Towards TMDs with contour deformations

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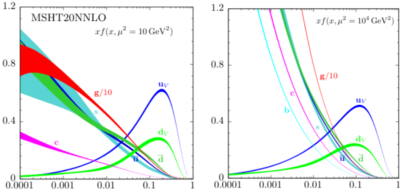
Hadrons on the Light Front

Goal: Use DSE/BSE to study hadrons on the light front, $x^+ = 0$.

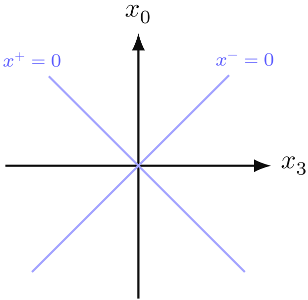
- Natural frame for defining parton distribution functions: PDFs, TMDs, ...

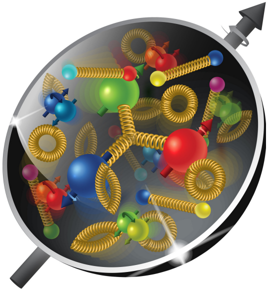


- Future: COMPASS/AMBER @ CERN EIC @ Brookhaven National Laboratory.

(AMBER: arXiv:1808.00848)

(EIC: Eur. Phys. J. A 52.9 (2016))

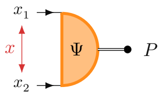




Hadronic quantities

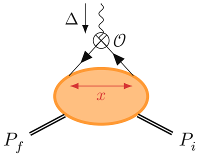
- Bethe-Salpeter Wavefunction

$$\langle 0|\, \mathrm{T}\Phi(x)\Phi(0)\, |P\rangle$$

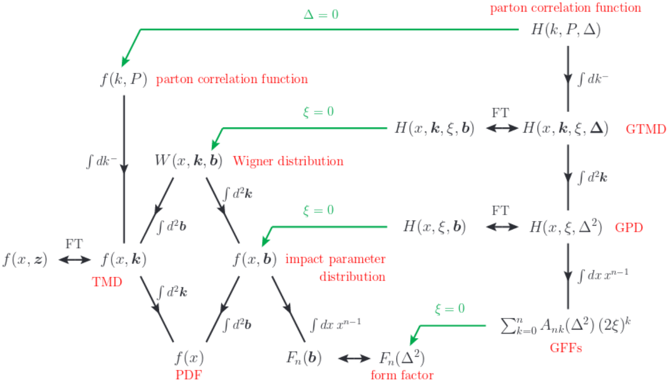


- Generic Correlator

$$\langle P_f|\, \mathrm{T}\Phi(x)\mathcal{O}\Phi(0)\, |P_i\rangle$$



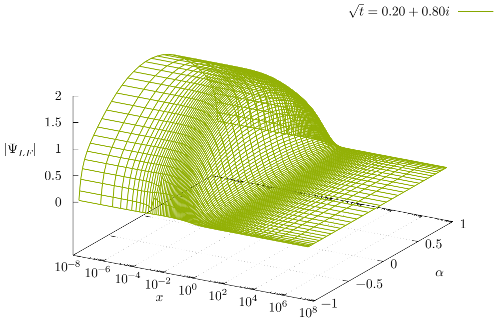
- With $x^+ = x^0 + x^3$, $x^- = x^0 - x^3$, $\vec{x}_\perp = \{x^1, x^2\}$.

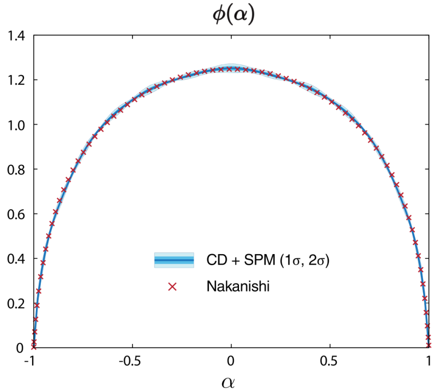


(Lorce, Pasquini, Vanderhaeghen; 2011), (Picture: Diehl, 2016); (Diehl, 2003); (Meissner, Goeke, Metz, Schlegel; 2008); (Meissner, Metz, Schlegel; 2009)

Light-Front WF and PDAs

- We explored successfully the calculation of LFWF and PDAs.





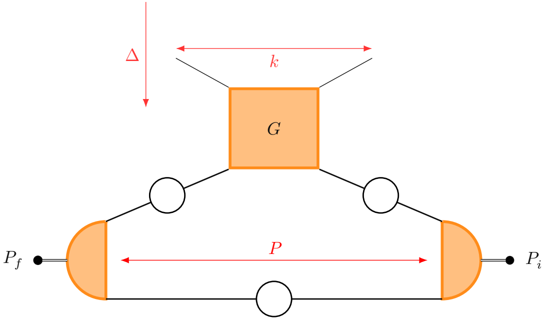
Definition of the TMD

$$\mathrm{TMD}(X,\alpha) \propto -2i\sqrt{X}\int_{-\infty}^{\infty} d\omega\, \mathcal{G}(X,\omega,t,\alpha)$$

G. Eichmann, EF, A. Stadler; Phys. Rev. D 105, 034009 (2022)

Writing the hadronic correlation

- **Main Goal:** Get partonic distribution functions from hadron-hadron correlations via **FUN ctional Methods**



- G is the four-point quark correlation function, calculated with scattering equation.
- The quark propagator is calculated via quark DSE.
- The BSWF is calculated via the meson BSE.

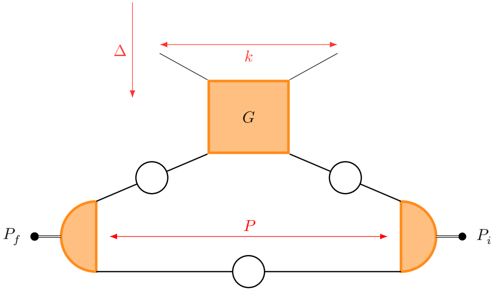
(Mezrag, arXiv:1507.05824); (Diehl, Gousset, 1998); (Tiburzi, Miller, 2003); (Mezrag, Chang, Moutarde, Roberts, Rodríguez-Quintero, Sabatié, Schmidt, 2015); (Cloët, Roberts, 2018), many many others, ...

$$\mathcal{G}^{[\Gamma]}(P,k,\Delta) = \frac{1}{2}\mathrm{Tr}\left[\int dk^- \int \frac{d^4z}{2\pi^4} e^{ik\cdot z} \langle P_f | \overline{\psi}(z)\mathcal{W}\Gamma\psi(0) | P_i \rangle\right]$$

- Partonic distributions are calculated by integrating the correlator in k^- and taking appropriate traces.

Triangle Diagram

- We start by solving a simple model, and gradually build up the complexity of the calculation.
  - Hadrons are on-shell: $P^2 = -M^2$

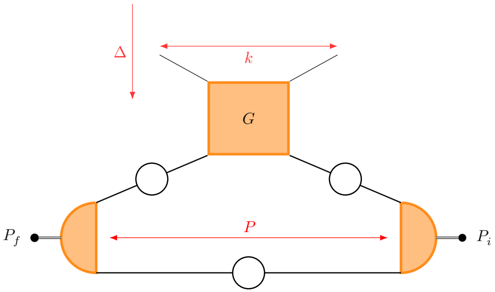


$$\Delta = 2M\sqrt{t}\begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad P = -iM\sqrt{1+t}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

$$k = xP + M\sqrt{R}\begin{pmatrix}0\\0\\\sqrt{1-Z^2}\\Z\end{pmatrix}$$

Triangle Diagram

- We start by solving a simple model, and gradually build up the complexity of the calculation.
  - Hadrons are on-shell: $P^2 = -M^2$
  - **Forward limit:** $\Delta \to 0 \implies t \to 0$ – We get the PDFs and TMDs



$$\Delta = 0 \qquad P = -iM \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$k = xP + M\sqrt{R} \begin{pmatrix} 0 \\ 0 \\ \sqrt{1 - Z^2} \\ Z \end{pmatrix}$$

$$\omega = \tilde{k} \cdot P \implies Z$$

Triangle Diagram

- We start by solving a simple model, and gradually build up the complexity of the calculation.
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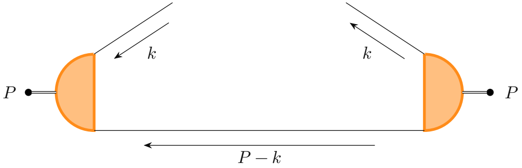


- Using tree-level propagators S
- The amplitudes Γ are calculated with the BSE
- Two diagrams:
  - Upper line spectating
  - Lower line spectating
- TMD obtained by projecting to the light-front (integration on k^-)

Definition of the TMD

$$\mathrm{TMD}(R,x) \propto -2i\sqrt{R}\int_{-\infty}^{\infty} d\omega\, \mathcal{G}(R,\omega,t,x)$$

G. Eichmann, EF, A. Stadler; Phys. Rev. D 105, 034009 (2022)

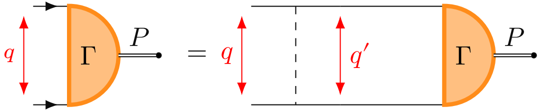
1 Triangle diagram

2 TMDs and PDFs

3 Four-point function

Scalar toy model

- Scalar model:
  - ϕ of mass m
  - χ of mass μ
- BS amplitude for bound-state of two ϕ:



$$q = k - \frac{1}{2}P \qquad q' = k' - \frac{1}{2}P$$

- Tree-level propagators
- Single χ exchange kernel

(Wick; 1954), (Cutkosky; 1954)

- The BSWF is a function of the kinematic invariants:

$$-M^2 = P^2 \qquad \frac{k^2}{M^2} = R \qquad Z = \hat{k} \cdot \hat{P}$$

$$\psi(R,Z,x) = \frac{M^4}{(2\pi)^3}\frac{1}{2}\int_0^\infty dR'\,R' \times \int_{-1}^{1} dZ'\sqrt{1-Z'^2}\,\mathbf{G_0}(R',Z',x) \times \int_{-1}^{1} dy\,\mathbf{K}(R,Z,R',Z',y)\psi(R',Z',x)$$

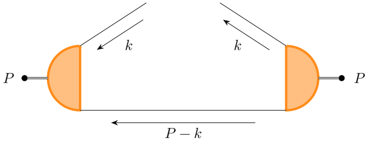
- Model parameters:

$$c = \frac{g^2}{16\pi^2 m^2} \qquad \beta = \frac{\mu}{m} \qquad \gamma = \frac{m}{M}$$

Light-front Projection

- The TMD is defined as:

$$\int dk^-$$



- In our kinematic variables:

$$k^- = -2iM\sqrt{R}Z - \frac{x}{2}M^2$$

x and $R = \frac{k^2}{M^2} = \frac{k_\perp^2}{M^2}$ are external variables

- Need the BSWF in $Z \in (-\infty, \infty)$.
- We use the Schlessinger method for analytic continuation:

$$f(Z) = \frac{a_0 + a_1 Z + a_2 Z^2 + \cdots + a_N Z^N}{1 + b_1 Z + b_1 Z^2 + \cdots + b_M Z^M}$$

(L. Schlessinger, 1968) (Tripolt et al., 2019) (D. Binosi, R-A. Tripolt; 2019)

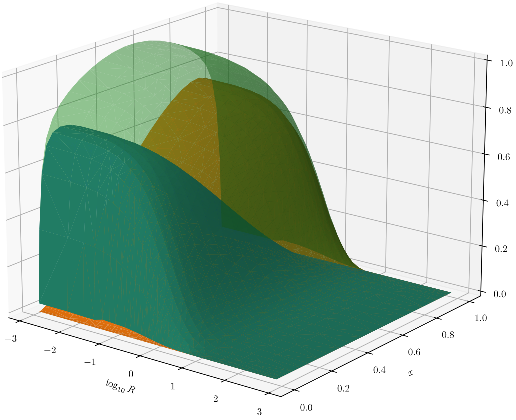
- $\{a_i, b_j\}$ obtained by imposing $f(Z_k) = \Gamma(Z_k)$
- $N - M$ is fixed, can control behaviour at very large Z.

Definition of the LFWF

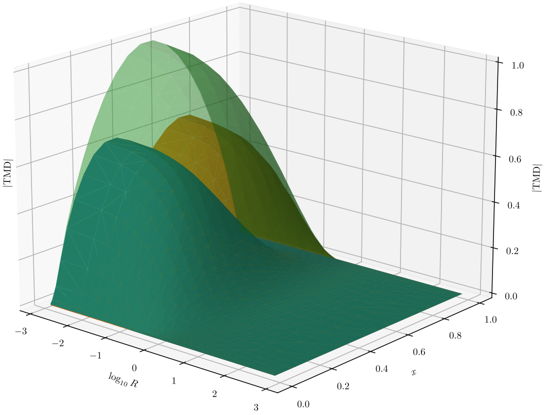
$$\text{TMD}(R, x) \propto -2i\sqrt{R}\int_{-\infty}^{\infty} dZ\, \mathcal{G}(R, Z, x)$$

TMD: Some results

- $\gamma = 1.5, \beta = 4, c = 1$



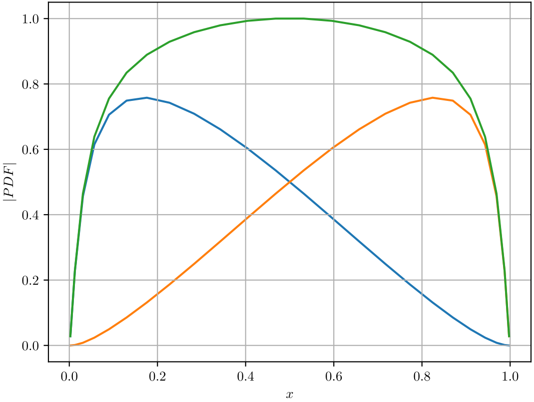
- $\gamma = 1, \beta = 1, c = 1$



- Upper line open
- Lower line open
- Sum of both

PDFs

- $\gamma = 1.5, \beta = 4, c = 1$



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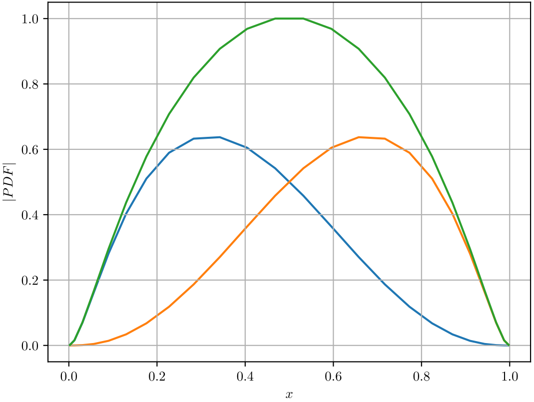

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PDFs

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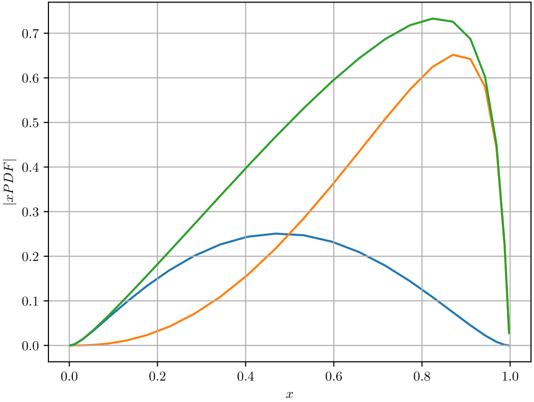

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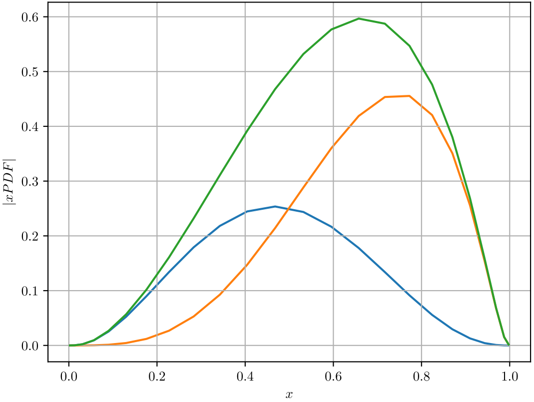


Upper line open

Lower line open

Sum of both

Analytic Structure $\operatorname{Re}\gamma < 1$ - *Implementation soon!*

$$\mathbf{G_0} = \frac{1}{\frac{k'^2}{M^2} + \gamma^2} \frac{1}{\frac{(P-k')^2}{M^2} + \gamma^2}$$

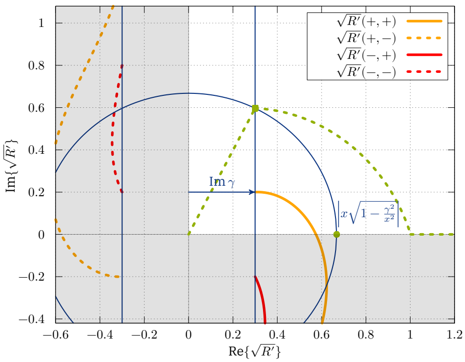
$$\mathbf{K} = \frac{16\pi^2\gamma^2 c}{\frac{(k-k')^2}{M^2} + \beta^2\gamma^2}$$

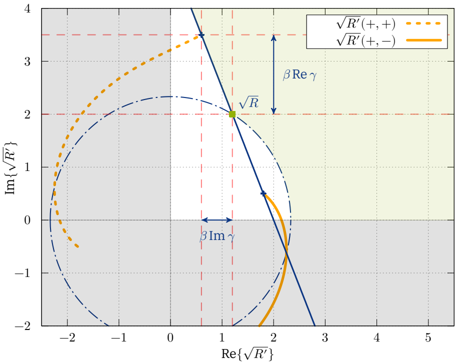
- Branch cuts in complex R' plane:

$$\sqrt{R'}^{\lambda}_{\pm} = \begin{Bmatrix} -ix \\ i(1-x) \end{Bmatrix} \left[Z' + i\lambda\sqrt{1 - Z'^2 + \frac{\gamma^2}{\{x^2; (1-x)^2\}}} \right]$$

- Branch cuts depend on path taken in R:

$$\sqrt{R'} = \sqrt{R}\left(\Omega \pm i\sqrt{1 - \Omega^2 + \frac{\beta^2\gamma^2}{R}}\right)$$

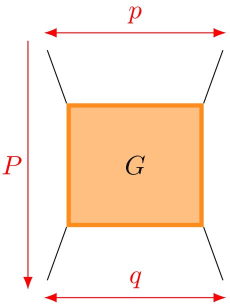




3 Four-point function

Quick detour: 4-point function

- 4- point function determined from scattering equation:
  $\mathbf{G} = \mathbf{G_0} + \mathbf{G_0 T G_0} \implies \mathbf{T} = \mathbf{K} + \mathbf{K G_0 T} \implies \mathbf{T} = (\mathbb{1} - \mathbf{K G_0})^{-1}\,\mathbf{K}.$



- Fully off-shell: 6 Lorentz invariants
  - 3 radial: X, t, R;
  - 3 angular: Y, Z, Q;

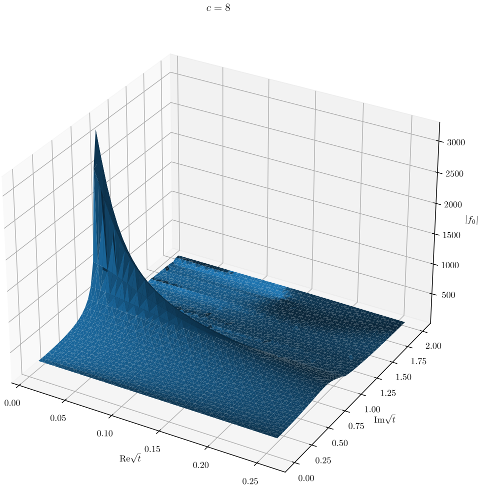
$$T(t,X,R,Z,Y,Q) = K(X,R,p\cdot q) + \frac{1}{2}\frac{m^4}{2\pi^4}\int_0^\infty dx\, x \int_{-1}^{1} dz\, \sqrt{1-z^2}G_0\,(x,z,t) \times \int_{-1}^{1} dy \int_0^{2\pi} d\Psi\, K(X,x,k\cdot q) T(t,x,y,z,R,Q)$$

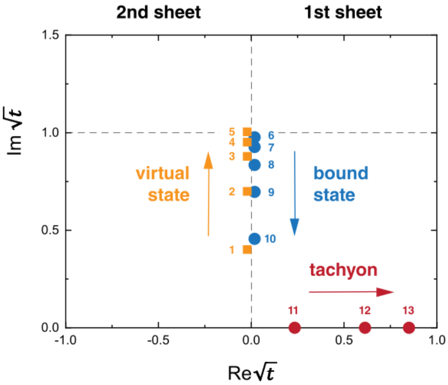
- Same G_0 and K as in the BSE
- Solved numerically as linear system
- Contains *all* dynamics of 2 ϕ particles
  - **Must produce bound state poles dynamically!**

(Eichmann, Duarte, Peña, Stadler; 2019)

4-point function results

- T is made fully on-shell
- Partial-wave expansion shows bound-state pole in the first Riemann sheet

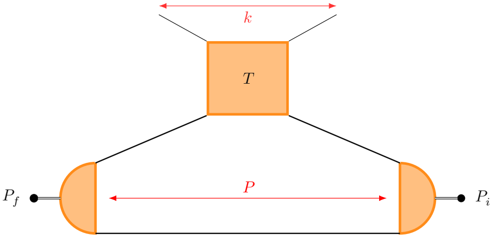




(Eichmann, Duarte, Peña, Stadler; 2019)

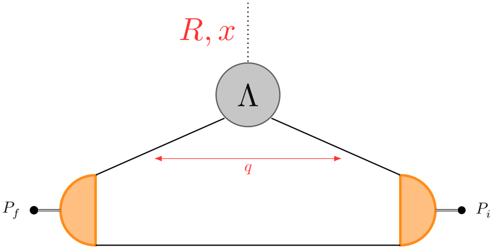
- Describes both long-range and short-range qq dynamics.

Including the four-point function

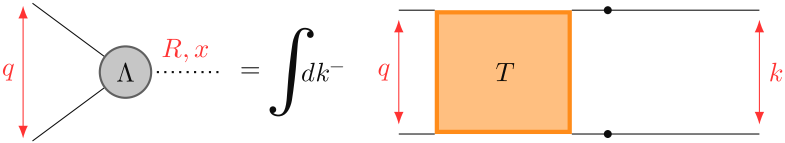


- *Close* the triangle.
- LF projection is now done in the open legs of T.

Including the four-point function



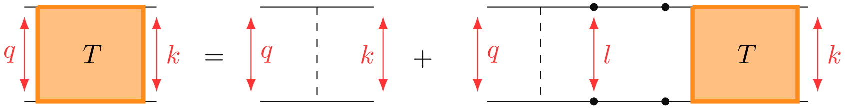
- *Close* the triangle.
- LF projection is now done in the open legs of T.
- Need a **light-front vertex.**
- Triangle is now a loop diagram
- Contains all dynamics between the two open legs:
  - Infinite ladder of χ exchanges.



- Can be calculated from the scattering equation.

Equation for the vertex

- Do the light-front projection in the scattering equation



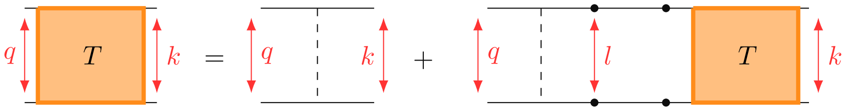
Scattering equation

$$\mathbf{T}(q,k) = \mathbf{K}(q,k) + \int d^4l \, \mathbf{K}(q,l)\mathbf{G_0}(l)\mathbf{T}(l,k)$$

(Bednar, Cloët, Tandy; 2020)

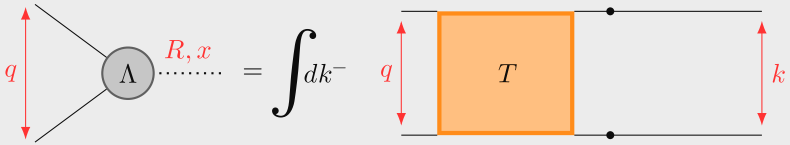
Equation for the vertex

- Do the light-front projection in the scattering equation



- Introduce the definition for Λ in the equation.

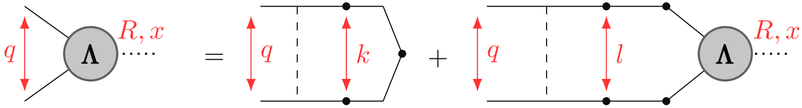
Definition of the vertex



(Bednar, Cloët, Tandy; 2020)

Equation for the vertex

- Do the light-front projection in the scattering equation



- LF projection happens in inhomogeneous term

Vertex equation

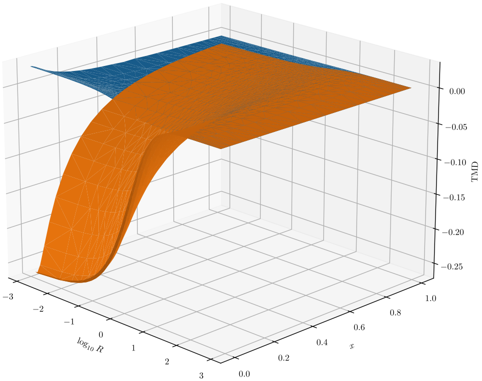
$$\mathbf{\Lambda}(q,R,x) = \int dk^- \; \mathbf{K}(q,k)\mathbf{G_0}(k) + \int d^4l \; \mathbf{K}(q,l)\mathbf{G_0}(l)\mathbf{\Lambda}(l,R,x)$$

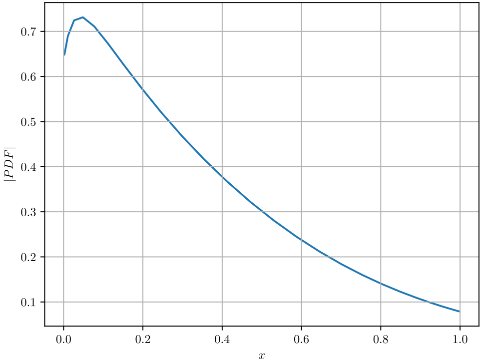
- Kernel and G_0 fully known – no extrapolation needed!

(Bednar, Cloët, Tandy; 2020)

TMDs/PDFs with vertex *Pre*

- $\gamma = 2, \beta = 2, c = 1$

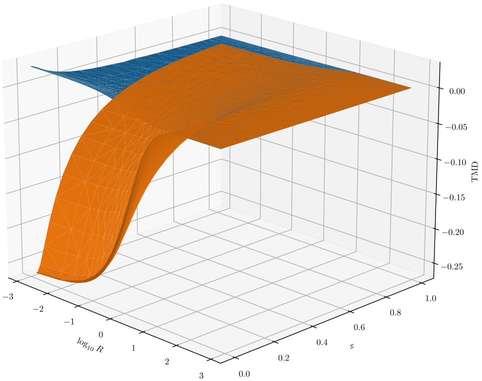


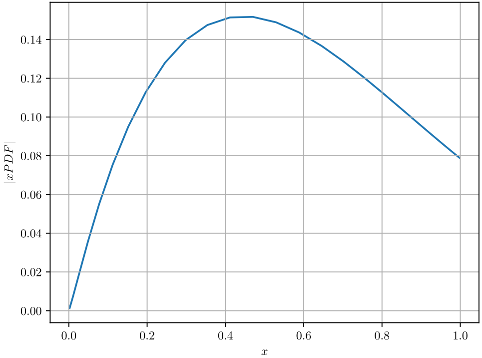


- No difference between upper leg or lower leg spectating
- Small correction
- Analytic result $\rightarrow$ divergence as $x \rightarrow 0$.

TMDs/PDFs with vertex *Pre*

- $\gamma = 2, \beta = 2, c = 1$





- No difference between upper leg or lower leg spectating
- Small correction
- Analytic result $\rightarrow$ divergence as $x \rightarrow 0$.

Conclusions and Outlook

- Method is fast and simple.
- Good results for TMDs and PDFs. Ready for application in QCD.
- No Mellin moments or Nakanishi representation.

Future Implementing contour deformations for entire range in γ – resonances.

Future $\Delta \neq 0$ – GPDs. T contribution can be very relevant due to bound-state contributions.

Future Article in preparation

Future QCD.