From Amplitude Evolution to NLL Parton Showers

Jack Holguin

In collaboration with Jeff Forshaw, Simon Plätzer.

Context

Parton Showers (PS) provide the backbone to analyses of high-energy collider experiments.

Recent years have seen a, mostly, concerted effort to systematically improve the accuracy of PS algorithms.

Progress in improving the PS accuracy

- **Assessing the logarithmic accuracy of a shower**
  Herwig [1904.11866, 2107.04051], Deductor [2011.04777], Forshaw, Holguin, Plätzer [2003.06400]
  PanScales [1805.09327, 2002.11114], Alaric [2110.05964], …
- **Triple collinear / double soft splittings**
  Dulat, Höche, Krauss, Gellersen, Prestel [1705.00982, 1705.00742, 1805.03757, 2110.05964]
  Li & Skands [1611.00013], Löschner, Plätzer, Simpson Dore [2112.14454], …
- Matching to fixed-order **see Alexander's talk**
  NLO; i.e. Frixione & Webber [0204244], Nason [0409146], …
  NNLO; i.e. UNNLOPS [1407.3773], MiNNLOps [1908.06987], Vincia [2108.07133], …
  NNNLO; Prestel [2106.03206], Bertone, Prestel [2202.01082]
- Colour (and spin) correlations **see Simon's talk**
  Forshaw, Holguin, Plätzer, Sjödahl [1201.0260, 1808.00332, 1905.08686, 2007.09648, 2011.15087]
  Deductor [0706.0017, 1401.6364, 1501.00778, 1902.02105], Herwig [1807.01955], Plätzer & Ruffa [2012.15215]
  PanScales [2011.10054, 2103.16526, 2111.01161], …
- Electroweak corrections
  Vincia [2002.09248, 2108.10786], Pythia [1401.5238], Herwig [2108.10817], …

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Disclaimer: list is not exhaustive

M. van Beekveld, Parton Showers for future e+e- colliders, CERN, 2023.

Context

Parton Showers (PS) provide the backbone to analyses of high-energy collider experiments.

Recent years have seen a, mostly, concerted effort to systematically improve the accuracy of PS algorithms.

In this talk, I will summarise the contributions I've been involved with and which are gradually being implemented in Herwig 7 and the CVolver library.

Outline

1. Amplitude Evolution

   De Angelis, Forshaw, J.H, Löschner, Martínez, Plätzer, Ruffa, Seymour, Sjödahl [1201.0260, 1802.08531, 1905.08686, 2007.09648, 2011.15087, 2012.15215, 2109.03665, 2112.13124, 2112.14454, 2204.06956]

2. From Amplitude Evolution to LC NLL Parton Showers

   Forshaw, J.H, Plätzer [2003.06400]

3. From Amplitude Evolution to sub-LC soft physics

   Forshaw, J.H, Plätzer [1808.00332, 2003.06399, 2011.15087, 2112.13124]

Amplitude Evolution Summarised

- Key to the formalism is the usage of amplitude level factorisation theorems, i.e. soft factorisation

$$\underbrace{|M_N\rangle}_{\text{Amplitude vector in a rep of } (SU(3)\times E(2))^N} = |\mathcal{M}(\underbrace{q_1,\ldots,q_N}_{\text{Soft}},\underbrace{p_1,\ldots p_n}_{\text{Hard}})\rangle \longrightarrow |M_N\rangle \simeq \underbrace{(g\mu^\epsilon)^N}_{\text{Leading poles}} \underbrace{\mathbf{J}(q_N)\cdots\mathbf{J}(q_1)}_{\text{Soft currents}}|M_0\rangle$$

$$\langle M_N|M_N\rangle = \sum_{\text{colours}} |M_N|^2$$

- Similar factorisations exist for collinear particles and the IR divergences from loops.

Amplitude Evolution Summarised

- We use these factorisation theorems to iteratively dress amplitudes with more radiation.
  - Important to this is the interference between amplitudes and the conjugate amplitude – this means density matrices are the fundamental object.

$$|M_N\rangle\langle M_N| = \mathbf{A}_n$$

$$\mathbf{A}_n(q_\perp;\{\tilde{p}\}_{n-1}\cup q_n) = \int \prod_{i=1}^{n_\mathrm{H}+n} \mathrm{d}^4 p_i \mathbf{V}_{q_\perp,q_{n\,\perp}} \mathbf{D}_n \mathbf{A}_{n-1}(q_{n\,\perp};\{p\}_{n-1})\mathbf{D}_n^\dagger \mathbf{V}^\dagger_{q_\perp,q_{n\,\perp}} \Theta(q_\perp \leq q_{n\,\perp}).$$

Integrate out momentum from before the latest emission.

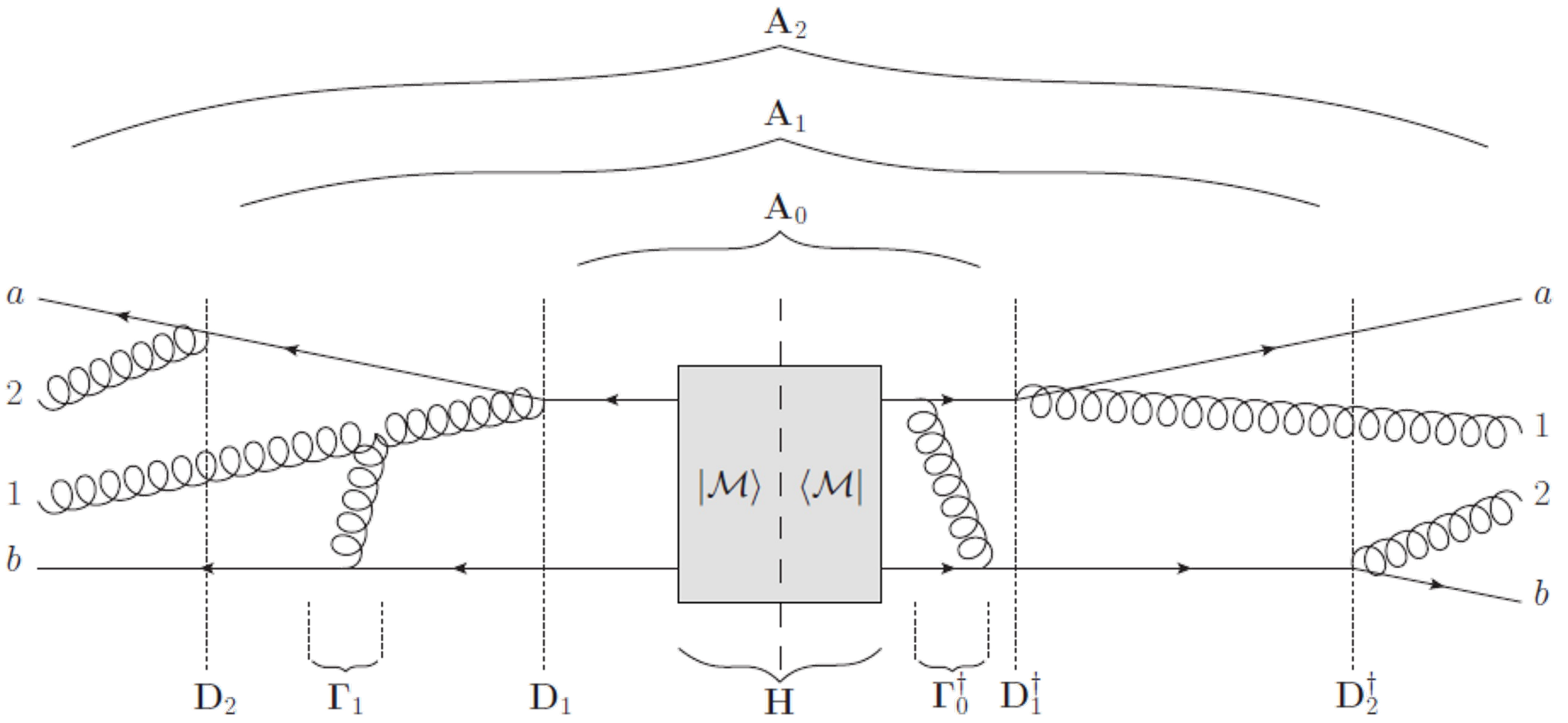
Operator for the addition of one real soft or collinear parton.

Exponentiated loops (anomalous dimensions). Related to $\mathbf{D}_n$ by the KLN theorem.

$$\mathbf{V}_{a,b} = \mathrm{Pexp}\left(-\int_a^b \frac{\mathrm{d}q_\perp}{q_\perp}\mathbf{\Gamma}_n(q_\perp)\right)$$

Amplitude Evolution Summarised

$$\mathbf{A}_n(q_\perp;\{\tilde{p}\}_{n-1}\cup q_n) = \int \prod_{i=1}^{n_{\rm H}+n} \mathrm{d}^4 p_i \mathbf{V}_{q_\perp,q_{n\perp}} \mathbf{D}_n \mathbf{A}_{n-1}(q_{n\perp};\{p\}_{n-1}) \mathbf{D}_n^\dagger \mathbf{V}^\dagger_{q_\perp,q_{n\perp}} \Theta(q_\perp \le q_{n\perp}).$$



Amplitude Evolution Summarised

- The formalism perhaps looks more familiar when expressed differentially, as an evolution equation for density matrices.

$$\mu \frac{\partial \mathbf{A}_n(\mu; \{p\}_n)}{\partial \mu} = \mathbf{\Gamma}_n(\mu)\, \mathbf{A}_n(\mu; \{p\}_n) + \mathbf{A}_n(\mu; \{p\}_n)\, \mathbf{\Gamma}_n^\dagger(\mu)$$

$$- \sum_{i=1}^{n} \int \mathrm{d}R_n^i\, \mathbf{D}_n^i(\mu_n)\, \mathbf{A}_{n-i}(\mu_n; \{p\}_{n-i})\, \mathbf{D}_n^{i\dagger}(\mu_n)\, \mu\, \delta(\mu - \mu_n).$$

Usual density matrix RG flow where anomalous dimensions are generators for evolution in the RG group.

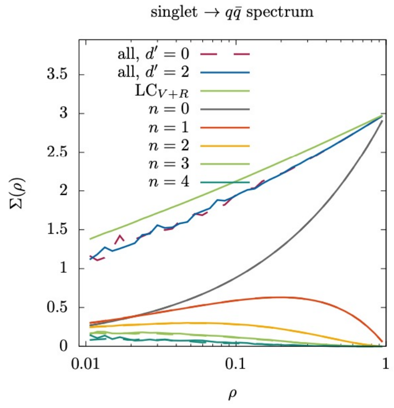
- Has the form of a renormalisation group equation through RG flows which do not conserve particle number. It becomes very reminiscent of recursive applications of SCET.

Becher, Caron-Huot, Forshaw, J.H, Neubert, Plätzer, Shao, Weigert [1501.03754, 2108.13439, 2108.13439, 2107.01212]

Amplitude Evolution Summarised

- Amplitude Evolution can be used to define amplitude-level PS algorithms. Such algorithms have been implemented into codes (CVolver, Deductor).

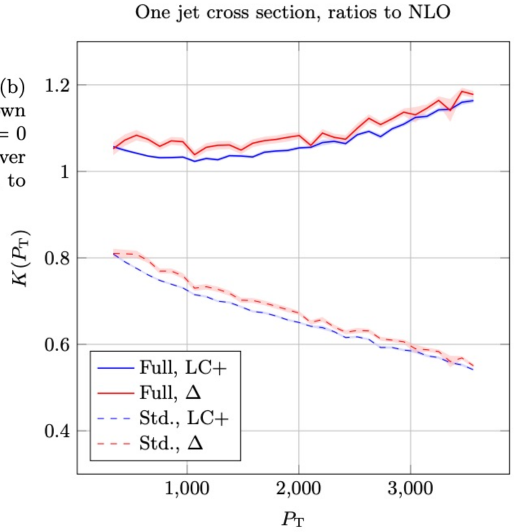
- However, they are also theoretical tools. Constructing a complete amplitude evolution puts one on a solid footing from which you can derive semi-classical results such as improvements to typical PS algorithms.



(b)

FIG. 7. The jet veto cross section in (a) $H \to gg$ and (b) $V \to q\bar{q}$ for different gluon multiplicities. Results are shown for $d = 0$ and $d = 2$. We show the contributions from $n = 0$ up to 4 emissions, however the complete result sums over all emissions (in practice we have limited the simulation to $n \leq 40$ emissions).

De Angelis, Forshaw, Plätzer [2007.09648]



Nagy, Soper [1902.02105]

FIG. 17. Jet cross section ratios with $N_\Delta^{\mathrm{thr.}} = 1$ and with $N_\Delta = N_{\mathrm{Re}} = N_{\mathrm{i}\pi} = 2$ for $t_0 < t < t_0 + 5$.

From Amplitude Evolution to LC NLL PS

- A classical PS is Markovian and unitary at each step of the evolution. Thus (simplifying things a little), to derive one we need only look at the second line in the evolution equation, responsible for the production of new partons.

$$\mu \frac{\partial \mathbf{A}_n(\mu;\{p\}_n)}{\partial \mu} = \mathbf{\Gamma}_n(\mu)\,\mathbf{A}_n(\mu;\{p\}_n) + \mathbf{A}_n(\mu;\{p\}_n)\,\mathbf{\Gamma}_n^\dagger(\mu)$$

This term would generate Sudakovs which are defined by unitarity.

$$- \sum_{i=1}^{n} \int \mathrm{d}R_n^i\, \mathbf{D}_n^i(\mu_n)\,\mathbf{A}_{n-i}(\mu_n;\{p\}_{n-i})\,\mathbf{D}_n^{i\,\dagger}(\mu_n)\,\mu\,\delta(\mu-\mu_n).$$

From Amplitude Evolution to LC NLL PS

- Parton shower algorithms can be summarised by a master equation for the emission probability:

$$\frac{\mathrm{d}\, P_{n-1\to n}(\mu)}{d \ln \mu} = K(\mu; n-1 \to n)$$

where K is the emission Kernel used by the shower, including whatever mechanisms are used for momentum conservation.

- Such a master equation can be derived with LC and NLL accuracy from the amplitude evolution

$$\mu \frac{\partial \mathbf{A}_n(\mu; \{p\}_n)}{\partial \mu} = -\sum_{i=1}^{n} \int \mathrm{d}R_n^i\, \mathbf{D}_n^i(\mu_n)\, \mathbf{A}_{n-i}(\mu_n; \{p\}_{n-i})\, \mathbf{D}_n^{i\,\dagger}(\mu_n)\, \mu\, \delta(\mu - \mu_n). \quad + \text{loops/Sudakov}$$

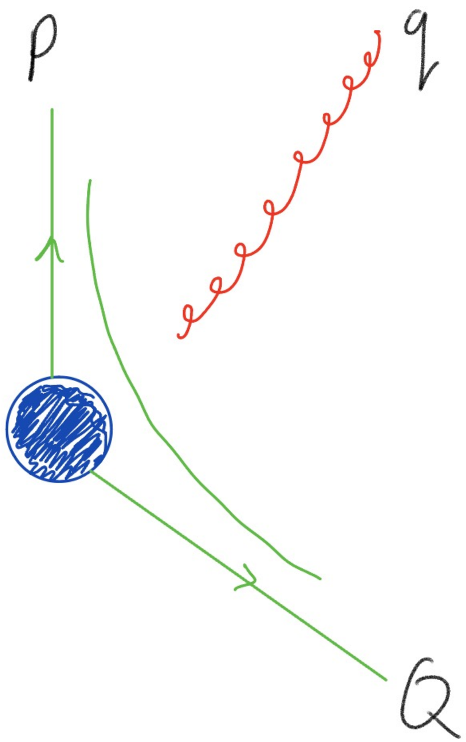
by expanding in colour, taking the trace so as to return probabilities, and factorising out the cross-sections. This returns a K with the interpretation of dipoles emitting partons.

From Amplitude Evolution to LC NLL PS

- Performing these manipulations was the core of Forshaw, J.H, Plätzer [2003.06400].

- There were two key results from this analysis, which can be summarised as follows:
  1. QCD coherence is an important property of how QCD amplitudes factorise. It requires that, for colour diagonal density matrices, partons widely separated in the Lab frame P.S. do not 'effect' each other (specifically the double and single IR poles in matrix elements factorise when partons are separated by parametrically large P.S. volumes). This constrains the mechanism used for momentum conservation. The LC density matrix is always diagonal.
  2. QCD coherence similarly constrains how one defines the 'parent' of a parton in a PS algorithm defined around a $1 \to 2$ parton cascade.

From Amplitude Evolution to LC NLL PS



$$q = \alpha P + \beta Q + k_\perp$$

What do these criteria mean?

1. Momentum conservation should be implemented such that P and Q are not 'substantially' deflected when q is wide-angle in the lab frame. See next slide for viable approaches.
2. If a parent must be assigned, then the parent should be the parton with the smallest angular separation to q in the frame where the observable will be defined – i.e. the lab frame. In this case P.

$$\frac{\mathrm{d}\, P_{n-1\to n}(\mu)}{d \ln \mu} = K(\mu; n-1 \to n)$$

From Amplitude Evolution to LC NLL PS

- These conclusions were independently arrived upon by the PanScales collaboration and are also embodied by the algorithms in the Deductor PS. They are the core of NLL parton showers:

$\beta = 0$ is $k_\perp$ ordering

+,- are the light-cone components of the emitted particle defined with respect to the parent dipole. When handled assymetrically, + is the parent side.

		Ordering	Kinematic map: Dipole-local	Kinematic map: Global
PanScales showers [2002.11114]	**PanLocal** (Dipole and antenna)	$0 < \beta < 1$	$+, -, \perp$	
	PanGlobal	$0 \leq \beta < 1$	$+, -$	$\perp$
Alaric [2208.06057]		$\beta = 0$	$+$	$-, \perp$
Deductor [2011.04777]	**Deductor k_t**	$\beta = 0$	$+$ (Also formulation with $+, -, \perp$)	$-, \perp$
	Deductor Λ	$\beta = 1$	$+$	$-, \perp$
Manchester-Vienna [2003.06400]		$\beta = 0$	$+$	$-, \perp$

Table from M. van Beekveld, Parton Showers for future e+e- colliders, CERN, 2023.

Notice the absence of currently available parton showers in the list here

Base for a new NLL Herwig 7 dipole PS

From Amplitude Evolution to sub-LC soft physics

- We can continue the approach taken to derive the LC NLL insights and look at sub-leading colour.

- Instead of expanding in colour, we can look at what manipulations can be made to the density matrices $\mathbf{A}_n$ such that they diagonalise. When $\mathbf{A}_n$ diagonalises it becomes more readily factorisable, and so taking the trace delivers an master equation for the PS emission probability with full colour accuracy.
  - This is the core of the coherent-branching/angular-ordered approach. In the two-jet limit, a density matrix $\mathbf{A}_n$ of $n-1$ collinear gluons and a single wide-angle gluon, dressing the jets, diagonalises when the resolution scale is an angular scale.
- Can this be generalised further?

From Amplitude Evolution to sub-LC soft physics

Two approaches:

1. Matrix Element corrections. Forshaw, Höche, JH, Plätzer, Reichelt, Sjödahl, Thoren [1808.00332, 2001.11492, 2003.06399]

2. Reconstruction of coherence beyond LC to rebuild some of the sub-leading diagonal colour structures. Forshaw, JH, PanScales, Plätzer [2011.10054, 2011.15087, 2112.13124]

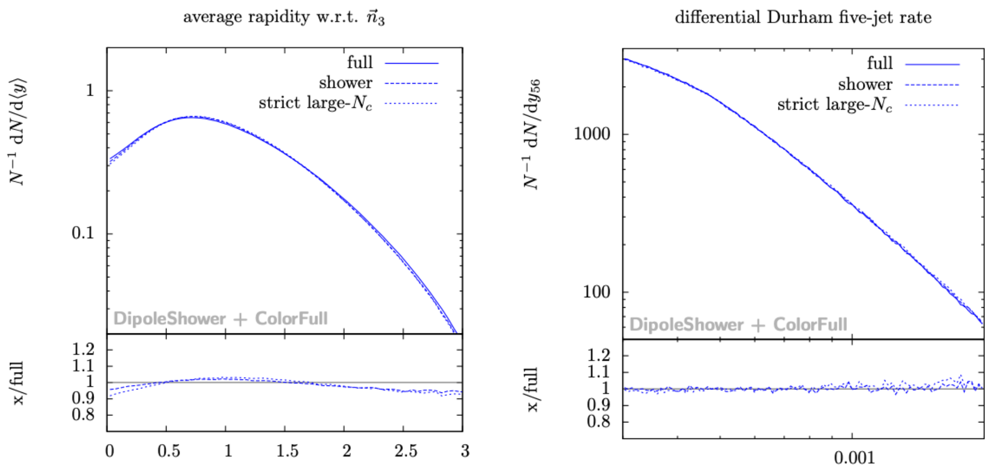
From Amplitude Evolution to sub-LC soft physics

Tree-level density matrix:

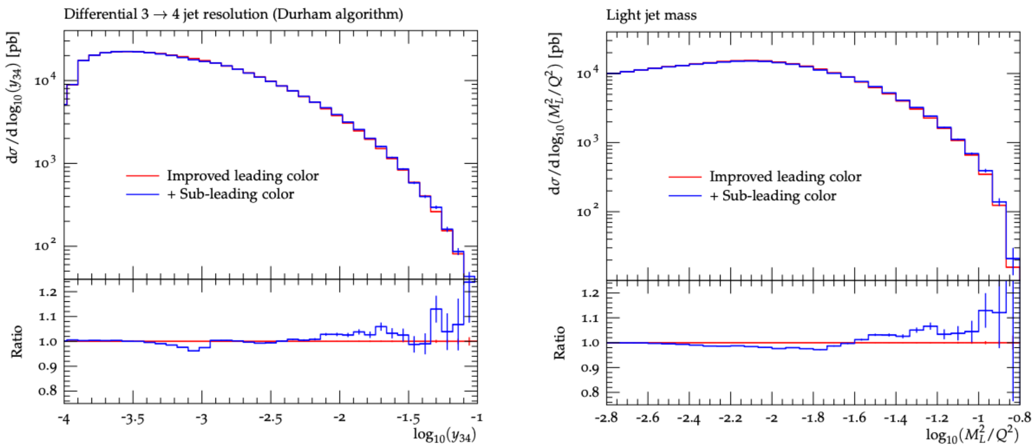
$$\mu \frac{\partial \mathbf{A}_n(\mu;\{p\}_n)}{\partial \mu} = -\sum_{i=1}^{n} \int \mathrm{d}R_n^i \, \mathbf{D}_n^i(\mu_n)\, \mathbf{A}_{n-i}(\mu_n;\{p\}_{n-i})\, \mathbf{D}_n^{i\dagger}(\mu_n)\, \mu\, \delta(\mu-\mu_n).$$

1. Matrix Element corrections.

Colour matrix element corrections compute the PS at tree-level with full colour (perhaps capped at a given multiplicity). *However, loops are still defined by Markovian Unitarity*. This breaks the colour accuracy of their resummation into Sudakov factors. It is observable dependent as to the accuracy achieved in this approach, however often the first sub-leading colour correction can be found.



Plätzer, Sjödahl [1808.00332]

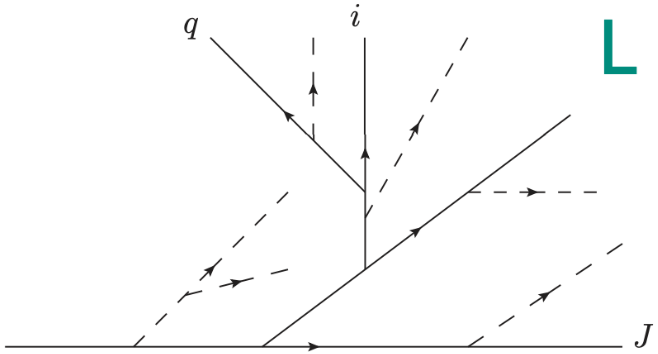


Höche , Reichelt, [2001.11492]

From Amplitude Evolution to sub-LC soft physics

2. Reconstruction of coherence beyond LC to rebuild some of the sub-leading diagonal colour structures.

Simplest approach is to define angular dependent colour factors which rebuild the colour structures from coherent branching. Forshaw, JH, PanScales, Plätzer [2011.10054, 2011.15087]

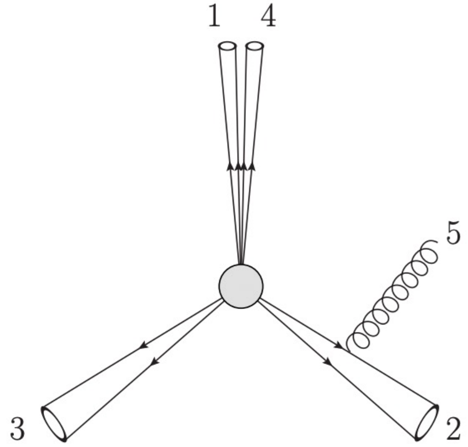


$$\mathcal{C}_{iJ}(\theta_{iq},\theta_{LJ}) = \left(C_F\delta_i^{(q)} + \frac{C_A}{2}\delta_i^{(g)}\right)\theta(\theta_{iq}<\theta_{LJ}) + \left(\frac{C_A}{2}\delta_J^{(g)} + C_F\delta_J^{(q)}\right)\theta(\theta_{iq}>\theta_{LJ})$$

From Amplitude Evolution to sub-LC soft physics

2. Reconstruction of coherence beyond LC to rebuild some of the sub-leading diagonal colour structures.

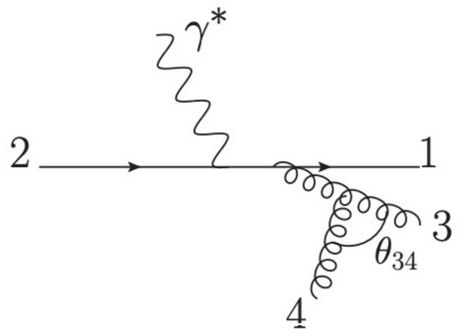
Another is to explore other shower kernels and evolution variables under which the density matrix diagonalises. Forshaw, JH, Plätzer [2112.13124]



$$\frac{\mathrm{d}(\cos\theta_{in})\mathrm{d}\phi_n}{1-\cos\theta_{in}}\Theta(\theta_{in}<\theta) \mapsto {}^{(n-1)}\mathrm{S}_i^{j,k}\, E_n^2\, \Theta(\theta_{in}<\theta)\, \mathrm{d}(\cos\theta_{in})\mathrm{d}\phi_n$$

$${}^{(n-1)}\mathrm{S}_i^{j,k} = \frac{s_{ij}s_{nk}+s_{ik}s_{nj}-s_{jk}s_{ni}}{2s_{ni}s_{nj}s_{nk}}$$

From Amplitude Evolution to sub-LC soft physics



$$\mathbf{A}_4 = \frac{2\alpha_s}{\pi}\left({}^{(3)}\mathrm{S}_1^{2,3}[1\cdot 1] + {}^{(3)}\mathrm{S}_2^{1,3}[2\cdot 2] + {}^{(3)}\mathrm{S}_3^{1,2}[3\cdot 3]\right)$$

$$\mathbf{A}_4\Big|_{n_1\to n_3} \approx \frac{2\alpha_s}{\pi}\Big[{}^{(3)}\mathrm{S}_1^{2,3}\Theta(\theta_{14} < \theta_{13})[1\cdot 1] + {}^{(3)}\mathrm{S}_2^{1,3}\Theta(\theta_{(1+3)4} > \theta_{13})[2\cdot 2] + {}^{(3)}\mathrm{S}_3^{1,2}\Theta(\theta_{34} < \theta_{13})[3\cdot 3]\Big]$$

$$^{(n-1)}\mathrm{S}_i^{j,k} = \frac{1}{2}\left(\omega_{ij} + \omega_{ik} - \omega_{jk}\right)$$

$$[i\cdot j] = \mathbf{T}_i \left|M_{n-1}\right\rangle \left\langle M_{n-1}\right| \mathbf{T}_j^\dagger$$

$$\sum_j \mathbf{T}_j = 0 \qquad \omega_{ij}(q_n) = \frac{q_i\cdot q_j}{q_n\cdot q_i\; q_n\cdot q_j}$$

Up to terms of the order $(\theta_{13}^0, \theta_{i4}^0)$ and in any frame.

This and the usual coherent branching result precisely agree when $\theta_{12} = \pi$ but for $\theta_{12} \neq \pi$ the above result has greater accuracy.

Outlook and Summary

- I have listed a lot directions along which PS algorithms in Monte Carlo event generators are being improved.
- Sticking my neck out a little, in the next 5 to 10 years we will likely see multiple significant updates to the accuracy of event generators. LL + LC → (N)NLL + sub-leading colour.
- Progress has come from multiple fronts
  - Amplitude evolution: built from the study of QCD amplitudes, SCET, soft theorems, and factorisation theorems. CVolver, Deductor
  - Complementary and sometimes faster progress has come from exhaustive phenomenological studies guided by insight from the resummation community. PanScales
- I haven’t discussed non-perturbative effects, however these are also an active area of study in the PS and amplitude evolution community. Gieseke, Kirchgaeßer, Plätzer, Siodmok [1808.06770, 2204.06956].