TMD factorization at next-to-leading power

Simone Rodini

In collaboration with Alexey Vladimirov

ÉCOLE POLYTECHNIQUE

INSTITUT POLYTECHNIQUE DE PARIS

Outline

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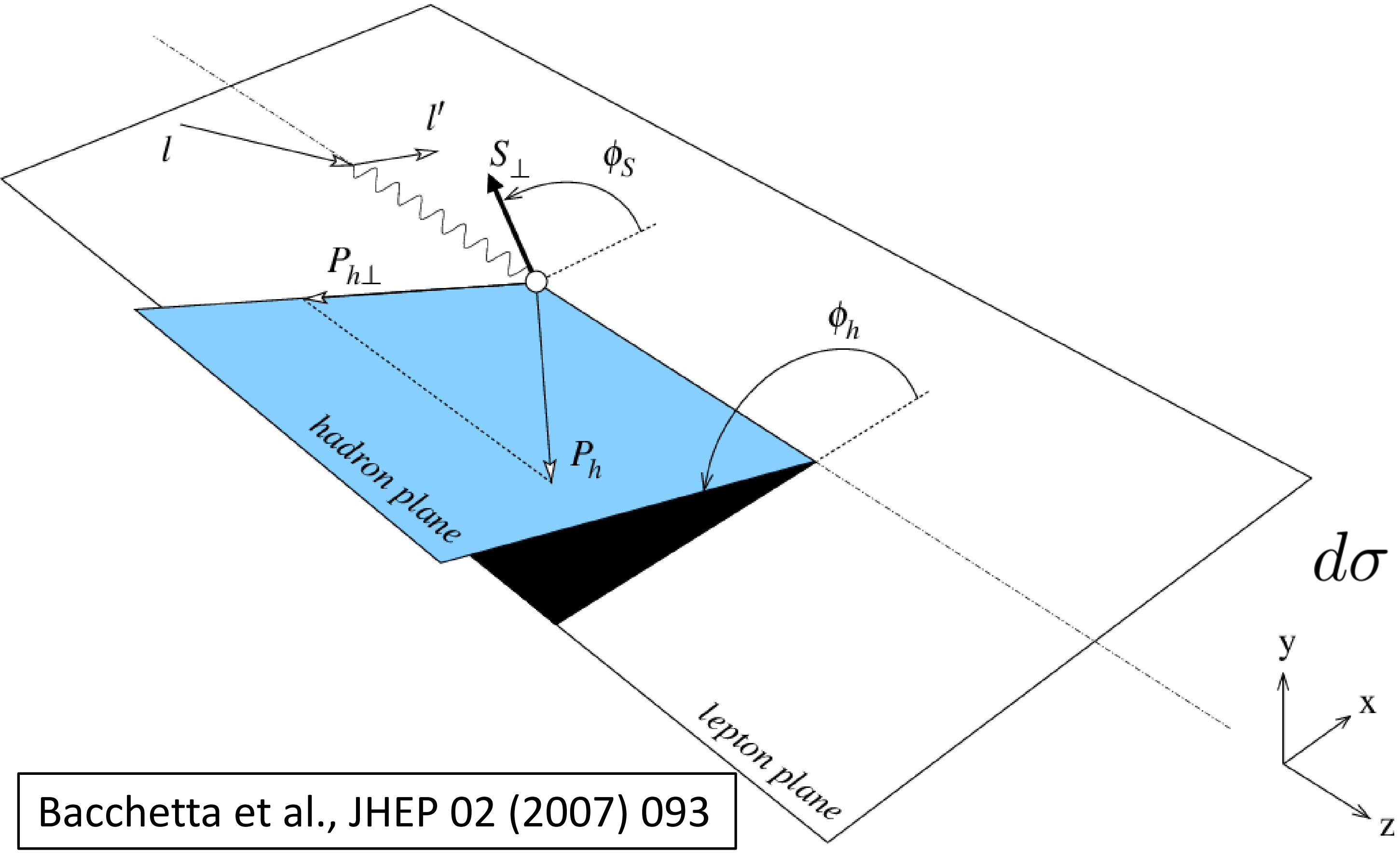
Full factorization theorem @ NLP/NLO in a ready-to-use form

NLP observables: SIDIS process

$$\ell(l) + N(P) \to \ell(l') + h(p_h) + X$$

LP and NLP contributes to different structure functions



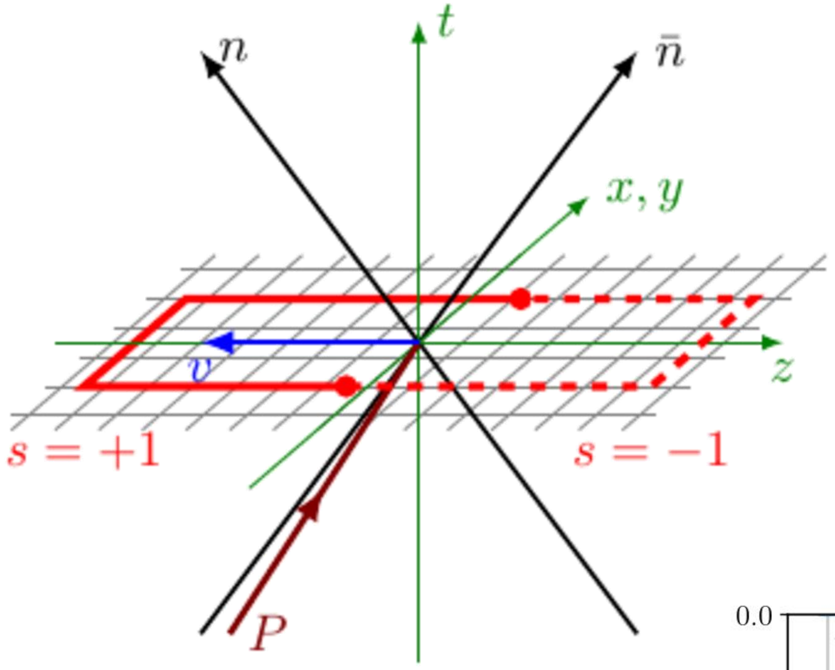
Bacchetta et al., JHEP 02 (2007) 093

All kinematic variables

expansion in $\frac{M}{Q}, \frac{|p_{h,\perp}|}{Q}$

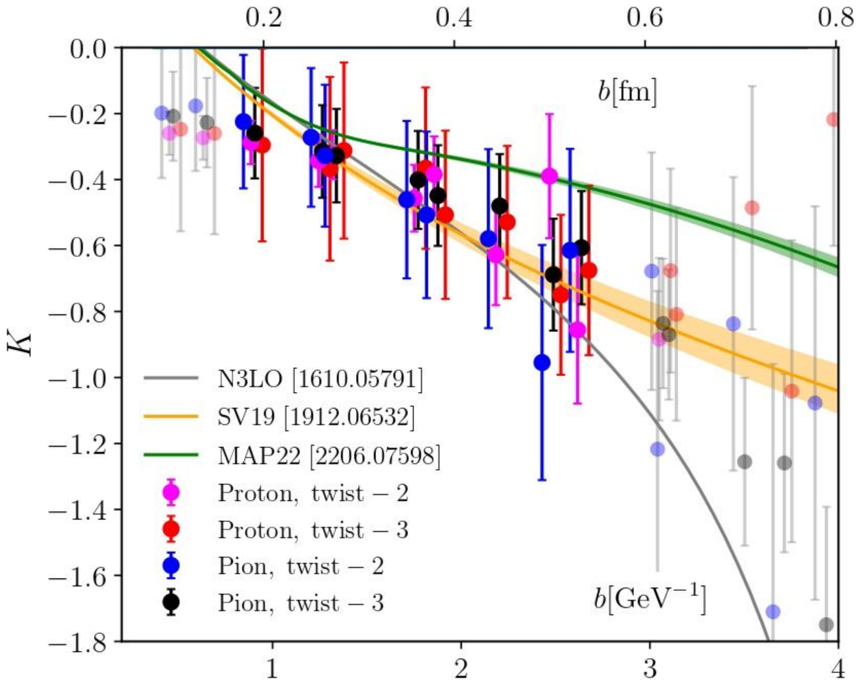
$$d\sigma = \frac{2}{s - M^2} \frac{\alpha_{\rm em}^2}{Q^4} \frac{d^3 l'}{2E'} \frac{d^3 p_h}{2E_h} L_{\mu\nu} W^{\mu\nu}$$

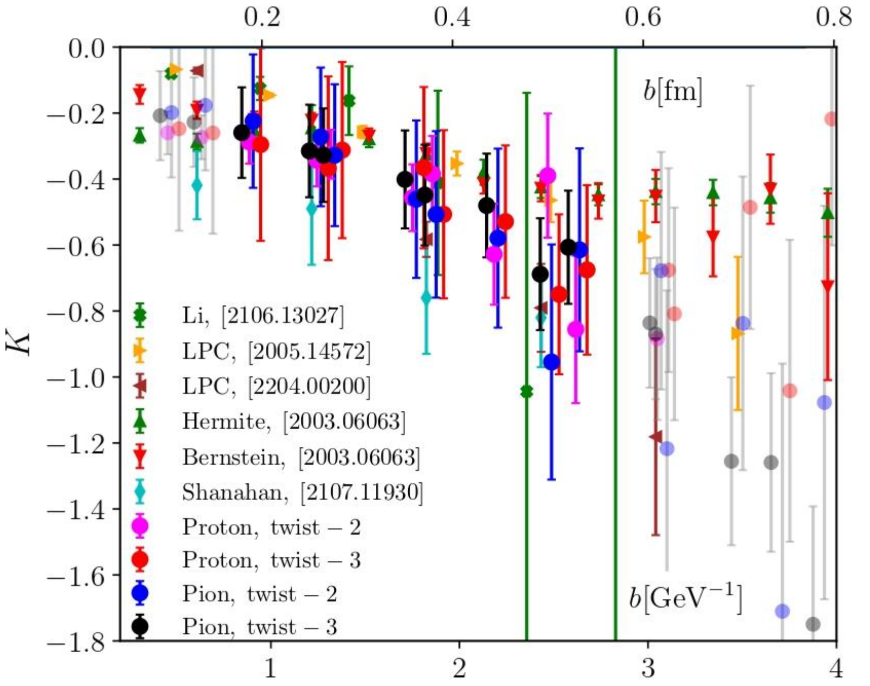
NLP observables: quasi-TMD on the lattice



Rapidity anomalous dimension is the same as LP

Test of universality

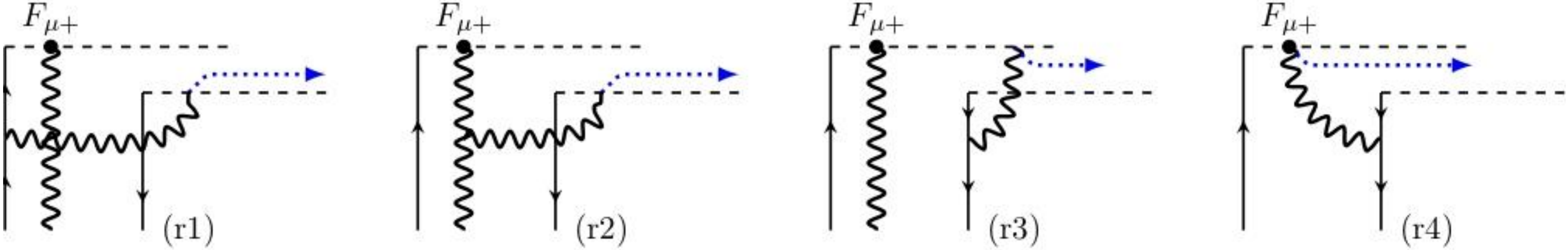




Shu et al., 2302.06502

NLP: theory

Emergence and cancellation of special rapidity divergences



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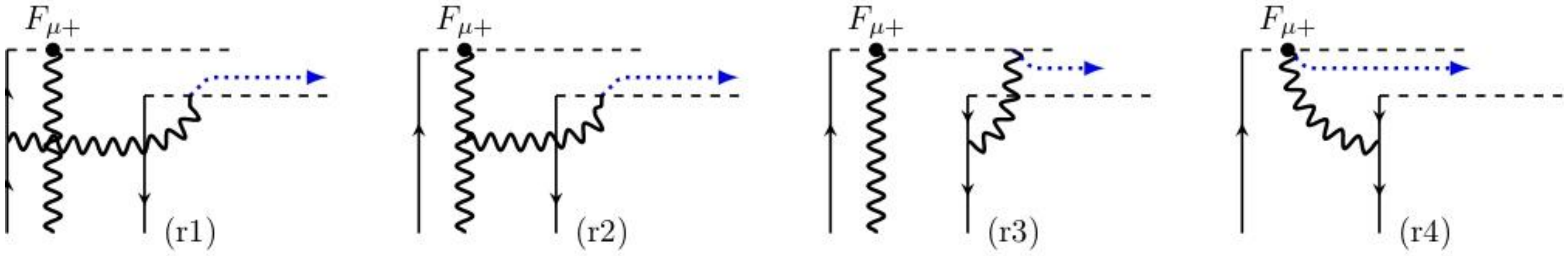

Already @ tree-level NLP coefficient function contains real and imaginary part

Imaginary part produces Qiu-Sterman-like contributions

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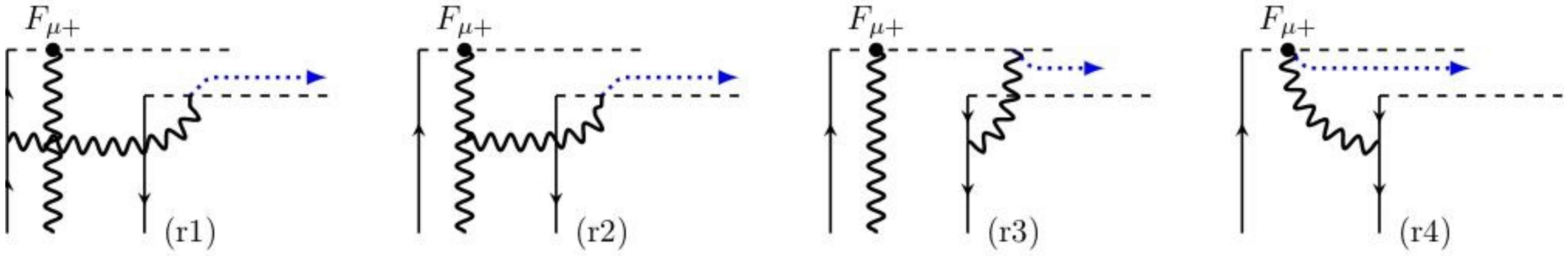

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Imaginary part produces Qiu-Sterman-like contributions

$$\lim_{x_g \to 0} \mathbb{C}_{\mathrm{NLP}}^{\mathrm{bare}} = \mathbb{C}_{LP}^{\mathrm{bare}}$$ Soft-gluon theorems?

First level: scaling and tree-level

We are interested in the TMD regime: $Q^2 \gg \boldsymbol{q}_T^2 = \text{fixed}$

$$W^{\mu\nu} = \int \frac{d^4y}{(2\pi)^4} e^{i(yq)} \langle P|J^{\mu,\dagger}(y)|p_h, X\rangle \langle p_h, X|J^{\nu}(0)|P\rangle$$

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Functional integration + background field approach

$$\phi(y) = \phi_{\bar{n}}(y) + \phi_n(y) + \psi(y)$$

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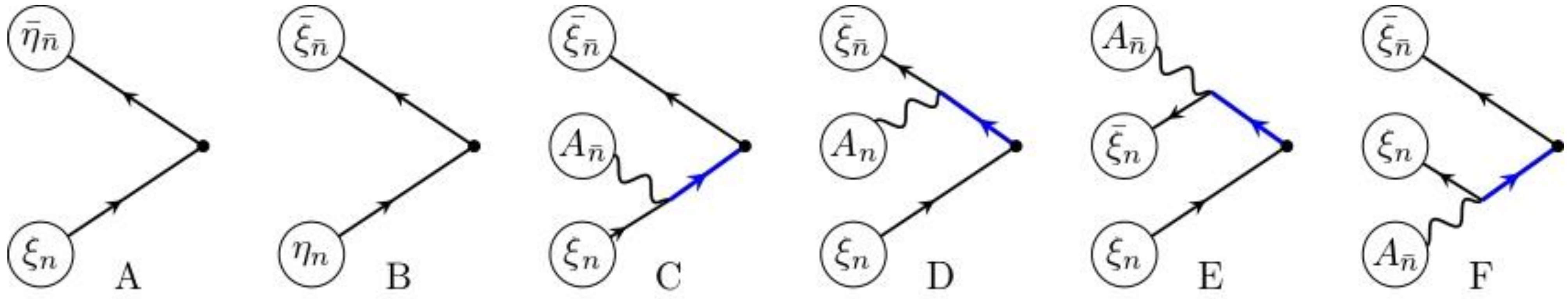
$$\phi(y) = \phi_{\bar{n}}(y) + \phi_n(y) + \psi(y)$$

$$\{\partial^+, \partial^-, \partial_T\}\phi_{\bar{n}} \lesssim Q\{1, \lambda^2, \lambda\}\phi_{\bar{n}} \qquad \xi_{\bar{n}} \sim \lambda \quad \eta_{\bar{n}} \sim \lambda^2 \quad A^{\mu}_{\bar{n}} \sim \begin{cases} 1 & \text{if } \mu = + \\ \lambda^2 & \text{if } \mu = - \\ \lambda & \text{if } \mu = T \end{cases}$$

$$\{y^+, y^-, y_T\} \sim Q^{-1}\{1, 1, \lambda^{-1}\}$$

At tree level we can simply expand the currents

$$J^\mu[\bar\psi + \bar q_{\bar n} + \bar q_n, ...] = \bar q_{\bar n}\gamma^\mu q_n + \bar q_n \gamma^\mu q_{\bar n} \\ + \bar\psi\gamma^\mu\psi + \bar q_{\bar n}\gamma^\mu\psi + \bar q_n\gamma^\mu\psi + \bar\psi\gamma^\mu q_{\bar n} + \bar\psi\gamma^\mu q_n \\ + \bar q_{\bar n}\gamma^\mu q_{\bar n} + \bar q_n\gamma^\mu q_n$$

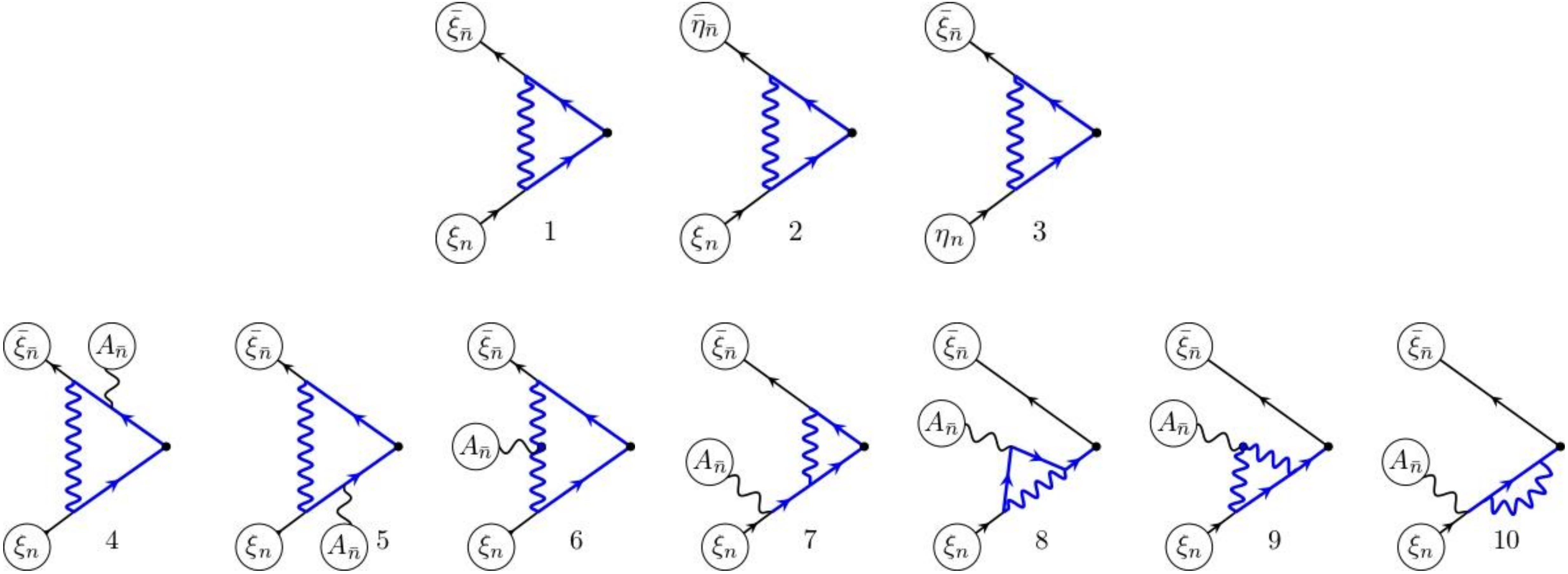


Vladimirov et al., JHEP 01 (2022) 110

Bacchetta et al., JHEP 02 (2007) 093

Second level: one-loop coefficient function

Exchange between the two currents is NNLP at least



$$
\begin{aligned}
J^\mu(y) \quad = \quad & P^+ p_h^- \int dx d\tilde{x}\, e^{ixP^+y^- + i\tilde{x}p_h^- y^+} C_1 J_{11}^\mu(x,\tilde{x},y_T) \\
& + (P^+)^2 p_h^- \int dx_1 dx_2 d\tilde{x}\, e^{i(x_1+x_2)P^+y^- + i\tilde{x}p_h^- y^+} C_2(x_{1,2}) J_{21}^\mu(x_{1,2},\tilde{x},y_T) \\
& + P^+ (p_h^-)^2 \int dx d\tilde{x}_1 d\tilde{x}_2\, e^{ixP^+y^- + i(\tilde{x}_1+\tilde{x}_2)p_h^- y^+} C_2(\tilde{x}_{1,2}) J_{12}^\mu(x,\tilde{x}_{2,1},y_T)
\end{aligned}
$$

$$
\begin{aligned}
J^\mu(y) \quad &= \quad P^+ p_h^- \int dx d\tilde{x}\, e^{ixP^+y^- + i\tilde{x}p_h^- y^+} C_1 J^\mu_{11}(x,\tilde{x},y_T) \\
&+ (P^+)^2 p_h^- \int dx_1 dx_2 d\tilde{x}\, e^{i(x_1+x_2)P^+y^- + i\tilde{x}p_h^- y^+} C_2(x_{1,2}) J^\mu_{21}(x_{1,2},\tilde{x},y_T) \\
&+ P^+ (p_h^-)^2 \int dx d\tilde{x}_1 d\tilde{x}_2\, e^{ixP^+y^- + i(\tilde{x}_1+\tilde{x}_2)p_h^- y^+} C_2(\tilde{x}_{1,2}) J^\mu_{12}(x,\tilde{x}_{2,1},y_T)
\end{aligned}
$$

$$
J_{21}(x_1,x_2,\tilde{x}) = \left(\frac{i\bar{n}^\mu}{p_h^- \tilde{x}} - \frac{in^\mu}{P_+(x_1+x_2)}\right) \frac{\bar{U}_{2\bar{n},\rho}(x_1,x_2)\gamma_T^\rho U_{1,n}(\tilde{x}) - \bar{U}_{1n}(\tilde{x})\gamma_T^\rho U_{2\bar{n},\rho}(x_2,x_1)}{x_2 - i0}
$$

$$
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J^\mu(y) \quad &= \quad P^+ p_h^- \int dx d\tilde{x}\, e^{ixP^+y^- + i\tilde{x}p_h^- y^+} C_1 J^\mu_{11}(x,\tilde{x},y_T) \\
&\quad + (P^+)^2 p_h^- \int dx_1 dx_2 d\tilde{x}\, e^{i(x_1+x_2)P^+y^- + i\tilde{x}p_h^- y^+} C_2(x_{1,2}) J^\mu_{21}(x_{1,2},\tilde{x},y_T) \\
&\quad + P^+ (p_h^-)^2 \int dx d\tilde{x}_1 d\tilde{x}_2\, e^{ixP^+y^- + i(\tilde{x}_1+\tilde{x}_2)p_h^- y^+} C_2(\tilde{x}_{1,2}) J^\mu_{12}(x,\tilde{x}_{2,1},y_T)
\end{aligned}
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$$

$$
\begin{aligned}
C_1(x,\tilde{x}) &= 2a_s C_F \frac{\Gamma(\varepsilon)\Gamma(-\varepsilon)\Gamma(2-\varepsilon)}{\Gamma(3-2\varepsilon)} \frac{2-\varepsilon+2\varepsilon^2}{(-2k^+k^- - i0)^\varepsilon} \\
C_2(x_1,x_2,\tilde{x}) &= 2a_s \frac{\Gamma(-\varepsilon)\Gamma(\varepsilon)\Gamma(1-\varepsilon)}{\Gamma(3-2\varepsilon)} \frac{1}{(-2k^+k^- - i0)^\varepsilon} \Bigg\{ C_F(1-\varepsilon)^2(2-\varepsilon) \\
&\quad + \left(C_F\varepsilon^2(1+\varepsilon) + C_A\varepsilon(1-\varepsilon-\varepsilon^2)\right) \frac{x_1+x_2}{x_1} \left(1 - \left(\frac{x_1+x_2-i0}{x_2-i0}\right)^\varepsilon\right) \\
&\quad - 2\left(C_F - \frac{C_A}{2}\right)(1-\varepsilon-\varepsilon^2)\frac{x_1+x_2}{x_2}\left(1 - \left(\frac{x_1+x_2-i0}{x_1-i0}\right)^\varepsilon\right) \Bigg\}
\end{aligned}
$$

Third level: recombination and cancellation of divergences

Renormalize the semicompact operators

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Take the product of the two currents

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Fiertz transformation to obtain TMD operator

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Fiertz transformation to obtain TMD operator

One obtains:

standard twist-(1,1) contributions

Kinematic corrections ~ derivatives of twist-(1,1)

Genuine corrections ~ quark-gluon-quark correlators

$$\Phi^{[\Gamma]}_{\mu,21}(x_1,x_2,x_3,b) = \int_{-\infty}^{\infty}\frac{d\lambda_1 d\lambda_2}{(2\pi)^2}e^{i(x_1\lambda_1+x_2\lambda_2)P^+}\langle P,s|\overline{U}_{\mu,2}(\lambda_1,\lambda_2;b)\frac{\Gamma}{2}U_1(0;0)|P,s\rangle$$

$$\Phi^{[\Gamma]}_{\mu,12}(x_1,x_2,x_3,b) = \int_{-\infty}^{\infty}\frac{d\lambda_1 d\lambda_2}{(2\pi)^2}e^{i(x_1\lambda_1+x_2\lambda_2)P^+}\langle P,s|\overline{U}_1(\lambda_1;b)\frac{\Gamma}{2}U_{\mu,2}(\lambda_2,0;0)|P,s\rangle$$

quark-gluon-quark correlators

$\Phi_{12,21}$

Have simple operator definition but undefined T-parity and complexity

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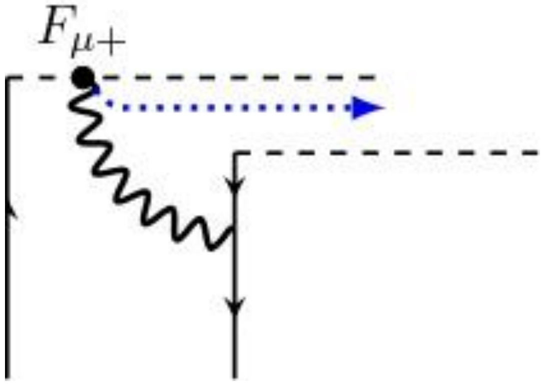

They also have uncompensated special rapidity divergences

$$\mathbf{\Phi}_{\mu,12}^{[\Gamma]}(x_1,x_2,x_3,b) = \Phi_{\mu,12}^{[\Gamma]}(x_1,x_2,x_3,b) - [\mathscr{R}_{12}\otimes\Phi_{11}]_{\mu}^{[\Gamma]}(x_1,x_2,x_3,b)$$
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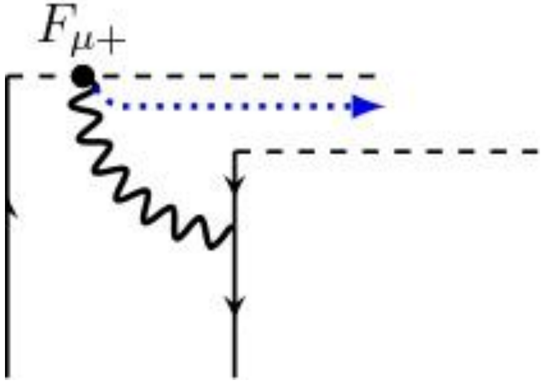

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$$\mathbf{\Phi}^{[\Gamma]}_{\mu,21}(x_1,x_2,x_3,b) = \Phi^{[\Gamma]}_{\mu,21}(x_1,x_2,x_3,b) - [\mathscr{R}_{21} \otimes \Phi_{11}]^{[\Gamma]}_{\mu}(x_1,x_2,x_3,b)$$

SPDs are cancelled in between terms:

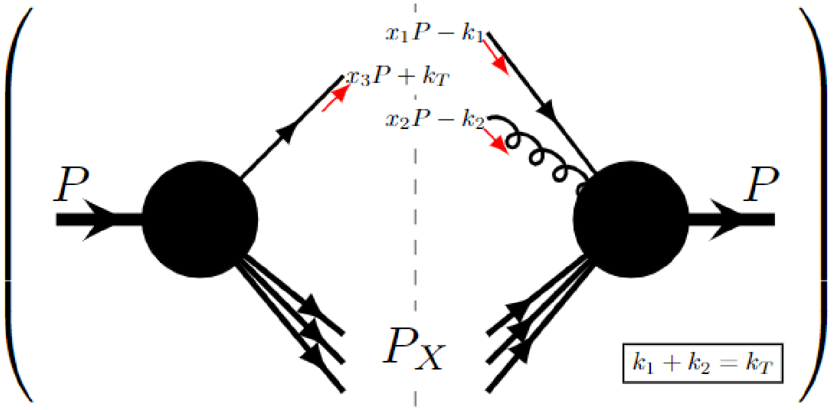
" Fragmentation functions in the n sector – TMDPDF in the nB sector"

Definite T-parity and complexity correlators

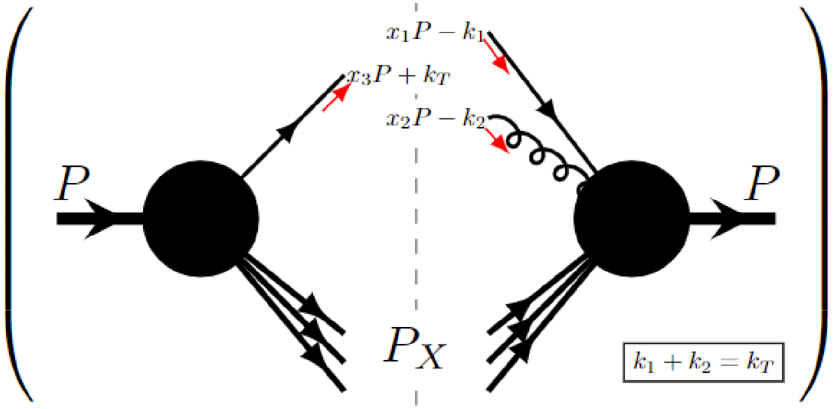
$$\mathbf{\Phi}^{[\Gamma]}_{\mu,\oplus}(x_1,x_2,x_3;b) \;=\; \frac{\mathbf{\Phi}^{[\Gamma]}_{\mu,21}(x_1,x_2,x_3;b)+\mathbf{\Phi}^{[\Gamma]}_{\mu,12}(-x_3,-x_2,-x_1;b)}{2}$$

$$\mathbf{\Phi}^{[\Gamma]}_{\mu,\ominus}(x_1,x_2,x_3;b) \;=\; i\,\frac{\mathbf{\Phi}^{[\Gamma]}_{\mu,21}(x_1,x_2,x_3;b)-\mathbf{\Phi}^{[\Gamma]}_{\mu,12}(-x_3,-x_2,-x_1;b)}{2}$$

$$\Phi_{\oplus}(x_1<0,x_2<0,x_3>0,k_T)\propto \mathrm{Re}$$

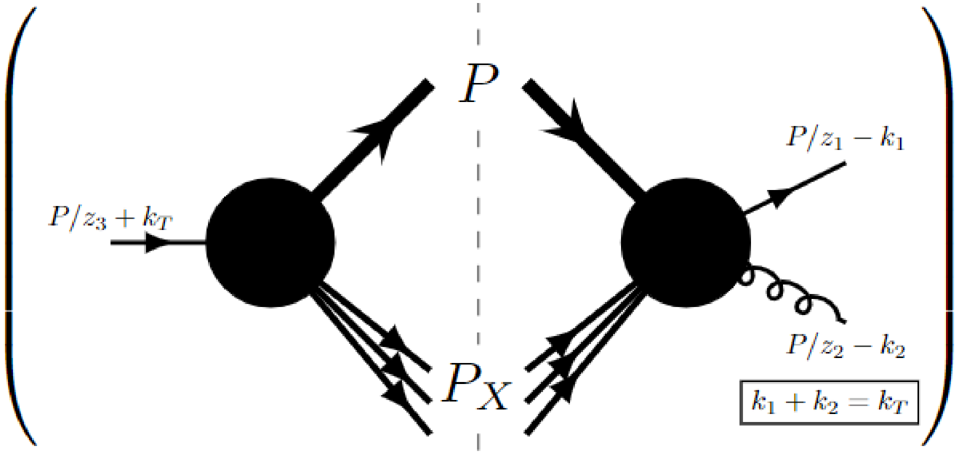


$$\Phi_{\ominus}(x_1<0,x_2<0,x_3>0,k_T)\propto \mathrm{Im}$$



$$0 < z_3 < 1 \qquad z_{1,2} < 0 \qquad \frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}=0$$

$$\Delta_{\oplus}(z_1, z_2, z_3, k_T) \propto \mathrm{Re}$$



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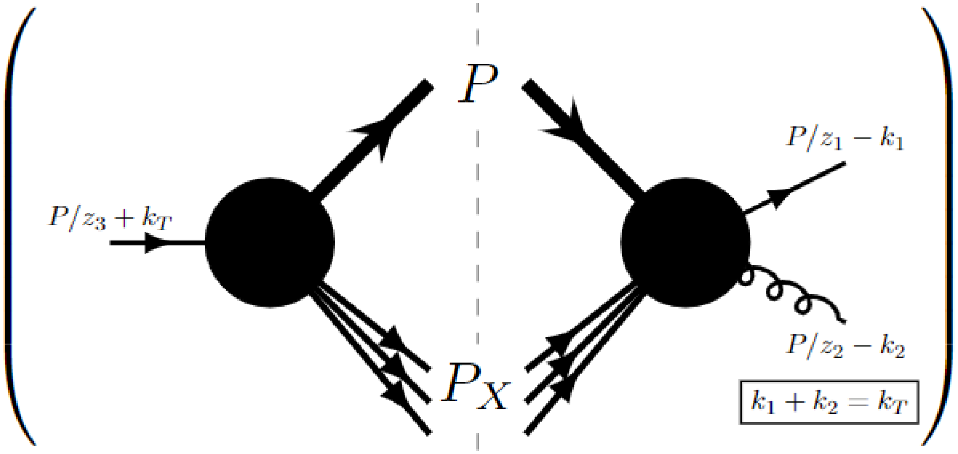

Using plus/minus correlators is also important because
It reveals that the cross section is real (as it must be)

Not a trivial statement @NLP, because we have complex coefficient functions

$$\frac{C^\dagger(\hat{u}_1, \hat{u}_2)C_1}{u_2 - i0} = \mathbb{C}_R(x, \hat{u}_2) + i\pi\mathbb{C}_I(x, \hat{u}_2)$$

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For Fragmentation functions no imaginary part
because the momentum fractions have fixed signs!

$$C_2^{\dagger}\left(\frac{1}{\hat{w}_1}, \frac{1}{\hat{w}_2}\right)C_1 = \frac{\mathbb{C}_2(z, \hat{w}_2)}{\hat{w}_2} \qquad 0 < z_3 < 1 \qquad z_{1,2} < 0 \qquad \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = 0$$

Factorized expression for the hadronic tensor

$$W^{\mu\nu} = \int d\tilde{x} d\tilde{z} \delta\left(\tilde{x} + \frac{q_+}{P_+}\right) \delta\left(\tilde{z} - \frac{p_h^-}{q_-}\right) \int \frac{d^2 b}{(2\pi)^2} e^{i(q_T b)} \; \frac{\tilde{z}}{2} \left[\widetilde{W}^{\mu\nu}_{\mathrm{LP}} + \widetilde{W}^{\mu\nu}_{\mathrm{kNLP}} + \widetilde{W}^{\mu\nu}_{\mathrm{gNLP}}\right]$$

$$\begin{aligned} \widetilde{\mathscr{W}}^{\mu\nu}_{\mathrm{kNLP}}(y) &= -i|C_V(\mu^2, Q^2)|^2 \sum_{n,m} \Bigg\{ \frac{\bar{n}^\mu \mathrm{Tr}[\gamma^\rho \bar{\Gamma}^+_m \gamma^\nu \bar{\Gamma}^-_n] + \bar{n}^\nu \mathrm{Tr}[\gamma^\mu \bar{\Gamma}^+_m \gamma^\rho \bar{\Gamma}^-_n]}{q_-} \Phi_{11}^{[\Gamma^+_n]} \left(\frac{\partial}{\partial b^\rho} - \frac{\partial_\rho \mathscr{D}}{2} \ln\left(\frac{\zeta}{\bar{\zeta}}\right)\right) \Delta_{11}^{[\Gamma^-_m]} \\ &+ \frac{n^\mu \mathrm{Tr}[\gamma^\rho \bar{\Gamma}^+_m \gamma^\nu \bar{\Gamma}^-_n] + n^\nu \mathrm{Tr}[\gamma^\mu \bar{\Gamma}^+_m \gamma^\rho \bar{\Gamma}^-_n]}{q_+} \Delta_{n11}^{[\Gamma^-_m]} \left(\frac{\partial}{\partial b^\rho} + \frac{\partial_\rho \mathscr{D}}{2} \ln\left(\frac{\zeta}{\bar{\zeta}}\right)\right) \Phi_{\bar{n}11}^{[\Gamma^+_n]} \Bigg\} + \mathrm{antiquark} \end{aligned}$$

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$$\begin{aligned} \widetilde{\mathscr{W}}^{\mu\nu}_{\text{kNLP}}(y) \quad = \quad & -i|C_V(\mu^2, Q^2)|^2 \sum_{n,m} \Bigg\{ \frac{\bar{n}^\mu \text{Tr}[\gamma^\rho \bar{\Gamma}_m^+ \gamma^\nu \bar{\Gamma}_n^-] + \bar{n}^\nu \text{Tr}[\gamma^\mu \bar{\Gamma}_m^+ \gamma^\rho \bar{\Gamma}_n^-]}{q_-} \Phi_{11}^{[\Gamma_n^+]} \left(\frac{\partial}{\partial b^\rho} - \boxed{\frac{\partial_\rho \mathcal{D}}{2} \ln\left(\frac{\zeta}{\bar{\zeta}}\right)}\right) \Delta_{11}^{[\Gamma_m^-]} \\ & + \frac{n^\mu \text{Tr}[\gamma^\rho \bar{\Gamma}_m^+ \gamma^\nu \bar{\Gamma}_n^-] + n^\nu \text{Tr}[\gamma^\mu \bar{\Gamma}_m^+ \gamma^\rho \bar{\Gamma}_n^-]}{q_+} \Delta_{n11}^{[\Gamma_m^-]} \left(\frac{\partial}{\partial b^\rho} + \boxed{\frac{\partial_\rho \mathcal{D}}{2} \ln\left(\frac{\zeta}{\bar{\zeta}}\right)}\right) \Phi_{\bar{n}11}^{[\Gamma_n^+]} \Bigg\} + \text{antiquark} \end{aligned}$$

Residue of the cancellation of special rapidity divergences.

Ensure boost-invariance!

$$
\begin{aligned}
\widetilde{W}_{\mathrm{gNLP}} \quad = \quad & i\sum_{n,m}\Bigg\{\int[d\hat{u}]\delta(\tilde{x}-\hat{u}_3)\Bigg[\\
& T_{-}^{\mu\nu\rho}(\bar{n},n)\left(\mathbb{C}_R(x,\hat{u}_2)\mathbf{\Phi}_{\rho,\oplus}^{[\Gamma_n^+]}\Delta_{11}^{[\Gamma_m^-]}+\pi\mathbb{C}_I(x,\hat{u}_2)\mathbf{\Phi}_{\rho,\ominus}^{[\Gamma_n^+]}\Delta_{11}^{[\Gamma_m^-]}\right) \\
& +iT_{+}^{\mu\nu\rho}(\bar{n},n)\left(\pi\mathbb{C}_I(x,\hat{u}_2)\mathbf{\Phi}_{\rho,\oplus}^{[\Gamma_n^+]}\Delta_{11}^{[\Gamma_m^-]}-\mathbb{C}_R(x,\hat{u}_2)\mathbf{\Phi}_{\rho,\ominus}^{[\Gamma_n^+]}\Delta_{11}^{[\Gamma_m^-]}\right) \\
& +T_{-}^{\mu\nu\rho}(n,\bar{n})\left(\mathbb{C}_R(x,\hat{u}_2)\overline{\mathbf{\Phi}}_{\rho,\oplus}^{[\Gamma_n^+]}\overline{\Delta}_{11}^{[\Gamma_m^-]}+\pi\mathbb{C}_I(x,\hat{u}_2)\overline{\mathbf{\Phi}}_{\rho,\ominus}^{[\Gamma_n^+]}\overline{\Delta}_{11}^{[\Gamma_m^-]}\right) \\
& +iT_{+}^{\mu\nu\rho}(n,\bar{n})\left(\pi\mathbb{C}_I(x,\hat{u}_2)\overline{\mathbf{\Phi}}_{\rho,\oplus}^{[\Gamma_n^+]}\overline{\Delta}_{11}^{[\Gamma_m^-]}-\mathbb{C}_R(x,\hat{u}_2)\overline{\mathbf{\Phi}}_{\rho,\ominus}^{[\Gamma_n^+]}\overline{\Delta}_{11}^{[\Gamma_m^-]}\right)\Bigg] \\
& +\int\frac{[d\hat{w}]}{|\hat{w}_1|}\delta(\tilde{z}-\hat{w}_3)\Bigg[\\
& T_{-}^{\mu\nu\rho}(\bar{n},n)\mathbb{C}_2(z,\hat{w}_2)\Phi_{11}^{[\Gamma_n^+]}\mathbf{\Delta}_{\rho,\oplus}^{[\Gamma_m^-]}-iT_{+}^{\mu\nu\rho}(\bar{n},n)\mathbb{C}_2(z,\hat{w}_2)\Phi_{11}^{[\Gamma_n^+]}\mathbf{\Delta}_{\rho,\ominus}^{[\Gamma_m^-]} \\
& +T_{-}^{\mu\nu\rho}(n,\bar{n})\mathbb{C}_2(z,\hat{w}_2)\overline{\Phi}_{11}^{[\Gamma_n^+]}\overline{\mathbf{\Delta}}_{\rho,\oplus}^{[\Gamma_m^-]}-iT_{+}^{\mu\nu\rho}(n,\bar{n})\mathbb{C}_2(z,\hat{w}_2)\overline{\Phi}_{11}^{[\Gamma_n^+]}\overline{\mathbf{\Delta}}_{\rho,\ominus}^{[\Gamma_m^-]}\Bigg]\Bigg\}
\end{aligned}
$$

One example: Cahn effect $F_{UU}^{\cos\phi}$

$$
|C_V|^2\Bigg\{\frac{|p_{h,\perp}|}{Qz}J_0[f_1 D_1]+\frac{|p_{h,\perp}|}{Qz}J_2[M m_h h_1^\perp H_1^\perp]
$$

$$
+\frac{2M}{Q}J_1\left[M\mathring{f}_1 D_1 - M f_1\mathring{D}_1 + M\mathring{\mathscr{D}}\log\frac{\zeta}{\bar{\bar{\zeta}}}\right]-\frac{2M}{Q}J_1\left[M^2 m_h b^2\mathring{h}_1^\perp H_1^\perp - M^2 m_h b^2 h_1^\perp\mathring{H}_1^\perp + M^2 m_h b^2\mathring{\mathscr{D}}\log\frac{\zeta}{\bar{\bar{\zeta}}}\right]\Bigg\}
$$

$$
-\mathbb{C}_2\frac{2m_h}{Q}J_1\left[m_h f_1(F_\oplus^\perp+G_\ominus^\perp)+2MH_\ominus h_1^\perp\right]
$$

$$
+\mathbb{C}_R\frac{2M}{Q}J_1\left[MD_1(f_\ominus^\perp-g_\oplus^\perp)+2m_h h_\ominus H_1^\perp\right]
$$

$$
-\mathbb{C}_I\frac{2M}{Q}J_1\left[MD_1(f_\oplus^\perp+g_\ominus^\perp)+2m_h h_\oplus H_1^\perp\right]
$$

One example: Cahn effect $F_{UU}^{\cos\phi}$

$$
|C_V|^2 \left\{ \frac{|p_{h,\perp}|}{Qz} J_0[f_1 D_1] + \frac{|p_{h,\perp}|}{Qz} J_2[M m_h h_1^\perp H_1^\perp] \right.
$$

$$
\left. + \frac{2M}{Q} J_1 \left[M \mathring{f}_1 D_1 - M f_1 \mathring{D}_1 + M \mathring{\mathcal{D}} \log \frac{\zeta}{\bar{\zeta}} \right] - \frac{2M}{Q} J_1 \left[M^2 m_h b^2 \mathring{h}_1^\perp H_1^\perp - M^2 m_h b^2 h_1^\perp \mathring{H}_1^\perp + M^2 m_h b^2 \mathring{\mathcal{D}} \log \frac{\zeta}{\bar{\zeta}} \right] \right\}
$$

$$
- \mathbb{C}_2 \frac{2m_h}{Q} J_1 \left[m_h f_1 (F_\oplus^\perp + G_\ominus^\perp) + 2M H_\ominus h_1^\perp \right]
$$

$$
+ \mathbb{C}_R \frac{2M}{Q} J_1 \left[M D_1 (f_\ominus^\perp - g_\oplus^\perp) + 2 m_h h_\ominus H_1^\perp \right]
$$

$$
- \mathbb{C}_I \frac{2M}{Q} J_1 \left[M D_1 (f_\oplus^\perp + g_\ominus^\perp) + 2 m_h h_\oplus H_1^\perp \right]
$$

All these combinations have
Tree-level matching to twist-4 PDFs

At one loop $f_\ominus^\perp - g_\oplus^\perp$
matches to twist-2 PDF

Conclusions

Full factorization theorem @ NLP/NLO for SIDIS

Emergence and cancellation of special rapidity divergences, restoration of boost-invariance

Definite T-parity correlators as real/imaginary part of 1->2 and 2->1 partonic interference processes

One-loop results for all LP and NLP structure functions

Only NNLP structure functions: $F_{UU,L}$ $F_{UT,L}^{\sin\phi-\phi_S}$