Kaon Oscillation and Decay in Finite Time Path Out-of-Equilibrium Field Theory

I. Dadić[1] research done in a collaboration with D. Klabučar[2]
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Abstract

We demonstrate that the finite-time-path field theory (FTPFT), is adequate tool for the calculation of kaon oscillation and decay. We apply a theory with mass-mixing Lagrangian by using the Gell-Mann - Pais like mixing matrix. The model is exactly solvable. The Dyson-Schwinger equations contribute to a pair K^0 and $\bar{K}^0$ an another pair of CP symmetric K_S and K_L kaons with different masses. This leads to oscillations. To the mixing matrix we add self-energies connected to 2π and 3π decays of K^S and K^L kaons,respectively. We don't need non-hermiticity of the hamiltonian hypotesis. The result is equivqlent to Gell-Mann - Pais formula.

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Kaon Decay Parameters

Mean lifetime for

$\tau_{\pm} = (1.23800.0020)10^{-8}s$

$\tau_S = (8.9540.004)10^{-11}s = .136 \times 10^{12}\bar{h}MeV^{-1}$

$\tau_L = (5.1160.021)10^{-8}s = .778 \times 10^{14}\bar{h}MeV^{-1}$

$1/\tau_S = 7.35 \times 10^{-12}MeV$

$1/\tau_L = 1.29 \times 10^{-14}MeV$

$\Delta = m_L - m_S = 3.484(6)10^{-12}MeV$ [10].

Gell-Mann-Pais theory

Murray Gell-Mann and Abraham Pais :

$$\psi(t) = U(t)\psi(0) = e^{iHt}\begin{pmatrix} a \\ b \end{pmatrix}, \quad H = \begin{pmatrix} M & \Delta \\ \Delta & M \end{pmatrix}$$

$$U_{K_S,K_0} = \frac{1}{2^{1/2}} = U_{K_S,\bar{K}_0} = U_{K_L,K_0} = -U_{K_L,\bar{K}_0}$$

$$|K_S >= \frac{1}{2^{1/2}}(|K_0 > +|\bar{K}_0 >), \quad |K_L >= \frac{1}{2^{1/2}}(|(K_0 > -|\bar{K}_0 >)$$

$$|K_L^0 >= (p|K_0 > -q|\bar{K}_0 >), \quad |K_S^0 >= (p|K^0 > +q|\bar{K}^0 >$$

$$|K_0 >= g_+(t)|K_0 > -\frac{q}{p}g_-(t)|\bar{K}_0 > , \quad |\bar{K}_0 >= -\frac{p}{q}g_-(t)|K_0 > +g_+(t)|\bar{K}_0 >$$

$$\frac{q}{p} = \frac{1-\varepsilon}{1+\varepsilon}., \quad |\varepsilon| = (2.228 \pm 0.011)10^{-3} \tag{1.1}$$

To get decays and CP-violation add complex valued widths to Δ

Direct and indirect CP-violation (ε and ε')

Serious failures:

1. it requires hamiltonian to be non-hermitian, to deal with CP nonconservation.

2. The model uses wave functions describing partcles on mass-shell.

Interest for non-hermitian hamiltonian . Our approach does not require non-hermiticity.

Finite Time path and Two Point Functions

$$F(x,y), \quad 0 \leq x_0, y_0 \leq t \tag{1.2}$$

Wigner transforms

$$F_{X_0}(p_o,\vec{p}) = \int_{-2X_0}^{2X_0} ds_0 \int d^3 s e^{-i(s_0 p_0 - \vec{s}\vec{p})} F(x,y)$$

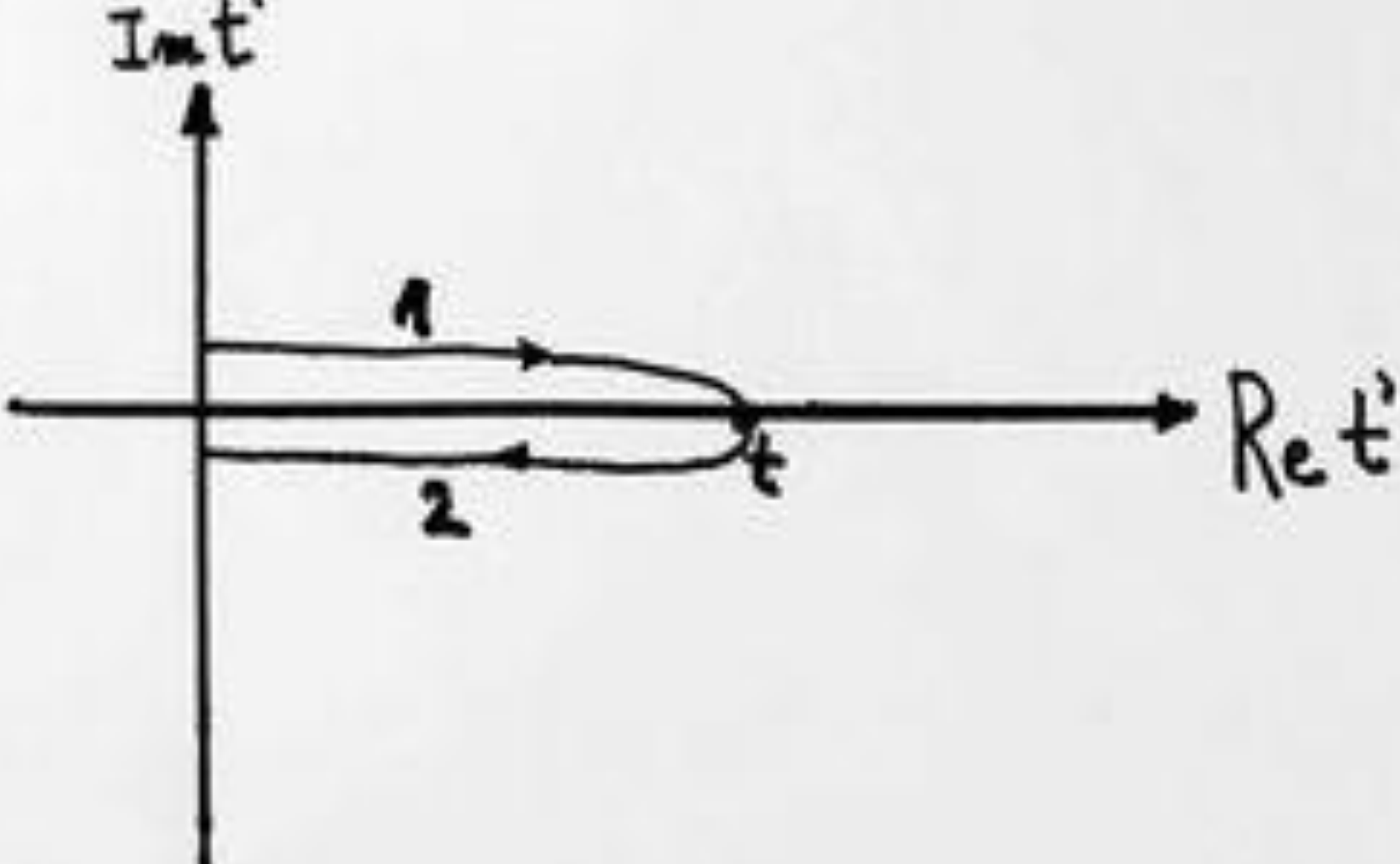
$$X = \frac{x+y}{2}, \; s = x - y, \tag{1.3}$$

Shift to ∞

If $F_\infty(p_0',\vec{p})$ exists then

$$F_{X_0}(p_0,\vec{p}) = \int_{-\infty}^{\infty} dp_0' P_{X_0}(p_0,p_0') F_\infty(p_0',\vec{p}),$$

$$P_{X_0}(p_0,p_0') = \frac{1}{\pi}\Theta(X_0)\frac{\sin\left(2X_0(p_0 - p_0')\right)}{p_0 - p_0'}, \quad \lim_{X_0\to\infty} P_{X_0}(p_0,p_0') = \delta(p_0 - p_0'), \tag{1.4}$$


Im t'
Re t'
1
2
t

Bare Propagators (1 for kaons , 2 for antikaons)

$$G_{\infty,1(2),R(A)}(p,m) \; = \; \frac{-i}{p^2-m^2 \pm 2ip_0\epsilon} \; ,$$

$$G_{\infty,1,K}(p,m) = G_{\infty,1,K,R}(p) - G_{\infty,1,K,A}(p) = 2\pi\delta(p^2-m^2)(1+2n(p_0,\vec{p})),$$

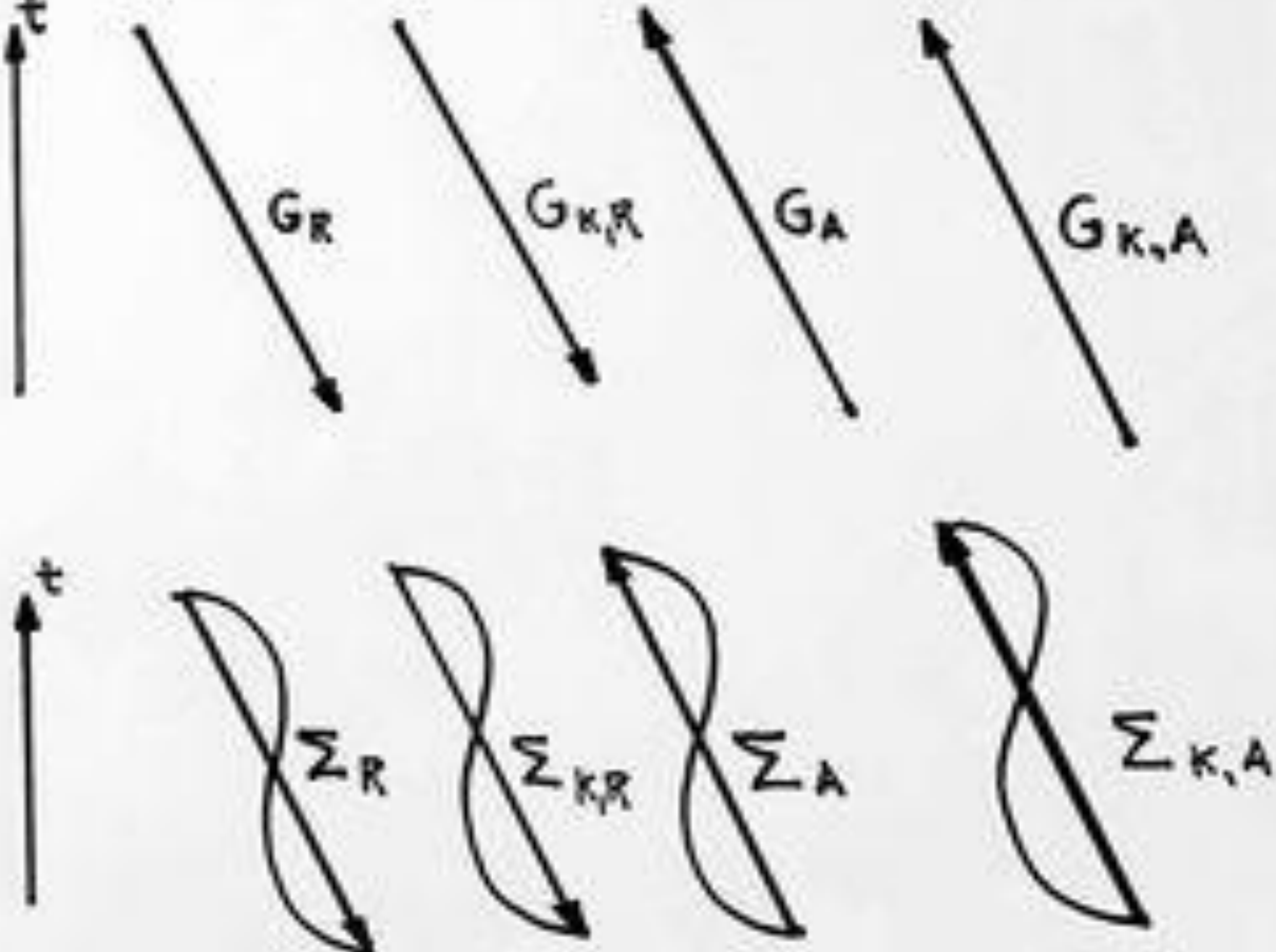
$$n(p_0,\vec{p}) = \Theta(p_0)n_1(\vec{p}) + \Theta(-p_0)n_2(-\vec{p}), \;\; \omega_p = \sqrt{\vec{p}^2+m^2}.$$

$$G_{\infty,1,K,R(A)}(p) = -\{[1+2n_1(\omega_p)]\frac{p_0+\omega_p}{2\omega_p} + [1+2n_2(\omega_p)]\frac{p_0-\omega_p}{2\omega_p}\, G_{\infty,1,R(A)}(p,m), \quad (1.5)$$

$n_1(\omega_p)$ is initial kaon distribution function, $n_2(\omega_p)$ is initial antikaon distribution function

Feynman propagator

$$G_F(p) = \frac{1}{2}[G_{\infty,1,R}(p) + G_{\infty,1,A}(p) - G_{\infty,1,K,n_1=n_2=0}(p)] \qquad (1.6)$$



t
G_R
$G_{K,R}$
G_A
$G_{K,A}$
t
Σ_R
$\Sigma_{K,R}$
Σ_A
$\Sigma_{K,A}$

Convolution Product of two-point functions [42]

$$C = A * B \Leftrightarrow C(x,y) = \int dz A(x,z)B(z,y) \tag{1.7}$$

$$C_{X_0}(p_0,\vec{p}) = \int dp_{01}dp_{02}P_{X_0}(p_0,\frac{p_{01}+p_{02}}{2})\frac{1}{2\pi}\frac{ie^{-iX_0(p_{01}-p_{02}+i\epsilon)}}{p_{01}-p_{02}+i\epsilon}A_\infty(p_{01},\vec{p})B_\infty(p_{02},\vec{p}). \tag{1.8}$$

Definition of the retarded (advanced) functions: (1) the function of p_0 is analytic above (below) the real axis, (2) the function goes to zero as $|p_0|$ approaches infinity in the upper (lower) semiplane. The choice above (below) and upper (lower) refers to R (A) components .

Under the assumption that A or B satisfies (1) and (2) (A as advanced or B as retarded) we obtain

$$C_{X_0}(p_0,\vec{p}) \;=\; \int dp_0' P_{X_0}(p_0,p_0')A_\infty(p_0',\vec{p})B_\infty(p_0',\vec{p}). \tag{1.9}$$

$$C_\infty(p_0,\vec{p}) \;=\; A_\infty(p_0',\vec{p})B_\infty(p_0',\vec{p}). \tag{1.10}$$

Particle Number as Average of Equal Time Limit of G_K

$$< N_{\vec{p}}(t) >= (2\pi)^3 d\mathcal{N}/(d^3x d^3p), \tag{1.11}$$

After the time t, the number of kaons (antikaon) is expressed through the average of equal-time limits (AETL) of the resummed Keldysh component $\tilde{G}_{\beta,\alpha,K}$ $(\beta = \alpha)$ of the Wigner transform of kaon propagator taken from above $(\delta > 0)$ and from below $(\delta < 0)$:

$$1 + \delta_{1,\alpha} \langle\, N_{K^0,\vec{p}}(t)\, \rangle\, + \delta_{2,\alpha} \langle\, N_{\bar{K}^0,\vec{p}}(t)\, \rangle$$

$$= \frac{\omega_p}{2\pi} \big[\lim_{0<\delta\to 0} + \lim_{0>\delta\to 0} \big] \int dp_0\, e^{-ip_0\delta}\, \tilde{G}_{\alpha,\alpha,K,t}(p)\,, \tag{1.12}$$

where $\delta = s_{01} - s_{02}$ and $t = X_0 = (s_{01} + s_{02})/2$.

AVERAGE - owing to the complex poles of $\tilde{G}_K$

the particle number can be known only approximately

other definitions of a particle number in the literature. more or less equivalent to our.

Our definition of particle number has some good properties :

1. It reproduces correctly the lowest order contribution (from $\S_K$).
2. for rising t it mildly approachess S-matrix theory results (e.g.for Compton scattering).

Lagrangian and Self-Energies

Lagrangian consists of five important pieces:

1. Free Lagrangian L_0, kaons K^0 and $\bar{K}^0$ free ,stable particles, mass m_{K^0}.

2. Mass mixing Lagrangian $(K_S, K_L) \leftrightarrow (K^0, \bar{K}^0)$

3. Couplings $K_L \leftrightarrow 3\pi$ generate self-energy $\Sigma_{L,R(A)} = \Sigma_{1,R(A)}$.

4. Couplings $K_S \leftrightarrow 2\pi$ generate self-energy $\Sigma_{S,R(A)} = \Sigma_{2,R(A)}$.

5. Cp violating Lagrangian, which causes $K_L \rightarrow 2\pi$ three selfenergies are associated:

self-energy $\Sigma^{\neq}_{1,R(A)}$ which describes the process $K_L \rightarrow 2\pi \rightarrow K_L$

hybrid selfenergy $\Sigma_{K_L \rightarrow K_S} \Sigma^{\neq}_{21,R(A)}$ which is associated to precesses where $\Sigma^{\neq}_{\infty,12,R(A)}$ in which first process is CP violating, while the second one $2\pi \rightarrow K_S$ is CP conserving

hybrid selfenergy $\Sigma^{\neq}_{21,R(A)}$, which describes the reversed process $K_S \rightarrow 2\pi \rightarrow K_L$.

Naively speaking, they should satisfy $Im\Sigma_{K_L \rightarrow 2\pi \rightarrow K_L} Im\Sigma_{K_S \rightarrow 2\pi \rightarrow K_S} = Im\Sigma_{K_L \rightarrow 2\pi \rightarrow K_S} Im\Sigma_{K_S \rightarrow 2\pi \rightarrow K_L}$. But, one cannot be sure that, the above mentioned $2\pi \rightarrow K_S$ which is CP conserving processis is the same as normal $2\pi \rightarrow K_S$ which describes fast K_S decay.

In the free part of Lagrangian the mass m is defined which is identical to $M_{K_0} = M_{\bar{K}_0}$.

Coupling parts of Lagrangian we shall not specify explicitly, as we shall need only self-energies taken close to the mass shell of kaon.The self-energies we shall assume to be known,at least in principle. Although, in the available literature, the tedious QCD calculations have been performed only partially, i.e. to determine decay rates. The self-energies will not be treated in their full complexity. Instead we shall approximate them with complex numbers $\Sigma_{R,L} \to R_L - iIL$ and $\Sigma_{R,S} \to R_S - iI_S$. we conststently replace $\Sigma_{A,L} \to R_L + iI_L$ and $\Sigma_{R,S} \to R_S + iI_S$. This will correspond to the pole approximation for resummed propagators $\tilde{G}_{R,S}$ and $\tilde{G}_{R,L}$.

Thus we have

$$\mathcal{L}(x) = \mathcal{L}^0(x) + \mathcal{L}^{mix}(x) + \mathcal{L}^{K_L \leftrightarrow 3\pi}(x) + \mathcal{L}^{K_S \leftrightarrow 2\pi}(x) + \mathcal{L}_{\neq}^{K_L \leftrightarrow 2\pi}(x) + \mathcal{L}_{\neq}^{K_S \leftrightarrow 2\pi}(x)$$

$$\mathcal{L}^0(x) = \sum_{\mu,\alpha} (\partial_\mu \Psi_\alpha)^*(x)(\partial^\mu \Psi_\alpha)(x) - M \sum_\alpha \Psi_\alpha^*(x)\Psi_\alpha(x)$$

$$\mathcal{L}^{mix}(x) = \sum_{\alpha,i} \bar{\Psi}_\alpha(x) U_{\alpha,i}^* M_{ij} U_{\beta,j} \Psi_\beta(x)$$

$$U_{K_S,K_0} = \frac{1}{2^{1/2}} = U_{K_S,\bar{K}_0} = U_{K_L,K_0} = -U_{K_L,\bar{K}_0}$$

$$M_{ij} = \delta_{ij}\Delta_i, \quad i,j = 1,2, \quad M = \begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{pmatrix} \tag{1.13}$$

$\alpha = 1$ forK_0 and $\alpha = 2$ for $\bar{K}_0$,
$i = 1$ for K_L, and $i = 2$ for K_S.

Dyson-Schwinger Equations and Solutions

The dynamics of kaon oscillation, decay, and CP violation is now determined by Dyson-Schwinger equations (DSE) satisfied by the Wigner transforms of bare and resummed kaon propagators:

In DSE there is $\tilde{M}_{R(A)}$ which is natrix M extended through retarded or advanced self-energies

$$\tilde{M}_{\infty,R(A)} = \begin{pmatrix} \Delta_1 + \Sigma_{1,\infty,R(A)} + \Sigma^{\neq}_{\infty,1,R(A)} & \Sigma^{\neq}_{\infty,12,R(A)} \\ \Sigma^{\neq}_{\infty,21,R(A)} & \Delta_2 + \Sigma_{\infty,2,R(A)} \end{pmatrix} \tag{1.14}$$

the self-energies $\Sigma^{\neq}$ are so small that they should be taken into account only linearly.

The self-energy for DSE is

$$\tilde{\Sigma}_{\infty,\beta,\alpha,R(A)} = \sum_i U^*_{\beta,i}\tilde{M}_{\infty,ij,R(A)}(p)U_{\alpha,i} \tag{1.15}$$

The Dyson-Schwinger equations for “oscillating” kaons are

$$\tilde{G}_{\beta,\eta,R} = G_{\beta,R}\delta_{\beta,\eta} + iG_{\beta,R} * \tilde{\Sigma}_{\beta,\alpha,R} * \tilde{G}_{\alpha,\eta,R}$$

$$\tilde{G}_{\beta,\eta,A} = G_{\beta,A}\delta_{\beta,\eta} + iG_{\beta,R} * \tilde{\Sigma}_{\beta,\alpha,A} * \tilde{G}_{\alpha,\eta,A}$$

$$\tilde{G}_{\beta,\eta,K} = G_{\beta,K}\delta_{\beta,\eta} + i[G_{\beta,R} * \tilde{\Sigma}_{\beta,\alpha,R} * \tilde{G}_{\alpha,\eta,K}$$

$$+G_{\beta,K} * \tilde{\Sigma}_{\beta,\alpha,A} * \tilde{G}_{\alpha,\eta,A} + G_{\beta,R} * \tilde{\Sigma}_{\beta,\alpha,K} * \tilde{G}_{\alpha,\eta,A}] \tag{1.16}$$

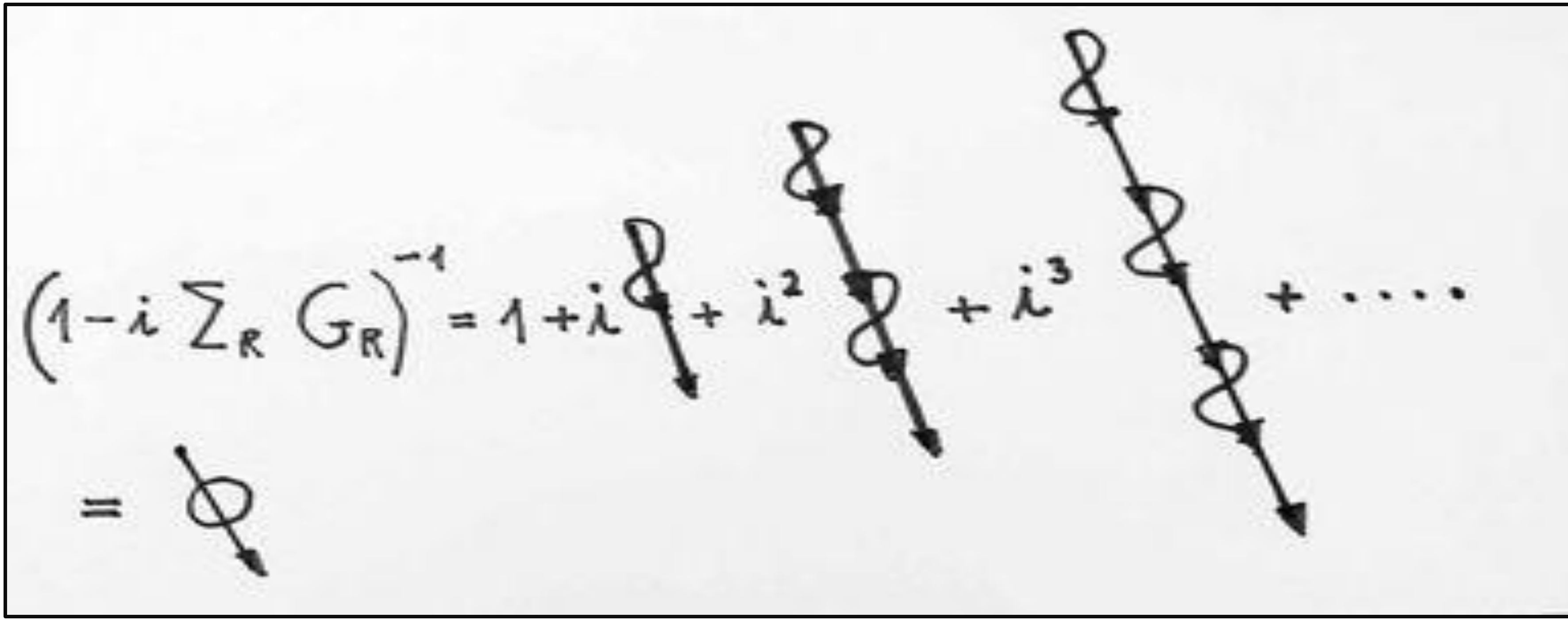
where $*$-symbol = convolution product between two Wigner functions The formal solution is (simplified by unitarity of U-matrix and $*$-product between RR factors becames algebraic)

$$\tilde{G}_{\beta,\eta,R} = \sum_i U^*_{\beta,i}[1 - i\tilde{\Sigma}_R G_R]^{-1} G_R U_{\eta,i}$$
$$\tilde{G}_{\beta,\eta,A} == \sum_i U^*_{\beta,i} G_A [1 - i\tilde{\Sigma}_A G_A]^{-1} U_{\eta,i} \tag{1.17}$$

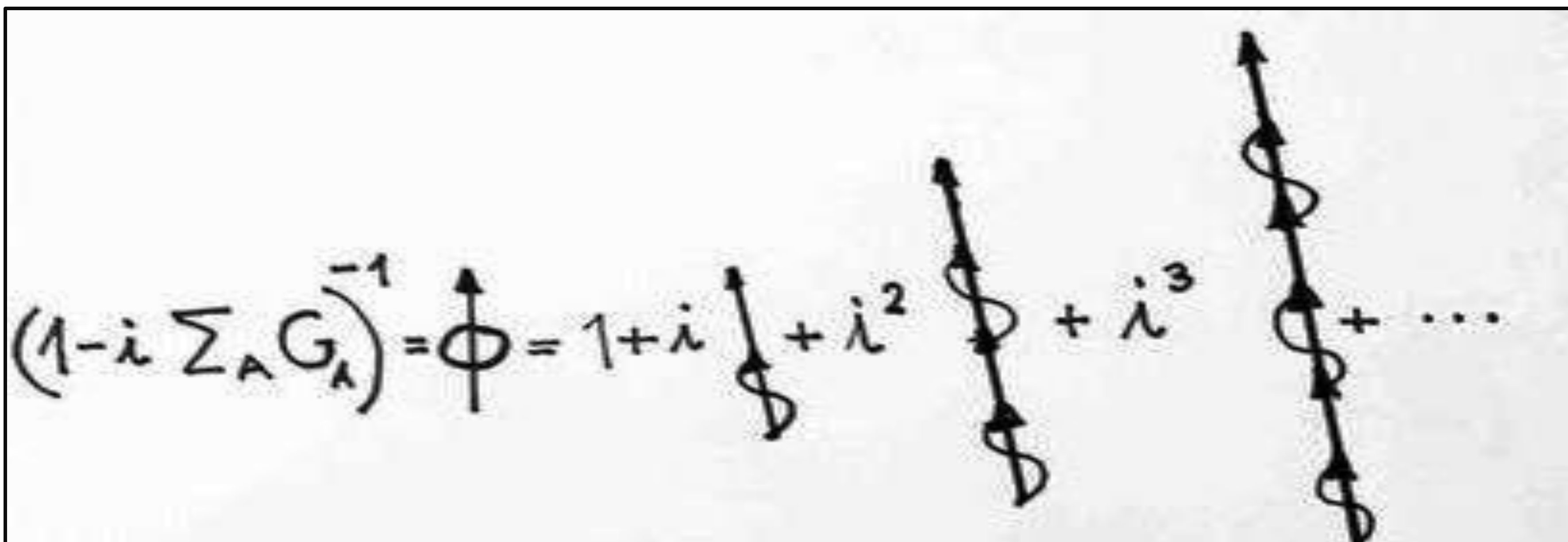
$$\tilde{G}_{\beta,\eta,K} = \tilde{G}_{\beta,\eta,K,alg} + \tilde{G}_{\beta,\eta,K,conv}$$

$$\tilde{G}_{\beta,\eta,K,alg} = -\sum_i U^*_{\beta,i} U_{\eta,i} G_{K,A}[1 - i\tilde{\Sigma}_{i,A} G_A]^{-1}$$
$$+\sum_i U^*_{\beta,i} U_{\eta,i}[1 - i\tilde{\Sigma}_{i,R} G_R]^{-1} G_{K,R}$$
$$\tilde{G}_{\beta,\eta,K,conv} = i\sum_{ij} U^*_{\beta,i} U\eta_i U^*_{\eta,j} U_{\beta,j}$$
$$[1 - i\tilde{\Sigma}_{i,R} G_R]^{-1}[G_{K,R} * \tilde{\Sigma}_{j,A} G_A - G_R \tilde{\Sigma}_{i,R} * G_{K,A}][1 - i\tilde{\Sigma}_{j,A} G_A]^{-1}. \tag{1.18}$$

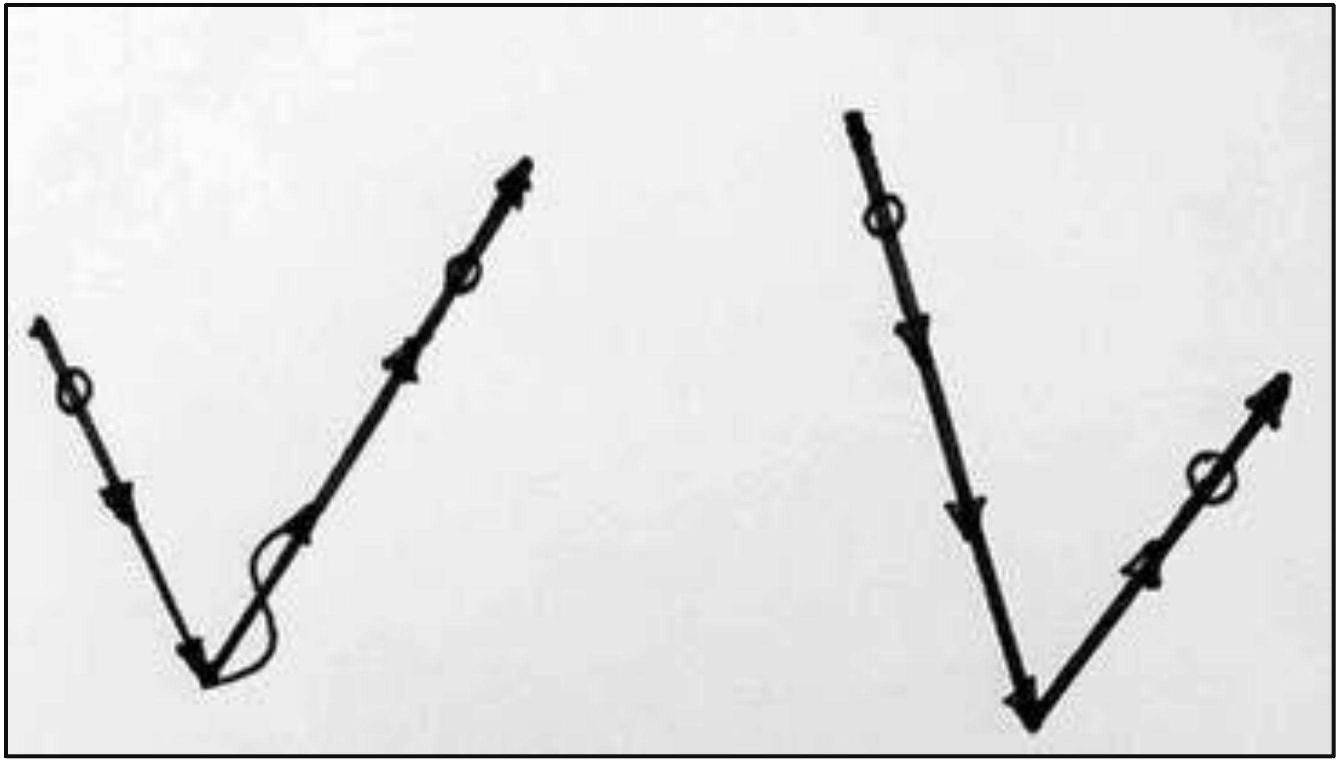
The solution of D-S equations is linear in G_K i.e. linear in n. It is strictly causal.



(1 − i Σ_R G_R)^{-1} = 1 + i + i^2 + i^3 + ...
=



(1 − i Σ_A G_A)^{-1} = = 1 + i + i^2 + i^3 + ...



Oscillating Kaons - Singularities of the Resummed $\tilde{G}_{\beta,\alpha,R}$

To proceed further we search forthe singularities of $(1 - iG_{i,R}\tilde{\Sigma}_{i,R})^{-1}G_{i,R}(p_0)$.
The symmetries of the self-energies are imortant

$$\tilde{\Sigma}_{K,R}(-p_0,\vec{p}) = -\tilde{\Sigma}^*_{K.R}(p_0,\vec{p}), \;\; \tilde{\Sigma}_R(-p_0,\vec{p}) = \tilde{\Sigma}^*_R(p_0,\vec{p}) \tag{1.19}$$

POLE APPROX.: REPLACE $\tilde{\Sigma}_{i,R}(p_0)$ by a constant $\tilde{\Sigma}_{i,R}(\bar{p}_0)$ taken at the $\bar{p}_0 = \pm\omega_p$.
Define $\tilde{\Sigma}_{i,R}(\pm\omega_p) = R_i \mp I_i$ dictated by the symmetry, negative I_i violates causality.
The approximation is valid in vicinity of the corresponding zero of $G^{-1}_{i,R}$.
The equation: $p_0^2 - \omega_p^2 - \mathcal{R}_i + i\mathcal{I}_i = 0$ with $\omega_p = (\vec{p}^2 + m^2)^{1/2}$, poses two solutions $p_{0,1,2} = \pm[\omega_p^2 + \mathcal{R}_i - i\mathcal{I}_i]^{1/2}$, but only one (with + sign) represents the pole near $+\omega_p$, the otner is near $-\omega_p$ where imaginary part of $\tilde{\Sigma}_{i,R}(p_0)$ has opposite sign, thus it is not the pole. The pole near $-\omega_p$ is obtained from the equation containing $-i\mathcal{I}_i$.

$$\omega_{i,\lambda_i} = \lambda_i[\omega_p^2 + \mathcal{R}_i - i\lambda_i\mathcal{I}_i]^{1/2}, \quad \lambda_i = \pm 1$$

$$\omega_{i,\lambda_i} \approx \lambda_i(\omega_p + \frac{\mathcal{R}_i}{2\omega_p}) - i\frac{\mathcal{I}_i}{2\omega_p} = \lambda_i\omega_i - \frac{im_i\Gamma_i}{\omega_i},$$

$$\omega_i^2 = \vec{p}^2 + m_i^2 - \Gamma_i^2, \quad m >> |\mathcal{R}_i|, \; |\mathcal{I}_i| \tag{1.20}$$

Near the poles the propagator is approximated as

$$[1 - iG_{i,R}\Sigma_{i,R})]^{-1}G_{i,R} \approx \frac{-i}{2\omega_{i,\lambda_i}(p_0 - \omega_{i,\lambda_i} + i\epsilon)}, \quad |p_0 - \omega_{i,\lambda_i}| << \omega_p \tag{1.21}$$

Final Result

Initial Kaon stream: $t = 0 \quad n_1(\omega_p), \quad n_2(\omega_p) = 0,$
Kaon number; $\quad \alpha = \beta = 1,$

$$N_{K_0}(t) = \frac{n_{\alpha+}(|\vec{p}|, \vec{p})}{4}[e^{-\frac{2m_1\Gamma_1}{\omega_p}t} + e^{-\frac{2m_2\Gamma_2}{\omega_p}t} + 2e^{-\frac{m_1\Gamma_1+m_2\Gamma_2}{\omega_p}t} \cos\frac{m_1^2 - m_2^2}{2\omega_p}t] \geq 0 \quad (1.22)$$

Antikaon number $\quad \alpha = \beta = 2.$

$$N_{\bar{K}_0}(t) = \frac{n_{\alpha+}(|\vec{p}|, \vec{p})}{4}[e^{-\frac{2m_1\Gamma_1}{\omega_p}))t} + e^{-\frac{2m_2\Gamma_2}{\omega_p}))t} - 2e^{-\frac{m_1\Gamma_1+m_2\Gamma_2}{\omega_p}))t} \cos\frac{m_1^2 - m_2^2}{2\omega_p}t] \quad (1.23)$$

At rest $\omega_p = M \approx m_1 \approx m_2$

$$\begin{aligned} N_{K_0}(t) &= \frac{n_{\alpha+}(|\vec{p}|, \vec{p})}{4}[e^{-2\Gamma_1 t} + e^{-2\Gamma_2 t} + 2e^{-(\Gamma_1+\Gamma_2)t}\cos\Delta t] \\ N_{\bar{K}_0}(t) &= \frac{n_{\alpha+}(|\vec{p}|, \vec{p})}{4}[e^{-2\Gamma_1 t} + e^{-2\Gamma_2 t} - 2e^{-(\Gamma_1+\Gamma_2)t}\cos\Delta t] \qquad (1.24) \end{aligned}$$

In the absence of CP violation, there is strict symmetry between K_0 and $\bar{K}_0$ decays.

Conclusions

1. Finite-Time-Path TFT is appropriate tool for the treatment of kaon decay, oscillation and CP-violation.

2. Within Toy model with Interaction Lagrangian built as mass mixing matrix and various process contributions , we calculate kaon decay, oscillation and CP-simmetry violations. The model is exactly solvable. No need for nonhermitian Hamilton operator

3. easy application to B^0 and D^0 decays and oscillations.

Model can predict ,at least in principle, some features

1. Oscillation and decay parameters

2. Energy difference $m_{K_S} - m_{K_L}$

3. relation among two parameters (ϵ and ϵ') for CP violation. This prediction relies heavily on the question whether the decay amplitude $K_S \to 2\pi$ is the same for true decay, and in the case where it assists the CP-violating amplitude $K_L \to 2\pi$ or not the same.

These predictions rely heavily on the available calculations of various self-energies. Not only imaginary parts of self-energies (decay rates) but also real parts of self-energies (mass shifts). In particular QCD self-energies should be translated to TOY-model self-energies.

The method have been applied succesfully to neutrino oscillations: predictions in agreement to Pontecorvo-Maki-Nakagawa-Sakata formula.

Further research can proceed along two main lines:

1. fitting the data and work on improving the model by taking into account the new data.

2. apply the formalism to the other oscillating and decay processes decay of positronium, damping rates insout of equilibrium many body problems, etc.

3. find the consequences of eventual neutrino decay

4. Further development of the formalism by considering higher order contributions to self-energies, which could contain diagrams with one or more minimal time vertices.