## QCD Evolution Workshop 2023

## single-hadron TMD*) fragmentation functions

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$\rightarrow$ FFs act as quark flavor-tagger and polarimeter

FF ... fragmentation function

## $e^{+} e^{-}$annihilation at Belle

- asymmetric beam-energy $e^{+} e^{-}$collider near and at $\Upsilon(4 \mathrm{~S})$ resonance $(10.58 \mathrm{GeV})$


## BELLE CsI ELECTROMAGNETIC CALORIMETER



# polarization effects 

despite unpolarized initial state

## hadron pairs: angular correlations

- angular correlations between nearly back-to-back hadrons used to tag transverse quark polarization -> Collins fragmentation functions
- RFO: one hadron as reference axis $\rightarrow \cos \left(2 \phi_{0}\right)$ modulation
- RF12: thrust (or similar) axis $\rightarrow \cos \left(\phi_{1}+\phi_{2}\right)$ modulation

- RFO and RF12: different convolutions over transverse momenta
- debatable: MC used to "correct" thrust axis to qā axis


## hadron pairs: angular correlations

- challenge: large modulations even without Collins effect (e.g., in PYTHIA MC)



## hadron pairs: angular correlations

- challenge: large modulations even without Collins effect (e.g., in PYTHIA MC)
- construct double ratio of normalized-yield distributions R12, e.g. unlike-/like-sign:

$$
\begin{aligned}
\frac{R_{12}^{U}}{R_{12}^{L}} & \simeq \frac{1+\left\langle\frac{\sin ^{2} \theta_{\mathrm{h}}}{1+\cos ^{2} \theta_{\mathrm{th}}}\right\rangle G^{U} \cos \left(\phi_{1}+\phi_{2}\right)}{1+\left\langle\frac{\sin ^{2} \theta_{\mathrm{h}}}{1+\cos ^{2} \theta_{\mathrm{th}}}\right\rangle G^{L} \cos \left(\phi_{1}+\phi_{2}\right)} \\
& \simeq 1+\left\langle\frac{\sin ^{2} \theta_{\mathrm{th}}}{1+\cos ^{2} \theta_{\mathrm{th}}}\right\rangle\left\{G^{U}-G^{L}\right\} \cos \left(\phi_{1}+\phi_{2}\right)
\end{aligned}
$$

- suppresses flavor-independent sources of modulations
- GU/L: specific combinations of FFs
- remaining MC asymmetries


## Collins asymmetries (RFO)

- first measurement of Collins asymmetries for charged pions by Belle [PRL 96 (2006) 232002, PRD 78 (2008) 032011, PRD 86 (2012) 039905(E)]
- significant asymmetries clearly rising with z
- used for first extractions of transversity parton distribution and Collins FF



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## Collins asymmetries - going further



- PT dependence for charged pions from BaBar \& BESIII
- typical rise with рт; turnover around 0.8 GeV


## Collins asymmetries - going further

[PRD 90 (2014) 052003]



- PT dependence for charged pions from $\mathrm{BaBar} \&$ BESIII
- typical rise with $\mathrm{p}_{\text {; }}$; turnover around 0.8 GeV
- ... now also from Belle in R12 frame:



## Collins asymmetries - going further

[PRD 100 (2019) 92008]

- ... as well as for neutral pion and eta

$$
\begin{aligned}
& R_{12}^{\pi^{0}}=\frac{R_{12}^{0 \pm}}{R_{12}^{L}}=\frac{\pi^{0} \pi^{+}+\pi^{0} \pi^{-}}{\pi^{+} \pi^{+}+\pi^{-} \pi^{-}} \\
& R_{12}^{\eta}=\frac{R_{12}^{\eta \pm}}{R_{12}^{L}}=\frac{\eta \pi^{+}+\eta \pi^{-}}{\pi^{+} \pi^{+}+\pi^{-} \pi^{-}}
\end{aligned}
$$

- no significant differences observed in this ( $z, P_{+}$)-binning
- again, rise with $P_{+}$in particular for larger z



## Collins asymmetries - going further

$$
\left.\begin{array}{rl}
R_{12}^{\pi^{0}}= & \frac{R_{12}^{0 \pm}}{R_{12}^{L}} \approx 1+\cos \left(\phi_{12}\right) \frac{\sin ^{2}(\theta)}{1+\cos ^{2}(\theta)} \\
\times & \left\{\frac{5\left(H_{1}^{\perp, f a v}+H_{1}^{\perp, \text { dis }}\right) \otimes\left(H_{1}^{\perp, f a v}+H_{1}^{\perp, d i s}\right)+4 H_{1, s \rightarrow \pi}^{\perp, \text { dis }} \otimes H_{1, s \rightarrow \pi}^{\perp, \text { dis }}}{\left.5\left(D_{1}^{f a v}+D_{1}^{d i s}\right) \otimes\left(D_{1}^{f a v}+D_{1}^{\text {dis }}\right)+4 D_{1, s \rightarrow \pi}^{d i s} \otimes D_{1, s \rightarrow \pi}^{d i s}\right)}\right. \\
& \left.-\frac{5\left(H_{1}^{\perp, f a v} \otimes H_{1}^{\perp, d i s}+H_{1}^{\perp, d i s} \otimes H_{1}^{\perp, f a v}\right)+2 H_{1, s \rightarrow \pi}^{\perp, \text { dis }} H_{1, s \rightarrow \pi}^{\perp, \text { dis }}}{5\left(D_{1}^{\text {fav }} \otimes D_{1}^{\text {dis }}+D_{1}^{\text {dis }} \otimes D_{1}^{\text {fav }}\right)+2 D_{1, s \rightarrow \pi}^{\text {dis }} \otimes D_{1, s \rightarrow \pi}^{\text {dis }}}\right\}
\end{array}\right\} \quad \text { isospin }=A_{12}^{U L}-A_{12}^{U C}
$$

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- consistency between neutral and charged pions
- typical rise with z also seen for neutral pions


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\end{array}\right\} \stackrel{\text { isospin } A_{12}^{U L}-A_{1}^{U C}}{ }
$$



- consistency between neutral and charged pions
- typical rise with z also seen for neutral pions
- ... while basically flat for eta


## Collins asymmetries - going further

- qualitative changes in 2019 Belle analysis w.r.t. previous Belle analyses of Collins asymmetries:
- no correction to qā axis;
" $\rightarrow$ rather to thrust axis, which is observable
- upper limit on opening angle imposed
- no correction for charm contribution; $\rightarrow$ provide charm fraction

the unpolarized case - baseline for asymmetries -


## hadron-pair production

- single-hadron production has low discriminating power for parton flavor
- can use $2^{\text {nd }}$ hadron in opposite hemisphere to "tag" flavor, transverse momentum, as well as polarization
- mainly sensitive to product of single-hadron FFs
- various definitions for scaling variable

- traditional z ("std"):

$$
\begin{array}{ll}
z_{i}=\frac{2 P_{i} \cdot q}{q^{2}} & (i=1,2) \\
z_{1}=\frac{2 P_{1} \cdot q}{q^{2}} & z_{2}=\frac{P_{1} \cdot P_{2}}{P_{1} \cdot q}
\end{array}
$$

- Altarelli et al. ("AEMP"): [Nucl. Phys. B160 (1979) 301]
- Mulders \& van Hulse ("MVH"): [PRD 100 (2019) 034011]

$$
z_{1}=\left(P_{1} \cdot P_{2}-\frac{M_{h 1}^{2} M_{h 2}^{2}}{P_{1} \cdot P_{2}}\right) \frac{1}{P_{2} \cdot q-M_{h 2}^{2} \frac{P_{1} \cdot q}{P_{1} \cdot P_{2}}}
$$

## light-meson pair production

- systematics-dominated over entire kinematic range
- strongly asymmetric systematics
- main contribution from Monte Carlo tune dependence

$z_{2}$


## light-meson pair production

- systematics-dominated over entire kinematic range
- clear flavor dependence
- suppression of kaons
- suppression of like-sign pairs
- more pronounced at large $z$ (stronger flavor sensitivity)



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- similar behavior for different $z$ definitions when imposing $T>0.8$

[PRD 101 (2020) 092004]


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- clear flavor dependence
- suppression of kaons
- suppression of like-sign pairs
- more pronounced at large z (stronger flavor sensitivity)
- similar behavior for different z definitions when imposing $T>0.8$
- larger suppression (low z) for fully inclusive pairs ("any hemisphere")

[PRD 101 (2020) 092004]


## single-hadron production

- very precise data for charged pions and kaons
- Belle data available up to very large $z(z<0.98)$
- included in several FF fits (e.g. DEHSS or MAPFF ) [cf. talk by Emanuele on Wed.]
- Belle radiative corrections "undone" in FF fits



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- data available also for (anti)protons
- not (yet) included in DEHSS or MAPFF, but, e.g., in NNFF [EPJC 77 (2017) 516]
- similar $z$ dependence as pions
- about $\sim 1 / 5$ of pion cross sections



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- about $\sim 1 / 5$ of pion cross sections
- Belle re-analysis presented in PRD 101 (2020) 092004



## ISR corrections - PRD 92 (2015) 092007



- relative fractions of hadrons as a function of $z$ originating from ISR or non-ISR events ( $\equiv$ energy loss less than 0.5\%)
- large non-ISR fraction at large $z$, as otherwise not kinematically reachable (remember: $z=E_{h} / 0.5 \sqrt{ }$ snominal )
- keep only fraction of the events -> strictly speaking not single-inclusive annihilation


## ISR corrections - PRD 101 (2020) 092004



- non-ISR / ISR fractions based on PYTHIA switch MSTP(11)
- PYTHIA model dependence; absorbed in systematics by variation of tunes


## comparison old\&new Belle single-hadron cross sections

- previous analysis



## comparison old\&new Belle single-hadron cross sections

- previous analysis

- updated analysis


## comparison old\&new Belle single-hadron cross sections



- updated analysis


## inclusive hadrons - transverse momentum

- quasi-inclusive hadron production gives access to transverse momentum in fragmentation
- transverse momentum measured with respect to thrust axis $n$
- involves sum over all final-state particles in event

- event selection and hadron distributions dependent on thrust value T required
- low thrust -> more spherical

$$
T \stackrel{\max }{=} \frac{\sum_{h}\left|\mathbf{P}_{h}^{\mathrm{CMS}} \cdot \hat{\mathbf{n}}\right|}{\sum_{h}\left|\mathbf{P}_{h}^{\mathrm{CMS}}\right|}
$$

- high thrust -> highly collimated


## inclusive hadrons - transverse momentum

- quasi-inclusive hadron production gives access to transverse momentum in fragmentation
- transverse momentum measured with respect to thrust axis $n$
- analysis performed differential in z \& $P_{h t}$, in various slices in thrust T (
- correction steps similar as for Pht-integrated cross sections


$$
T \stackrel{\max }{=} \frac{\sum_{h}\left|\mathbf{P}_{h}^{\mathrm{CMS}} \cdot \hat{\mathbf{n}}\right|}{\sum_{h}\left|\mathbf{P}_{h}^{\mathrm{CMS}}\right|}
$$

- Gaussian fits to transverse-momentum distribution provided for all hadrons in ( $z, T$ )-bins


## thrust distribution: process contributions



- large contribution from BB at lower thrust
- large thrust dominated by uds and charm fragmentation (at very large $T$ significant $\tau$ contribution for pions, not visible here)
- will concentrate mainly on $0.85<T<0.9$ bin, though others available as well


## transverse-momentum distributions

- lowest T bin -> rather spherical events
- transverse momenta almost uniformly distributed in medium-z bins
- faster drop for heavier hadrons



## transverse-momentum distributions

- $0.7<T<0.8$-> particles already more collimated
- transverse momenta more Gaussian distributed
- large-z region with large uncertainties



## transverse-momentum distributions

- $0.8<T<0.85$
- transverse momenta mostly Gaussian distributed
- possible deviations for large-Pht tails [but also larger uncertainties]



## transverse-momentum distributions

- $0.85<T<0.9$
- transverse momenta mostly Gaussian distributed; widths narrowing
- possible deviations for large-Pht tails [but also larger uncertainties]



## transverse-momentum distributions

## - 0.9<T<0.95

- transverse momenta mostly Gaussian distributed; widths even narrower
- possible deviations for large-Pht tails [but also larger uncertainties]



## transverse-momentum distributions

- $0.95<T<1.0$
- transverse momenta mostly Gaussian distributed
- widths very narrow as particles now very collimated



## transverse-momentum: Gaussian widths

- $0.85<T<0.90$
- fit Gauss to low-Pht data
- mostly well described with possible exception at high z
- deviation from Gauss at large $P_{h T}$
- clear increase of width with $z$ for low values of $z$


| $\bullet$ | $\pi^{ \pm}$ | $0.15<z<0.20$ |
| :---: | :---: | :---: |
| HCB | $\pi^{ \pm}$ | $0.25<z<0.30$ |
| 4 | $\pi^{ \pm}$ | $0.35<z<0.40$ |
| $\checkmark$ | $\pi^{ \pm}$ | $0.45<z<0.50$ |
| 8888 | $\pi^{ \pm}$ | $0.55<z<0.60$ |
| ¢¢ロ́c) | $\pi^{ \pm}$ | $0.65<z<0.70$ |
| $\triangle$ | $\pi^{ \pm}$ | $0.75<z<0.80$ |

## transverse-momentum: Gaussian widths

## - $0.85<T<0.90$

- fit Gauss to low-Pht data
- mostly well described with possible exception at high z
- deviation from Gauss at large $P_{h T}$
- clear increase of width with $z$ for low values of $z$
- Gaussian widths as function of $z$
- general increase with z with turnover at larger values of $z$ for mesons
- protons with smaller width and a more linear rise with z




## transverse-momentum: Gaussian widths

- $0.85<T<0.90$
- fit Gauss to low-Pht data
- mostly well described with possible exception at high z
- deviation from Gauss at large $P_{h T}$
- clear increase of width with $z$ for low values of $z$

- Gaussian widths depend on $z$ and $T$
- general increase with $z$ with turnover at larger values of $z$
- clear decrease of widths with increase of T
- particles more and more collimated


- $e^{+} e^{-}$annihilation is powerful laboratory for hadronization studies
- in two-hadron production, observing a "back-to-back" hadron allows for tagging transverse momenta, quark flavor as well as polarization
- Collins effect allows for the study of quark-polarisation dependence of hadronization
- previous charged-pion analyses supplemented with transverse-momentum dependence and analysis of neutral-pion and eta mesons in latest Belle Collins analysis
- results for neutral \& charged pions consistent
- no significant difference between neutral pions and eta seen
- re-analysis of unpolarized fragmentation
- inclusion of alternative variable choices for two-hadron cross sections
- updated ISR correction; now consistent ISR treatment in all Belle unpolarized Xsec's
- non-trivial hadron and thrust dependent transverse momentum distributions
- clearly non-zero transverse $\Lambda$-hyperon self-polarization at Belle [not shown here; cf. talk by Marco on Monday]

