Curious phenomena in axion electrodynamics

András Patkós

Institute of Physics, Eötvös University

ACHT 2023, September 27-29, Leibnitz

Outline

- Reminder on axion hypothesis
  Non-existent CP-violation in QCD: Peccei-Quinn mechanism

- Effective Electro-Magneto Dynamics of axions

- Self-sustaining homogeneous axion-electromagnetic configuration in EEMD *

- Electromagnetic Radiation damping in axion stars † ‡

- Outlook

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*arXiv:2309.05523, MPLA to appear
†Phys. Rev. D**107** 055017 (2023)
‡Symmetry **14** 1113 (2022)

Reminder on axion hypothesis

Non-abelian gluon-dynamics with CP-odd completion

$$L = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}, \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^\mu - gf^{abc}A^b_\mu A^c_\nu.$$

$$\Delta L = \Theta\frac{g^2}{64\pi^2}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}, \qquad \tilde{F}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{a\rho\sigma}.$$

Unnatural smallness: $|\Theta_{exp}| < 10^{-10}$.

Peccei-Quinn mechanism

$U_{PQ}(1)$ spontanously broken at scale f_a provides pseudo-Goldstone, pseudoscalar $a(x)$:

$$\Theta \to \Theta_{eff} = \Theta + \xi\frac{\langle a(x)\rangle}{f_a} \approx 0.$$

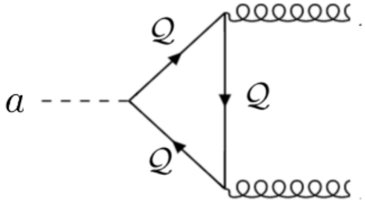
Model: axion-gauge coupling mediated by heavy fermions

Axion-coupling to electromagnetic field:

$$L_{EM+a} = \frac{1}{2}\left[(\partial_\mu a(x))^2 - m_a^2 a(x)^2\right] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j_\mu A^\mu + \frac{1}{4}g_{a\gamma\gamma}a(x)F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x).$$

$F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$ $\qquad$ $\partial_\mu F^{\mu\nu} + g_{a\gamma\gamma}\partial_\mu a(x)\tilde{F}^{\mu\nu} = j_e^\nu$

$a\gamma\gamma$ coupling might arise from PQ-charged heavy quarks



Axion electrodynamics

In non-relativistic notation

$F^{0i} = -E_i, \qquad F^{ij} = -\epsilon^{ijk} B_k \qquad \mathbf{B} = \nabla \times \mathbf{A}, \qquad \mathbf{E} = -\dot{\mathbf{A}} - \nabla A_0.$

Homogeneous Maxwell-equations follow:

$\nabla \mathbf{B} = 0, \qquad \nabla \times \mathbf{E} = -\dot{\mathbf{B}}.$

$$\nabla \mathbf{E} + g_{a\gamma\gamma} \mathbf{B} \nabla a(x) = \rho_e,$$

$$-\dot{\mathbf{E}} + \nabla \times \mathbf{B} - g_{a\gamma\gamma}(\dot{a}\mathbf{B} + \nabla a(x) \times \mathbf{E}) = \mathbf{j}_e,$$

$$L = \frac{1}{2}\left(\epsilon \mathbf{E}^2(x) - \frac{1}{\mu}\mathbf{B}^2(x)\right) - g_{a\gamma\gamma} a(x) \mathbf{E}(x) \cdot \mathbf{B}(x)$$

$$+\frac{1}{2}\left((\partial_t a(x))^2 - (\partial_i a(x))^2 - m_a^2 a^2\right) - j_0 A_0(x) + \mathbf{j}(x)\mathbf{A}(x),$$

Axion equation

$$\ddot{a}(\mathbf{x}, t) - \nabla^2 a(\mathbf{x}, t) + m_a^2 a(\mathbf{x}, t) = g_{a\gamma\gamma} \mathbf{E} \cdot \mathbf{B}.$$

Review: Paul Sikivie, Rev. Mod. Phys. **93** (2020) 15004

Axion Electro-Magneto Dynamics

PQ-charged fermions could be dyons (Sokolov and Ringwald, 2022)

Dual (magnetic) vector potential C^μ introduced in addition to A^μ:

$$G^{\mu\nu} = \partial^\mu C^\nu - \partial^\nu C^\mu, \qquad \tilde{G}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}$$

$$\partial_\mu G^{\mu\nu} = j_m^\nu, \qquad \partial_\mu \tilde{G}^{\mu\nu} = 0 \quad \text{Bianchi-identity},$$

Constraint enforcing unique electromagnetic field strength in field equations

$$\tilde{F}^{\mu\nu}|_{\mathbf{B},\mathbf{E}} = G^{\mu\nu}|_{\mathbf{B},\mathbf{E}}.$$

$$F^{0i} = -E_i, \qquad F^{ij} = -\epsilon^{ijk} B_k, \qquad G^{0i} = -B_i, \qquad G^{ij} = \epsilon^{ijk} E_k.$$

Two new axion-photon couplings

$$\Delta L_{aEMD} = -\frac{1}{8}a(x)\left(g_{EE}F_{\mu\nu}\tilde{F}^{\mu\nu} + g_{MM}G_{\mu\nu}\tilde{G}^{\mu\nu} + 2g_{EM}F_{\mu\nu}\tilde{G}^{\mu\nu}\right).$$

$$\partial_\mu F^{\mu\nu} + g_{EE}\partial_\mu a(x)\tilde{F}^{\mu\nu} + g_{EM}\partial_\mu a(x)\tilde{G}^{\mu\nu} = j_e^\nu$$
$$\partial_\mu G^{\mu\nu} + g_{MM}\partial_\mu a(x)\tilde{G}^{\mu\nu} + g_{EM}\partial_\mu a(x)\tilde{F}^{\mu\nu} = j_m^\nu.$$

In terms of field strengths

$$\nabla\mathbf{E} + g_{EE}\mathbf{B}\nabla a(x) - g_{EM}\mathbf{E}\nabla a(x) = \rho_e,$$
$$\nabla\mathbf{B} - g_{MM}\mathbf{E}\nabla a(x) + g_{EM}\mathbf{B}\nabla a(x) = \rho_m,$$
$$-\dot{\mathbf{E}} + \nabla\times\mathbf{B} - g_{EE}(\dot{a}\mathbf{B} + \nabla a(x)\times\mathbf{E}) + g_{EM}(\dot{a}\mathbf{E} - \nabla a(x)\times\mathbf{B}) = \mathbf{j}_e,$$
$$\dot{\mathbf{B}} + \nabla\times\mathbf{E} - g_{MM}(\dot{a}\mathbf{E} - \nabla a(x)\times\mathbf{B}) + g_{EM}(\dot{a}\mathbf{B} + \nabla a(x)\times\mathbf{E}) = -\mathbf{j}_m.$$

Axion field equation

$$\ddot{a}(\mathbf{x},t) - \nabla^2 a(\mathbf{x},t) + m_a^2 a(\mathbf{x},t) = (g_{EE} - g_{MM})\mathbf{E}\cdot\mathbf{B} - g_{EM}(\mathbf{E}^2 - \mathbf{B}^2).$$

Axion Electro-Magneto Dynamics

Energy-balance equation (Poynting-theorem)

$$-\int d^3x\,[(\mathbf{j}_e+\mathbf{j}_{axion,e})\cdot\mathbf{E}(t,\mathbf{x})+(\mathbf{j}_m+\mathbf{j}_{axion,m})\cdot\mathbf{B}(t,\mathbf{x})]$$
$$=\int d\mathbf{F}\cdot(\mathbf{E}\times\mathbf{B})+\frac{d}{dt}\int d^3x\frac{1}{2}\Big(\mathbf{E}^2(t,\mathbf{x})+\mathbf{B}^2(t,\mathbf{x})\Big)\,.$$

With the explicit expressions of axionic currents ($\mathbf{j}_e=\mathbf{j}_m=0$)

$$-\int d^3x\,[\mathbf{j}_{axion,e}\cdot\mathbf{E}(t,\mathbf{x})+\mathbf{j}_{axion,m}\cdot\mathbf{B}(t,\mathbf{x})]$$
$$=-\int d^3x\dot{a}\Big[(g_{aEE}-g_{aMM})\mathbf{E}\cdot\mathbf{B}-g_{aEM}(\mathbf{E}^2-\mathbf{B}^2)\Big]\,,$$

by the axion equation equals the rate of change of the axion energy

$$-\frac{d}{dt}\int d^3x\frac{1}{2}\left(\dot{a}^2+(\nabla a)^2+m_a^2a^2\right).$$

Self-sustaining homogeneous axion-electromagnetic field configuration

$\mathbf{E}(t), \mathbf{B}(t), a(t)$ initiated by external uniform magnetic field $\mathbf{B}_0$

$$\dot{\mathbf{E}}(t) = -g_{EE}\dot{a}_0(t)(\mathbf{B}_0 + \mathbf{B}(t)) + g_{EM}\dot{a}_0(t)\mathbf{E}(t)$$
$$\dot{\mathbf{B}}(t) = g_{MM}\dot{a}_0(t)\mathbf{E}(t) - g_{EM}\dot{a}_0(t)(\mathbf{B}_0 + \mathbf{B}(t)).$$

Ansätze

$$\mathbf{E} = f_E(a_0(t))\mathbf{B}_0, \qquad \mathbf{B} = f_B(a_0(t))\mathbf{B}_0$$

Initial conditions

$$f_E(a_0(t_0) = a_i) = 0, \qquad f_B(a_0(t_0) = a_i) = 0.$$

Solution with arbitrary $a_0(t)$ $\qquad [\lambda^2 = g_{EM}^2 - g_{EE}g_{MM}]$

$$f_E(a_0) = -\frac{g_{EE}}{\lambda}\sinh[\lambda(a_0(t) - a_i)],$$
$$f_B(a_0) = \cosh[\lambda(a_0(t) - a_i)] - 1 - \frac{g_{EM}}{\lambda}\sinh[\lambda(a_0(t) - a_i)].$$

Axion equation emerging after the "elimination" of electromagnetic fields

$$\ddot{a}_0(t) + m_a^2 a_0(t) =$$
$$= [(g_{EE} - g_{MM}) f_E(a_0)(f_B(a_0) + 1) - g_{EM}(f_E(a_0)^2 - (f_B(a_0) + 1)^2)] B_0^2$$

completes the stationary $a_0(t), \mathbf{E}(a_0), \mathbf{B}(a_0)$ configuration induced by $\mathbf{B}_0$.

Small amplitude motion $|\lambda(a_0(t) - a_i)| << 1$ results in a mass-shift of axions:

$$\ddot{a}_0(t) + M_a^2 a_0(t) = 0, \qquad M_a^2 = m_a^2 + [(g_{EE} - g_{aMM}) g_{EE} + 2g_{EM}^2] B_0^2.$$

EEMD theory of an axion star

Free axions embedded in $\mathbf{B}_0$-induced medium with gravitational interaction

$$U_{eff} = \frac{1}{2}\left(\dot{a}^2 + (\nabla a)^2 + M_a^2 a^2\right) + U_{grav}$$
$$U_{grav} = -\frac{G_N}{2}\int d^3x \int d^3y \frac{\rho_{axion}(\mathbf{x})\rho_{axion}(\mathbf{y})}{|x-y|}.$$

Approximate lowest energy configuration searched in the form $\psi(\mathbf{x},t) = e^{i\mu_g t}\phi(\mathbf{x})$. μ_g: specific gravitational binding energy

$$a(\mathbf{x},t) = \frac{1}{\sqrt{2M_a}}\left(e^{-iM_a t}\psi(\mathbf{x},t) + e^{iM_a t}\psi^*(\mathbf{x},t)\right),$$
$$\int d^3x|\psi(\mathbf{x},t)|^2 = N_{axion}, \quad \rho_{axion} = (M_a - \mu_g)|\psi(\mathbf{x},t)|^2$$

Variational estimate: $\mu_g/M_a \approx 10^2(G_N M_a^2 N)^2 << 1$ for $N \sim 10^{61}$.

$$a(\mathbf{x},t) = \sqrt{\frac{2}{M_a}}wF(\xi)\cos(M_a t), \qquad w \sim \sqrt{N}, \qquad \xi = \frac{|\mathbf{x}|}{R}$$

EEMD electromagnetic radiation from an axion star

$$\Box A^\mu = j^\mu_{axion,e}[\phi, \mathbf{B_0}], \qquad \Box C^\mu = j^\mu_{axion,m}[\phi, \mathbf{B}_0],$$
$$\mathbf{e} = -\dot{\mathbf{A}} - \nabla A^0, \qquad \mathbf{b} = -\dot{\mathbf{C}} - \nabla C^0$$

$$\int d^3x \left(\mathbf{j}_{axion,e} \cdot \mathbf{e}(t,\mathbf{x}) + \mathbf{j}_{axion,m} \cdot \mathbf{b}(t,\mathbf{x})\right)$$
$$= -\int d^3x \int d^3x' \frac{1}{|\mathbf{x}-\mathbf{x}'|}\Big[\mathbf{j}_{axion,e}(\mathbf{x},t) \cdot \frac{\partial}{\partial t}\mathbf{j}_{axion,e}(\mathbf{x}', t - |\mathbf{x}-\mathbf{x}'|)$$
$$+ \frac{\partial}{\partial t}\rho_{axion,e}(\mathbf{x},t)\rho_{axion,e}(\mathbf{x}', t - |\mathbf{x}-\mathbf{x}'|)\Big]$$
$$-\int d^3x \int d^3x' \frac{1}{|\mathbf{x}-\mathbf{x}'|}\Big[\mathbf{j}_{axion,m}(\mathbf{x},t) \cdot \frac{\partial}{\partial t}\mathbf{j}_{axion,m}(\mathbf{x}', t - |\mathbf{x}-\mathbf{x}'|)$$
$$+ \frac{\partial}{\partial t}\rho_{axion,m}(\mathbf{x},t)\rho_{axion,m}(\mathbf{x}', t - |\mathbf{x}-\mathbf{x}'|)\Big].$$

EEMD electromagnetic radiation from an axion star

$$\overline{\frac{dE_{ax}}{dt}}^T = -\frac{NB_0^2X^3}{C_2}\Big[(g_{EE}^2 + g_{EM}^2)I_{mag}^2 - \frac{1}{3X^2}(g_{EE}^2 - g_{EM}^2 - 2g_{MM}^2)I_{el}^2\Big],$$

where $X = M_a R$ gives the size of the star in units of M_a^{-1}.

$$I_{mag} = 4\pi\int d\xi\xi^2\frac{\sin(X\xi)}{X\xi}F(\xi),$$

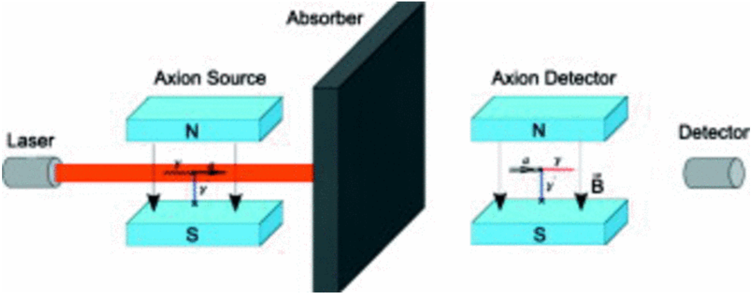
$$I_{el} = 4\pi\int d\xi\xi^2\left(\frac{\sin(X\xi)}{(X\xi)^2} - \frac{\cos(X\xi)}{X\xi}\right)F'(\xi).$$

Reinterpretation of $\overline{\frac{dE_{ax}}{dt}}^T$ as $M_a\overline{\frac{dN(t)}{dt}}^T$ yields $N(t) \sim t^{-1/5}$.

Summary

- Charge-0 axion under specific conditions could carry effective electric charge and current densities
- In electro-magneto dynamics self-sustaning homogeneous axion+electomagnetic configurations might appear
- Gravitationally self-bound axion stars in EEMD emit electromagnetic radiation at slow rate

Enjoyable playground



for retraining yourself in classical electrodynamics

Some further interesting phenomena

- Coupled propagation of axionic and electromagnetic waves in external magnetic field (Primakoff-effect)
- Rotation of polarization plane of electromagnetic radiation propagating through inhomogeneous axion medium
- Echo of electromagnetic radiation from axion cloud via induced axion decay into two photons
- Parametric electromagnetic instabilities of axion stars