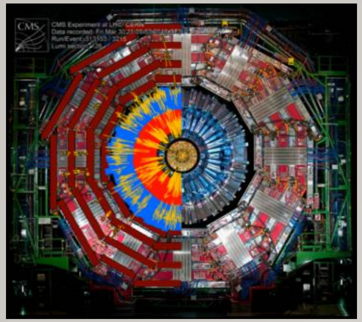

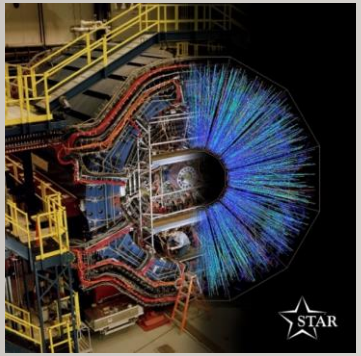

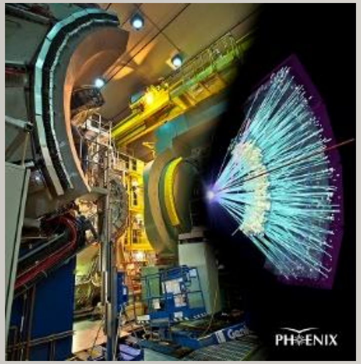

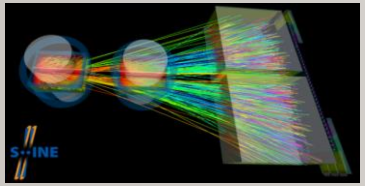

FEMTOSCOPY WITH LÉVY DISTRIBUTIONS FROM SPS TO LHC

MÁTÉ CSANÁD (EÖTVÖS UNIVERSITY)

AUSTRIA-CROATIA-HUNGARY TRIANGLE (ACHT) 2023 WORKSHOP, SCHLOSS RETZHOF

DER RETZHOF

CONTENTS OF THIS TALK

- Basics of femtoscopy and Lévy sources
- First thorough Lévy HBT analysis in AA by PHENIX
- Recent phenomenological updates
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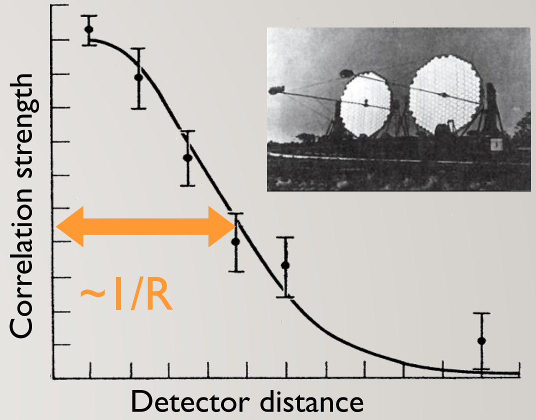
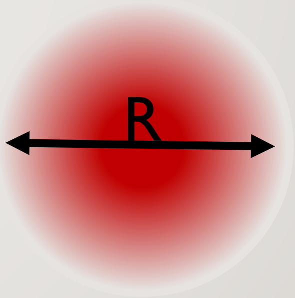
FEMTOSCOPY IN HIGH ENERGY PHYSICS

- R. Hanbury Brown, R. Q. Twiss - observing Sirius with radio telescopes
  - Intensity correlations vs detector distance ⇒ source size
  - Measure the sizes of apparently point-like sources!
- Goldhaber et al: applicable in high energy physics
- Understanding: Glauber, Fano, Baym, …
  Phys. Rev. Lett. 10, 84; Rev. Mod. Phys. 78 1267, …
  - Momentum correlation $C(q)$ related to source $S(r)$

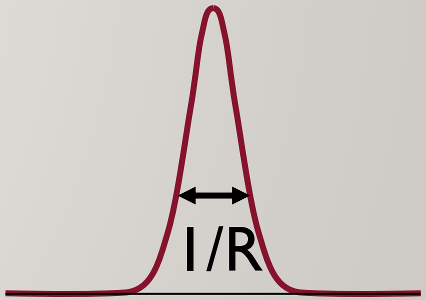
    $C(q) \cong 1 + \left| \int S(r) e^{iqr} dr \right|^2$
    (under some assumptions)
  - Also with distance distribution $D(r)$:

    $C(q) \cong 1 + \int D(r) e^{iqr} dr$
  - Neglected: pair reconstruction, final state interactions, multi-particle correlations, coherence, …
- What is the source shape? Can be explored via femtoscopy





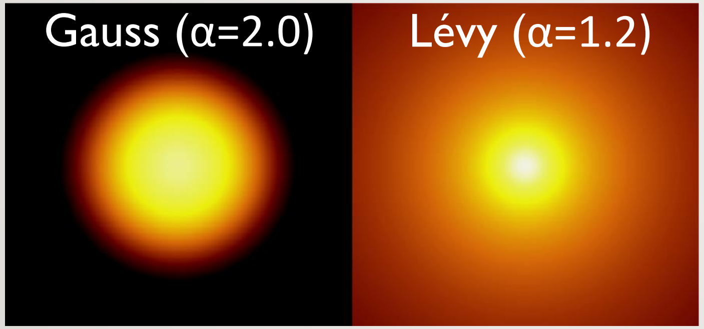
source function S(r)



correlation funct. C(q)

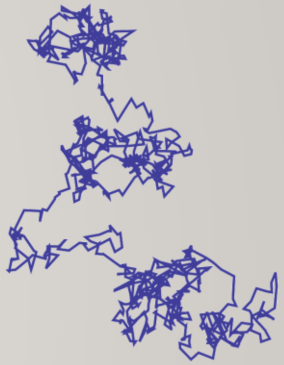
LÉVY DISTRIBUTIONS IN HEAVY ION PHYSICS

- Central limit theorem (diffusion) and thermodynamics lead to Gaussians
- Measurements suggest phenomena beyond Gaussian distribution
- Lévy-stable distribution: $\mathcal{L}(\alpha, R; r) = (2\pi)^{-3} \int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$
  - From generalized central limit theorem, power-law tail ~ $r^{-(1+\alpha)}$
  - Special cases: α = 2 Gaussian, α = 1 Cauchy

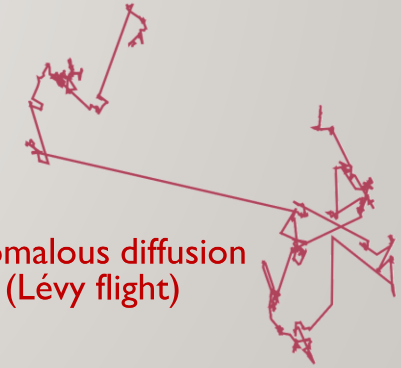


- Shape of the correlation functions with Lévy source:
  - $C_2(q) = 1 + \lambda \cdot e^{-|qR|^\alpha}$; $\alpha = 2$: Gaussian; $\alpha = 1$: exponential
    Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67-78
- A possible reason for Lévy source: anomalous diffusion, many others

Normal diffusion

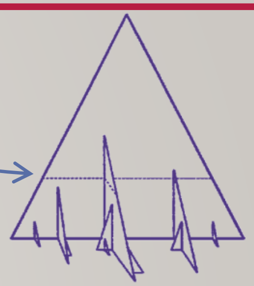


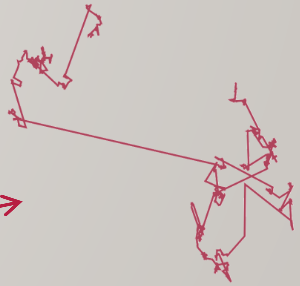
Anomalous diffusion (Lévy flight)



WHY DO LÉVY SHAPES APPEAR, WHY IS IT IMPORTANT?

- A more comprehensive list of possible reasons:
  - Jet fragmentation (Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon. B36 (2005) 329-337)
    - See also talk by Yacine Mehtar-Tani at ExploreQGP workshop in Belgrade
  - Critical phenomena (Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828 (2006) no.1, 525-532)
  - Direction averaging and non-sphericality (Cimerman et al., Phys.Part.Nucl. 51 (2020) 282)
  - Event averaging (Cimerman et al., Phys.Part.Nucl. 51 (2020) 282)
  - Resonance decays (Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002;
    Kincses, Stefaniak, Csanád, Entropy 24 (2022) 308)
  - Hadronic rescattering, Lévy flight (Braz.J.Phys. 37 (2007) 1002; Entropy 24 (2022) 308)

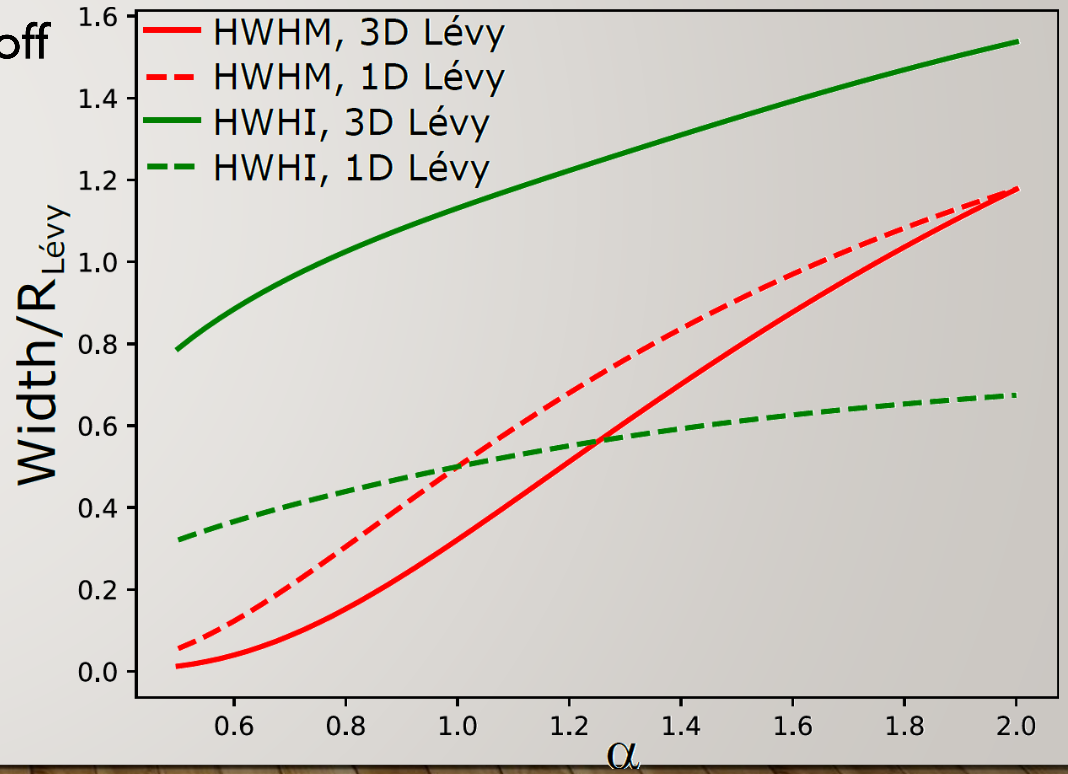




- Importance of utilizing Lévy sources:
  - Measuring α and R
    - Order of quark-hadron transition, critical point search, understanding source dynamics
  - Measuring λ also requires correct shape assumption
    - In-medium mass modification, coherent pion production

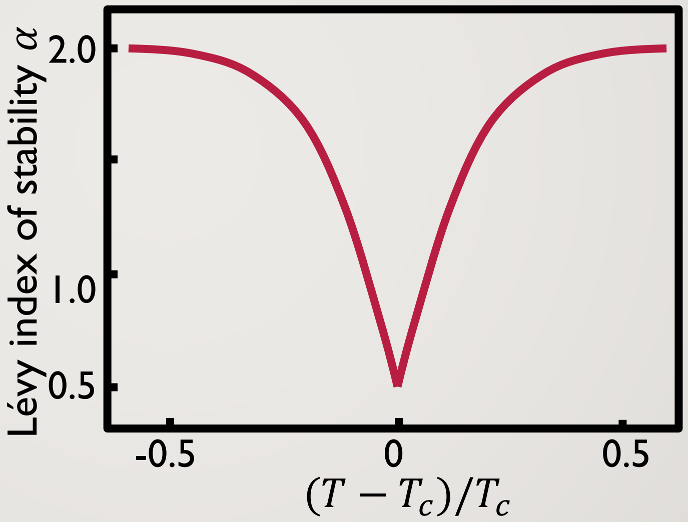
WHAT IS THE TRUE SIZE OF THE SOURCE?

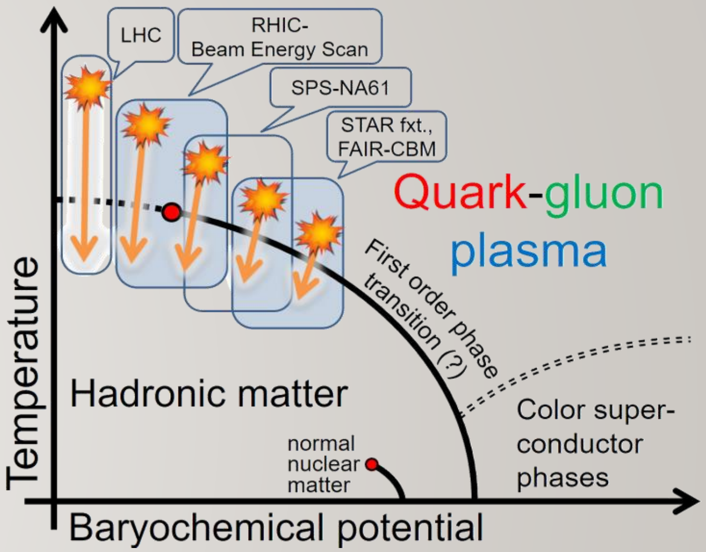
- No tail if $\alpha = 2$, power law if $\alpha < 2$; tail depends on α
- If $S(r)$ Lévy, $D(r)$ Lévy with same α and $R \to 2^{1/\alpha} R$
- In principle, RMS $= \infty$ if $\alpha < 2$, practice: depends on cutoff
- What do Gaussian HBT radii mean?
- Alternative measures:
  - HWHI: (half) width at half integral
  - HWHM: (half) width at half max
  - Large difference between 1D and 3D relative width
  - Width (normalized by R) nontrivially depends on α
  - If $\alpha = 2$ or $\alpha = 1$ assumed: deviation from true scale



LÉVY INDEX AS A CRITICAL EXPONENT?

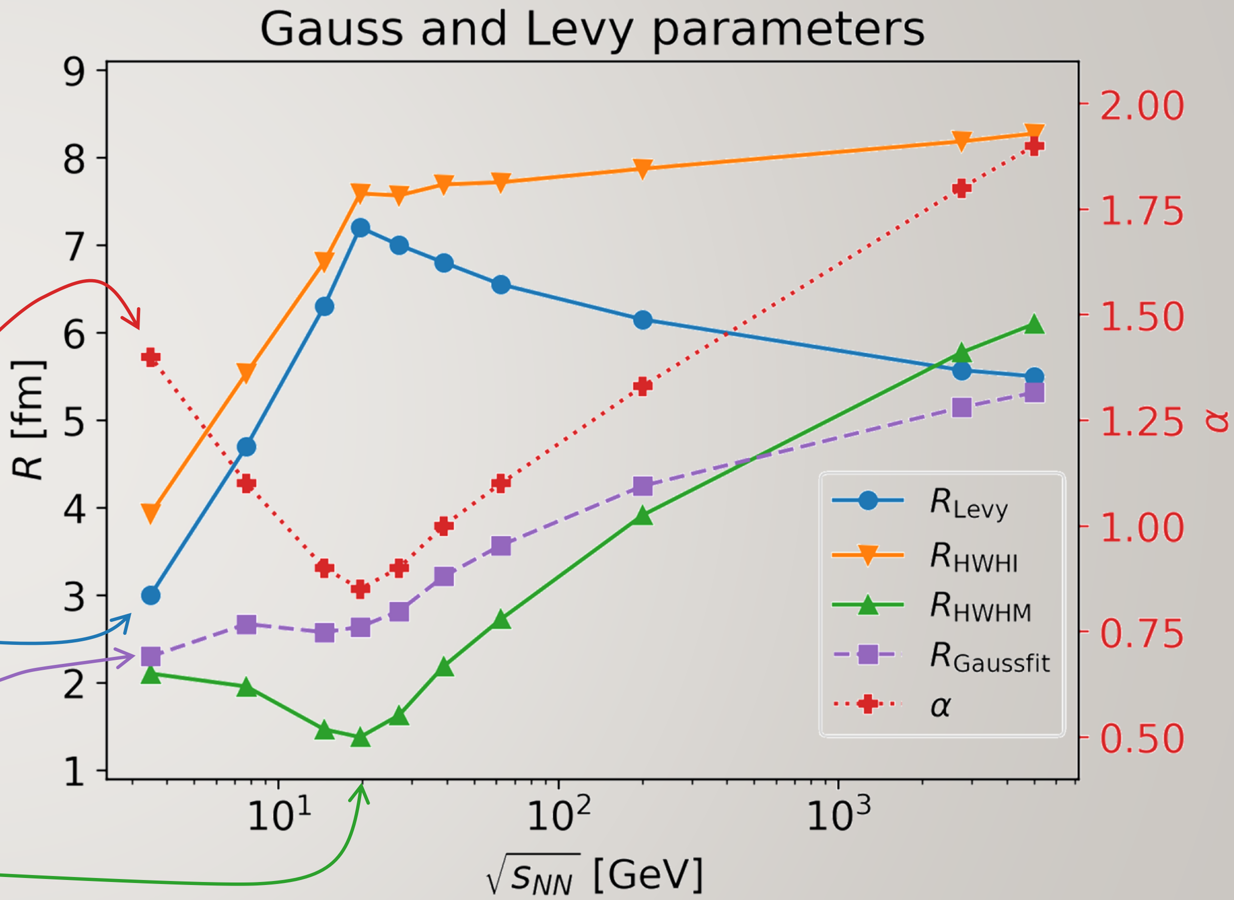
- Critical spatial correlation: $\sim r^{-(d-2+\eta)}$; Lévy source: $\sim r^{-(1+\alpha)}$; $\alpha \Leftrightarrow \eta$?
  Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67
- QCD universality class ↔ 3D Ising
  Halasz et al., Phys.Rev.D58 (1998) 096007
  Stephanov et al., Phys.Rev.Lett.81 (1998) 4816
- At the critical point:
  - Random field 3D Ising: η = 0.50±0.05
    Rieger, Phys.Rev.B52 (1995) 6659
  - 3D Ising: η = 0.03631(3)
    El-Showk et al., J.Stat.Phys.157 (4-5): 869
- Motivation for precise Lévy HBT!
- Change in α_{Levy} proximity of CEP?
- Finite size/time & non-equilibrium effects → what does power-law tail mean?





SOURCE SIZE MEASURES AROUND THE CRITICAL POINT?

- Main Lévy source parameters: R_{Levy}, α
- Other source size measures:
  - R_{Gaussfit}: $C(Q; R_{\text{Levy}}, \alpha)$ fitted with α = 2 fixed
  - R_{HWHM}: half width at half maximum
  - R_{HWHI}: half width at half integral
- **Simulated scenario:**
  - minimum in α vs. s_{NN}
  - maximum in R_{Levy} vs. s_{NN}
- **Observation:**
  - R_{Gaussfit}: approximately monotonic increase
  - Minimum in R_{HWHM}!
  - Trend change in R_{HWHI}!



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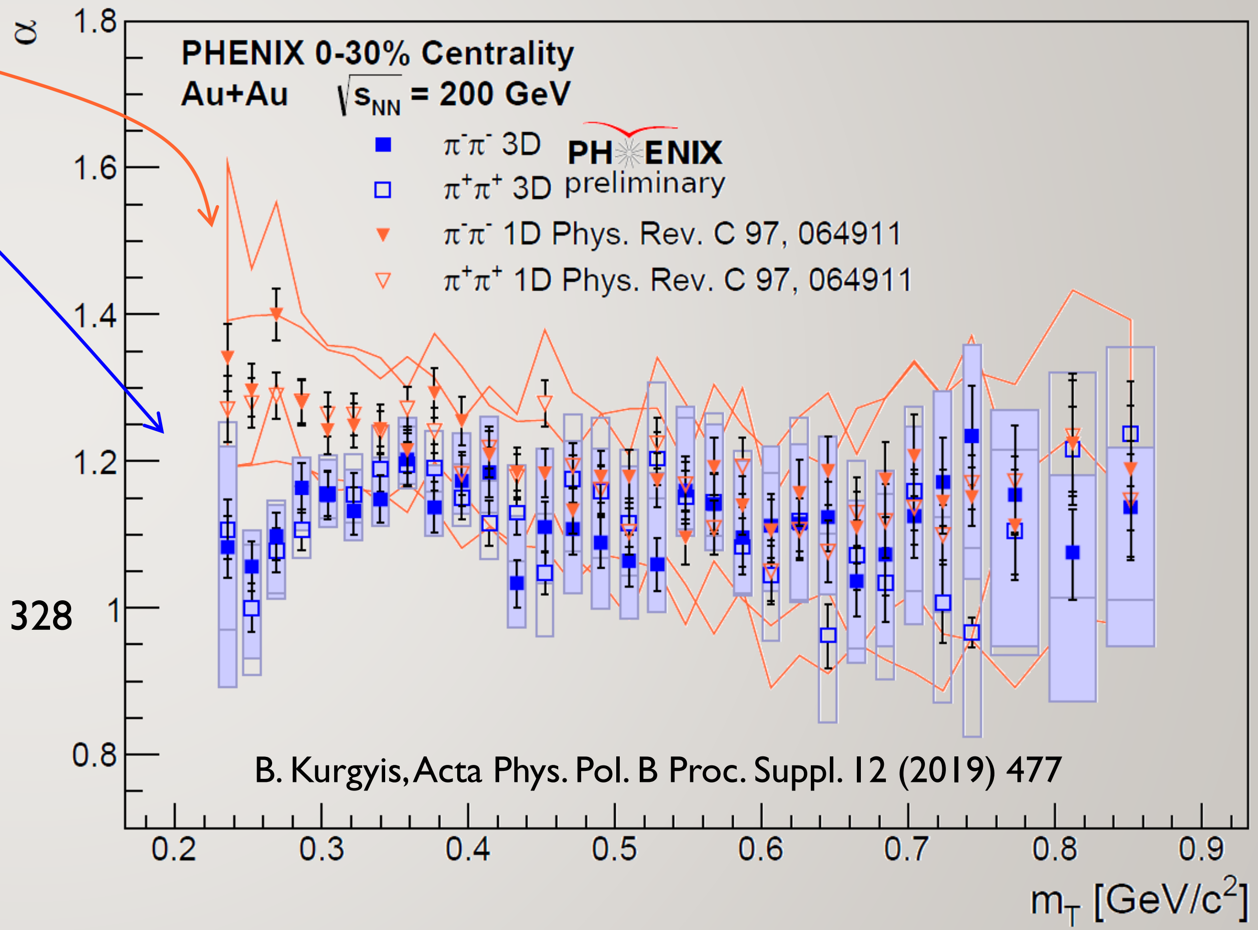
LÉVY EXPONENT VERSUS TRANSVERSE MASS, 1D AND 3D

- Lévy exponent α in 3D close to 1D result
- On average still far from 2
- Observable differences at low m_T
  - Maybe due to lack of spherical symmetry?
- Coulomb effect for non-spherical sources?
  - Approximation possible

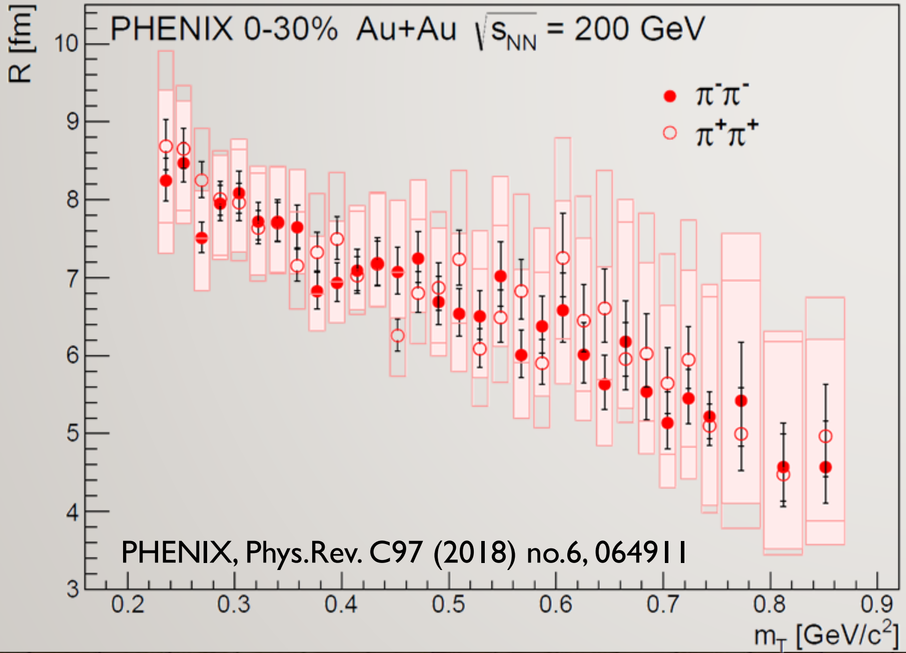
    Kurgyis, Kincses, Csanád, Nagy, Universe 9 (2023) 7, 328
  - If spherical in LCMS, radius in PCMS:

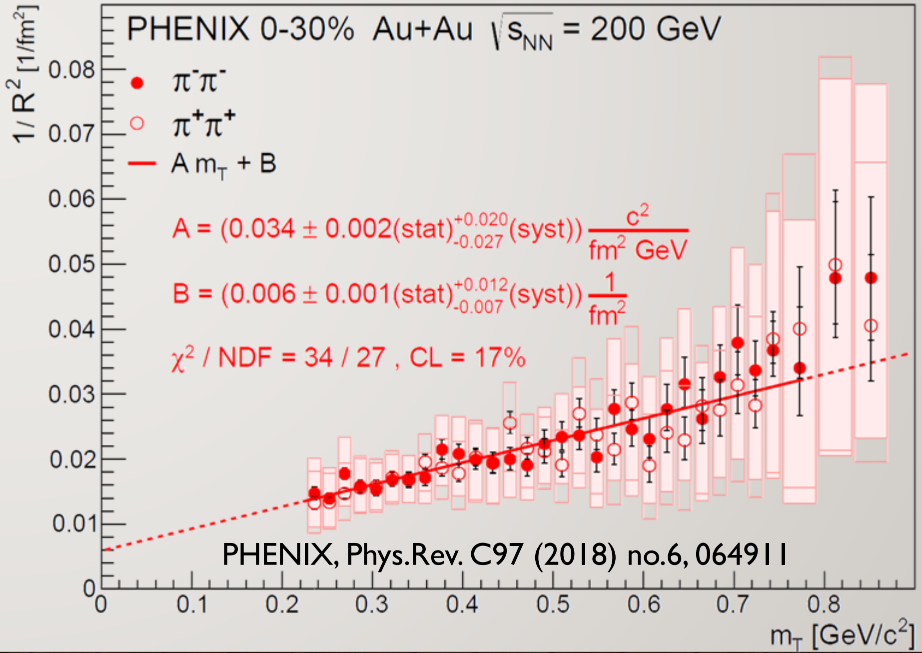
$$R_{PCMS} = \sqrt{\frac{1-2\beta_T^2/3}{1-\beta_T^2}} \cdot R_{LCMS}$$



LÉVY SCALE PARAMETER R AT RHIC

- Similar decreasing trend as Gaussian HBT radii, but it is not an RMS!
  - RMS of a Lévy source: in principle infinity, obtained value depends on cutoff
- What do model calculations, simulations say about this?
- Hydro behavior ($1/R^2 \sim m_T$) not invalid; but: predicted for Gaussian case only!

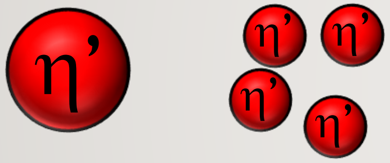




CORRELATION STRENGTH λ: IN-MEDIUM MASS?

- Connection to chiral restoration
  - Decreased η’ mass → more η’ produced → more decay pions → λ decreases
  - Kinematics: η’ → ππππ with low m_T → decreased $\lambda(m_T)$ specifically at low m_T
  - Dependence on in-medium η’ mass?

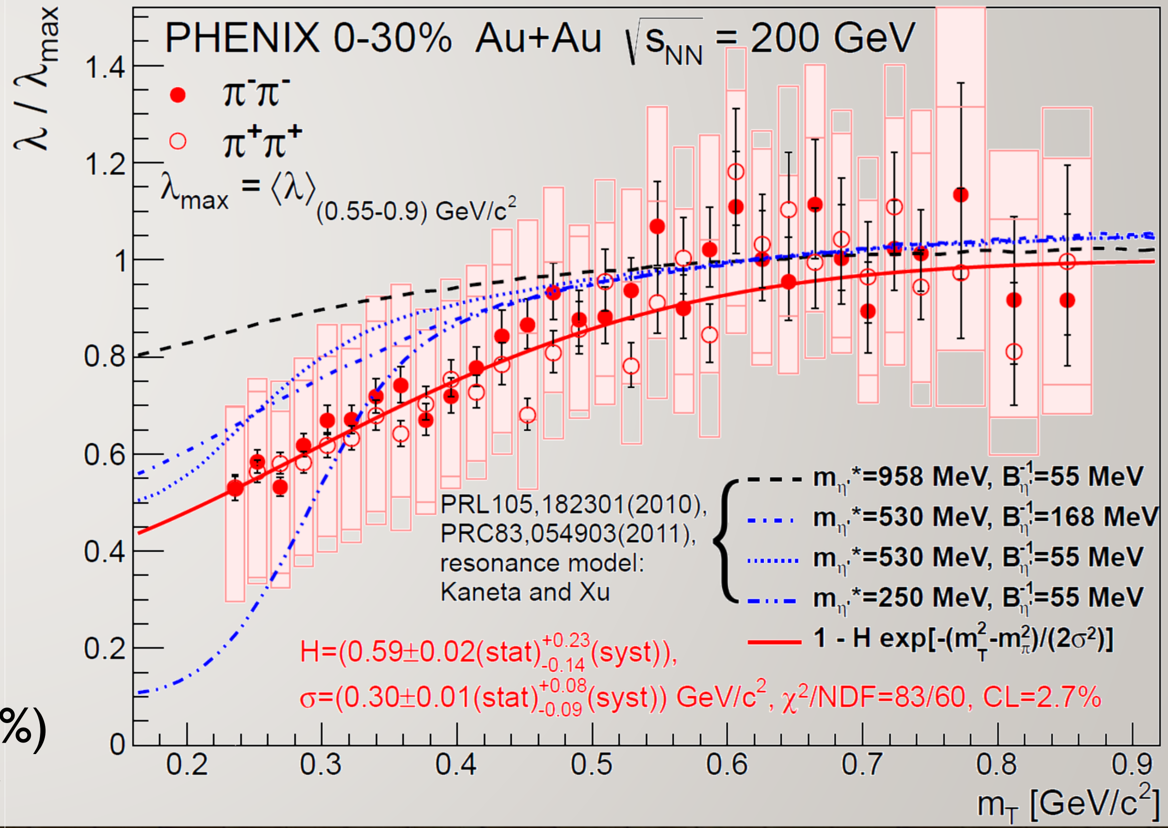
Kapusta, Kharzeev, McLerran, PRD53 (1996) 5028
Vance, Csörgő, Kharzeev, PRL 81 (1998) 2205
Csörgő, Vértesi, Sziklai, PRL105 (2010) 182301



$$T < T^*$$
$$m_{\eta'} > m^*_{\eta'}$$
$$N_{\eta'} < N^*_{\eta'}$$
$$\lambda > \lambda^*$$

- Results not incompatible with this
- Recall: 3D results similar to 1D
- Would need direct check with photons ($\eta' \to \gamma\gamma$, 2.3%)
- Centrality dependent analysis in collaboration review

PHENIX, Phys.Rev. C97 (2018) no.6, 064911



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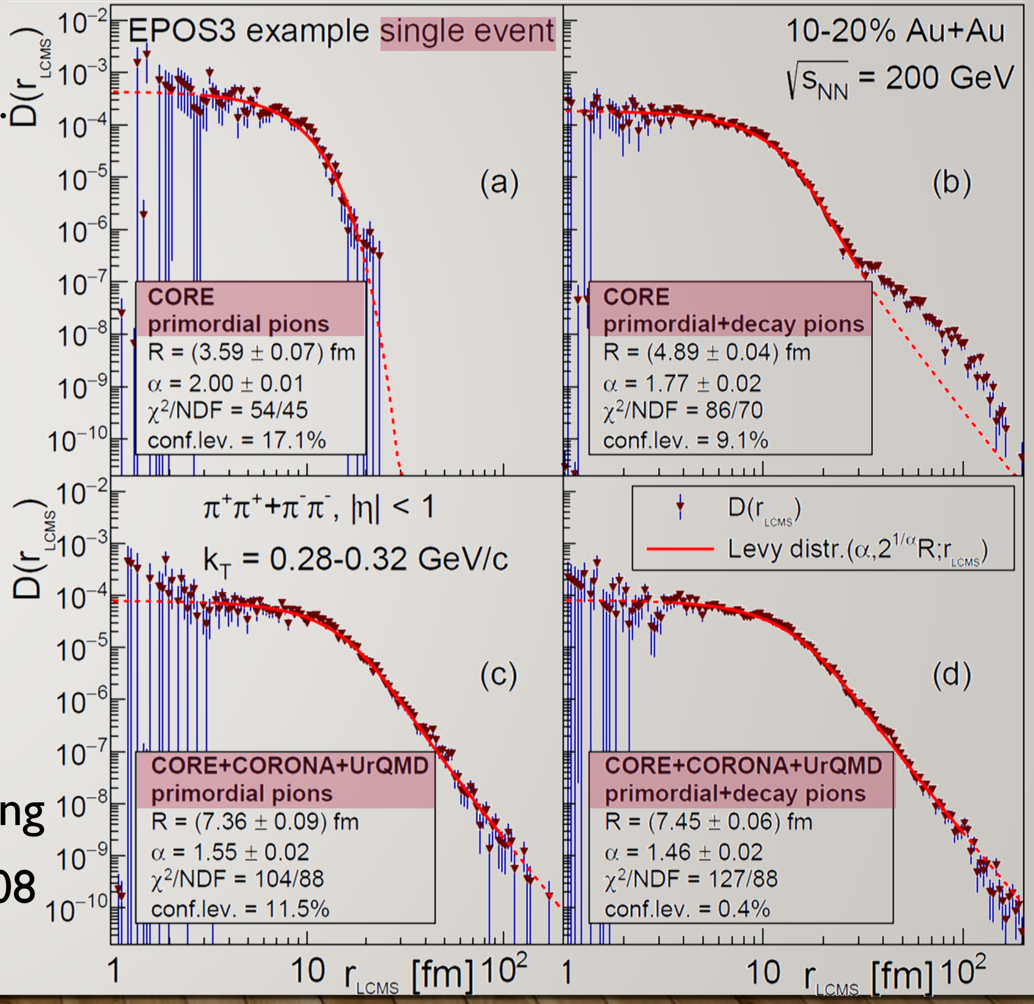
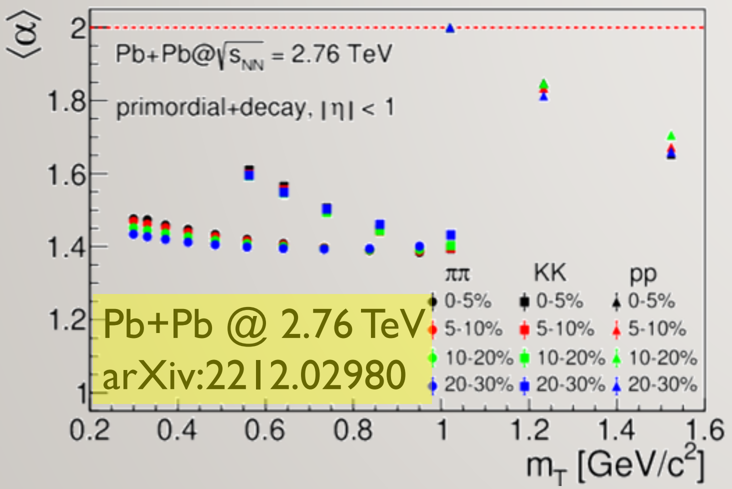
EVENT BY EVENT SHAPE ANALYSIS WITH EPOS

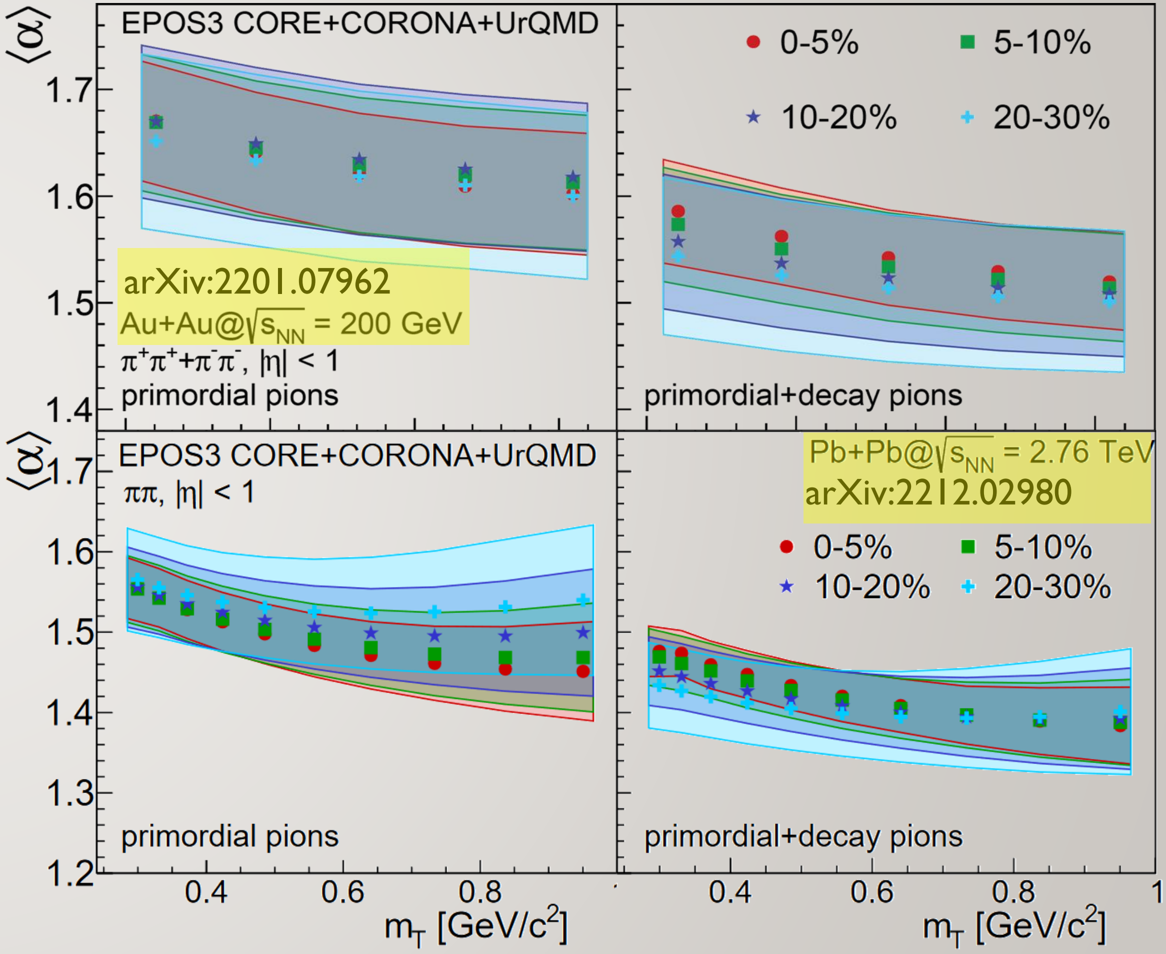
- EPOS model: parton-based Gribov-Regge theory (PBGRT)
  - Werner et al., PRC82 (2010) 044904, PRC89 (2014) 064903, …
  - Core-Corona, viscous hydro (vHLLE), cascades, UrQMD
- Pair distribution calculated: $D(r_{LCMS}) = \int d\Omega dt D(t, r_x, r_y, r_z)$
  - Angle-averaged radial source distribution of like-sign pion pairs
- Investigated cases:
  - a) CORE, primordial pions: close to Gaussian
  - b) CORE, with decay products: power-law structures
  - c) CORE+CORONA+UrQMD, primordial pions: Lévy shape
  - d) CORE+CORONA+UrQMD, with decay products: Lévy shape
- Lévy shape in single events; source size versus m_T: hydro scaling
  - 200 GeV AuAu: Kincses, Stefaniak, Cs., Entropy 24 (2022) 308
  - 2.76 TeV PbPb: Kórodi, Kincses, Cs., arXiv:2212.02980



AVERAGE LÉVY EXPONENT VS TRANSVERSE MASS

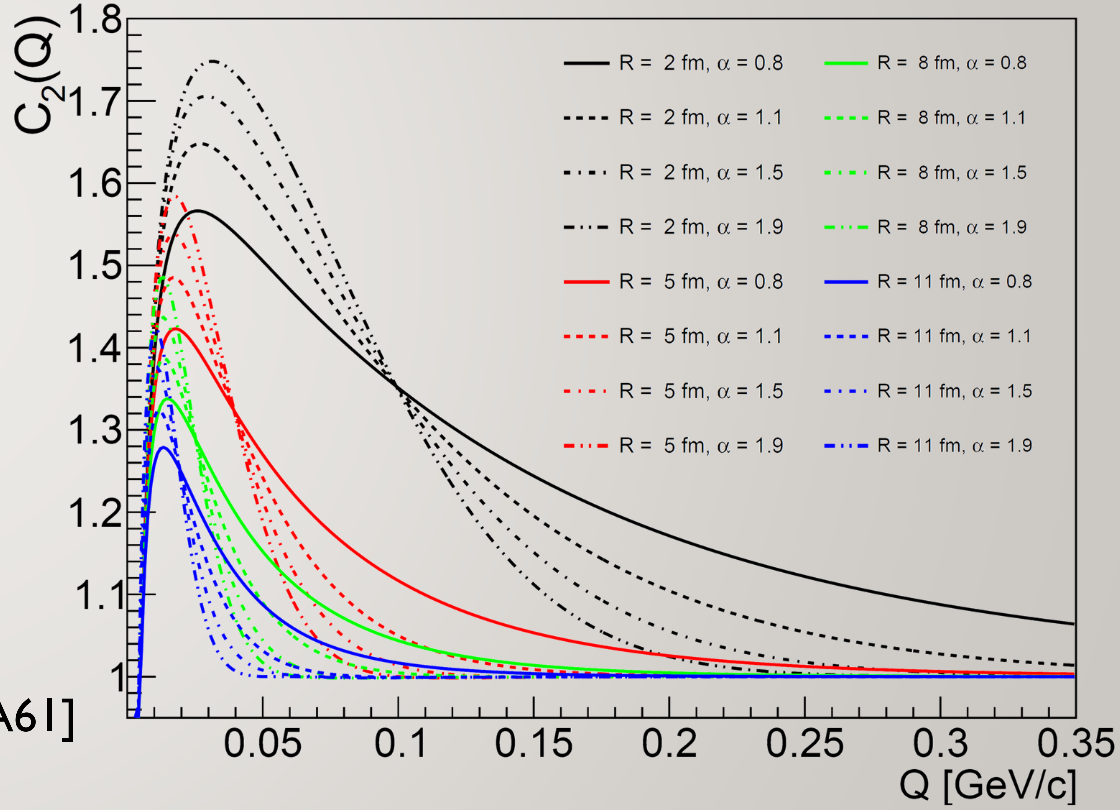
- $\langle\alpha\rangle$ versus m_T and centrality: small dependence
  - 200 GeV Au+Au: Entropy 24 (2022) 308
  - 2.76 TeV Pb+Pb: arXiv:2212.02980
- With or without decays at RHIC: $\alpha_{\text{EPOS}} > \alpha_{\text{measured}}$
  - Opposite at LHC energies, see arXiv:2212.02980
- Similar analysis at 2.76 TeV [arXiv:2212.02980]: particle type dependence as well





HOW TO CALCULATE THE COULOMB EFFECT

- Calculating correlation functions with the Coulomb effect included: time consuming in the past
- Method used in early analyses: Coulomb correction calculated for fixed radius and shape
  - For example, fixing $R = 5$ fm and $\alpha = 2$
- More consistent method: correlation function with Coulomb FSI precalculated in a tabular form
  - Iterative fitting, convergence in 2-3 iterations usually, see e.g., PHENIX Coll., Phys.Rev.C 97 (2018) 6, 064911
- Convenient, but somewhat restricted method: interpolating functional form, in a limited R, α range
  - See Csanád, Lökös, Nagy, Phys.Part.Nucl. 51 (2020) 238, used in arXiv:2306.11574 [CMS], arXiv:2302.04593 [NA61]



A NOVEL METHOD FOR LÉVY SHAPES WITH COULOMB FSI

- New mathematical development:

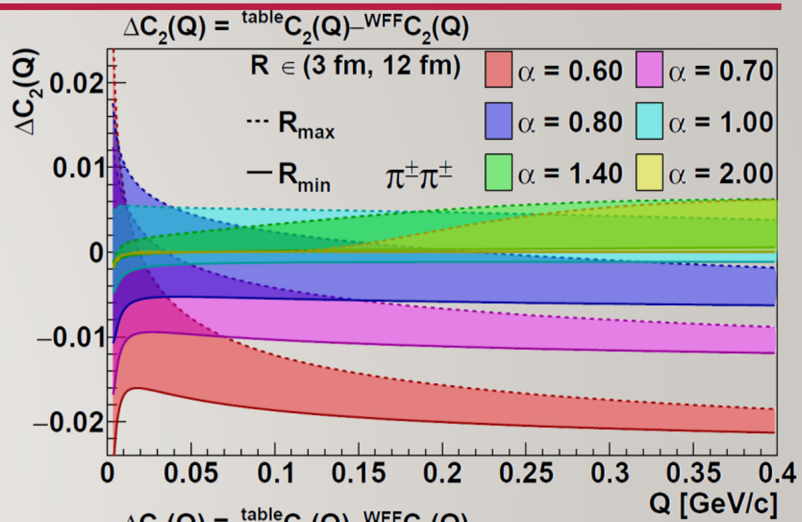
  Coulomb integral $C_2(Q) = \int d^3r \left|\psi_Q(r)\right|^2 D(r)$ can be performed

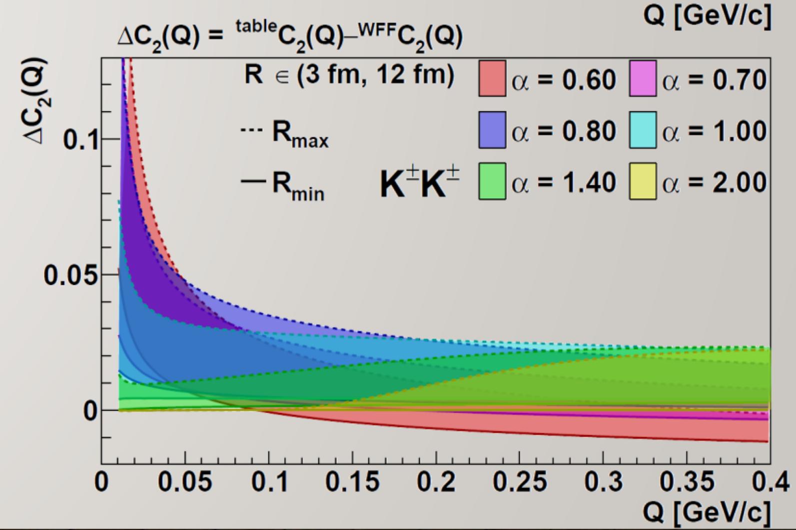
  - $D(r)$ is expressible as a Fourier transform:
    $D(r) = \int d^3q e^{iqr} f(q)$, for example $D(r)$ Lévy: $f(q) = e^{-|qR|^\alpha}$
  - Integrals $\int d^3r$ and $\int d^3q$ unfortunately cannot be exchanged
  - Calculation can still be performed via Lebesgue and Fubini theorems

- Result: $C_2(Q) = |\mathcal{N}|^2 \left(1 + f(Q) + \frac{\eta}{\pi}[A_{1s}[f](Q) + A_{2s}[f](Q)]\right)$,

  where $|\mathcal{N}|^2 = \frac{2\pi\eta}{e^{2\pi\eta}-1}$ (Gamow), $\eta = \frac{mc^2\alpha}{\hbar c Q}$, A_{ns} functionals

  - Few percent difference to numerical (tabularized) values used earlier
  - Details in Nagy, Purzsa, Csanád, Kincses arXiv:2308.10745,
    code at github.com/csanadm/CoulCorrLevyIntegral





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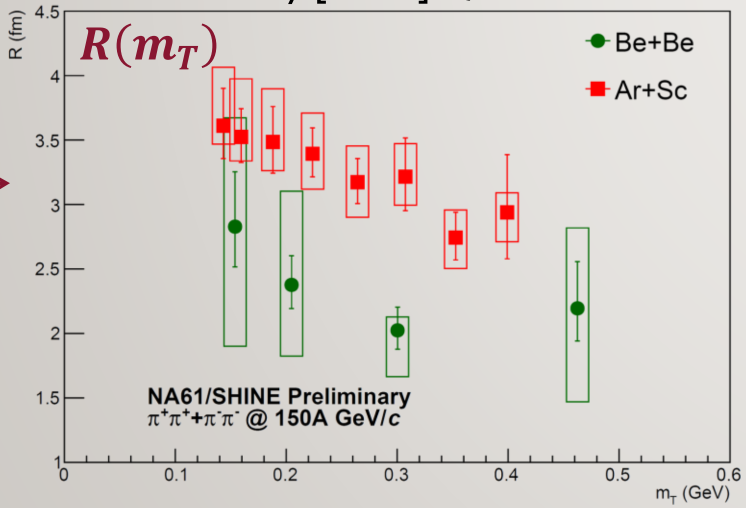
PION ANALYSIS AT SPS NA61/SHINE

- Lévy scale R of Ar+Sc [prelim.: Universe 9 (2023) 7, 298] and Be+Be [arXiv:2302.04593, EPJC accepted]:
  - Compatible with initial geometry factor 1.6 between Ar+Sc and Be+Be
  - Decrease with m_T due to transverse flow?
- No m_T dependence in λ, in contrast to RHIC result – can be turned off?
- Lévy index α: significant difference

B. Pórfy [NA61], QM 23 & WPCF 22



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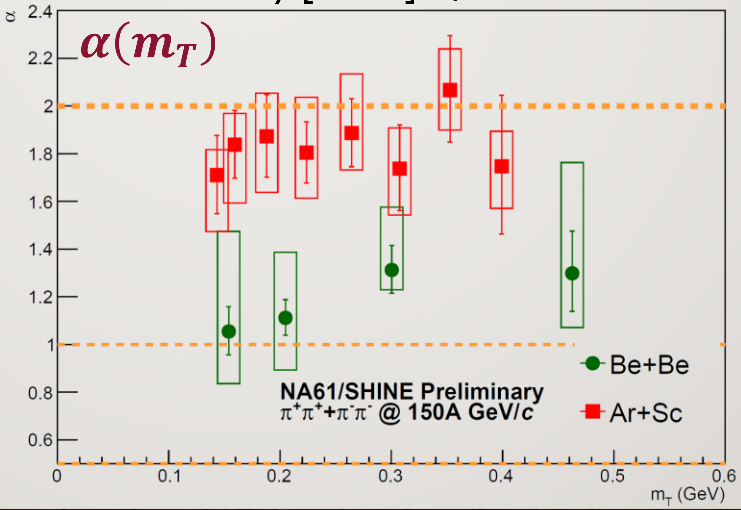

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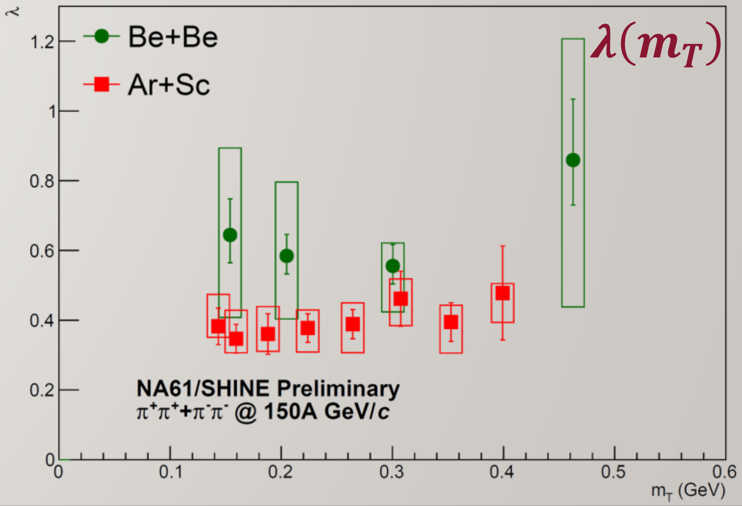
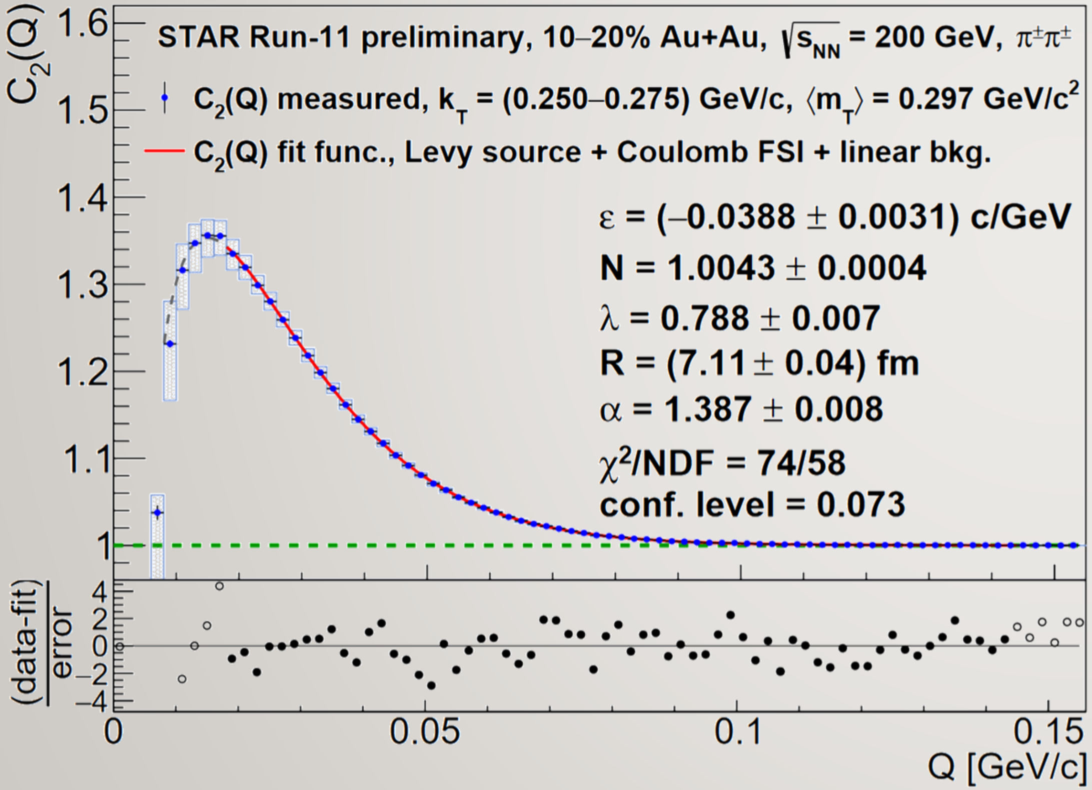
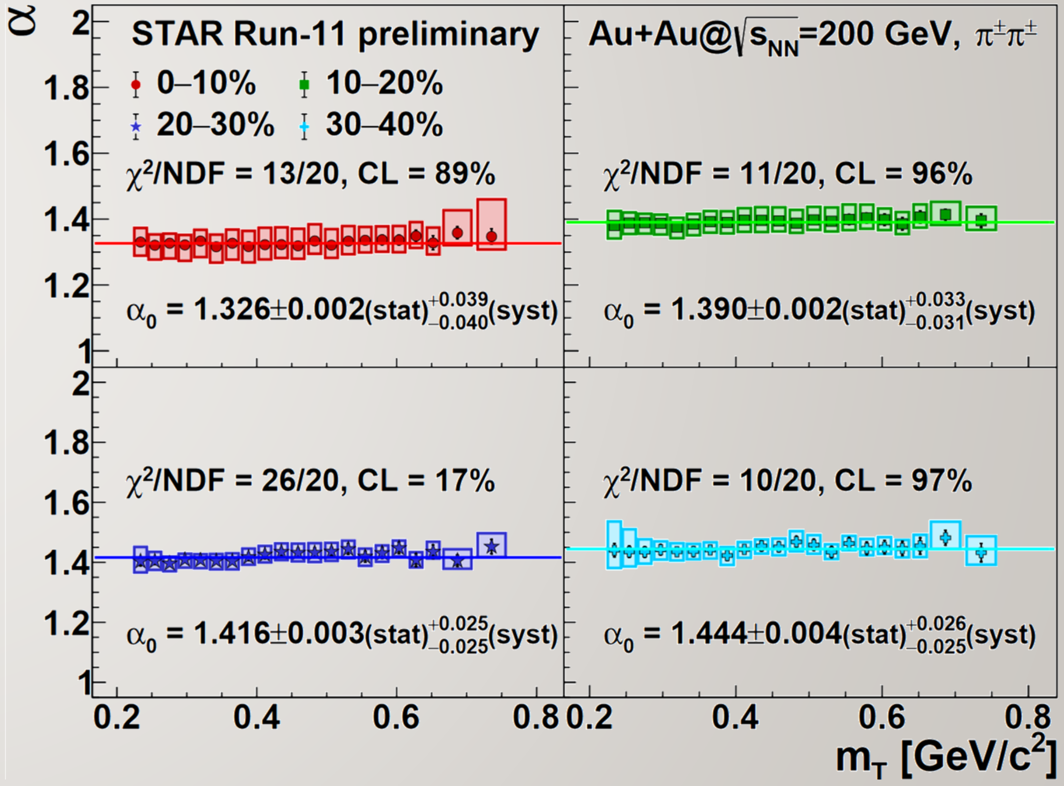


PION ANALYSIS WITH LÉVY SOURCES AT STAR

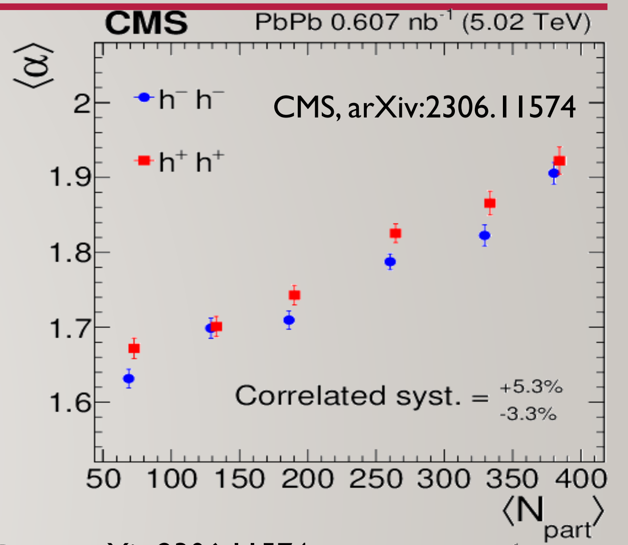
- Run-11 Au+Au at 200 GeV, ~ 550 M events, PID by TPC+TOF, 21 m_T and 4 centrality bins
- Source far from Gaussian, constant in m_T, slight increase of α for peripheral collisions $(1.326 \rightarrow 1.444)$

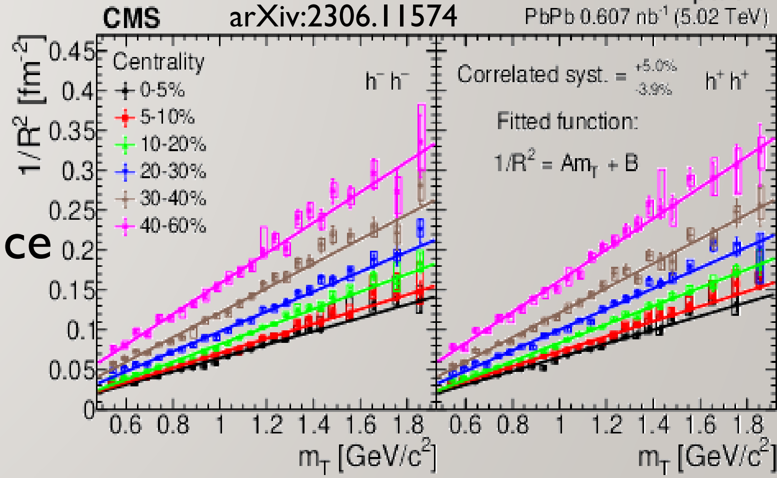




CHARGED HADRON ANALYSIS IN 5 TEV PB+PB

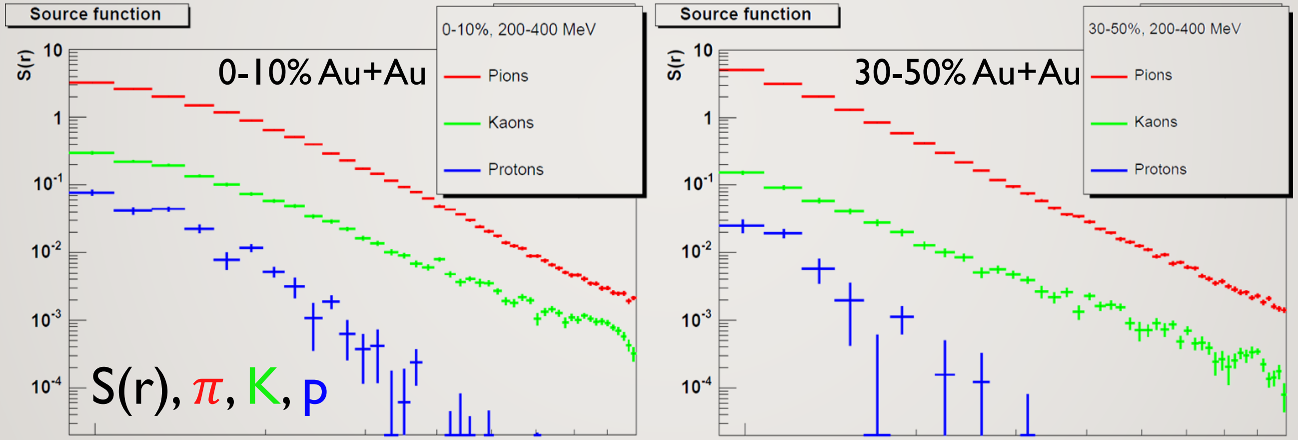
- Lévy index α measured:
  - Far from Cauchy
  - Not exactly Gaussian
  - Closer to Gaussian for large N_{part}, unlike RHIC
- Lévy scale R: hydro scaling confirmed
  - In every centrality class, despite non-Gaussianity
  - Hubble coefficient can be extracted: 0.12-0.18 c/fm, larger than at RHIC
- Correlation strength λ also analyzed
- Low-Q deviation cross-checked with Monte-Carlo: two-track acceptance
- Final CMS result: HIN-21-011, arXiv:2306.11574 [under review]
  - Preliminary results in proceedings: B. Kórodi, Universe 9 (2023) 7, 318





THE IMPORTANCE OF A KAON ANALYSIS

- Kaons: smaller cross-section, larger mean free path
- Mean free path increases more during a time-step → heavier power-law tail?
- Prediction for π, K, p based on Humanic's Resonance Model (HRM): anomalous diffusion due to rescattering
  Humanic, Int.J.Mod.Phys. E15 (2006) 197 [nucl-th/0510049]
  Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002 [hep-ph/0702032]

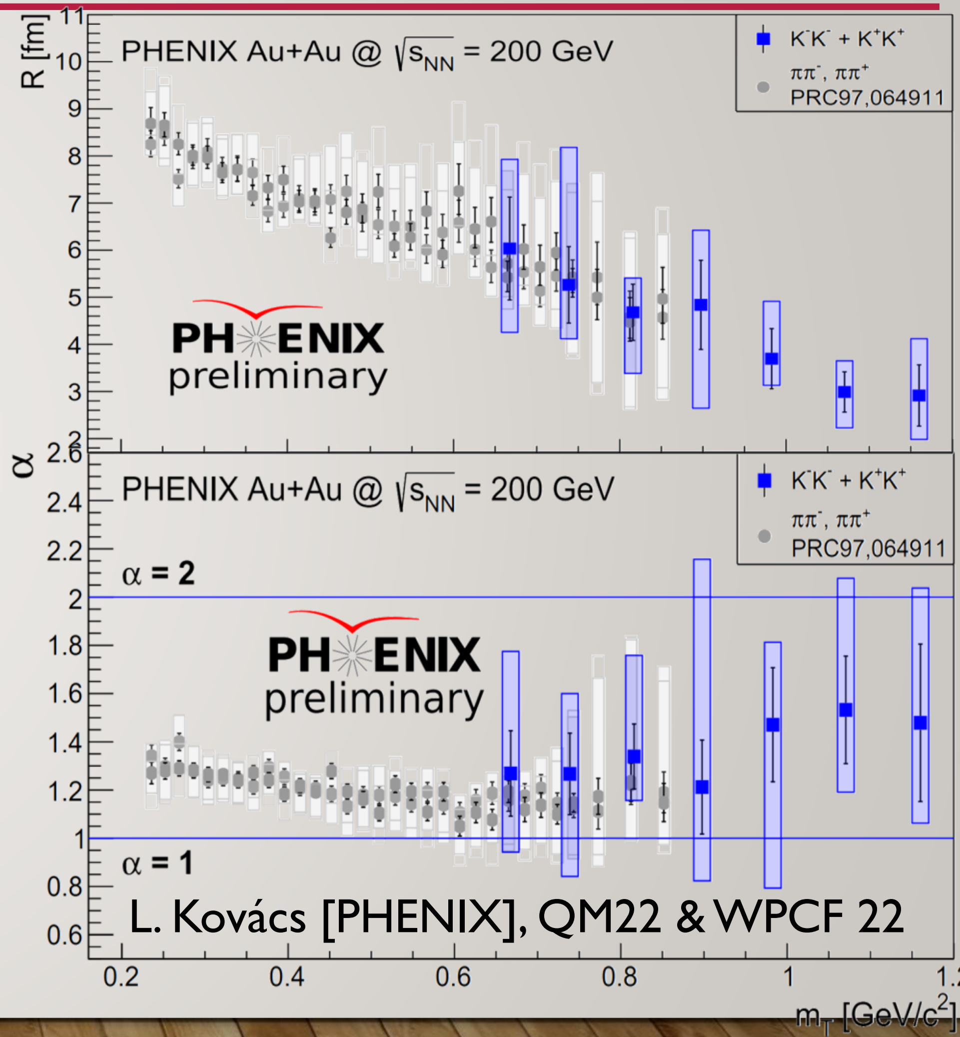


Braz.J.Phys. 37(2007)1002

- Kaon HBT radii: m_T scaling or its violation for Lévy scale R?
- Prediction: $\alpha(p) > \alpha(\pi) > \alpha(K)$

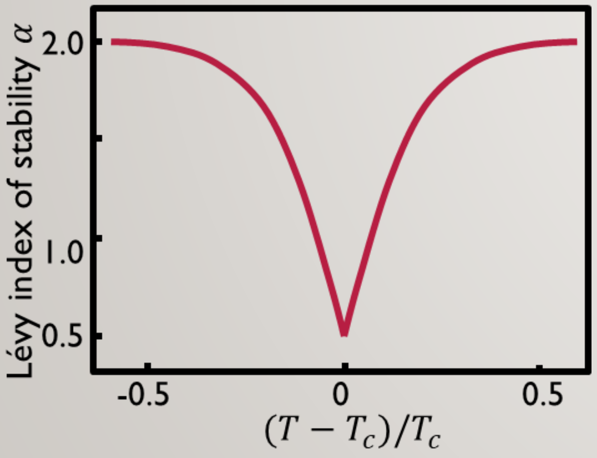
KAON ANALYSIS AT PHENIX AND STAR

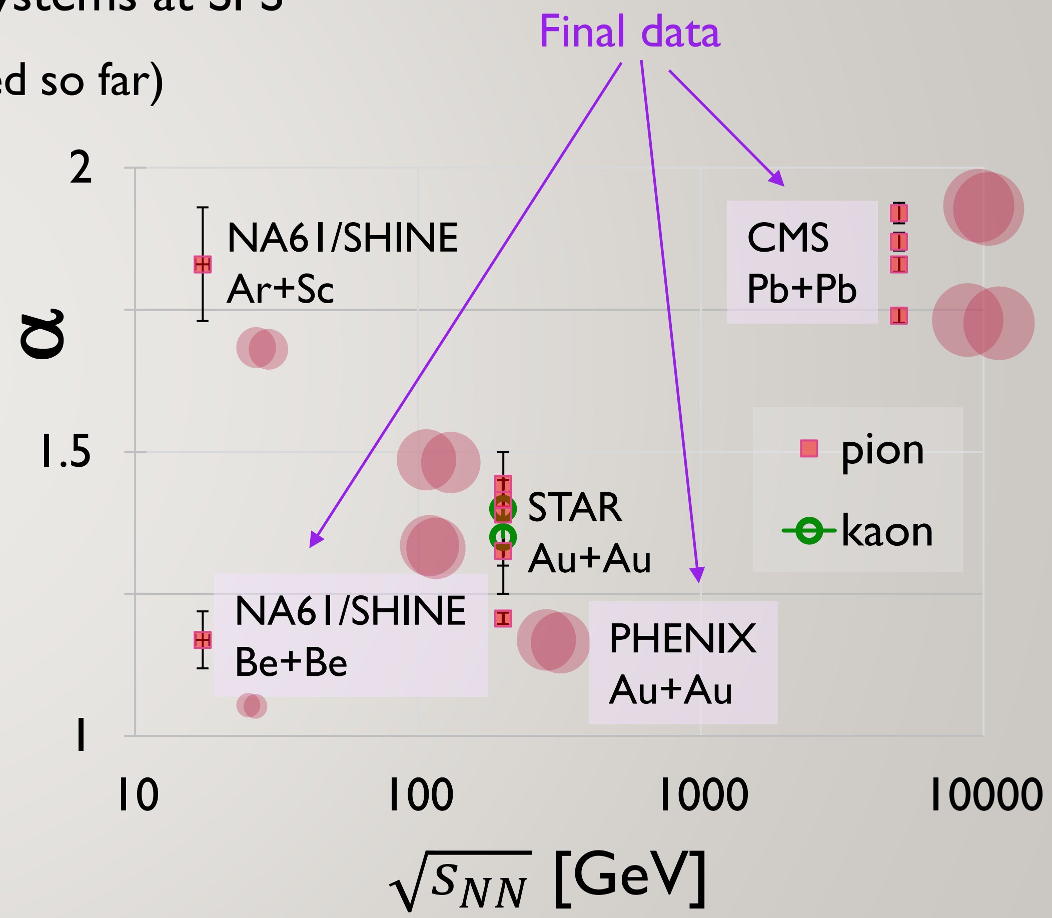
- Preliminary analysis performed at PHENIX and STAR
- Kaon and pion data seem compatible at the same m_T
- Lévy scale R shows hydro type of scaling with m_T
  - R depending on m_T but not on particle type separately
- $\alpha(K) \geq \alpha(\pi)$, but anomalous diffusion suggests opposite
- Dominant mechanism creating Lévy source?
  - Not only rescattering?
  - Anomalous hydro at the sQGP stage?
- PHENIX prelim. results: L. Kovács, Universe 9 (2023) 7, 336
- STAR prelim. results: A. Mukherjee, Universe 9 (2023) 7, 300



STABILITY PARAMETER α FROM SPS TO LHC

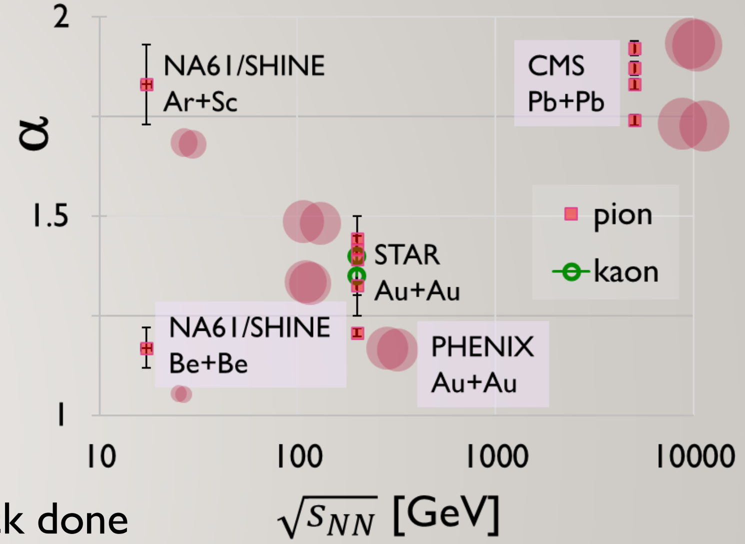
- Different values for small (Be+Be) & medium (Ar+Sc) systems at SPS
  - Also true for Pb+Pb and p+p at LHC? (pp: $\alpha = 1$ assumed so far)
- Medium and large systems: non-monotonic trend
- Compare to expectation cartoon based on Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67





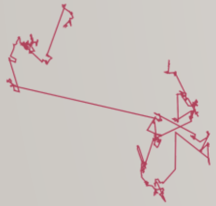
CONTENTS OF THIS TALK

CONCLUSIONS AND OUTLOOK

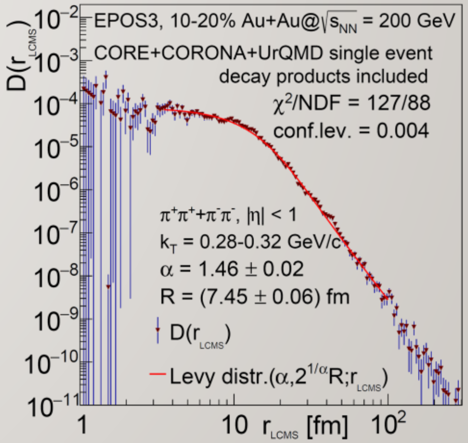


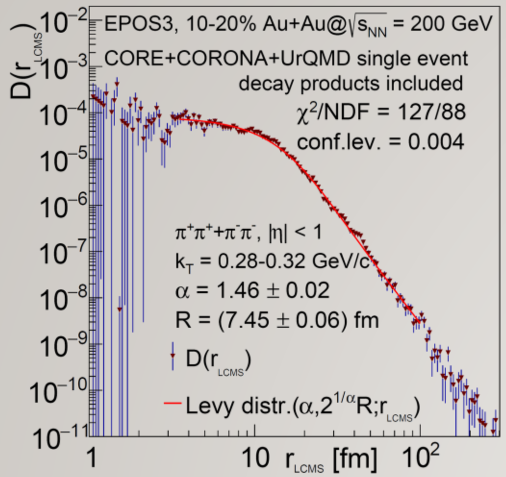
- Lévy sources from SPS to RHIC and LHC
  - **Lévy α**: between 1 and 2, increases with $\sqrt{s_{NN}}$?
    - Contrary to expectations, $\alpha(K) \geq \alpha(\pi)$
  - **Lévy R**: hydro scaling, despite not Gaussian
  - **Lévy λ**: signs of η' in-medium mass modification

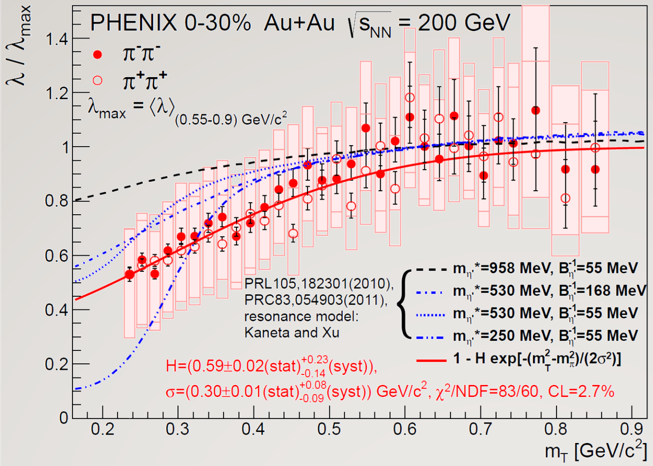
- Possible reasons:
  - Jet fragmentation → not dominant in AA collisions
  - **Critical phenomena** → maybe at lowest RHIC energies and SPS
  - Directional averaging → source is (approx.) spherical in LCMS, 3D cross-check done
  - Event averaging → event-by-event simulations show Lévy
  - **Resonance decays** → part of the reason, not enough alone
  - **Hadronic rescattering, Lévy flight** → $\alpha(K) \geq \alpha(\pi)$ puzzling

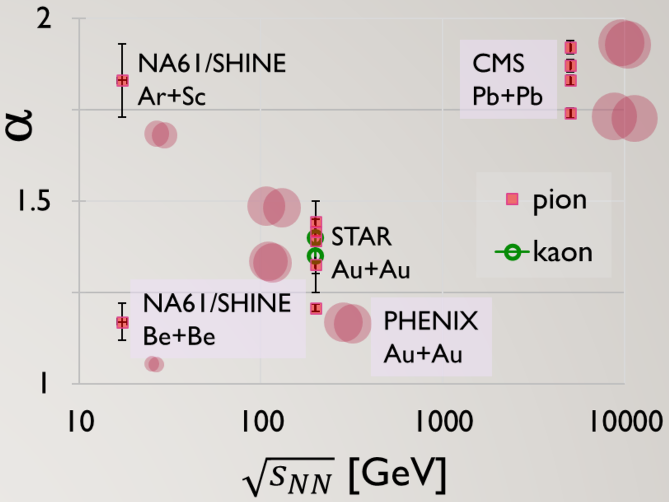


- Questions to be answered:
  - When measuring α, what effects need to be considered?
  - Can there be anomalous diffusion in the quark stage?
  - What is the role of finite size and finite time?









THANK YOU FOR YOUR ATTENTION

And if you are interested in these topics:

: https://agenda.infn.it/event/33324/

http://zimanyischool.kfki.hu/23/