

# AZIMUTHAL ASYMMETRY IN $J/\psi$ PRODUCTION IN ELECTRON-PROTON COLLISION AT EIC

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The 3-D nucleon structure in momentum space is still not known yet!

Parton intrinsic transverse momentum?

Spin and  $k_{\perp}$  correlations?

Angular momentum of partons?

Spatial distribution?

ACHT-2023 Workshop  
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# PLAN OF TALK

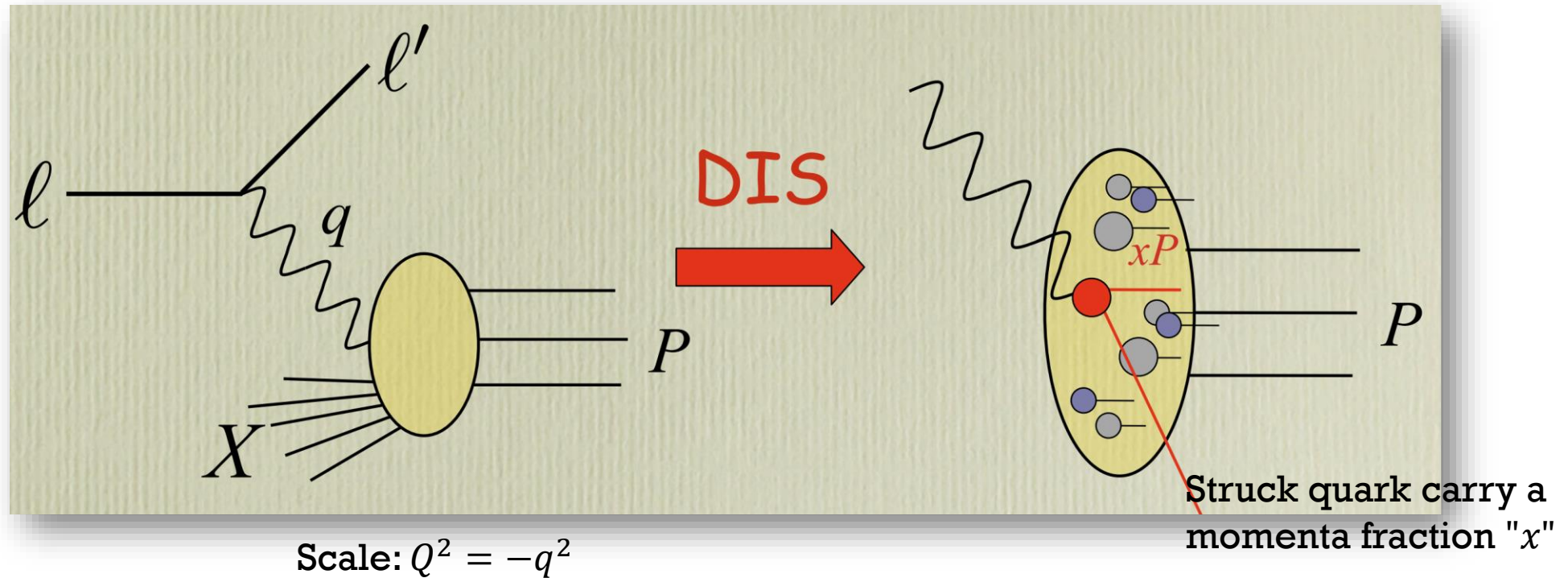
Parton Distribution function: PDFs, TMDs (Gluon TMDs)

Azimuthal asymmetries in  $J/\psi - jet$  and  $J/\psi - \gamma$  pair production in  $ep$  scattering at EIC

Numerical estimates

Conclusion

**DIS process:** usual way of exploring the proton structure



Naïve parton model: 
$$d\sigma^{lp \rightarrow l'X} = \sum_q f_q(x) \otimes d\sigma^{lq \rightarrow l'q}$$

Within the Parton model of nucleons we defined parton distribution functions (PDFs).

- Collinear parton distribution functions,  $f_a(x, Q^2)$ , gives the number density of partons with momentum fraction  $x$  and that depends on scale  $Q^2$ , gives the 1-D picture of a nucleon.

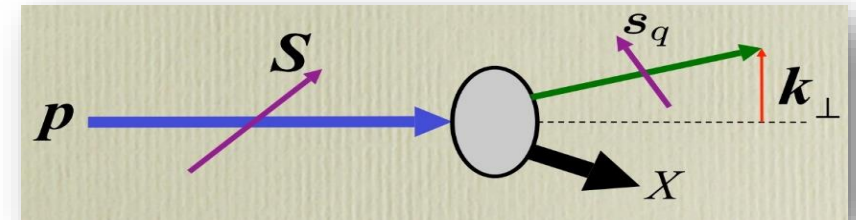
# Transverse momentum-dependent distribution and fragmentation functions

➤ Introducing a new concept: Transverse momentum dependent, distribution functions (TMD-PDFs) and fragmentation functions (TMD-FFs).

□ TMD-PDFs (Transverse Momentum Dependent Parton Distribution Functions):  $f(x, k_{\perp}, Q^2)$  gives the number density of partons, with their intrinsic transverse motion and spin, inside a nucleon.

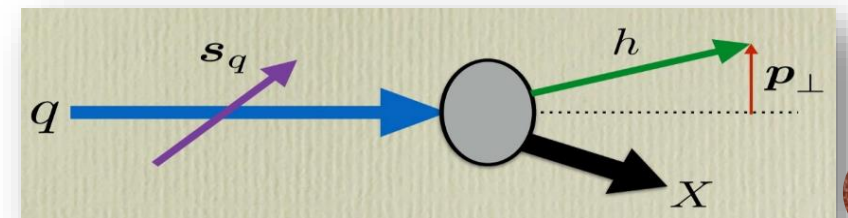
$$\mathbf{S} \cdot (\mathbf{p} \times \mathbf{k}_{\perp}) : \text{Sivers effect}$$

$$\mathbf{s}_q \cdot (\mathbf{p} \times \mathbf{k}_{\perp}) : \text{Boer-Mulders effect}$$



□ TMD-FFs (Transverse Momentum Dependent Fragmentation Functions):  $D(z, p_{\perp})$  gives the number density of hadrons, with their momentum, originated in the fragmentation of a parton.

$$\mathbf{s}_q \cdot (\mathbf{p}_q \times \mathbf{p}_{\perp}) : \text{Collins effect}$$





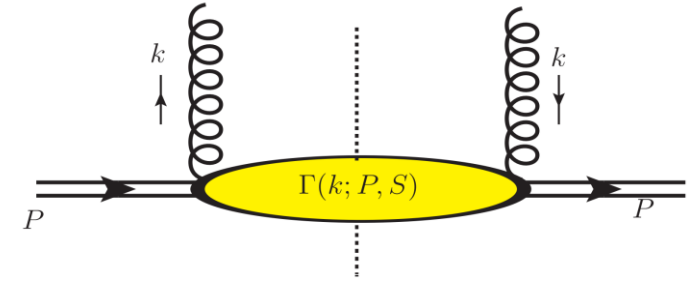
# Gluon TMDs

- Gluon-gluon correlator at leading twist

$$\Gamma^{+i;+j}(x, \mathbf{k}_T; P, S) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ik \cdot \xi} \langle P, S | \text{Tr}[F^{+i}(0) W_{[0,\xi]} F^{+j}(\xi) W_{[\xi,0]}] | P, S \rangle_{|\xi^+=0}$$

Gluon field tensor

Gauge links



$$W_{[0,\xi]} = \mathcal{P} \exp \left( -ig \int_0^\xi dz_\mu A^\mu(z) \right)$$

- Parameterizations: for the unpolarized (U) and transversely polarized (T) target

$$\Gamma_U^{ij}(x, \mathbf{k}_T^2) = \frac{x}{2} \left\{ -g_T^{ij} f_1^g(x, \mathbf{k}_T^2) + \left( \frac{k_T^i k_T^j}{M_p^2} + g_T^{ij} \frac{\mathbf{k}_T^2}{2M_p^2} \right) h_1^{\perp g}(x, \mathbf{k}_T^2) \right\}$$

$$\Gamma_T^{ij}(x, \mathbf{k}_T^2) = \frac{x}{2} \left\{ -g_T^{ij} \frac{\epsilon_T^{\rho\sigma} k_{T\rho} S_{T\sigma}}{M_p} f_{1T}^{\perp g}(x, \mathbf{k}_T^2) + \dots \right\}$$

		Gluon Polarization			
		U	Circular	Linear	
Hadron Pol.	U	$f_1^g$		$h_1^{\perp g}$	BOER-MULDERS
	L		$g_{1L}^g$	$h_{1L}^{\perp g}$	KOTZINIAN-MULDERS
	T	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_{1T}^g, h_{1T}^{\perp g}$	PRETZELOSITY
		SIVERS	HELICITY	WORM-GEAR	

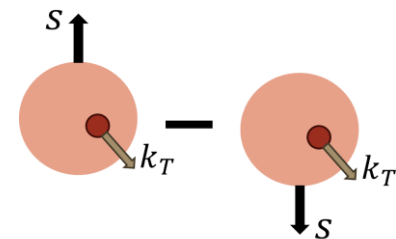
# GLUON TMDs: (BOER-MULDERS) $h_1^{\perp g}(x, k_T^2)$ AND (GSF) $\Delta^N f_{g/P^\uparrow}(x, k_T^2)$

- Gluon Boer-Mulders function is associated with a symmetric tensor structure

$$\frac{1}{2} \left( \frac{2k_T^i k_T^j}{k_T^2} - \delta^{ij} \right) = \frac{1}{2} \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix}$$

- Interpret  $h_1^{\perp g}(x, k_T^2)$  as "azimuthal correlated" gluon distribution function.
- It affects the unpolarized cross section and cause **azimuthal asymmetries**:  $\langle \cos(2\phi) \rangle, \langle \cos(4\phi) \rangle$
- GSF describes the number density of unpolarized gluon inside a transversely polarized nucleon.

$$\Delta^N f_{g/P^\uparrow}(x, \mathbf{k}_T) = \hat{f}_g(x, \mathbf{k}_T; \mathbf{S}_T) - \hat{f}_g(x, \mathbf{k}_T; -\mathbf{S}_T) \equiv \Delta^N f_g(x, k_T) (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_T) \cdot \hat{\mathbf{S}}_T$$

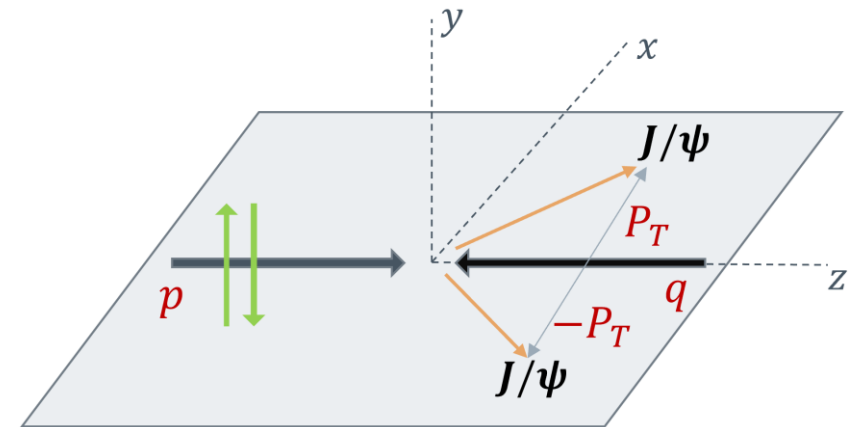


# SINGLE SPIN ASYMMETRY (SSA) AND AZIMUTHAL ASYMMETRY

The TMDs can be studied in Drell-Yan (DY) ( $pp \rightarrow ll'$ ), semi-inclusive deep inelastic scattering (SIDIS) ( $ep \rightarrow e'hX$ ) and electron-positron ( $e^+e^- \rightarrow h$ ) annihilation processes.

- Transverse SSA in a hadronic scattering process:  $A + B^\uparrow \rightarrow C + X$

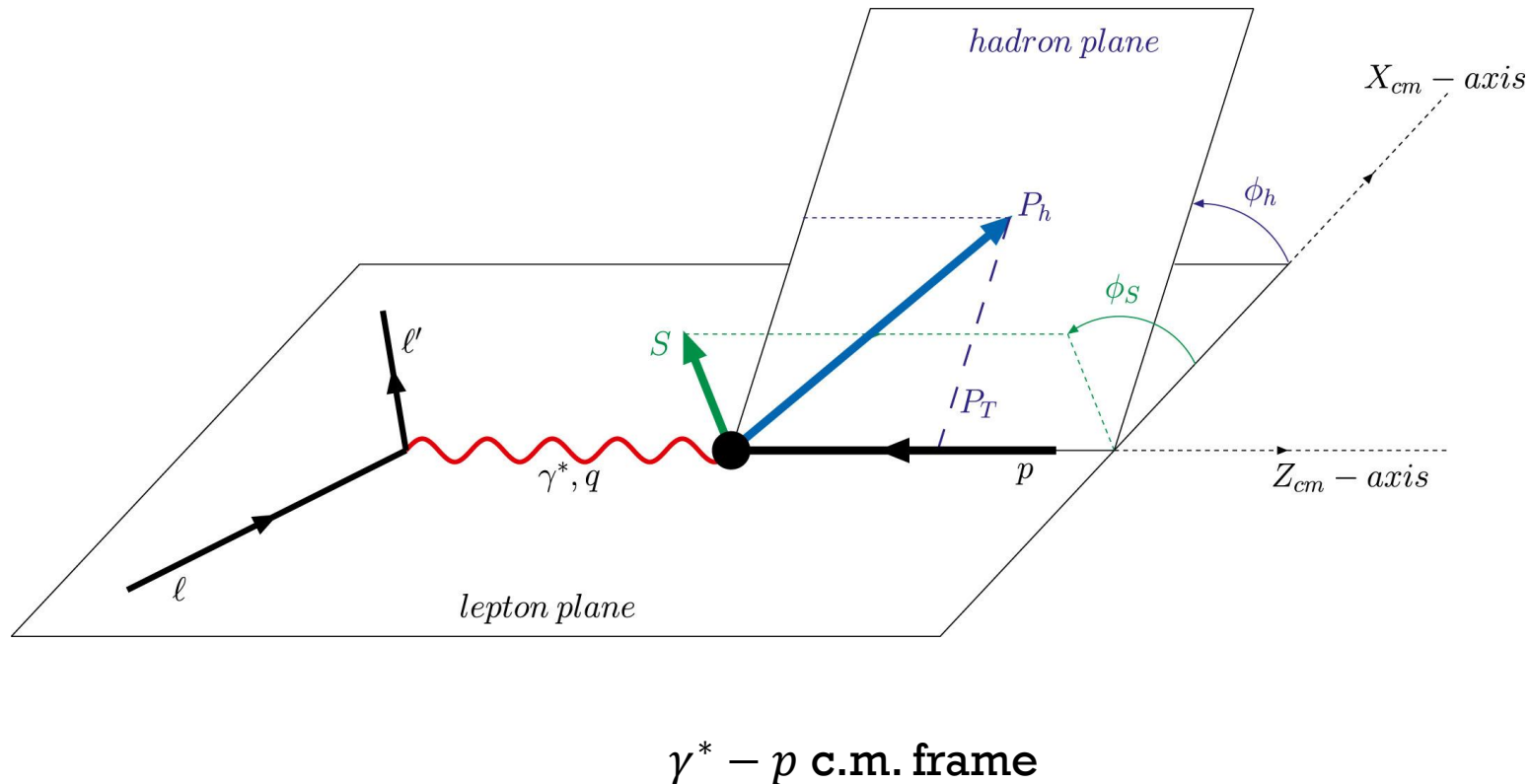
$$A_N = \frac{d\sigma^\uparrow(\mathbf{P}_T) - d\sigma^\downarrow(\mathbf{P}_T)}{d\sigma^\uparrow(\mathbf{P}_T) + d\sigma^\downarrow(\mathbf{P}_T)} \equiv \frac{d\sigma^\uparrow(\mathbf{P}_T) - d\sigma^\uparrow(-\mathbf{P}_T)}{2d\sigma^{\text{unp}}(\mathbf{P}_T)}$$



- Azimuthal asymmetry:  $\langle \cos(2\phi_h) \rangle = \frac{\int d\phi_h \cos(2\phi_h) d\sigma}{\int d\phi_h d\sigma}$

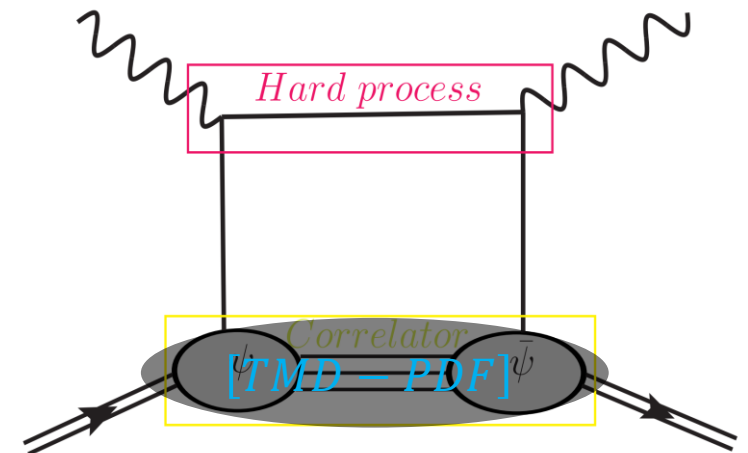
# $J/\psi$ PRODUCTION IN $ep$ SCATTERING

➤ Consider the electroproduction processes:  $e(l) + p(P) \rightarrow e(l') + J/\psi(P_\psi) + X$



$$z \left( = \frac{P \cdot P_\psi}{P \cdot q} \right) = 1 \text{ (LO)}$$

$z$  is fraction of virtual photon energy carried by  $J/\psi$  in proton rest frame.

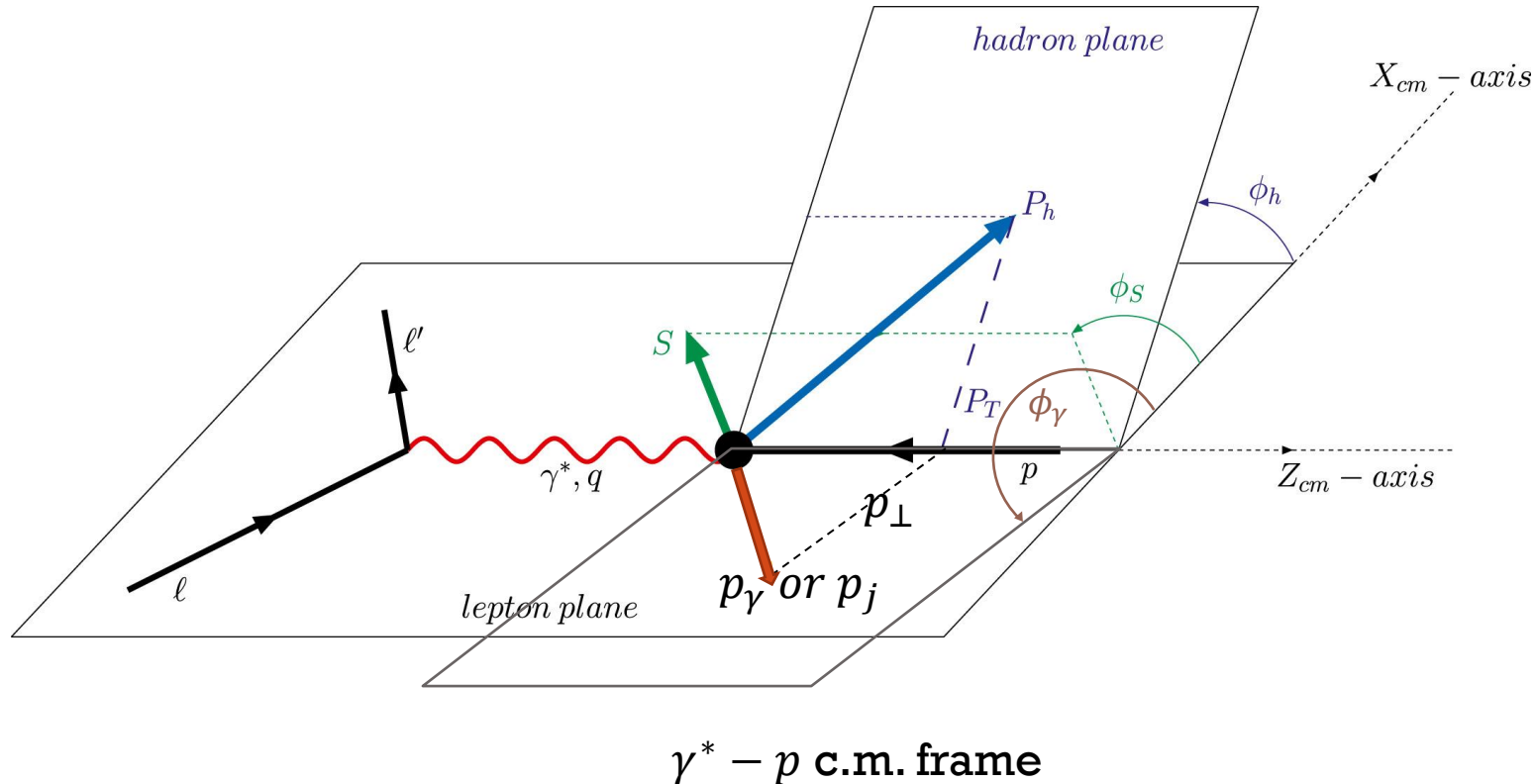


$$d\sigma \propto [TMD - PDF] \otimes d\hat{\sigma} \otimes [Hadronization]$$



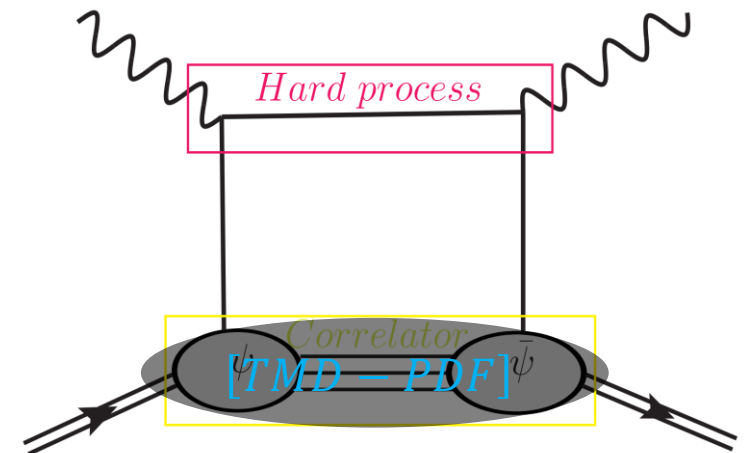
# A BACK-TO-BACK $J/\psi - jet$ AND $J/\psi - \gamma$ PRODUCTION IN $ep$ SCATTERING

- Consider the electroproduction processes:  $e(l) + p(P) \rightarrow e(l') + J/\psi(P_\psi) + \gamma(p_\gamma) + X$   
 $e(l) + p(P) \rightarrow e(l') + J/\psi(P_\psi) + jet(p_j) + X$



$$z \left( = \frac{P \cdot P_\psi}{P \cdot q} \right) < 1 \text{ (LO)}$$

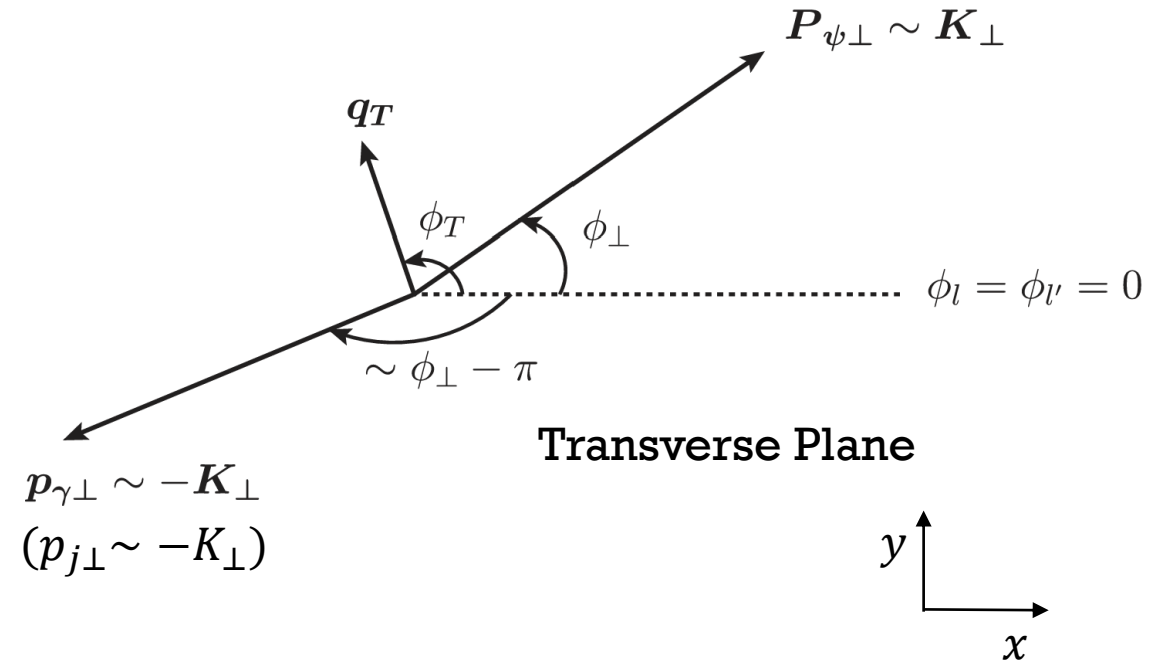
$z$  is fraction of virtual photon energy carried by  $J/\psi$  in proton rest frame.



$$d\sigma \propto [TMD - PDF] \otimes d\hat{\sigma} \otimes [Hadronization]$$

# A BACK-TO-BACK $J/\psi - jet$ AND $J/\psi - \gamma$ PRODUCTION IN $ep$ SCATTERING

- Assume TMD factorization.
- $p - \gamma^*$  center of mass frame which move along  $z$  direction
- $P_{\psi\perp}$  and  $P_{j\perp}$  are transverse momentum of  $J/\psi$  and  $jet$  respectively in the plane orthogonal to the proton momentum.
- We define sum and difference of transverse momenta



$$q_T = P_{\psi\perp} + P_{j\perp} \quad , \quad K_{\perp} = \frac{P_{\psi\perp} - P_{j\perp}}{2} \quad \phi_T \text{ denotes azimuthal angle of } q_T$$

- In the case where  $|q_T| \ll |K_{\perp}|$ , the  $J/\psi$  and  $jet$  are almost back-to-back in the transverse plane.

TMD Factorization

# CROSS SECTION: $ep \rightarrow e + J/\psi + jet + X$

$$d\sigma = \frac{1}{2s} \frac{d^3 l'}{(2\pi)^3 2E_{l'}} \frac{d^3 P_\psi}{(2\pi)^3 2E_\psi} \frac{d^3 P_j}{(2\pi)^3 2E_j} \int dx d^2 p_T (2\pi)^4 \delta^4(q + p_g - P_\psi - P_j) \times$$

$$\frac{1}{Q^4} L^{\mu\mu'}(l, q) \Phi_g^{\nu\nu'}(x, p_T^2) M_{\mu\nu}^{g\gamma^* \rightarrow J/\psi g} M_{\mu'\nu'}^{*g\gamma^* \rightarrow J/\psi g}$$

Lepton tensor:  $L^{\mu\mu'}(l, q) = e^2(-g^{\mu\mu'} Q^2 + 2(l^\mu l'^{\mu'} + l'^{\mu} l^\mu))$

Parameterization of gluon correlator for **unpolarized proton** target at 'Leading Twist'

$$\Phi_g^{\nu\nu'}(x, p_T^2) = \frac{1}{2x} [-g_\perp^{\nu\nu'} f_1^g(x, p_T^2) + \left( \frac{p_T^\nu p_T^{\nu'}}{M_p^2} + g_\perp^{\nu\nu'} \frac{p_T^2}{2M_p^2} \right) h_1^{\perp g}(x, p_T^2)]$$

Unpolarized gluon distribution

Linearly polarized gluon distribution

# QUARKONIUM PRODUCTION

- Quarkonium is a bound state of heavy quark and anti-quark ( $Q\bar{Q}$ )

Describes conversion of  $Q\bar{Q}[n]$  states into final quarkonium state.

NRQCD factorization

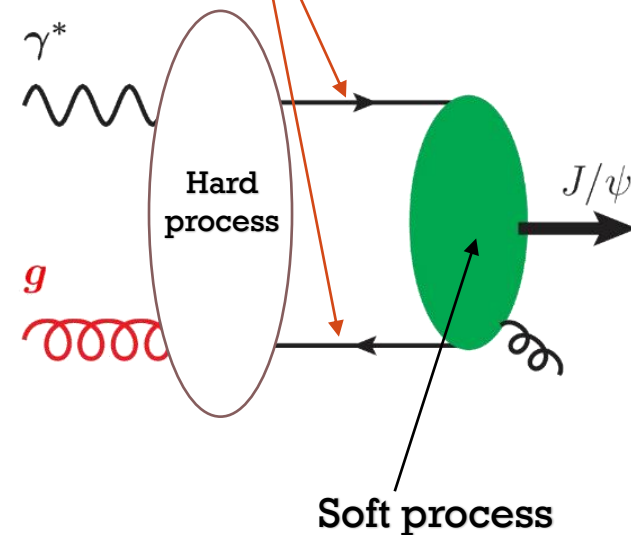
Non-perturbative; long distance matrix elements (LDMEs)

$$d\sigma^{ab \rightarrow J/\psi} = \sum_n d\hat{\sigma}[ab \rightarrow c\bar{c}(n)] \langle 0 | \mathcal{O}_n^{J/\psi} | 0 \rangle$$

Perturbative short distance coefficient

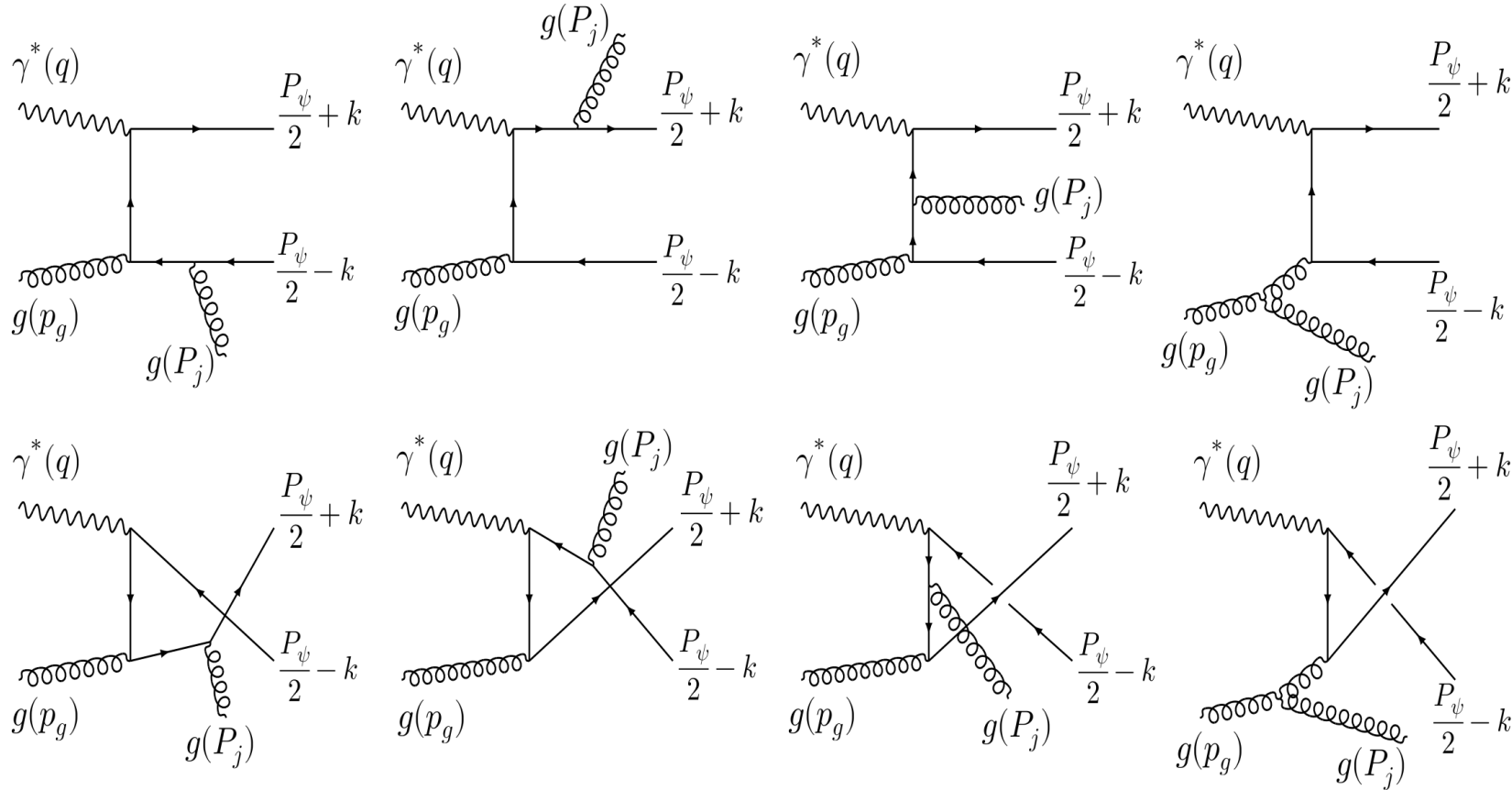
Cross section in particular color, angular momentum and spin state "n":  ${}^{2S+1}L_J$ , calculated by perturbative QCD

$Q\bar{Q}$  pair with  $[{}^{2S+1}L_J^{(1,8)}]$  quantum number



# FEYNMAN DIAGRAMS

Gluon initiated hard process:  $\gamma^* g \rightarrow Q \bar{Q} g$ , contributes significantly over the quark(anti-quark) initiated hard process:  $\gamma^* q(\bar{q}) \rightarrow Q \bar{Q} q(\bar{q})$ , in the small- $x$  domain.



Tree level Feynman diagrams for the hard process:  $\gamma^* + g \rightarrow c + \bar{c} + g$

# AMPLITUDE CALCULATIONS USING NRQCD

The amplitude can be written as

$$\begin{aligned}
 & M(\gamma^* g \rightarrow Q\bar{Q} [{}^{2S+1}L_J^{(1,8)}](P_\psi) + g(P_j)) \\
 &= \sum_{L_z S_z} \int \frac{d^3 k}{(2\pi)^3} \Psi_{LL_z}(k) \langle LL_z; SS_z | JJ_z \rangle \text{Tr}[\mathcal{O}(q, p, P_\psi, k) \mathcal{P}_{SS_z}(P_\psi, k)]
 \end{aligned}$$

D. Boer and C. Pisano (2012)

$\mathcal{O}(q, p, P_\psi, k)$ : amplitude for production of  $Q\bar{Q}$  pair.

$$\mathcal{O}(q, p, P_\psi, k) = \sum_{m=1}^8 C_m \mathcal{O}_m(q, p, P_\psi, k)$$

The spin projection operator,  $\mathcal{P}_{SS_z}(P_\psi, k)$ , projects the spin triplet and spin singlet states of  $Q\bar{Q}$  pair

$$\begin{aligned}
 \mathcal{P}_{SS_z}(P_\psi, k) &= \sum_{s_1 s_2} \left\langle \frac{1}{2} s_1; \frac{1}{2} s_2 \middle| SS_z \right\rangle v \left( \frac{P_\psi}{2} - k, s_1 \right) \bar{u} \left( \frac{P_\psi}{2} + k, s_2 \right) \\
 &= \frac{1}{4M_\psi^{3/2}} (-\not{P}_\psi + 2\not{k} + M_\psi) \Pi_{SS_z} (\not{P}_\psi + 2\not{k} + M_\psi) + O(k^2)
 \end{aligned}$$

$$\Pi_{SS_z} = \gamma^5 \text{ for spin singlet } (S = 0)$$

$$\Pi_{SS_z} = \epsilon_{S_z}^\mu (P_\psi) \gamma_\mu \text{ for spin triplet } (S = 1)$$



# Amplitude Calculations

Since,  $k \ll P_h$ , amplitude expanded in Taylor series about  $k = 0$

First term in the expansion gives the S-states ( $L = 0, J = 0, 1$ ). The linear term in  $k$  gives the P-states ( $L = 1, J = 0, 1, 2$ ).

The S-states amplitude : 
$$M[{}^{2S+1}S_J^{(1,8)}](P_\psi, k) = \frac{1}{\sqrt{4\pi}} R_0(0) \text{Tr}[\mathcal{O}(q, p, P_\psi, k) \mathcal{P}_{SS_z}(P_\psi, k)] \Big|_{k=0}$$

The P-states  
amplitude :

$$M[{}^{2S+1}P_J^{(1,8)}](P_\psi, k) = -i \sqrt{\frac{3}{4\pi}} R'_1(0) \sum_L \epsilon_{L_z}^\alpha(P_\psi) \langle LL_z; SS_z | JJ_z \rangle \text{Tr}[\mathcal{O}_\alpha(0) \mathcal{P}_{SS_z}(0) + \mathcal{O}(0) \mathcal{P}_{SS_z\alpha}(0)]$$

$$\mathcal{O}_\alpha(0) = \frac{\partial}{\partial k^\alpha} \mathcal{O}(q, p, P_\psi, k) \Big|_{k=0} \qquad \mathcal{P}_{SS_z\alpha}(0) = \frac{\partial}{\partial k^\alpha} \mathcal{P}_{SS_z}(q, p, P_\psi, k) \Big|_{k=0}$$

Contribution:  ${}^3S_1^{(1)}, {}^3S_1^{(8)}, {}^1S_0^{(8)}, {}^3P_{j(=0,1,2)}^{(8)}$

$R_0$  and  $R'_1$  are related with the LDMEs

# FINAL CROSS SECTION: $ep \rightarrow e + J/\psi + jet + X$ OR $ep \rightarrow e + J/\psi + \gamma + X$

$$\frac{d\sigma}{dzdydx_B d^2\mathbf{q}_T d^2\mathbf{K}_\perp} \equiv d\sigma(\phi_S, \phi_T, \phi_\perp) = d\sigma^U(\phi_T, \phi_\perp) + d\sigma^T(\phi_S, \phi_T, \phi_\perp)$$

$$d\sigma^U = \mathcal{N} \left[ (\mathcal{A}_0 + \mathcal{A}_1 \cos \phi_\perp + \mathcal{A}_2 \cos 2\phi_\perp) f_1^g(x, \mathbf{q}_T^2) + (\mathcal{B}_0 \cos 2\phi_T + \mathcal{B}_1 \cos(2\phi_T - \phi_\perp) \right. \\ \left. + \mathcal{B}_2 \cos 2(\phi_T - \phi_\perp) + \mathcal{B}_3 \cos(2\phi_T - 3\phi_\perp) + \mathcal{B}_4 \cos(2\phi_T - 4\phi_\perp)) \frac{\mathbf{q}_T^2}{M_p^2} h_1^{\perp g}(x, \mathbf{q}_T^2) \right]$$

$$d\sigma^T = \mathcal{N} |\mathcal{S}_T| \left[ \sin(\phi_S - \phi_T) (\mathcal{A}_0 + \mathcal{A}_1 \cos \phi_\perp + \mathcal{A}_2 \cos 2\phi_\perp) \frac{|\mathbf{q}_T|}{M_p} f_{1T}^{\perp g}(x, \mathbf{q}_T^2) \right. \\ \left. + \cos(\phi_S - \phi_T) (\mathcal{B}_0 \sin 2\phi_T + \mathcal{B}_1 \sin(2\phi_T - \phi_\perp) + \mathcal{B}_2 \sin 2(\phi_T - \phi_\perp) \right. \\ \left. + \mathcal{B}_3 \sin(2\phi_T - 3\phi_\perp) + \mathcal{B}_4 \sin(2\phi_T - 4\phi_\perp)) \frac{|\mathbf{q}_T|^3}{M_p^3} h_{1T}^{\perp g}(x, \mathbf{q}_T^2) \right. \\ \left. + (\mathcal{B}_0 \sin(\phi_S + \phi_T) + \mathcal{B}_1 \sin(\phi_S + \phi_T - \phi_\perp) + \mathcal{B}_2 \sin(\phi_S + \phi_T - 2\phi_\perp) \right. \\ \left. + \mathcal{B}_3 \sin(\phi_S + \phi_T - 3\phi_\perp) + \mathcal{B}_4 \sin(\phi_S + \phi_T - 4\phi_\perp)) \frac{|\mathbf{q}_T|}{M_p} h_{1T}^g(x, \mathbf{q}_T^2) \right],$$

**Coefficients:**  $A_i \sim A_i^{\gamma g \rightarrow c\bar{c}[n]g} \langle 0 | O_n^{J/\psi} | 0 \rangle$  and  $B_i \sim B_i^{\gamma g \rightarrow c\bar{c}[n]g} \langle 0 | O_n^{J/\psi} | 0 \rangle$  comes from partonic processes.

# ASYMMETRY CALCULATIONS

Weighted azimuthal asymmetry: 
$$A^{W(\phi_S, \phi_T)} \equiv 2 \frac{\int d\phi_S d\phi_T d\phi_\perp W(\phi_S, \phi_T) d\sigma(\phi_S, \phi_T, \phi_\perp)}{\int d\phi_S d\phi_T d\phi_\perp d\sigma(\phi_S, \phi_T, \phi_\perp)}$$

U. D'Alesio (2019)

Azimuthal modulations probe Boer-Mulder gluon TMD:

$$A^{\cos 2\phi_T} = \frac{\mathbf{q}_T^2}{M_p^2} \frac{\mathcal{B}_0}{\mathcal{A}_0} \frac{h_1^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)} \quad A^{\cos 2(\phi_T - \phi_\perp)} = \frac{\mathbf{q}_T^2}{M_p^2} \frac{\mathcal{B}_2}{\mathcal{A}_0} \frac{h_1^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

Azimuthal modulations which can be exploited to extract polarized TMDs  $f_{1T}^{\perp g}$ ,  $h_1^g$  and  $h_{1T}^{\perp g}$

$$A^{\sin(\phi_S - \phi_T)} = \frac{|\mathbf{q}_T|}{M_p} \frac{f_{1T}^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

$$A^{\sin(\phi_S - 3\phi_T)} = -\frac{|\mathbf{q}_T|^3}{2M_p^3} \frac{\mathcal{B}_0}{\mathcal{A}_0} \frac{h_{1T}^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

$$A^{\sin(\phi_S + \phi_T)} = \frac{|\mathbf{q}_T|}{M_p} \frac{\mathcal{B}_0}{\mathcal{A}_0} \frac{h_1^g(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

# GAUSSIAN PARAMETERIZATION (GP)

$f_1^g(x, \mathbf{q}_T^2)$  and  $h_1^{\perp g}(x, \mathbf{q}_T^2)$  are assumed to be factorized as function of  $x$ , i.e. collinear PDFs and a Gaussian function of the transverse momentum  $q_T$ .

$$f_1^g(x, \mathbf{q}_T^2) = f_1^g(x, \mu) \frac{1}{\pi \langle q_T^2 \rangle^2} e^{-\frac{q_T^2}{\langle q_T^2 \rangle}}$$

$$h_1^{\perp g}(x, \mathbf{q}_T^2) = \frac{M_P^2 f_1^g(x, \mu)}{\pi \langle q_T^2 \rangle^2} \frac{2(1-r)}{r} e^{-\frac{q_T^2}{r \langle q_T^2 \rangle}}$$

$r$  ( $0 < r < 1$ ) and  $\langle q_T^2 \rangle$  are parameters

We took,  $r = 1/3$  and  $\langle q_T^2 \rangle = 0.25$

$$\begin{aligned} \Delta^N f_{g/p^\uparrow}(x, q_T) &= \left( -\frac{2|\mathbf{q}_T|}{M_P} \right) f_{1T}^{\perp g}(x, q_T) \\ &= 2 \frac{\sqrt{2}e}{\pi} \mathcal{N}_g(x) f_{g/p}(x) \sqrt{\frac{1-\rho}{\rho}} q_T \frac{e^{-q_T^2/\rho \langle q_T^2 \rangle}}{\langle q_T^2 \rangle^{3/2}} \end{aligned}$$

$$\mathcal{N}_g(x) = N_g x^\alpha (1-x)^\beta \frac{(\alpha + \beta)^{(\alpha + \beta)}}{\alpha^\alpha \beta^\beta}$$

$$N_g = 0.25, \quad \alpha = 0.6, \quad \beta = 0.6, \quad \rho = 0.1$$

U. D'Alesio (2019)

Gluon TMDs satisfy positivity bounds:

$$\frac{q_T^2}{2M_P} \left| h_1^{\perp g}(x, \mathbf{q}_T^2) \right| \leq f_1^g(x, \mathbf{q}_T^2)$$

$$\frac{|\mathbf{q}_T|}{M_P} \left| f_{1T}^{\perp g}(x, \mathbf{q}_T^2) \right| \leq f_1^g(x, \mathbf{q}_T^2)$$

D. Boer and C. Pisano (2012)

# PARAMETERIZATION: SPECTATOR MODEL (SM)

A. Bacchetta, F. G. Celiberto, M. Radici, and P. Taelis (2020)

Nucleon is assumed to emit a gluon and the remaining is treated as a single on-shell particle called spectator particle.

Mass of the spectator particle is allowed to take a continuous values described by a spectral function,  $\rho_X(M_X)$

$$\text{Gluon TMDs: } F^g(x, q_t^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}^g(x, q_t^2; M_X) \quad \rho_X(M_X) = \mu^{2a} \left[ \frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right]$$

where,  $A, B, C, D, a, b, \sigma$  are free parameters

At leading-twist, T-even unpolarized and linearly polarized gluon TMDs can be written as

$$\hat{f}_1^g(x, \mathbf{q}_t^2; M_X) = \left[ (2Mxg_1 - x(M + M_X)g_2)^2 \left[ (M_X - M(1 - x))^2 + \mathbf{q}_t^2 \right] + 2\mathbf{q}_t^2(\mathbf{q}_t^2 + xM_X^2)g_2^2 + 2\mathbf{q}_t^2M^2(1 - x)(4g_1^2 - xg_2^2) \right] \times \left[ (2\pi)^3 4xM^2(L_X^2(0) + \mathbf{q}_t^2)^2 \right]^{-1}$$

$$\hat{h}_1^{\perp g}(x, \mathbf{q}_t^2; M_X) = \left[ 4M^2(1 - x)g_1^2 + (L_X^2(0) + \mathbf{q}_t^2)g_2^2 \right] \times \left[ (2\pi)^3 x(L_X^2(0) + \mathbf{q}_t^2)^2 \right]^{-1}$$

Where  $g_{1,2}(p^2)$  are model-dependent form factors, given as:  $g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2(1-x)^2}{(q_t^2 + L_X^2(\Lambda_X^2))^2}$

$p^2$  is gluon momentum,  $\kappa_{1,2}$  and  $\Lambda_X$  are normalization and cut-off parameters respectively, and  $L_X^2(\Lambda_X^2) = xM_X^2 + (1 - x)\Lambda_X^2 - x(1 - x)M^2$

# RESULTS: $J/\psi - jet$ PRODUCTION

PRD 106.034009

$\cos 2\phi_t$  azimuthal asymmetry in

(A) Gaussian Parameterization

(B) Spectator Model

Kinematics:  $\sqrt{s} = 140$  GeV

$0.1 < y < 1$ ,  $0 < q_t < 1$  GeV

$$Q = \sqrt{M_\psi^2 + K_t^2}$$

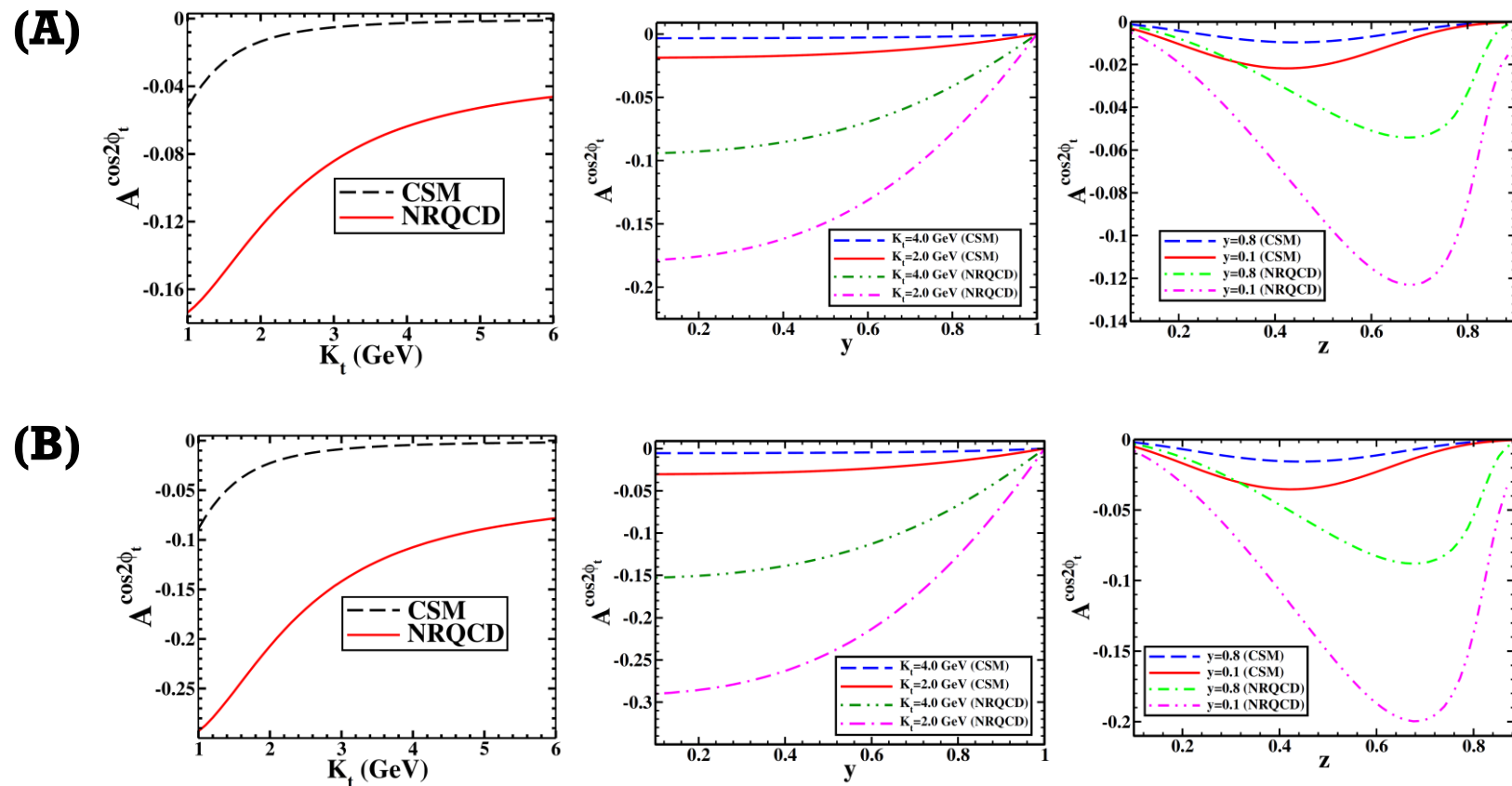
Plots as function of  $K_t$  and  $y$  are at fixed  $z = 0.7$

Significant contribution to  $A^{\cos 2\phi_t}$  coming from color octet states.

Asymmetry hardly change with  $\sqrt{s}$

We used CSMWZ set of LDME

Chao (2012)



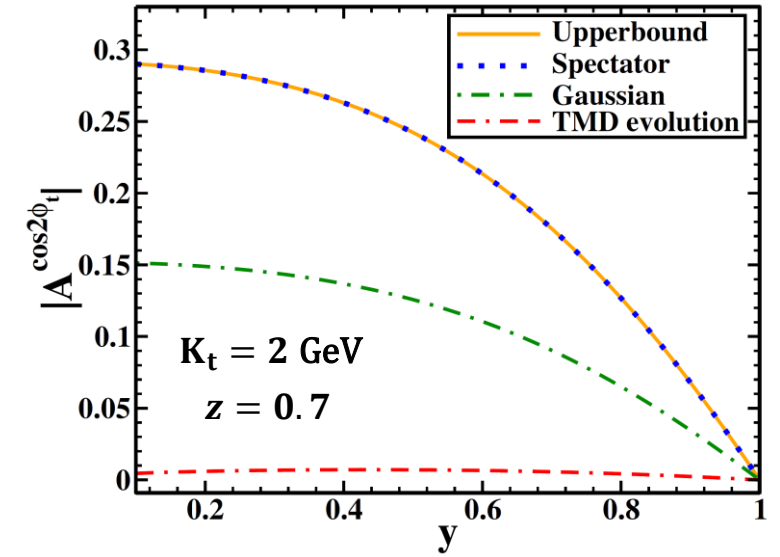
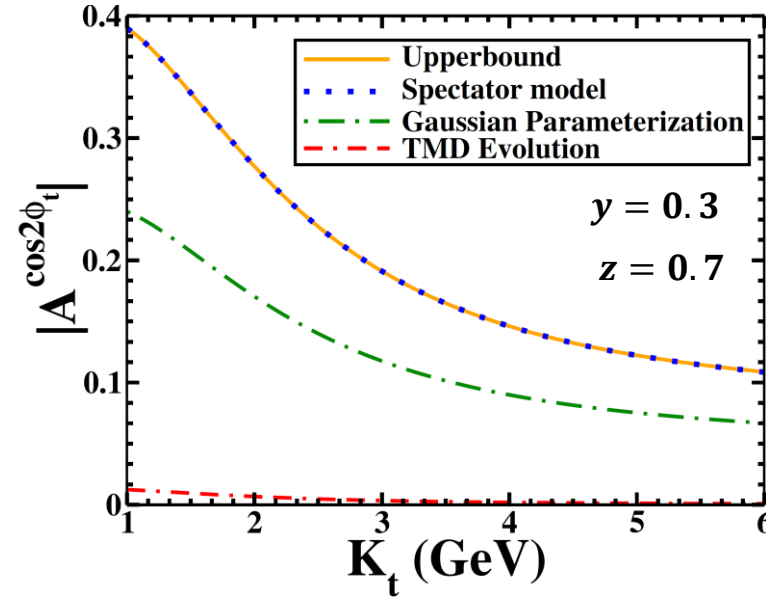


# RESULTS: $J/\psi - jet$ PRODUCTION

PRD 106.034009

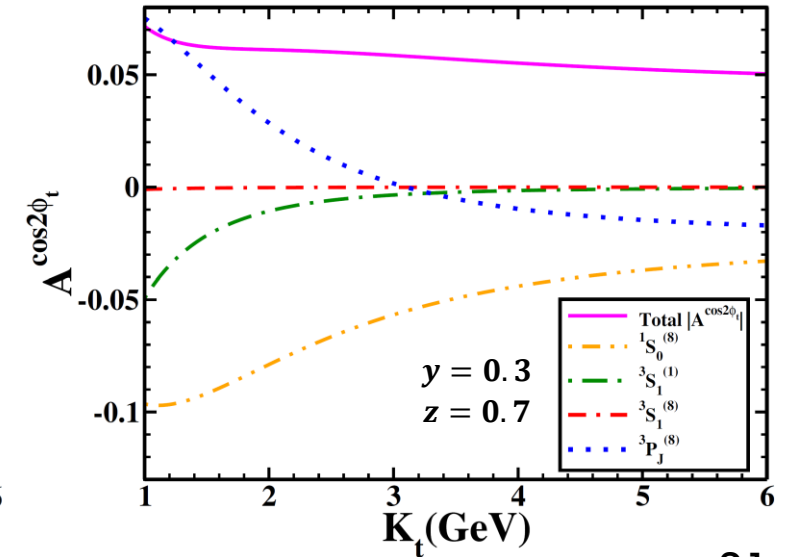
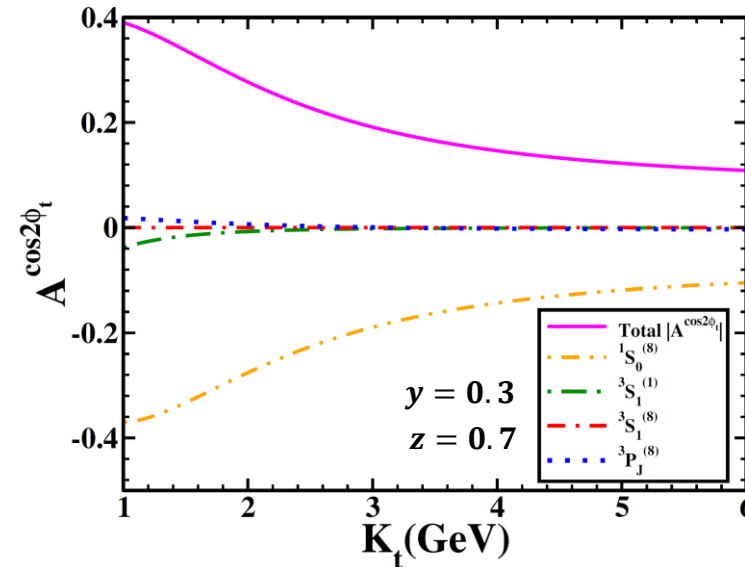
(A) Comparing  $|A^{\cos 2\phi_t}|$  calculated in SM, GP and TMD evolution with the upper bound on the asymmetry.

(A)



(B) Contribution to  $|A^{\cos 2\phi_t}|$  from all the color singlet and color octet states for two sets of LDMEs, CMSWZ (left) and SV (right).

(B)



Chao (2012)

Sharma (2013)

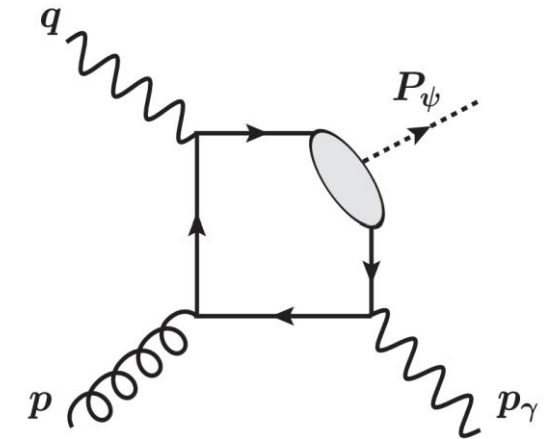
# AMPLITUDE CALCULATIONS USING NRQCD

$$\triangleright e(l) + p(P) \rightarrow e(l') + J/\psi(P_\psi) + \gamma(p_\gamma) + X$$

The amplitude can be written as

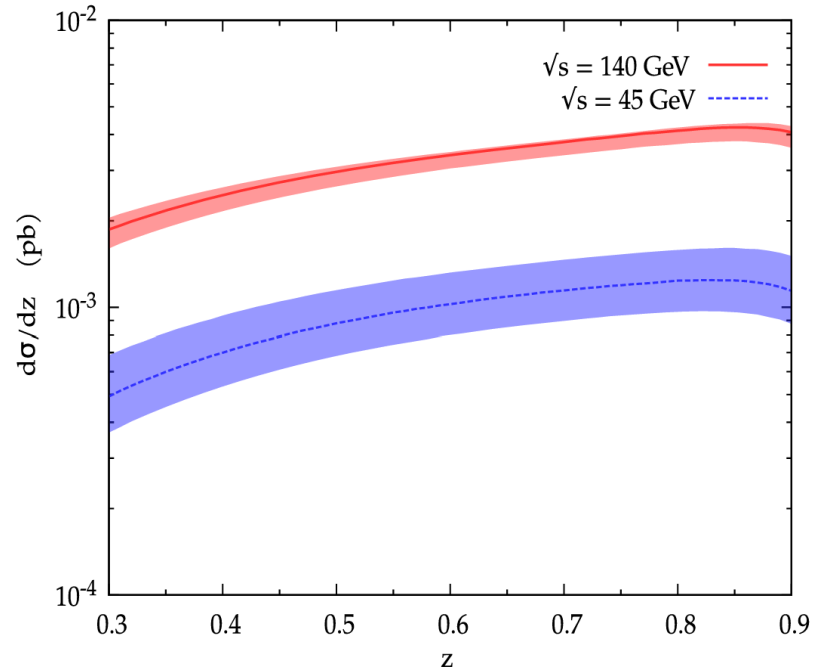
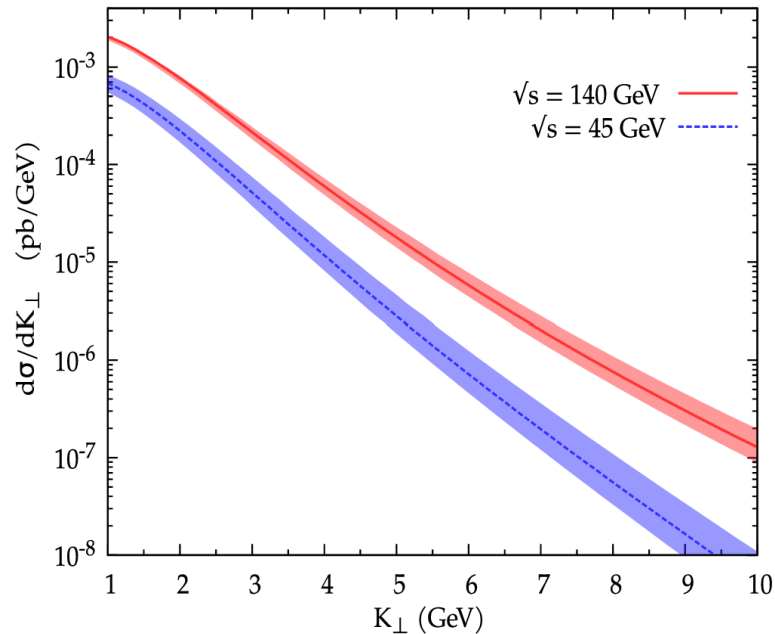
$$\begin{aligned} M(\gamma^* g \rightarrow Q\bar{Q}[{}^{2S+1}L_J^{(1,8)}](P_\psi) + \gamma(p_\gamma)) \\ = \sum_{L_z S_z} \int \frac{d^3 k}{(2\pi)^3} \Psi_{LL_z}(k) \langle LL_z; SS_z | JJ_z \rangle \text{Tr}[\mathcal{O}(q, p, P_\psi, k) \mathcal{P}_{SS_z}(P_\psi, k)] \end{aligned}$$

D. Boer and C. Pisano (2012)



Contribution:  ${}^3S_1^{(8)}$

# UNPOLARIZED CROSS SECTION: $J\psi - \gamma$ PRODUCTION



$Z < 0.9$  : avoid contribution from diffractive process and prevents hitting ultraviolet divergences.

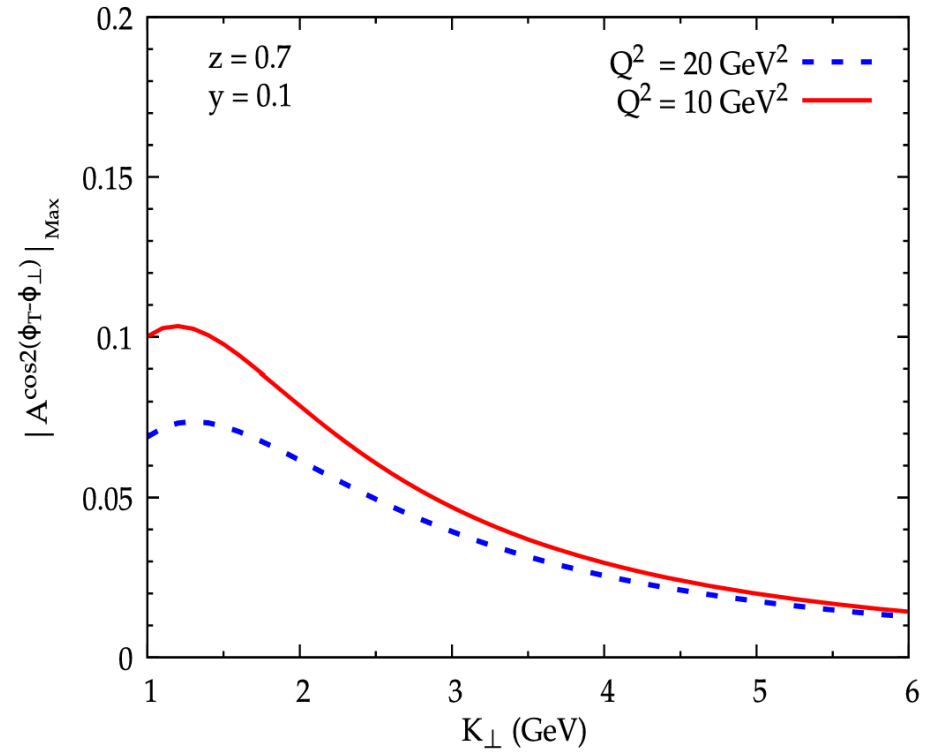
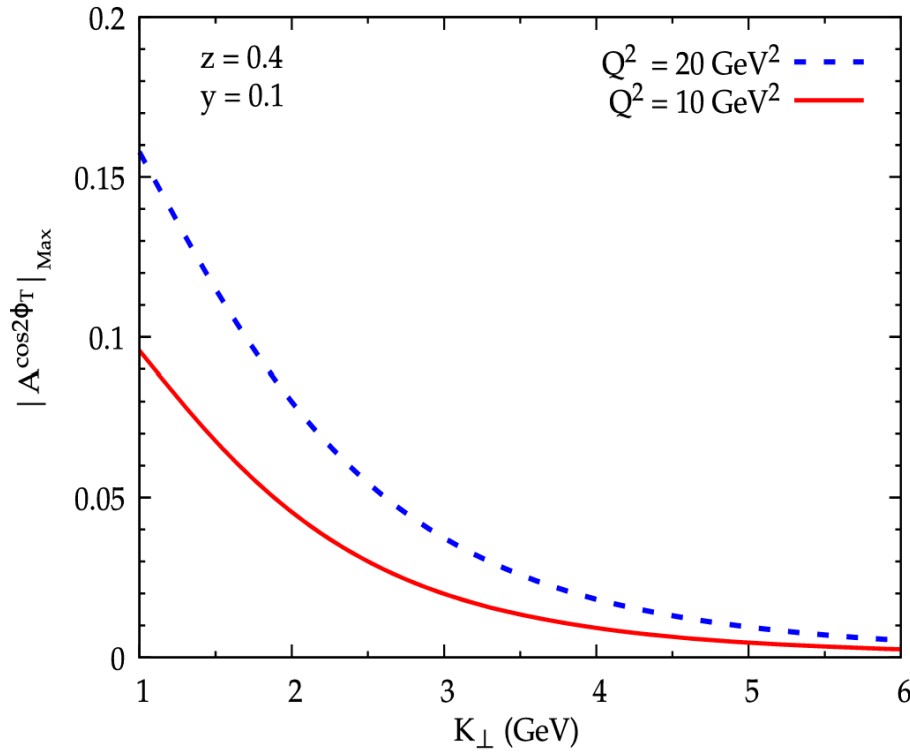
$0.3 < Z$  : avoid contribution via resolved-photon channel

A sizable cross section, expected to be detected at upcoming EIC.

Its measurement can provide a clean probe CO mechanism within NRQCD framework.

Provide clean extraction of a CO LDME

# ASYMMETRY AT EIC: $J/\psi - \gamma$ PRODUCTION



Model independent Upper bounds on the asymmetries:

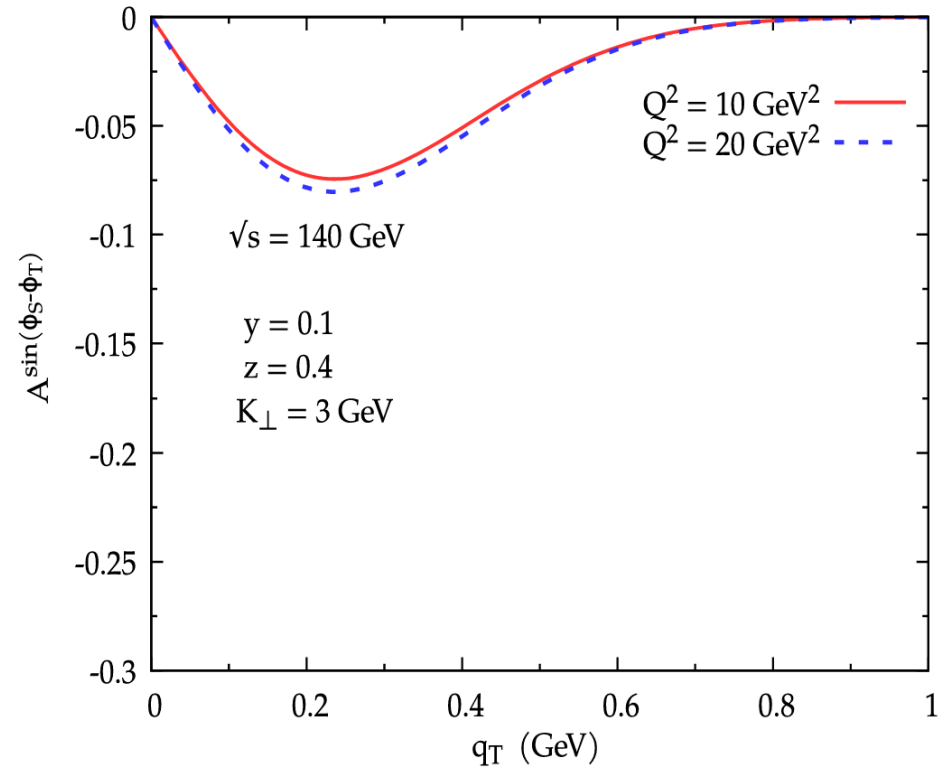
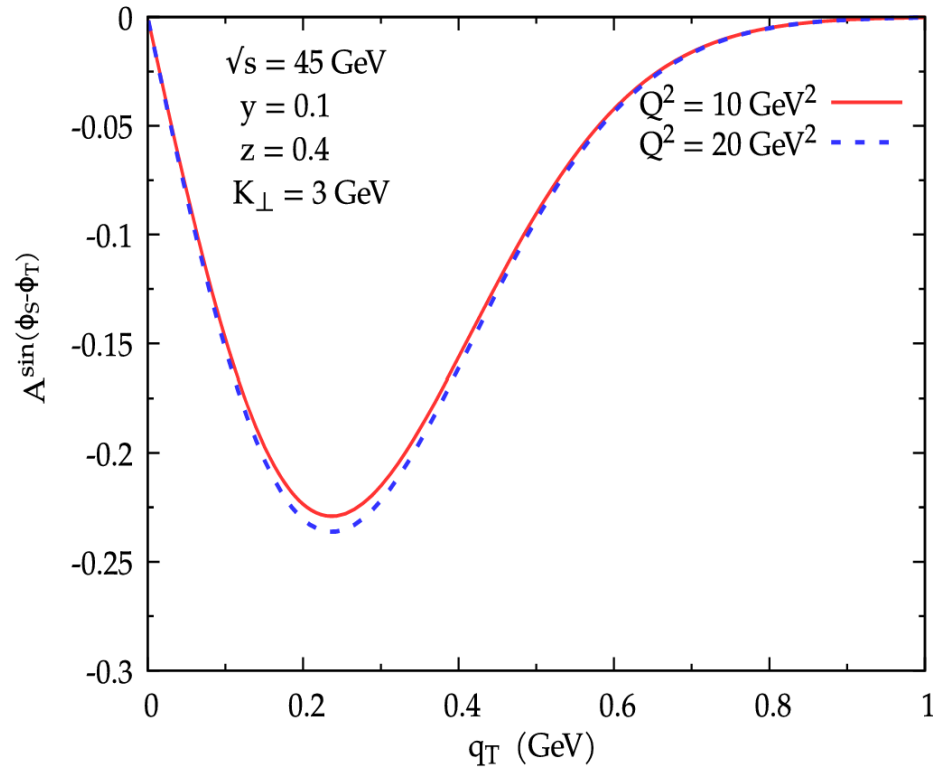
$$|A^{\cos 2\phi_T}| \leq 2 \frac{|\mathcal{B}_0|}{\mathcal{A}_0}, \quad |A^{\cos 2(\phi_T - \phi_{\perp})}| \leq 2 \frac{|\mathcal{B}_2|}{\mathcal{A}_0}$$

Asymmetries are independent of center of mass energy.

Asymmetries are independent of LDMEs.

D. Chakrabarti, RK, A. Mukherjee, R. Sangem, PRD.107.014008

# ASYMMETRY AT EIC: $J/\psi - \gamma$ PRODUCTION



Upper bounds on the Sivers asymmetry is 1.

$q_T$  dependent of Sivers asymmetry is obtained using Gaussian parameterization. Asymmetry depends on center of mass energy. Asymmetry hardly depends on virtuality.

# CONCLUSION

We calculated the azimuthal asymmetry in  $J/\psi - jet$  and  $J/\psi - \gamma$  electroproductions where the pair produced in a almost back-to-back in the transverse plane.

We used NRQCD framework for  $J/\psi$  production.

$A^{\cos(2\phi_T)}$  and  $A^{\cos 2(\phi_T - \phi_\perp)}$  azimuthal asymmetries probes the linearly polarized gluon TMD, where as the  $A^{\sin(\phi_S - \phi_T)}$  can probe gluon Sivers function.

We show the numerical estimates of the asymmetries in a *model dependent parameterizations of the TMDs* and *model independent upper bounds on these asymmetries using the positivity bounds on TMDs*. **We found a sizable azimuthal asymmetries both in  $J/\psi - jet$  and  $J/\psi - \gamma$  productions.**

Back-to-back  $J/\psi - jet$  and  $J/\psi - \gamma$  electroproduction could be a promising channel to probe poorly known gluon TMDs at the future proposed EIC.

**Thank You**