Luka Leskovec

A lattice QCD study of $B \to \rho \ell \bar{\nu}$

ACHT 2023
Non-perturbative aspects of
Nuclear, Particle and Astroparticle Physics
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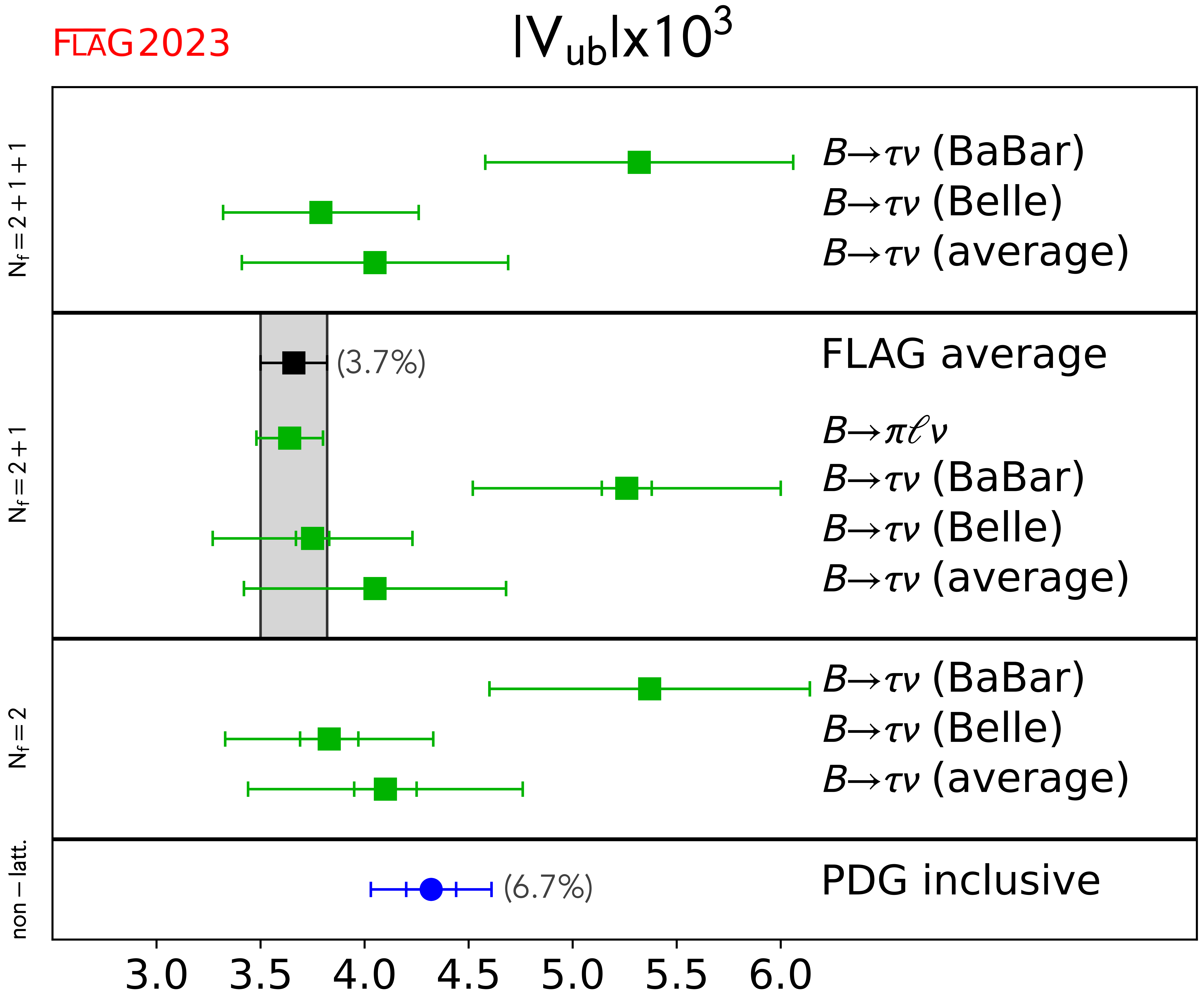
University *of Ljubljana*
Faculty *of Mathematics and Physics*

in collaboration with:
S. Meinel, M. Petschlies, C. Alexandrou,
J.W. Negele, S. Paul, A. Pochinsky

Institut
"Jožef Stefan"
Ljubljana, Slovenija

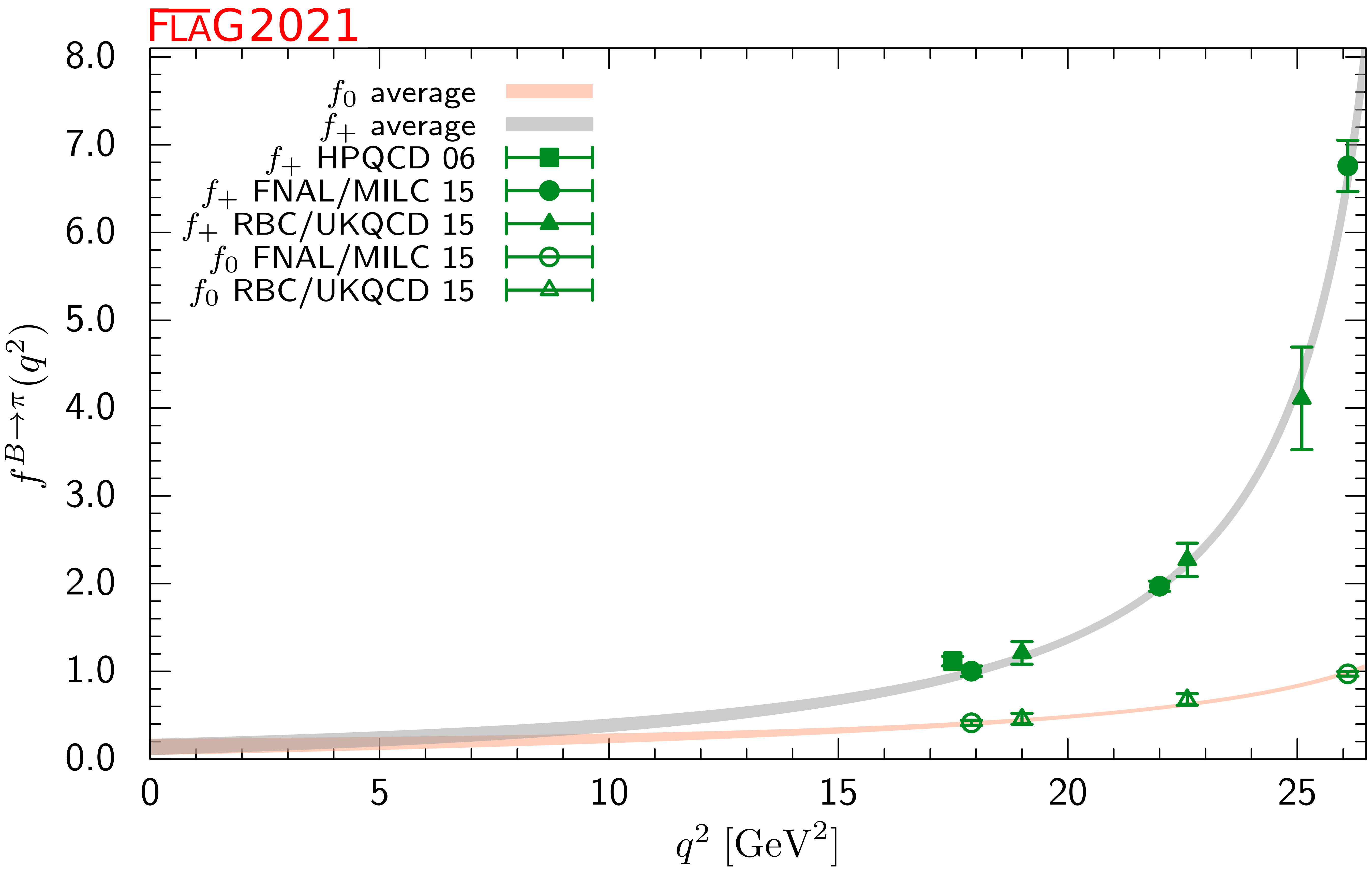

u
c
t
d
s
b
Quarks
e
μ
τ
ν e
ν μ
ν τ
Leptons
Higgs boson
W+
γ
Z0
g
W-
Forces
CERN
ACCELERATING SCIENCE

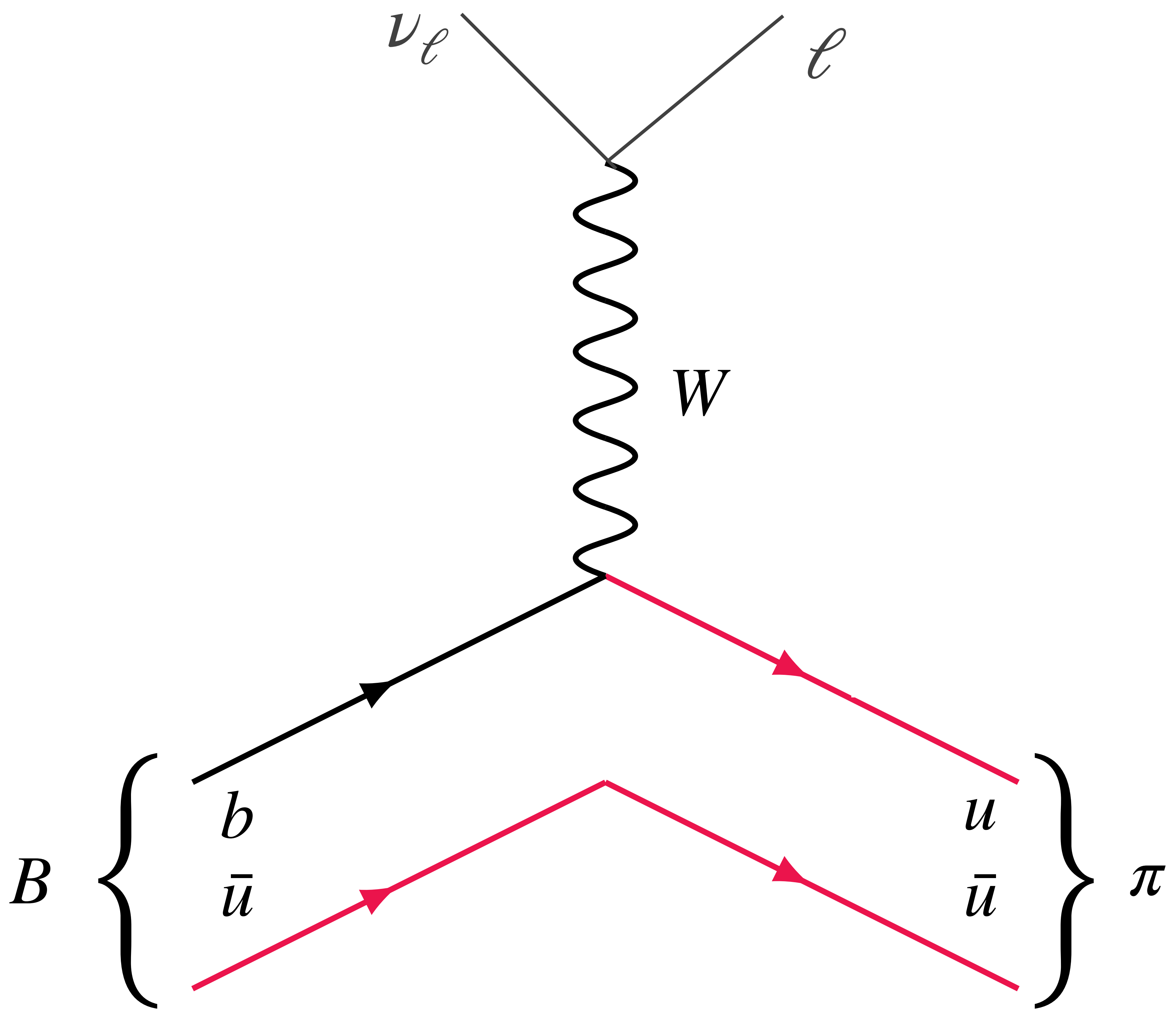


FLAG2023
|Vub|x10^3
Nf=2+1+1
B→τν (BaBar)
B→τν (Belle)
B→τν (average)
Nf=2+1
(3.7%)
FLAG average
B→πℓν
B→τν (BaBar)
B→τν (Belle)
B→τν (average)
Nf=2
B→τν (BaBar)
B→τν (Belle)
B→τν (average)
non−latt.
(6.7%)
PDG inclusive
3.0
3.5
4.0
4.5
5.0
5.5
6.0

example: $B \to \pi \ell \nu$

$$\langle \pi | J^{\mu}_{V-A} | B \rangle = f_+(q^2)\left[p_B^\mu + p_\pi^\mu + \frac{M_B^2 - m_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{M_B^2 - m_\pi^2}{q^2} q^\mu$$

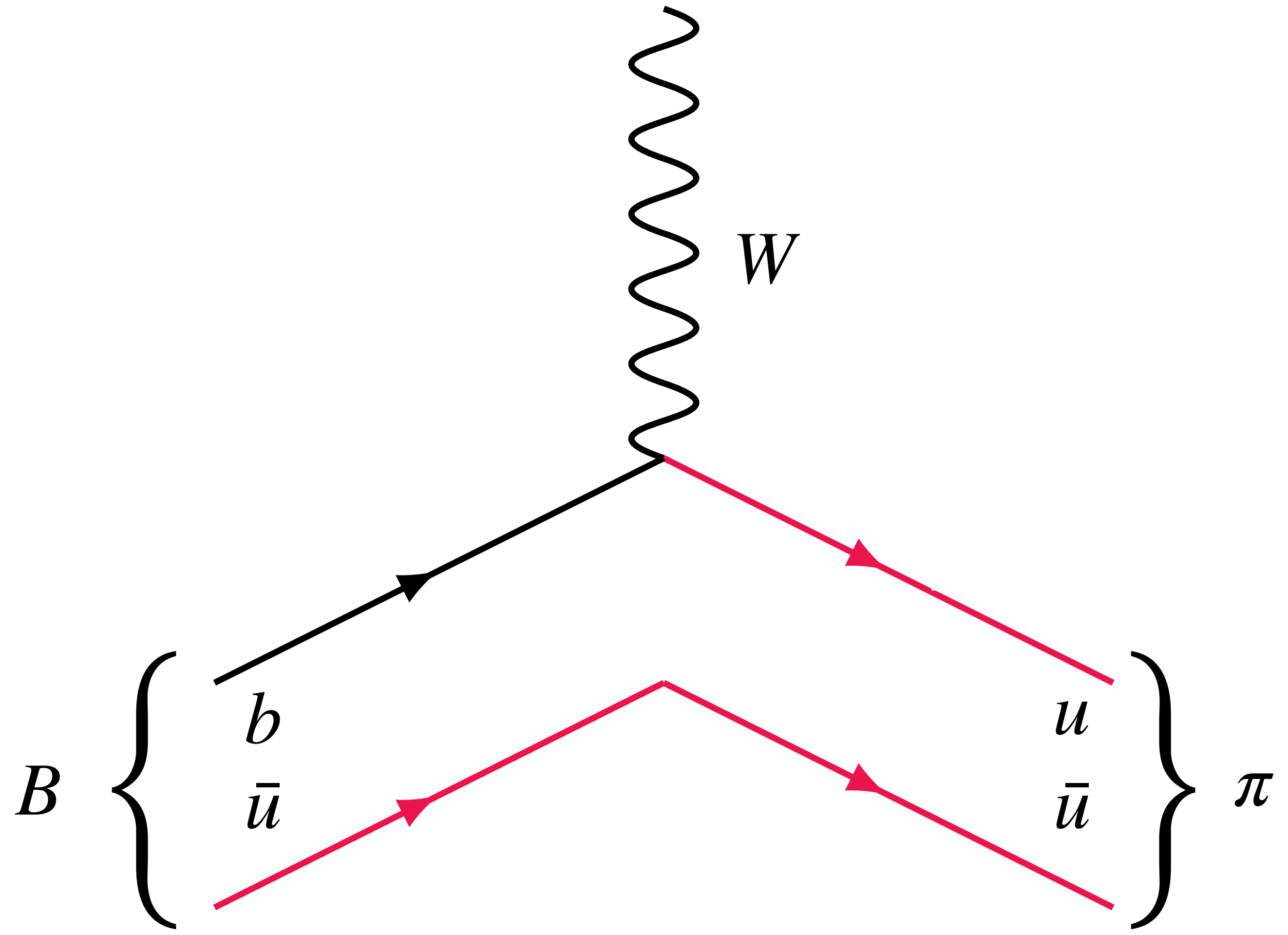




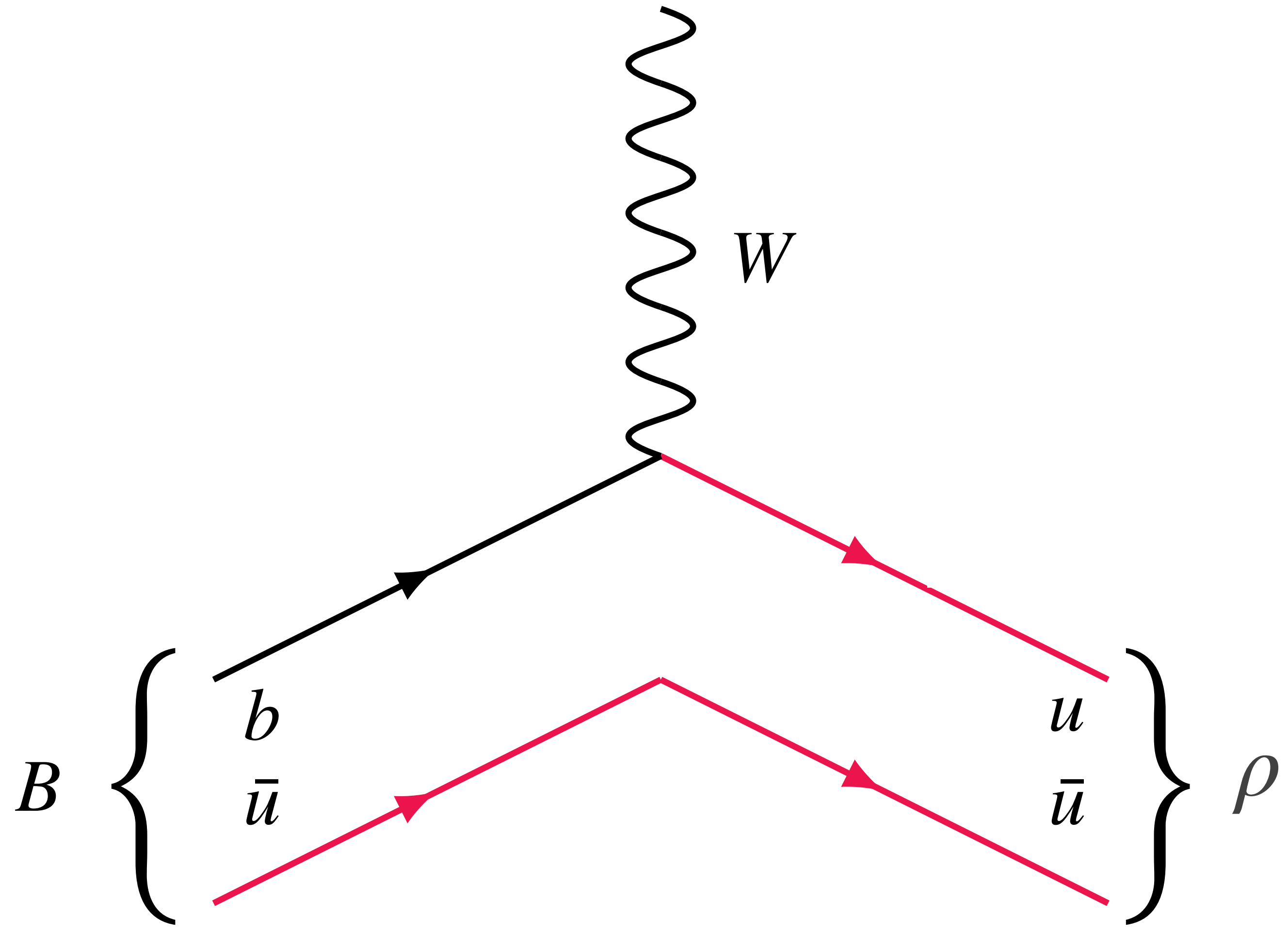
$$\frac{d\mathbf{B}}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3 q^2} \vec{p}_\pi^{\,3} |f_+(q^2)|^2$$

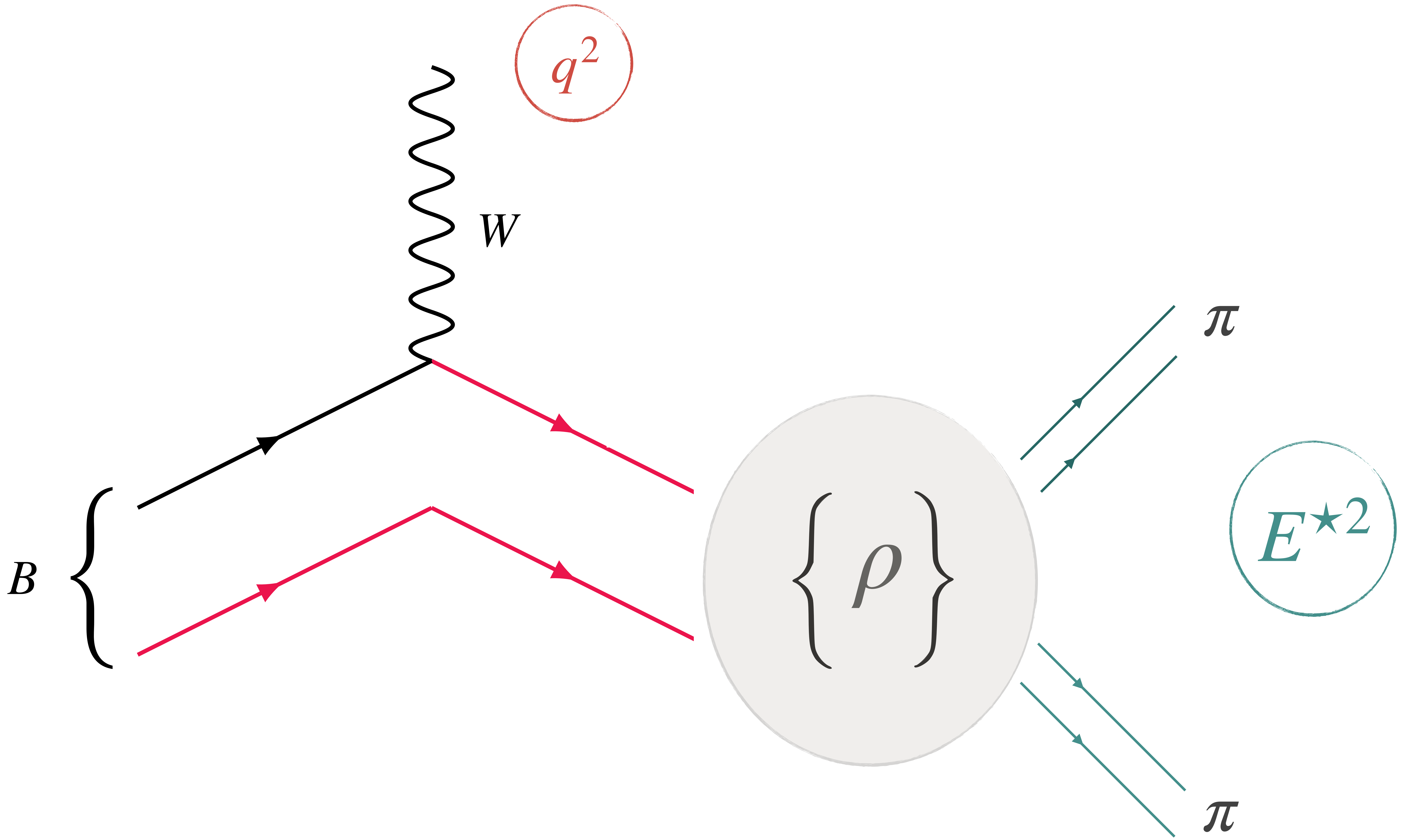
a novel process: $B \to \rho \ell \bar{\nu}$?

- $B \to \pi \ell \nu$
  - f_+, f_0
  - established
  - very precise



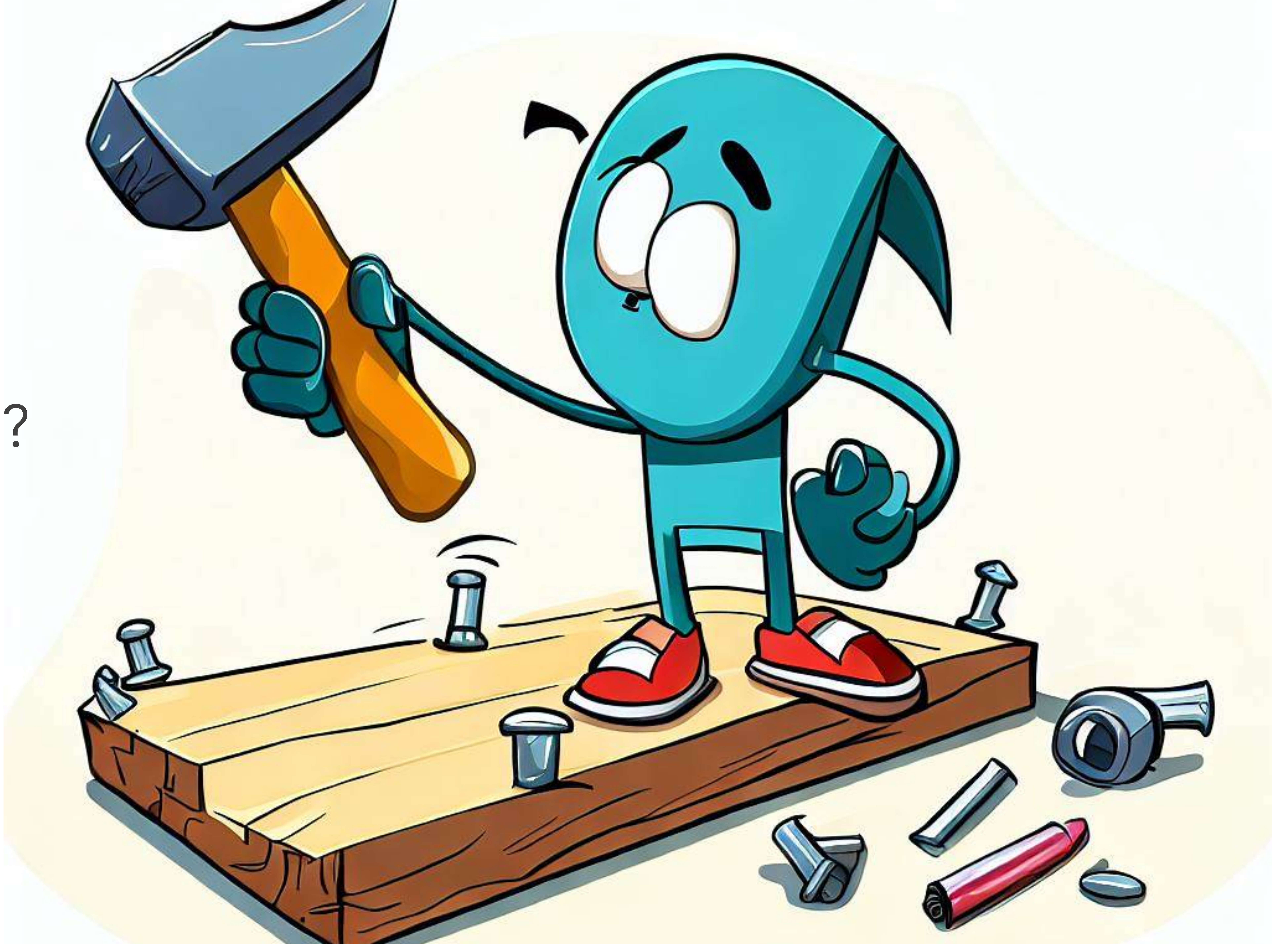
- $B \to \rho \ell \nu$
  - V, A_0, A_1, A_2
  - new





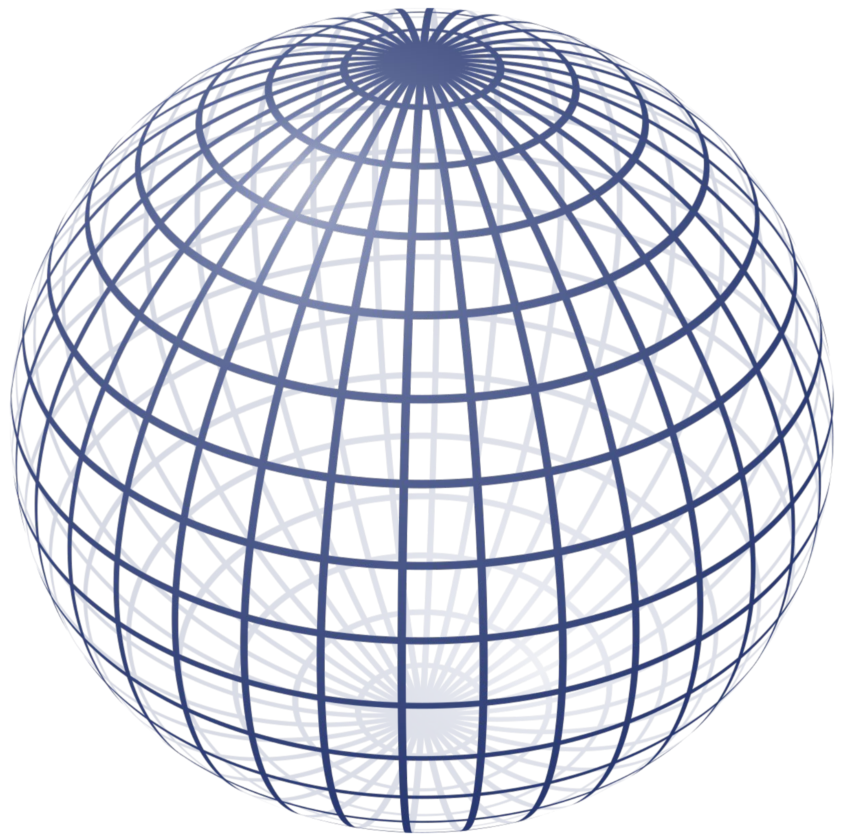
q^2
W
B
$\{\rho\}$
π
$E^{\star 2}$
π

how to
achieve this?

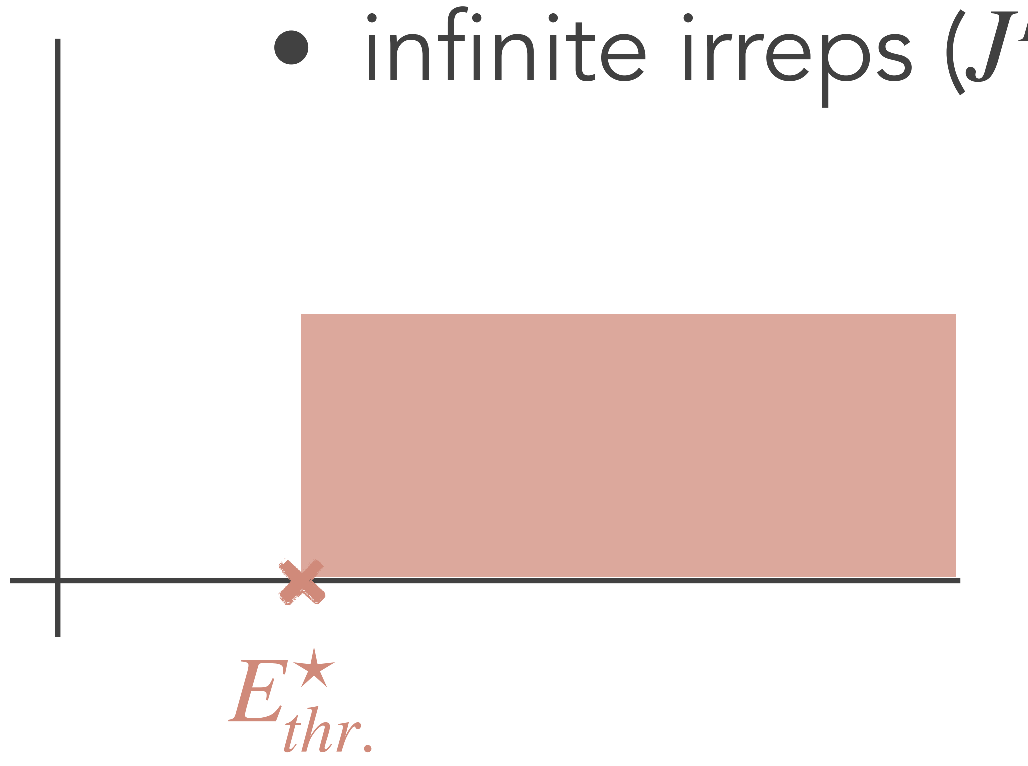


scattering on the lattice

infinite volume:



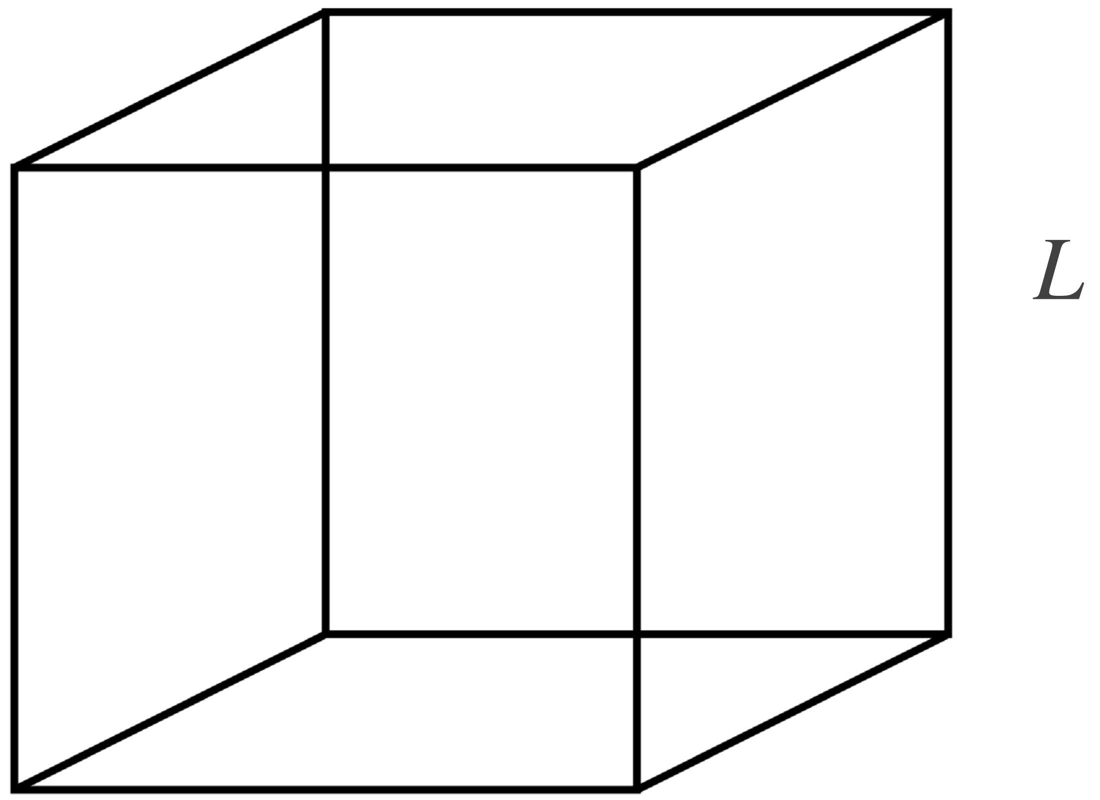
- $O(3)$ symmetry
- infinite irreps (J^P)



$\langle p_1 \; p_2 \rangle = 2E\, \delta(p_1 - p_2)$

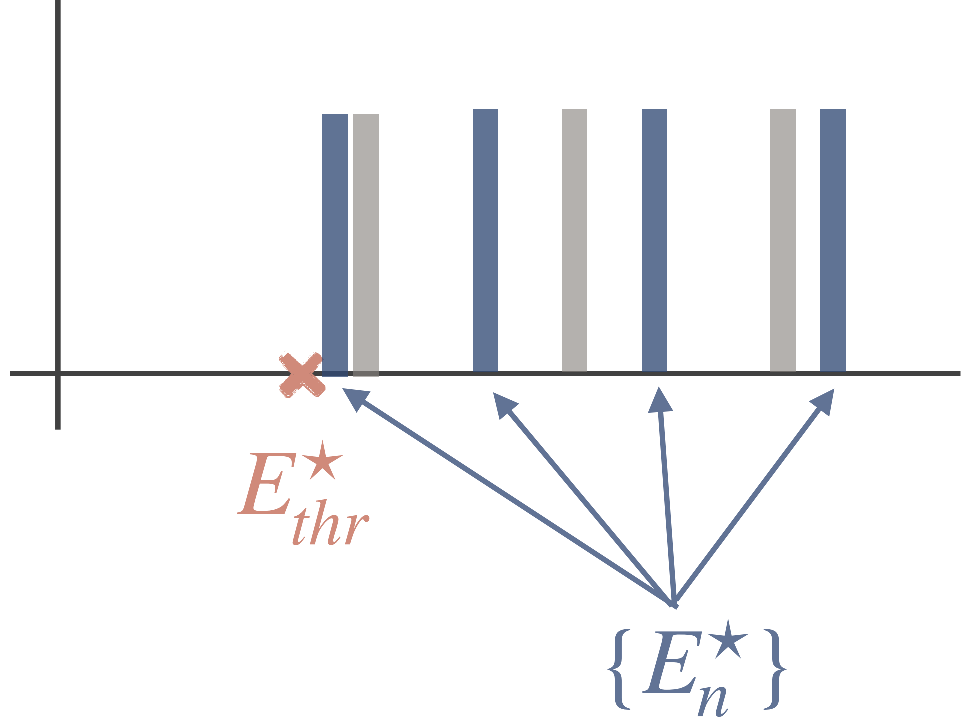
many-to-one

finite volume:



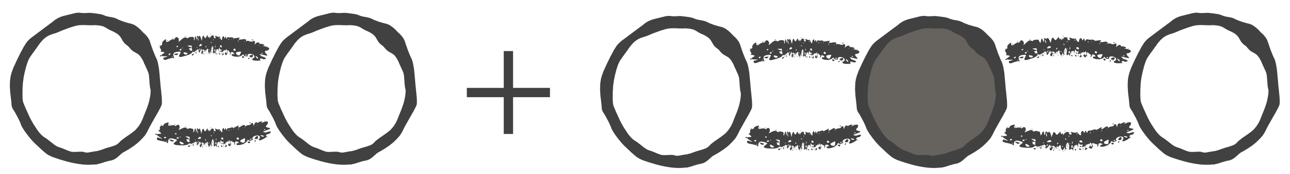
- discrete symmetries, Λ

$\vec{p}_i = \frac{2\pi}{L}\vec{m} \quad \vec{m} \in Z^3$



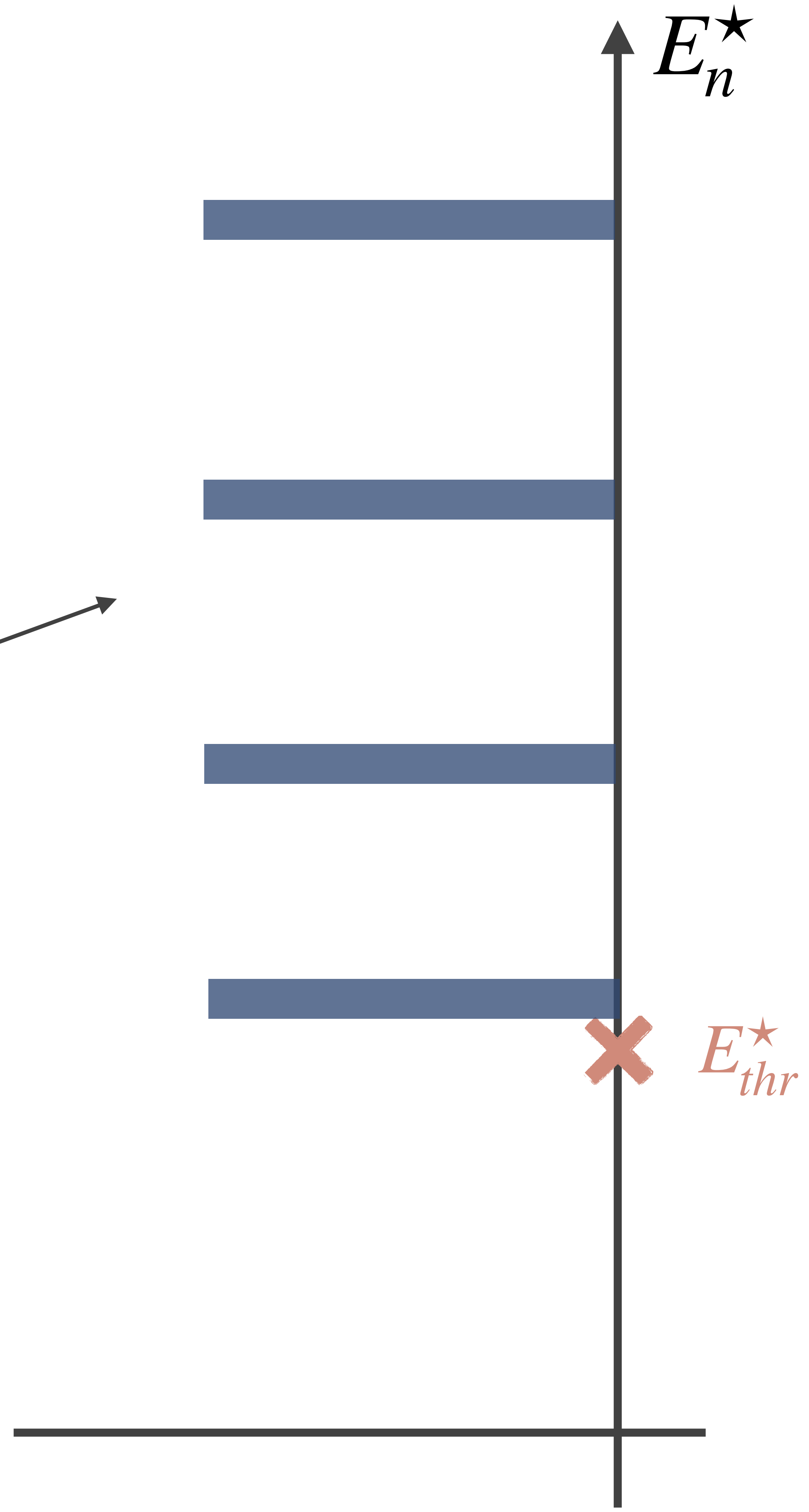
$\langle p_1 \; p_2 \rangle \neq 2E\, \delta(p_1 - p_2)$

lattice states: energies

$C_L^{(2)} =$  $+ \ldots$

$$C_L^{(2)} = C_\infty^{(2)} - A' \frac{1}{F^{-1}(E^\star) + T(E^\star)} A$$

poles!

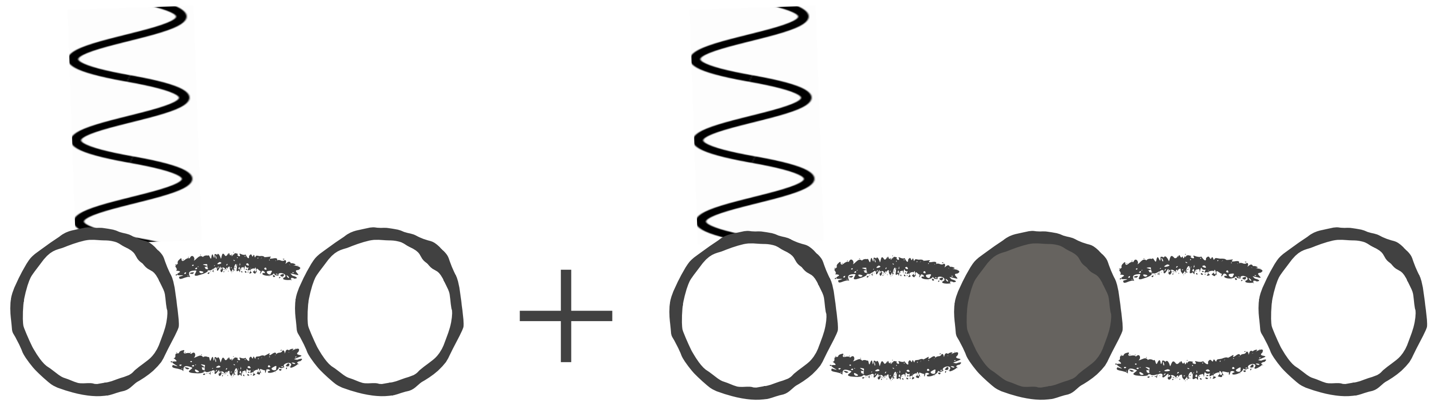


discrete spectrum where:

$$\det\left[F^{-1}(E^\star) + T(E^\star)\right] = 0$$

Luscher NPB354
Rummukainen, Gottlieb hep-lat/9503028
Kim, Sharpe, Sachrajda hep-lat/0507006
Leskovec, Prelovsek hep-lat/1202.2145
Briceno 1401.3312
Briceno, Dudek, Young 1706.06223
Woss, Wilson, Dudek 2001.08474
[and many more]

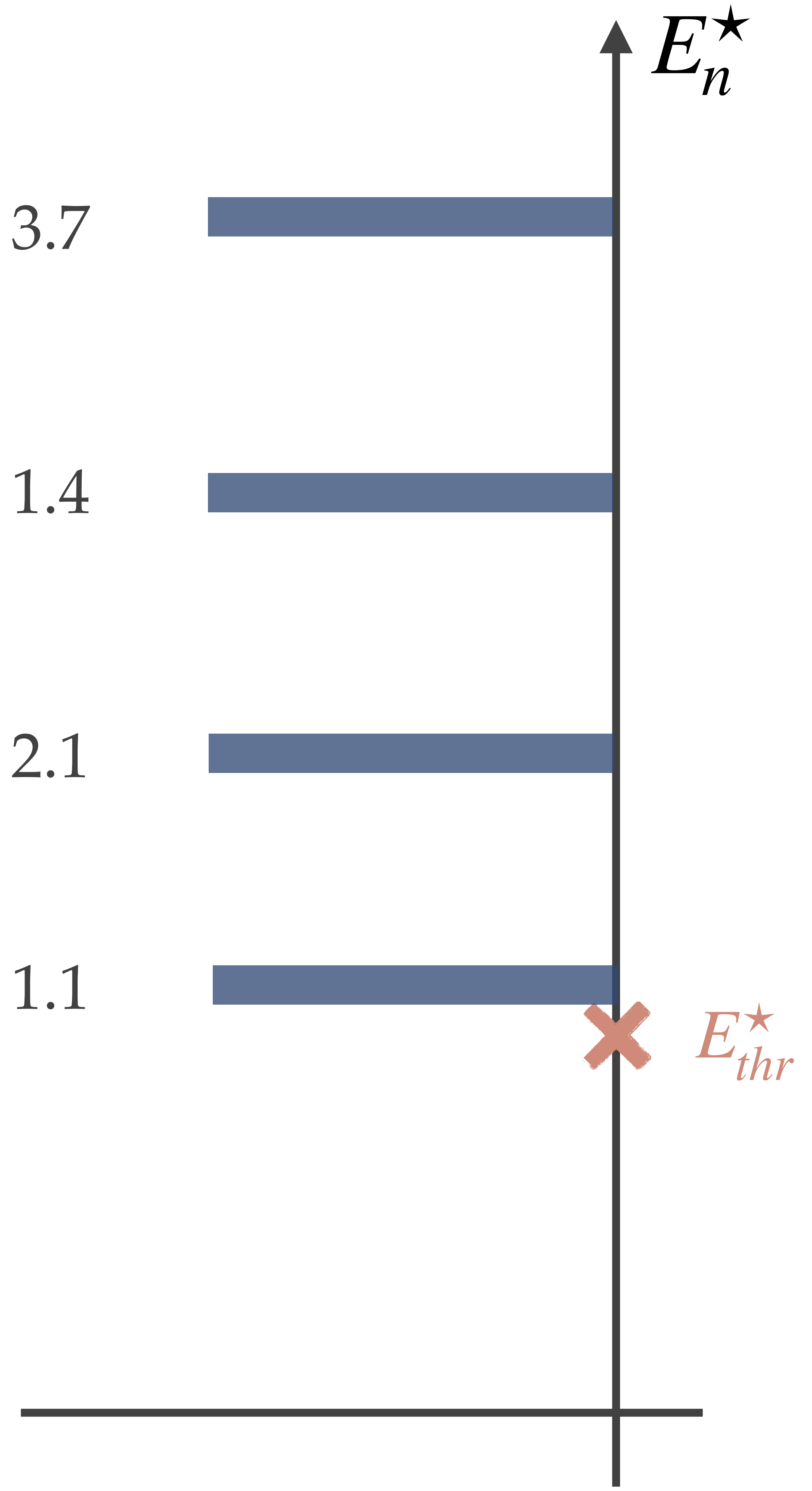
lattice states: normalization

$C_L^{(3)} =$  $+ \ldots$

$$C_L^{(3)} = C_\infty^{(3)} - A'RA$$

$$R = \lim_{E \to E_n} \frac{E - E_n}{F^{-1} + T}$$

residue of pole!



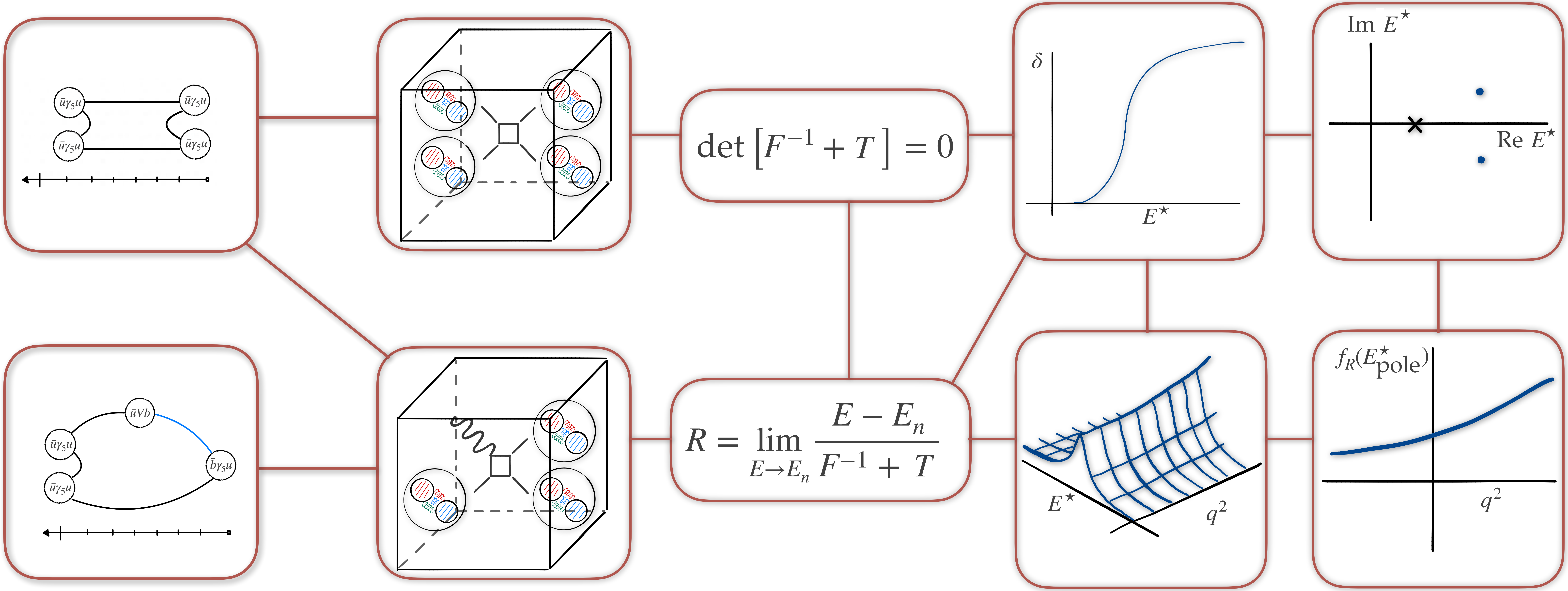
normalization of finite-volume states

$$E_n^{\Lambda\star}\rangle_L \sim \sqrt{R}\;\; p_1 p_2(E^\star = E_n^{\Lambda\star})\rangle_\infty$$

the "Lellouch-Lüscher" factor

Lellouch, Luscher hep-lat/0003023
Lin, Sachrajda, Testa hep-lat/0104006
…
Briceno, Hansen, Walker-Loud 1406.5965
Briceno, Hansen 1502.04314
Briceno, Dudek, LL 2105.02017

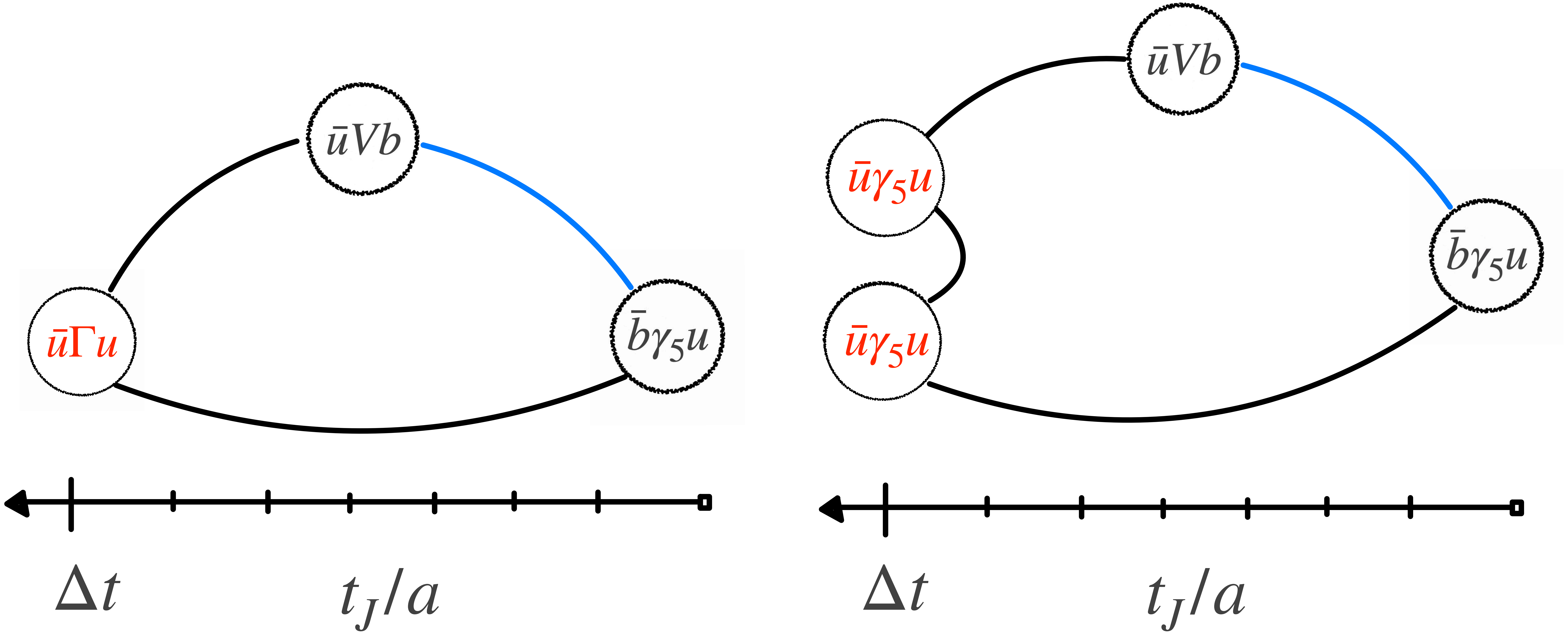
a cartoon scheme of $B \to \pi\pi\ell\bar{\nu}$



Briceño-Hansen-Walker-Loud approach

3-point functions

$$C_{3,i} = \langle\ O_i(\vec{p}, \Lambda)\ \ V^\mu\ \ O_B(\vec{p}_B)\ \rangle$$



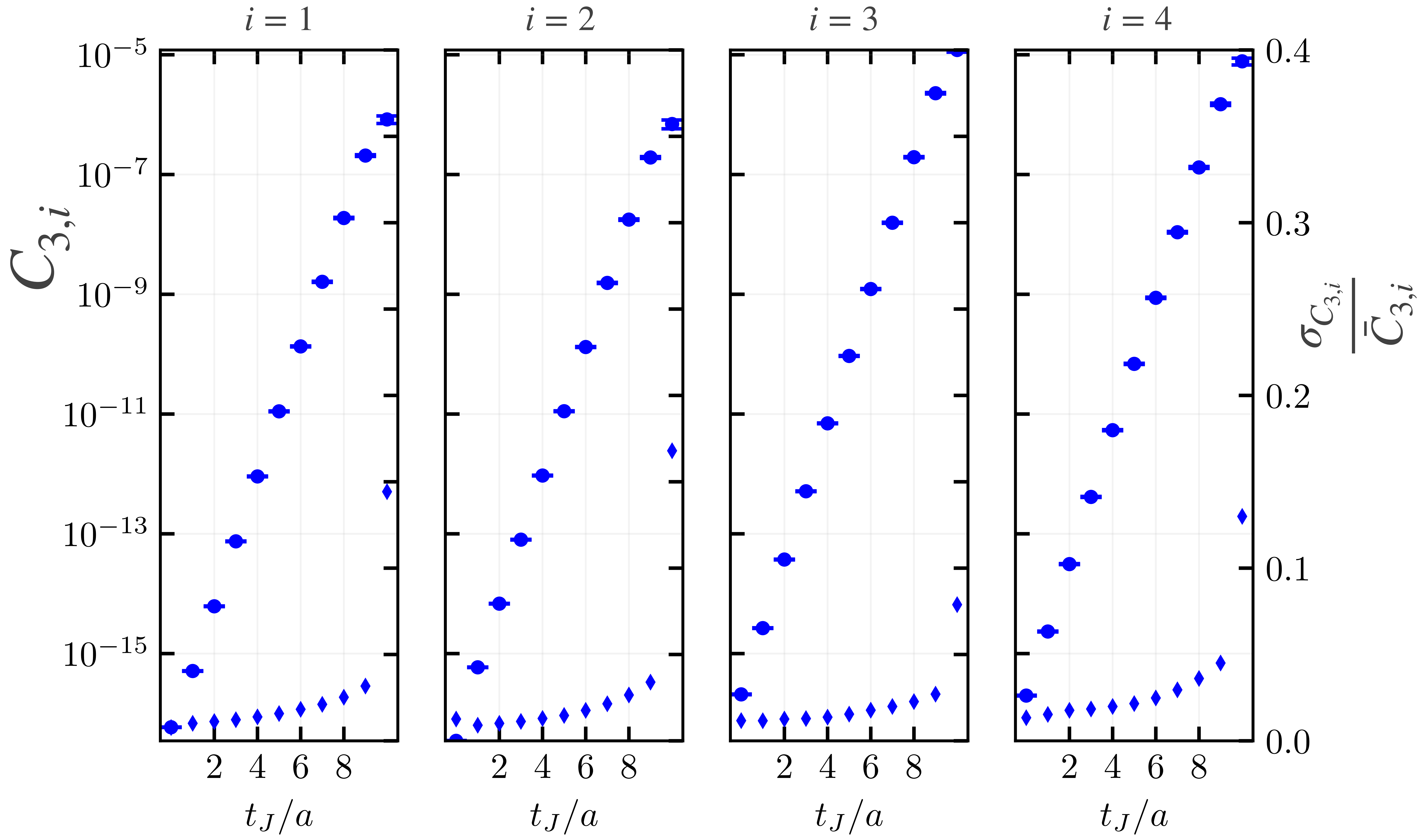
3-point function lattice data

$\vec{p} = \frac{2\pi}{L}[1,0,1]$
$\Lambda = B_2$

$\mu = z$

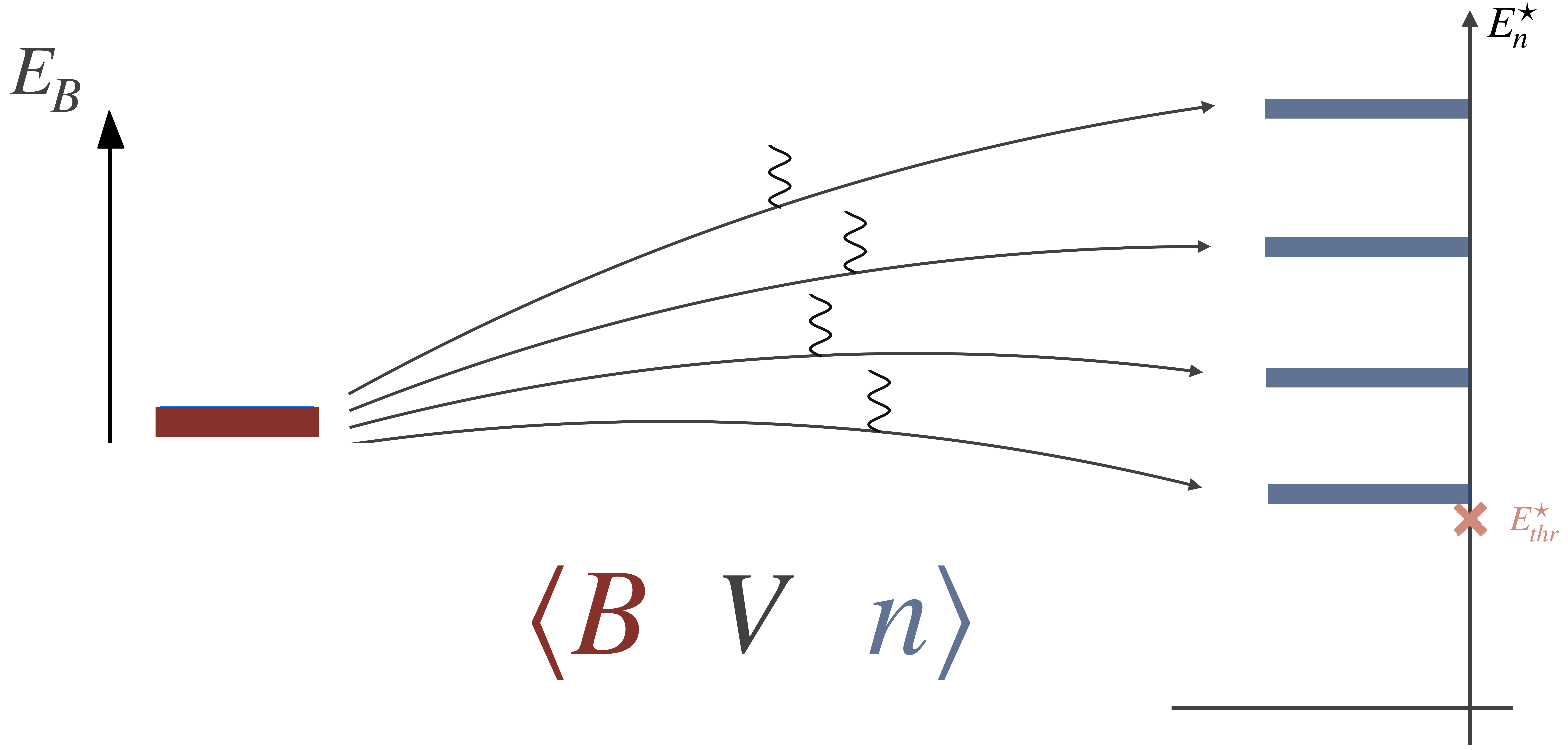
$\vec{p}_B = \frac{2\pi}{L}[0,0,1]$

$$C_{3,i} = < O_i(\vec{p}, \Lambda)\ V^\mu\ O_B(\vec{p}_B)\rangle$$



$$O_1 = \bar{u}\Gamma_{B_1}u$$
$$O_2 = \bar{u}\gamma_t\Gamma_{B_1}u$$
$$O_3 = \pi(\vec{p}_1)\pi(\vec{p}_2)$$
$$O_4 = \pi(\vec{p}_1)\pi(\vec{p}_2)$$

matrix elements



$\langle B \; V \; n \rangle$

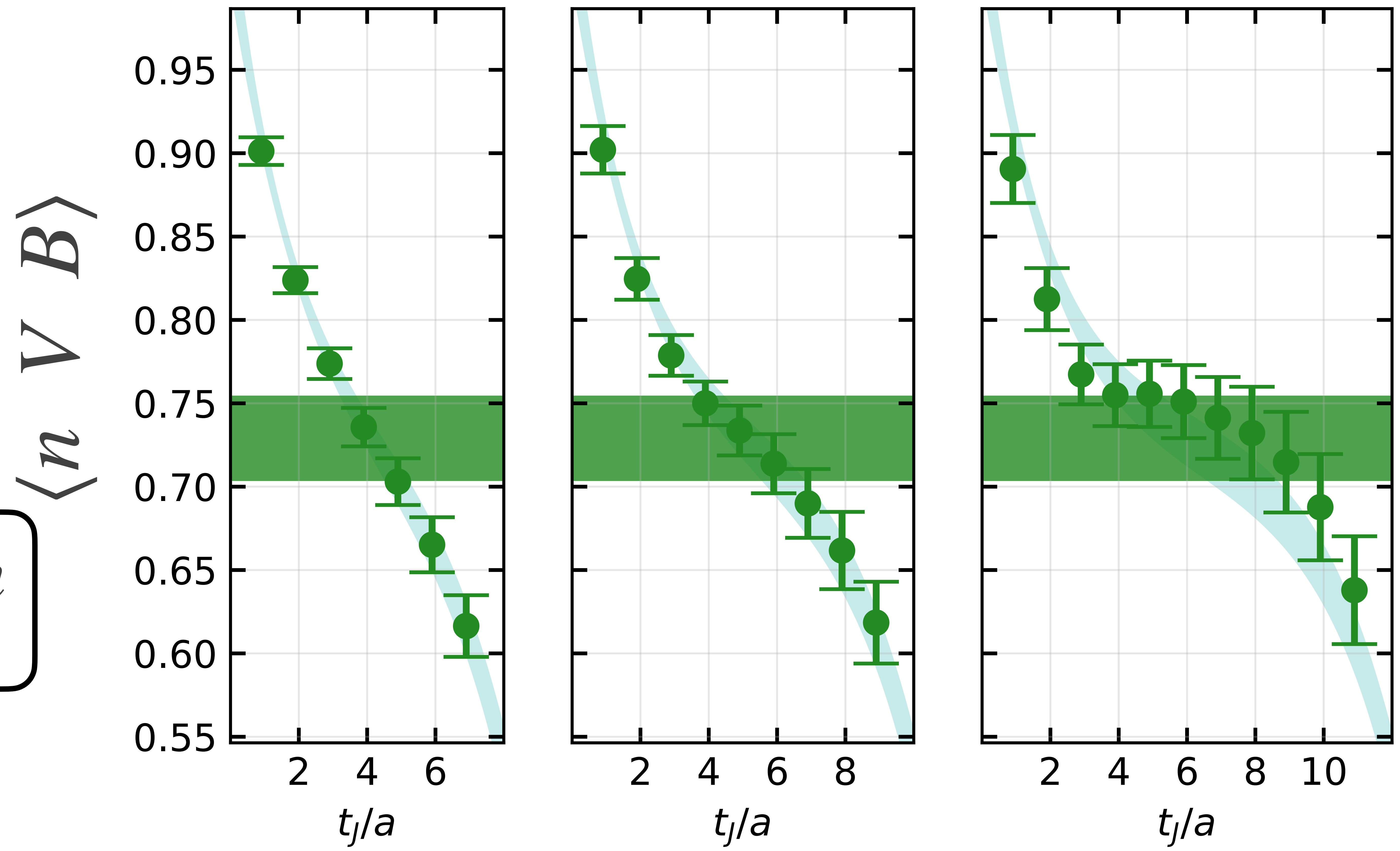
state projection

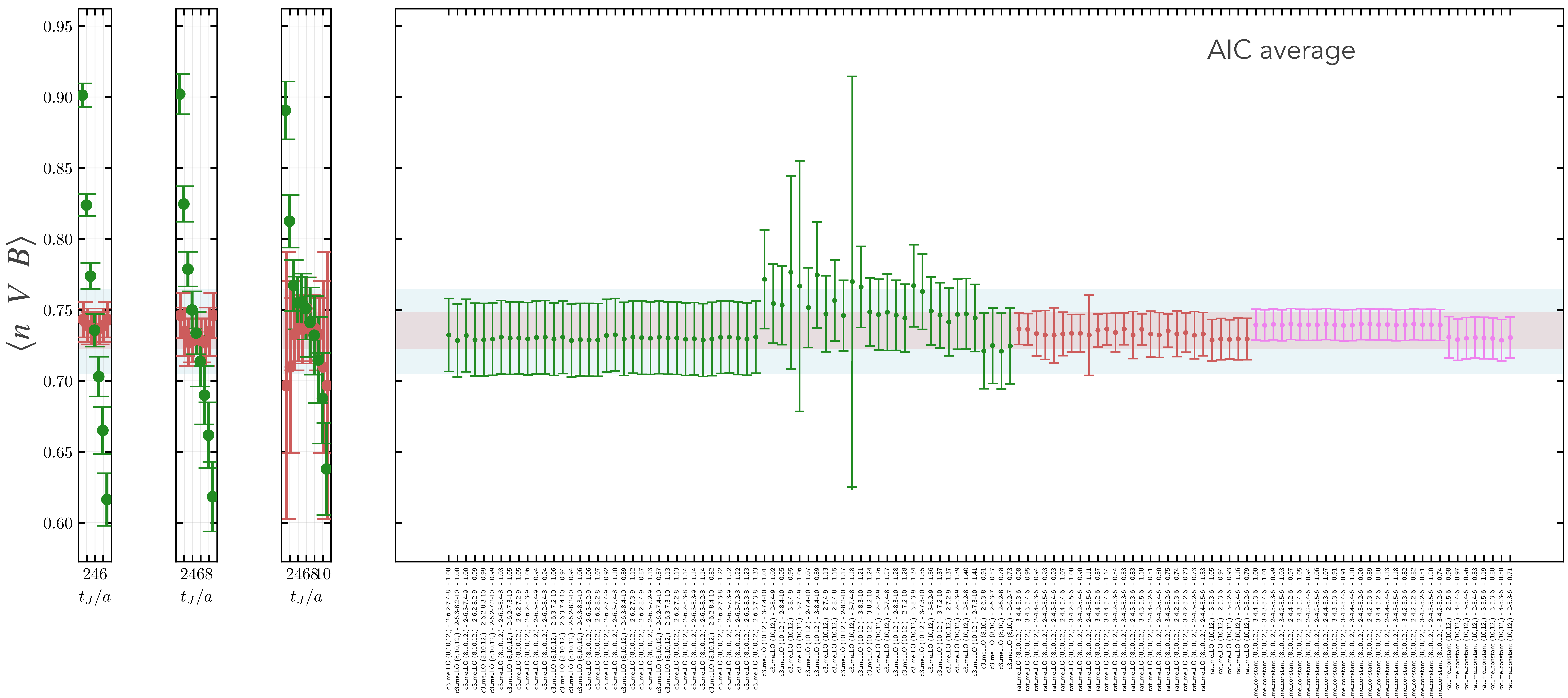
$$C_{3,i} = \sum_{n\in[\pi\pi]} Z_i^n \langle B \;\; V \;\; n\rangle Z_B \frac{e^{-E_n(\Delta t - t)} e^{-E_B t}}{2E_n 2E_B}$$

$$C_3^n = u_i^n C_{3,i}$$

because in GEVP

$$u_i^{n\star} Z_i^m = 2E_n \; e^{E_n t_0} \; \delta^{nm}$$





a multitude of models…

Jay & Neil, 2008.01069

transition amplitude

Boyd, Grinstein, Lebed hep-ph/9412324
Bourrely, Caprini, Lellouch 0807.2722
Alexandrou, LL, Meinel et al. 1807.08357

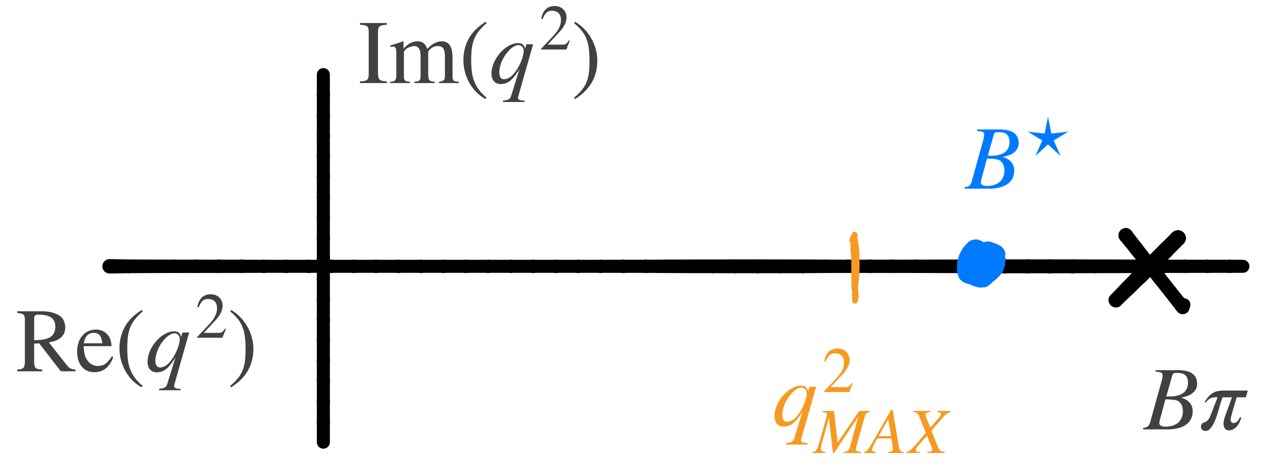
$q = p_f - p_i$

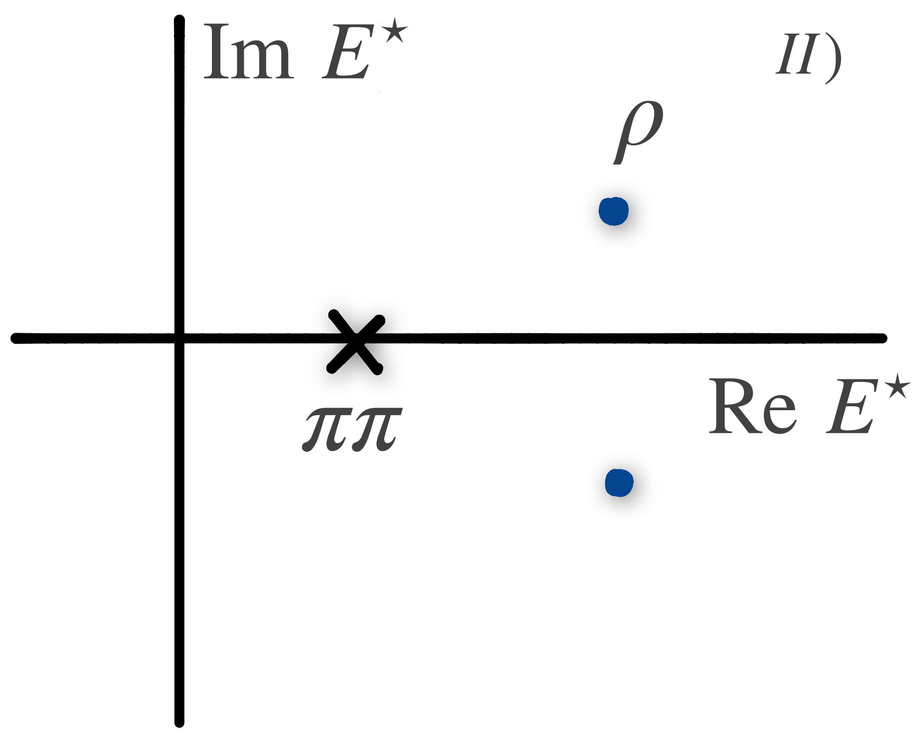
$E^\star = 2\sqrt{m_\pi^2 + k^2}$

$$\langle \pi\pi, E^\star \;\; V \;\; B, p_B \rangle_\infty = \frac{2iV(E^\star, q^2)}{m_B + 2m_\pi} \epsilon_{\mu\nu\alpha\beta}\, \varepsilon^{\nu *}\, p^\alpha p_B^\beta$$

$$V(E^\star, q^2) = F(E^\star, q^2)\frac{T(E^\star)}{k}$$

$$T = \frac{E^\star \Gamma_i}{m_R^2 - E^{\star 2} - iE^\star \Gamma_i}$$





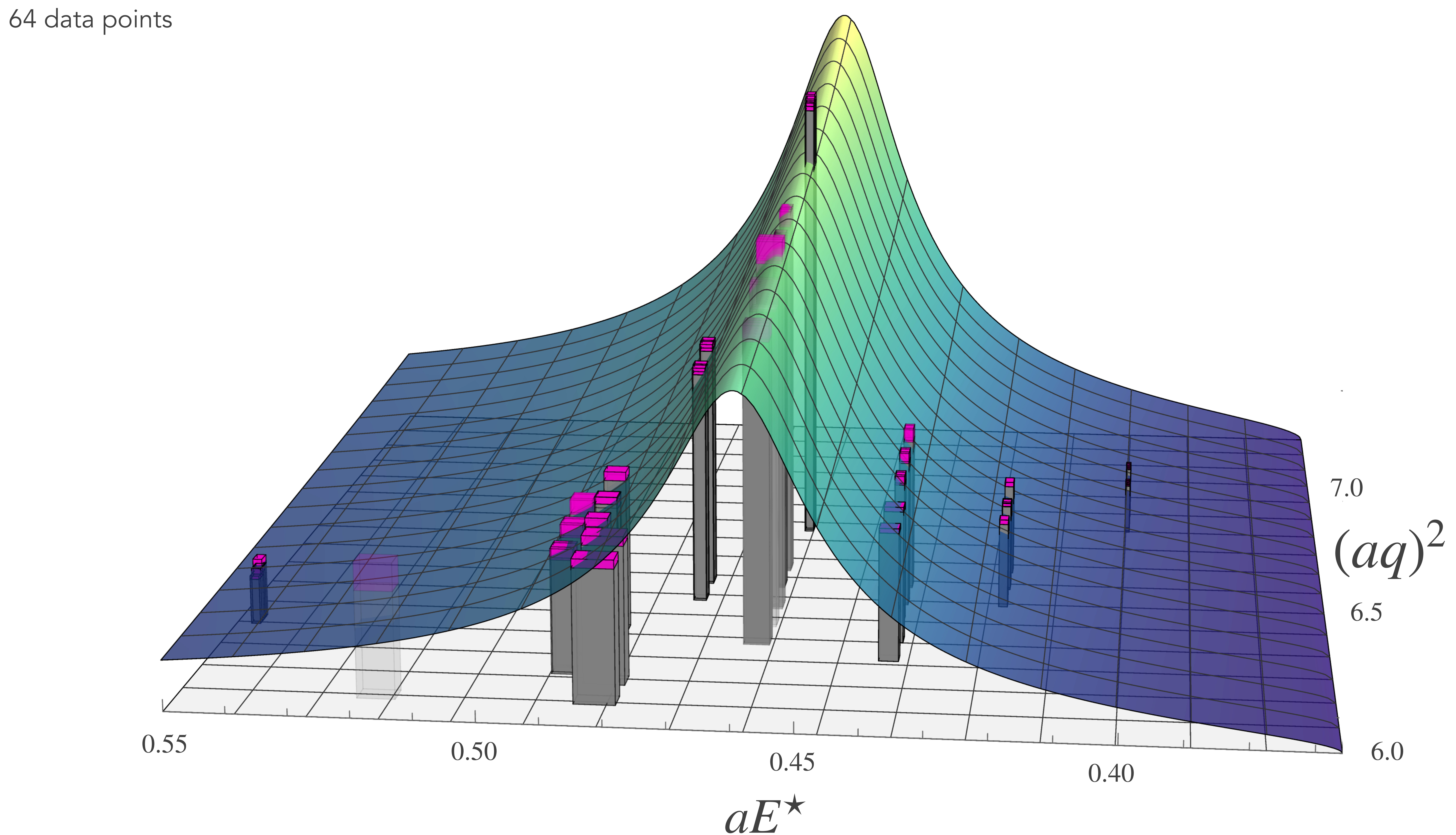
$$\Gamma_I = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{p^3}{E^{\star 2}}$$

$$\Gamma_{II} = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{p^3}{E^{\star 2}}\frac{1 + (k_R r_0)^2}{1 + (k r_0)^2}$$

$$F(E^\star, q^2) = \frac{1}{1 - \frac{q^2}{m_P^2}} \sum_{n,m} A_{n,m}\, z^n(q^2)\, (E^{\star 2} - E_{thr}^2)^m$$

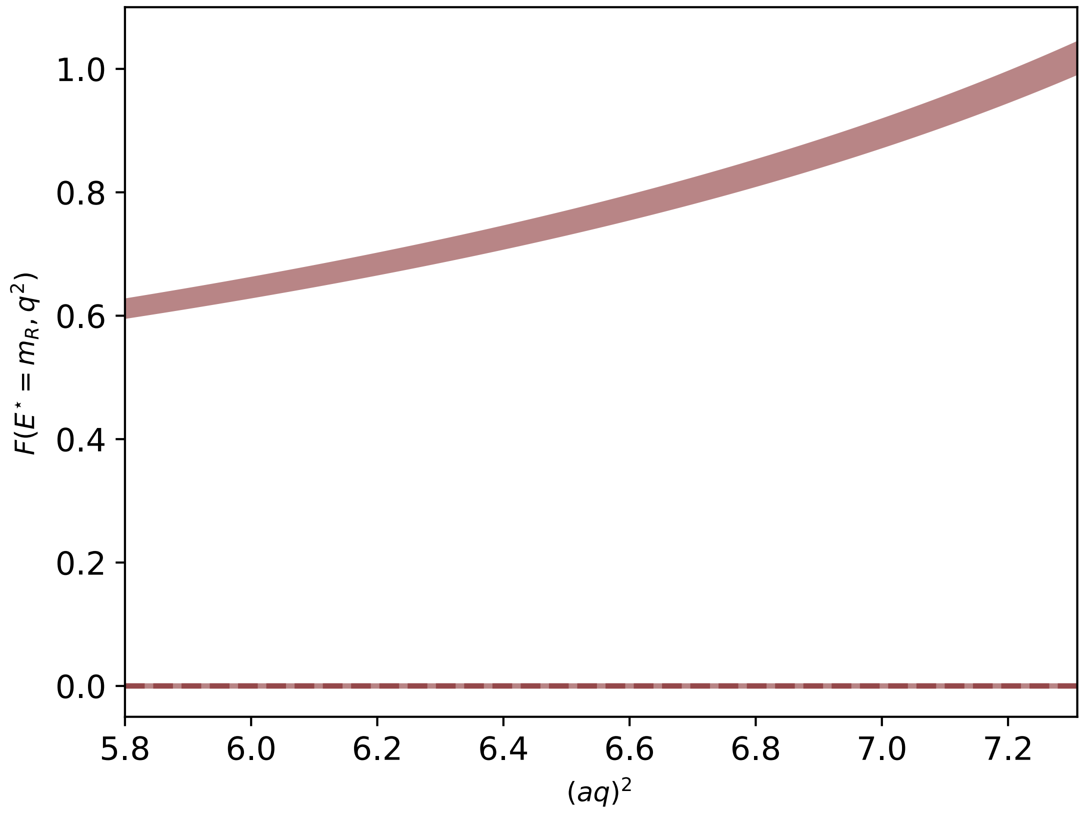
$$\langle B \;\; V \;\; n \rangle_L = \sqrt{R_n}\, \langle B, p_B \;\; V \;\; \pi\pi, E^\star \rangle_\infty$$

"Lellouch-Lüscher" factor

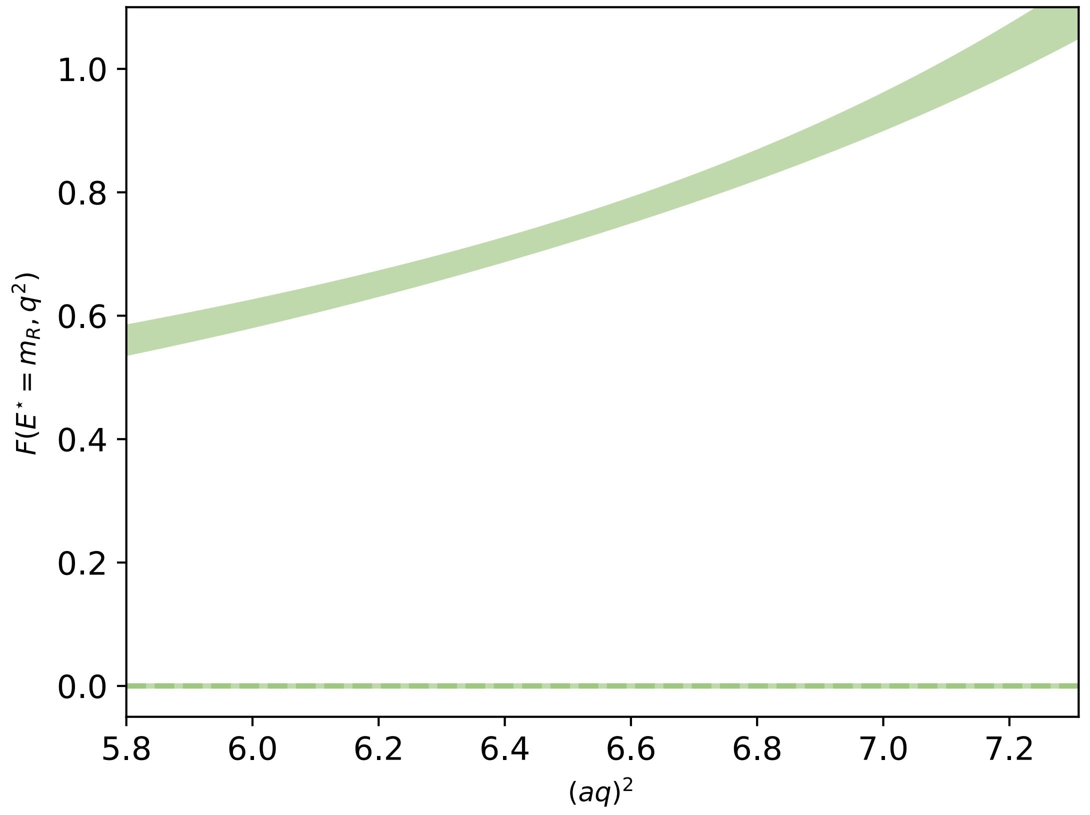

64 data points
7.0
$(aq)^2$
6.5
6.0
0.55
0.50
0.45
0.40
$aE^\star$

details I

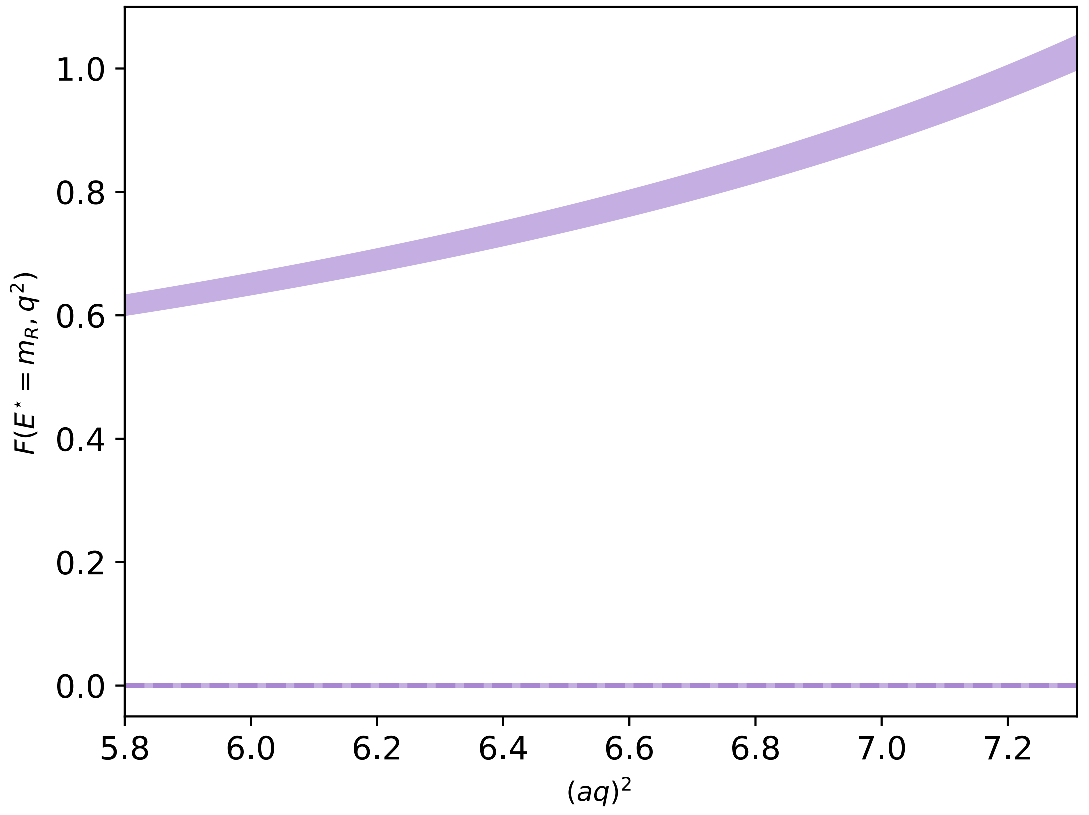
- FF3N0M0_TBWI
  - $\chi^2/\text{dof} = 74.3\,/\,63\ = 1.179$
  - $A_{0,0} = 0.2413(66)$



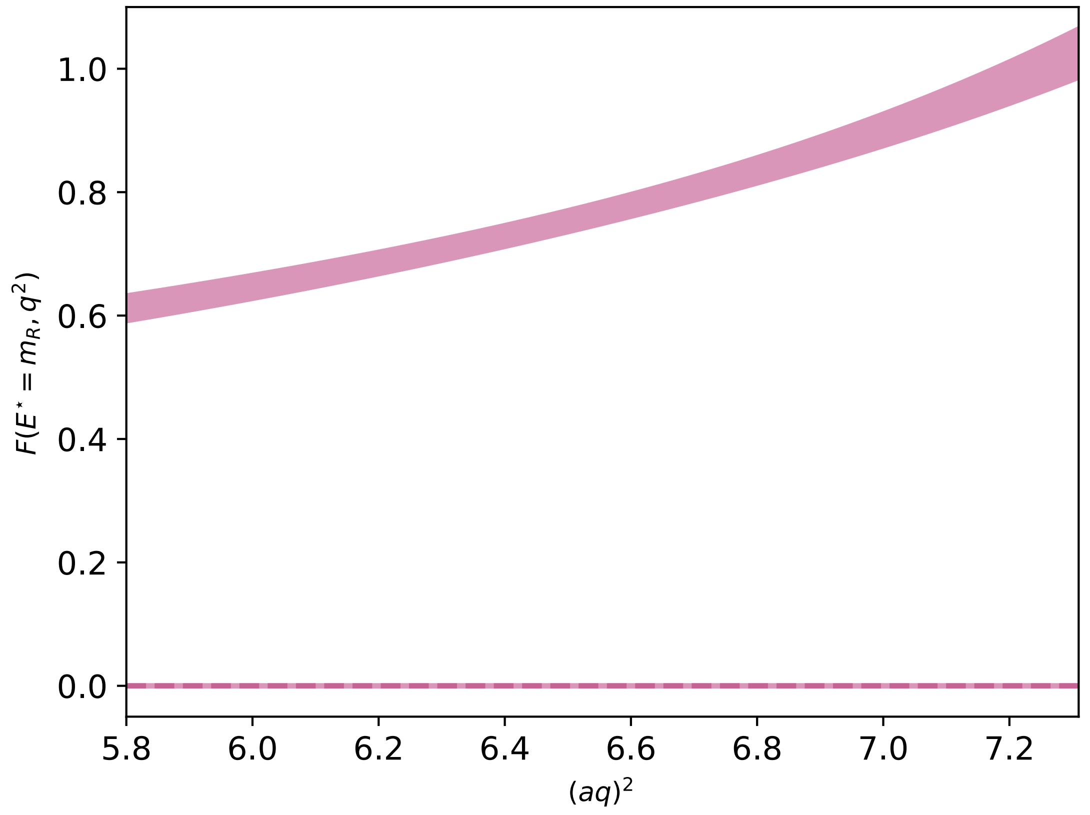
- FF3N1M0_TBWI
  - $\chi^2/\text{dof} = 66.8\,/\,62 = 1.078$
  - $A_{0,0} = 0.2256(87)$
  - $A_{1,0} = -0.41(18)$



- FF3N0M1_TBWI
  - $\chi^2/\text{dof} = 39.1/62 = 0.630$
  - $A_{0,0} = 0.279(12)$
  - $A_{0,1} = -0.061(13)$

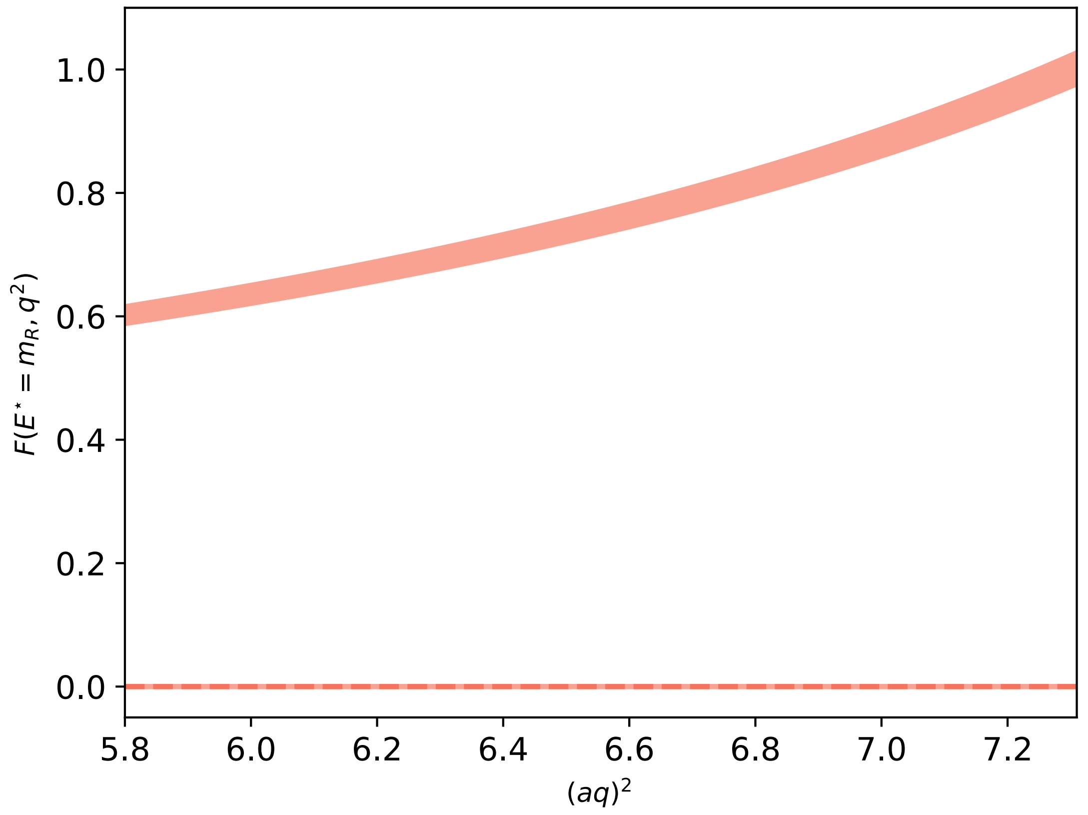


- FF3N1M1_TBWI
  - $\chi^2/\text{dof} = 29.9\,/\,60 = 0.499$
  - $A_{0,0} = 0.260(16)$
  - $A_{1,0} = -0.88(50)$
  - $A_{0,1} = -0.031(21)$
  - $A_{1,1} = 1.46(82)$

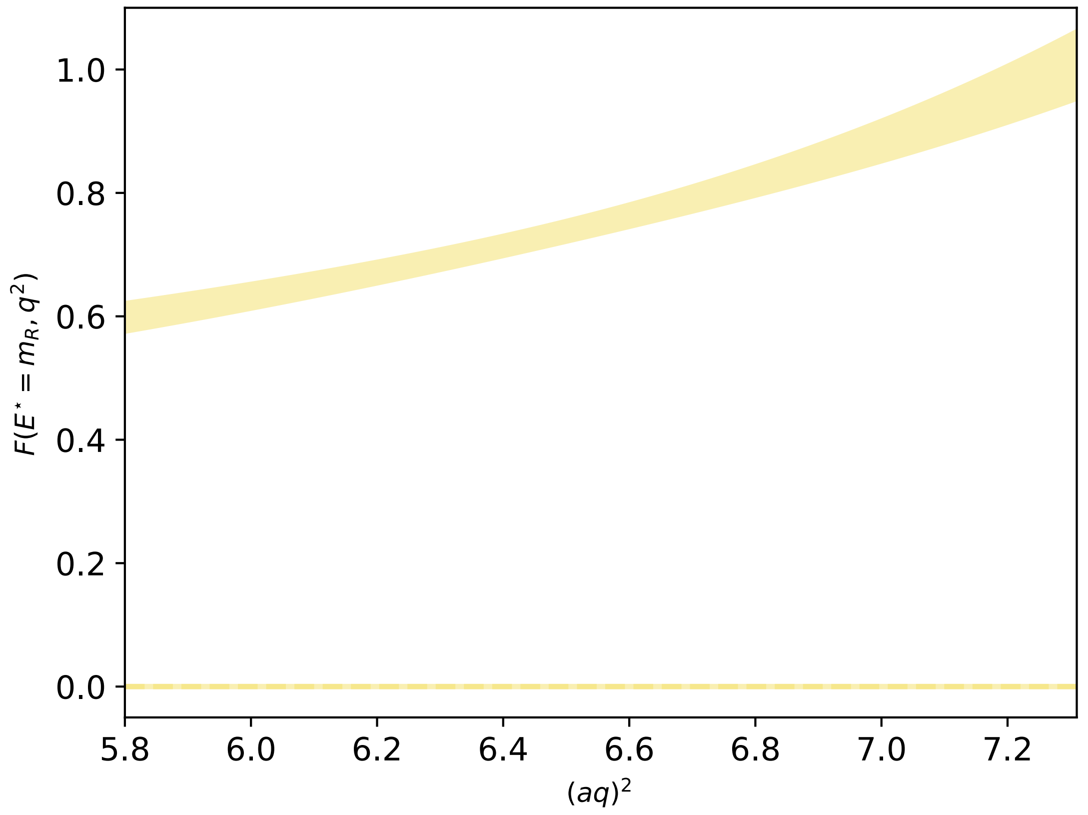


details II

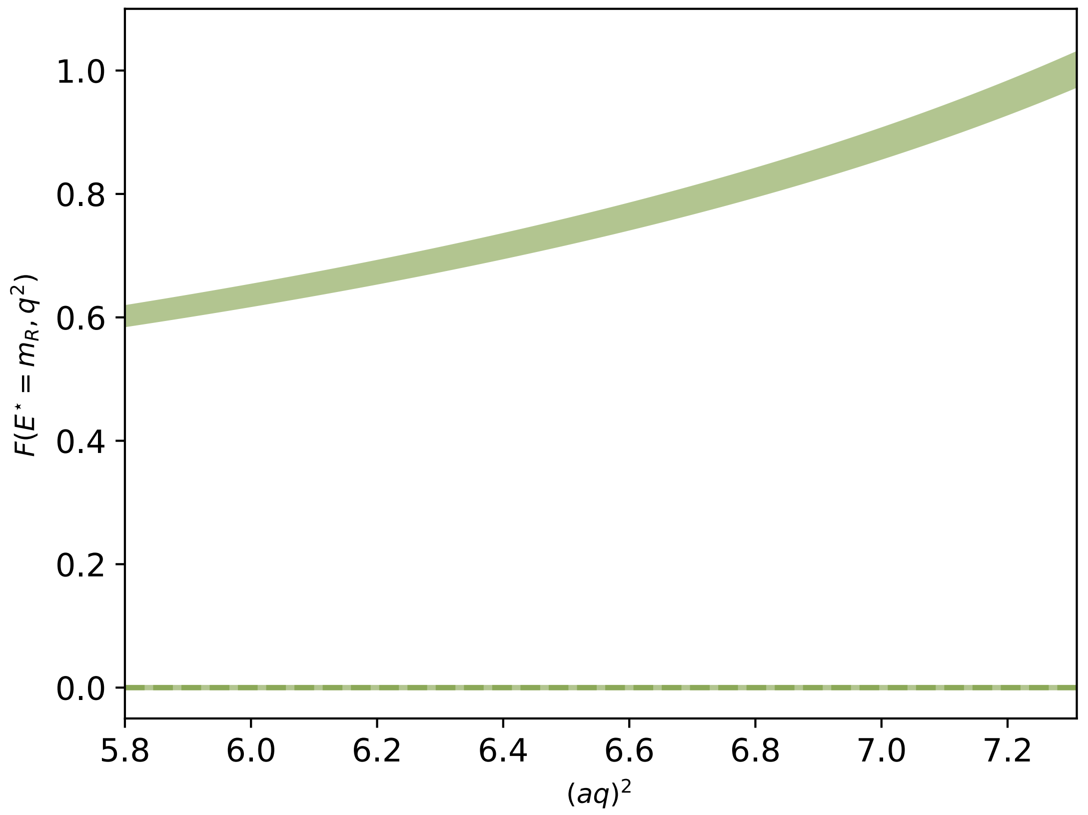
- FF3N0M0_TBWII
  - $\chi^2/\mathrm{dof} = 34.3\,/\,63 = 0.545$
  - $A_{0,0} = 0.2376(71)$



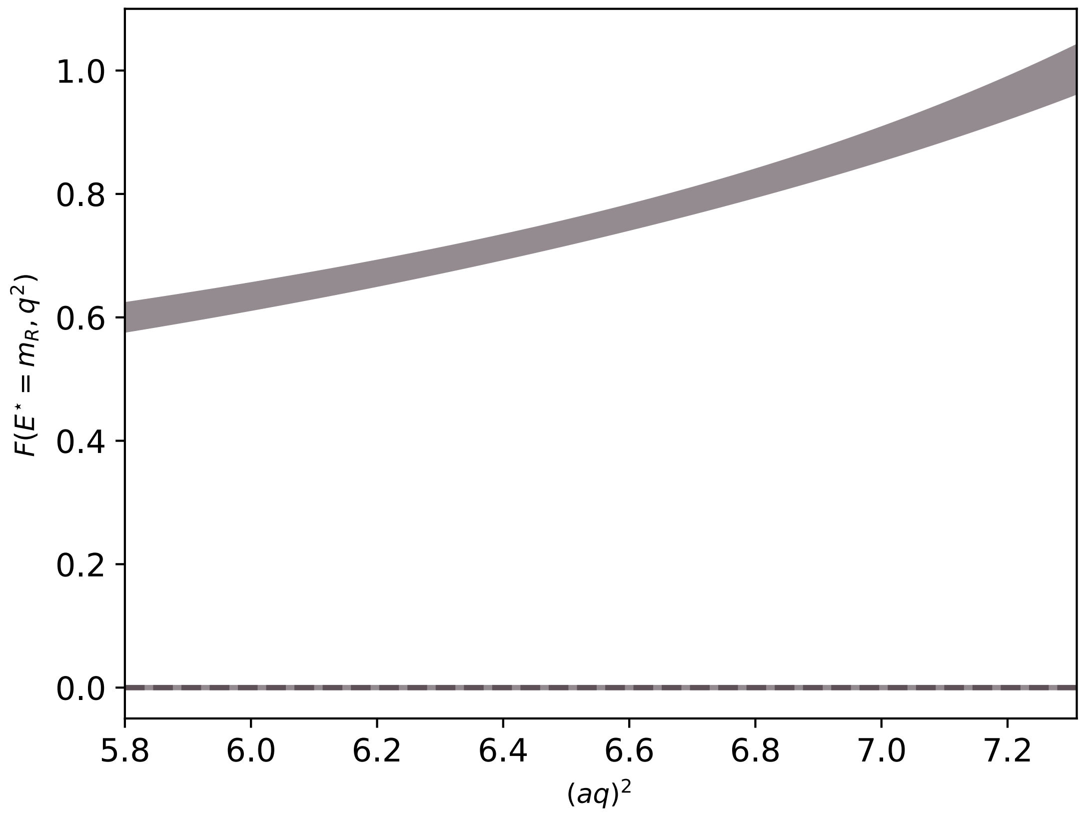
- FF3N1M0_TBWII
  - $\chi^2/\mathrm{dof} = 34.0\,/\,62 = 0.549$
  - $A_{0,0} = 0.2365(89)$
  - $A_{1,0} = -\,0.03(22)$



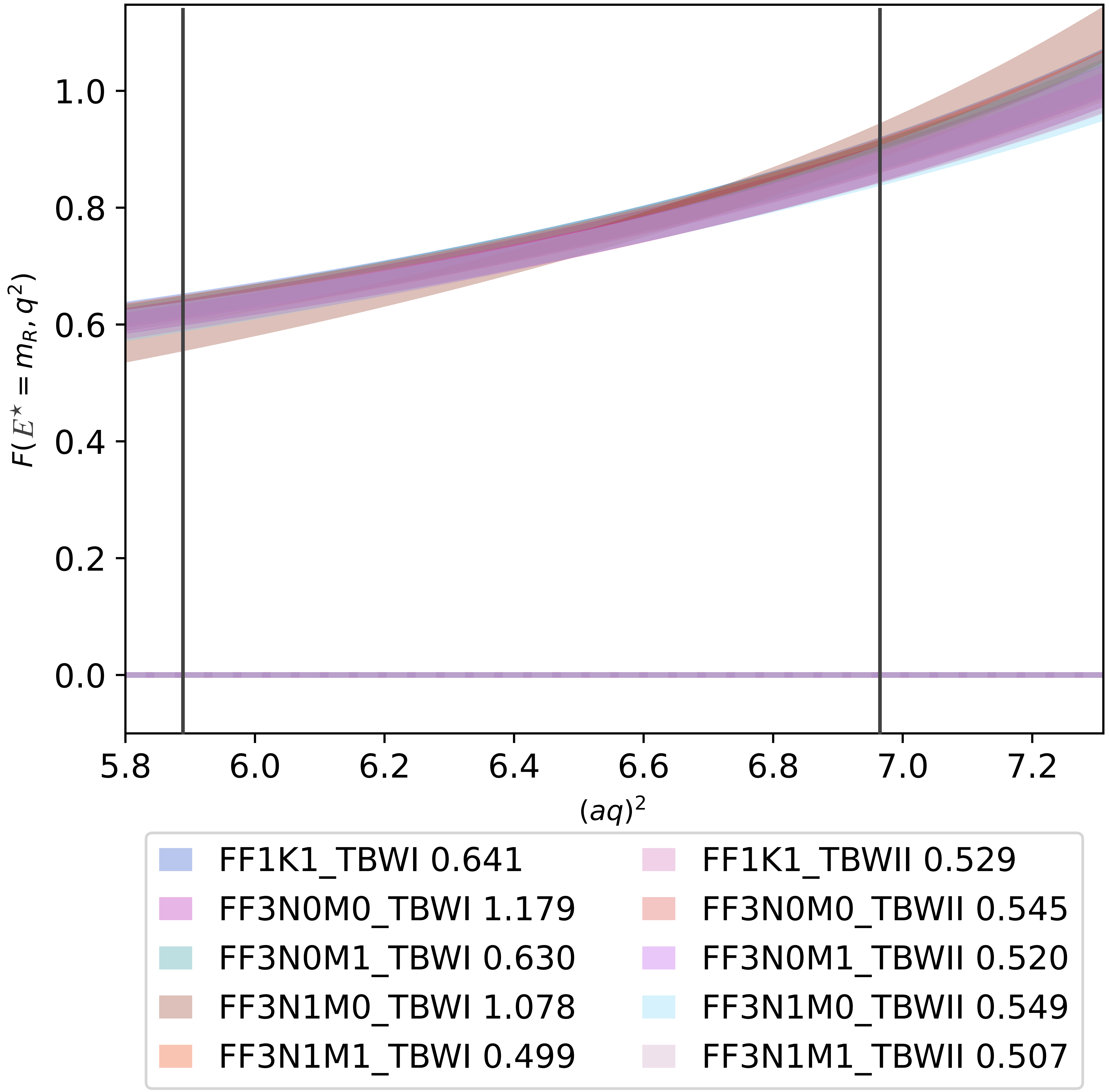
- FF3N0M1_TBWII
  - $\chi^2/\mathrm{dof} = 32.2\,/\,62 = 0.520$
  - $A_{0,0} = 0.240(23)$
  - $A_{0,0} = -\,0.003(33)$



- FF3N1M1_TBWII
  - $\chi^2/\mathrm{dof} = 30.4\,/\,60 = 0.507$
  - $A_{0,0} = 0.232(20)$
  - $A_{1,0} = -\,0.27(60)$
  - $A_{0,1} = 0.008(28)$
  - $A_{1,1} = 0.45(1.02)$

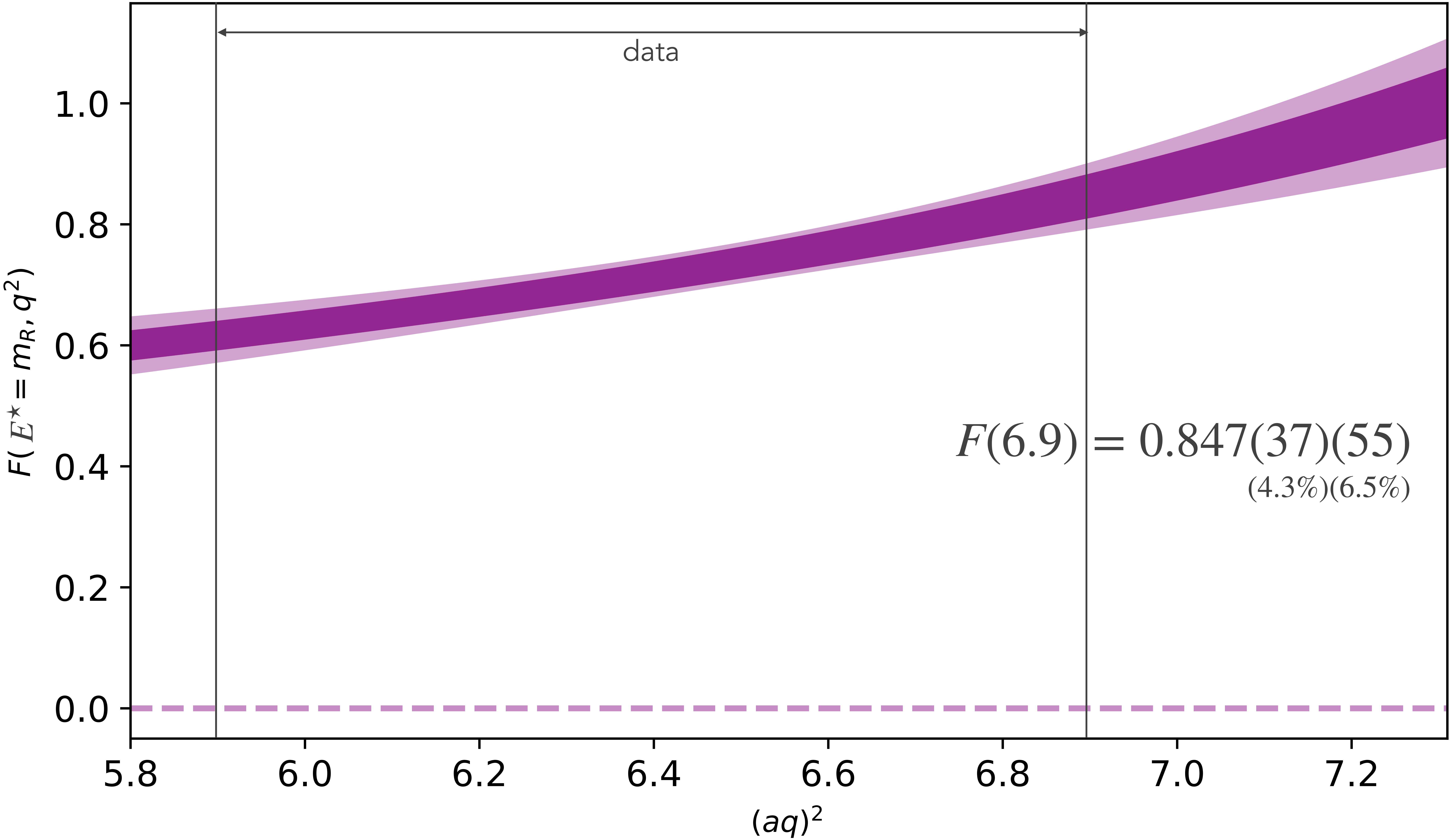


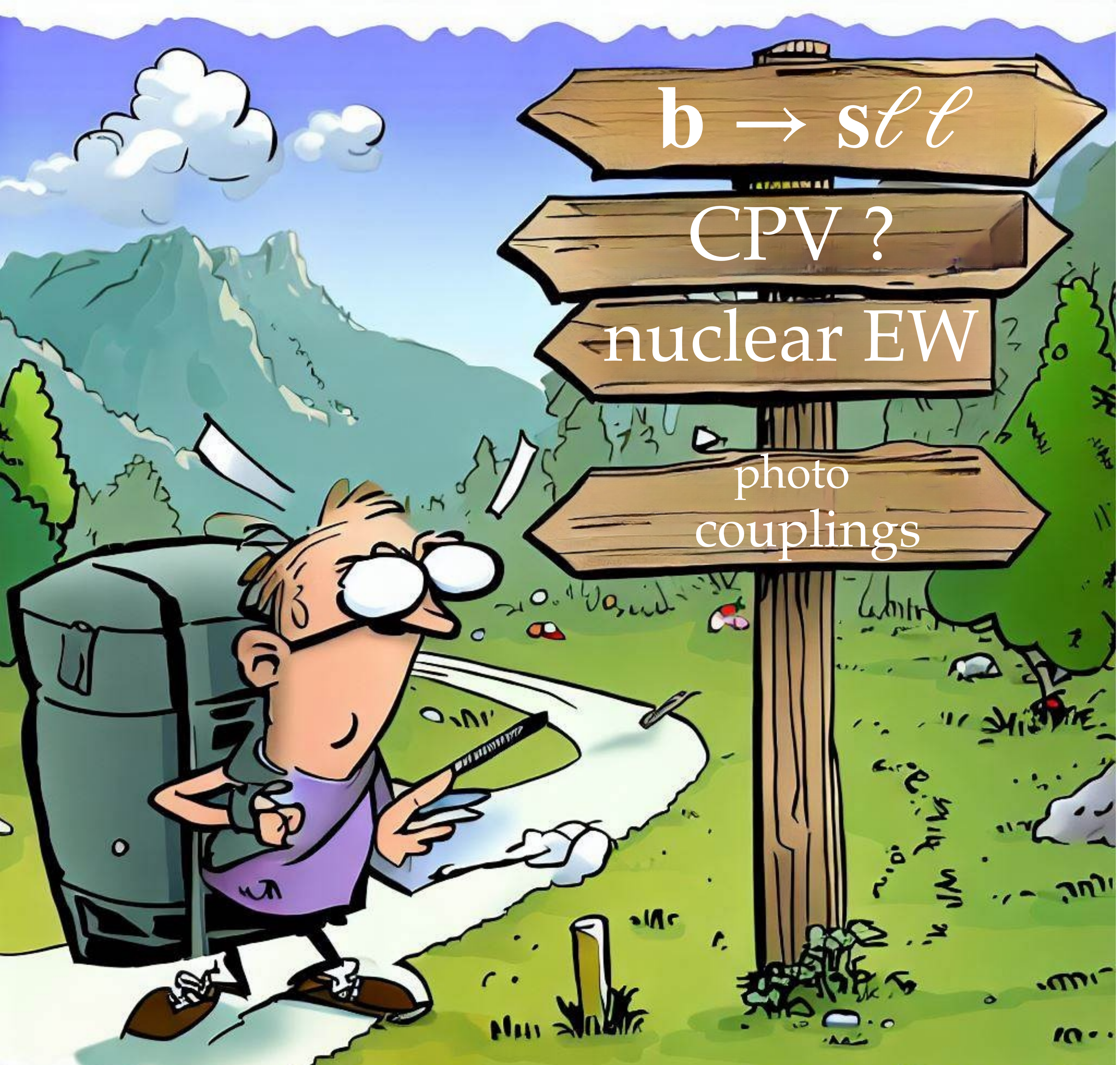
parameterization dependence



- $E^\star$ dependence
- different $E^\star$ dependence in BWI and BWII
- **FF3N1M1_TBWII - central**

combining the parameterizations





- Vector transition amplitude example
- similar uncertainties as FLAG!
- a great start to a (more) complete understanding of SM
- todo:
  - m_π continuation to physics!
  - $a \rightarrow 0$ limit!