Identifying the Time Scales in Electron-Positron Production from Ultra-Strong Electric Fields

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[1]based on work with
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Outline

1. Motivation: Time scales in the quantum world
2. Basic Aspects of the Sauter-Schwinger Pair Production
3. Dirac-Heisenberg-Wigner Approach to Sauter-Schwinger Effect
   - Formalism
   - Selected Numerical Results
4. Physics of Adiabatic Particle Number
5. Time Scales of Particle Formation
6. Summary

Time scales in the quantum world

- Vastly different time scales for quantum phenomena, e.g., lifetime of
  - Z_0 resonance: $\tau \approx 3 \times 10^{-25}$ s
  - ^{124}Xe nucleus: $\tau \approx 6 \times 10^{+29}$ s

  Long time scales due to tunnel effect, resp., evanescent waves
- "Textbook" Quantum Field Theory:
  In S-matrix *only* asymptotic *in*- and *out*-states at $t_{i,f} \to \mp\infty$.
- Understanding of time scales in quantum systems? ☹
  Atomic ionisation: Expressions for "tunneling times" experimentally falsified,
  see, e.g., A.S. Landsman and U. Keller, Phys. Reports **547** (2015) 1.
- Non-equilibrium Quantum Field Theory:
  One short time scale in simple systems and/or pert. processes
  vs.
  several time scales in complex quantum systems and/or non-perturbative processes???

Time scales in the quantum world

What about quantum phenomena in time-dependent backgrounds?
As, e.g.,

- particle production in an expanding universe,
- Hawking radiation of Black Holes (grav. collapse!), or
- e^+e^- pair production in ultra-strong crossed laser beams
  ($\leftrightarrow$ Sauter-Schwinger effect).

Fundamental conundrum:
The time interval for extracting the particle number should be short enough to restrict the variation of the background field to a negligible amount. However, Heisenberg's uncertainty principle leads for short time intervals to a large spread in energy, and thus to the possibility of a large number of virtual pairs, and a correspondingly undetermined particle number.

Time scales in the quantum world

Is the question for time scales in such processes well-defined?

If so, are there several distinct time scales?

To which sub-processes can they be attributed?

How long do the sub-processes and the whole process take?

What are the consequences for our understanding of quantum systems?

Sauter-Schwinger Pair Production

. . . a long-standing prediction . . .

More than 90 years ago: Sauter's calculation

Über das Verhalten eines Elektrons im homogenen elektrischen Feld nach der relativistischen Theorie Diracs.

Von **Fritz Sauter** in München.

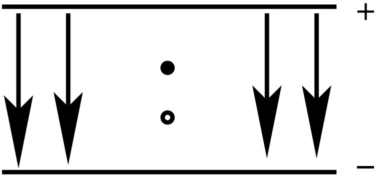
Mit 6 Abbildungen. (Eingegangen am 21. April 1931.)

Es werden die Lösungen der Diracgleichung mit dem Potential $V = vx$ angegeben und ihr Verhalten diskutiert. Zu dem Funktionsverlauf, der auch bei nichtrelativistischer Rechnung auftritt, kommt in der Diracschen Theorie noch ein Gebiet hinzu, in dem Elektronenimpuls und -geschwindigkeit entgegengesetztes Vorzeichen besitzen. Im Anschluß daran wird für ein Elektron die Wahrscheinlichkeit berechnet, aus dem Gebiet „positiven Impulses" in das mit „negativem Impuls" überzugehen. Es ergibt sich, daß die Durchgangswahrscheinlichkeit erst dann endliche Werte annimmt, wenn die Größe des Potentialanstieges auf einer Strecke gleich der Comptonwellenlänge vergleichbar wird mit der Ruheenergie des Elektrons. Die von O. Klein berechneten großen Werte für die Durchgangswahrscheinlichkeit durch einen Potentialsprung von der Größenordnung der doppelten Ruheenergie sind in dem Sinne als Grenzwerte im Falle unendlich steilen Potentialanstieges zu verstehen.

Vor einiger Zeit erschien eine interessante Arbeit von O. Klein* über

Schwinger's formula

1931: First calculation [F. Sauter, Z. Phys. **69** (1931) 742]
1932: Discovery of the positron [C.D. Anderson, Phys. Rev. **43** (1933) 491]
1936: Theoretical description of pair production from fields
[W. Heisenberg and H. Euler, Z. Phys. **98** (1936) 714 [arXiv:physics/0605038]]
1950/51: quantum field theoretical calculation
[J. Schwinger, Phys. Rev. **82** (1951) 664.]
1971: alternating field [E. Brezin, C. Itzykson, Phys. Rev. **D2** (1970) 1191.]



Static & spatially uniform electric field $\Longrightarrow$ "Vacuum" decays

Schwinger's formula

Full one-loop calculation in background of classical electric field for (boson/fermion) pair production provides
vacuum persistence probability / volume · time
(Schwinger's formula):

$$P_0 = \frac{e^2 E^2}{4\pi^3 c\hbar^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-n\pi m_e^2 c^3 / \hbar e E}$$

- Due to tunneling of n e^+e^- pairs out of "vacuum".
- $n = 1$ term dominates ...

Schwinger's formula

Estimate of scales:

electron Compton wavelength $\frac{\lambda_e}{2\pi} = \frac{\hbar}{m_e c}$ = 386 fm

rest energy of e^+e^- pair $2m_e c^2$=1.022 MeV

work of field on charge e over Compton wavelength = rest energy

$$eE_c \frac{\lambda_e}{2\pi} = m_e c^2 \qquad \Longrightarrow E_c = \frac{m_e^2 c^3}{e\hbar} \approx 1.3 \cdot 10^{18} \frac{\mathrm{V}}{\mathrm{m}}$$

$E \ll E_c$: pair production is a quantum tunneling process

$\Longrightarrow$ amplitude $\propto \exp\left(-\pi E_c/E\right) \propto \exp\left(-\frac{1}{e}\right)$

is **NON-perturbative!**[2]

[2]Not strictly for pulses of finite duration and extension ...

Pair Production in Crossed Laser Beams?

In the last two years:

Multi-Petawatt Physics Prioritisation (MP3) Workshops
to discuss physics opportunities for future ultra-strong lasers,
cf. arXiv:2211.13187.

October this year:

White papers for the CNRS Apollon Research Infrastructure / Apollon Laser Facility at CEA Saclay

Dirac-Heisenberg-Wigner formalism

Phase-Space formulation of Schwinger effect: $\{\vec{x}, \vec{p}, t\}$

- Generalisation of Quantum Kinetic Theory needed for inhomogeneous electric and/or magnetic fields.
- Momentum $\vec{k}$ conjugate variable of $\vec{x}$ $\rightarrow$ No direct generalization!

Approach: Dirac-Heisenberg-Wigner (DHW) function

see, e.g., F. Hebenstreit, PhD thesis, April 2011
F. Hebenstreit, R. A., H. Gies, Phys.Rev.D **82** (2010) 105026;
Phys.Rev.Lett. **107** (2011) 180403
D. Berényi, P. Lévai, S. Varró, V.V. Skokov, *e.g.*, Phys. Lett. **B749** (2015) 210

- Based on gauge-invariant density operator for Dirac fields
$\hat{\mathcal{C}}_{\alpha\beta}(r,s) = \mathcal{U}(A,r,s)\ \left[\bar{\psi}_\beta(r-s/2), \psi_\alpha(r+s/2)\right]$

Dirac-Heisenberg-Wigner formalism

- Gauge-invariant density operator for Dirac fields
  $\hat{\mathcal{C}}_{\alpha\beta}(r,s) = \mathcal{U}(A,r,s)\ \left[\bar{\psi}_\beta(r-s/2), \psi_\alpha(r+s/2)\right]$
- Wilson line factor $\mathcal{U}(A,r,s) = \exp\left(\mathrm{i}\ e\ s\int_{-1/2}^{1/2} d\xi\ A(r+\xi s)\right)$
- Hartree approximation: Mean field $F^{\mu\nu}(x) \approx \langle \hat{F}^{\mu\nu}(x)\rangle$
- DHW function $\mathcal{W}_{\alpha\beta}(r,p) = \langle\Phi|\frac{1}{2}\int d^4s\ \mathrm{e}^{ips}\ \hat{\mathcal{C}}_{\alpha\beta}(r,s)\,|\Phi\rangle.$

- E.g., equation for vanishing magnetic field ($\vec{B}=0$):

$$D_t\mathcal{W}_{\alpha\beta} = -\frac{1}{2}\nabla\left[\gamma^0\vec{\gamma},\mathcal{W}\right]_{\alpha\beta} - i\left[m\gamma^0,\mathcal{W}\right]_{\alpha\beta} - i\left\{\gamma^0\vec{\gamma}\vec{p},\mathcal{W}\right\}_{\alpha\beta}$$

$$\text{with}\quad D_t = \partial_t + e\int_{-1/2}^{1/2} d\lambda\vec{E}(\vec{x}+i\lambda\partial_p,t)\partial_p$$

$\mathbf{E}(\mathbf{x}, t)$ and $\mathbf{B}(\mathbf{x}, t)$: DHW Formalism

C. Kohlfürst, PhD thesis, 2015, arXiv:1512.06082

- full equations including electric and magnetic fields
- 3+1, 2+1 and 1+1 dimensions
- selected symmetries as *e.g.* cylindrically symmetric fields
- Quantum Kinetic Theory in homogeneous limit
- most efficient numerical solution by pseudo-spectral methods (check for convergence at late time)
- calculations of observables from Wigner components straightforward
- onset of (semi-)classical propagation

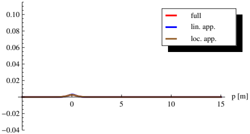
Numerical Results: Single Pulse in 1+1 Dimension

Electric Field: $E(x,t) = E_0 \operatorname{sech}^2(t/\tau)\exp(-x^2/2\lambda^2)$

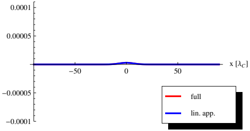
- Spatial: $\sim \exp(-x^2/2\lambda^2)$
- Temporal: $\sim \operatorname{sech}^2(t/\tau)$
- 3 relevant scales: E_0, τ, λ
- Simplification: Consider QED_{1+1} instead of QED_{3+1}
- Particle momentum density : $f_p(p,t) = \int [dx] f(x,p,t)$
- Particle space density: $f_x(x,t) = \int [dp] f(x,p,t)$
- Charge momentum density: $q_p(p,t) = \int [dx] q(x,p,t)$
- Charge space density: $q_x(x,t) = \int [dp] q(x,p,t)$

Time evolution: Small $\lambda = 10\lambda_C$

$f_p(p, t)$

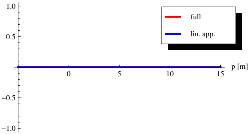


$f_x(x, t)$

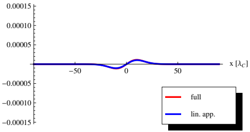


$t = -2\tau$

$q_p(p, t)$

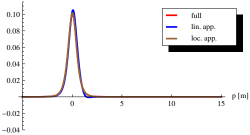


$q_x(x, t)$

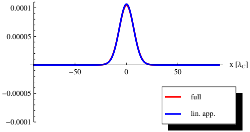


Time evolution: Small $\lambda = 10\lambda_C$

$f_p(p,t)$

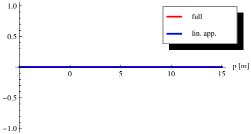


$f_x(x,t)$

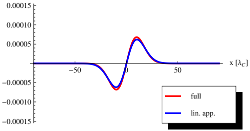


$t = -\tau$

$q_p(p,t)$

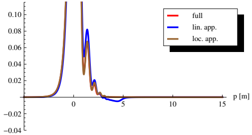


$q_x(x,t)$

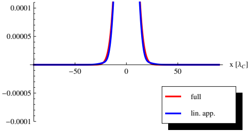


Time evolution: Small $\lambda = 10\lambda_C$

$f_p(p, t)$

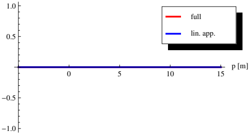


$f_x(x, t)$

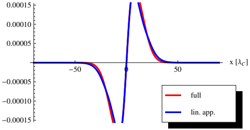


$t = 0$

$q_p(p, t)$

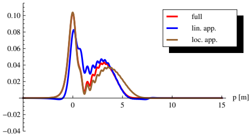


$q_x(x, t)$

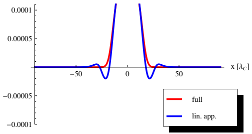


Time evolution: Small $\lambda = 10\lambda_C$

$f_p(p, t)$

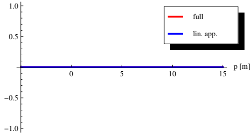


$f_x(x, t)$

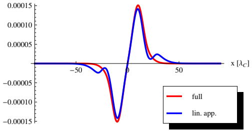


$t = \tau$

$q_p(p, t)$

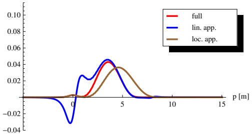


$q_x(x, t)$

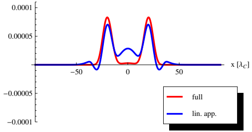


Time evolution: Small $\lambda = 10\lambda_C$

$f_p(p,t)$

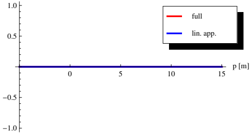


$f_x(x,t)$

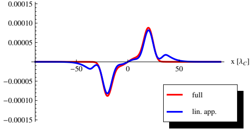


$t = 2\tau$

$q_p(p,t)$

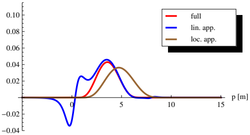


$q_x(x,t)$

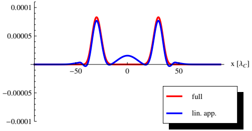


Time evolution: Small $\lambda = 10\lambda_C$

$f_p(p,t)$

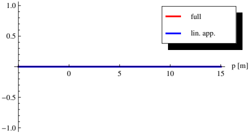


$f_x(x,t)$

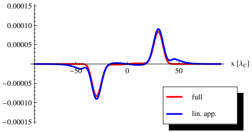


$t = 3\tau$

$q_p(p,t)$

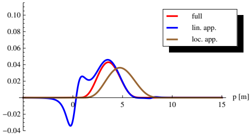


$q_x(x,t)$

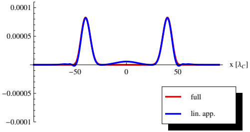


Time evolution: Small $\lambda = 10\lambda_C$

$f_p(p, t)$

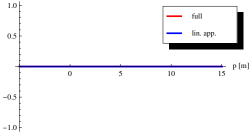


$f_x(x, t)$

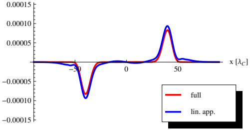


$t = 4\tau$

$q_p(p, t)$

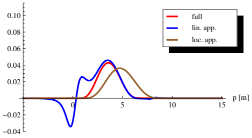


$q_x(x, t)$

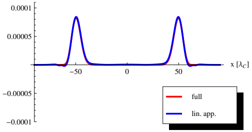


Time evolution: Small $\lambda = 10\lambda_C$

$f_p(p, t)$

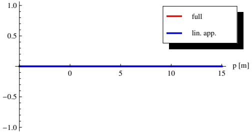


$f_x(x, t)$

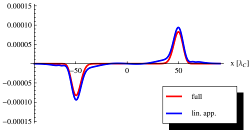


$t = 5\tau$

$q_p(p, t)$



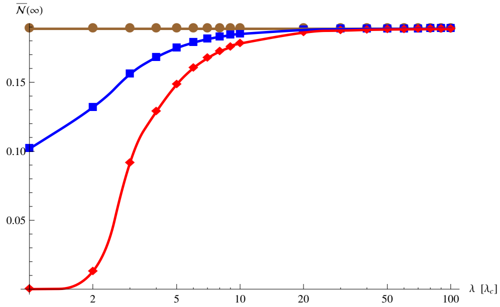
$q_x(x, t)$



Total number of created particles

- Trivial scaling effect $\to$ reduced number of created particles

$$\overline{N}(\infty) = \frac{N(\infty)}{\lambda}$$



local density appr.
linear appr.
full solution

- Need to include higher derivatives for small λ!
- Sharp drop for small $\lambda \to$ particle creation terminates because field energy becomes less than $2m$ — **only** seen in full solution!

$\mathbf{E}(\mathbf{x}, t)$ and $\mathbf{B}(\mathbf{x}, t)$: Model for electromagnetic field

C. Kohlfürst and R.A., Phys.Lett. **B756** (2016) 371 [arXiv:1512.06668]

- Superposition of left- and right-running pulses in 2+1 dim.:

$$\vec{A}(z,t) = \varepsilon\,\tau \left(\tanh\left(\frac{t}{\tau}+1\right) - \tanh\left(\frac{t}{\tau}-1\right)\right) \exp\left(-\frac{z^2}{2\lambda^2}\right)\,\vec{e}_x .$$

  - ε maximal electric field strength
  - τ temporal extent (difference of Sauter pulses)
  - λ spatial extent (Gaußian)
- homogeneous Maxwell eqs. fulfilled by construction
- electric field: double-peak structure, antisymmetric in time
- magnetic field: maximal strength $\varepsilon\tau/\lambda^2$
- Field energy in
  - electric field for $\tau/\lambda \ll 1$
  - magnetic field for $\tau/\lambda \gg 1$

for more details and other model fields see:

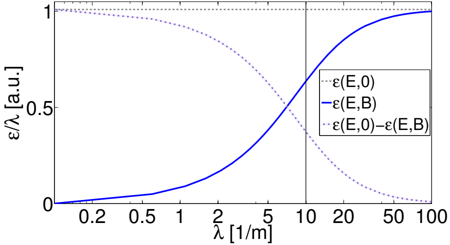
Chapter 7 of C. Kohlfürst, PhD thesis

Numerical Results for Model Field

- pseudoscalar Lorentz invariant $\tilde{F}_{\mu\nu} F^{\mu\nu} \propto \vec{E}\vec{B} = 0$
- scalar Lorentz invariant $F_{\mu\nu} F^{\mu\nu} \propto \vec{E}^2 - \vec{B}^2 \ \left(\begin{smallmatrix}>\\<\end{smallmatrix}\right)\ 0$
- pair production only for regions in which $E^2(t,z) - B^2(t,z) > 0$

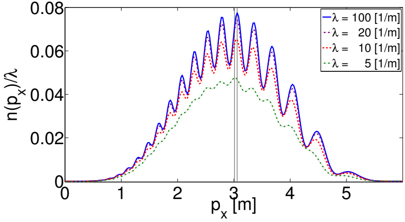
- Effectively available field energy

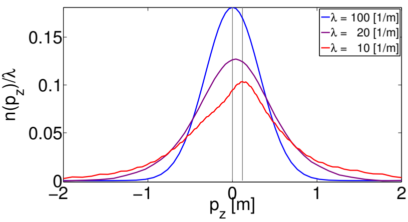
$$\mathcal{E}[\vec{E},\vec{B}] = \int dtdz \ \left(E^2(t,z) - B^2(t,z)\right) \Theta\left(E^2(t,z) - B^2(t,z)\right)$$



Numerical Results for Model Field

Reduced particle density as function of p_x and p_z
($\varepsilon = 0.707\ E_c$, $\tau = 5/m$):





$\lambda \gg \tau, 1/m$: homogeneous limit

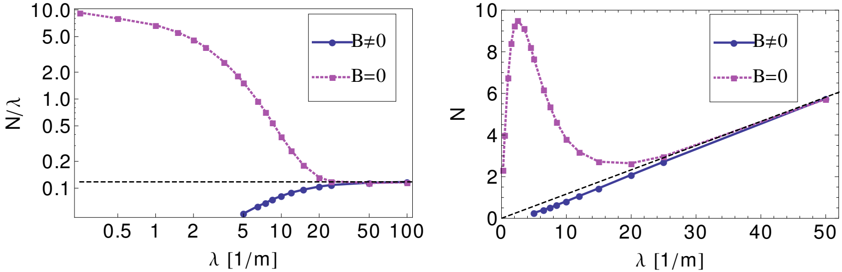
$\lambda \approx \tau$: deflection due to magnetic field

Suppression of pair production?

Numerical Results for Model Field

Comparison to calculation with magnetic field neglected

Reduced total particle number, resp., total particle number
(ε = 0.707 E_c, $\tau = 10/m$):



Significantly overestimated particle number for small λ!

NB: Regions with negative "particle distribution"!

Physics of Adiabatic Particle Number

If there is no well-defined particle number at finite times:
How can extracted time scales have physical meaning?

A. Ilderton, Phys.Rev. **D 105** (2022) 016021 [arXiv:2108.13885];
M. Diez, RA and C. Kohlfürst, Phys. Lett. **B 844** (2023) 138063 [arXiv:2211.07510]

- Time-dependent background $\Rightarrow$ time-dependent “Dirac vacuum"
- Adiabatic (i.e., instantaneous) eigenstates of the Hamiltonian: Preferred basis
- At $t < \tau$ (pulse length):
  Many more “adiabatic particles” than asymptotic particles
- “Adiabatic particles” unphysical?
- Gedankenexperiment: Shut off the background field rapidly
  e.g., $E(t, x) \to E(t, x)\Theta_{reg}(t_0 - t)$
  $\Rightarrow$ calculated spectrum for $t = t_0$ accurately represents the then measured spectrum.

Physics of Adiabatic Particle Number

This Gedankenexperiment

- relates *virtual quantum fluctuations* to *real particles* (= localized wave-packets build from asymptotic states),
- explains why the intermediate adiabatic particle number generically exceeds strongly asymptotic one,
  (shutoff field contains high-frequency modes $\rightarrow$ multiphoton pair production)
- disproves semi-classical expectations
  ("intuitive" picture of gradually forming and then accelerated particles),
- points towards multi-structure self-interfering particle phase-space distributions, and
- allows to identify sub-processes and corresponding multiple well-defined time scales.

Time Scales of Particle Formation

Matthias Diez, RA and Christian Kohlfürst, Phys. Lett. **B 844** (2023) 138063
[arXiv:2211.07510]

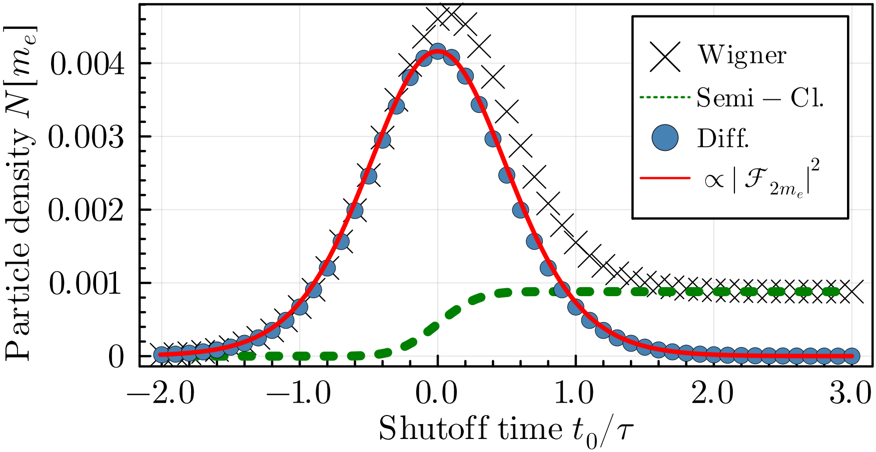
Model field

$$|e|\mathbf{E}(t,x) = \varepsilon\, m_e^2\, \mathrm{sech}^2(t/\tau)\, \exp\left(-\frac{x^2}{2\lambda^2}\right) \mathbf{e}_x$$

in Schwinger regime: $0.1 \leq \varepsilon \leq 0.5$, $\tau \geq 10/m_e$ and $\lambda \geq 10/m_e$

- as in previous numerical studies,
- minimal quantum interference, and
- seperation of charge carriers easily recognised.

Time Scales of Particle Formation



DHW particle number $N(t = t_0)$ = semi-classical Schwinger
+ perturbative "shut-off"
particle numbers

Time Scales of Particle Formation

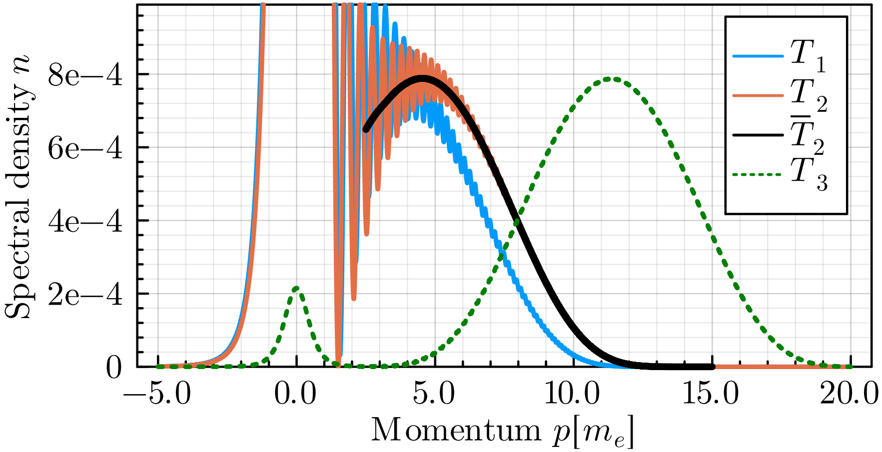
- Semi-classical Sauter-Schwinger particle creation at all times (green dashed curve)
- Perturbative particle creation due to “shut-off” high-frequency component scales with Fourier transform of $E(t)\Theta(t_0 - t)$ (full red curve)
- Full numerical DHW solution for adiabatic particle number (black crosses)
  coincides with sum of both

Expectation based on Gedankenexperiment verified ☺

Shut-off & measurement $\Leftrightarrow$ quantum interferences w.o. measurement

Time Scales of Particle Formation

Analysis of spectrum as function of kinetic momentum $p(t)$ reveals four subprocesses and **three time scales**:



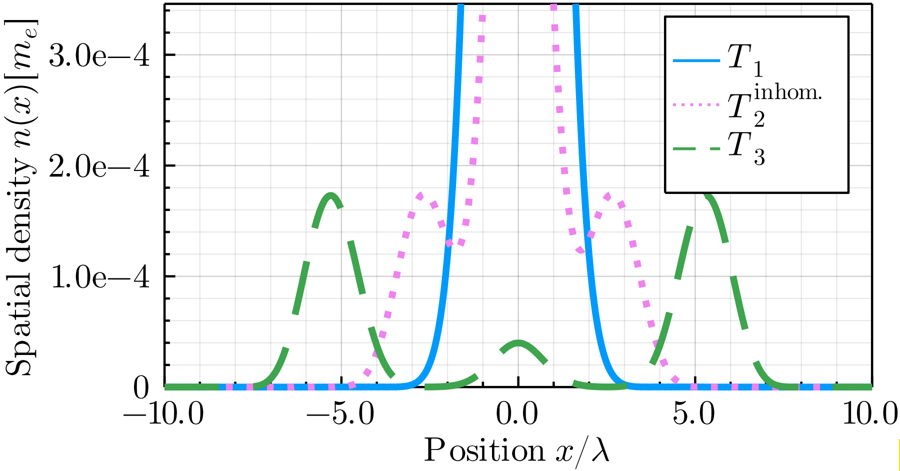
Time Scales of Particle Formation

- Early build-up of narrow perturbative peak at $p = 0$
- Self-interference $\Rightarrow$ "side" peak at $p \neq 0$ appears at $T_1 \approx 0.65\,\tau^{1/4}\,t_c^{3/4}/\varepsilon^2$, in central peak destructive interference starts to dominate
- Partial wave packet starts to follow classical trajectory at $T_2 = T_1 + 0.06\tau^{3/4}t_c^{1/4}/\varepsilon^{3/2}$ ($\rightarrow$ pre-particle)
- Perturbative peak fades away (below 1% of maximal value) at $T_3 \approx 1.8\tau$.

- ▶ Pre-particle number according to semi-class. expectation, accelerated by electric field.
- ▶ Perturbative peak height scales with ε^2 (power-like!), **not** accelerated by electric field.

Time Scales of Particle Formation

Creation of pre-electron and pre-positron at a mutual distance $d \approx 5\lambda$:



Summary

- *X*-ray FELs and/or multi-petawatt lasers (ELI, MP3, Apollon, . . .) may test for the first time **strong-field non-perturbative QED** under controlled conditions.
- Strong interferences: Pair creation and *annihilation* will happen!
- Asymptotic particle production: $-1\,\mathrm{MeV} \lesssim p_\parallel \lesssim 1\,\mathrm{MeV}$ and $k_\perp \lesssim 1\,\mathrm{MeV}$
- Solution within DHW formalism for spatially inhomogeneous electric fields! Shown here: Results in 1+1 dimensions!
- Solution within DHW formalism for inhomog. time-dep. electro**magnetic** fields! (Model field: eff. 2+1 dimensional)

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Summary

- First results for particle formation time scales!
- For the employed model field in the sub-critical Schwinger regime there is
  1. a time scale $T_1 \approx 0.65\, \tau^{1/4}\, t_c^{3/4} / \varepsilon^2$ for the appearance of a peak in the particle distribution related to a pre-particle;
  2. a time scale $T_2 = T_1 + 0.06 \tau^{3/4} t_c^{1/4} / \varepsilon^{3/2}$ at which the pre-particle identifiable follows a classical trajectory; and
  3. a time scale $T_3 = 1.8\tau$ at which quantum fluctuations (quantum interference and the 'perturbative' peak related to a would-be shutoff) fade out.

Outlook

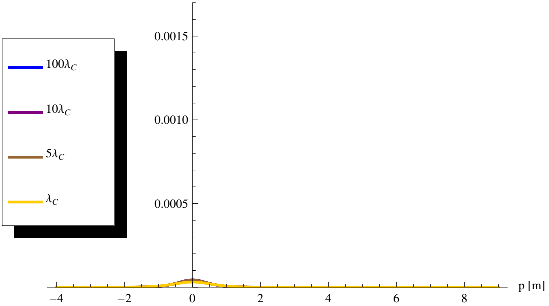
- Include
  - first classical and
  - then quantum

  back-reaction and study Sauter-Schwinger effect at critical and super-critical field strengths:
  - Sub-processes of particle production?
  - Generic patterns for the time scales?
  - Formation of a QED cascade and fundamental limitation for physically achievable field strenghts?

Reduced particle number density

Reduced particle momentum distribution: $\bar{f}_p(p,t) \equiv \dfrac{f_p(p,t)}{\lambda}$

$$t = -2\tau$$



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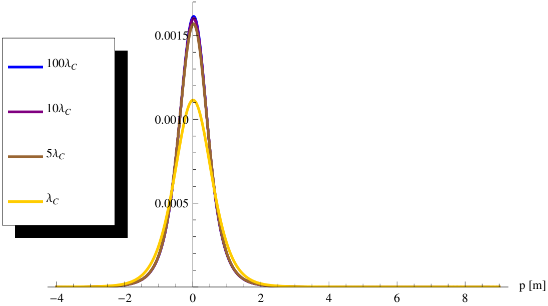

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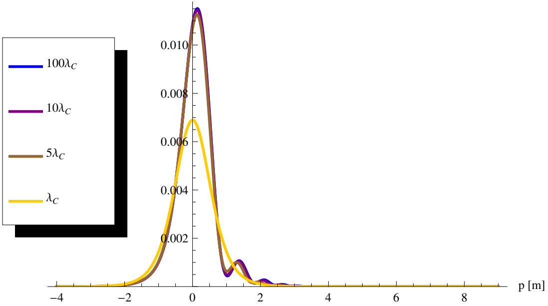

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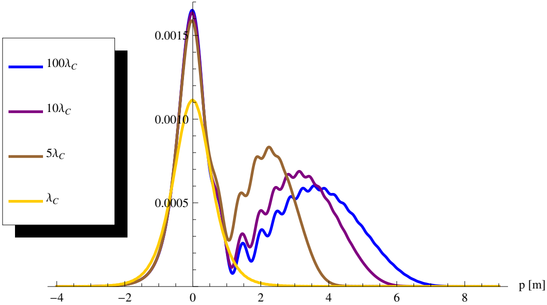

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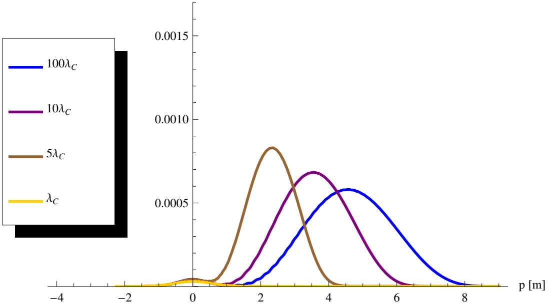

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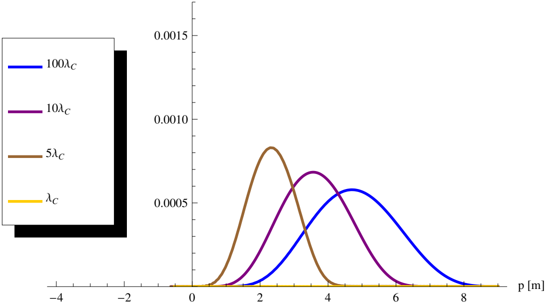

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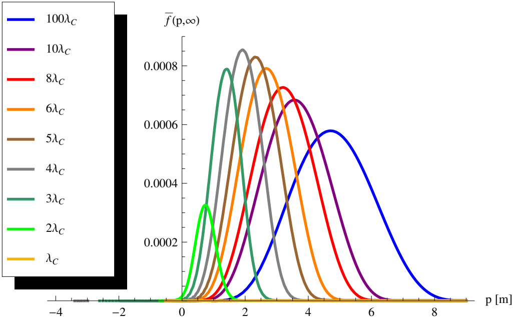

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