Aspects of non-Abelian gauge theories with fundamentally charged fermions

Georg Wieland
georg.wieland@edu.uni-graz.at

Institute of Physics
University of Graz

September 28, 2023
ACHT 2023

Motivation

Non-Abelian quantum gauge field theories are a cornerstone of particle physics

- Essential part of the SM (QCD and EW unification)
- Bridge to "unexplored" areas (DM, GUTs, ...)

The formulation of an effective field theory rests on two principles

- Decoupling theorem
- Wilson's renormalization group

Objectives

- Solve the full fermion-gauge boson vertex for non-zero current masses to investigate the behavior of the χS and χSB dressing functions for large fermion masses ($M \gtrsim \Lambda_{YM}$)
- Test the decoupling theorem using non-perturbative methods
- Establish numerical methodologies to enable $N_f > 0$ calculations for various gauge groups in light of future investigations

Non-Abelian gauge theory in Landau gauge

Lagrangian density

$$\mathcal{L} = \underbrace{\frac{1}{2}\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}\right)}_{\text{gauge bosons}} + \underbrace{\sum_f \bar{\psi}_f \left(\not{D} + m_f\right)\psi_f}_{\text{fermions}} + \underbrace{\mathcal{L}_{GF}}_{\text{ghosts}}$$

- Field strength tensor: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig\left[A_\mu, A_\nu\right]$
- Covariant derivative: $D_\mu = \partial_\mu + igA_\mu$
- Color structure: $A_\mu = A_\mu^a t^a$

Non-commutative nature $\Longrightarrow$ self-interactions between charged force carriers

Matter particles $\Longrightarrow$ dynamical generation of fermion mass

Strongly interacting $\Longrightarrow$ non-perturbative functional methods

Setup and outline

Yang-Mills sector

- Gauge boson propagator
- Ghost propagator
- Three-gauge boson vertex
- Ghost-gauge boson vertex

Use as input → $N_f = 0$

Fermion sector

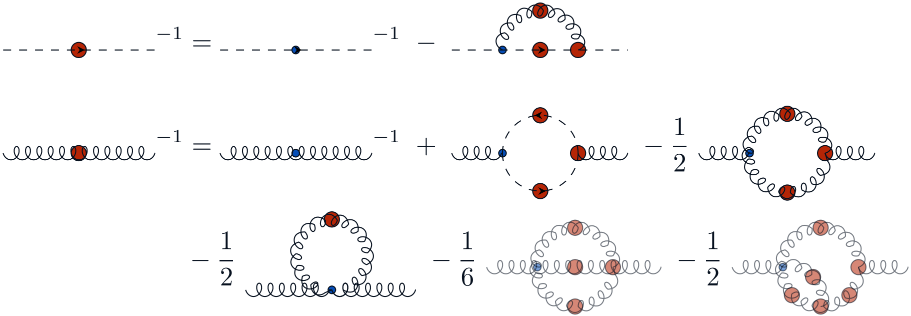
- Fermion propagator
- Fermion-gauge boson vertex

$\Longrightarrow$ dynamical mass generation

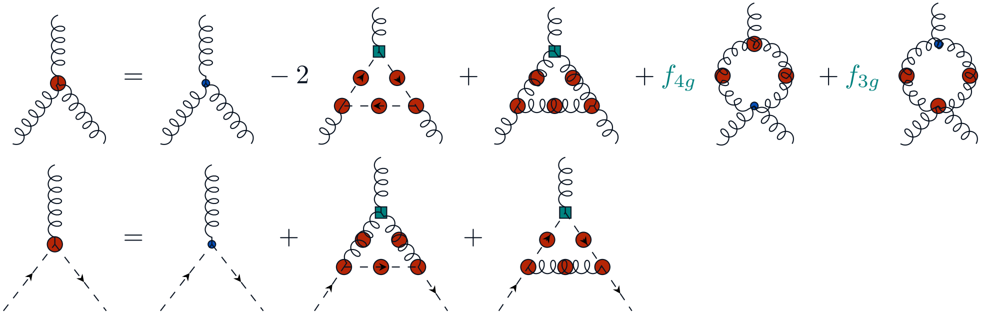
Disclaimer

- The main results are presented for an SU(3) gauge group without external scale setting
- A general terminology is used to emphasize the differences to QCD

Yang-Mills propagator DSEs



EoMs of the Yang-Mills vertices

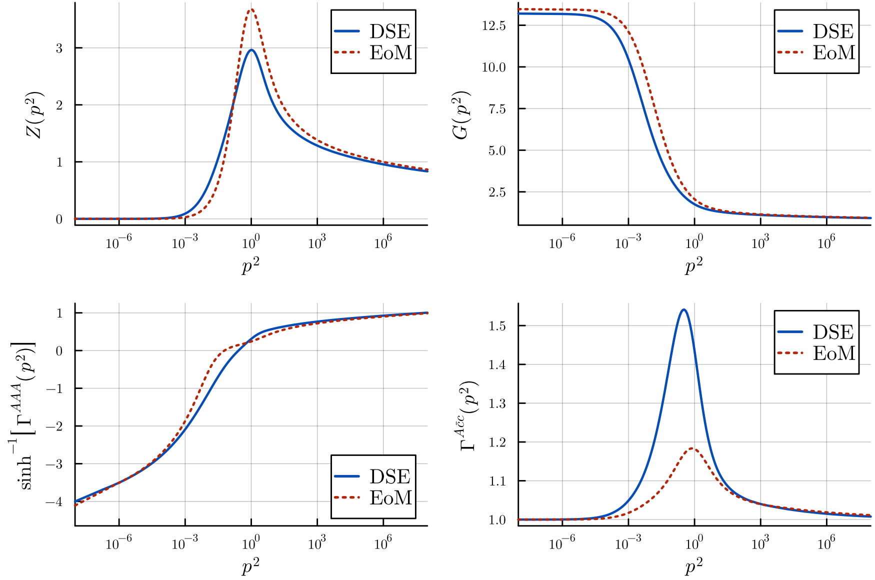


Truncation scheme

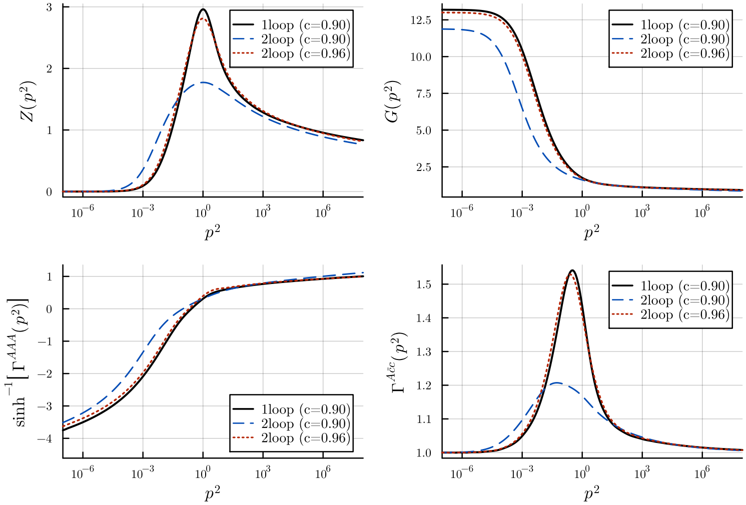
- Three- and four-point functions: $\Gamma^{\mu\cdots}(p,\dots) \longrightarrow \Gamma\left(\bar{s}^2\right)\cdot T^{\mu\cdots}_{\text{tree}}$
- Four-gauge boson vertex dressing: $\Gamma^{AAAA}(\bar{s}^2) \longrightarrow G^2(\bar{s}^2)/Z(\bar{s}^2)$

Huber, Phys. Rev. D **101** (2020)
Eichmann, Pawlowski, Silva, Phys. Rev. D **104** (2021)
Aguilar, Ferreira, Papavassiliou, Santos, Eur. Phys. J. C **83** (2023)

Quenched Yang-Mills theory



Effect of two-loop diagrams



Parameter $c \in (0, 1]$ used to modify the renormalization constant of the three-gauge boson vertex

$$Z_1 \to c\,Z_1 = c\,\frac{Z_3}{\tilde{Z}_3}$$

System of equation converges for values $c \leq c_{\text{max}}$

Allows for estimating the error of the truncation

Eichmann, Pawlowski, Silva, Phys. Rev. D **104** (2021)

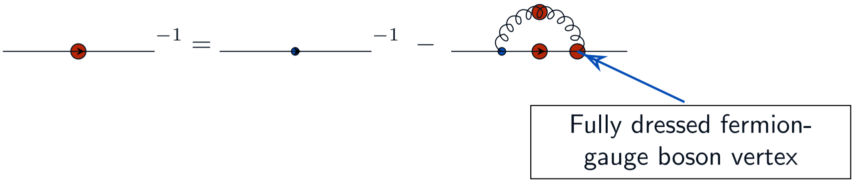
Fermion propagator

Fermion mass is not constant $\implies$ dynamical mass generation

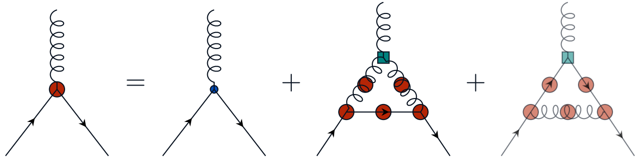
Inverse fermion propagator

$$\left(S^{(0)}(p)\right)^{-1} = Z_2\left(i\not{p} + Z_m m_R\right) \quad \longrightarrow \quad S^{-1}(p) = A(p^2)\left(i\not{p} + M(p^2)\right) = iA(p^2)\not{p} + B(p^2)$$

Fermion propagator Dyson-Schwinger equation



Fermion-gauge boson vertex



Fully dressed fermion-gauge boson vertex

$$\Gamma_\mu^{A\bar\psi\psi,a,ij}(k;-p,q) = igt^{a,ij}\sum_{i=1}^{8}\Gamma^{(i),A\bar\psi\psi}(k^2,\bar p^2,k\cdot\bar p)R_\mu^{(i)}(k;\bar p)$$

Transverse basis which renders the dressings free of kinematic singularities

χS: $R^{(1),\mu} = \mathcal{T}_k^{\mu\nu}\gamma_\nu$, $R^{(4),\mu} = \frac{1}{6}\left[\gamma^\mu, \not{\bar p}, \not k\right]$, $R^{(6),\mu} = \mathcal{T}_k^{\mu\nu}\,\bar p_\nu \not{\bar p}$, $R^{(7),\mu} = (\bar p\cdot k)t_{k\bar p}^{\mu\nu}\gamma_\nu$,

χSB: $R^{(2),\mu} = \frac{i}{2}(\bar p\cdot k)\mathcal{T}_k^{\mu\nu}\left[\gamma_\nu, \not{\bar p}\right]$, $R^{(3),\mu} = \frac{i}{2}\left[\gamma^\mu, \not k\right]$, $R^{(5),\mu} = i\,\mathcal{T}_k^{\mu\nu}\,\bar p_\nu$, $R^{(8),\mu} = \frac{i}{2}t_{k\bar p}^{\mu\nu}\left[\gamma_\nu, \not{\bar p}\right]$

Williams, Eur. Phys. J. A **51** (2015)
Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog. Part. Nucl. Phys. **91** (2016)

Charge conjugation

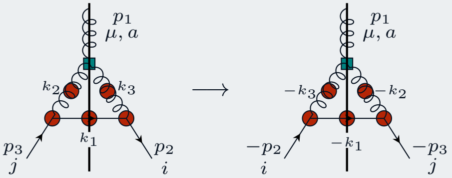
The dressed propagator and vertex must undergo the same transformation as the bare propagator and vertex

$$C^{-1}S(p)C = \left(S(-p)\right)^{\top}, \qquad C^{-1}\Gamma_{\mu}^{A\bar{\psi}\psi}(k;\bar{p})C = -\left(\Gamma_{\mu}^{A\bar{\psi}\psi}(k;-\bar{p})\right)^{\top}$$

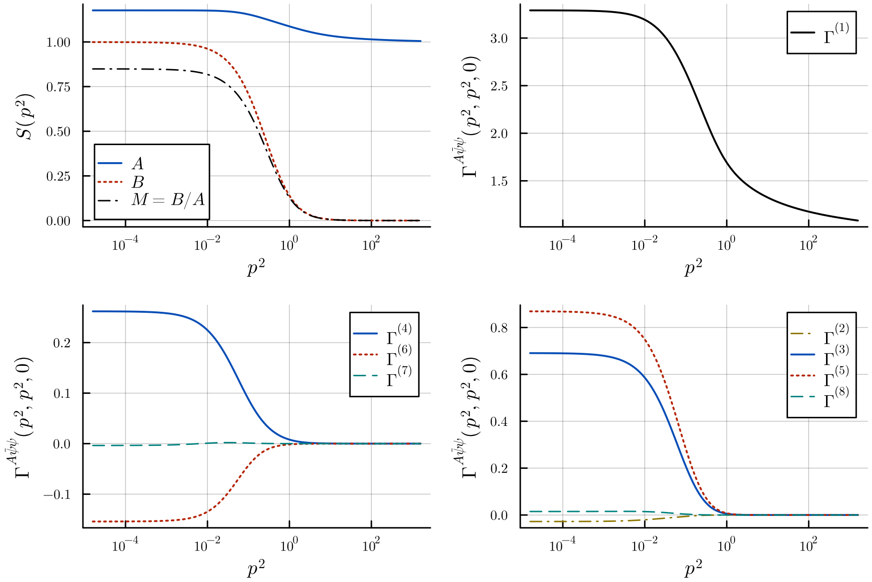
Crossing symmetry of the non-Abelian diagram

- A redefinition in momenta is reflected by sign changes in angular variables
- Establishes relations between parts of the kernels

$$V_{ij}^{(k)}(w,z,t) = V_{ji}^{(k)}(-w,-z,t)$$



Chiral limit $m_R = 0$

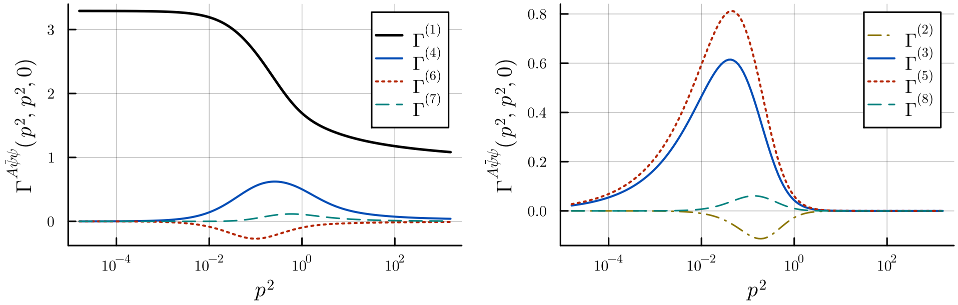


Basis for solving the vertex numerically

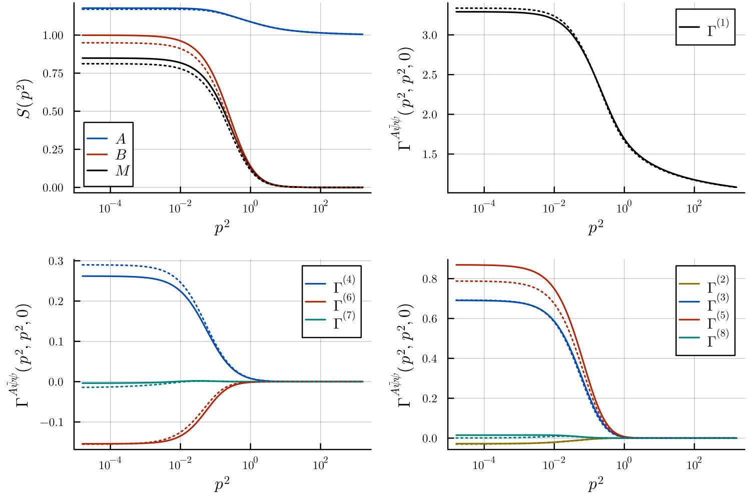
For the numerical computation, it is advantageous to employ a basis that results in the majority of dressing functions being zero in both the IR and UV

Therefore, the basis is adjusted by normalizing all momenta

$$\Gamma^{A\bar{\psi}\psi,\mu}(k;-p,q) = \sum_{i=1}^{8} \Gamma_{\mathcal{N}}^{(i),A\bar{\psi}\psi}(k^2,\bar{p}^2,k\cdot\bar{p}) R_{\mathcal{N}}^{(i),\mu}(k;\bar{p})$$



Effect of the Abelian diagram



For SU(n) gauge groups, the Abelian diagram is suppressed by n^2

Maximum discrepancy of the dressing functions remains below 10%

Mass function M experiences minor alterations in the IR below 5%

Including massive fermions

Generalize calculations for current masses $m_R > 0$

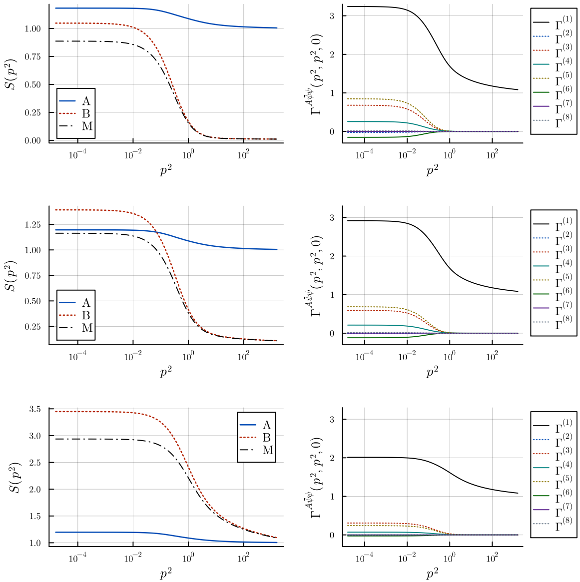
Only difference in fermion propagator equations

$$A(p^2) = 1 + \Pi_A(p^2) - \Pi_A(\Lambda_f^2)$$
$$B(p^2) = m_R + \Pi_B(p^2) - \Pi_B(\Lambda_f^2)$$

Choose $m_R \in [10^{-3}, 10^3]$

Extrapolate according to

$$B(p^2 \to \infty) = a_B \ln(p^2)^{f_B}$$



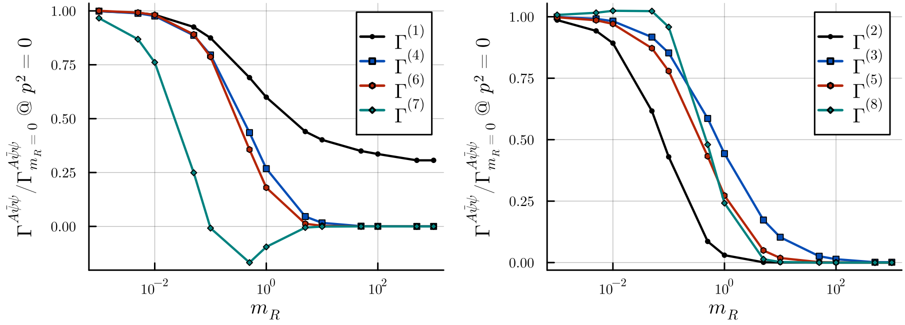
Decoupling for $m_R \gg \Lambda_{YM}$

Suppression mechanism related to the fermion propagator $S(p)$

For large m_R, $M(p^2) \approx m_R$ and $M(p^2) \gg A(p^2)$ leading to

$$S(p) \approx M^{-1}(p^2) \quad \text{for} \quad p^2 \ll m_R^2$$

Observed suppression is related to the dressing $B(p^2)$



Summary

Efficient approach for solving the two- and three-point functions of a quenched non-Abelian gauge theory in a self-consistent manner

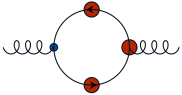
Fully dressed fermion-gauge boson vertex for various current masses shows decoupling behavior of fermions with $m_R \gg \Lambda_{YM}$

- Suppression mechanism of the fermion propagator cannot be offset by any quantity in the fermion sector
- Fermionic contributions to the Yang-Mills theory are suppressed by powers of M
- For $m_R \gg \Lambda_{YM}$, the Yang-Mills behavior is recovered

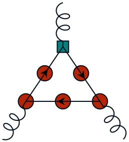
Outlook

Calculate fermionic contributions to the Yang-Mills sector for $M \lesssim \Lambda_{YM}$

fermion loop in the gauge boson propagator DSE



fermion triangle in the three-gauge boson vertex EoM



Move beyond quenched approximation

- Coupled set of equations for $N_f > 0$ fermions
- Large-N_f calculations (conformal window, N_f^{crit}, ...)

Long-term: BSE calculations in QCD beyond rainbow latter with full quark-gluon vertex

Backup slides

Renormalization

Momentum subtraction scheme for propagator equations

$$Z_\mu = Z(p^2 = \mu^2), \qquad G_0 = G(p^2 = 0), \qquad \{1, m_R\} = \{A, B\}(p^2 = \Lambda_f^2)$$

The renormalization constants of higher n-point functions are fixed via their STIs in Landau gauge, where $\tilde{Z}_1 = 1$

$$Z_1 = Z_3/\tilde{Z}_3, \qquad Z_4 = Z_3/\tilde{Z}_3^2, \qquad Z_{1F} = Z_2/\tilde{Z}_3$$

Redefine dressings Z and G and all quantities that renormalize like Z and G

$$Z(p^2) \to \frac{Z(p^2)}{Z_\mu}, \qquad G(p^2) \to \frac{G(p^2)}{G_0}$$

Define "coupling" parameter α, which appears in each diagram

$$\alpha := \frac{g^2}{4\pi} Z_\mu G_0^2$$

Eichmann, Pawlowski, Silva, Phys. Rev. D **104** (2021)

Renormalization

Rewrite all equations into dimensionless quantities by redefining all external and internal momenta

$$x := \frac{p^2}{\beta\mu^2}, \qquad y := \frac{q^2}{\beta\mu^2}$$

This necessitates the redefinition of the fermion propagator dressing B and the renormalized mass m_R

$$B^{\text{dimless}}(x) := \frac{B(x)}{\sqrt{\beta\mu^2}}, \qquad m_R^{\text{dimless}} := \frac{m_R}{\sqrt{\beta\mu^2}}$$

The fully dressed fermion-gauge boson vertex must scale in accordance with its tree-level structure $\Longrightarrow$ redefinition of dressings via powers of $\beta\mu^2$

Lastly, redefine dressing function G once more

$$G(x) \to \sqrt{\alpha}G(x)$$

Eichmann, Pawlowski, Silva, Phys. Rev. D **104** (2021)

Renormalization

Final set of equations

$$Z^{-1}(x) = 1 + \hat{\Pi}_Z(x) - \hat{\Pi}_Z(1/\beta), \qquad F^{AAA}(x) = Z_1 + \Pi^{AAA}(x)$$

$$G^{-1}(x) = 1/\sqrt{\alpha} + \Pi_G(x) - \Pi_G(0), \qquad F^{A\bar{c}c}(x) = 1 + \Pi^{A\bar{c}c}(x)$$

$$\{A, B\}(x) = \{1, m_R\} + \Pi_{\{A,B\}}(x) - \Pi_{\{A,B\}}(L_f), \qquad F^{A\bar{\psi}\psi}(x) = Z_{1F} + \Pi^{A\bar{\psi}\psi}(x)$$

$$Z_3 = 1 - \hat{\Pi}_Z(1/\beta), \qquad \tilde{Z}_3 = 1/\sqrt{\alpha} - \Pi_G(0), \qquad Z_2 = 1 - \Pi_A(L_f)$$

Remove quadratic divergences by decomposing the self-energy in the gauge boson propagator equation using suitable projectors

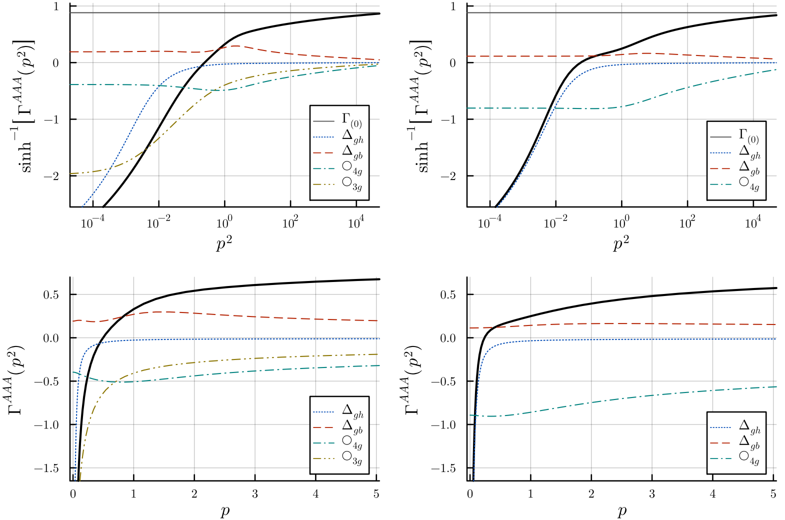
$$\hat{\Pi}_Z(x) = \Pi_Z(x) + \frac{\tilde{\Pi}_Z(x)}{x}$$

$\Pi_Z(x)$ diverges only logarithmically, spurious divergences only appear in $\tilde{\Pi}_Z(x)$

Eichmann, Pawlowski, Silva, Phys. Rev. D **104** (2021)

Three-gauge boson vertex in detail

DSEs

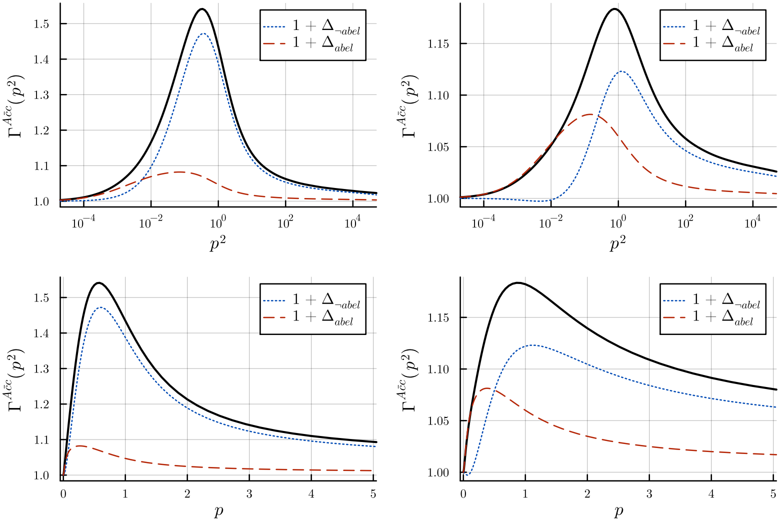


3PI EoMs

In qualitative agreement with Eichmann, Williams, Alkofer, Vujinovic, Phys. Rev. D **89** (2014) and Williams, Fischer, Heupel, Phys. Rev. D **93** (2016)

Ghost-gauge boson vertex in detail

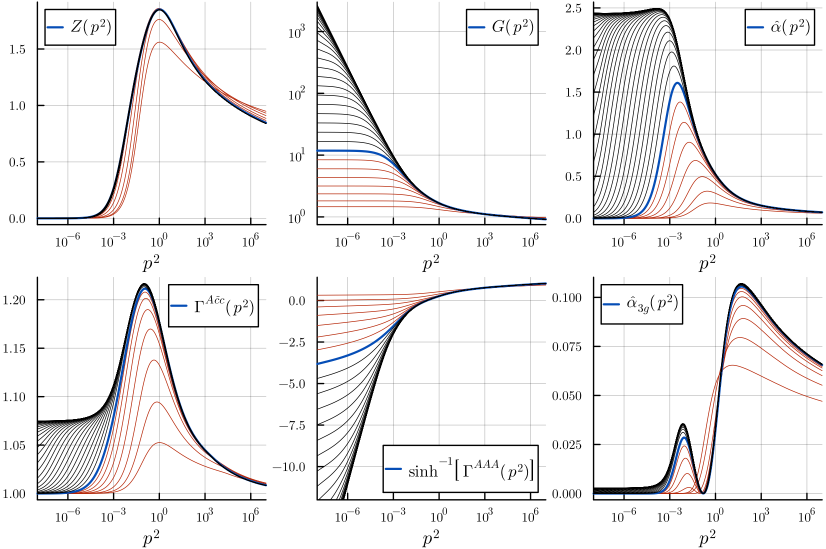
DSEs



3PI
EoMs

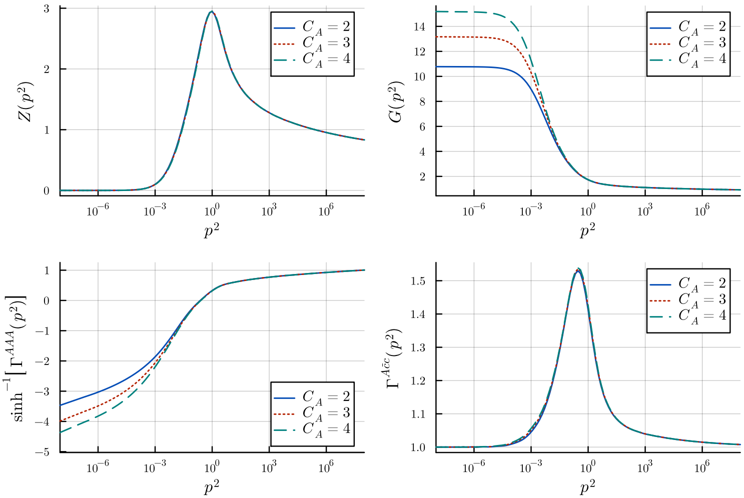
In qualitative agreement with Huber, Phys. Rev. D **101** (2020)
and Zierler, Master thesis, 2019

Different values of the "coupling" parameter α



In agreement with Eichmann, Pawlowski, Silva, Phys. Rev. D **104** (2021)

Yang-Mills sector for different values of C_A



Define "coupling" α by including the color pre-factor C_A

$$\alpha \propto \frac{g^2}{4\pi} C_A$$

The differences among various gauge groups are effectively encoded in the coupling α

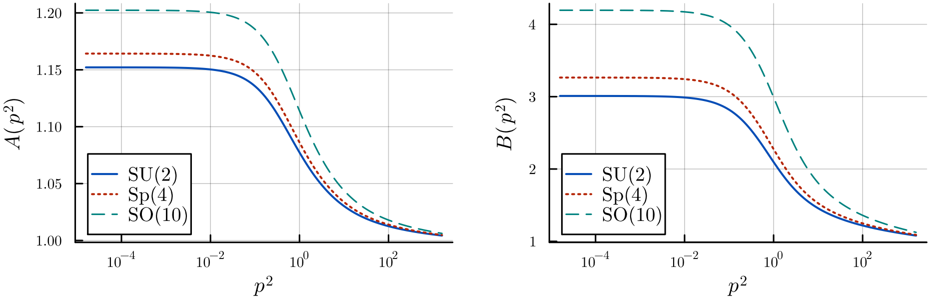
Fermion sector for selected gauge groups

Define remaining color factor in the quenched approximation

$$C'_F := C_F/C_A$$

Final set of renormalized equations in the fermion sector

$$\{A,B\}(x) = \{1, m_R\} + C'_F \Pi_{\{A,B\}}(x) - C'_F \Pi_{\{A,B\}}(L_f),$$
$$F^{A\bar{\psi}\psi}(x) = Z_{1F} + 1/2\, \Pi^{A\bar{\psi}\psi}_{\neg abel}(x) + (1/2 - C'_F)\, \Pi^{A\bar{\psi}\psi}_{abel}(x)$$



Supported by findings in Llanes-Estrada, Salas-Berárdez, Commun. Theor. Phys. **71** (2019)

Fermion-gauge boson vertex for several gauge groups

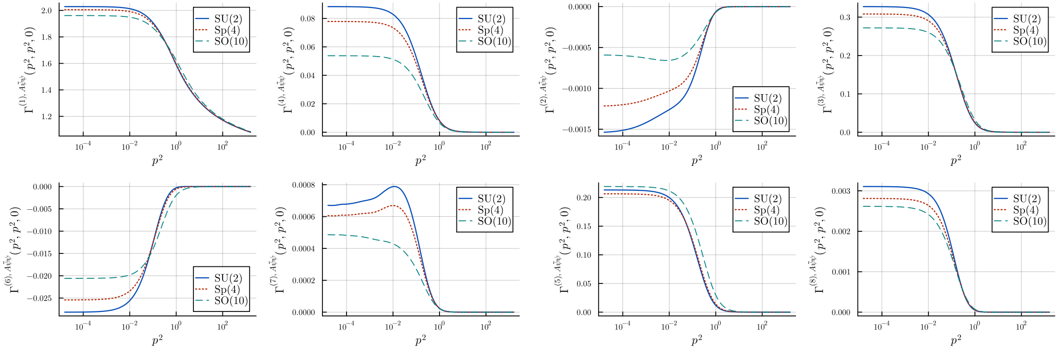
Final set of renormalized equations

$$Z^{-1}(x) = 1 + \hat{\Pi}_Z(x) - \hat{\Pi}_Z(1/\beta), \qquad F^{A\bar{c}c}(x) = 1 + \Pi^{A\bar{c}c}(x),$$

$$G^{-1}(x) = 1/\sqrt{\alpha} + \Pi_G(x) - \Pi_G(0), \qquad F^{AAA}(x) = Z_1 + \Pi^{AAA}(x),$$

$$\{A, B\}(x) = \{1, m_R\} + C_F' \Pi_{\{A,B\}}(x) - C_F' \Pi_{\{A,B\}}(L_f),$$

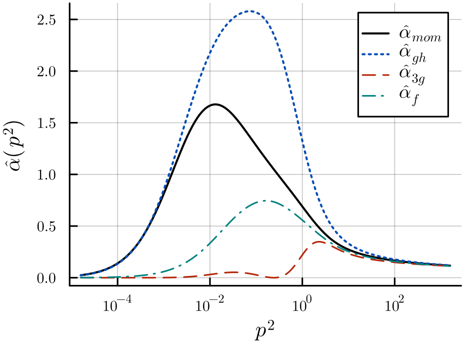
$$F^{A\bar{\psi}\psi}(x) = Z_{1F} + 1/2\,\Pi^{A\bar{\psi}\psi}_{\neg abel}(x) + \left(1/2 - C_F'\right) \Pi^{A\bar{\psi}\psi}_{abel}(x)$$

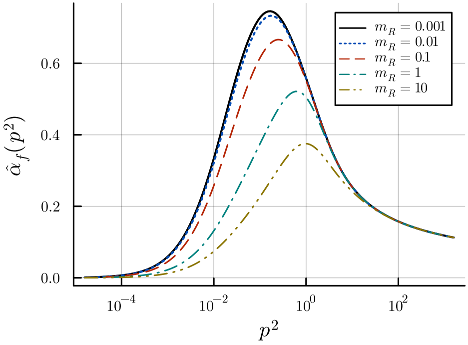


Running couplings

RG invariant couplings deviate for non-perturbative scales but exhibit good agreement in the UV regime above $p^2 = 10^1$

Agreement can be improved by choosing a more inclusive truncation, see, e.g., Huber, Phys. Rev. D **101** (2020) for the Yang-Mills theory





Constructing the basis

Complete basis free of kinematic singularities (dressing functions $\tilde{\Gamma}^{(i)}$ go to constants for $k \to 0$ and $\bar{p} \to 0$) with $t^{kk}_{\mu\nu} = k^2\delta_{\mu\nu} - k_\mu k_\nu$ and $t^{k\bar{p}}_{\mu\nu} = (k\cdot\bar{p})\delta_{\mu\nu} - \bar{p}_\mu k_\nu$

$$\tilde{G}^{(1)}_\mu = \gamma_\mu, \qquad \tilde{R}^{(1)}_\mu = t^{kk}_{\mu\nu}\gamma^\nu, \qquad \tilde{R}^{(5)}_\mu = i\, t^{kk}_{\mu\nu}\,\bar{p}^\nu,$$

$$\tilde{G}^{(2)}_\mu = \bar{p}_\mu\,\bar{\not{p}}, \qquad \tilde{R}^{(2)}_\mu = \frac{i}{2}(\bar{p}\cdot k)t^{kk}_{\mu\nu}\left[\gamma^\nu, \bar{\not{p}}\right], \qquad \tilde{R}^{(6)}_\mu = t^{kk}_{\mu\nu}\,\bar{p}^\nu\bar{\not{p}},$$

$$\tilde{G}^{(3)}_\mu = i\,\bar{p}_\mu, \qquad \tilde{R}^{(3)}_\mu = \frac{i}{2}\left[\gamma_\mu, \not{k}\right], \qquad \tilde{R}^{(7)}_\mu = (\bar{p}\cdot k)t^{k\bar{p}}_{\mu\nu}\gamma^\nu,$$

$$\tilde{G}^{(4)}_\mu = \frac{i}{2}(\bar{p}\cdot k)\left[\gamma_\mu, \not{k}\right], \qquad \tilde{R}^{(4)}_\mu = \frac{1}{6}\left[\gamma_\mu, \bar{\not{p}}, \not{k}\right], \qquad \tilde{R}^{(8)}_\mu = \frac{i}{2}t^{k\bar{p}}_{\mu\nu}\left[\gamma^\nu, \bar{\not{p}}\right].$$

Relations between dressing functions

$$\Gamma^{(1)} = k^2\,\tilde{\Gamma}^{(1),T} + \tilde{\Gamma}^{(1),G}, \qquad \Gamma^{(5)} = k^2\,\tilde{\Gamma}^{(5),T} + \tilde{\Gamma}^{(3),G},$$

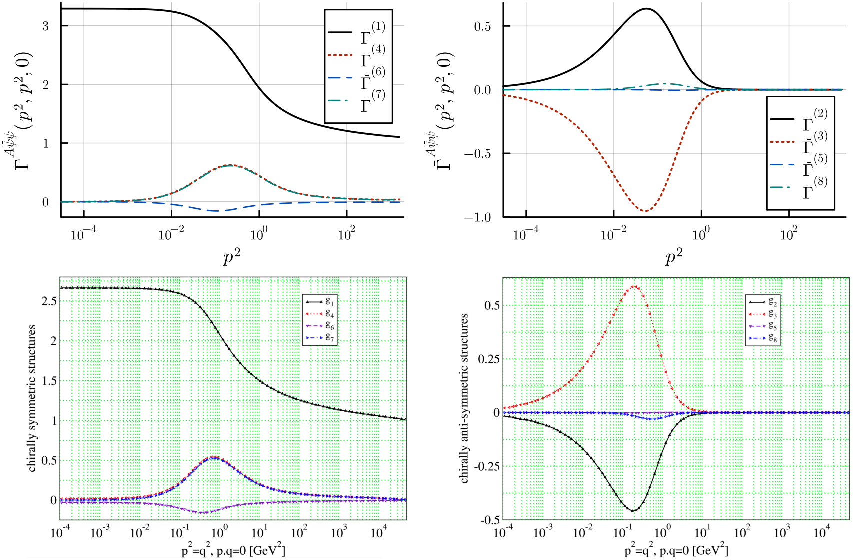
$$\Gamma^{(2)} = k^2\,\tilde{\Gamma}^{(2),T}, \qquad \Gamma^{(6)} = k^2\,\tilde{\Gamma}^{(6),T} + \tilde{\Gamma}^{(2),G},$$

$$\Gamma^{(3)} = \tilde{\Gamma}^{(3),T} + (\bar{p}\cdot k)\,\tilde{\Gamma}^{(4),G}, \qquad \Gamma^{(7)} = \tilde{\Gamma}^{(7),T},$$

$$\Gamma^{(4)} = \tilde{\Gamma}^{(4),T}, \qquad \Gamma^{(8)} = \tilde{\Gamma}^{(8),T}.$$

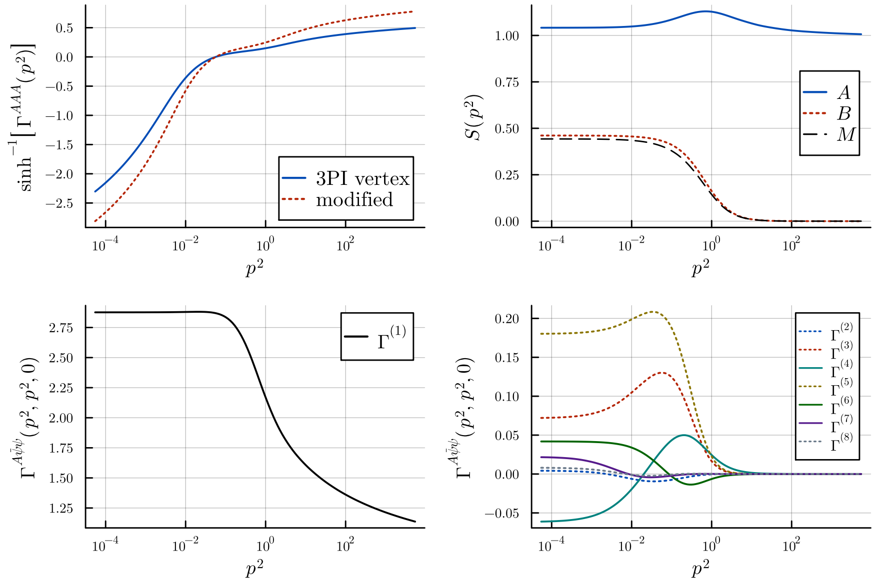
Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog. Part. Nucl. Phys. **91** (2016)

Comparing to previous results

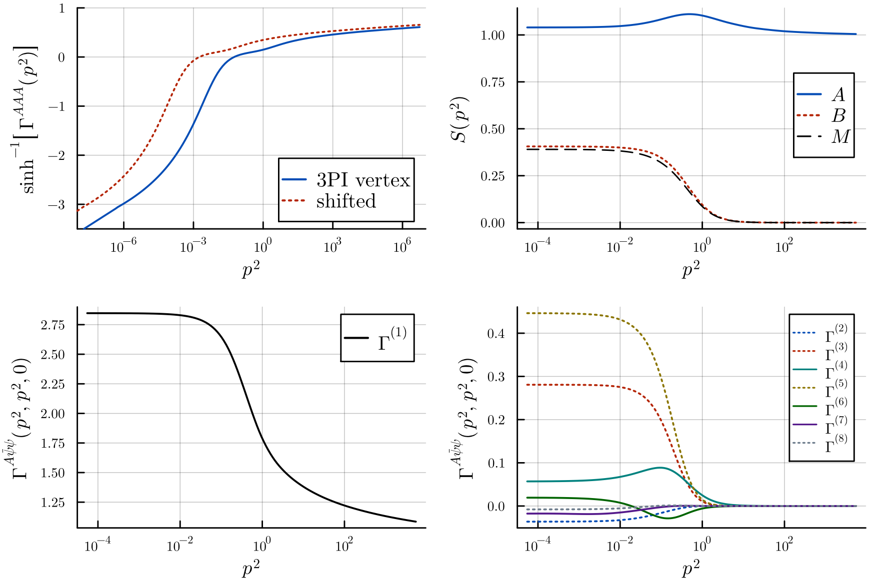


Windisch, PhD thesis, 2014

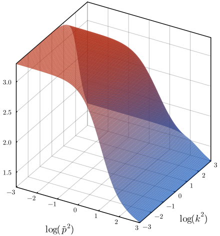
Fermion sector 3PI EoMs - $1.671 \cdot \Gamma^{AAA}$



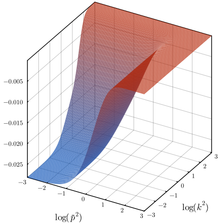
Fermion sector 3PI EoMs - shifted Γ^{AAA}



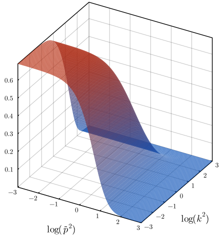
Multivariate dressings



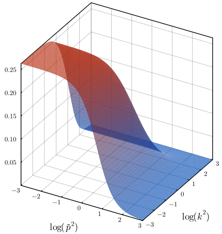
a) $\Gamma^{(1),A\bar{\psi}\psi}(k^2,\bar{p}^2,0.5)$



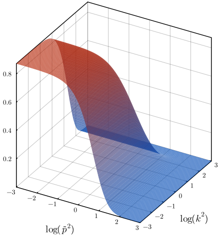
b) $\Gamma^{(2),A\bar{\psi}\psi}(k^2,\bar{p}^2,0.5)$



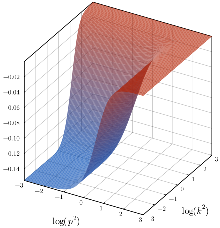
c) $\Gamma^{(3),A\bar{\psi}\psi}(k^2,\bar{p}^2,0.5)$



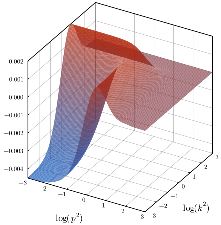
d) $\Gamma^{(4),A\bar{\psi}\psi}(k^2,\bar{p}^2,0.5)$



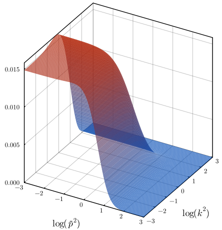
e) $\Gamma^{(5),A\bar{\psi}\psi}(k^2,\bar{p}^2,0.5)$



f) $\Gamma^{(6),A\bar{\psi}\psi}(k^2,\bar{p}^2,0.5)$



g) $\Gamma^{(7),A\bar{\psi}\psi}(k^2,\bar{p}^2,0.5)$



h) $\Gamma^{(8),A\bar{\psi}\psi}(k^2,\bar{p}^2,0.5)$