UNIVERSITÀ DEGLI STUDI DI GENOVA

INFN

Next-to-Leading Order virtual correction to Higgs-induced DIS

QCD Evolution Workshop 2023

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Small-x resummation:
How does it work?

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- Resummation $\Longleftrightarrow$ Factorization

We need some factorisation properties

- Mellin Transform

$$g(N,Q^2) = \int_0^1 dx\, x^N g\left(x,Q^2\right) \qquad \ln^k(x) \rightarrow \frac{1}{N^{k+1}}$$

Collinear factorization theorem

$$\sigma(N,Q^2) = \sum_{i=q,g} C_i\left(N, \alpha_s(Q^2)\right) f_i\left(N, Q^2\right)$$

Coefficient function

Parton distribution function (PDF)

$\ln(x)$

Our goal: resum NLL terms in the coefficient function

High energy factorization theorem

$$\sigma(N, Q^2) = \int_0^\infty dk_\perp^2\, \mathscr{C}\left(N, k_\perp^2, Q^2, \alpha_s\right)\, \mathscr{F}_g\left(N, k_\perp^2\right)$$

Off-shell coefficient function

Unintegrated PDF

$$\mathscr{F}_g\left(N, k_\perp^2\right) = \mathscr{U}\left(N, k_\perp^2, Q^2\right) f_g(N, Q^2)$$

$$\boxed{C_g(N, \alpha_s) = \int_0^\infty dk_\perp^2\, \mathscr{C}\left(N, k_\perp^2, Q^2, \alpha_s\right)\, \mathscr{U}\left(N, k_\perp^2, Q^2\right)}$$

Bonvini, Marzani and Peraro (**2016**)

Coefficient function

We want to resum NLL terms in the coefficient function

$$C_g(N,\alpha_s) = \int_0^\infty dk_\perp^2\, \mathscr{C}\left(N, k_\perp^2, Q^2, \alpha_s\right)\, \mathscr{U}\left(N, k_\perp^2, Q^2\right)$$

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We have to compute the one-loop off-shell coefficient function

Higgs induced DIS

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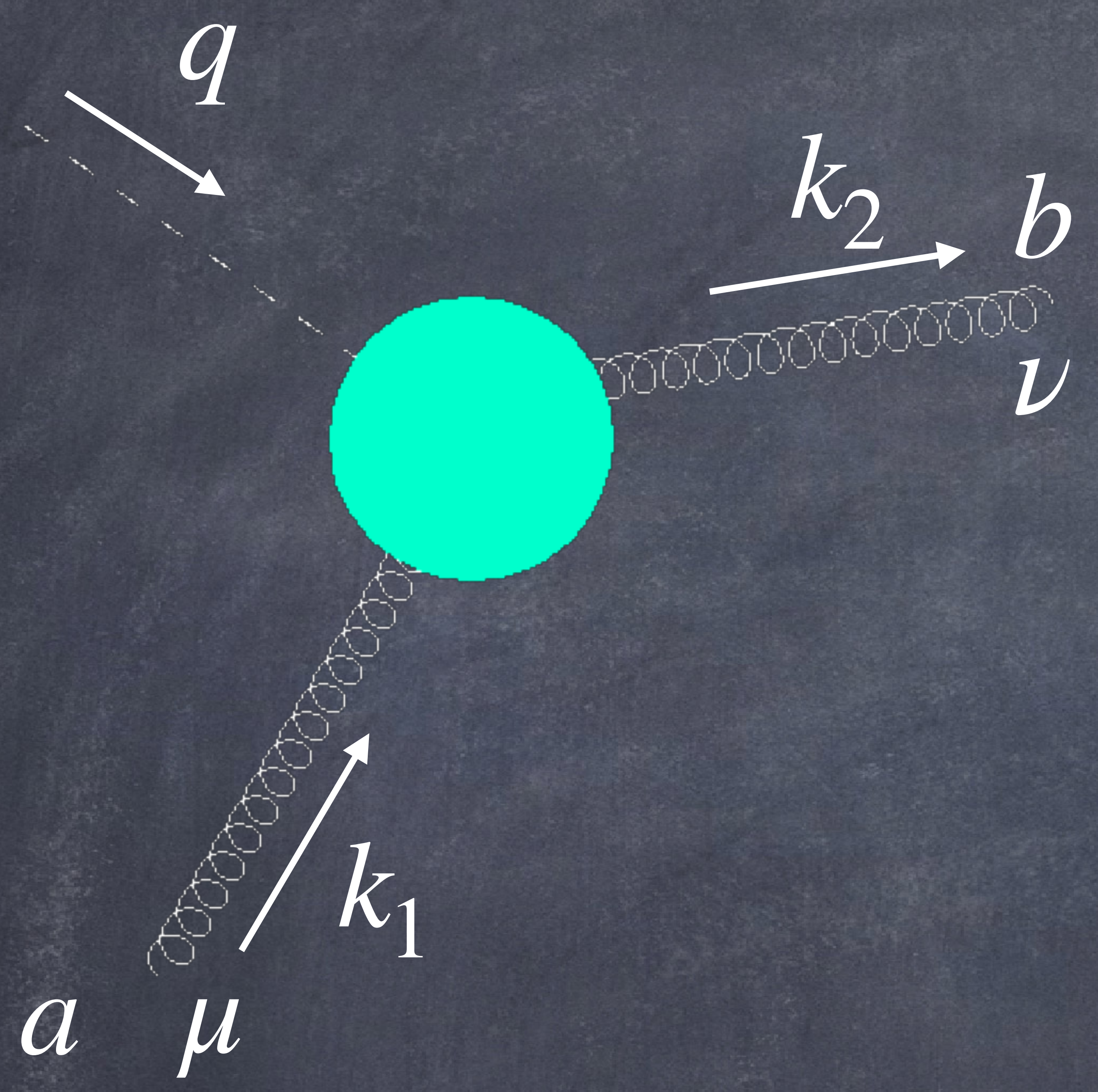


- $n_f = 0$

- Higgs gluon effective vertex:

$$M^{\mu\nu} = i\, c\, \delta_a^b \left[k_2^\mu k_1^\nu - g^{\mu\nu} k_1 \cdot k_2 \right]$$

- Off-shell coefficient function

$$k_1^2 = -\vec{k}_1^{\,2}$$

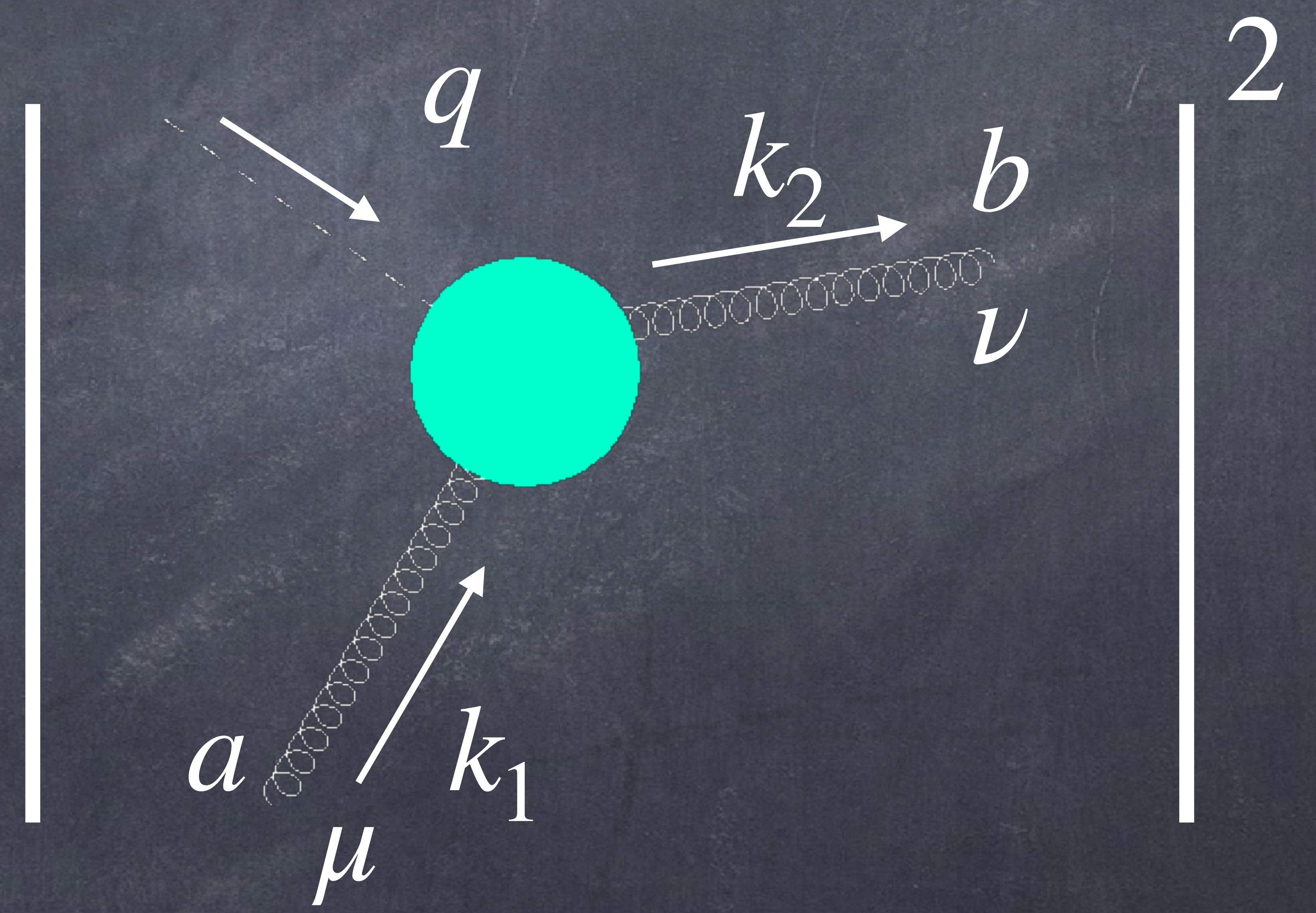
Key points

1. We have to work in axial gauge: $A \cdot n = 0$

The off-shell coefficient function is free from logs if we work in axial gauge

Catani and Hautmann (**1994**)

2. We have to understand the "sum over polarisation" of an off-shell gluon at NLL



Axial gauge

- Growing number of terms due to gauge choice

$$\Pi^{\mu\nu}_{a,b}(k,n) = i\,\delta_{a,b}\left[\frac{-g^{\mu\nu}}{k^2} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k^2\,k\cdot n}\right]$$

- Non covariant loop integrals

$$I = \int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2(k-k_1)^2(k-k_2)^2(k\cdot n)}$$

- Spurious poles

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Non covariant loop integrals

$$I_n = \int \frac{d^d k}{(2\pi)^d} \frac{f(k\cdot n)}{\underbrace{D_1 D_2 \ldots D_n}_{\text{Covariant denominators}}}$$

Non-covariant part:

$$\frac{1}{(k\cdot n)}$$

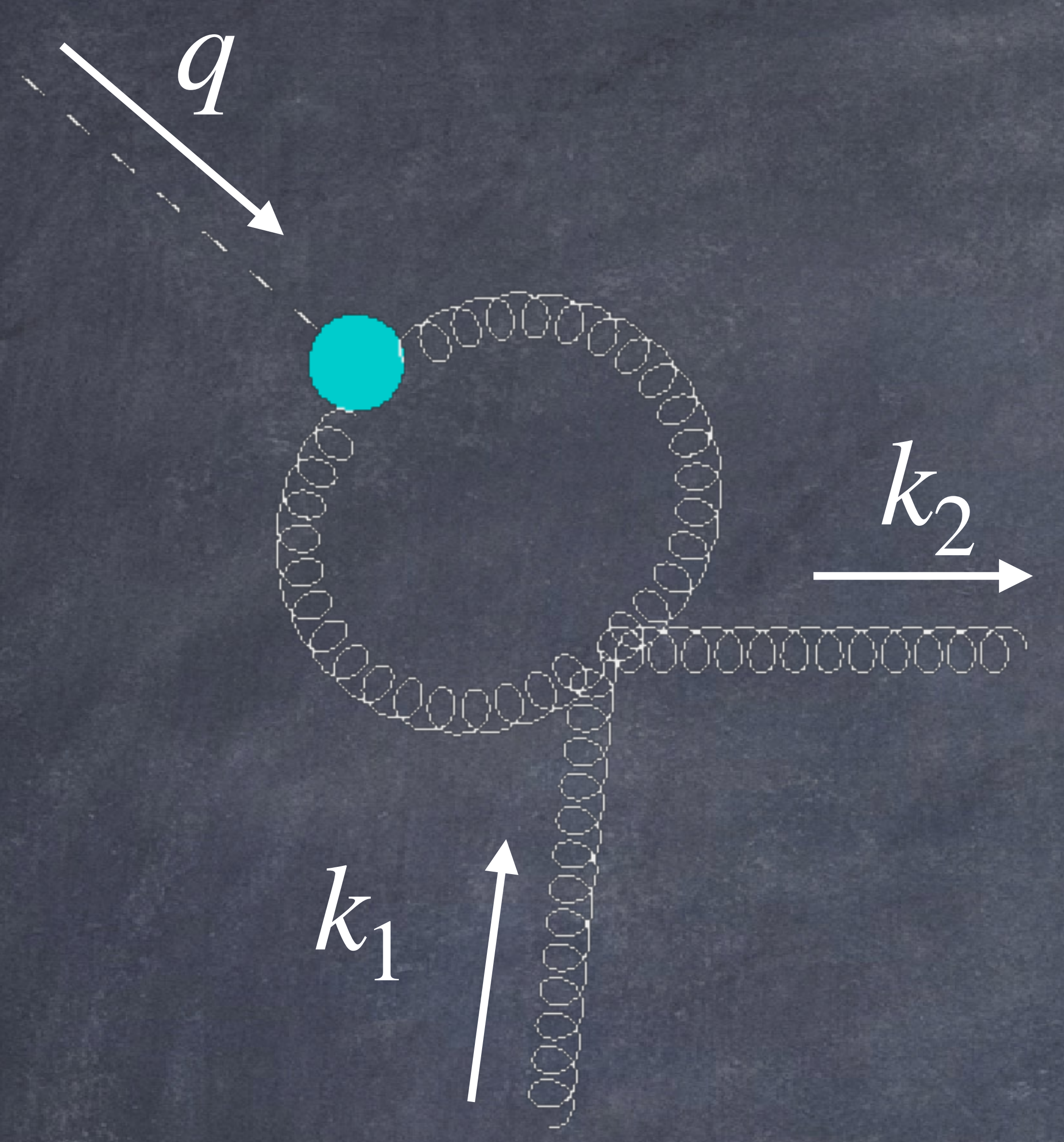
$$\downarrow$$

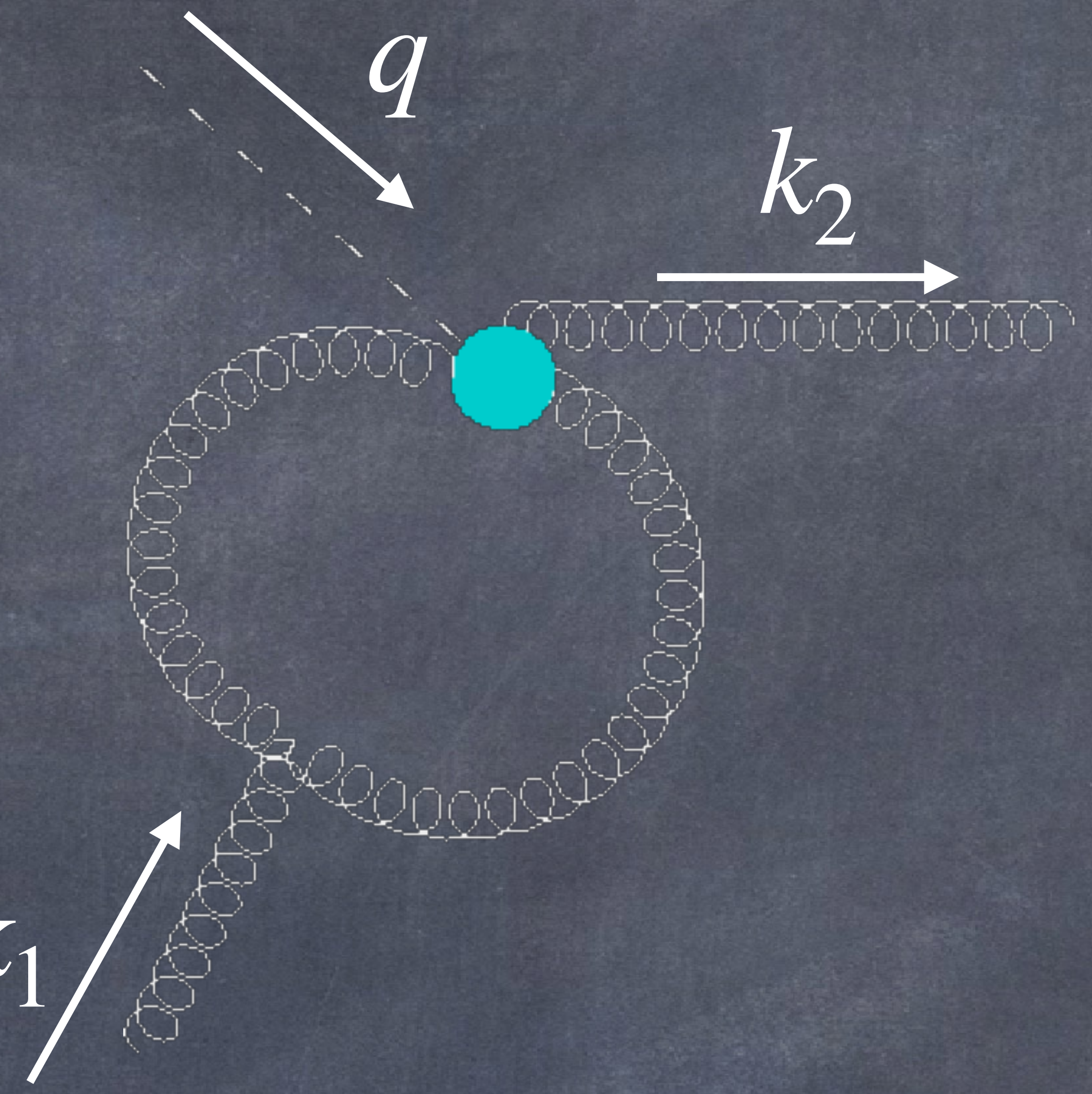
Principal value prescription

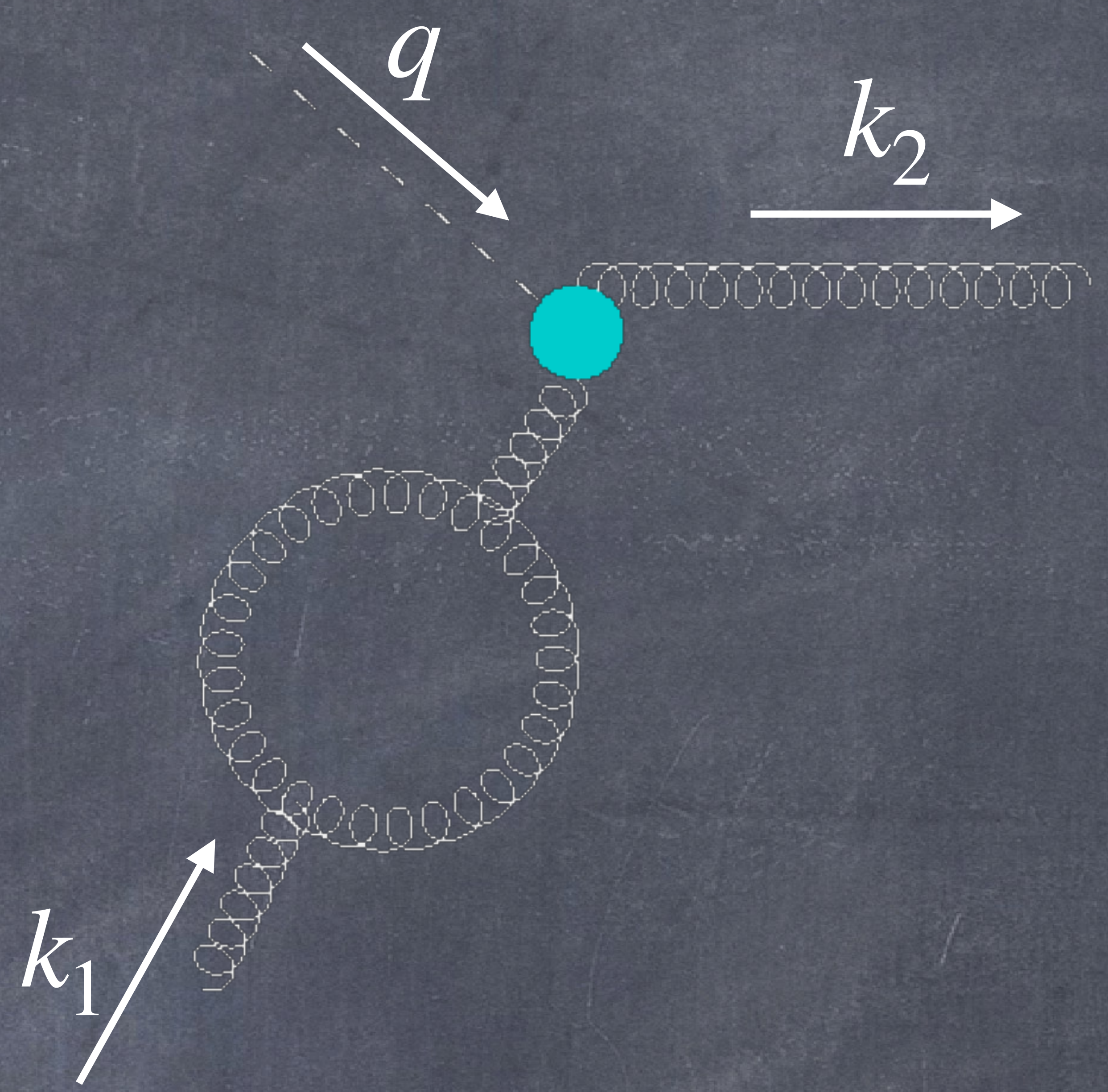
$$\frac{1}{(k\cdot n)} \to \frac{k\cdot n}{(k\cdot n)^2 + \delta^2}$$

Curci, Furmanski and Petronzio (**1980**)

Non covariant loop integrals



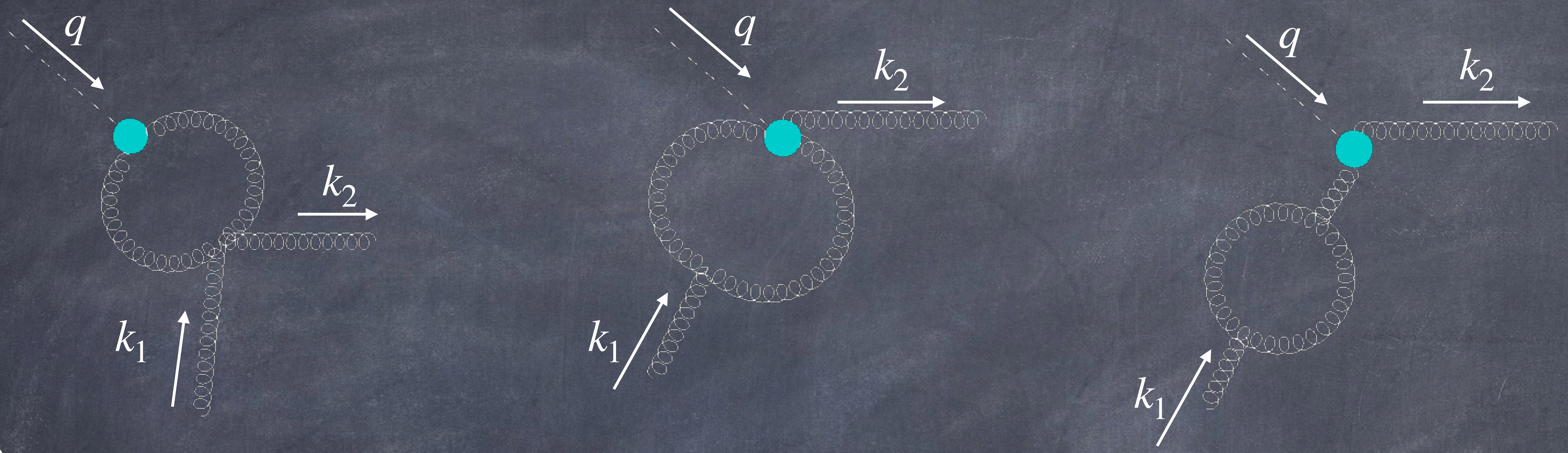




$$k_1^2 \neq 0$$

$$I_2 = \int \frac{d^d k}{(2\pi)^d} \frac{f\left(k \cdot n\right)}{k^2 (k-l)^2}$$

Non covariant loop integrals

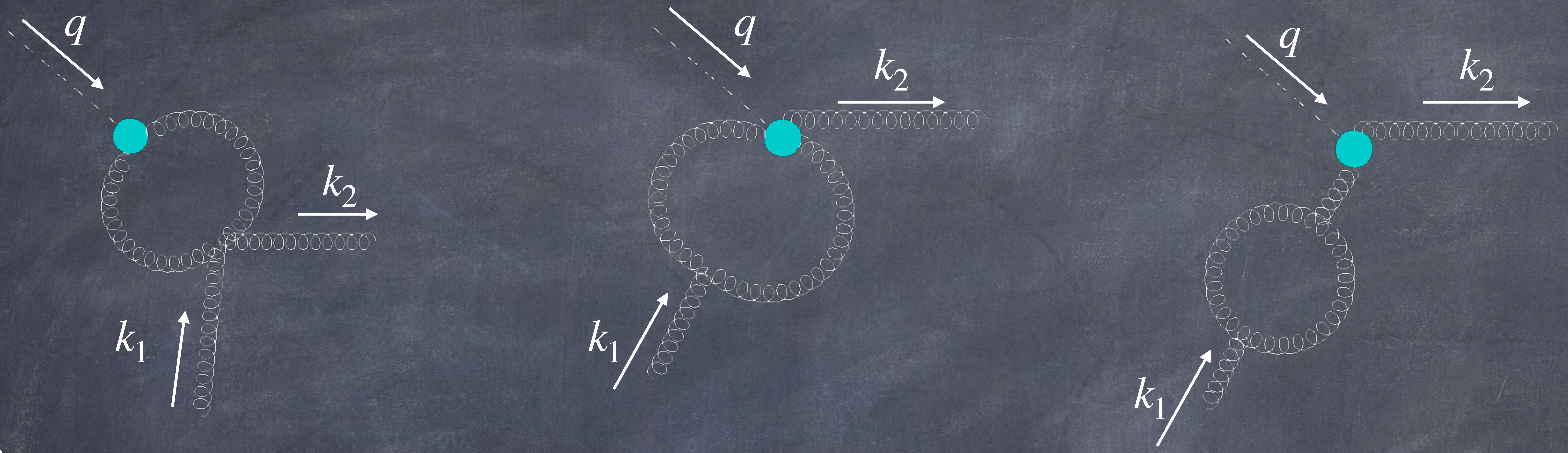


$k_1^2 \neq 0$

$$I_2 = \int \frac{d^dk}{(2\pi)^d} \frac{f\left(k \cdot n\right)}{k^2(k-l)^2} = \frac{i}{16\pi^2}\left(\frac{4\pi}{-l^2}\right)^{\epsilon} \Gamma(\epsilon) \int_0^1 dz\, f\left(l_+ z\right) z^{-\epsilon}(1-z)^{-\epsilon}$$

$$l_+ = l \cdot n$$

Non covariant loop integrals



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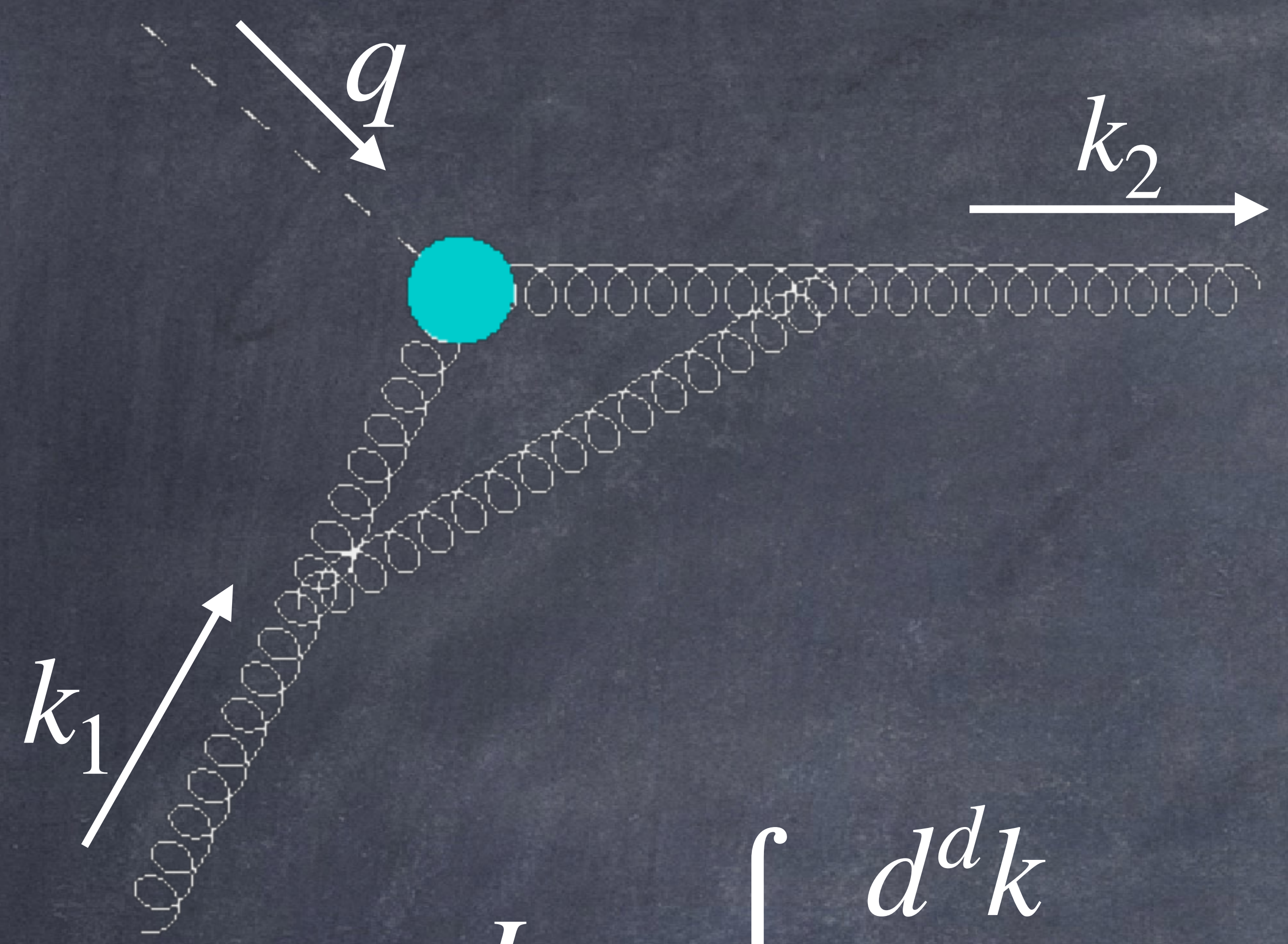

$$x = \frac{k_2 \cdot n}{k_1 \cdot n} \qquad b = -\frac{k_1^2}{2k_1 \cdot k_2}$$

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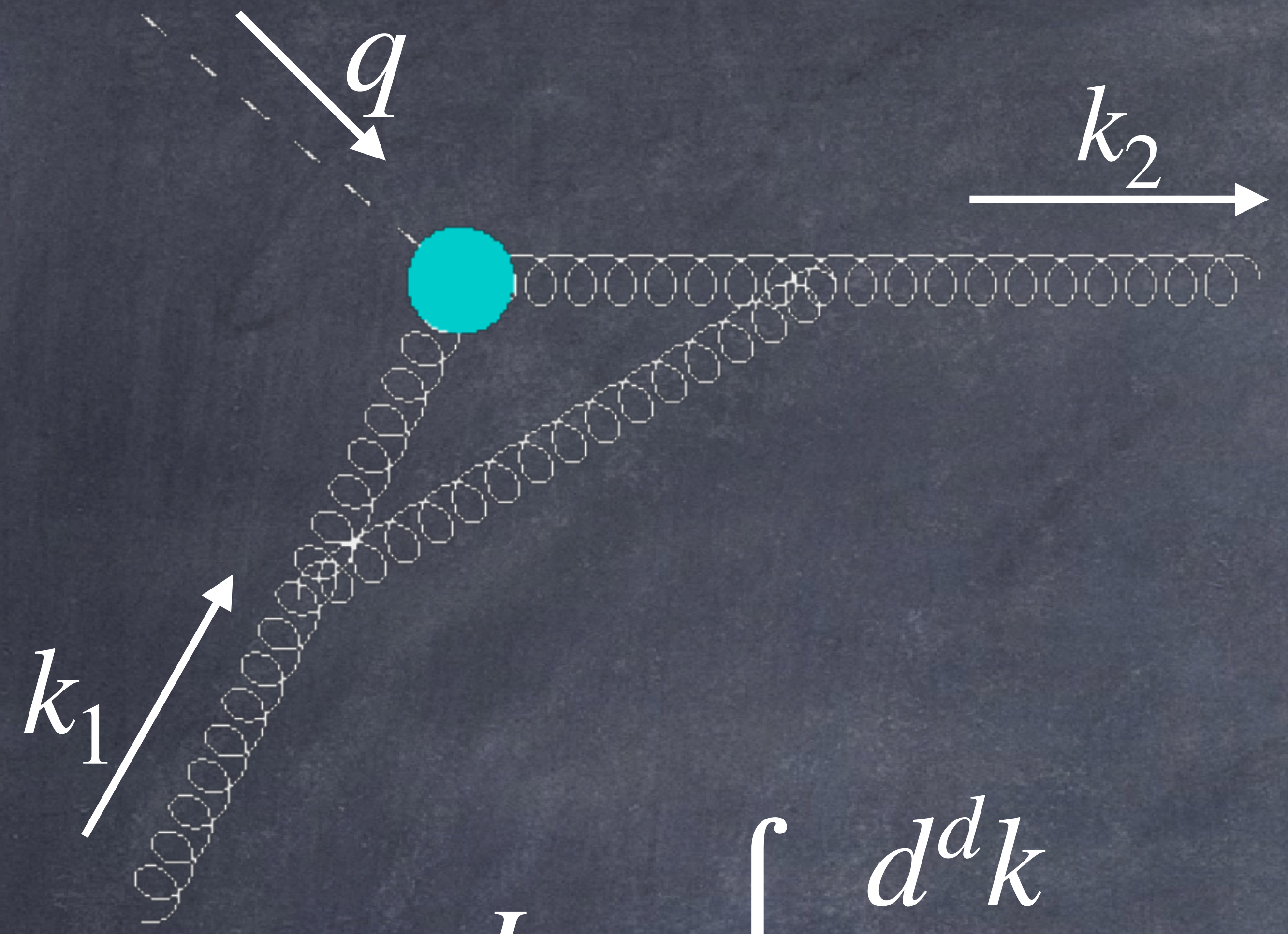
Non covariant loop integrals



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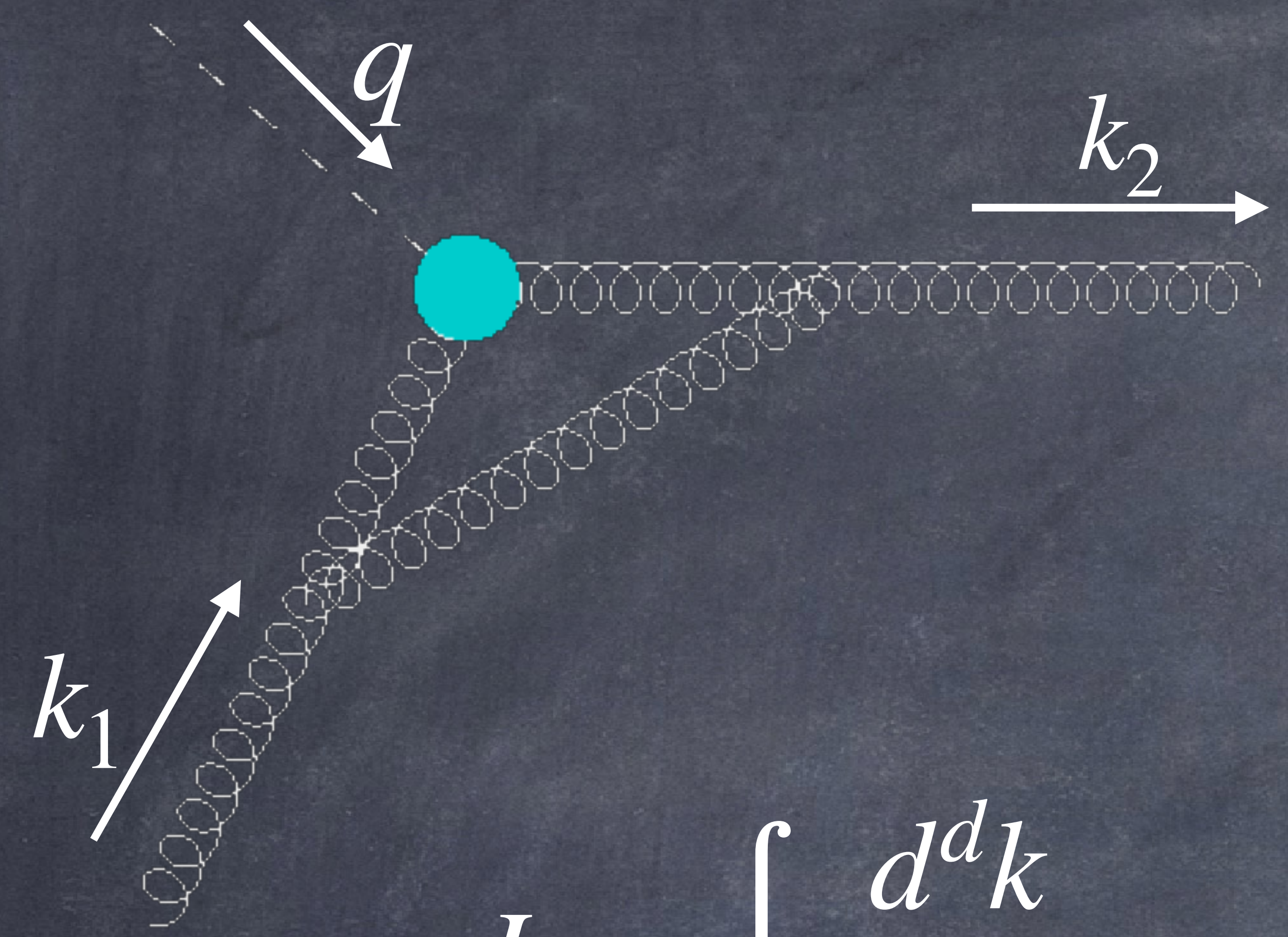
$$\left[x^{\epsilon} \int_0^1 dz\, f\left(l_+ z\right)\; z^{-\epsilon} (b\,x - z)^{-1-\epsilon}\, {}_2F_1\left(1+\epsilon, -\epsilon; 1-\epsilon; \frac{(b\,x-1)z}{b\,x - z}\right)\right.$$

$$\left. -x^{\epsilon}(1-x)^{\epsilon} \int_x^1 dy\, f\left(l_+ y\right) (y-x)^{-\epsilon} (b\,x - y)^{-1-\epsilon}\, {}_2F_1\left(1+\epsilon, -\epsilon; 1-\epsilon; \frac{(y-x)(b\,x-1)}{(b\,x-y)(1-x)}\right)\right]$$

Non covariant loop integrals



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Axial gauge

- Growing number of terms due to gauge choice ✓

$$\Pi^{\mu\nu}_{a,b}(k,n) = i\,\delta_{a,b}\left[\frac{-g^{\mu\nu}}{k^2} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k^2\,k\cdot n}\right]$$

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- Spurious poles

Spurious poles

$$I_2 = \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2(k-l)^2} \frac{1}{k \cdot n}$$

Power counting: UV finite

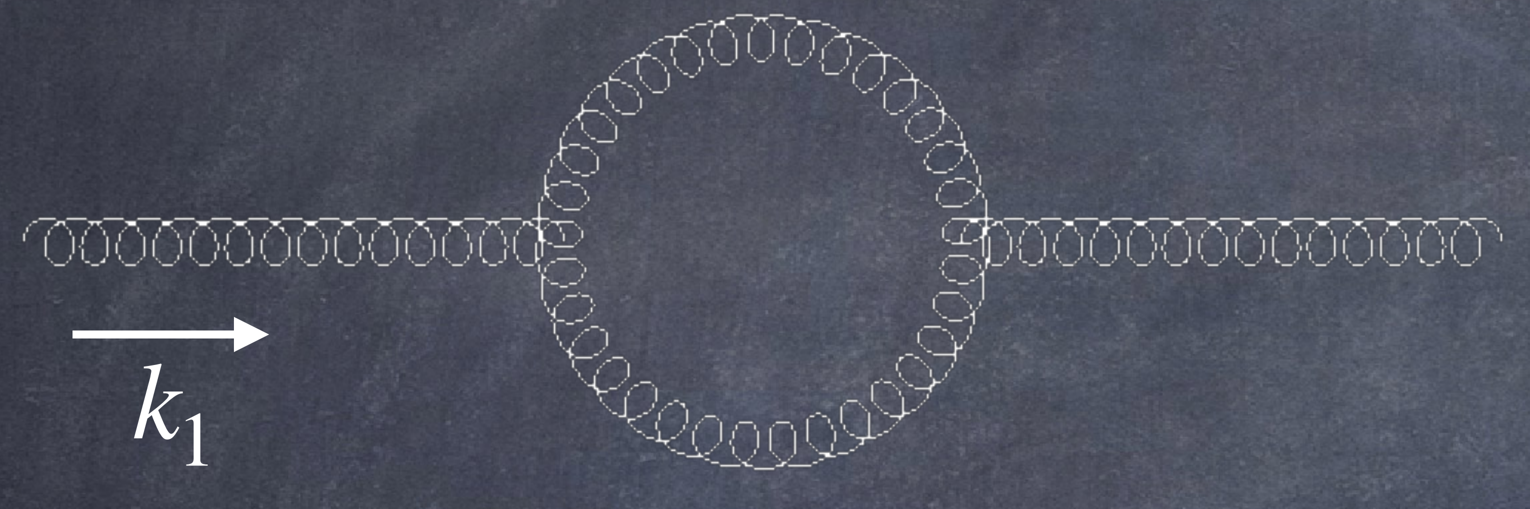
$$\sim \Gamma(\epsilon) \int_0^1 dz\, f\left(l_+ z\right) z^{-\epsilon}(1-z)^{-\epsilon}$$

UV spurious pole

Define gauge-dependent counterterms that cancel all the UV poles

Pritchard and Stirling (**1979**)
Curci, Furmanski and Petronzio (**1980**)

Counterterms in axial gauge: gluon propagator



$n_f = 0$

Feynman gauge:

$$\Pi^{\mu\nu}(k_1, n) = \mathrm{i}\frac{\alpha_s}{4\pi}\frac{C_A\delta_{a,b}}{\epsilon}Z_A\left(k_1^\mu k_1^\nu - k_1^2 g^{\mu\nu}\right)$$

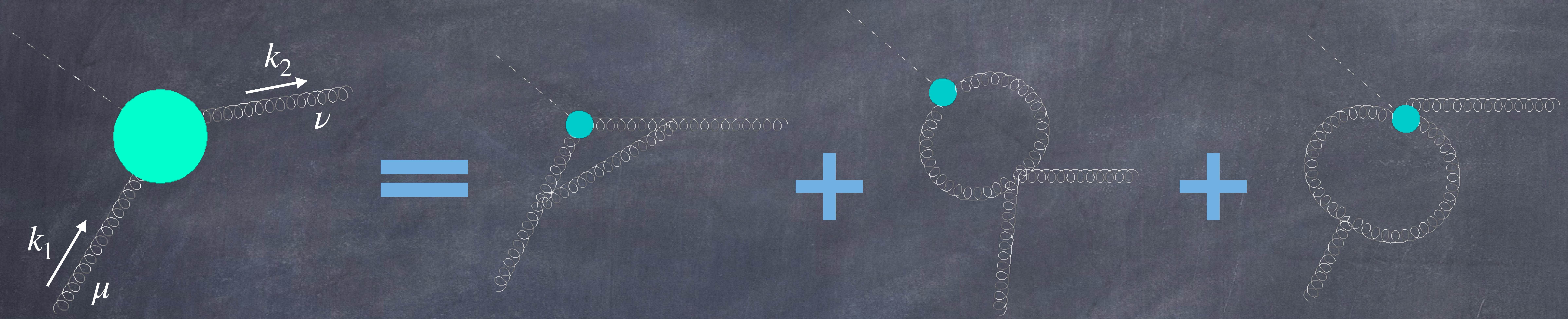
$$I_0 = \int_0^1 \frac{du\, u}{u^2+\delta^2} = -\ln(\delta)$$

Axial gauge:

$$\Pi^{\mu\nu}(k_1, n) = -\mathrm{i}\frac{\alpha_s}{4\pi}\frac{C_A\delta_{a,b}}{\epsilon}\Bigg[\left(\frac{11}{3} - 4I_0\right)\left(k_1^\mu k_1^\nu - k_1^2 g^{\mu\nu}\right)$$

$$-4\left(1 - I_0\right)\left(k_1^\mu k_1^\nu - \frac{k_1^2}{k_1\cdot n}(k_1^\mu n^\nu + k_1^\nu n^\mu) + \frac{k_1^4}{(k_1\cdot n)^2}n^\mu n^\nu\right)\Bigg]$$

Counterterms in axial gauge: effective vertex



$$\mathrm{CT}^{\mu\nu}(k_1,k_2,n) = -\left[D_1^{\mu\nu}(k_1,k_2,n)\,|_{UV} + D_2^{\mu\nu}(k_1,k_2,n)\,|_{UV} + D_3^{\mu\nu}(k_1,k_2,n)\,|_{UV}\right]$$

$k_1^2 \neq 0$

$k_2^2 \neq 0$

$$D_1^{\mu\nu}\,|_{UV} = C_{k_1k_1}(k_1,k_2,n)k_1^\mu k_1^\nu + C_{k_2k_2}(k_1,k_2,n)k_2^\mu k_2^\nu$$
$$+C_{k_1k_2}(k_1,k_2,n)k_1^\mu k_2^\nu + C_{k_2k_1}(k_1,k_2,n)k_2^\mu k_1^\nu$$
$$+C_g(k_1,k_2,n)g^{\mu\nu} + \ldots$$

Axial gauge

- Growing number of terms due to gauge choice ✓

$$\Pi^{\mu\nu}_{a,b}(k,n) = i\,\delta_{a,b}\left[\frac{-g^{\mu\nu}}{k^2} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k^2\, k\cdot n}\right]$$

- Non covariant loop integrals ✓

$$I = \int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2(k-k_1)^2(k-k_2)^2(k\cdot n)}$$

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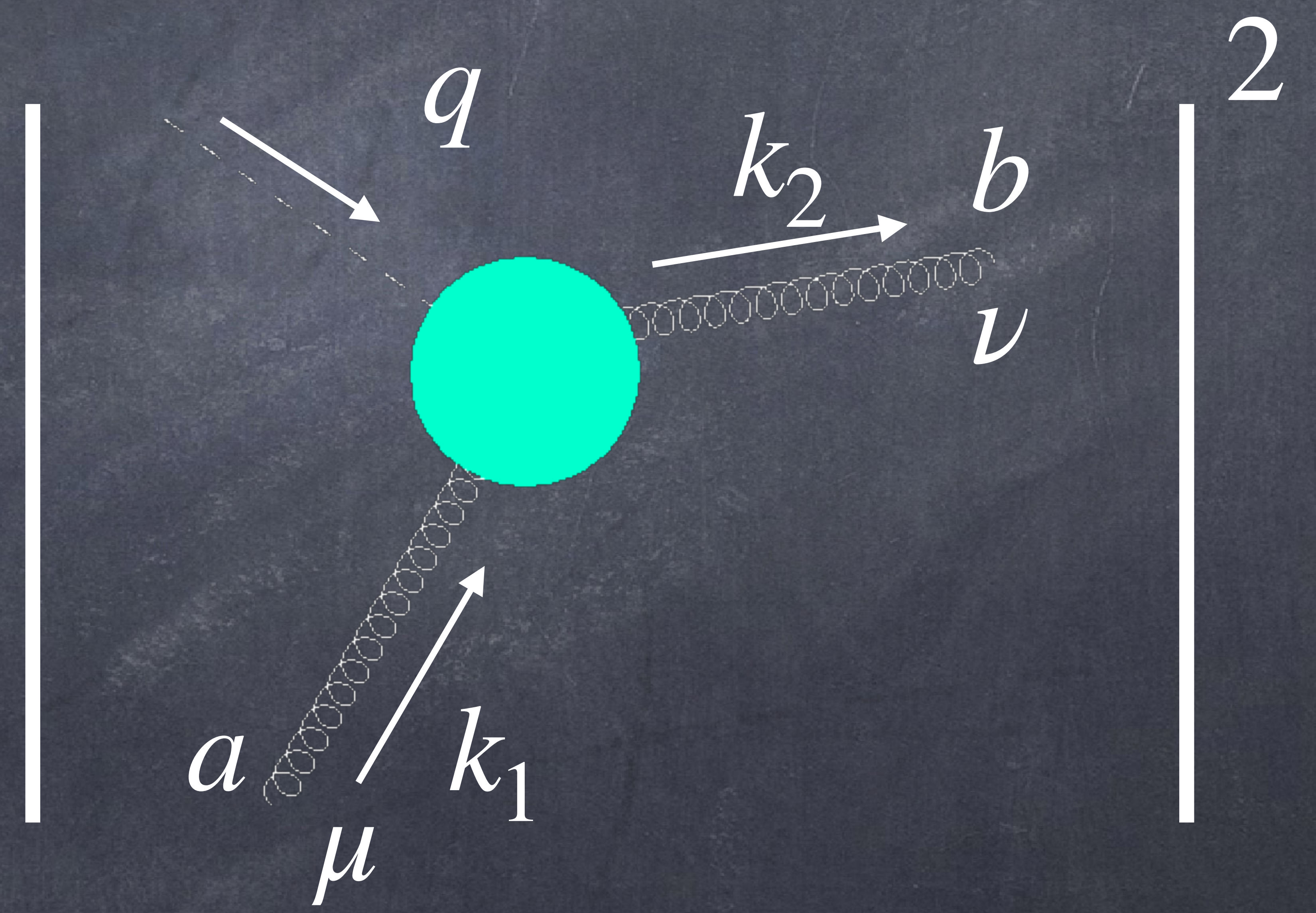

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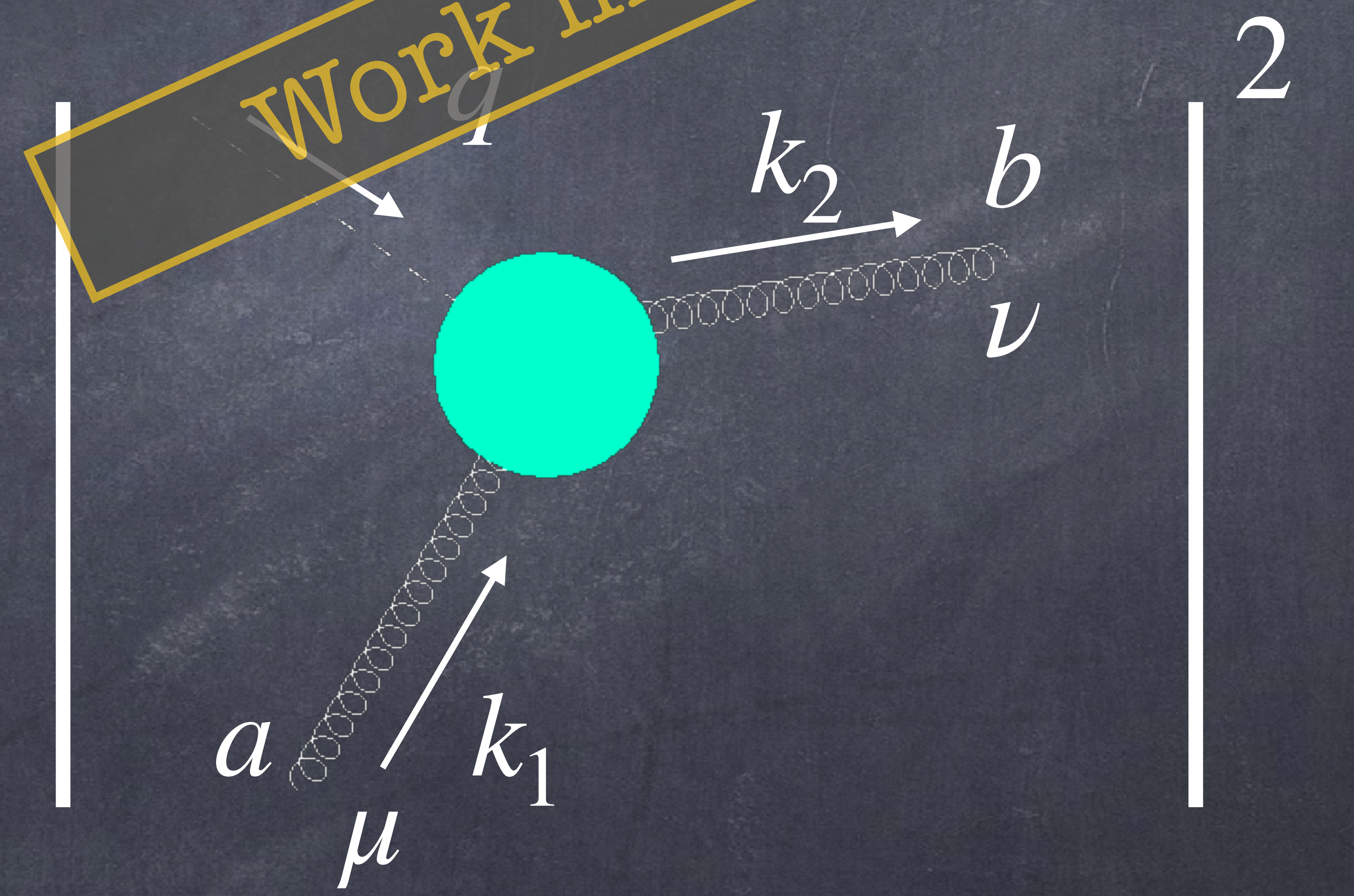
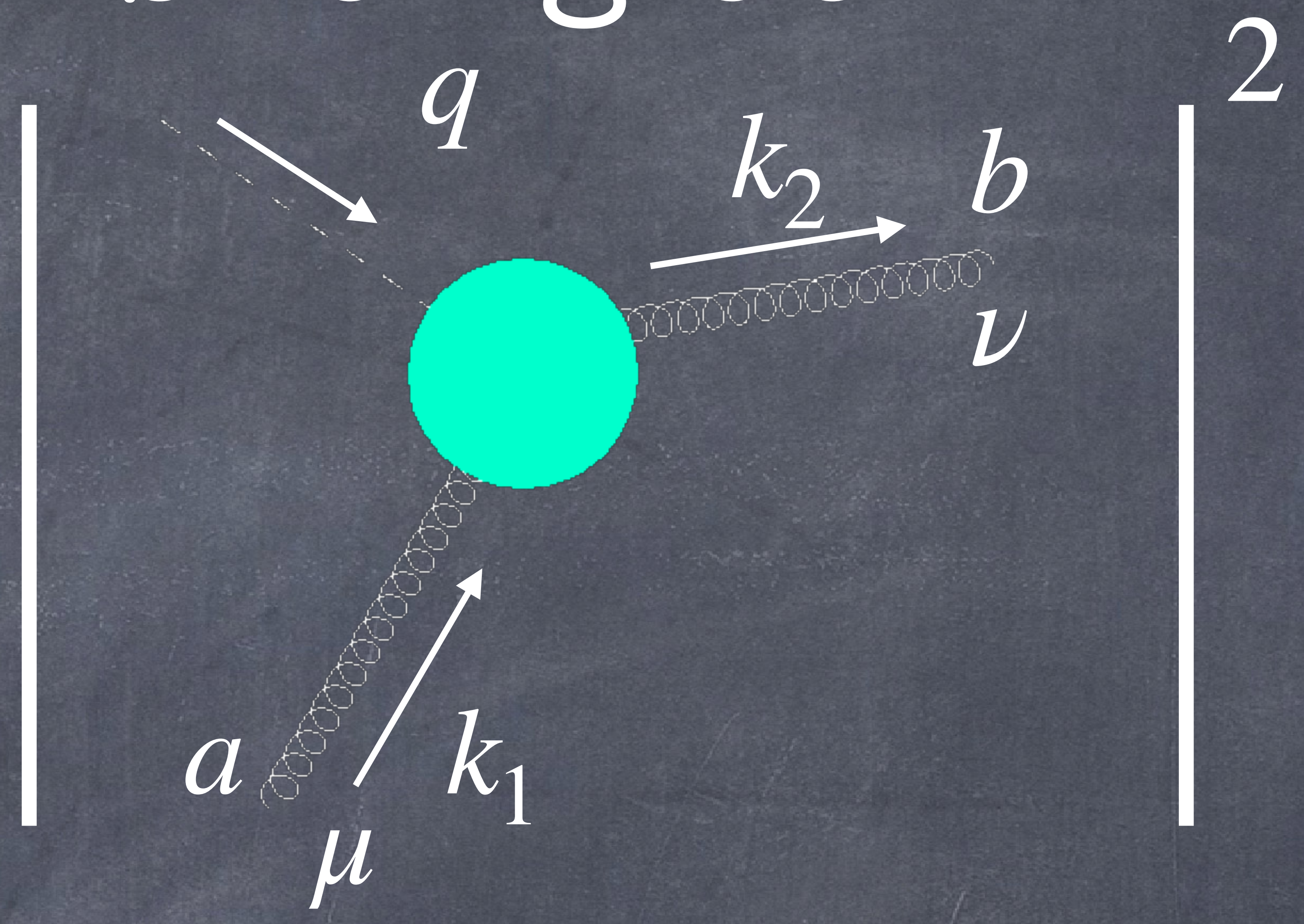


Sum over polarisation of an off-shell gluon

$$k_1^\mu = k^\mu + k_\perp^\mu$$
$$k_1^2 = k_\perp^2$$

$$d_{CH}^{\mu\nu} = (d-2)\frac{k_\perp^\mu k_\perp^\nu}{\vec{k}_\perp^{\,2}}$$



1. $d_{\mathrm{CH}}^{\mu\nu}$ selects the dominant part of the amplitude in the leading logarithm approximation

2. $\lim_{\vec{k}_\perp^{\,2} \to 0} \langle d_{CH}^{\mu\nu} \rangle = g_\perp^{\mu\nu}$

Catani and Hautmann (**1994**)

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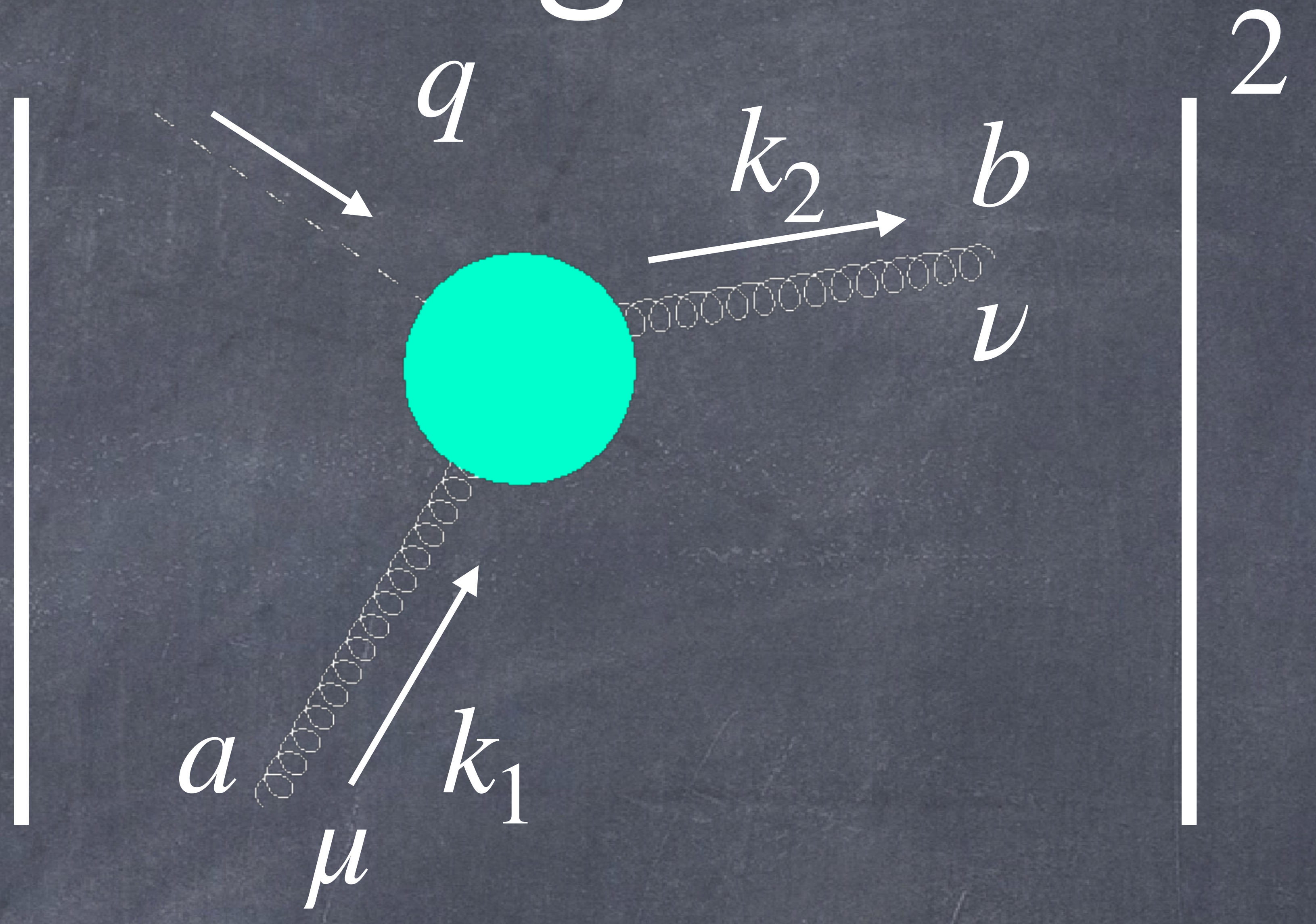

1. $d_{\mathrm{CH}}^{\mu\nu}$ selects the dominant part of the amplitude in the leading logarithm approximation

Still true at NLL?

2. $\lim_{\vec{k}_\perp^{\,2}\to 0} \langle d_{CH}^{\mu\nu} \rangle = g_\perp^{\mu\nu}$

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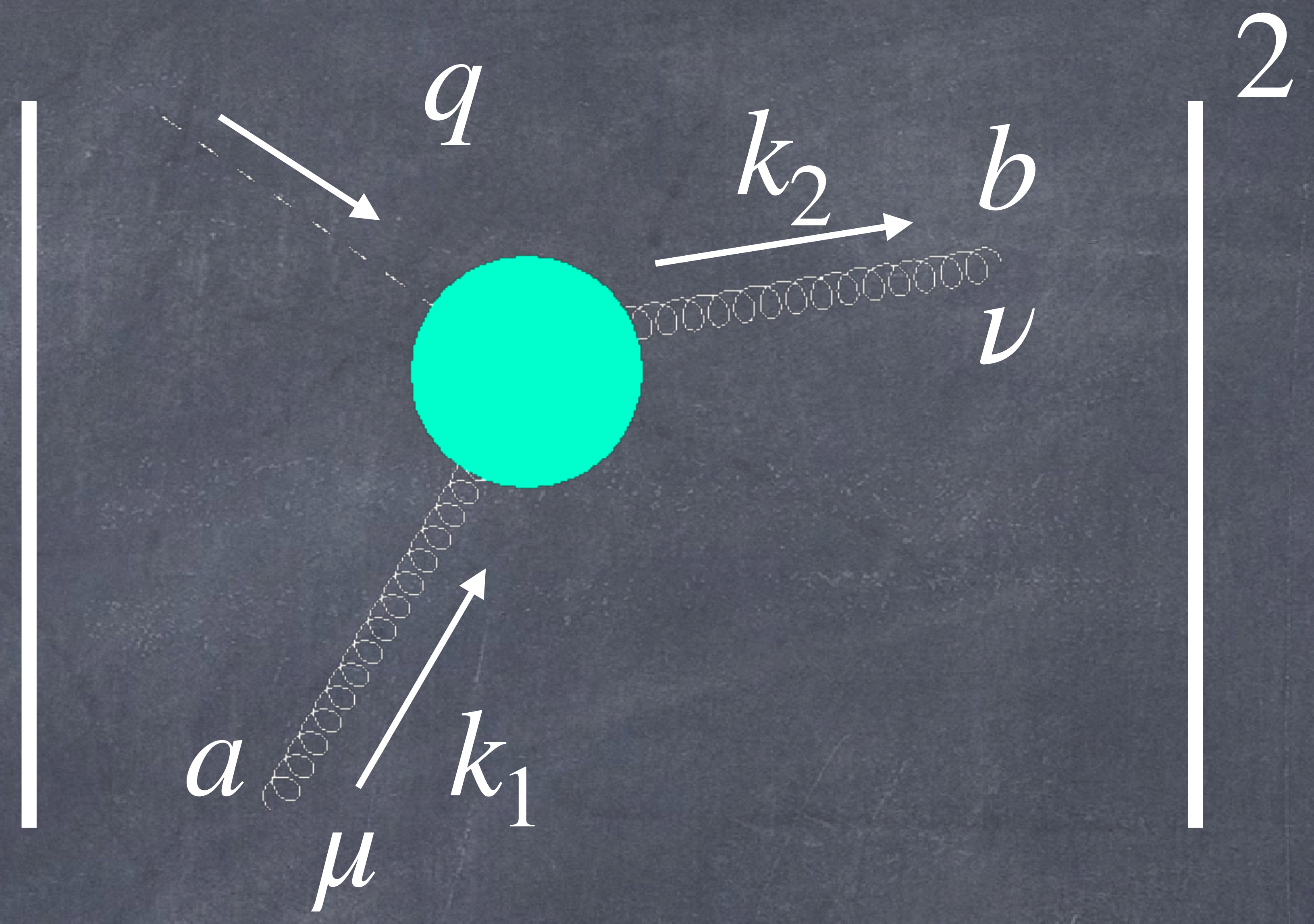
Virtual one-loop coefficient function

$$|M|^2 = \mathscr{M}^{\mu\nu}(k_1, k_2, n)\, d_{\mu\nu}(k_1, n)$$



Off-shell gluon emitted by a quark:

$$d_{\mu\nu}(k_1, n) = -g_{\mu\nu} + \frac{k_{1\,\mu} n_\nu + k_{1\,\nu} n_\mu}{k_1 \cdot n}$$

$$\mathscr{C}^{(1)} = -\frac{2\alpha_s^3 \sqrt{2} G_F Q^2}{3\,\pi^3}\left[\frac{\ln(\xi) - \ln(1+\xi)}{\epsilon}\frac{\left(\xi^2+2\xi+2\right)\left(\xi^3-\xi^2+\xi-1\right)}{4(\xi+1)^2} + \frac{\xi\left(2\xi^3-2\xi-1\right)}{(\xi+1)^3}\ln^2(\delta) + \frac{\xi^3\ln(1+\xi)}{2(\xi+1)^2}\ln(\delta) + \ldots\right]$$

$$\xi = \frac{\vec{k}_\perp^{\,2}}{Q^2}$$

AR arXiv:2212.02959

Conclusions

Where are we?

$$C_g(N, \alpha_s) = \int_0^\infty dk_\perp^2 \, \mathscr{C}\left(N, k_\perp^2, Q^2, \alpha_s\right) \mathscr{U}\left(N, k_\perp^2, Q^2\right)$$

- Solved main issues due to the choice of axial gauge
- Virtual contribution ✓
- Real contribution
- Cross checks $\boxed{\text{Work in progress}}$

Thank you!