

# TMD factorization at next-to-leading power

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# Outline

Next-to-leading power observables

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**First level:** scaling and tree-level

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**Third level:** recombination and cancellation of divergences

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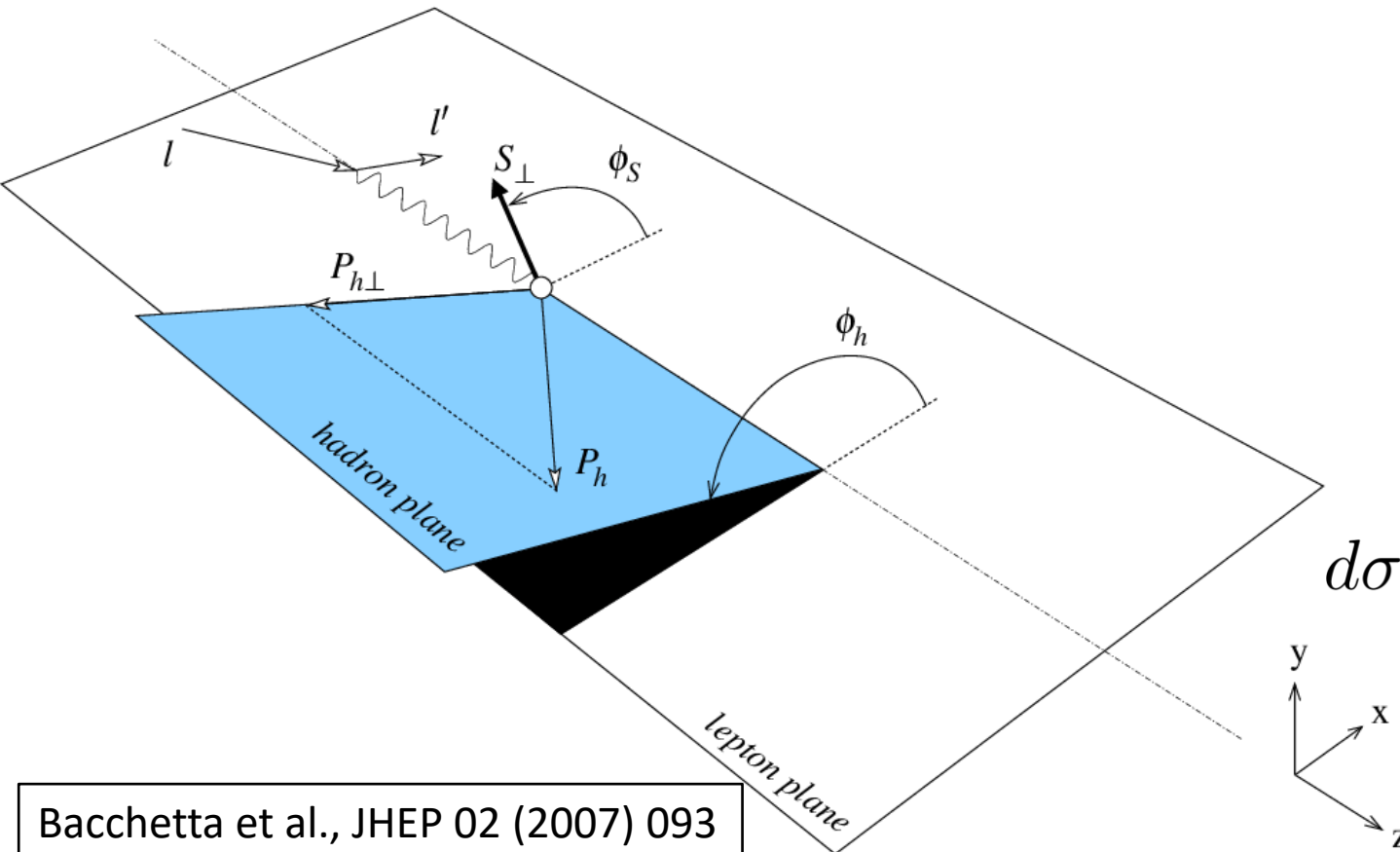
**Third level:** recombination and cancellation of divergences

Full factorization theorem @ NLP/NLO in a ready-to-use form

# NLP observables: SIDIS process

$$\ell(l) + N(P) \rightarrow \ell(l') + h(p_h) + X$$

LP and NLP contributes to different structure functions



All kinematic variables

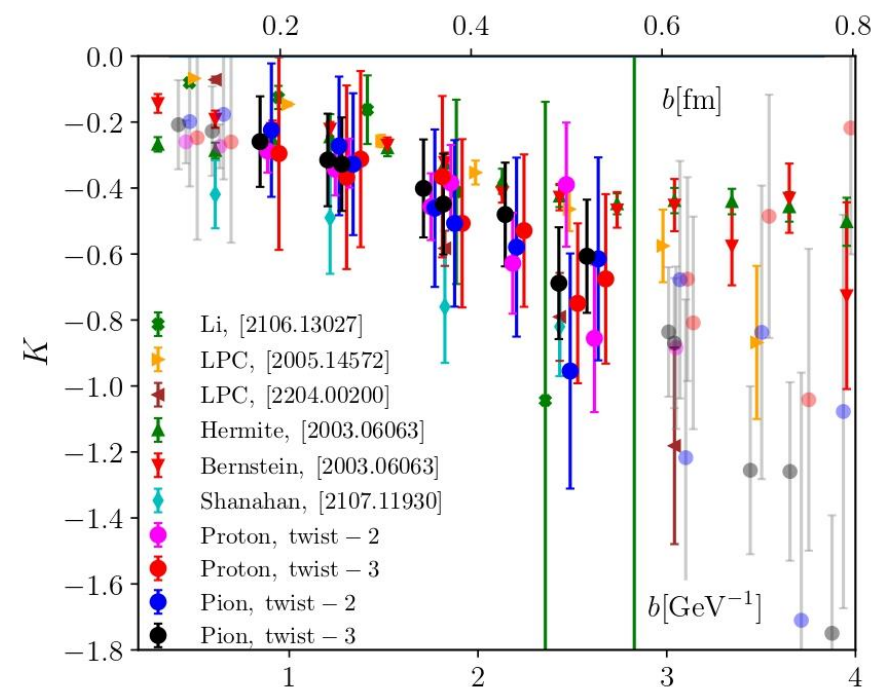
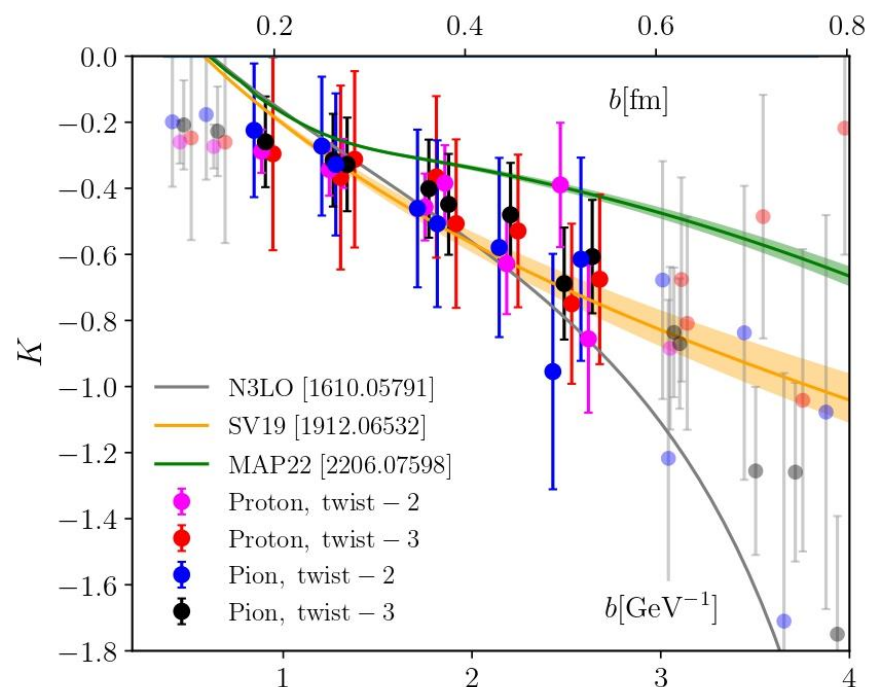
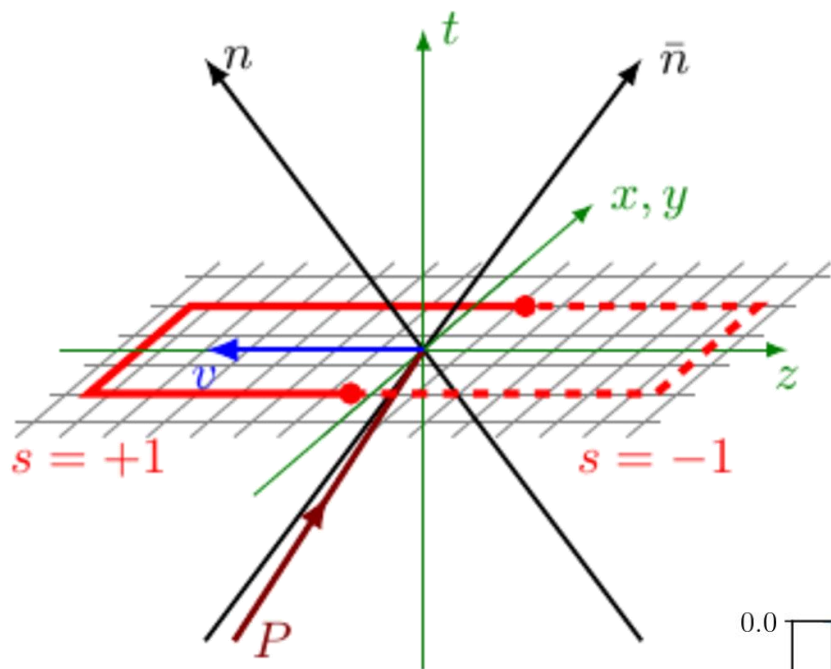
expansion in  $\frac{M}{Q}, \frac{|p_{h,\perp}|}{Q}$

$$d\sigma = \frac{2}{s - M^2} \frac{\alpha_{\text{em}}^2}{Q^4} \frac{d^3 l'}{2E'} \frac{d^3 p_h}{2E_h} L_{\mu\nu} W^{\mu\nu}$$

# NLP observables: quasi-TMD on the lattice

Rapidity anomalous dimension is the same as LP

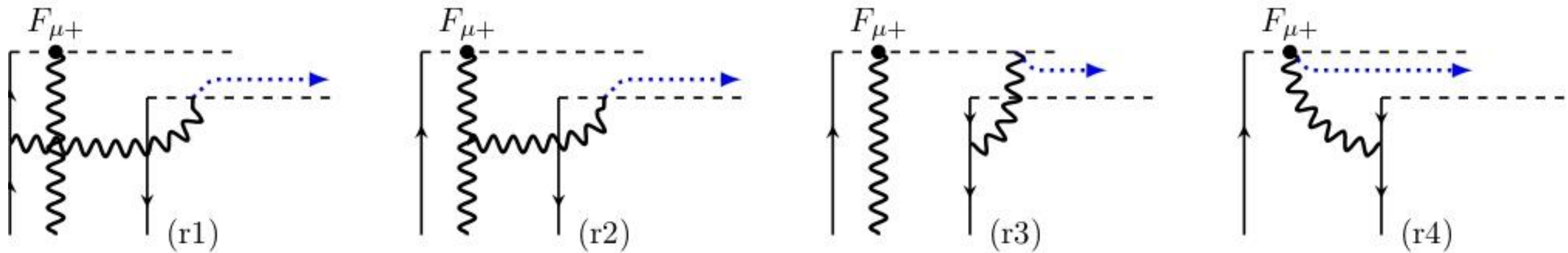
Test of universality





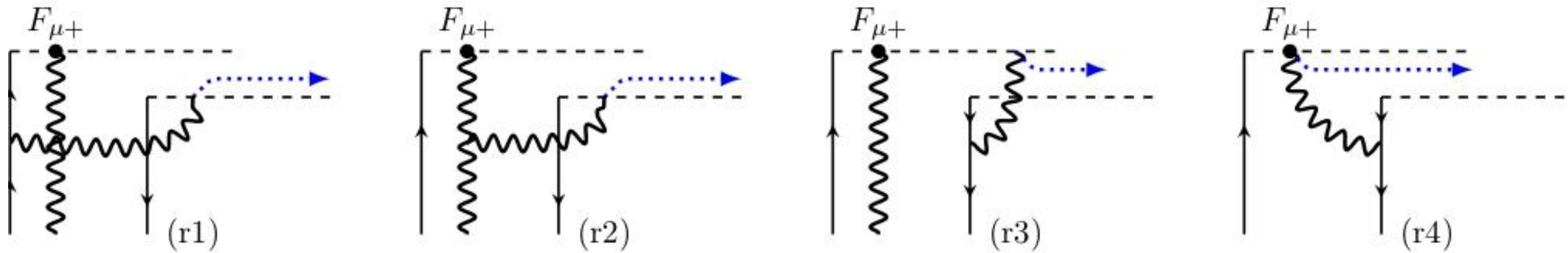
# NLP: theory

Emergence and cancellation of special rapidity divergences



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## Emergence and cancellation of special rapidity divergences

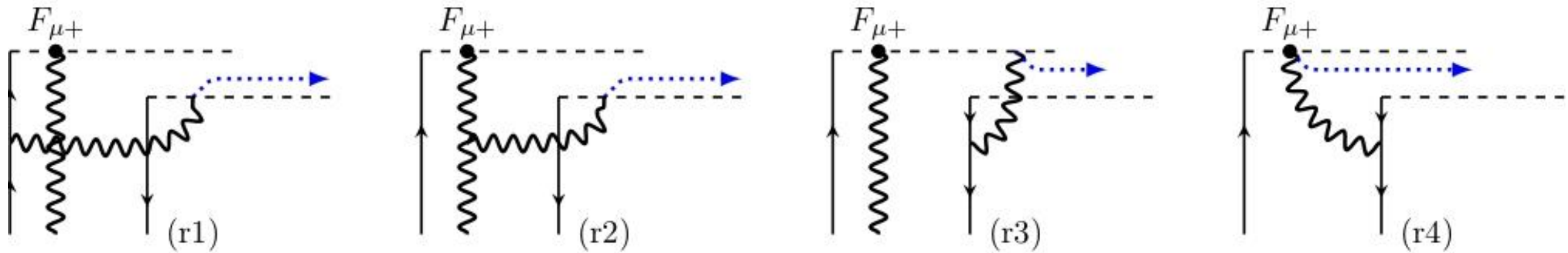


Already @ tree-level NLP coefficient function contains real and imaginary part

Imaginary part produces Qiu-Sterman-like contributions

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Imaginary part produces Qiu-Sterman-like contributions

$$\lim_{x_g \rightarrow 0} \mathbb{C}_{\text{NLP}}^{\text{bare}} = \mathbb{C}_{LP}^{\text{bare}} \quad \text{Soft-gluon theorems?}$$

## First level: scaling and tree-level

We are interested in the TMD regime:  $Q^2 \gg \mathbf{q}_T^2 = \text{fixed}$

$$W^{\mu\nu} = \int \frac{d^4 y}{(2\pi)^4} e^{i(yq)} \langle P | J^{\mu,\dagger}(y) | p_h, X \rangle \langle p_h, X | J^\nu(0) | P \rangle$$

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Functional integration + background field approach

$$\phi(y) = \phi_{\bar{n}}(y) + \phi_n(y) + \psi(y)$$

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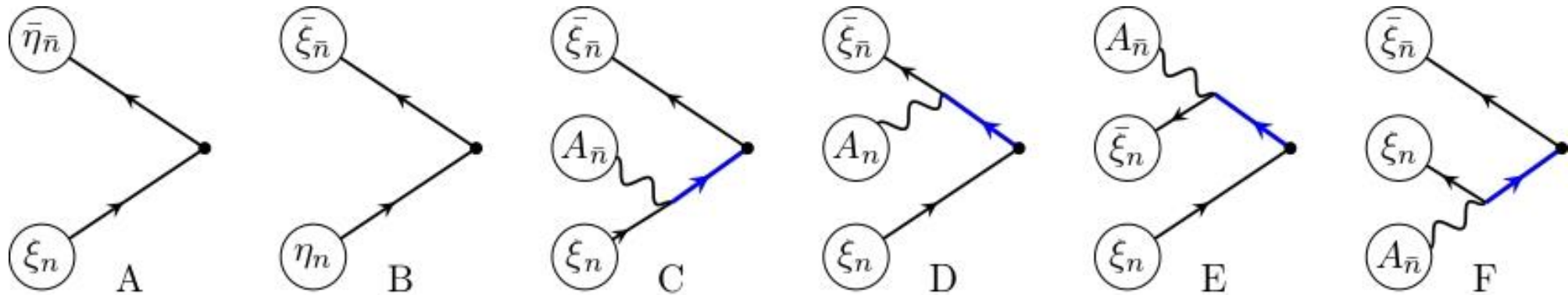
$$\phi(y) = \phi_{\bar{n}}(y) + \phi_n(y) + \psi(y)$$

$$\{\partial^+, \partial^-, \partial_T\} \phi_{\bar{n}} \lesssim Q \{1, \lambda^2, \lambda\} \phi_{\bar{n}} \quad \xi_{\bar{n}} \sim \lambda \quad \eta_{\bar{n}} \sim \lambda^2 \quad A_{\bar{n}}^\mu \sim \begin{cases} 1 & \text{if } \mu = + \\ \lambda^2 & \text{if } \mu = - \\ \lambda & \text{if } \mu = T \end{cases}$$

$$\{y^+, y^-, y_T\} \sim Q^{-1} \{1, 1, \lambda^{-1}\}$$

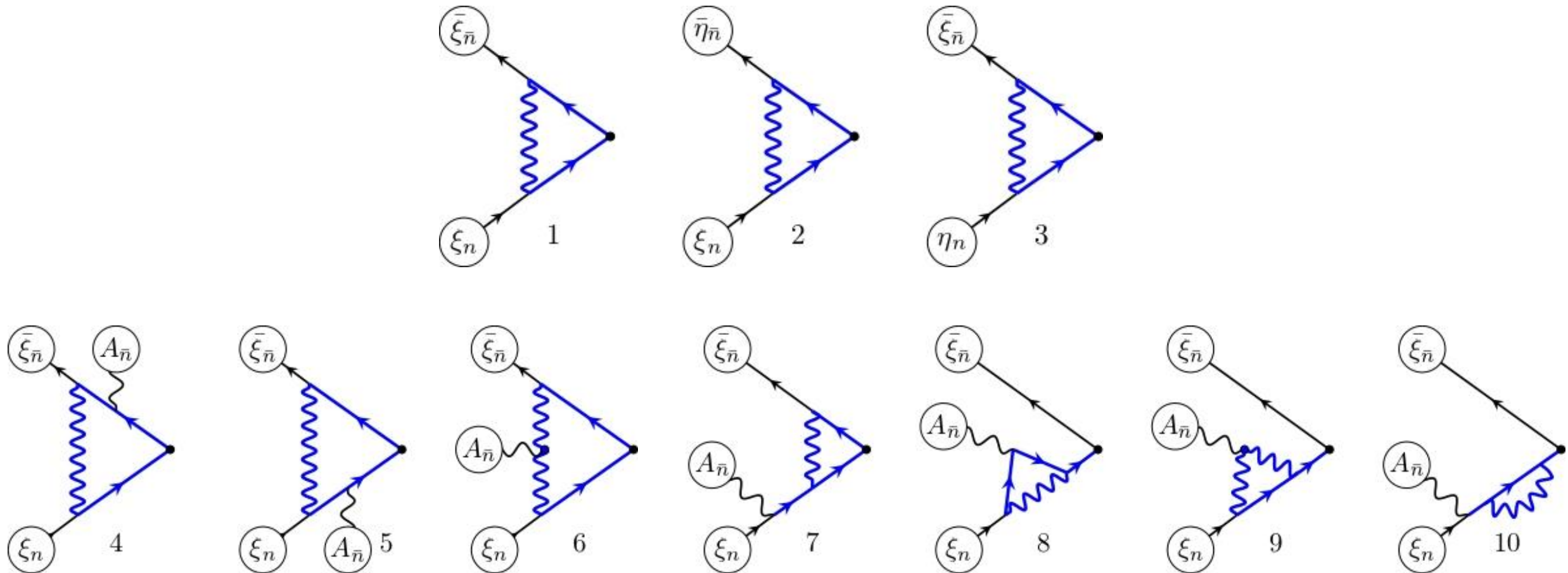
At tree level we can simply expand the currents

$$\begin{aligned}
 J^\mu[\bar{\psi} + \bar{q}_{\bar{n}} + \bar{q}_n, \dots] = & \bar{q}_{\bar{n}}\gamma^\mu q_n + \bar{q}_n\gamma^\mu q_{\bar{n}} \\
 & + \bar{\psi}\gamma^\mu\psi + \bar{q}_{\bar{n}}\gamma^\mu\psi + \bar{q}_n\gamma^\mu\psi + \bar{\psi}\gamma^\mu q_{\bar{n}} + \bar{\psi}\gamma^\mu q_n \\
 & + \bar{q}_{\bar{n}}\gamma^\mu q_{\bar{n}} + \bar{q}_n\gamma^\mu q_n
 \end{aligned}$$



## Second level: one-loop coefficient function

Exchange between the two currents is NNLP at least





$$\begin{aligned}
J^\mu(y) &= P^+ p_h^- \int dx d\tilde{x} e^{ixP^+ y^- + i\tilde{x}p_h^- y^+} C_1 J_{11}^\mu(x, \tilde{x}, y_T) \\
&+ (P^+)^2 p_h^- \int dx_1 dx_2 d\tilde{x} e^{i(x_1+x_2)P^+ y^- + i\tilde{x}p_h^- y^+} C_2(x_{1,2}) J_{21}^\mu(x_{1,2}, \tilde{x}, y_T) \\
&+ P^+ (p_h^-)^2 \int dx d\tilde{x}_1 d\tilde{x}_2 e^{ixP^+ y^- + i(\tilde{x}_1+\tilde{x}_2)p_h^- y^+} C_2(\tilde{x}_{1,2}) J_{12}^\mu(x, \tilde{x}_{2,1}, y_T)
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\end{aligned}$$

$$J_{21}(x_1, x_2, \tilde{x}) = \left( \frac{i\bar{n}^\mu}{p_h^- \tilde{x}} - \frac{in^\mu}{P_+(x_1 + x_2)} \right) \frac{\bar{U}_{2\bar{n},\rho}(x_1, x_2) \gamma_T^\rho U_{1,n}(\tilde{x}) - \bar{U}_{1n}(\tilde{x}) \gamma_T^\rho U_{2\bar{n},\rho}(x_2, x_1)}{x_2 - i0}$$

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$$\begin{aligned}
C_1(x, \tilde{x}) &= 2a_s C_F \frac{\Gamma(\varepsilon)\Gamma(-\varepsilon)\Gamma(2-\varepsilon)}{\Gamma(3-2\varepsilon)} \frac{2-\varepsilon+2\varepsilon^2}{(-2k^+k^- - i0)^\varepsilon} \\
C_2(x_1, x_2, \tilde{x}) &= 2a_s \frac{\Gamma(-\varepsilon)\Gamma(\varepsilon)\Gamma(1-\varepsilon)}{\Gamma(3-2\varepsilon)} \frac{1}{(-2k^+k^- - i0)^\varepsilon} \left\{ C_F(1-\varepsilon)^2(2-\varepsilon) \right. \\
&+ (C_F\varepsilon^2(1+\varepsilon) + C_A\varepsilon(1-\varepsilon-\varepsilon^2)) \frac{x_1+x_2}{x_1} \left( 1 - \left( \frac{x_1+x_2-i0}{x_2-i0} \right)^\varepsilon \right) \\
&\left. - 2 \left( C_F - \frac{C_A}{2} \right) (1-\varepsilon-\varepsilon^2) \frac{x_1+x_2}{x_2} \left( 1 - \left( \frac{x_1+x_2-i0}{x_1-i0} \right)^\varepsilon \right) \right\}
\end{aligned}$$

**Third level:** recombination and cancellation of divergences

Renormalize the semicompact operators

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Take the product of the two currents

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Fiertz transformation to obtain TMD operator

One obtains:

standard twist-(1,1) contributions

Kinematic corrections  $\sim$  derivatives of twist-(1,1)

Genuine corrections  $\sim$  quark-gluon-quark correlators

$$\begin{aligned}\Phi_{\mu,21}^{[\Gamma]}(x_1, x_2, x_3, b) &= \int_{-\infty}^{\infty} \frac{d\lambda_1 d\lambda_2}{(2\pi)^2} e^{i(x_1 \lambda_1 + x_2 \lambda_2)P^+} \langle P, s | \bar{U}_{\mu,2}(\lambda_1, \lambda_2; b) \frac{\Gamma}{2} U_1(0; 0) | P, s \rangle \\ \Phi_{\mu,12}^{[\Gamma]}(x_1, x_2, x_3, b) &= \int_{-\infty}^{\infty} \frac{d\lambda_1 d\lambda_2}{(2\pi)^2} e^{i(x_1 \lambda_1 + x_2 \lambda_2)P^+} \langle P, s | \bar{U}_1(\lambda_1; b) \frac{\Gamma}{2} U_{\mu,2}(\lambda_2, 0; 0) | P, s \rangle\end{aligned}$$

## quark-gluon-quark correlators

$\Phi_{12,21}$

Have simple operator definition but  
undefined T-parity and complexity

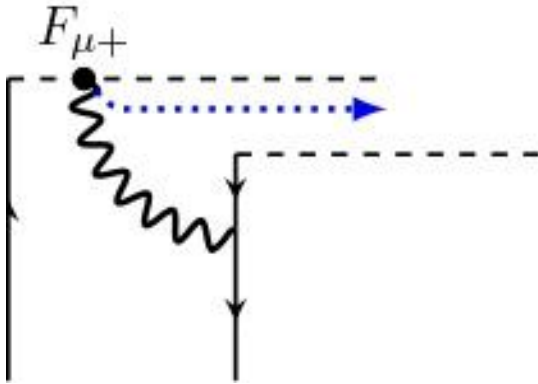


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$$\Phi_{\mu,12}^{[\Gamma]}(x_1, x_2, x_3, b) = \Phi_{\mu,12}^{[\Gamma]}(x_1, x_2, x_3, b) - [\mathcal{R}_{12} \otimes \Phi_{11}]_{\mu}^{[\Gamma]}(x_1, x_2, x_3, b)$$

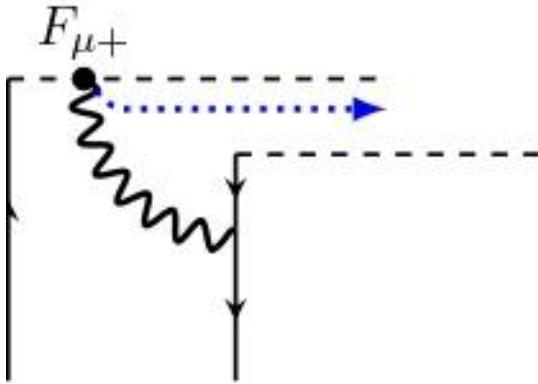
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SPDs are cancelled in between terms:

“ Fragmentation functions in the n sector – TMDPDF in the nB sector”

# Definite T-parity and complexity correlators

$$\Phi_{\mu,\oplus}^{[\Gamma]}(x_1, x_2, x_3; b) = \frac{\Phi_{\mu,21}^{[\Gamma]}(x_1, x_2, x_3; b) + \Phi_{\mu,12}^{[\Gamma]}(-x_3, -x_2, -x_1; b)}{2}$$

$$\Phi_{\mu,\ominus}^{[\Gamma]}(x_1, x_2, x_3; b) = i \frac{\Phi_{\mu,21}^{[\Gamma]}(x_1, x_2, x_3; b) - \Phi_{\mu,12}^{[\Gamma]}(-x_3, -x_2, -x_1; b)}{2}$$

$$\Phi_{\oplus}(x_1 < 0, x_2 < 0, x_3 > 0, k_T) \propto \text{Re} \left( \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right)$$

Diagram 1: Two black circular vertices connected by a vertical dashed line labeled  $P_X$ . The left vertex has an incoming horizontal arrow from the left labeled  $P$  and three outgoing arrows pointing down and to the right. The right vertex has an outgoing horizontal arrow to the right labeled  $P$  and three incoming arrows pointing down and to the left. A red arrow labeled  $x_3 P + k_T$  points from the left vertex to the right vertex. Two red arrows labeled  $x_1 P - k_1$  and  $x_2 P - k_2$  point from the right vertex to the left vertex. A box labeled  $k_1 + k_2 = k_T$  is in the bottom right.

$$\Phi_{\ominus}(x_1 < 0, x_2 < 0, x_3 > 0, k_T) \propto \text{Im} \left( \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right)$$

Diagram 2: Identical to Diagram 1, showing the same vertices, arrows, and labels, including the box  $k_1 + k_2 = k_T$ .

$$0 < z_3 < 1 \qquad z_{1,2} < 0 \qquad \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = 0$$

$$\Delta_{\oplus}(z_1, z_2, z_3, k_T) \propto \text{Re} \left( \begin{array}{c} \text{Diagram 1} \end{array} \right)$$

$$\Delta_{\ominus}(z_1, z_2, z_3, k_T) \propto \text{Im} \left( \begin{array}{c} \text{Diagram 2} \end{array} \right)$$

The diagrams show two complex conjugate processes separated by a vertical dashed line. In both, an incoming line from the left with momentum  $P/z_3 + k_T$  enters a black circular vertex. From this vertex, a line with momentum  $P$  goes up and to the right, and a multi-line bundle with momentum  $P_X$  goes down and to the right. On the right side of the dashed line, another black circular vertex receives the  $P$  line from the left and the  $P_X$  bundle from the left. From this vertex, a line with momentum  $P/z_1 - k_1$  goes up and to the right, and a wavy line with momentum  $P/z_2 - k_2$  goes down and to the right. A box at the bottom right of each diagram contains the equation  $k_1 + k_2 = k_T$ .

Using plus/minus correlators is also important because  
It reveals that the cross section is real (as it must be)

Not a trivial statement @NLP, because we have complex coefficient functions

$$\frac{C^\dagger(\hat{u}_1, \hat{u}_2)C_1}{u_2 - i0} = \mathbb{C}_R(x, \hat{u}_2) + i\pi\mathbb{C}_I(x, \hat{u}_2)$$

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For Fragmentation functions no imaginary part  
 because the momentum fractions have fixed signs!

$$C_2^\dagger \left( \frac{1}{\hat{w}_1}, \frac{1}{\hat{w}_2} \right) C_1 = \frac{\mathbb{C}_2(z, \hat{w}_2)}{\hat{w}_2} \quad 0 < z_3 < 1 \quad z_{1,2} < 0 \quad \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = 0$$

# Factorized expression for the hadronic tensor

$$W^{\mu\nu} = \int d\tilde{x} d\tilde{z} \delta\left(\tilde{x} + \frac{q_+}{P_+}\right) \delta\left(\tilde{z} - \frac{p_h^-}{q_-}\right) \int \frac{d^2b}{(2\pi)^2} e^{i(q_T b)} \frac{\tilde{z}}{2} \left[ \widetilde{W}_{\text{LP}}^{\mu\nu} + \widetilde{W}_{\text{kNLP}}^{\mu\nu} + \widetilde{W}_{\text{gNLP}}^{\mu\nu} \right]$$

$$\begin{aligned} \widetilde{\mathcal{W}}_{\text{kNLP}}^{\mu\nu}(y) = & -i|C_V(\mu^2, Q^2)|^2 \sum_{n,m} \left\{ \frac{\bar{n}^\mu \text{Tr}[\gamma^\rho \bar{\Gamma}_m^+ \gamma^\nu \bar{\Gamma}_n^-] + \bar{n}^\nu \text{Tr}[\gamma^\mu \bar{\Gamma}_m^+ \gamma^\rho \bar{\Gamma}_n^-]}{q_-} \Phi_{11}^{[\Gamma_n^+]} \left( \frac{\partial}{\partial b^\rho} - \frac{\partial_\rho \mathcal{D}}{2} \ln \left( \frac{\zeta}{\bar{\zeta}} \right) \right) \Delta_{11}^{[\Gamma_m^-]} \right. \\ & \left. + \frac{n^\mu \text{Tr}[\gamma^\rho \bar{\Gamma}_m^+ \gamma^\nu \bar{\Gamma}_n^-] + n^\nu \text{Tr}[\gamma^\mu \bar{\Gamma}_m^+ \gamma^\rho \bar{\Gamma}_n^-]}{q_+} \Delta_{n11}^{[\Gamma_m^-]} \left( \frac{\partial}{\partial b^\rho} + \frac{\partial_\rho \mathcal{D}}{2} \ln \left( \frac{\zeta}{\bar{\zeta}} \right) \right) \Phi_{\bar{n}11}^{[\Gamma_n^+]} \right\} + \text{antiquark} \end{aligned}$$

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$$\begin{aligned} \widetilde{\mathcal{W}}_{\text{kNLP}}^{\mu\nu}(y) = & -i|C_V(\mu^2, Q^2)|^2 \sum_{n,m} \left\{ \frac{\bar{n}^\mu \text{Tr}[\gamma^\rho \bar{\Gamma}_m^+ \gamma^\nu \bar{\Gamma}_n^-] + \bar{n}^\nu \text{Tr}[\gamma^\mu \bar{\Gamma}_m^+ \gamma^\rho \bar{\Gamma}_n^-]}{q_-} \Phi_{11}^{[\Gamma_n^+]} \left( \frac{\partial}{\partial b^\rho} - \boxed{\frac{\partial_\rho \mathcal{D}}{2} \ln \left( \frac{\zeta}{\bar{\zeta}} \right)} \right) \Delta_{11}^{[\Gamma_m^-]} \right. \\ & \left. + \frac{n^\mu \text{Tr}[\gamma^\rho \bar{\Gamma}_m^+ \gamma^\nu \bar{\Gamma}_n^-] + n^\nu \text{Tr}[\gamma^\mu \bar{\Gamma}_m^+ \gamma^\rho \bar{\Gamma}_n^-]}{q_+} \Delta_{n11}^{[\Gamma_m^-]} \left( \frac{\partial}{\partial b^\rho} + \boxed{\frac{\partial_\rho \mathcal{D}}{2} \ln \left( \frac{\zeta}{\bar{\zeta}} \right)} \right) \Phi_{\bar{n}11}^{[\Gamma_n^+]} \right\} + \text{antiquark} \end{aligned}$$

Residue of the cancellation of special rapidity divergences.  
Ensure boost-invariance!



$$\begin{aligned}
\widetilde{W}_{\text{gNLP}} = & i \sum_{n,m} \left\{ \int [d\hat{u}] \delta(\tilde{x} - \hat{u}_3) \left[ \right. \\
& T_-^{\mu\nu\rho}(\bar{n}, n) \left( \mathbb{C}_R(x, \hat{u}_2) \boldsymbol{\Phi}_{\rho, \oplus}^{[\Gamma_n^+]} \Delta_{11}^{[\Gamma_m^-]} + \pi \mathbb{C}_I(x, \hat{u}_2) \boldsymbol{\Phi}_{\rho, \ominus}^{[\Gamma_n^+]} \Delta_{11}^{[\Gamma_m^-]} \right) \\
& + iT_+^{\mu\nu\rho}(\bar{n}, n) \left( \pi \mathbb{C}_I(x, \hat{u}_2) \boldsymbol{\Phi}_{\rho, \oplus}^{[\Gamma_n^+]} \Delta_{11}^{[\Gamma_m^-]} - \mathbb{C}_R(x, \hat{u}_2) \boldsymbol{\Phi}_{\rho, \ominus}^{[\Gamma_n^+]} \Delta_{11}^{[\Gamma_m^-]} \right) \\
& + T_-^{\mu\nu\rho}(n, \bar{n}) \left( \mathbb{C}_R(x, \hat{u}_2) \overline{\boldsymbol{\Phi}}_{\rho, \oplus}^{[\Gamma_n^+]} \overline{\Delta}_{11}^{[\Gamma_m^-]} + \pi \mathbb{C}_I(x, \hat{u}_2) \overline{\boldsymbol{\Phi}}_{\rho, \ominus}^{[\Gamma_n^+]} \overline{\Delta}_{11}^{[\Gamma_m^-]} \right) \\
& \left. + iT_+^{\mu\nu\rho}(n, \bar{n}) \left( \pi \mathbb{C}_I(x, \hat{u}_2) \overline{\boldsymbol{\Phi}}_{\rho, \oplus}^{[\Gamma_n^+]} \overline{\Delta}_{11}^{[\Gamma_m^-]} - \mathbb{C}_R(x, \hat{u}_2) \overline{\boldsymbol{\Phi}}_{\rho, \ominus}^{[\Gamma_n^+]} \overline{\Delta}_{11}^{[\Gamma_m^-]} \right) \right] \\
& + \int \frac{[d\hat{w}]}{|\hat{w}_1|} \delta(\tilde{z} - \hat{w}_3) \left[ \right. \\
& T_-^{\mu\nu\rho}(\bar{n}, n) \mathbb{C}_2(z, \hat{w}_2) \Phi_{11}^{[\Gamma_n^+]} \boldsymbol{\Delta}_{\rho, \oplus}^{[\Gamma_m^-]} - iT_+^{\mu\nu\rho}(\bar{n}, n) \mathbb{C}_2(z, \hat{w}_2) \Phi_{11}^{[\Gamma_n^+]} \boldsymbol{\Delta}_{\rho, \ominus}^{[\Gamma_m^-]} \\
& \left. + T_-^{\mu\nu\rho}(n, \bar{n}) \mathbb{C}_2(z, \hat{w}_2) \overline{\Phi}_{11}^{[\Gamma_n^+]} \overline{\boldsymbol{\Delta}}_{\rho, \oplus}^{[\Gamma_m^-]} - iT_+^{\mu\nu\rho}(n, \bar{n}) \mathbb{C}_2(z, \hat{w}_2) \overline{\Phi}_{11}^{[\Gamma_n^+]} \overline{\boldsymbol{\Delta}}_{\rho, \ominus}^{[\Gamma_m^-]} \right] \left. \right\}
\end{aligned}$$

One example: Cahn effect  $F_{UU}^{\cos \phi}$

$$\begin{aligned}
& |C_V|^2 \left\{ \frac{|p_{h,\perp}|}{Qz} J_0[f_1 D_1] + \frac{|p_{h,\perp}|}{Qz} J_2[M m_h h_1^\perp H_1^\perp] \right. \\
& + \frac{2M}{Q} J_1 \left[ M \dot{f}_1 D_1 - M f_1 \dot{D}_1 + M \dot{\mathcal{D}} \log \frac{\zeta}{\bar{\zeta}} \right] - \frac{2M}{Q} J_1 \left[ M^2 m_h b^2 \dot{h}_1^\perp H_1^\perp - M^2 m_h b^2 h_1^\perp \dot{H}_1^\perp + M^2 m_h b^2 \dot{\mathcal{D}} \log \frac{\zeta}{\bar{\zeta}} \right] \Big\} \\
& - \mathbb{C}_2 \frac{2m_h}{Q} J_1 [m_h f_1 (F_\oplus^\perp + G_\ominus^\perp) + 2M H_\ominus h_1^\perp] \\
& + \mathbb{C}_R \frac{2M}{Q} J_1 [M D_1 (f_\ominus^\perp - g_\oplus^\perp) + 2m_h h_\ominus H_1^\perp] \\
& - \mathbb{C}_I \frac{2M}{Q} J_1 [M D_1 (f_\oplus^\perp + g_\ominus^\perp) + 2m_h h_\oplus H_1^\perp]
\end{aligned}$$

# One example: Cahn effect $F_{UU}^{\cos \phi}$

$$\begin{aligned}
 |C_V|^2 & \left\{ \frac{|p_{h,\perp}|}{Qz} J_0[f_1 D_1] + \frac{|p_{h,\perp}|}{Qz} J_2[M m_h h_1^\perp H_1^\perp] \right. \\
 & + \frac{2M}{Q} J_1 \left[ M \dot{f}_1 D_1 - M f_1 \dot{D}_1 + M \dot{\mathcal{D}} \log \frac{\zeta}{\bar{\zeta}} \right] - \frac{2M}{Q} J_1 \left[ M^2 m_h b^2 \dot{h}_1^\perp H_1^\perp - M^2 m_h b^2 h_1^\perp \dot{H}_1^\perp + M^2 m_h b^2 \dot{\mathcal{D}} \log \frac{\zeta}{\bar{\zeta}} \right] \Big\} \\
 & - \mathbb{C}_2 \frac{2m_h}{Q} J_1 [m_h f_1 (F_\oplus^\perp + G_\ominus^\perp) + 2M H_\ominus h_1^\perp] \\
 & + \mathbb{C}_R \frac{2M}{Q} J_1 [M D_1 (f_\ominus^\perp - g_\oplus^\perp) + 2m_h h_\ominus H_1^\perp] \\
 & - \mathbb{C}_I \frac{2M}{Q} J_1 [M D_1 (f_\oplus^\perp + g_\ominus^\perp) + 2m_h h_\oplus H_1^\perp] \Big]
 \end{aligned}$$

All these combinations have  
Tree-level matching to twist-4 PDFs

At one loop  $f_\ominus^\perp - g_\oplus^\perp$   
matches to twist-2 PDF

# Conclusions

Full factorization theorem @ NLP/NLO for SIDIS

Emergence and cancellation of special rapidity divergences,  
restoration of boost-invariance

Definite T-parity correlators as real/imaginary part of  
1->2 and 2->1 partonic interference processes

One-loop results for all LP and NLP structure functions

Only NNLP structure functions:  $F_{UU,L}$   $F_{UT,L}^{\sin \phi - \phi_S}$