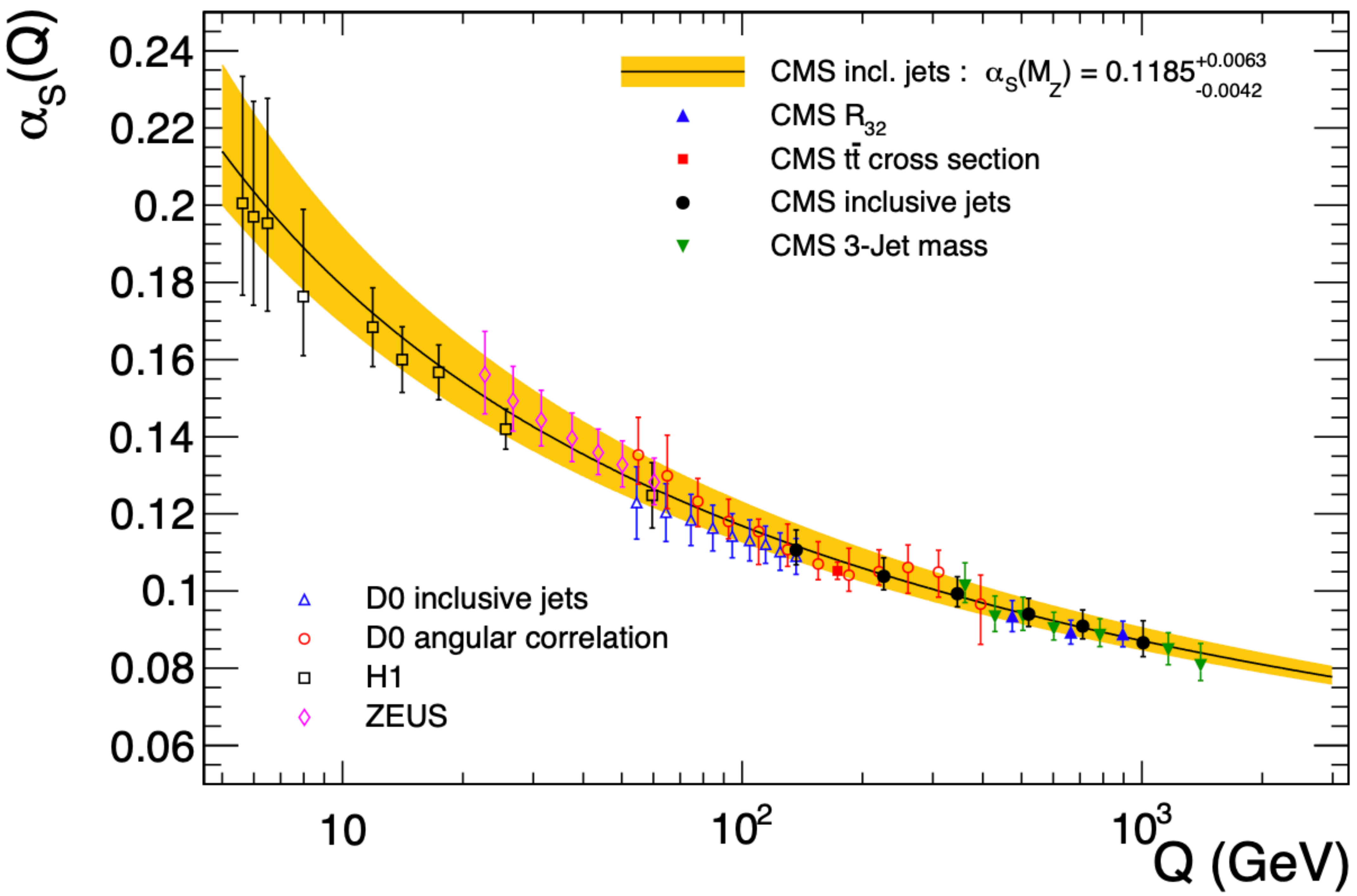




- Like in QCD, energetic dark quarks radiate dark gluons which in turn mainly radiate further dark gluons but occasionally dark quarks, leading to a showering of dark quarks and gluons known as a "dark jet".
- This shower eventually will hadronise and form bound states, such as dark pions. A proportion of dark hadrons will decay to SM particles through the mediator.
- Which gives a jet with a mixture of stable dark hadrons and SM decay products of unstable dark hadrons; typically these high MET events are known as "dark showers".

arXiv:1503.00009 - T. Cohen et al.

Modelling dark sector signatures



arXiv:1512.05194 - D. d'Enterria et al.

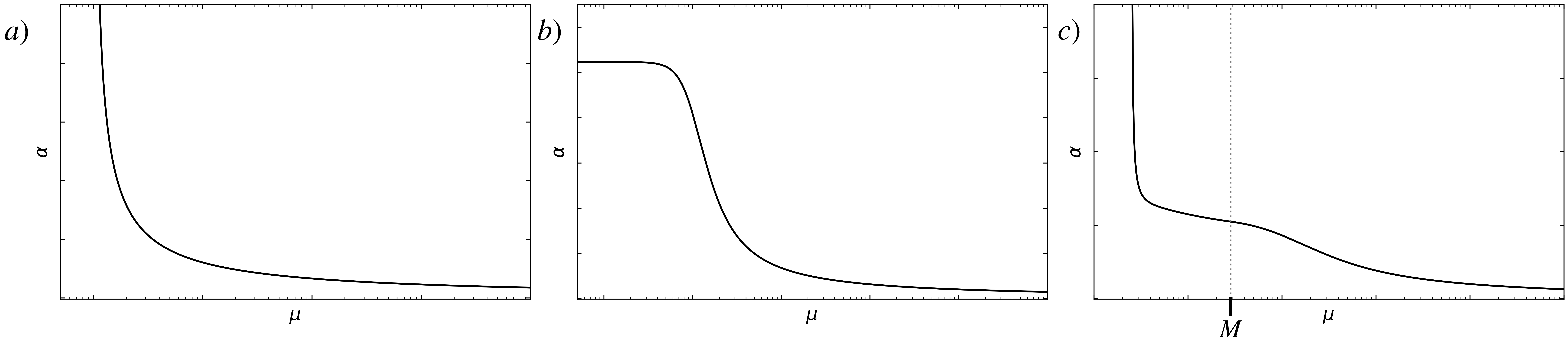
- Properties of α, such as asymptotic freedom, ensure the formation of jets and govern jet properties, such as shape and multiplicity.

- The β function governs how α, the running coupling, varies under the renormalisation group. Given at two-loop order by,

$$\beta(\alpha) = \mu^2 \frac{d\alpha}{d\mu^2} = -\alpha^2 \left(\beta_0 + \beta_1 \alpha\right)$$

- Different phases within the $\frac{N_f}{N_c}$ space of such theories give different running behaviours. We need to understand and map these phases onto the signature space.

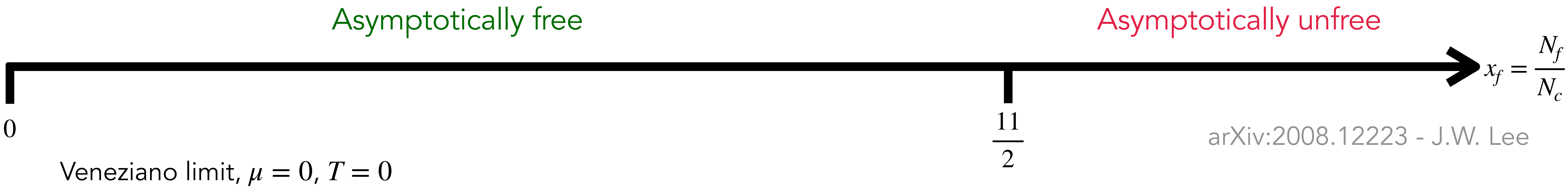
Modelling dark sector signatures



- The Monte Carlo generator Pythia can currently simulate the jet signatures of Hidden Valley theories that are QCD-like; theories which display asymptotic freedom and confinement.
- Conformal theories do not confine and so do not form bound states, therefore their signatures are purely missing energy and can not be modelled within MC generators.
- Consider mass-split Hidden Valleys, which although still confine, have a more complicated running coupling structure. Will we need to expand the scope of available simulation tools in order to model these theories?

What is asymptotic freedom?

- Consider the phase structure of a sector analogous to QCD with an $SU\left(N_c\right)$ gauge symmetry with N_f flavours of fundamental massless fermions.



Veneziano limit, $\mu = 0$, $T = 0$

- From the first coefficient of $\beta\left(\alpha\right)$ it can be shown that for $\frac{N_f}{N_c} < \frac{11}{2}$, α displays 'asymptotic freedom', the decrease of interaction strength with energy scale.
- $\frac{N_f}{N_c} = \frac{11}{2}$ is the upper end of the "conformal window" in which no confinement takes place - pure missing E_T jet signature.

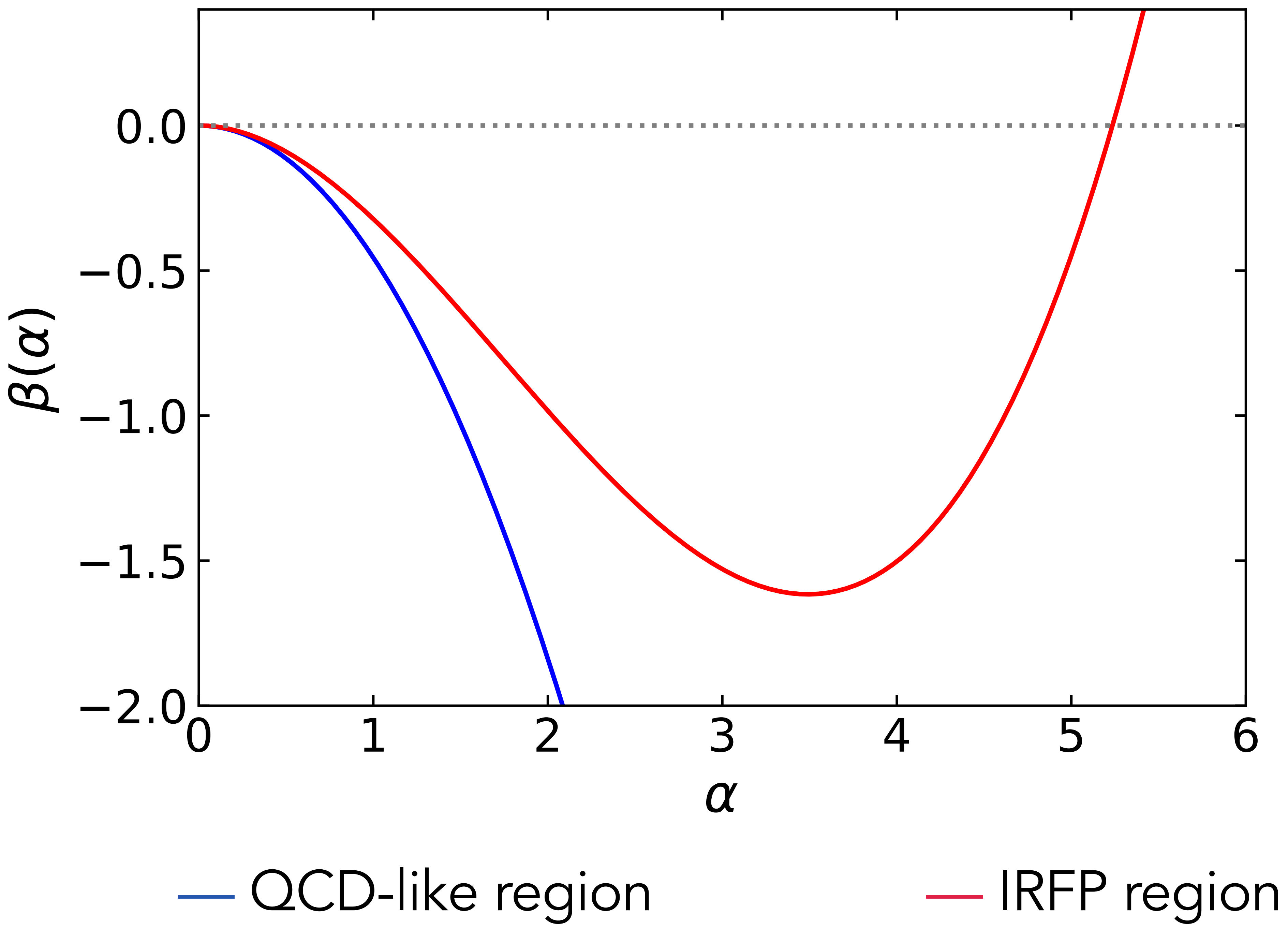
What is the conformal window?

- The lower-end of this conformal window is defined to be where chiral symmetry is no longer broken. Above x_f^c there is no longer any confinement.

- At some critical number of flavours, chiral symmetry is restored and the running coupling of such massless conformal theories will flow toward an infrared (IR) fixed point.

- Lattice calculations place this critical number anywhere between $\frac{N_f}{N_c}$ = 3 - 4.

What can we see perturbatively?



- Two-loop running coupling flows to a perturbative IR fixed-point (IRFP) when $\alpha_* > 0$, the Banks-Zaks fixed point. This is the first non-trivial zero of $\beta(\alpha)$.

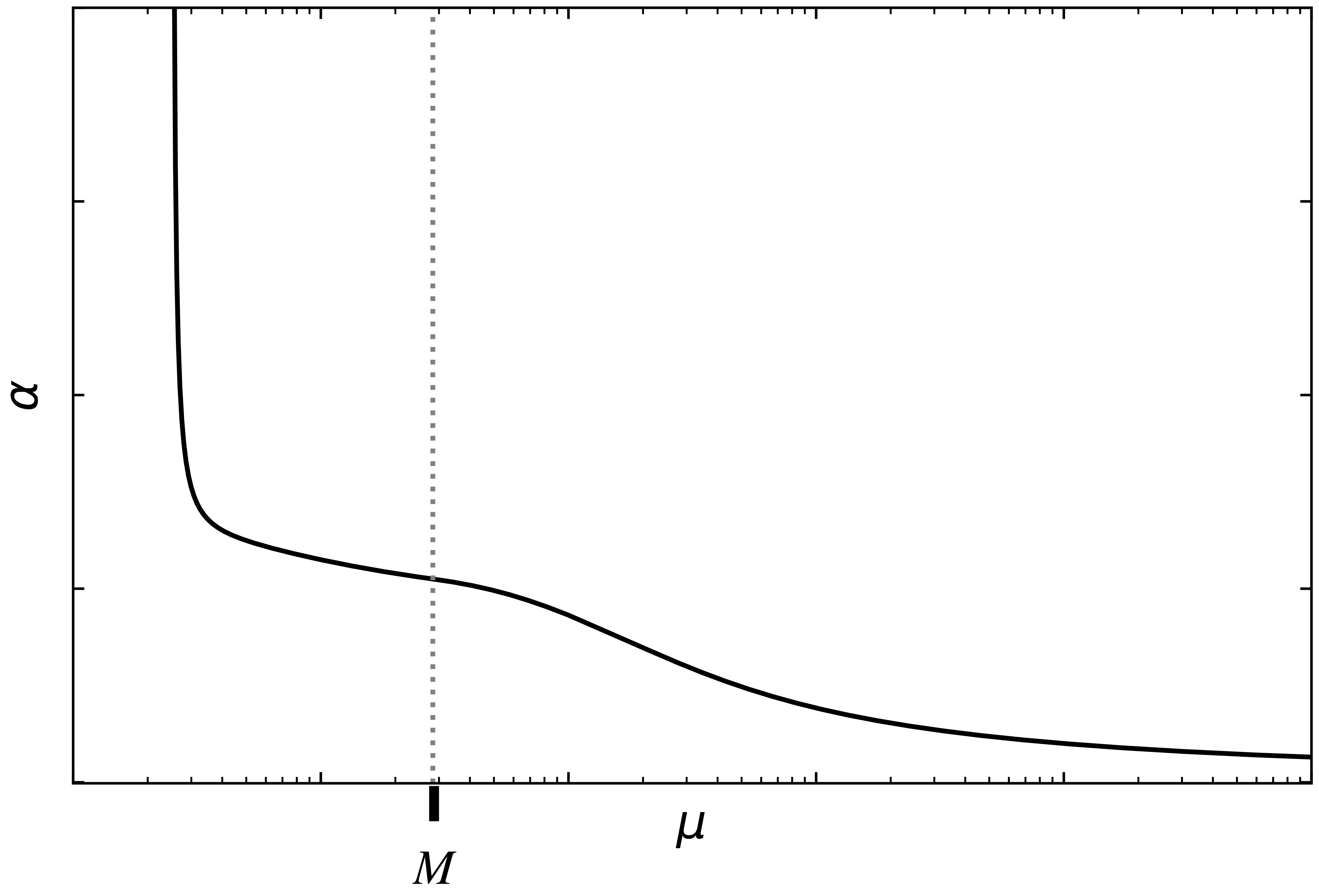
$$\beta(\alpha) = \mu^2 \frac{d\alpha}{d\mu^2} = -\alpha^2 \left(\beta_0 + \beta_1 \alpha\right)$$

- Appearance of two-loop IRFPs at x_f^{FP} provides an approximation of the true IRFPs appearance at x_f^c. Two-loop running coupling with IRFPs provides a perturbative approximation of behaviour near and around the conformal window.

Veneziano limit, $\mu = 0$, $T = 0$

arXiv:2008.12223 - J.W. Lee

Mass-split theories; a useful example



- Consider a dark sector with 4 light dark quarks and 8 heavy dark quarks at some scale M; a mass-split dark sector.
- For $\mu > M$, the number of active flavours is $N_f = 12$ - this would be a conformal theory with the resulting running coupling beginning to approach a fixed-point.
- For $\mu < M$, the number of active flavours is $N_f = 4$ - this would be a QCD-like theory. Running coupling slows down and appears to "walk" over a large range of energies.

- Light quark production within the jet is affected by running of α which experiences contributions from the heavier quarks, will need to account for such contributions if we want to model mass-split theories appropriately in Pythia.

Running coupling - current procedure

- The one-loop running coupling is parameterised by a scale Λ, defined to be the divergence of the running coupling; below this scale the perturbative expansion breaks down.

$$\alpha = \frac{1}{\beta_0 \ln\left(\frac{\mu^2}{\Lambda^2}\right)}$$

W.-M. Yao et al., Review of Particle Physics (2006) , arXiv:0607209 - Prosperi et al.

- At two loops, we obtain an implicit equation from integrating the RGE,

$$\beta_0 \ln\left(\frac{\mu^2}{\mu_0^2}\right) = \left(\frac{1}{\alpha} - \frac{1}{\alpha_0}\right) + \frac{1}{\alpha_*} \ln\left(\frac{1 - \frac{\alpha_*}{\alpha}}{1 - \frac{\alpha_*}{\alpha_0}}\right) \quad ; \qquad \alpha_* = -\frac{\beta_0}{\beta_1}$$

- Define Λ in such a way that absorbs the arbitrary reference scale and coupling, μ_0 and $\alpha\left(\mu_0\right)$. "Two-loop exact" solution solvable through special functions, not true at higher-loop order.

Running coupling - what do we need?

- Choose Λ to be scale of divergence in α, the Landau pole of the theory, where our perturbative expansion breaks down in the infrared (IR).

$$\beta_0 \ln\left(\frac{\Lambda^2}{\mu_0^2}\right) = -\frac{1}{\alpha_0} - \frac{1}{\alpha_*}\ln\left(1 - \frac{\alpha_*}{\alpha_0}\right)$$

- One can substitute this definition back into the RGE and through the expansion of special functions obtain a form for α. Such a definition of Λ and α does not work in the IRFP region, when α_* changes sign.

- One way of avoiding this is assuming the logarithmic terms dominate over the magnitude of the fixed point, allowing for problematic terms to be safely neglected,

$$\beta_0 \ln\left(\frac{\mu^2}{\Lambda^2}\right) > |\alpha_*|$$

arXiv:0607209, Prosperi et al.

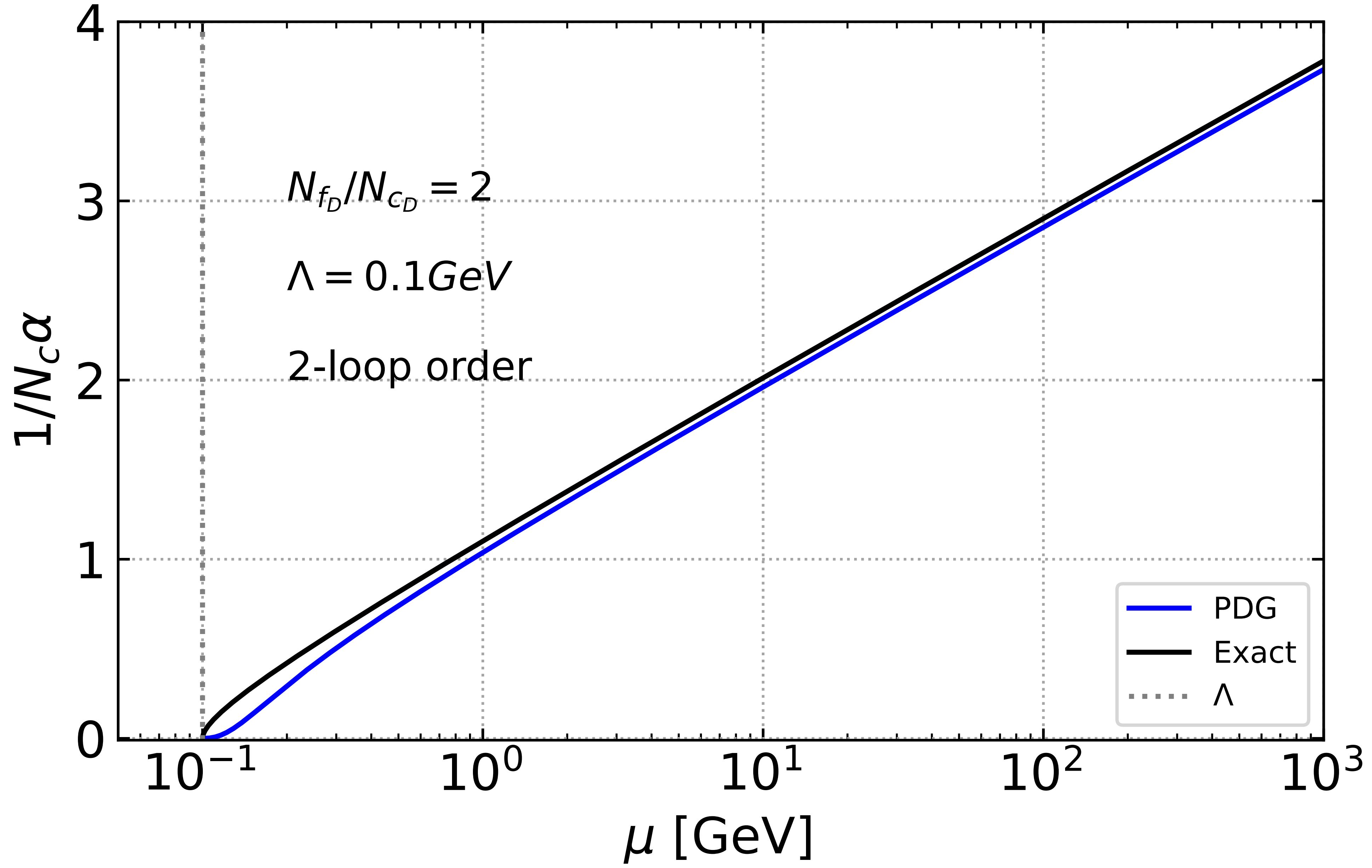
Running coupling - current procedure

- Considering this expansion again, one can derive the two-loop correction to the running coupling currently used by Pythia and the Particle Data Group (PDG),

$$\alpha = \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2)} \left[1 + \frac{1}{\alpha_*} \frac{\ln[\ln(\mu^2/\Lambda^2)]}{\beta_0 \ln(\mu^2/\Lambda^2)} \right]$$

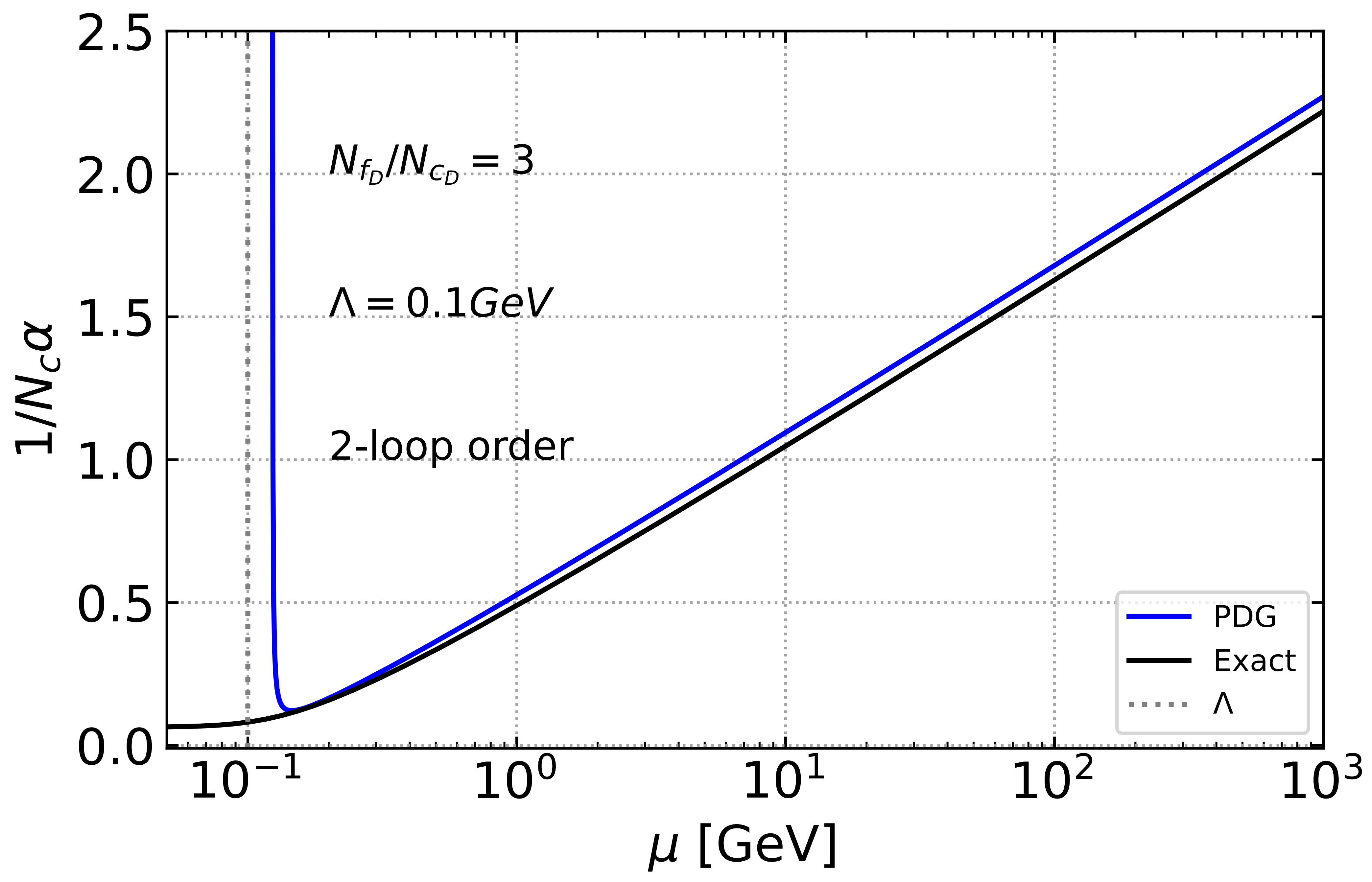
W.-M. Yao et al., Review of Particle Physics (2006) , arXiv:0607209

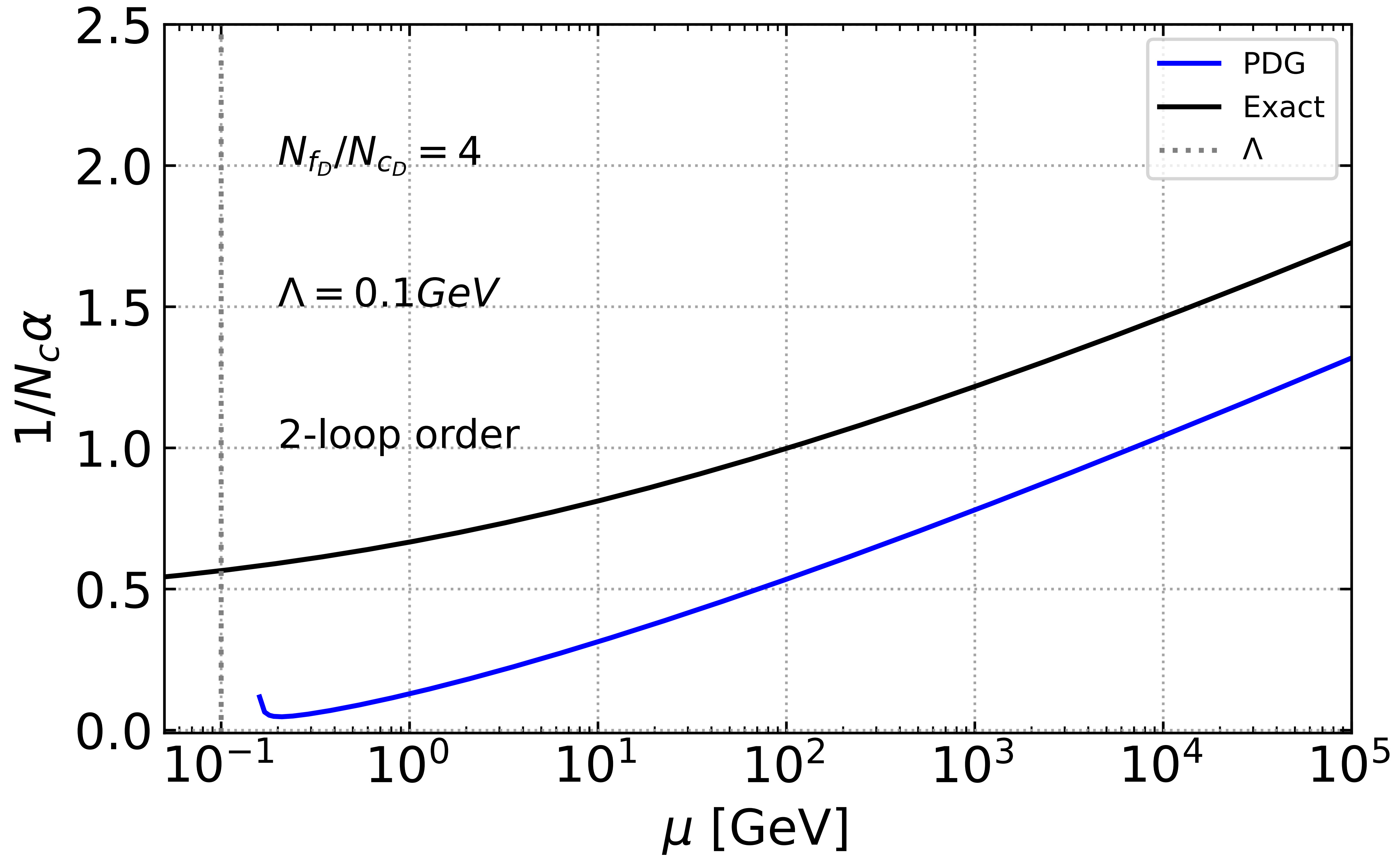
- Does this provide good modelling of showering? The QCD-like region is shown below.



Running coupling - current procedure

- In the IRFP region, PDG approximation can no longer widely be used at large $\frac{N_f}{N_c}$.
- There is unphysical turning behaviour caused by the changing of sign of α_*, significantly affecting MC event generation.
- Both features mean we can not model mass-split theories or accurately account for threshold effects.





Finding a scale in the IRFP region

- In general, the scale Λ describes a cross-over between two regions, below which perturbative expansion is invalid. Unlike the QCD-like region, the low energy behaviour of running in the IRFP region takes on a power-law form,

$$\alpha - \alpha_* \sim \left(\frac{\mu^2}{\mu_0^2}\right)^{\beta_0 \alpha_*}$$

- Then we can define Λ_{FP} as the transition between the asymptotic free $\sim \frac{1}{log}$ and power-law behaviour. The exact scale below which the power-law dominates can be found to be,

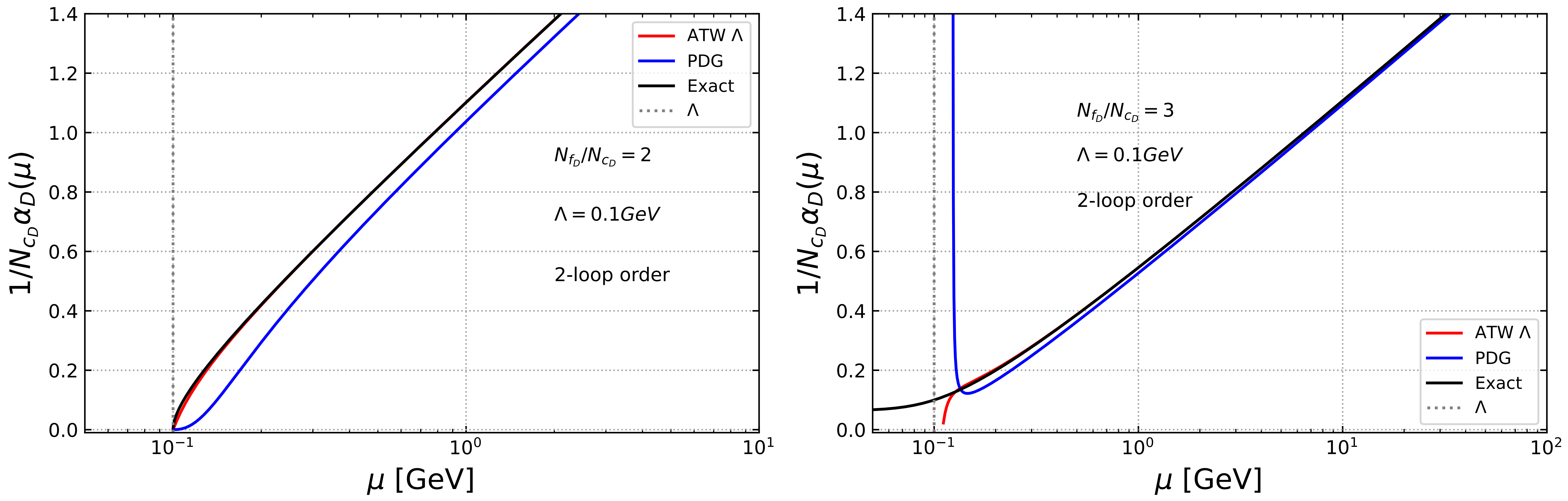
$$\beta_0 \ln\left(\frac{\Lambda_{FP}^2}{\mu_0^2}\right) = -\frac{1}{\alpha_0} - \frac{1}{\alpha_*} \ln\left(\frac{\alpha_*}{\alpha_0} - 1\right)$$

arxiv:9602385,
arxiv:9806409 - T. Appelquist et al.
arxiv:9810192 - E. Gardi et al.

- This can be seen as an analytic continuation of the QCD-like definition of Λ.

Monte Carlo implementation

- Substituting Λ and Λ_{FP} back into the RGE, we obtain two approximate forms of running coupling for both regions, by expanding for large μ. We shall refer to these as the ATW solutions from Appelquist, Terning, Wijewardena; arxiv:9806409

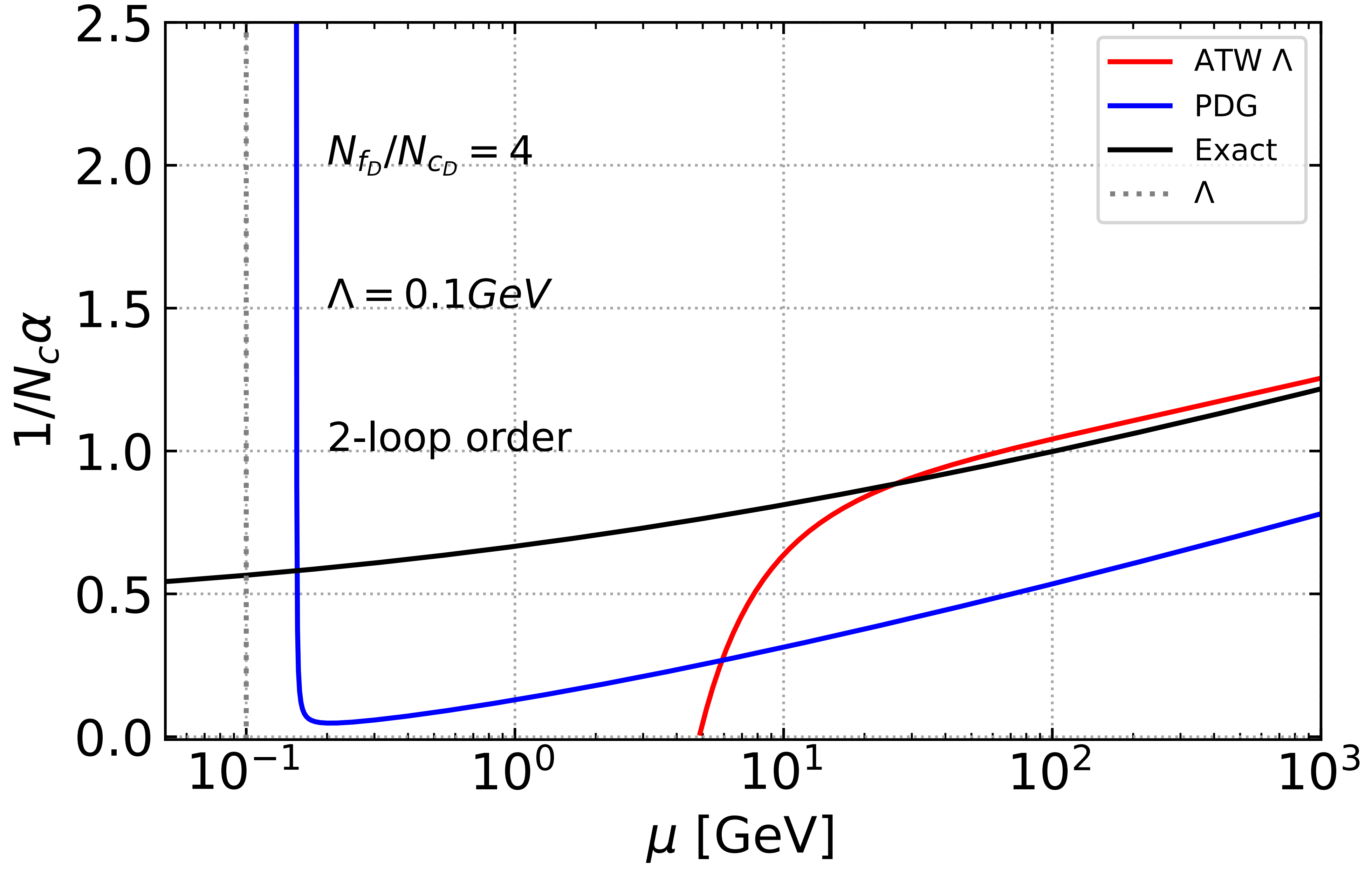


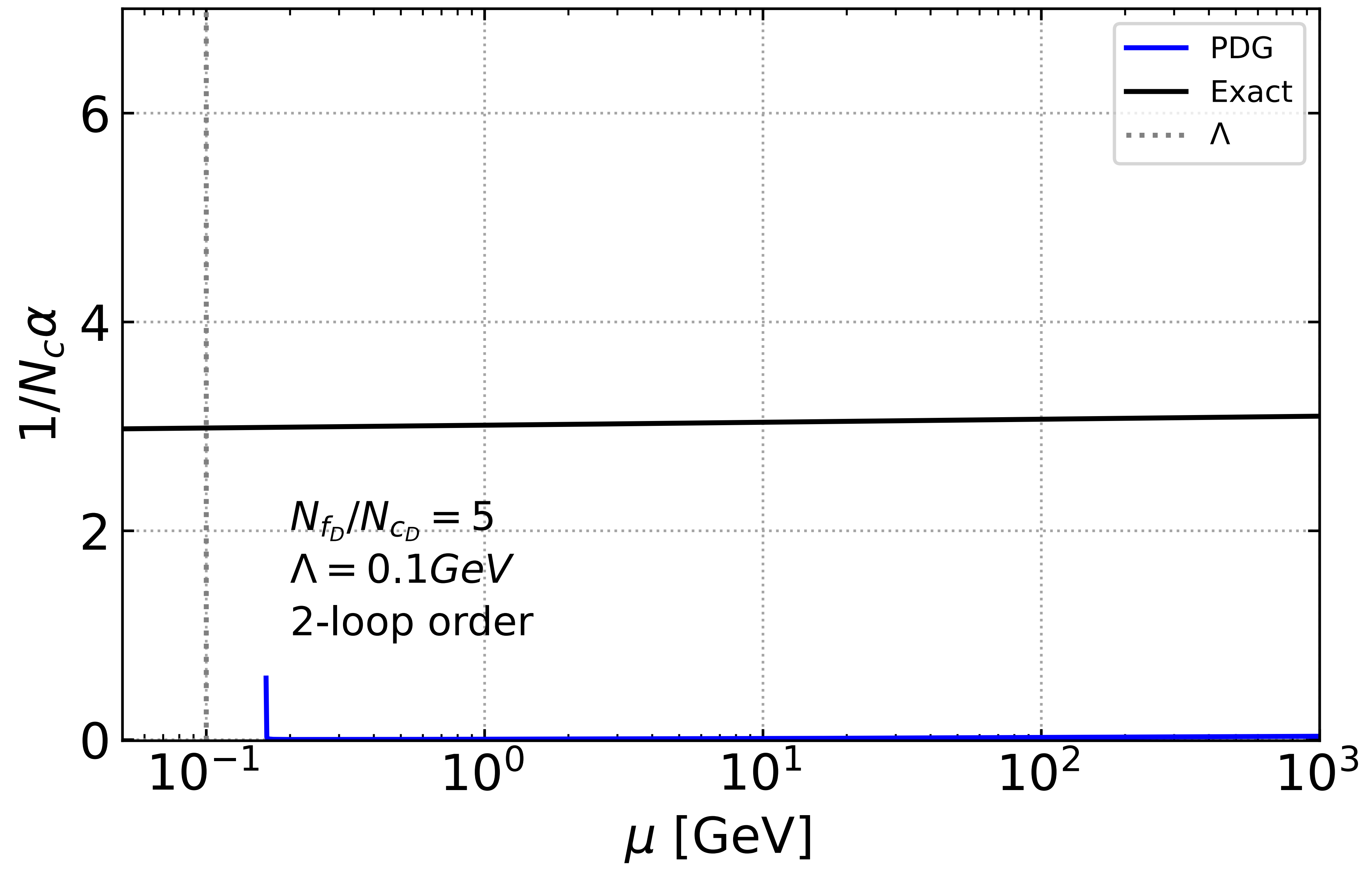
- Excellent match, even in the QCD-like region! No unphysical turning like in the PDG approximation.

- Solutions display 2% difference with shortest distance measurement (compared to PDG approximation's 13%)

Monte Carlo implementation

- The PDG approximation was found to not work in the IRFP region, how far can our approximations go?





- As $\frac{N_f}{N_c}$ is increased, the energy range over which the ATW approximation proves to be reliable begins to decrease.
- In this region, there are not many cases in which the ATW approximation, even when expanding to higher orders, proves to be reliable. To get consistently reliable results an entirely new method is needed.

What's next?

- A more complete investigation of the expansion parameters involved in the ATW and PDG approximations.

- Banks-Zaks expansion; are we applying the ATW approximation within areas they are not valid because of the small size of α_* at high $\frac{N_f}{N_c}$?

- Implementation in Pythia (and other Monte Carlo generators). Simulations of dark showers for a variety of mass-split theories.

- Three-loop investigations. Difficulties arise from scheme-dependence; fixed-points appear at lower N_f in some schemes. Power-law and subsequent definition of Λ not so easy to see.

Conclusion

- Current implementation of confining Hidden Valley theories is not sufficient nor accurate enough to simulate mass-split theories and account for its threshold effects.

- To properly define a scale within both QCD-like and IRFP regions requires two different definitions - Λ and Λ_{FP}. Importantly both scales describe the cross-over between differing scaling behaviours of the theory.

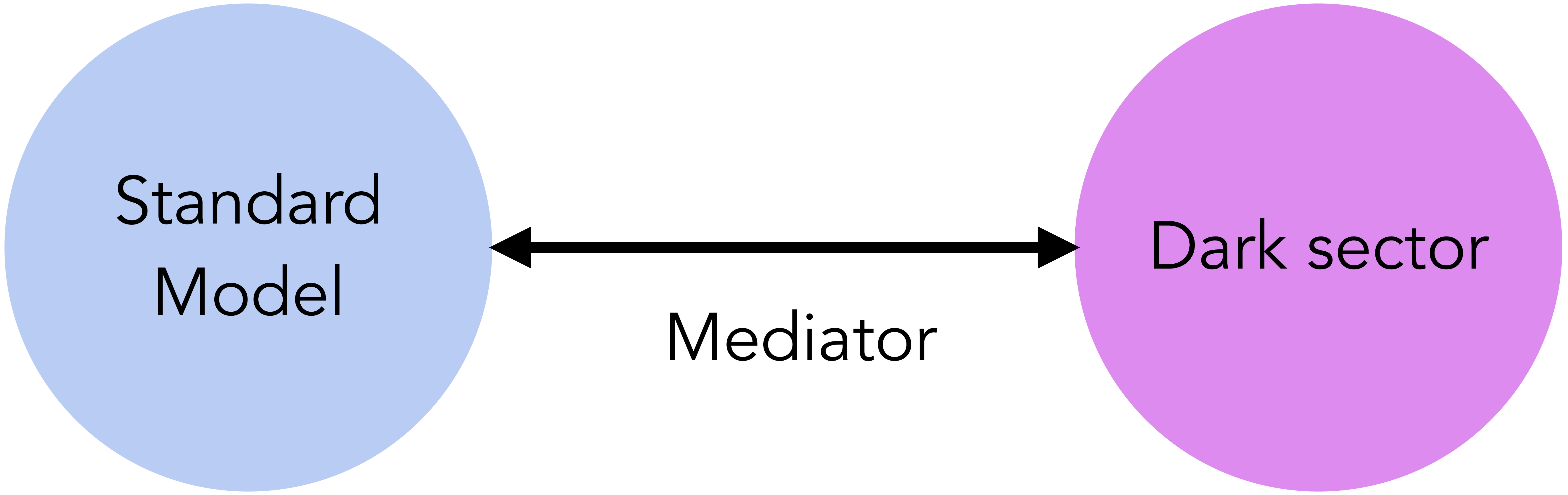
- One finds that the resulting approximation, the ATW approximation, has a wide range of applicability for simulating mass-split theories but a reduced range in theories with $\dfrac{N_f}{N_c} \gtrsim 4$, where new methods clearly must be used.

Thank you! Any questions?

Back-up

The dark sector

- Analogously to QCD, the dark "quarks" confine, producing bound states such as dark "pions". These can be light and long-lived; dark matter candidates?



arxiv:0604261
arxiv:0712.2041
arxiv:0806.2385

- A Hidden Valley; current constraints do not rule out DS relic density mechanisms and many areas of parameter space. arXiv:2012.01875

- Unique signatures; abundant phenomenology leads to novel signatures, e.g. dark-showers

Beta functions

- Beta coefficients for arbitrary gauge group and representation.

$$4\pi\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F N_f$$

$$(4\pi)^2\,\beta_1 = \frac{34}{3}C_A^2 - 4C_F T_F N_f - \frac{20}{3}C_A T_F N_f$$

$$(4\pi)^3\,\beta_2 = \frac{2857}{54}C_A^3 + 2C_F^2 T_F N_f - \frac{205}{9}C_F C_A T_F N_f - \frac{1415}{27}C_A^2 T_F N_f + \frac{44}{9}C_F T_F^2 N_f^2 + \frac{158}{27}C_A T_F^2 N_f^2$$

- $SU\left(N_c\right)$ with N_f fundamental fermions.

$$4\pi\beta_0 = \frac{11}{3}N_c - \frac{2}{3}N_f$$

$$(4\pi)^2\,\beta_1 = \frac{34}{3}N_c^2 - 2\frac{N_c^2 - 1}{2N_c}N_f - \frac{10}{3}N_c N_f$$

Two scales, two regions

- When $\beta_1 = 0$, the β function reduces to its one-loop variety and thus the scale Λ must reduce to its one-loop equivalent.

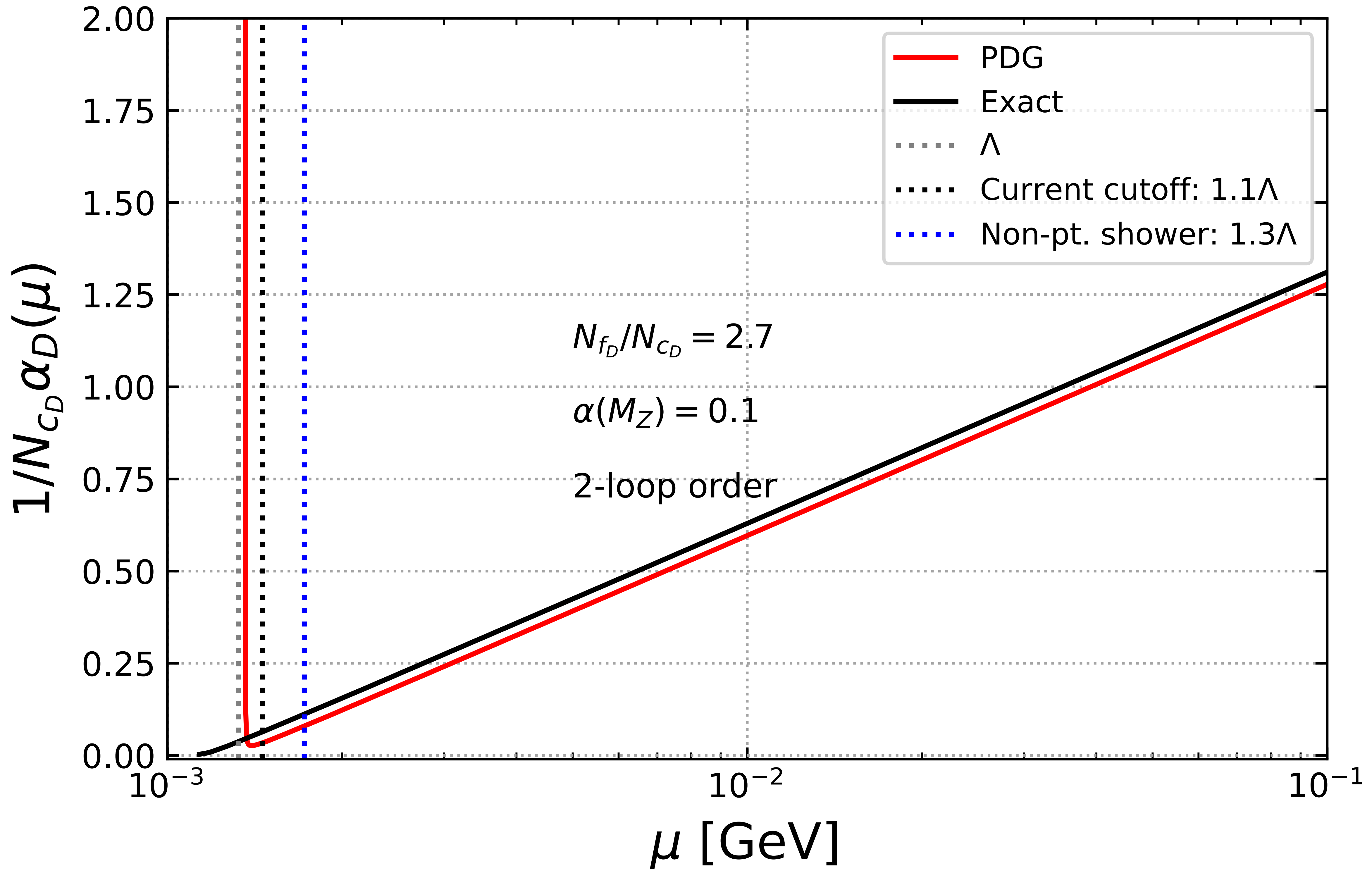
$$\Lambda\Big|_{\beta_1=0} = \Lambda_{1-loop}$$

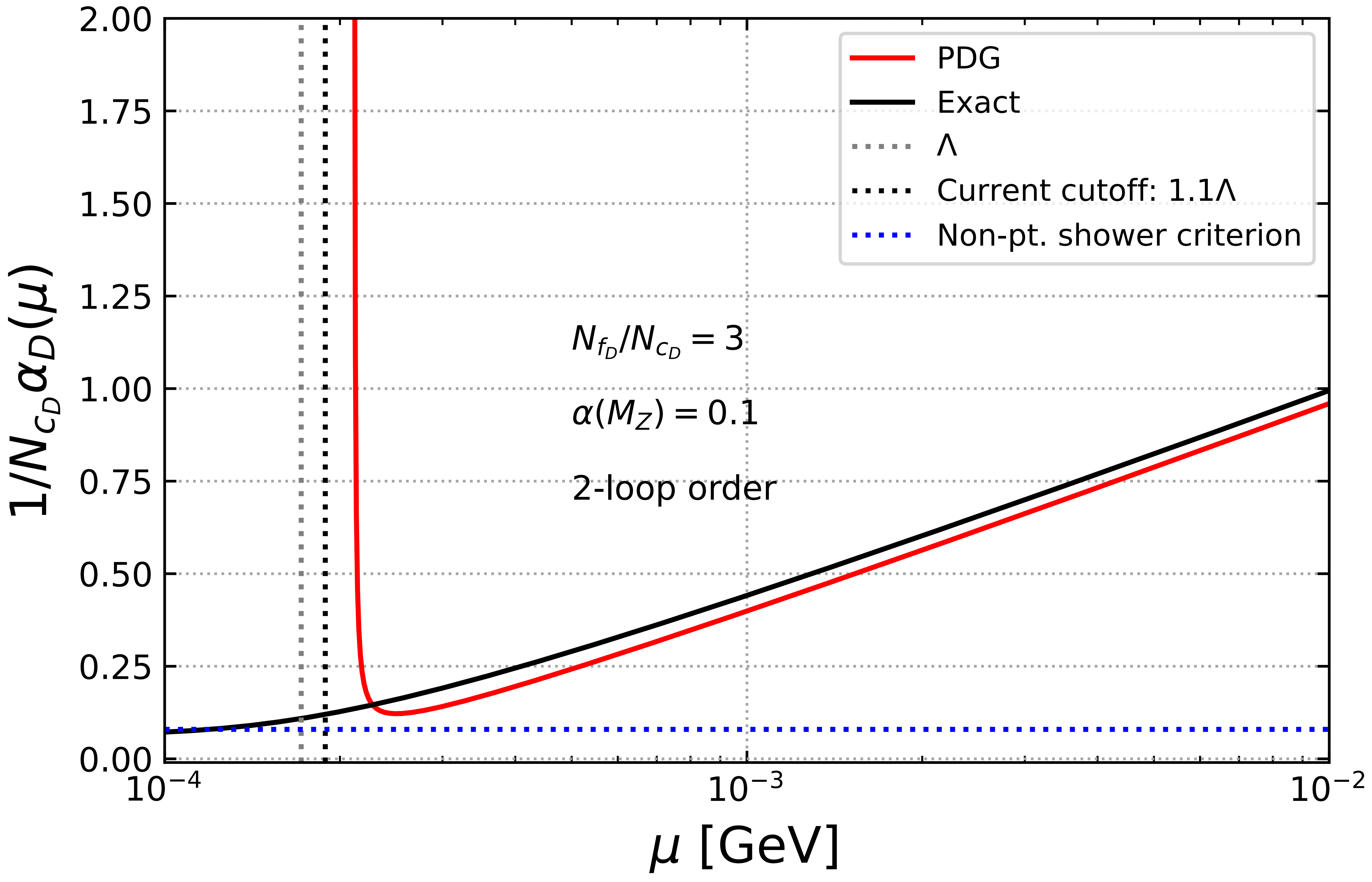
- In IRFP region, close to fixed-point, running coupling scales with a power-law relation, indicating that Λ within this region parameterises the transition to IRFP power-law behaviour.

$$\alpha - \alpha_* \sim \left(\frac{\mu^2}{\Lambda^2}\right)^{\beta_0 \alpha_*}$$

- Taking this into account, we found that a single $\hat{\alpha}$ can not simultaneously satisfy being close to QCD-like scale of divergence and the above criteria without becoming unphysical. Both regions have their own associated scales that characterise the respective behaviour within each region.

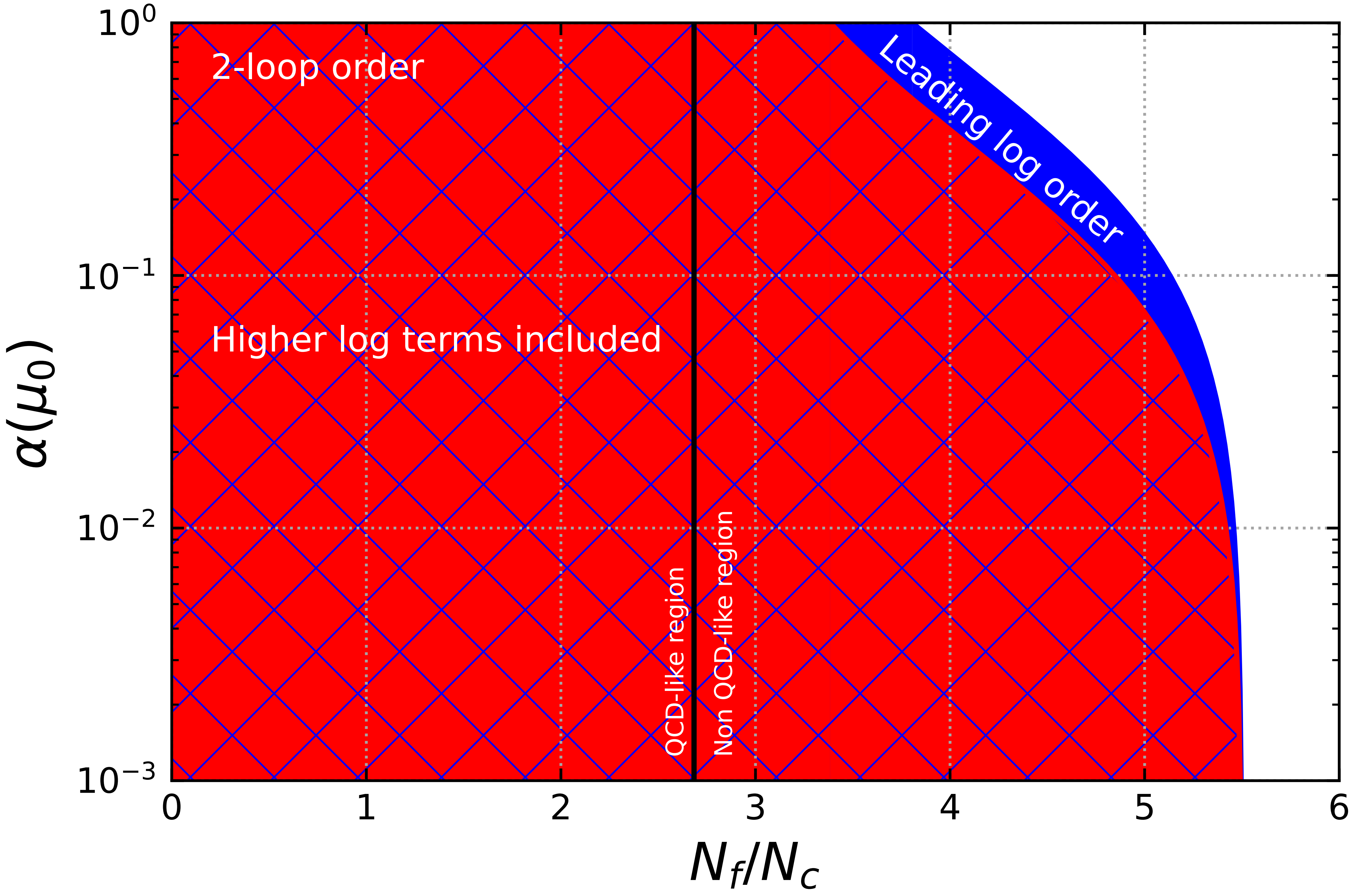
PDG for fixed reference scale





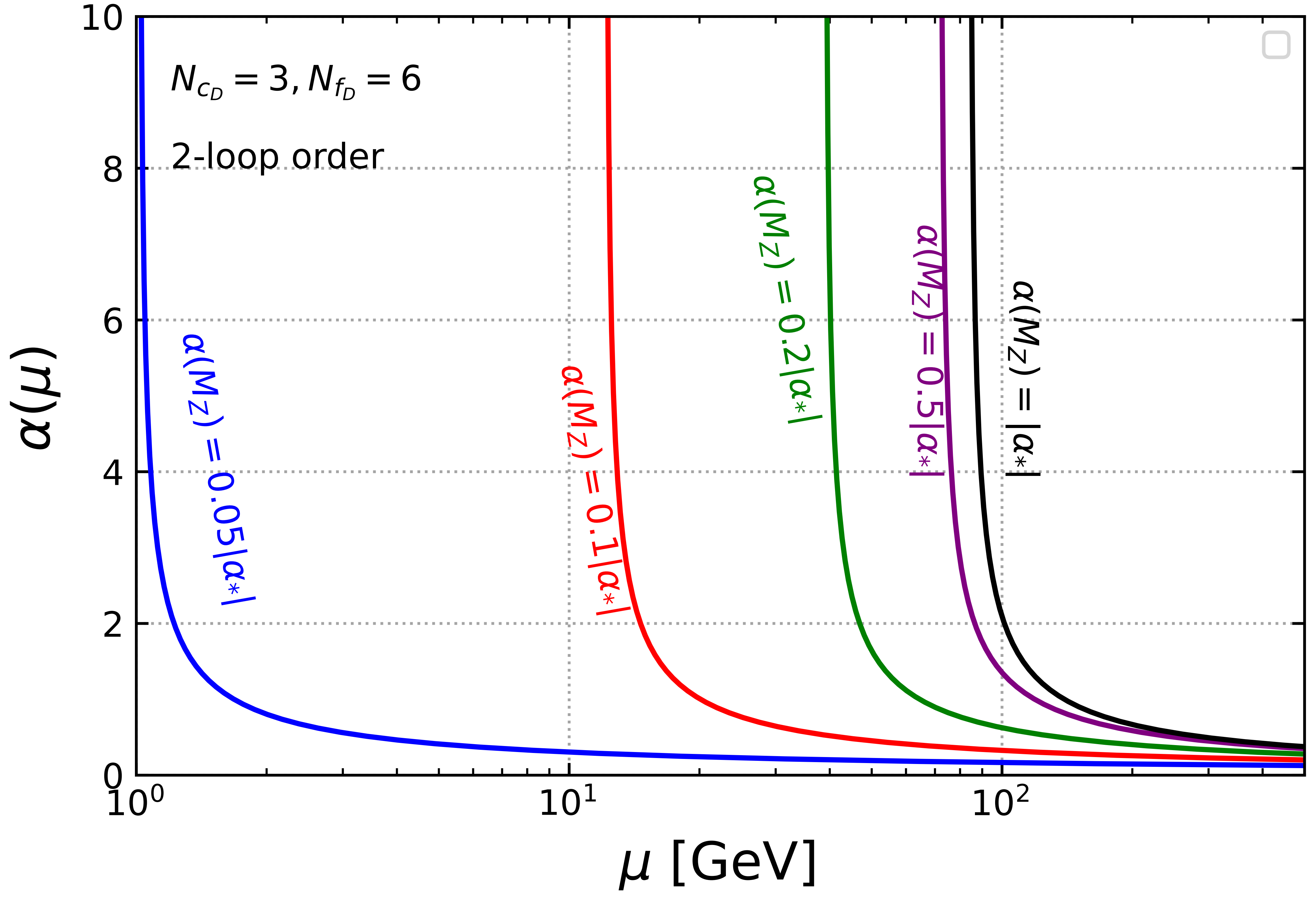
Choosing reference scales

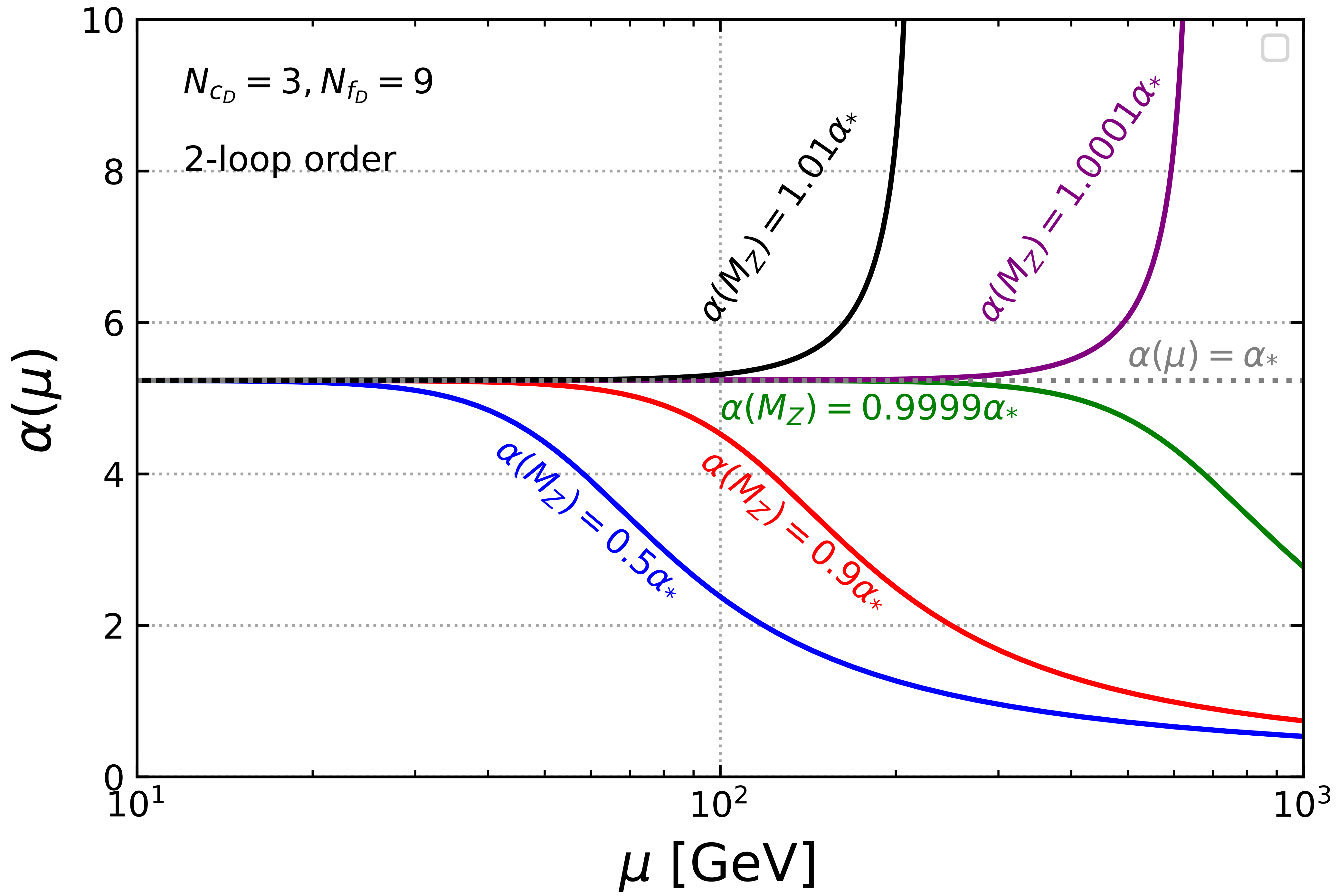
- Allowed values of $\alpha\left(\mu_0\right)$ for a given $\dfrac{N_f}{N_c}$.



Choosing reference scales

- Behaviour of running couplings in both regions for a variety of $\alpha(\mu_0)$.





Point of power-law domination

- Power-law dominates when $\alpha_{true} = 2\alpha_{power-law}$, where,

$$\alpha = \frac{\alpha_*}{1 + \frac{1}{e}\left(\frac{\mu^2}{\Lambda^2}\right)^{\beta_0 \alpha_*}}$$

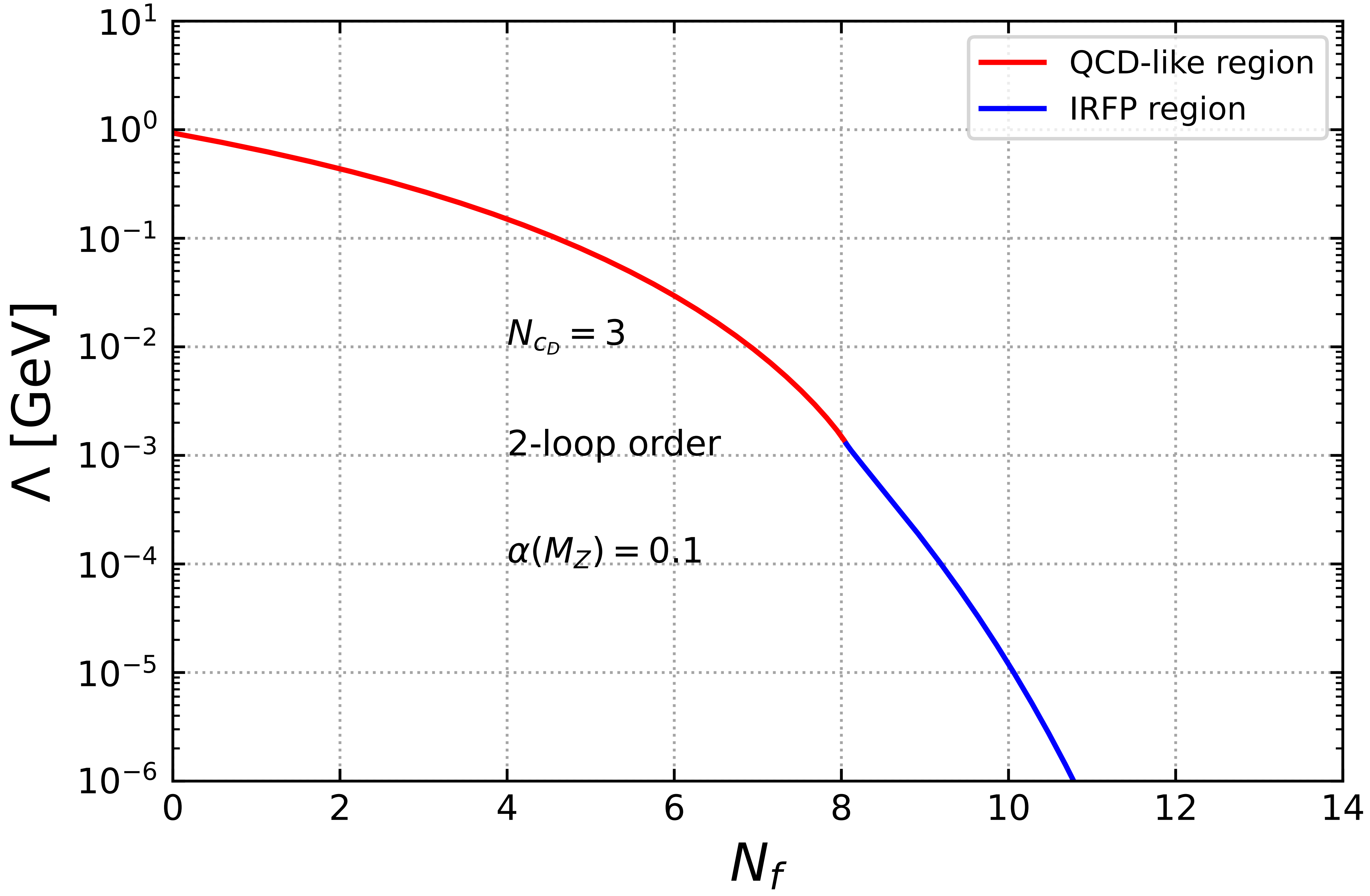
- Substituting this into the integral of the RGE and expanding the RHS, we can implictly solve this equation.

$$\beta_0 \ln\left(\frac{\Lambda^2}{\mu^2}\right) = -\frac{1}{\alpha} - \frac{1}{\alpha_*}\ln\left(\frac{\alpha_*}{\alpha} - 1\right)$$

- This yields solutions of $\hat{\alpha} = 0.78\alpha_*$ and $\mu_{power-law-domination} = \Lambda$.

From one region to another

- Importantly one can show that there is continuity between the two definitions; $\Lambda_{QCD-like}\Big|_{\beta_1=0} = \Lambda_{IRFP}\Big|_{\beta_1=0}$. One should expect this as the $\beta(\alpha)$ reduces to its one-loop form when $\beta_1 = 0$.



Monte Carlo implementation

- Can substitute the definitions of Λ and Λ_{FP} back into the RGE, obtaining the following implicit equations for the running coupling.

$$\beta_0 \ln\left(\frac{\mu^2}{\Lambda^2}\right) = \frac{1}{\alpha} + \frac{1}{\alpha_*}\ln\left(1 - \frac{\alpha_*}{\alpha}\right) \quad ; \quad \beta_0 \ln\left(\frac{\mu^2}{\Lambda_{FP}^2}\right) = \frac{1}{\alpha} + \frac{1}{\alpha_*}\ln\left(\frac{\alpha_*}{\alpha} - 1\right)$$

- These can be arranged in the explicit form in terms of the two real branches of the Lambert W function,

$$\alpha = \alpha_* \left[W_{-1}\left(-\frac{1}{e}\left(\frac{\mu^2}{\Lambda^2}\right)^{\beta_0\alpha_*}\right) + 1\right]^{-1} \quad ; \quad \alpha = \alpha_* \left[W_{0}\left(\frac{1}{e}\left(\frac{\mu^2}{\Lambda_{FP}^2}\right)^{\beta_0\alpha_*}\right) + 1\right]^{-1}$$

- For large μ, we can use the expansion of,

$$W(x) = L_1 - L_2 + \frac{L_2}{L_1} + \mathcal{O}\left(\left[\frac{L_2}{L_1}\right]^2\right)$$

- Where $L_1 = \ln(x)$, $L_2 = \ln\left(\ln(x)\right)$ for $W_0(x)$ and $L_1 = \ln(-x)$, $L_2 = \ln\left(-\ln(-x)\right)$ for $W_{-1}(x)$,

Monte Carlo implementation

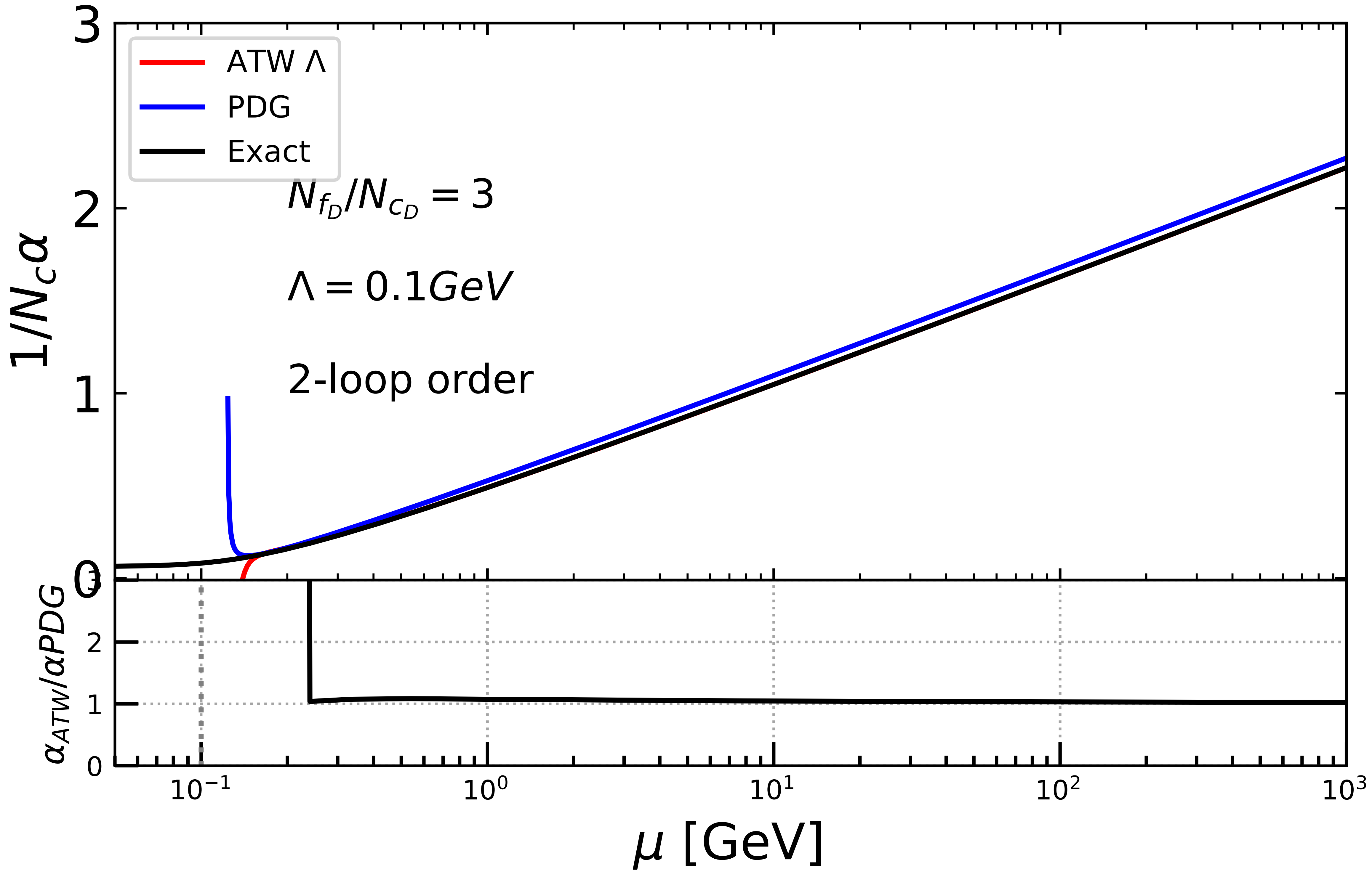
- Gives following form of ATW in the QCD-like region of,

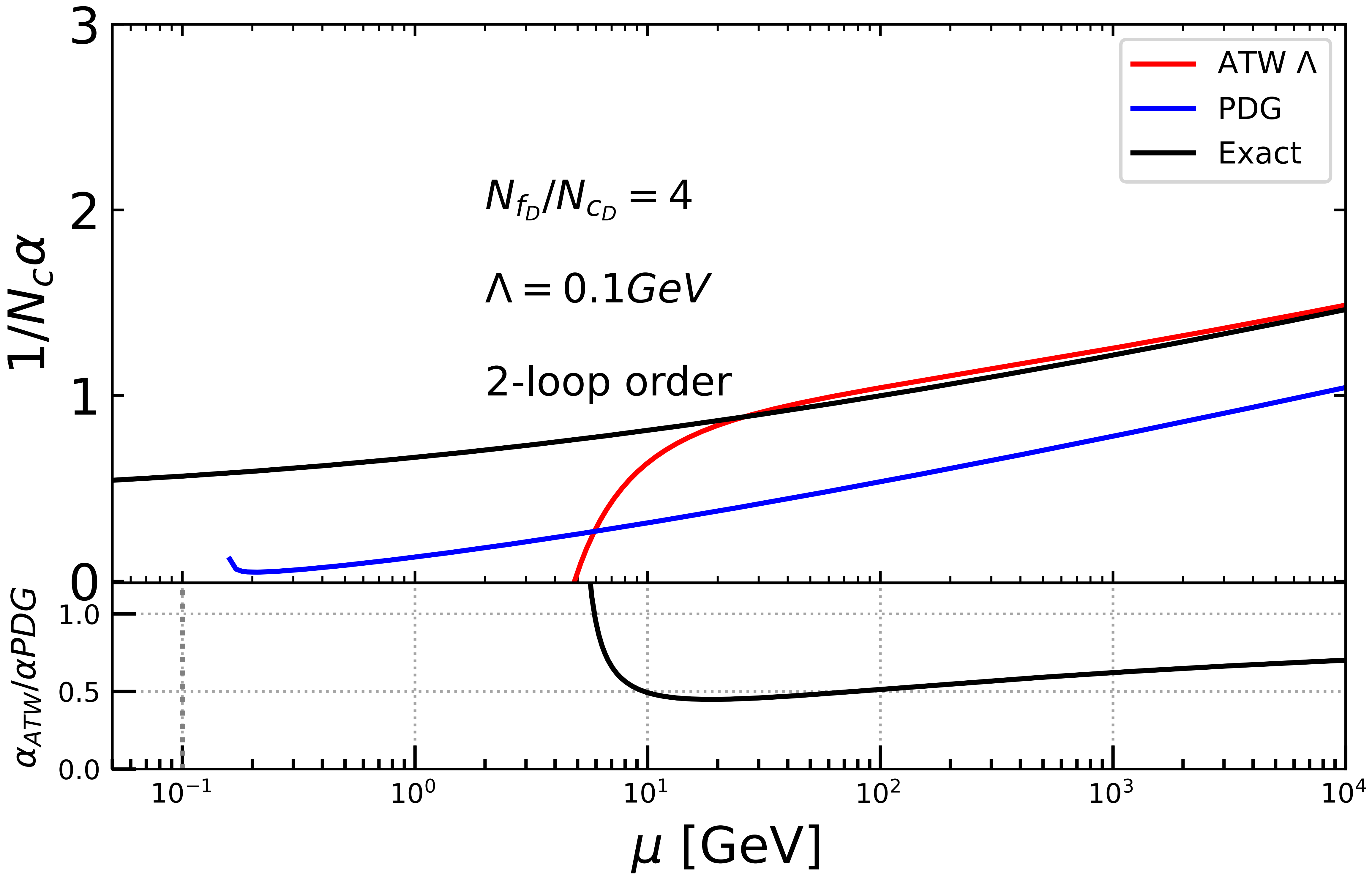
$$\frac{1}{\alpha} = \beta_0 \ln\left(\frac{\mu^2}{\Lambda^2}\right) - \frac{1}{\alpha_*}\ln\left(1 - \beta_0\alpha_* \ln\left(\frac{\mu^2}{\Lambda^2}\right)\right) + \frac{1}{\alpha_*}\frac{\ln\left(1 - \beta_0\alpha_* \ln\left(\frac{\mu^2}{\Lambda^2}\right)\right)}{\beta_0\alpha_* \ln\left(\frac{\mu^2}{\Lambda^2}\right) - 1}$$

- And in the IRFP region of,

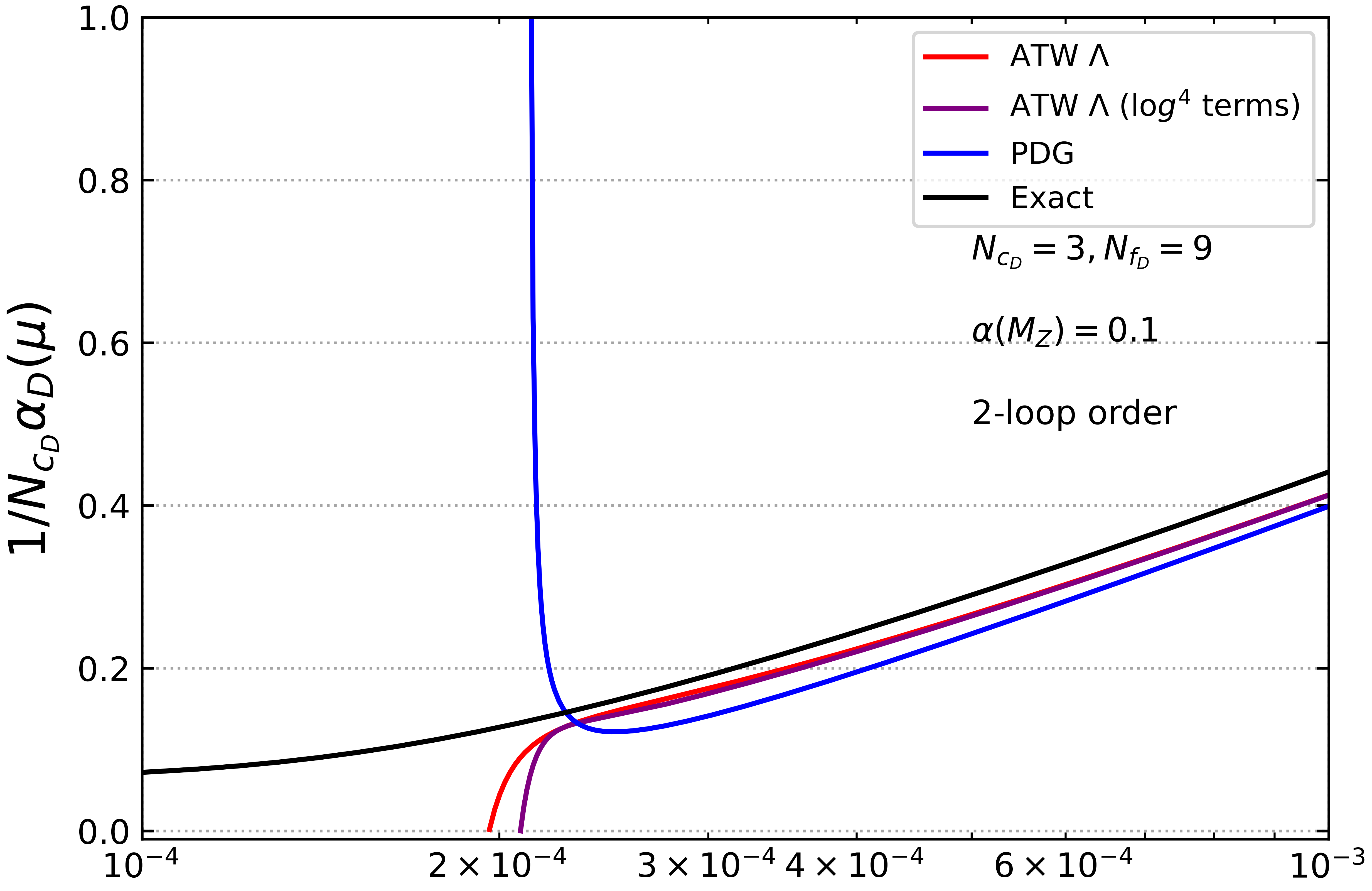
$$\frac{1}{\alpha} = \beta_0 \ln\left(\frac{\mu^2}{\Lambda^2}\right) - \frac{1}{\alpha_*}\ln\left(\beta_0\alpha_* \ln\left(\frac{\mu^2}{\Lambda^2}\right) - 1\right) + \frac{1}{\alpha_*}\frac{\ln\left(\beta_0\alpha_* \ln\left(\frac{\mu^2}{\Lambda^2}\right) - 1\right)}{\beta_0\alpha_* \ln\left(\frac{\mu^2}{\Lambda^2}\right) - 1}$$

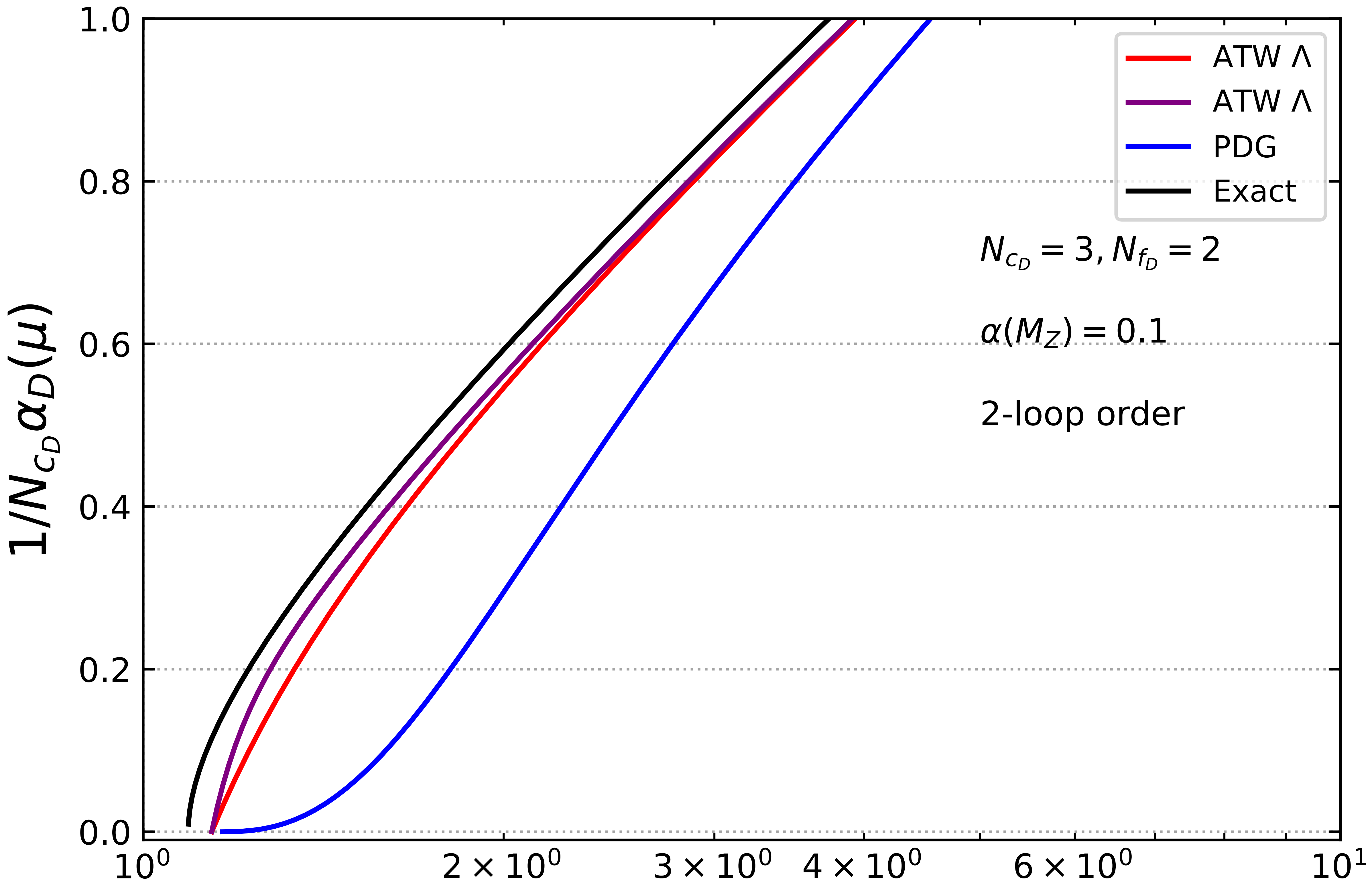
High Nf/Nc investigations - ratio plots



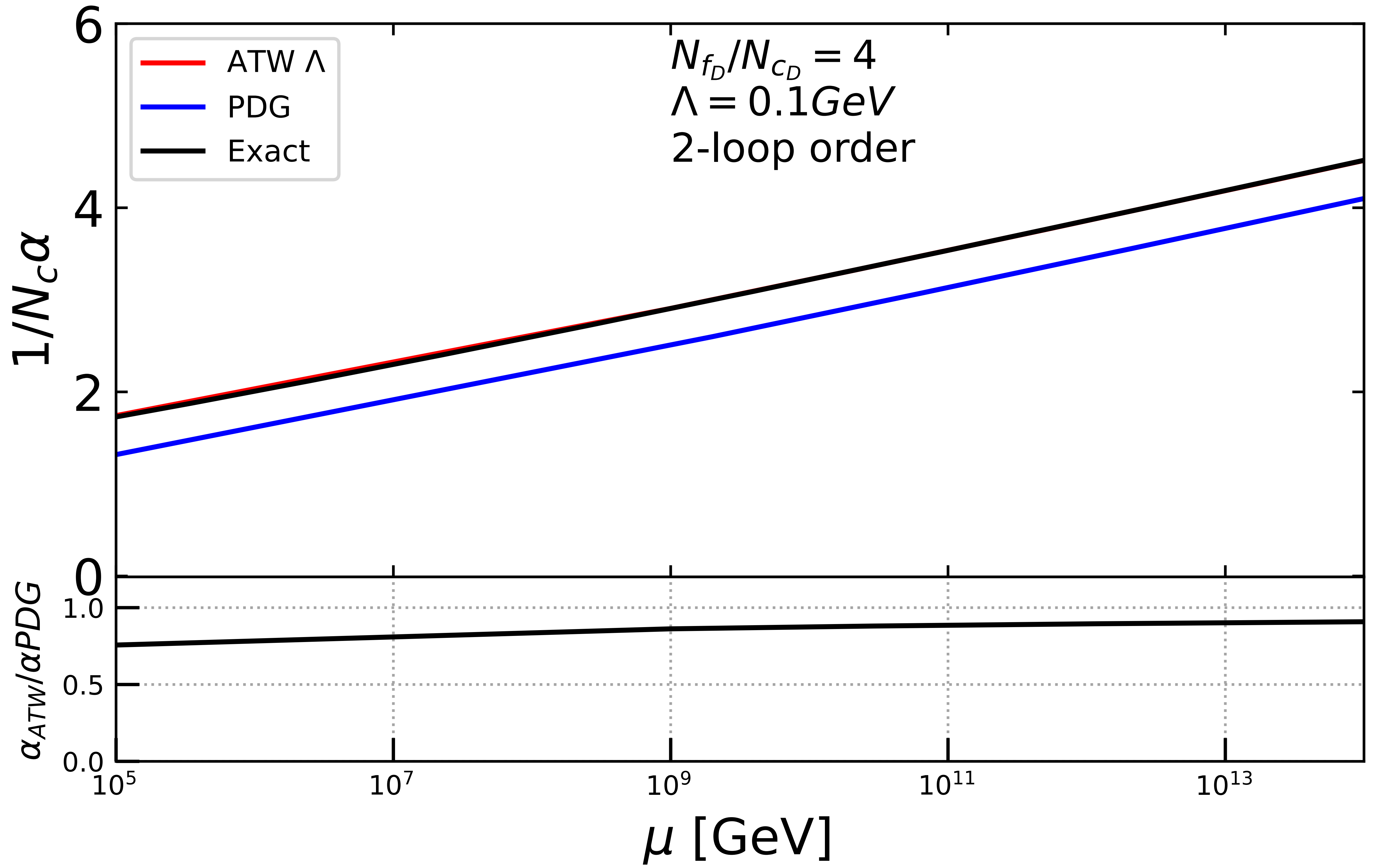


ATW - Higher order expansion





High Nf investigations - higher scales



- Extending our investigation to higher energy scales; PDG approximation never approaches ATW approximation.