High-energy resummation in Higgs-plus-jet production

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Outline

Introduction

BFKL approach

- Reggeization
- BFKL in the LLA
- BFKL in the NLLA

NLO impact factors: Higgs case

- Real corrections
- Virtual corrections
- Cancellation of divergences

Outline

Introduction

- Record energies in the center-of-mass reachable by modern and future colliders allow us to study Quantum Chromodynamics (QCD) in its least well understood "final frontier"
- **Semi-hard** collision process $\to$ stringent *scale hierarchy*

$$s \gg Q^2 \gg \Lambda^2_{\rm QCD}, \qquad Q^2 \text{ a hard scale,}$$

$\nearrow$

Regge kinematic region

$$\alpha_s(Q^2)\ \ln\left(\frac{s}{Q^2}\right) \sim 1 \implies \text{all-order } \textbf{resummation} \text{ needed}$$

- **Linear regime** of high-energy QCD

The **BFKL** (Balitsky, Fadin, Kuraev, Lipatov) approach

i. Leading-Logarithmic-Approximation (**LLA**): $(\alpha_s \ln s)^n$

ii. Next-to-Leading-Logarithmic-Approximation (**NLLA**): $\alpha_s(\alpha_s \ln s)^n$

iii. Progress on **next-to-NLLA**

[Falcioni, Gardi, Maher, Milloy, Vernazza (2022)]

[Caola, Chakraborty, Gambuti, Manteuffel, Tancredi (2022)]

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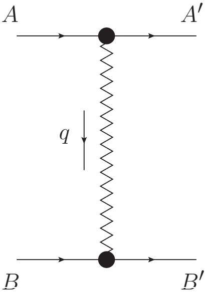
The Reggeized gluon in pQCD

- Elastic scattering process $A + B \longrightarrow A' + B'$

  i. ***Gluon quantum numbers*** in the t-channel

  ii. ***Regge limit*** $\longrightarrow s \simeq -u \to \infty$, $t = q^2$ fixed (i.e not growing with s)

  iii Valid in **LLA** ($\alpha_s^n \ln^n s$ resummed) and **NLLA** ($\alpha_s^{n+1} \ln^n s$ resummed)



$$(\mathcal{A})_{AB}^{A'B'} = \Gamma^c_{A'A} \left[\left(\frac{-s}{-t} \right)^{j(t)} - \left(\frac{s}{-t} \right)^{j(t)} \right] \Gamma^c_{B'B}$$

$$j(t) = 1 + \omega(t), \qquad j(0) = 1$$

$j(t)$-***Reggeized gluon trajectory***

$$\Gamma^c_{A'A} = g \langle A' | T^c | A \rangle \, \Gamma_{A'A}$$

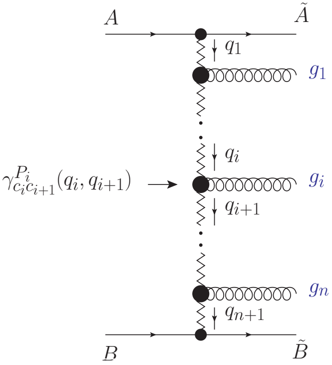
T^c- fundamental(quarks) or adjoint(gluons)

- LLA [Lipatov (1976)]

$$\Gamma^{(0)}_{A'A} = \delta_{\lambda_{A'}\lambda_A}, \quad \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{(D-1)}} \frac{N}{2} \int \frac{d^{D-2}k_\perp}{k_\perp^2 (q-k)_\perp^2} = -g^2 \frac{N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} (\vec{q}^{\,2})^\epsilon$$

BFKL in LLA

- Inelastic scattering process $A + B \longrightarrow \tilde{A} + \tilde{B} + n$ in the LLA



i. ***Leading-logarithm resummation***

$\downarrow$

Multi-Regge kinematics (MRK)

ii. Exchange of fermions suppressed in LLA

iii. Vertical gluons become Reggeized due to loop radiative corrections

iv. $\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \rightarrow$ ***Lipatov vertex***

- ***Multi-Regge form of inelastic amplitudes***

$$\Re \mathcal{A}^{\tilde{A}\tilde{B}+n}_{AB} = 2s\Gamma^{c_1}_{\tilde{A}A} \left(\prod_{i=1}^{n} \gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \left(\frac{s_i}{s_0}\right)^{\omega(t_i)} \frac{1}{t_i} \right) \frac{1}{t_{n+1}} \left(\frac{s_{n+1}}{s_0}\right)^{\omega(t_{n+1})} \Gamma^{c_{n+1}}_{\tilde{B}B}$$

Multi-Regge kinematics

- ***Sudakov decomposition***

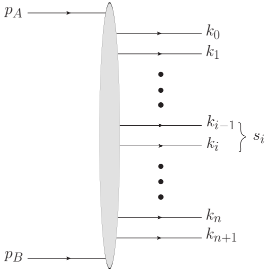
$$k_i = z_i p_A + \lambda_i p_B + k_{i\perp} \qquad p_A^2 = p_B^2 = 0$$

- ***Multi-Regge kinematics (MRK)***

$$z_0 \gg z_1 \gg \ldots \gg z_n \gg z_{n+1}$$

$$\lambda_{n+1} \gg \lambda_n \gg \ldots \gg \lambda_1 \gg \lambda_0$$

$$k_{0\perp} \sim k_{1\perp} \sim \ldots \sim k_{n\perp} \sim k_{n+1\perp}$$



- Cutkosky rules

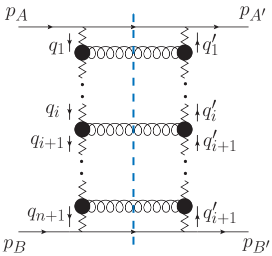
$$\Im \mathcal{A}_{AB}^{A'B'} = \frac{1}{2} \sum_n \int d\Phi_{\tilde{A}\tilde{B}+n} \mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} \left(\mathcal{A}_{A'B'}^{\tilde{A}\tilde{B}+n} \right)^*$$

- Integration over phase space

Each integration over s_i (or z_i)

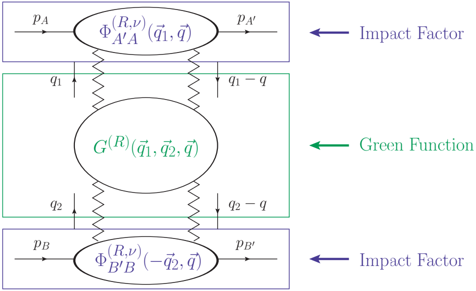
$\downarrow$

One energy logarithm



BFKL resummation

- Diffusion $A + B \longrightarrow A' + B'$ in the ***Regge kinematical region***
- BFKL factorization for $\Im\mathcal{A}_{AB}^{A'B'} \to$ convolution of a ***Green function*** (process independent) with the ***Impact factors*** of the colliding particles (process dependent)



$$\Im\mathcal{A}_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}(\vec{q}_1 - \vec{q}\,)^{\,2}} \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2 - \vec{q}\,)^{\,2}}$$

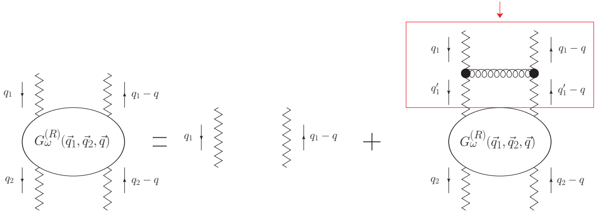
$$\times \sum_{\nu} \Phi^{(R,\nu)}_{A'A}(\vec{q}_1, \vec{q}, s_0) \int \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G^{(R)}_{\omega}(\vec{q}_1, \vec{q}_2; \vec{q}\,) \right] \Phi^{(R,\nu)}_{B'B}(-\vec{q}_2, \vec{q}, s_0)$$

- $\mathcal{R} = 1^{+}(\text{singlet}), 8^{-}(\text{octect}), \ldots$

BFKL resummation

- $G_\omega^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,)$-Mellin transform of the Green function for the Reggeon-Reggeon scattering

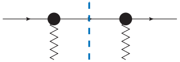
$$\omega G_\omega^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,) = \vec{q}_1^{\,2}(\vec{q}_1 - \vec{q}\,)^2 \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int \frac{d^{D-2}q_1'}{\vec{q}_1^{\,\prime 2}(\vec{q}_1^{\,\prime} - \vec{q}\,)^2} \mathcal{K}^{(R)}(\vec{q}_1, \vec{q}_1^{\,\prime}; \vec{q}\,) G_\omega^{(R)}(\vec{q}_1^{\,\prime}, \vec{q}_2; \vec{q}\,)$$



- ***BFKL equation*** ($\vec{q}^{\,2} = 0$ and singlet color state representation)
  [Balitsky, Fadin, Kuraev, Lipatov (1975-1978)]

- $\Phi_{P'P}^{(R,\nu)}$- LO impact factor in the t-channel color state (R, ν)

$$\Phi_{PP'}^{(R,\nu)} = \langle cc'|\hat{\mathcal{P}}|\nu\rangle \sum_{\{f\}} \int \frac{ds_{PR}}{2\pi} d\rho_f \Gamma^c_{\{f\}P} (\Gamma^{c'}_{\{f\}P'})^*$$



BFKL at NLLA in a nutshell

- Simple factorized form of inelastic amplitudes

$\downarrow$ [Fadin, Lipatov (1989)]

Straightforward program of computations

- Resummation of subleading logarithms means a ***new kinematics***

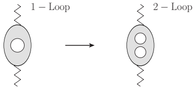
  i. *Multi-Regge kinematics (MRK)*

  ii. *Quasi multi-Regge kinematics (QMRK)*

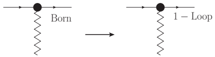
- ***Multi-Regge kinematics***

  Previous quantity must be calculated at higher loops (one α_s more)

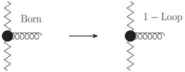
i. $\omega^{(1)}(t) \longrightarrow \omega^{(2)}(t)$



ii. $\Gamma^{c(0)}_{P'P} \longrightarrow \Gamma^{c(1)}_{P'P}$



iii. $\gamma^{G_i(0)}_{c_i c_{i+1}} \longrightarrow \gamma^{G_i(1)}_{c_i c_{i+1}}$

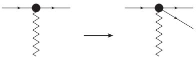


BFKL at NLLA in a nutshell

- ***Quasi Multi-Regge kinematics***

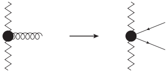
  A pair of particles (but only one!) may have longitudinal Sudakov variables of the same order (one logarithm less)

i. $\Gamma^{c(0)}_{P'P} \longrightarrow \Gamma^{c(0)}_{\{f\}P}$



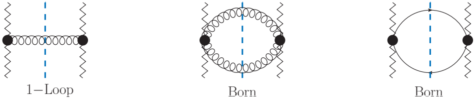
ii. $\gamma^{G(0)}_{c_i c_{i+1}} \longrightarrow \gamma^{GG(0)}_{c_i c_{i+1}}$

iii. $\gamma^{G(0)}_{c_i c_{i+1}} \longrightarrow \gamma^{Q\bar{Q}(0)}_{c_i c_{i+1}}$



- 3 ***new contributions to the real kernel***

$$\mathcal{K}_r\left(\vec{q}_1, \vec{q}_2\right) = \mathcal{K}^{(1)}_{RRG}\left(\vec{q}_1, \vec{q}_2\right) + \mathcal{K}^{(0)}_{RRGG}\left(\vec{q}_1, \vec{q}_2\right) + \mathcal{K}^{(0)}_{RRQ\bar{Q}}\left(\vec{q}_1, \vec{q}_2\right).$$



BFKL at NLLA in a nutshell

- Separating MRK and QMRK → Introduction of s_Λ parameter
- ***QMRK*** $(s_{ij} < s_\Lambda)$

  In the ***two-gluon contribution to the kernel*** the invariant mass should be constrained

$$\mathcal{K}_r(\vec{q}_1,\vec{q}_2) = \frac{\langle c_1 c_1' | \hat{\mathcal{P}}_0 | c_2 c_2' \rangle}{2} \sum_{\{f\}} \int \frac{ds_{RR}}{(2\pi)^D} d\rho_f \; \gamma_{c_1 c_2}^{\{f\}}(q_1,q_2) \left(\gamma_{c_1' c_2'}^{\{f\}}(q_1,q_2) \right)^* \theta(s_\Lambda - s_{RR})$$

- ***MRK*** $(s_{ij} > s_\Lambda)$

  The lower bound of integration over invariant masses is s_Λ

$$-\frac{1}{2} \int d^{D-2}q' \; \vec{q}_1^{\,2} \vec{q}_2^{\,2} \mathcal{K}_r^{(0)}(\vec{q}_1, \vec{q}\,') \mathcal{K}_r^{(0)}(\vec{q}\,', \vec{q}_2) \ln \left(\frac{{s_\Lambda}^2}{(\vec{q}\,' - \vec{q}_1)^2 (\vec{q}\,' - \vec{q}_2)^2} \right)$$

- Similarly, for the ***impact factors***

$$\Phi_{AA}(\vec{q}_1; s_0) = \left(\frac{s_0}{\vec{q}_1^{\,2}} \right)^{\omega(-\vec{q}_1^{\,2})} \sum_{\{f\}} \int \theta(s_\Lambda - s_{AR}) \frac{ds_{AR}}{2\pi} \; d\rho_f \; \Gamma_{\{f\}A}^{c} \left(\Gamma_{\{f\}A}^{c'} \right)^* \langle c c' | \hat{\mathcal{P}}_0 | 0 \rangle$$

$$-\frac{1}{2} \int d^{D-2}q_2 \; \frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \; \Phi_{AA}^{(0)}(\vec{q}_2) \; \mathcal{K}_r^{(0)}(\vec{q}_2, \vec{q}_1) \; \ln \left(\frac{{s_\Lambda}^2}{s_0 (\vec{q}_2 - \vec{q}_1)^2} \right)$$

- Dependence on s_Λ disappears in the combination

Outline

Factorization scheme for hadronic impact factors

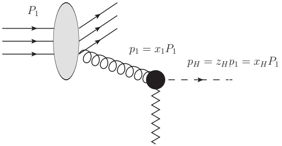
- Infrared safety of impact factor for colorless particle

[Fadin, Martin (1999)]

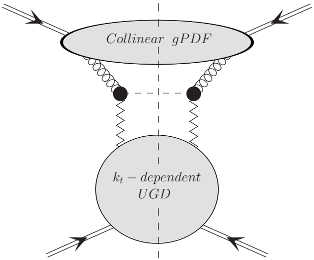
- Impact factors of colored particles afflicted by ***infrared singularities***

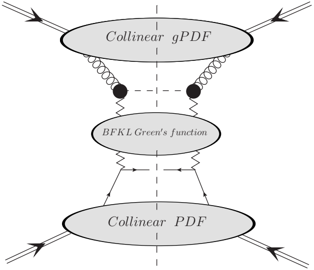
$$p_H = z_H p_1 + \frac{m_H^2 + \vec{p}_H^{\,2}}{z_H s} p_2 + p_{H,\perp}$$

$$\frac{d\Phi_{PP}^{H}}{dx_H d^2\vec{p}_H} = \int_{x_H}^{1} \frac{dz_H}{z_H} f_g\left(\frac{x_H}{z_H}\right) \frac{d\Phi_{gg}^{H}}{dz_H d^2\vec{p}_H}$$



- **Hybrid factorization(s)**

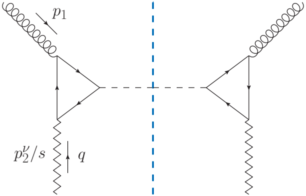




[Mueller, Navelet (1987)]

LO Higgs impact factor

- Gluon-Reggeon $\longrightarrow$ Higgs (through the top quark loop)
- Off-shell t-channel gluon with effective p_2^ν/s polarization



- **LO impact factor**

$$\frac{d\Phi_{PP}^{\{H\}(0)}}{dx_H d^2\vec{p}_H} = \frac{\alpha_s^2}{v^2}\frac{\vec{q}^{\,2}|\mathcal{F}(m_t, m_H, \vec{q}^{\,2})|^2}{128\pi^2\sqrt{2(N^2-1)}} f_g(x_H)\delta^{(2)}(\vec{p}_H - \vec{q})$$

$\Big\downarrow$ *Infinite top-mass limit*

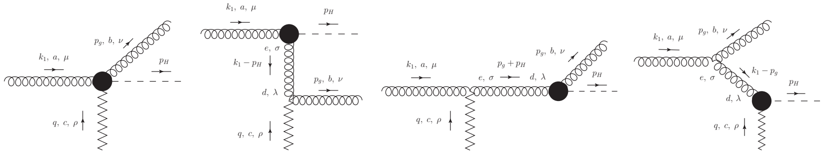
$$\frac{d\Phi_{PP}^{\{H\}(0)}}{dx_H d^2\vec{p}_H} = \frac{g_H^2\vec{q}^{\,2} f_g(x_H)\delta^{(2)}(\vec{q} - \vec{p}_H)}{8\sqrt{N^2-1}}$$

- The study can be upgraded to **Next-to-Leading Order (NLO)**, in the limit $m_t \to \infty$, by using the effective lagrangian

$$\mathcal{L}_{\mathbf{ggH}} = -\frac{\mathbf{1}}{\mathbf{4}}\mathbf{g_H F^a_{\mu\nu} F^{\mu\nu,a} H} \qquad g_H = \frac{\alpha_s}{3\pi v} + \mathcal{O}(\alpha_s^2)$$

NLO Higgs impact factor: Real corrections

- **Gluon initiated contribution**



$$d\Phi_{gg}^{\{Hg\}} \sim \left\{ \frac{\vec{q}^{\,2} z_H}{(1-z_H)\vec{r}^{\,2}} + \frac{\vec{q}^{\,2}}{\vec{r}^{\,2}} \left[z_H(1-z_H) + 2(1-\epsilon)\frac{1-z_H}{z_H}\frac{(\vec{q}\cdot\vec{r})^2}{\vec{q}^{\,2}\vec{r}^{\,2}} \right] \right\}$$

$$\times\, \theta\left(s_\Lambda - \frac{(1-z_H)m_H^2 + \vec{\Delta}^2}{z_H(1-z_H)} \right) + \text{finite}$$

- Divergences

Rapidity divergence $\Longrightarrow$ s_Λ still present

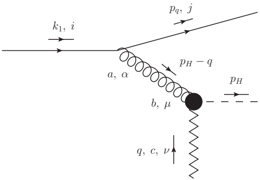
$\vec{\Delta} = \vec{p}_H - z_H \vec{q}$

Soft divergence: $z_H \to 1$, $\vec{r} \to \vec{0}$

Collinear divergence: $\vec{r} \to \vec{0}$

NLO Higgs impact factor: Real corrections

- **Quark initiated contribution**



$$d\Phi_{qq}^{\{Hq\}} \sim \left[\frac{4(1-z_H)\,(\vec{r}\cdot\vec{q}\,)^2 + z_H^2 \vec{q}^{\,2}\vec{r}^{\,2}}{z_H\,(\vec{r}^{\,2})^2} \right]$$

- Divergences

  Rapidity divergence absent $\implies s_\Lambda \to \infty$

  Collinear divergence: $\vec{r} \equiv (\vec{q} - \vec{p}_H) \to \vec{0}$

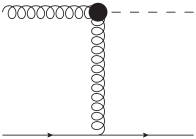
- Agreement with calculation within Lipatov effective action framework

  [Hentschinski, Kutak, Van Hameren (2021)]

NLO Higgs impact factor: Virtual corrections

- 1-loop ggH effective vertex

$$\Gamma^{ac(1)}_{\{H\}g}(q) = \Gamma^{ac(0)}_{\{H\}g}(q)\,[1 + \delta_{\rm NLO}]$$

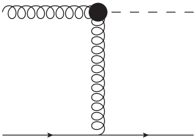


- General strategy: Comparison of a suitable amplitude (in the high-energy limit) with the ***Regge form***

$$\mathcal{A}^{(8,-)}_{gq\to Hq} = \Gamma^{ac}_{\{H\}g}\frac{s}{t}\left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)}\right]\Gamma^{c}_{qq} \approx \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq}$$

$$+\Gamma^{ac(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right) + \ln\left(\frac{-s}{-t}\right)\right]\Gamma^{c(0)}_{qq} + \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(1)}_{qq} + \Gamma^{ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq}$$

NLO Higgs impact factor: Virtual corrections



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$$+\Gamma^{ac(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right)+\ln\left(\frac{-s}{-t}\right)\right]\Gamma^{c(0)}_{qq} + \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(1)}_{qq} + \Gamma^{ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq}$$

- **Virtual corrections** to the impact factor

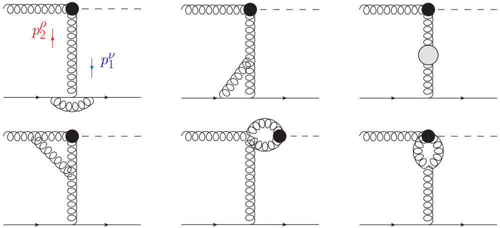
$$\frac{d\Phi^{\{H\}(1)}_{gg}}{dz_H d^2\vec{p}_H} = \frac{d\Phi^{\{H\}(0)}_{gg}}{dz_H d^2\vec{p}_H}\,\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\left[-\frac{C_A}{\epsilon^2}+\frac{11C_A-2n_f}{6\epsilon}\right.$$

$$\left.-\frac{C_A}{\epsilon}\ln\left(\frac{\vec{q}^{\,2}}{s_0}\right)-\frac{5n_f}{9}+C_A\left(2\,\Re\left(\mathrm{Li}_2\left(1+\frac{m_H^2}{\vec{q}^{\,2}}\right)\right)+\frac{\pi^2}{3}+\frac{67}{18}\right)+11\right]$$

- Agreement with [Nefedov (2019)]

NLO Higgs impact factor: Virtual corrections

- Single gluon in the t-channel diagrams



Gribov's prescription: $g^{\rho\nu} = g_{\perp\perp}^{\rho\nu} + 2\frac{p_1^\rho p_2^\nu + p_1^\nu p_2^\rho}{s} \to 2s\,\frac{p_1^\nu}{s}\,\frac{p_2^\rho}{s}$

NLO Higgs impact factor: Virtual corrections

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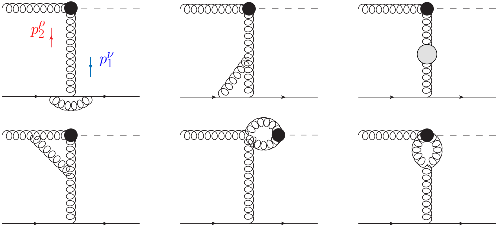
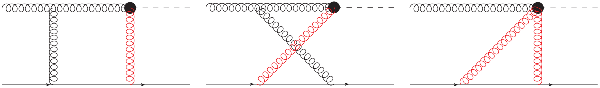


Gribov's prescription: $g^{\rho\nu} = g_{\perp\perp}^{\rho\nu} + 2\frac{p_1^\rho p_2^\nu + p_1^\nu p_2^\rho}{s} \to 2s\,\frac{p_1^\nu}{s}\,\frac{p_2^\rho}{s}$

- Two gluons in the t-channel diagrams



Dimension-5 operator in $\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{ggH}$ $\to$ **Gribov's trick modification**

$$g^{\rho\nu} = g_{\perp\perp}^{\rho\nu} + 2\frac{p_1^\rho p_2^\nu + p_1^\nu p_2^\rho}{s} \to 2s\,\frac{p_1^\nu}{s}\,\frac{p_2^\rho}{s} + g_{\perp\perp}^{\rho\nu}$$

Showing cancellation of divergences

- Perturbative expansion of the Kernel: $\hat{K} = \bar{\alpha}_s \hat{K}^0 + \bar{\alpha}_s^2 \hat{K}^1$

$$\hat{1} = \left(\omega - \hat{K}\right) \hat{G}_\omega \implies \hat{G}_\omega = \left(\omega - \hat{K}\right)^{-1}$$

$$\hat{G}_\omega \simeq \left(\omega - \bar{\alpha}_s \hat{K}^0\right)^{-1} + \left(\omega - \bar{\alpha}_s \hat{K}^0\right)^{-1} \left(\bar{\alpha}_s^2 \hat{K}^1\right) \left(\omega - \bar{\alpha}_s \hat{K}^0\right)^{-1}$$

- Eigenfunctions of the LO kernel

$$\hat{K}^0 \left|n,\nu\right\rangle = \chi(n,\nu) \left|n,\nu\right\rangle \ , \qquad \left\langle \vec{q} \,|n,\nu\right\rangle = \frac{1}{\pi\sqrt{2}} (\vec{q}^{\,2})^{i\nu - \frac{1}{2}} e^{in\phi}$$

Alternative: NLO eigenfunctions [G. A. Chirilli, Y. V. Kovchegov (2013)]

- BFKL cross-section

$$d\sigma_{AB} = \frac{1}{(2\pi)^{D-2}} \sum_{n,n'} \int d\nu \int d\nu' \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^\omega$$

$$\times \left\langle \frac{d\Phi_{AA}}{\vec{q}_1^{\,2}} |n,\nu\right\rangle \left\langle n,\nu|\hat{G}_\omega|n',\nu'\right\rangle \left\langle n'\nu'|\frac{d\Phi_{BB}}{\vec{q}_2^{\,2}}\right\rangle$$

- **Projection** onto the eigenfunction of the BFKL kernel

$$\left\langle \frac{d\Phi_{AA}}{\vec{q}^{\,2}} |n,\nu\right\rangle = \int \frac{d^{2-2\epsilon}q}{\pi\sqrt{2}} (\vec{q}^{\,2})^{i\nu - \frac{3}{2}} e^{in\phi} d\Phi_{AA}(\vec{q}\,) \equiv d\Phi_{AA}(n,\nu)$$

Showing cancellation of divergences

- **Rapidity** divergences → removed by the BFKL counterterm

$$d\Phi_{PP}^{\{Hg\}} \longrightarrow d\tilde{\Phi}_{PP}^{\{Hg\}} = d\Phi_{PP}^{\{Hg\}} - d\Phi_{PP}^{\{H\}} \otimes \mathcal{K}_r^{(0)} \ln s_\Lambda$$

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- Surviving **collinear** divergences → gPDF renormalization

$$f_g(x,\mu) = f_g(x,\mu_F) - \frac{\alpha_s(\mu_F)}{2\pi}\left(-\frac{1}{\epsilon} - \ln(4\pi e^{-\gamma_E}) + \ln\left(\frac{\mu_F^2}{\mu^2}\right)\right)$$
$$\times \int_x^1 \frac{dz}{z}\left[P_{gq}(z)\sum_{a=q\bar{q}} f_a\left(\frac{x}{z},\mu_F\right) + P_{gg}(z) f_g\left(\frac{x}{z},\mu_F\right)\right]$$

Showing cancellation of divergences

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- Complete final expression

Gaussian hypergeometric functions ${}_2F_1(a,b,c;z)$

Showing cancellation of divergences

- UV counterterm $d\Phi_{PP}^{\{H\}}\Big|_{\alpha_s\ c.t.} = d\Phi_{PP}^{\{H\}(0)} \frac{\bar{\alpha}_s}{2\pi}\left[-\frac{\beta_0}{\epsilon}\right] + \text{finite}$
- gPDF counterterm

$$d\Phi_{PP}^{\{H\}}\Big|_{\mathrm{P_{qg}\ c.t.}} = \frac{d\Phi_{PP}^{\{H\}(0)}}{f_g(x_H)} \frac{\bar{\alpha}_s}{2\pi}\left[\frac{1}{\epsilon}P_{gq}\otimes\sum_{a=q\bar{q}} f_a\right] + \text{finite}$$

$$d\Phi_{PP}^{\{H\}}\Big|_{\mathrm{P_{gg}\ c.t.}} = \frac{d\Phi_{PP}^{\{H\}(0)}}{f_g(x_H)} \frac{\bar{\alpha}_s}{2\pi}\left[\frac{1}{\epsilon}\tilde{P}_{gg}\otimes f_g + \frac{1}{2}\frac{\beta_0}{\epsilon}f_g(x_H)\right] + \text{finite}$$

- Real quark contribution

$$d\Phi_{PP}^{\{Hg\}}\Big|_{\mathrm{quark}} = \frac{d\Phi_{PP}^{\{H\}(0)}}{f_g(x_H)} \frac{\bar{\alpha}_s}{2\pi}\left[-\frac{1}{\epsilon}P_{gq}\otimes\sum_{a=q\bar{q}} f_a\right] + \text{finite}$$

- Real gluon contribution (BFKL counterterm subtracted)

$$d\Phi_{PP}^{\{Hq\}}\Big|_{\mathrm{gluon}} = \frac{d\Phi_{PP}^{\{H\}(0)}}{f_g(x_H)} \frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}\left[\left(\frac{C_A}{\epsilon^2} + \frac{C_A}{\epsilon}\ln\left(\frac{\vec{p}_H^{\,2}}{s_0}\right)\right)f_g(x_H) - \frac{1}{\epsilon}\tilde{P}_{gg}\otimes f_g\right] + \text{finite}$$

- Virtual corrections contribution

$$d\Phi_{PP}^{\{H\}}\Big|_{\mathrm{virtual}} = d\Phi_{PP}^{\{H\}(0)} \frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}\left[-\frac{C_A}{\epsilon^2} - \frac{C_A}{\epsilon}\ln\left(\frac{\vec{p}_H^{\,2}}{s_0}\right) + \frac{1}{\epsilon}\frac{\beta_0}{2}\right] + \text{finite}$$

Summary and outlook

Summary

- Higgs plus jet production at large difference of rapidity has been investigated within partial NLLA in the BFKL approach

  [Celiberto, Ivanov, Mohammed, Papa (2020)]

  [Andersen, Hassan, Maier, Paltrinieri, Papaefstathiou, Smillie (2022)]

- **NLO corrections to the forward Higgs boson impact factor** has been obtained both in q_T and (n, ν)-space in the $m_t \to \infty$ limit
- *Gribov's prescription* for high-energy computations in QCD needs to be modified in the present case

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Outlook

- Full **NLL matched to NLO** Higgs plus jet production
- **Finite top-mass corrections**
- Extension at full NLL of other interesting processes: J/ψ plus jet, Drell-Yan plus jet, heavy-light dijet

  [Boussarie, Ducloué, Szymanowski, Wallon (2018)]

  [Golec-Biernat, Motyka, Stebel (2018)]

  [Bolognino, Celiberto, M. F., Ivanov, Papa (2021)]

Thanks for your attention!

Backup

"*Higgs impact factor at NLO*"

Finite part of the result in the (n, ν)-space

- The complete finite result is obtained in terms of hypergeometric functions and integrals of hypergeometric functions (with some shrewdness!), i.e,

$$I_2(\gamma_1, n, \nu) = \int \frac{d^{2-2\epsilon}\vec{q}}{\pi\sqrt{2}} (\vec{q}^{\,2})^{i\nu-\frac{3}{2}} e^{in\phi} (\vec{q}^{\,2})^{-\gamma_1} \frac{1}{\left[(\vec{q}-\vec{p}_H)^2\right]\left[(1-z_H)m_H^2 + (\vec{p}_H - z_H\vec{q})^2\right]}$$

$$= \frac{(\vec{p}_H^{\,2})^{\frac{n}{2}} e^{in\phi_H}}{z_H^2\sqrt{2}\pi^{\epsilon}} \left[\frac{\Gamma\left(\frac{5}{2}+\gamma_1+\frac{n}{2}-i\nu+\epsilon\right)\Gamma\left(-\frac{1}{2}-\gamma_1+\frac{n}{2}+i\nu-\epsilon\right)}{\Gamma(1+n-\epsilon)}\right]$$

$$\times \int_0^1 d\Delta \left(\Delta + \frac{(1-\Delta)}{z_H}\right)^n \left[\left(\Delta + \frac{(1-\Delta)}{z_H^2}\right)\vec{p}_H^{\,2} + \frac{(1-\Delta)(1-z_H)m_H^2}{z_H^2}\right]^{-\frac{5}{2}-\gamma_1+i\nu-\frac{n}{2}-\epsilon}$$

$$\times\ {}_2F_1\left(-\frac{1}{2}-\gamma_1+\frac{n}{2}+i\nu-\epsilon, \frac{5}{2}+\gamma_1-i\nu+\frac{n}{2}+\epsilon, 1+n-\epsilon, \zeta\right), \qquad \zeta \xrightarrow{\Delta\to 1} 1$$

- Extracting singular part

$$I_{2,\mathrm{as}}(\gamma_1, n, \nu) = \frac{(\vec{p}_H^{\,2})^{-\frac{3}{2}-\gamma_1+i\nu-\epsilon} e^{in\phi_H}\Gamma(1+\epsilon)}{(1-z_H)\sqrt{2}\pi^{\epsilon}} \frac{1}{\left(m_H^2 + (1-z_H)\vec{p}_H^{\,2}\right)} \int_0^1 d\Delta (1-\Delta)^{-\epsilon-1}$$

$$= -\frac{1}{\epsilon} \frac{(\vec{p}_H^{\,2})^{-\frac{3}{2}-\gamma_1+i\nu-\epsilon} e^{in\phi_H}\Gamma(1+\epsilon)}{(1-z_H)\sqrt{2}\pi^{\epsilon}} \frac{1}{\left(m_H^2 + (1-z_H)\vec{p}_H^{\,2}\right)}$$

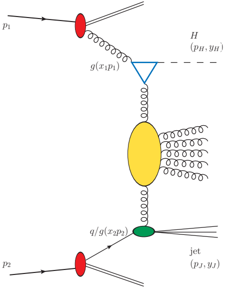
- Replacement: $I_2 = I_{2,as} + (I_2 - I_{2,\mathrm{as}}) \equiv I_{2,as} + I_{2,\mathrm{reg}}$

Higgs plus jet as a paradigm

- Inclusive **Higgs plus jet** production in proton-proton collision

  i. Full NLL Green function + Partial NLO impact factors (full m_t-dep.)
  [Celiberto, Ivanov, Mohammed, Papa (2021)]

  ii. Same process in HEJ framework (full m_t,m_b-dep.)
  [Andersen et al. (2022)]



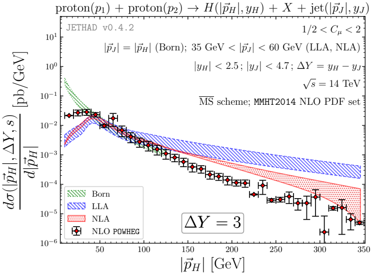
$$\frac{d\sigma_{\rm pp}}{dy_H\, dy_J\, d|\vec{p}_H|\, d|\vec{p}_J|\, d\phi_1\, d\phi_2} = \frac{1}{(2\pi)^2}$$
$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} \left(\mathcal{V}_H^{(g)}(\vec{q}_1, s_0, x_1, \vec{p}_H) \otimes f_g(x_1)\right)$$
$$\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_H x_J s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$
$$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} \left(\sum_r \mathcal{V}_J^{(p)}(\vec{q}_2, s_0, x_2, \vec{p}_J) \otimes f_r(x_2)\right)$$

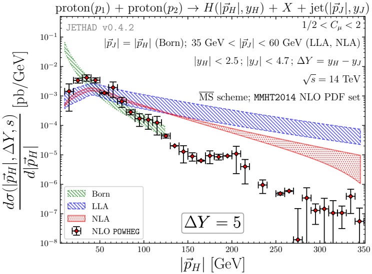
- Hadronic cross section expanded in **azimuthal coefficients**

$$\frac{d\sigma_{\rm pp}}{dy_H\, dy_J\, d|\vec{p}_H|\, d|\vec{p}_J|\, d\phi_1\, d\phi_2} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + 2\sum_{n=1}^{\infty} \cos(n\varphi)\mathcal{C}_n\right] \qquad \varphi = \phi_1 - \phi_2 - \pi$$

- ***Higgs p_T-distribution***

$$\frac{d\sigma\left(|\vec{p}_H|, \Delta Y, s\right)}{d|\vec{p}_H| d\Delta Y} = \int_{p_J^{min}}^{p_J^{max}} d|\vec{p}_J| \int_{y_H^{min}}^{y_H^{max}} dy_H \int_{y_J^{min}}^{y_J^{max}} dy_J \delta\left(y_H - y_J - \Delta Y\right) C_0$$





Higgs p_T-distribution

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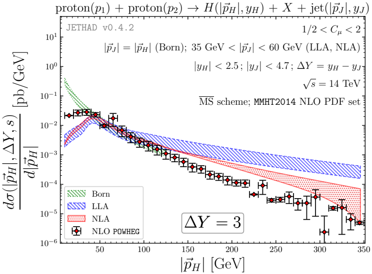
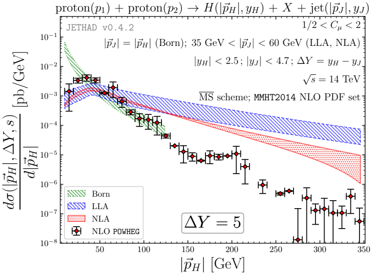




- **Additive matching procedure**

[Celiberto, Delle Rose, M.F., Gatto, Papa (to appear)]

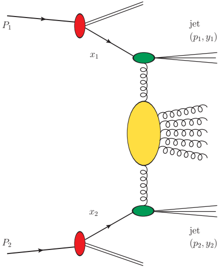
$$d\sigma^{\text{NLL/NLO}}(\Delta Y,s) = \underbrace{d\sigma^{\text{NLO}}(\Delta Y,s)}_{\text{fixed order}} + \underbrace{d\sigma^{\text{NLL}}(\Delta Y,s)}_{\text{BFKL}} - \underbrace{\Delta d\sigma^{\text{NLL/NLO}}(\Delta Y,s)}_{\text{NLO double counting}}$$

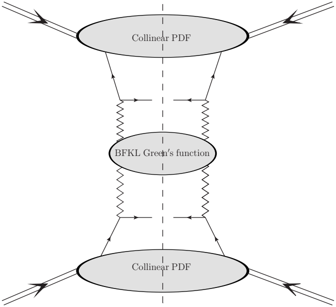
"*Hybrid factorization: Theory vs Experiments*"

Hybrid collinear/high-energy factorization

Mueller-Navelet jets

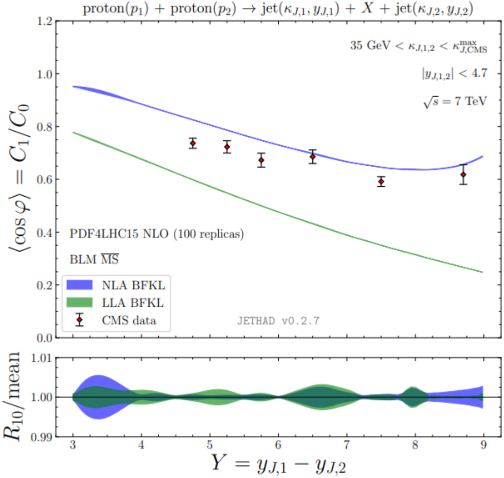
- Inclusive two jet production in proton-proton collision
- Large p_T and large rapidity separation
- Large energy logarithms → BFKL resummed partonic cross section
- Moderate values of parton x → collinear PDFs





- **Hybrid** formalism: can be extended to several type of semi-hard reactions

Muller-Navelet: Theory vs Experiment



[B. Ducloué, L. Szymanowski, S. Wallon (2013)]

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]

In this slide: [F.G. Celiberto (2021)]

Mueller-Navelet: Theory vs Experiment

- CMS at 7 Tev with symmetric p_T-cuts, only!

[CMS collaboration (2016)]

- Asymmetric cuts would provide a better discrimination between DGLAP and BFKL
- LHC kinematic **domain** in between the sectors described by BFKL and DGLAP approaches

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- Strong manifestation of higher-order **instabilities** via scale variation

NLA BFKL corrections to cross section with opposite sign with respect to the leading order (LO) result and large in absolute value...

- ◇ ...call for some optimization procedure...
- ◇ ...choose scales to mimic the most relevant subleading terms

- **BLM** [S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]
  - ✓ preserve the conformal invariance of an observable...
  - ✓ ...by making vanish its β_0-dependent part

* "Exact" BLM:

suppress NLO IFs + NLO Kernel β_0-dependent factors