# Revealing the nucleon structure through exclusive meson production

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# Outline

- Introduction (EMFFs, PDFs)
- Generalized Parton Distributions (GPDs)

  through deeply virtual processes
- Photon-meson photoproduction
- Summary and outlook

in collaboration with G. Duplančić, S. Nabeebaccus, K. Passek-K., B. Pire, L. Szymanowski, S. Wallon

# Introduction

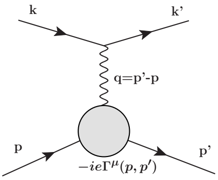
SCATTERING

$$
\left.
\begin{array}{ll}
\rightarrow \text{elastic} & (e^- p \rightarrow e^- p) \\
\rightarrow \text{inelastic} & (e^- p \rightarrow e^- \pi p)
\end{array}
\right\} \text{exclusive}
$$

$$
(e^- p \rightarrow e^- X) \quad \} \quad \text{inclusive}
$$

# Nucleon Electromagnetic Form Factors (EMFFs)

ELASTIC SCATTERING on composite particle




$$\mathcal{M} = e^2 \bar{u}(p')\Gamma^\mu(p,p')u(p)L_\mu(k,k')$$

$$\Rightarrow \Gamma^\mu(p,p') = \gamma^\mu F_1(q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2M}F_2(q^2)$$

$$G_E(q^2) = F_1(q^2) + \tau F_2(q^2) \overset{FT}{\rightarrow} \text{charge distr. } \rho$$

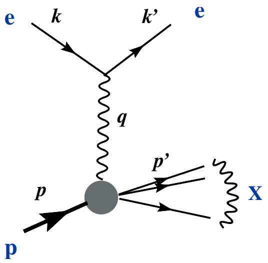
$$G_M(q^2) = F_1(q^2) + F_2(q^2) \overset{FT}{\rightarrow} \text{ magnetic distr. } \mu$$

$(\tau = -q^2/4M^2)$

$$\frac{d\sigma}{d\Omega_{lab}} = \frac{\alpha^2}{4E^2\sin^4\theta/2}\frac{E'}{E}\left\{\frac{G_E^2(q^2)+\tau G_M^2(q^2)}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2(q^2)\sin^2\frac{\theta}{2}\right\}_{ep\text{-}}$$

[Rosenbluth]

# Inelastic inclusive scattering




$$q = (\nu, \vec{q})$$

scalars often used:

$E'$, $\theta$ (exp.)

$q^2$, $\nu = \frac{q \cdot p}{M} = E - E'$ (teor.)

$q^2$, $x = \frac{-q^2}{2q \cdot p}$ (teor.)

$$W^{\mu\lambda} = -W_1 g^{\mu\lambda} + \frac{W_2}{M^2} p^\mu p^\lambda + \frac{W_4}{M^2} q^\mu q^\lambda + \frac{W_5}{M^2} (p^\mu q^\lambda + p^\lambda q^\mu)$$

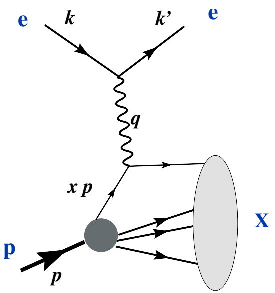
$$q_\mu W^{\mu\lambda} = q_\lambda W^{\mu\lambda} = 0$$

$$d\sigma \sim L^e_{\mu\lambda} \, W^{\mu\lambda}$$

$$\sim \left\{ W_2(q^2, \nu) \cos^2 \frac{\theta}{2} + 2 W_1(q^2, \nu) \sin^2 \frac{\theta}{2} \right\}_{ep \to eX}$$

$W_1$, $W_2$ . . . structure functions

# Deep inelastic scattering (DIS)




**Bjorken limit:**

$$q^2 \to \infty \qquad x = x_B = \frac{-q^2}{2q \cdot p}$$

$$\nu \to \infty \qquad = cte.$$

Partons almost free

$$\Downarrow$$

sum of elastic $e^-$-parton scatterings

structure functions: Bjorken scaling

$$M\ W_1(q^2, x) \quad \to \quad F_1(x)$$

$$-\frac{q^2}{2Mx} W_1(q^2, x) \quad \to \quad F_2(x)$$

# Scaling violation in DIS

structure functions:

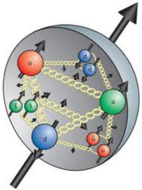
$$F_1(x) \rightarrow F_1(x, Q^2)$$
$$F_2(x) \rightarrow F_2(x, Q^2)$$

$$\downarrow$$

ln $Q^2$ dependence ($Q^2 = -q^2$)

$$\Uparrow$$

parton interactions



# PDFs and factorization of DIS

- asymptotic freedom
- factorization

$$\Downarrow$$

structure functions:

$$F_i(x, Q^2) = \sum_a \int dz \; C_i^a(x/z, Q^2/\mu^2) \; q_a(z, \mu^2)$$

$\mu^2$ ... factorization scale
$a$ ... parton type

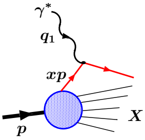
$C_i^a(x/z, Q^2/\mu^2)$ ... coefficient functions $\rightarrow$ pQCD ($\alpha_S$ exp.)

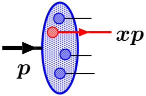
$q_a(z, \mu^2)$ ... parton distribution functions (PDFs)

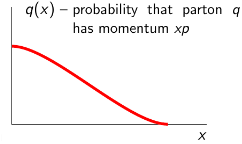
$\rightarrow$ nonpert. input + DGLAP evolution equation (pQCD)

# Nuclear structure from EMFFs and PDFs

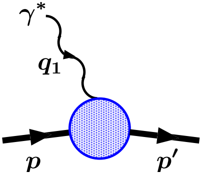
- Deeply inelastic scattering $-q_1^2 \equiv Q^2 \to \infty,\ x_{BJ} \equiv \frac{-q_1^2}{2p\cdot q_1} \to \text{cte.}$ (Bjorken limit)

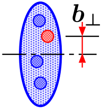


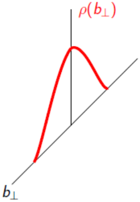



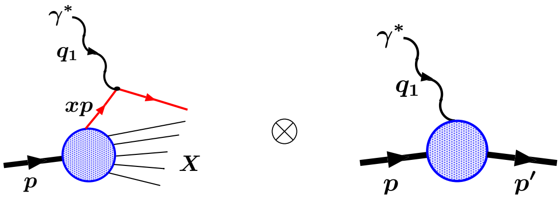

- Form factors → charge distribution $\rho(b_\perp) \sim \int dq_1\, e^{-iq_1\cdot b_\perp} F_1(t = q_1^2)$

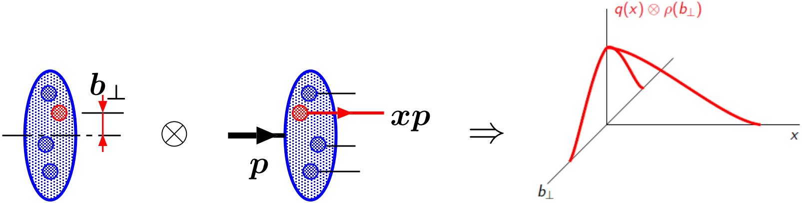


# Nuclear structure from EMFFs and PDFs



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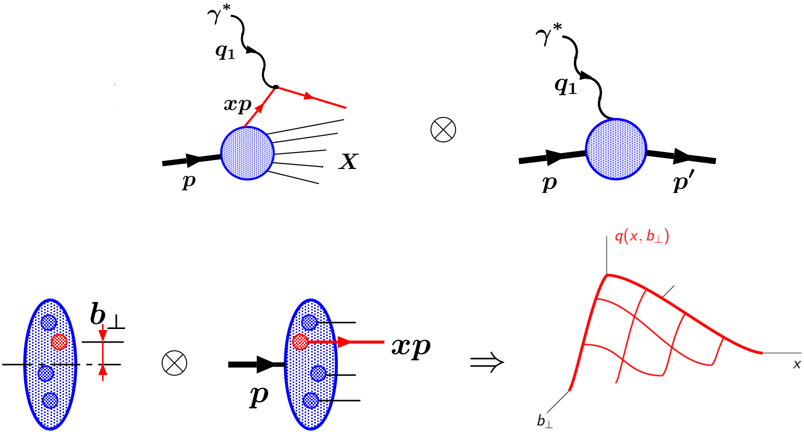
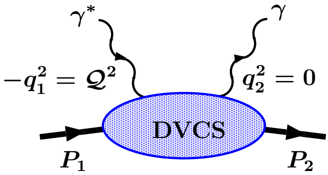


# Deeply virtual Compton scattering (DVCS)




$$P = P_1 + P_2$$
$$\Delta = P_2 - P_1$$
$$q = (q_1 + q_2)/2$$

[Müller '92, et al. '94]

generalized Bjorken limit:

$$-q^2 \equiv Q^2 \to \infty \qquad \eta = -\frac{\Delta \cdot q}{P \cdot q} \overset{\text{DVCS}}{=} \xi \quad \text{(skewness)}$$

$$\xi = \frac{-q^2}{2P \cdot q} \to \text{const } (as\ x_B) \qquad t = (P_2 - P_1)^2 = \Delta^2 \overset{\text{DVCS}}{\ll} Q^2$$

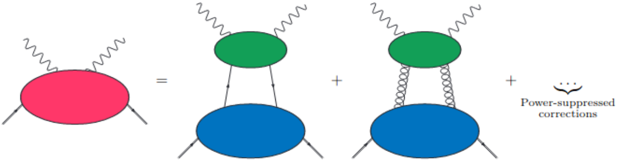
$$\sigma \quad \propto \quad |\mathcal{A}(\gamma^* p \to \gamma p)|^2$$

# Exclusive processes and factorization

exclusive hard-scattering: $\exists$ large scale(s) $\to$ factorization:

| hard scattering amplitude | = | elementary hard-scattering amplitude | $\otimes$ | hadron wave functions (GPDs, DAs) |
|---|---|---|---|---|

pQCD | evolution, input



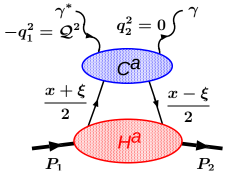

[ Mezrag 22']

DA . . . distribution amplitude (meson, nucleon)
GPD . . . generalized parton distributions

# Factorization of DVCS $\rightarrow$ GPD

$\rightarrow$ cross-section can be expressed in terms of
Compton form factors: $\mathcal{H}(\xi, t, Q^2)$, $\mathcal{E}(\xi, t, Q^2)$, $\tilde{\mathcal{H}}(\xi, t, Q^2)$, $\tilde{\mathcal{E}}(\xi, t, Q^2)$, ...

[Collins and Freund '99]



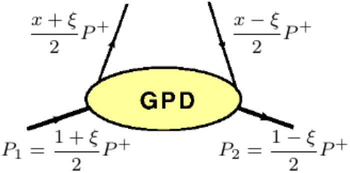

$a = q, G$

- Compton form factor is a convolution:

$$^a\mathcal{H}(\xi, t, Q^2) = \int_{-1}^{1} \mathrm{d}x \; C^a(x, \xi, Q^2/\mu^2) \; H^a(x, \eta = \xi, t, \mu^2)$$

- $H^a(x, \eta, t, \mu^2)$ — Generalized parton distribution (GPD)

# Definition of GPDs




[Müller '92, et al. '94, Ji, Radyushkin '96]

$$P = P_1 + P_2$$
$$\Delta = P_2 - P_1$$

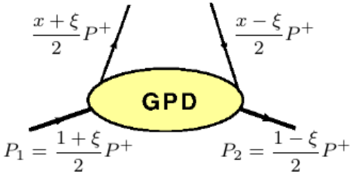
$x^\mu = (x^0, x^1, x^2, x^3) \to x^\pm = x^0 \pm x^3$

$$F^a(x, \xi, t = \Delta^2; \mu) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2|\mathcal{O}^a(z)|P_1\rangle\Big|_{z^+=0,\, z_\perp=0}$$

$a \in q, G$ , $\mu \ldots$ factorization scale

- $x$ : parton's average longitudinal momentum fraction
- $\xi = -\frac{\Delta^+}{P^+}$ : longitudinal momentum transfer (skewness), $\xi = \frac{x_B}{2-x_B}$
- $t = \Delta^2$ : momentum transfer squared (Mandelstam variable)

# Definition of GPDs

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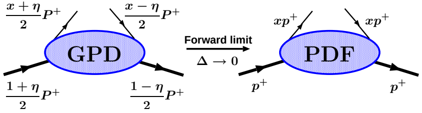
$a \in q, G$ , $\mu \ldots$ factorization scale

- vector ($H^a$,$E^a$) and axial vector GPDs ($\tilde{H}^a$,$\tilde{E}^a$)
  $\rightarrow$ chiral-even
  $\mathcal{O}^q(z) = \bar{q}(z)\gamma^+(\gamma^+\gamma_5)q(-z)$
- transversity GPDs ($H^a_T$,$E^a_T$,$\tilde{H}^a_T$,$\tilde{E}^a_T$)
  $\rightarrow$ chiral-odd
  $\mathcal{O}^q(z) = \bar{q}(z)i\sigma^{+i}q(-z)$

# Properties of GPDs

- Forward limit ($\Delta \to 0$, $\xi \to 0$): $H$-GPDs $\to$ PDFs




$P = P_1 + P_2$

$\Delta = P_2 - P_1,\ \Delta^2 = t$

$\xi = -\frac{\Delta^+}{P^+}$

- Sum rules: GPD $\to$ form factors

$$\sum_{q=u,d} Q_q \int_{-1}^{1} dx \begin{cases} H^q(x,\xi,t) \\ E^q(x,\xi,t) \end{cases} = \begin{cases} F_1(t) \\ F_2(t) \end{cases}$$

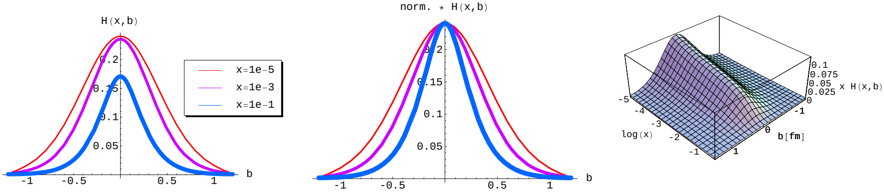
- Polinomiality and positivity constraints
- Possibility of solution of proton spin problem

$$\frac{1}{2}\int_{-1}^{1} dx\, x\Big[H^q(x,\xi,t) + E^q(x,\xi,t)\Big] = J^q(t)$$ [Ji '96]

# Three-dimensional image of a proton

- Fourier transform of GPD for $\xi = 0$ can be interpreted as probability density depending on $x$ and transversal distance $b$ [Burkardt '00, '02]

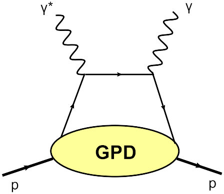
$$q(x, \vec{b}) = \int \frac{d^2\vec{\Delta}}{(2\pi)^2}\, e^{-i\vec{b}\cdot\vec{\Delta}} H(x, \xi = 0, \Delta^2 = -\vec{\Delta}^2)$$




[Kumerički,Müller,Passek-K. '08]

# Selected exclusive processes of interest

Deeply vitual Comptom scattering (DVCS)



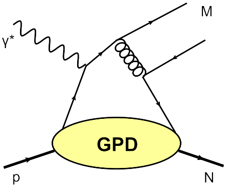

$$\boxed{\gamma^* p \to \gamma p}$$

factorization: [Collins, Freund '99]

$H^q, E^q, \tilde{H}^q, \tilde{E}^q$

$H^G, E^G, \tilde{H}^G, \tilde{E}^G$ (NLO)

Deeply virtual meson production (DVMP)




$$\boxed{\gamma^* p \to MN}$$

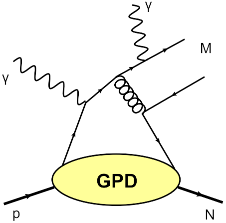
factorization:

[Collins, Frankfurt, Strikman '97]

$H^{q_i}, E^{q_i}; H^G, E^G$ ($V_L$)

$\tilde{H}^{q_i}, \tilde{E}^{q_i}$ (PS)

Photon-meson photoproduction




$$\boxed{\gamma p \to \gamma MN}$$

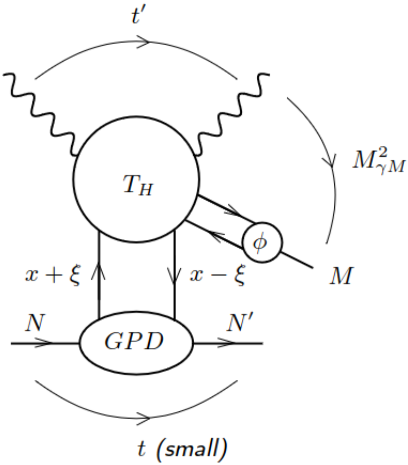
factorization: [Qiu, Yu '22]

$H^a, E^a, \tilde{H}^a, \tilde{E}^a$

$H^a_T, E^a_T, \tilde{H}^a_T, \tilde{E}^a_T$

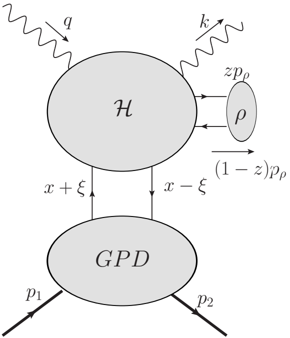
# Photon-meson photoproduction

$$\gamma + N \to \gamma + M(\rho, \pi, \dots) + N'$$



# Kinematics

$$\gamma^*(q) + N(p1) \to \gamma(k) + M(p_M) + N'(p_2)$$




$$u' = (p_M - q)^2 \gg$$
$$t' = (k - q)^2 \gg$$
$$s' = M^2_{\gamma M} = (k + p_M)^2 \gg$$

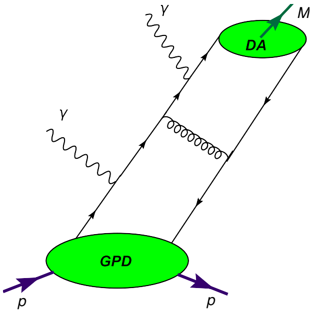
$$t = (p_2 - p_1)^2 \ll$$
$$s = S^2_{\gamma N} = (q + p_1)^2$$

$$\xi = \frac{\tau}{2-\tau}, \ \tau = \frac{M^2_{\gamma M}}{S^2_{\gamma N} - M^2}$$

amplitudes depend on $\xi$, $-t$, $s$, $\alpha = -u'/s$

## Photon-meson photoproduction

$$\gamma^* q \to \gamma q (q\bar{q})$$




LO $\rho$ mesons: [Boussarie, Pire, Szymanowsky, Wallon '16]

LO $\pi^\pm$ mesons: [Duplančić, Passek-K, Pire, Szymanowski, Wallon '18]

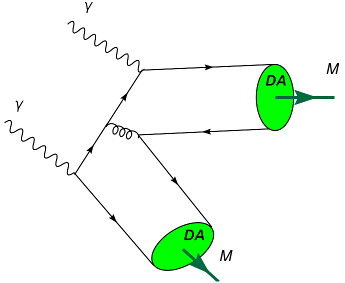
$$\pi^\pm : H^q, E^q, \widetilde{E}^q, \tilde{H}^q$$

$$\rho^0_L : H^q, E^q, \widetilde{E}^q, \tilde{H}^q$$

$$\rho^0_T : H^q_T, E^q_T, \widetilde{E}^q_T, \tilde{H}^q_T$$

## Meson pair production

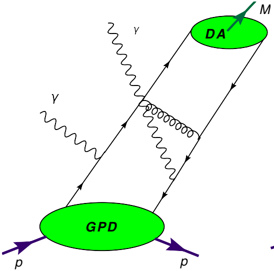
$$\gamma^* \gamma \to (q\bar{q})(q\bar{q})$$

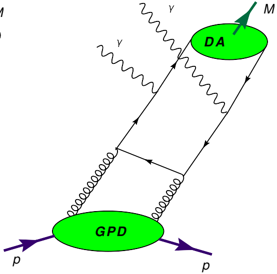



NLO: [Nižić '87, Duplančić, Nižić '06]

## Photon-$\pi^0$ photoproduction

$$\gamma q \to \gamma(q\bar{q})q,\ \gamma g \to \gamma(q\bar{q})g$$




20 diagrams

24 diagrams

$\widetilde{H}^q, \widetilde{E}^q$
$H^q, E^q$

$\widetilde{H}^G, \widetilde{E}^G$
$H^G, E^G$
$F_T^G, \widetilde{F}_T^G$

## $(M)\pi^0$ photoproduction

$$\gamma\gamma \to (q\bar{q})(q\bar{q})$$
$$\gamma\gamma \to (gg)(q\bar{q})$$

$\gamma\gamma \to (PS)\pi^0 \quad \to \widetilde{H}^q, \widetilde{E}^q$

$\gamma\gamma \to (S)\pi^0 \quad \to H^q, E^q$

$\gamma\gamma \to (PS)_g\pi^0 \quad \to \widetilde{H}^G, \widetilde{E}^G$

$\gamma\gamma \to (S)_g\pi^0 \quad \to H^G, E^G$

$\gamma\gamma \to (T)_g\pi^0 \quad \to F_T^G, \widetilde{F}_T^G$

LO: [Bayer, Grozin '85]

# Factorization

$$^{a}\mathcal{M} = \int_{-1}^{1} \mathrm{d}x \int_{0}^{1} \mathrm{d}z \; T^{a}(x, \xi, z, \mu_F, \mu_\varphi; s, \alpha) \; F^{a}(x, \xi, t, \mu_F) \; \phi_M(z, \mu_\varphi)$$

$a = q, g$  $\mu_F, \mu_\varphi$ ... factorization scales

- $T^a$: subprocess hard-scattering amplitudes → pQCD

  more complicated expressions than in the case of DVCS and DVMP
  ⇒ more demanding integrations with $F^a$ and $\phi_M$

- $F^a$: GPD
  → different models based on [Radyushkin '98],[Goloskokov, Kroll '10]

- $\phi_M$: meson distribution amplitude (DA)

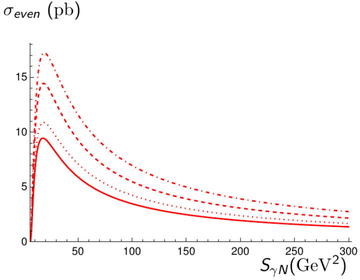
  → models:

$\phi(z) = 6z(1-z)$ ... asymptotic DA

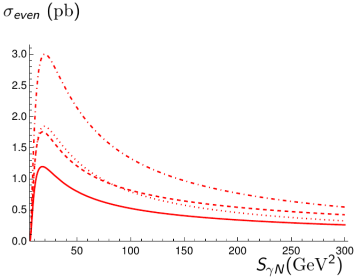
$\phi(z) = \frac{8}{\pi}\sqrt{z(1-z)}$ ... holographic DA [Brodsky, de Teramond '06]

$\phi(z, \mu_\varphi) = 6z(1-z)\left[1 + \sum_n a_n(\mu_\varphi) C_n^{3/2}(2z-1)\right]$

... expansion in Gegenbauer pol. ⇒ evolution




(proton target)




(neutron target)

[Duplančić, Passek-K, Nabeebaccus, Pire, Szymanowski, Wallon '22]

| GPD models | $\phi_{\mathrm{as}}(z) = 6z(1-z)$ | $\phi_{\mathrm{hol}}(z) = \frac{8}{\pi}\sqrt{z(1-z)}$ |
|---|---|---|
| "valence" | solid | dashed |
| "standard" | dotted | dash-dotted |

$\rightarrow$ complex numerical integration

# Recent work: $\gamma\pi^0$ photoproduction

quark contributions

- obtained simpler closed expression for sum of diagrams suited for improved numerical integrations and inclusion of different DA models
- PV formalism enables efficient treatment of poles;
  example, mixed term:

$$\frac{1}{z\bar{z}\,(y\bar{z}\,-\alpha z\bar{y}\,+i\epsilon)} \to \frac{1}{\alpha\, z\bar{z}\,(\alpha+\bar{\alpha}\,y)}\left(\mathrm{PV}\frac{1}{z-\frac{y}{\alpha+\bar{\alpha}\,y}}+i\pi\;\delta\left(z-\frac{y}{\alpha+\bar{\alpha}\,y}\right)\right)$$

$y=\frac{\xi+x}{2\xi}, z \ldots$ partons momentum fractions , $\alpha=-u'/s'$, and $\bar{a}\equiv 1-a$

gluon contributions

- determined and closed expressions for the sum of diagrams obtained
- due gg projector with factor $\frac{1}{(y+i\epsilon)(\bar{y}\,+i\epsilon)}$, contributions

$$\frac{1}{(y+i\epsilon)(\bar{y}\,+i\epsilon)}\frac{N(y,z,\alpha)}{z\bar{z}\,(y-i\epsilon)(\bar{y}\,+i\epsilon)(y\bar{z}\,-\alpha z\bar{y}\,+i\epsilon)}$$

demand additional attention

$\Rightarrow$ additional regularization?$(k_\perp)$, breakdown of factorization?

$\to$ work in progress

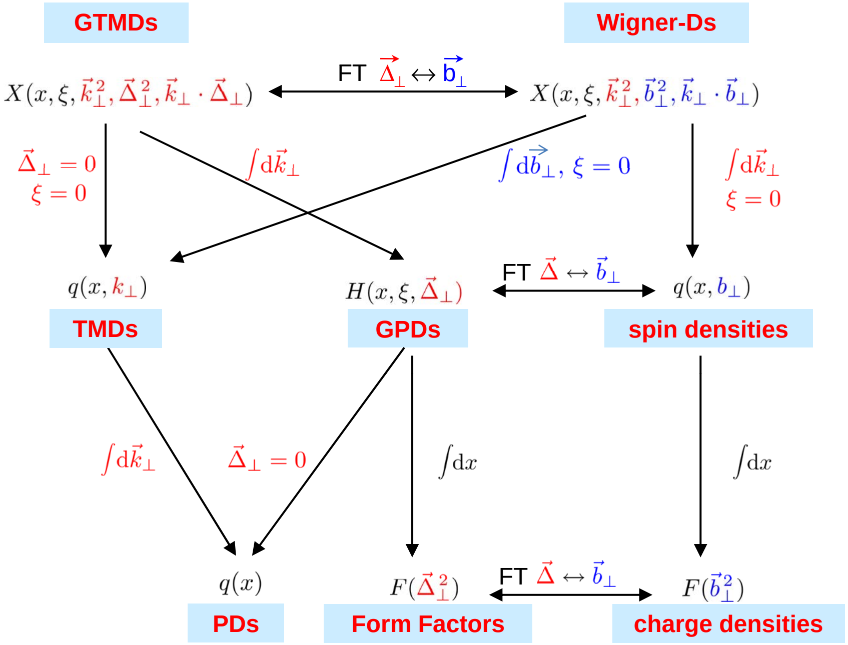
# Summary

- Hard-exclusive processes described in terms of GPDs offer challenging but promising tool for resolving hadron structure
- A vast amount of experimental data (HERA, JLab, COMPASS) and more is yet to come (EIC in 2030 or proposed LHeC, EicC)

$\gamma N \to (\gamma M) N'$

- Provides additional channel for extracting GPDs and access to transversity GPDs ($F_T$)
- Recent proof of factorisation (for quarks)
- Expected good statistics in various experiments, particularly at JLab
- $(\gamma\pi^0)$: access to gluon GPDs (including $F_T^G$)
- quark contributions improved, gluon contributions challenging

Thank you!


GTMDs
Wigner-Ds
FT $\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$
$X(x,\xi,\vec{k}_\perp^2,\vec{\Delta}_\perp^2,\vec{k}_\perp\cdot\vec{\Delta}_\perp)$
$X(x,\xi,\vec{k}_\perp^2,\vec{b}_\perp^2,\vec{k}_\perp\cdot\vec{b}_\perp)$
$\vec{\Delta}_\perp = 0$
$\xi = 0$
$\int d\vec{k}_\perp$
$\int d\vec{b}_\perp,\ \xi = 0$
$\int d\vec{k}_\perp$
$\xi = 0$
$q(x, k_\perp)$
TMDs
$H(x,\xi,\vec{\Delta}_\perp)$
GPDs
FT $\vec{\Delta} \leftrightarrow \vec{b}_\perp$
$q(x, b_\perp)$
spin densities
$\int d\vec{k}_\perp$
$\vec{\Delta}_\perp = 0$
$\int dx$
$\int dx$
$q(x)$
PDs
$F(\vec{\Delta}_\perp^2)$
Form Factors
FT $\vec{\Delta} \leftrightarrow \vec{b}_\perp$
$F(\vec{b}_\perp^2)$
charge densities

# Definition of GPDs

- without helicity flip (chiral-even Γ matrices): 8 chiral-even GPDs:

$$F^q(x,\eta,t=\Delta^2) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2 | \bar{q}(-z)\gamma^+ q(z) | P_1 \rangle \Big|_{z^+=0,\, z_\perp=0}$$

$$F^g(x,\eta,t=\Delta^2) = \frac{2}{P^+} \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2 | G_a^{+\mu}(-z) G_{a\mu}{}^{+}(z) | P_1 \rangle \Big|_{\dots}$$

$x^\mu = (x^0, x^1, x^2, x^3) \to x^\pm = x^0 \pm x^3$

- Lorentz decompozition:

$$F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2) i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \qquad a = q, g$$

[Müller '92, et al. '94, Ji, Radyushkin '96]

# Definition of GPDs

- without helicity flip (chiral-even Γ matrices): 8 chiral-even GPDs:

$$\widetilde{F}^q(x,\eta,t=\Delta^2) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+\gamma_5 q(z)|P_1\rangle\Big|_{z^+=0,\, z_\perp=0}$$

$$\widetilde{F}^g(x,\eta,t=\Delta^2) = \frac{4}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|G^{+\mu}(-z)\widetilde{G}_\mu{}^+(z)|P_1\rangle\Big|_{\dots}$$

- Lorentz decompozition:

$$\widetilde{F}^a = \frac{\bar{u}(P_2)\gamma^+\gamma_5 u(P_1)}{P^+}\widetilde{H}^a + \frac{\bar{u}(P_2)i\sigma^{+\nu}\gamma_5 u(P_1)\Delta_\nu}{2MP^+}\widetilde{E}^a \qquad a = q, g$$

[Müller '92, et al. '94, Ji, Radyushkin '96]

# Definition of GPDs

- With helicity flip (chiral-odd $\Gamma$ matrices): 8 chiral-odd GPDs (transversity GPD):

$$F_T^q(x,\eta,t=\Delta^2) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z) i\sigma^{+i} q(z)|P_1\rangle\Big|_{z^+=0,\, z_\perp=0}$$

- Lorentz decompozition:

$$F_T^q = \frac{\bar{u}(P_2) i\sigma^{+i} u(P_1)}{P^+} H_T^q + \frac{\bar{u}(P_2)(P^+\Delta^i - \Delta^+ P^i)u(P_1)}{M^2P^+}\widetilde{H}_T^q + \\ + \frac{\bar{u}(P_2)(\gamma^+\Delta^i - \Delta^+\gamma^i)u(P_1)}{2MP^+} E_T^q + \frac{\bar{u}(P_2)(\gamma^+P^i - P^+\gamma^i)u(P_1)}{MP^+}\widetilde{E}_T^q$$

[Müller '92, et al. '94, Ji, Radyushkin '96]

# Definition of GPDs

- With helicity flip (chiral-odd Γ matrices): 8 chiral-odd GPDs (transversity GPD):

$$F_T^g(x,\eta,t=\Delta^2) = \frac{1}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-}\langle P_2|\mathbf{S}\,G^{+i}(-z)G^{j\,+}(z)|P_1\rangle\Big|_{z^+=0,\,}$$

- Lorentz decompozition:

$$F_T^g = \mathbf{S}\frac{(P^+\Delta^j - \Delta^+P^j)}{2MP^+}\frac{\bar{u}(P_2)}{P^+}\Big[i\sigma^{+i}H_T^g + \frac{(P^+\Delta^i - \Delta^+P^i)}{M^2}\widetilde{H}_T^g + \\ + \frac{(\gamma^+\Delta^i - \Delta^+\gamma^i)}{2M}E_T^g + \frac{(\gamma^+P^i - P^+\gamma^i))}{M}\widetilde{E}_T^g\Big]u(P1)$$

[Müller '92, et al. '94, Ji, Radyushkin '96]

# Polynomiality and positivity constraints of GDP

- Polynomiality

  Lorentz covariance $\Rightarrow \int_{-1}^{1} dx x^m H^q(x,\xi,t) = \sum_{k=0}^{m} \xi^k A^q_{m,k}(t)$

- Positivity

  Positivity of Hilbert space norm $\Rightarrow H^q(x,\xi,t) \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}$

# DAs

$$\phi_{\pi^0}(x,\mu_F) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle 0|\bar{q}(-z)\gamma^+\gamma_5 q(z)|\pi^0(P)\rangle\Big|_{z^+=0,\, z_\perp=0}$$

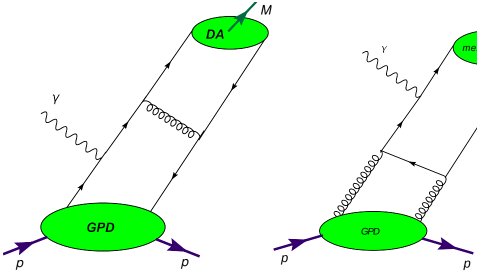
$$\phi_{\pi^0}(x,\mu_F) = 6x\,(1-x)\left[1 + \sum_{n=2,4,\ldots} a_{\pi^0,n}(\mu_F)\; C_n^{3/2}(2x-1)\right]$$

$$a_{M,n}(\mu_F) = a^{LO}_{M,n}(\mu_F) + \frac{\alpha_S(\mu_F)}{4\pi} a^{NLO}_{M,n}(\mu_F)$$

$$a^{LO}_{M,n}(\mu_F) \;\; = \;\; \left(\frac{\alpha_S(\mu_O)}{\alpha_S(\mu_F)}\right)^{\gamma_n/\beta_0} a^{LO}_{M,n}(\mu_O) \qquad (\leq a^{LO}_{M,n}(\mu_O))$$

# DVMP

$$\gamma^* q \to (q\bar{q})q,\ \gamma^* g \to (q\bar{q})g$$



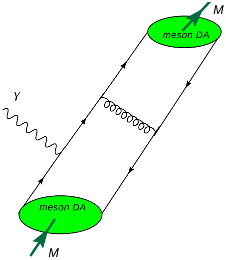

NLO DV PS$^+$ prod.: [Belitsky and Müller '01]

NLO DV $V_L$ prod.: [Ivanov et al '04,]

NLO DV $V_L$ (corr.), PS, (S, $PV_L$) prod.: [Duplančić, Müller, Passek-K. '17]

# Meson em form factor

$$\gamma^*(q\bar{q}) \to (q\bar{q})$$




NLO: [..., Melić et al '99]

$$\gamma_L^*(M^\pm) \to M^\pm,$$
$$\gamma_L^*(S) \to V_L,\ \gamma_L^*(V_L) \to S$$
$$\gamma_L^*(PV_L) \to PS,\ \gamma_L^*(PS) \to PV_L$$
$$\Rightarrow \text{DVMP}$$

## (D)DVCS

$$\gamma^* q \to \gamma^{(*)} q,\ \gamma^* g \to \gamma^{(*)} g$$

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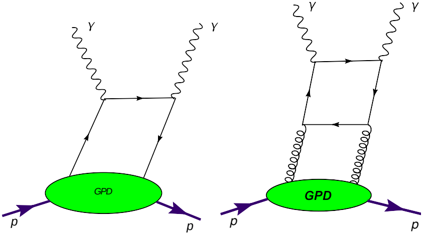


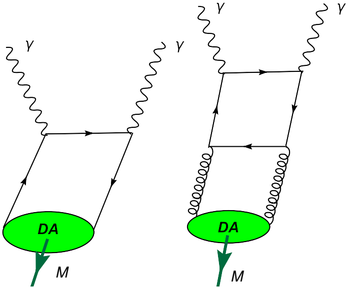

NLO: [Ji, Belitsky et al, Mankiewicz et al, '97]
[Pire, Szymanowski, Wagner '11]

$\beta_0$ proportional NNLO: [Belitsky, Schäfer '98]

NNLO from conf. sym: [Müller '05, Kumerički, Müller, Passek-K '07]

## Meson transition form factor

$$\gamma^* \gamma^{(*)} \to (q\bar{q}),\ \gamma^* \gamma^{(*)} \to (gg)$$




NLO: [..., Kroll, Passek-K '02] [Kroll, Passek-K '19]

$\beta_0$ proportional NNLO: [Melić, Nižić, Passek '01]

NNLO from conf. sym: [Melić, Müller, Passek '02]

# Status

- experiment
  - DVCS, vector ($\rho$, $J/\Psi$, $\phi$) and pseudoscalar ($\pi$, $\eta$) meson production measured by H1, ZEUS, HERMES (HERA, DESY), COMPASS (SPS, CERN), CLAS, Hall-A,C (JLab) ...
  - LHeC, EicC proposed
  - EIC (Electron Ion Collider at Brokhaven, 2030) under construction (luminosity 100-1000 times HERA)