High-Energy factorisation and matching to NLO for (inclusive) quarkonium production [1]

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[1] Based on JHEP **05** (2022) 083 and ongoing work

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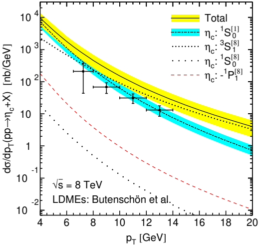
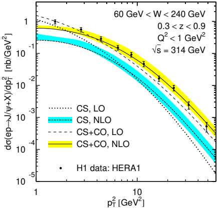
Colour-singlet dominance in quarkonium production

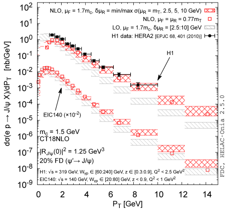
The η_c **hadroproduction** was found to be dominated by the $c\bar{c}\left[{}^1S_0^{(1)}\right]$ state **for all values of p_T:**



[Butenschön, He, Kniehl, '15] This is a problem for NRQCD factorization, because roughly the same contribution of color-octet states **at $p_T \gg M$** as for J/ψ was expected.

The **inclusive J/ψ photoproduction** is fairly reasonably described by $c\bar{c}\left[{}^3S_1^{(1)}\right]$ contribution at NLO, with $\leq 50\%$ admixture of CO contributions:





[Butenschön, He, Kniehl, '11; Colpani Serri *et al.*, '22]

Another CS-dominated observable: J/ψ pair production with $M_{\psi\psi} \gtrsim 2M_\psi$ (talk of Alice).

Perturbative instability of quarkonium total cross sections

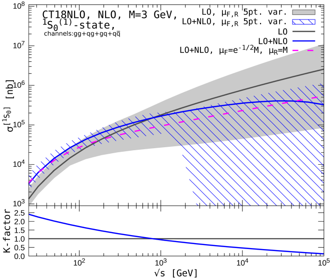
Inclusive η_c-hadroproduction (CSM)

[Mangano *et.al.*, '97, ..., Lansberg, Ozcelik, '20]

$$p+p \to c\bar{c}\left[{}^1S_0^{[1]}\right]+X,\ \text{LO: } g(p_1)+g(p_2) \to c\bar{c}\left[{}^1S_0^{[1]}\right],$$

$$\sigma(\sqrt{s_{pp}}) = f_i(x_1,\mu_F) \otimes f_j(x_2,\mu_F) \otimes \hat{\sigma}(z),$$

where $z = \frac{M^2}{\hat{s}}$ with $\hat{s} = (p_1+p_2)^2$.



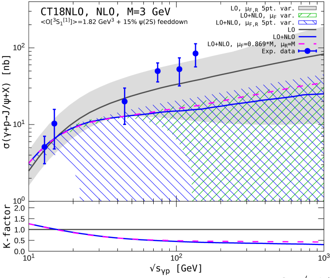
Inclusive J/ψ-photoproduction (CSM)

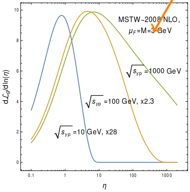
[Krämer, '96, ...,Colpani Serri *et.al*, '21]

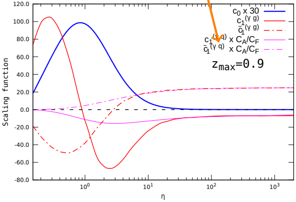
$$\gamma+p \to c\bar{c}\left[{}^3S_1^{[1]}\right]+X,\ \text{LO: } \gamma(q)+g(p_1) \to c\bar{c}\left[{}^3S_1^{[1]}\right]+g,$$

$$\sigma(\sqrt{s_{\gamma p}}) = f_i(x_1,\mu_F) \otimes \hat{\sigma}(\eta),$$

where $\eta = \frac{\hat{s}-M^2}{M^2}$ with $\hat{s} = (q+p_1)^2$, $z = \frac{pP}{qP}$.



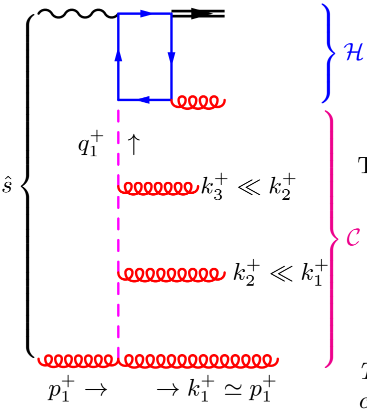




High-Energy Factorization (J/ψ photoproduction)

The **LLA** ($\sum_n \alpha_s^n \ln^{n-1}$) formalism [Collins, Ellis, '91; Catani, Ciafaloni, Hautmann, '91,'94]

Physical picture in the **LLA** for photoproduction:



The LLA in $\ln \frac{1}{\xi} = \ln \frac{p_1^+}{q_1^+} \sim \ln(1+\eta)$:

$$\hat{\sigma}_{\rm HEF}^{\ln(1/\xi)}(\eta) \propto \int\limits_{1/z}^{1+\eta} \frac{dy}{y} \int\limits_0^\infty d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{y}{1+\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \mathcal{H}(y, \mathbf{q}_{T1}^2),$$

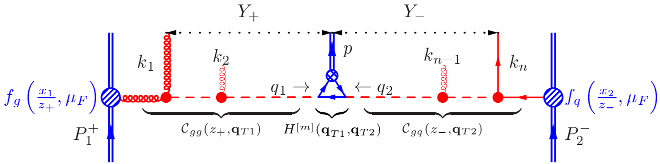
The **strict LLA** in $\ln(1+\eta) = \ln \frac{\hat{s}}{M^2}$:

$$\hat{\sigma}_{\rm HEF}^{\ln(1+\eta)}(\eta) \propto \int\limits_0^\infty d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{1}{1+\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \int\limits_{1/z}^\infty \frac{dy}{y} \mathcal{H}(y, \mathbf{q}_{T1}^2).$$

The LLA($\ln(1/\xi)$) contains some (N..)NLLA contributions relative to the LLA($\ln(1+\eta)$).

The coefficient function $\mathcal{H}$ has been calculated at LO[Kniehl, Vasin, Saleev, '06] and decreases as $1/y^2$ for $y \gg 1$.

High-Energy Factorization (η_c hadroproduction)



Small parameter: $z = \frac{M^2}{\hat{s}}$, LLA in $\alpha_s^n \ln^{n-1} \frac{1}{z}$:

$$\hat{\sigma}_{ij}^{[m],\ \mathrm{HEF}}(z, \mu_F, \mu_R) = \int\limits_{-\infty}^{\infty} d\eta \int\limits_{0}^{\infty} d\mathbf{q}_{T1}^2 d\mathbf{q}_{T2}^2\ \mathcal{C}_{gi}\left(\frac{M_T}{M}\sqrt{z}e^{\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right)$$

$$\times \mathcal{C}_{gj}\left(\frac{M_T}{M}\sqrt{z}e^{-\eta}, \mathbf{q}_{T2}^2, \mu_F, \mu_R\right) \int\limits_{0}^{2\pi} \frac{d\phi}{2} \frac{H^{[m]}(\mathbf{q}_{T1}^2, \mathbf{q}_{T2}^2, \phi)}{M_T^4}$$

The coefficient functions $H^{[m]}$ are known at LO in α_s [Hagler *et.al*, 2000; Kniehl, Vasin, Saleev 2006] for $m = {}^1S_0^{(1,8)}, {}^3P_J^{(1,8)}, {}^3S_1^{(8)}$.
The $H^{[m]}$ is a tree-level “squared matrix element” of the $2 \rightarrow 1$-type process:

$$R_+(\mathbf{q}_{T1}, q_1^+) + R_-(\mathbf{q}_{T2}, q_2^-) \rightarrow c\bar{c}[m].$$

LLA evolution w.r.t. $\ln 1/\xi$

In the LL($\ln 1/\xi$)-approximation, the $Y = \ln 1/\xi$-evolution equation for *collinearly un-subtracted* $\tilde{\mathcal{C}}$-factor has the form:

$$\tilde{\mathcal{C}}(\xi, \mathbf{q}_T) = \delta(1-\xi)\delta(\mathbf{q}_T^2) + \hat{\alpha}_s \int\limits_{\xi}^{1} \frac{dz}{z} \int d^{2-2\epsilon}\mathbf{k}_T K(\mathbf{k}_T^2, \mathbf{q}_T^2) \tilde{\mathcal{C}}\left(\frac{\xi}{z}, \mathbf{q}_T - \mathbf{k}_T\right)$$

with $\hat{\alpha}_s = \alpha_s C_A/\pi$ and

$$K(\mathbf{k}_T^2, \mathbf{q}_T^2) = \frac{1}{\pi(2\pi)^{-2\epsilon}\mathbf{k}_T^2} + \delta^{(2-2\epsilon)}(\mathbf{k}_T)\ 2\omega_g(\mathbf{q}_T^2),$$

where $\omega_g(\mathbf{q}_T^2)$ – 1-loop Regge trajectory of a gluon. It is convenient to go from $(z, \mathbf{q}_T)$-space to $(N, \mathbf{x}_T)$-space:

$$\tilde{\mathcal{C}}(N, \mathbf{x}_T) = \int d^{2-2\epsilon}\mathbf{q}_T\ e^{i\mathbf{x}_T\mathbf{q}_T} \int\limits_0^1 dx\ x^{N-1}\ \tilde{\mathcal{C}}(x, \mathbf{q}_T),$$

because:

- Mellin convolutions over z turn into products: $\int \frac{dz}{z} \rightarrow \frac{1}{N}$
- Large logs map to poles at $N = 0$: $\boxed{\alpha_s^{k+1} \ln^{k} \frac{1}{\xi} \rightarrow \frac{\alpha_s^{k+1}}{N^{k+1}}}$
- All *collinear divergences* are contained inside $\mathcal{C}$ in $\mathbf{x}_T$-space.

Exact LL solution and the DLA

In $(N, \mathbf{q}_T)$-space, subtracted $\mathcal{C}$, which resums all terms $\propto (\hat{\alpha}_s/N)^n$ (complete LLA) has the form [Collins, Ellis, '91; Catani, Ciafaloni, Hautmann, '91,'94]:

$$\mathcal{C}(N, \mathbf{q}_T, \mu_F) = R(\gamma_{gg}(N,\alpha_s)) \frac{\gamma_{gg}(N,\alpha_s)}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2}\right)^{\gamma_{gg}(N,\alpha_s)},$$

where $\gamma_{gg}(N, \alpha_s)$ is the solution of [Jaroszewicz, '82]:

$$\frac{\hat{\alpha}_s}{N}\chi(\gamma_{gg}(N,\alpha_s)) = 1, \text{ with } \chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma),$$

where $\psi(\gamma) = d\ln\Gamma(\gamma)/d\gamma$ – Euler's ψ-function. The first few terms:

$$\gamma_{gg}(N,\alpha_s) = \underbrace{\underbrace{\frac{\hat{\alpha}_s}{N}}_{\text{DLA [Blümlein, '95]}} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{N^6} + \dots}_{\text{LLA}}$$

$\frac{\hat{\alpha}_s}{N} \leftrightarrow P_{gg}(z \to 0) = \frac{2C_A}{z} + \dots$

The function $R(\gamma)$ is

$$R(\gamma_{gg}(N,\alpha_s)) = 1 + O(\alpha_s^3).$$

Fixed-order asymptotics from HEF

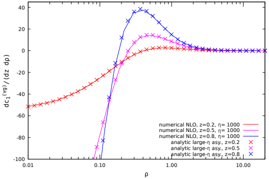
When expanded up to $O(\alpha_s)$ the HEF resummation should predict the $\hat{s} \gg M^2$ asymptotics of the CF coefficient function $\hat{\sigma}$

For the $g + g \to c\bar{c}\left[{}^1S_0^{(1)}, {}^3P_0^{(1)}, {}^3P_2^{(1)}\right]$ the NLO and NNLO($\alpha_s^2 \ln(1/z)$) terms in $\hat{\sigma}$ are predicted [M.N., Lansberg, Ozcelik '22]:

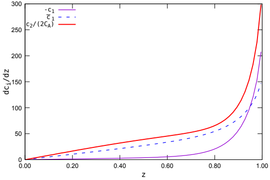
State	$A_0^{[m]}$	$A_1^{[m]}$	$A_2^{[m]}$	$B_2^{[m]}$
${}^1S_0$	1	$\mathbf{-1}$	$\frac{\pi^2}{6}$	$\frac{\pi^2}{6}$
${}^3S_1$	0	1	0	$\frac{\pi^2}{6}$
${}^3P_0$	1	$\mathbf{-\frac{43}{27}}$	$\frac{\pi^2}{6} + \frac{2}{3}$	$\frac{\pi^2}{6} + \frac{40}{27}$
${}^3P_1$	0	$\frac{5}{54}$	$-\frac{1}{9}$	$-\frac{2}{9}$
${}^3P_2$	1	$\mathbf{-\frac{53}{36}}$	$\frac{\pi^2}{6} + \frac{1}{2}$	$\frac{\pi^2}{6} + \frac{11}{9}$

$$\hat{\sigma}_{gg}^{[m]}(z \to 0) = \sigma_{\rm LO}^{[m]} \left\{ A_0^{[m]} \delta(1-z) + \frac{\alpha_s}{\pi} 2C_A \left[A_1^{[m]} + A_0^{[m]} \ln \frac{M^2}{\mu_F^2} \right] + \left(\frac{\alpha_s}{\pi}\right)^2 \ln \frac{1}{z} \cdot C_A^2 \left[2A_2^{[m]} + B_2^{[m]} + 4A_1^{[m]} \ln \frac{M^2}{\mu_F^2} + 2A_0^{[m]} \ln^2 \frac{M^2}{\mu_F^2} \right] + O(\alpha_s^3) \right\},$$

For the $\gamma + g \to c\bar{c}\left[{}^3S_1^{(1)}\right] + g$ we have computed $\eta \to \infty$ limit of the z and $\rho = \mathbf{p}_T^2/M^2$-differential NLO "scaling functions" in closed analytic form,



and obtained numerical results for NNLO "scaling function" c_2 in front of $\alpha_s \ln(1+\eta)$.



Inverse Error Weighting (InEW) matching

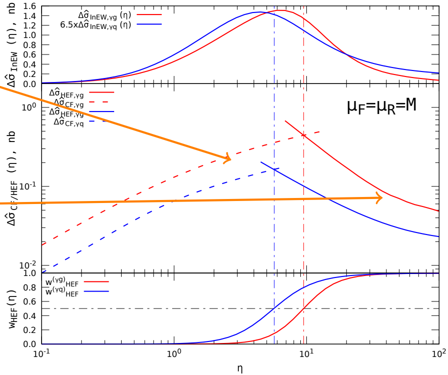
Development of an idea from [Echevarria *et al.*, 18'] :

$$\hat{\sigma}(\eta) = w_{\rm CF}(\eta)\hat{\sigma}_{\rm CF}(\eta) + (1 - w_{\rm CF}(\eta))\hat{\sigma}_{\rm HEF}(\eta),$$

the weights are determined through the estimates of “errors”:

$$w_{\rm CF}(\eta) = \frac{\Delta\hat{\sigma}^{-2}_{\rm CF}(\eta)}{\Delta\hat{\sigma}^{-2}_{\rm CF}(\eta) + \Delta\hat{\sigma}^{-2}_{\rm HEF}(\eta)}, \quad w_{\rm HEF}(\eta) = 1 - w_{\rm CF}(\eta).$$

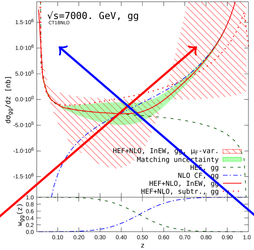
- $\Delta\hat{\sigma}_{\rm CF}(\eta)$ is due to **missing higher orders and large logarithms,** it can be estimated from the α_s expansion of $\hat{\sigma}_{\rm HEF}(\eta)$.
- $\Delta\hat{\sigma}_{\rm HEF}(\eta)$ is (mostly) due to **missing power corrections in** $1/\eta$: $\Delta\hat{\sigma}_{\rm HEF}(\eta) \sim A\eta^{-\alpha_{\rm HEF}}$. We determine A and $\alpha_{\rm HEF}$ from behaviour of $\hat{\sigma}_{\rm CF}(\eta) - \hat{\sigma}_{\rm CF}(\infty)$ at $\eta \gg 1$.



Matching with NLO

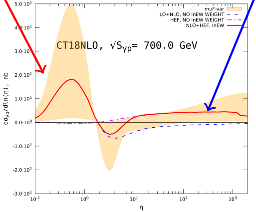
The HEF is valid in the **leading-power** in $M^2/\hat{s}$, so for $\hat{s} \sim M^2$ we match it with NLO CF by the *Inverse-Error Weighting Method* [Echevarria *et.al.*, 18'].

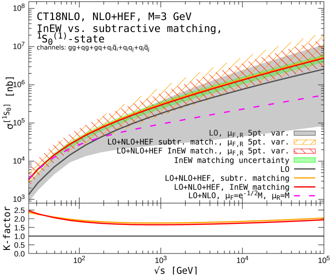
η_c**-hadroproduction**,
$z = M^2/\hat{s}$:

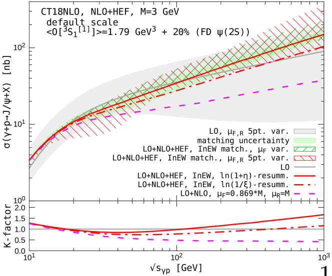


NLO **HEF**

J/ψ**-photoproduction**,
$\eta = (\hat{s} - M^2)/M^2$:





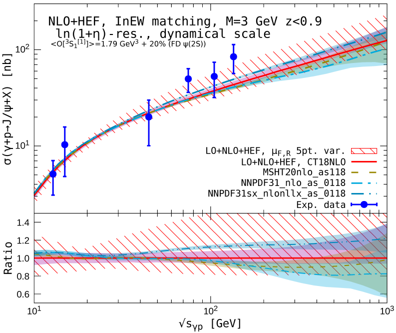


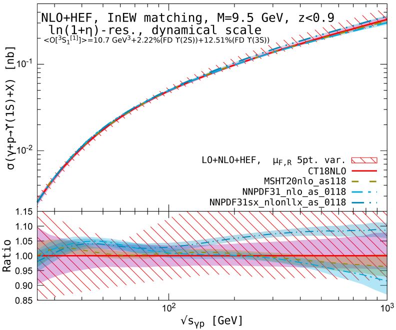
Vector quarkonium photoproduction: dynamical scale

Matched results for J/ψ photoproduction can be further improved by noticing that in the LO process:

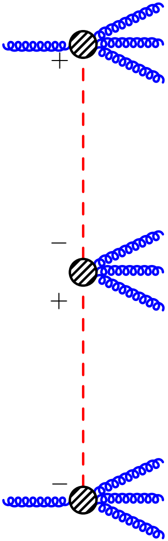
$$\gamma(q) + g(p_1) \to Q\bar{Q}\left[{}^3S_1^{[1]}\right] + g,$$

the emitted gluon can not be soft, so that $\langle \hat{s} \rangle_{\rm LO}$ (~ 25 GeV^2 at high $\sqrt{s_{\gamma p}}$ for J/ψ) rather than M^2 can be taken as a default value of μ_F^2 and μ_R^2:





Beyond DLA: The Gauge-Invariant High-Energy EFT



Reggeized gluon fields $R_\pm$ carry $(k_\pm, \mathbf{k}_T, k_\mp = 0)$.
Induced interactions of particles and Reggeons [Lipatov '95, '97; Bondarenko, Zubkov '18]:

$$\frac{i}{g_s}\,\mathrm{tr}\left[R_+\partial_\perp^2\partial_-\left(W\left[A_-\right]-W^\dagger\left[A_-\right]\right)+(+\leftrightarrow -)\right],$$

with $W_{x_\mp}[x_\pm, \mathbf{x}_T, A_\pm] = P\exp\left[\frac{-ig_s}{2}\int\limits_{-\infty}^{x_\mp} dx'_\mp A_\pm(x_\pm, x'_\mp, \mathbf{x}_T)\right] = \left(1+ig_s\partial_\pm^{-1}A_\pm\right)^{-1}$. Expansion of the Wilson line generates **induced vertices**:

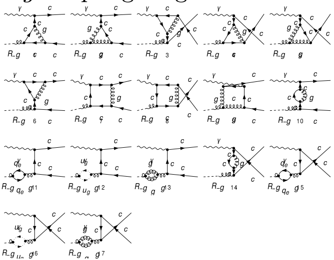
$$\mathrm{tr}\left[R_+\partial_\perp^2 A_- + (-ig_s)(\partial_\perp^2 R_+)(A_-\partial_-^{-1}A_-)\right.$$
$$\left.+(-ig_s)^2(\partial_\perp^2 R_+)(A_-\partial_-^{-1}A_-\partial_-^{-1}A_-)+O(g_s^3)+(+\leftrightarrow -)\right].$$

The *Eikonal propagators* $\partial_\pm^{-1} \to -i/(k^\pm + i\varepsilon)$ lead to **rapidity divergences**, which are regularized by tilting the Wilson lines from the light-cone [Hentschinski, Sabio Vera, Chachamis *et. al.*, '12-'13; M.N. '19]:

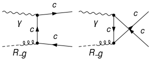
$$n_\pm^\mu \to \tilde{n}_\pm^\mu = n_\pm^\mu + r n_\mp^\mu,\ r \ll 1.$$

Beyond DLA: $R\gamma \to c\bar{c}\left[{}^1S_0^{(8)}\right]$ @ 1 loop

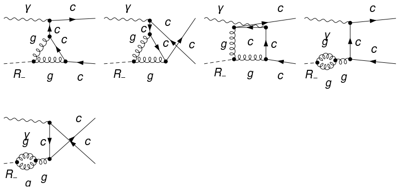
Rg-coupling diagrams:



Interference with LO:



Induced Rgg coupling diagrams:

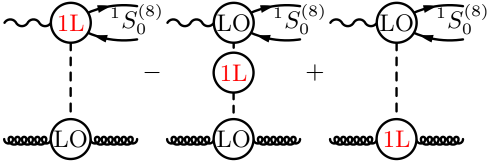


- Diagrams had been generated using custom `FeynArts` model-file, projector on the $c\bar{c}\left[{}^1S_0^{(8)}\right]$-state is inserted
- heavy-quark momenta $= p_{\mathcal{Q}}/2 \Rightarrow$ need to resolve linear dependence of quadratic denominators in some diagrams before IBP
- IBP reduction to master integrals has been performed using `FIRE`
- Rapidity-divergent scalar one-loop integrals with linear denominators had been computed for massless case with up to 2 external scales [Hentschinski, Sabio Vera, Chachamis *et. al.*, '12-'13; M.N. '19] . Scalar RD integrals with massive denominators arise, but they can be dealt with using:

$$\frac{1}{((\tilde{n}_+ l)+k_+)(l^2-m^2)} = \frac{1}{((\tilde{n}_+ l)+k_+)(l+\kappa\tilde{n}_+)^2} + \frac{\cancel{2\kappa\left[(\tilde{n}_+ l)+\frac{m^2+\tilde{n}_+^2\kappa^2}{2\kappa}\right]}}{\cancel{((\tilde{n}_+ l)+k_+)}(l+\kappa\tilde{n}_+)^2(l^2-m^2)}.$$

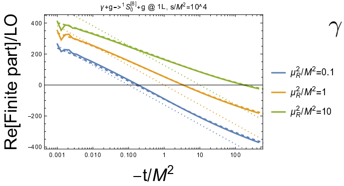
Beyond DLA: $R\gamma \to c\bar{c}\left[{}^1S_0^{(8)}\right]$ @ 1 loop, cross-checks

- The double logarithms $\ln^2 r$ cancel between diagrams, the single logarithm RD is proportional to the one-loop Regge trajectory of a gluon: $\omega_g(\mathbf{q}_T^2)\ln r$.
- In the combination of 1-loop results in the EFT:



the $\ln r$ cancels and it should reproduce the the Regge limit$(s \gg -t)$ of the *real part* of the $2 \to 2$ 1-loop QCD amplitude:

$$\gamma + g \to c\bar{c}\left[{}^1S_0^{(8)}\right] + g.$$



Solid lines – QCD, dashed lines – EFT, dotted lines – $-2C_A\ln(-t/\mu_R^2)\ln(s/M^2)$

- The $2 \to 2$ QCD 1-loop amplitude can be computed numerically using `FormCalc` (with some tricks, due to Coulomb divergence)
- The Regge limit of $1/\epsilon$ divergent part agrees with the EFT result
- For the finite part agreement within few % is reached, need to push to higher s

Beyond DLA: the "Monster logs" at small $\mathbf{q}_T$ are not scary

$$\hat{\sigma}_{\rm HEF}(\eta) \propto \int\limits_0^{1+\eta} \frac{dy}{y} \int\limits_0^{\infty} d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{y}{1+\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \mathcal{H}(y, \mathbf{q}_{T1}^2).$$

At NLO for $\mathcal{H}$ one typically encounters corrections $\propto \alpha_s \ln^n \frac{M^2}{\mathbf{q}_T^2}$ at $\mathbf{q}_T^2 \ll M^2$ with $n = 1, 2$. Let's study their effect in N-space (note that $\gamma_N = \hat{\alpha}_s/N$):

$$\int\limits_0^{\mu_F^2} d\mathbf{q}_T^2\, \mathcal{C}_{\rm DLA}(N, \mathbf{q}_T^2, \mu_F^2) \times \hat{\alpha}_s \ln^n \frac{\mu_F^2}{\mathbf{q}_T^2} = \hat{\alpha}_s \gamma_N \int\limits_0^{\mu_F^2} \frac{d\mathbf{q}_T^2}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2}\right)^{\gamma_N} \ln^n \frac{\mu_F^2}{\mathbf{q}_T^2}$$

$$= \hat{\alpha}_s \frac{(-1)^n n!}{\gamma_N^n} = \begin{cases} -N & \text{for } n=1 \\ \frac{2N^2}{\hat{\alpha}_s} & \text{for } n=2 \end{cases} \xrightarrow[\text{Mellin transform}]{} \begin{cases} -\delta'(\eta) & \text{for } n=1 \\ \frac{2}{\hat{\alpha}_s}\delta''(\eta) & \text{for } n=2 \end{cases}$$

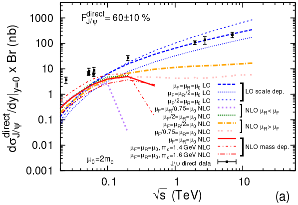
So these contributions *do not belong to NLA in* $\eta = (\hat{s} - M^2)/M^2 \gg 1$ and *will be removed by the matching!*

Conclusions and outlook

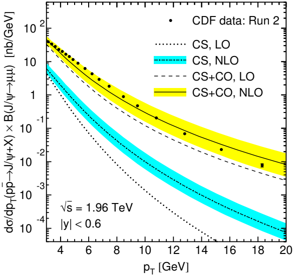
- The perturbative instability of p_T-integrated quarkonium production cross sections at NLO comes from the region $\hat{s} \gg M^2$
- The problem can be solved via matching of NLO calculation at $\hat{s} \sim M^2$ and LLA HEF calculation at $\hat{s} \gg M^2$
- The *Inverse-Error Weighting(InEW) method* is an efficient matching prescription without free parameters. The uncertainties due to matching are smaller than residual scale uncertainties
- The LLA HEF has to be truncated down to DLA for resummation factors, to be consistent with NLO DGLAP evolution
- The inclusive η_c hadroproduction and J/ψ photoproduction have been considered as examples
- The *next-to-DLA* calculation is needed to further reduce scale-uncertainties. Both virtual and real corrections to HEF coefficient function can be computed within the *High-Energy EFT* formalism
- The logarithms $\ln M^2/\mathbf{q}_T^2$ for $\mathbf{q}_T^2 \ll M^2$ in the NLO HEF coefficient function will not be a problem for the matching calculation!

There is a lot to do even at DLA+NLO!

[Y.Feng, *et al.*, '15]



[Butenschön, He, Kniehl, '11]



Mini workshop on overlap between QCD resummations

- 3 day mini-workshop (14–17 Jan. 2024) in "Centre Paul Langevin" in Aussois (France), right after **"Quarkonia as tools 2024"**
- Indico: `https://indico.cern.ch/event/1290502/`
- `maxim DOT nefedov AT desy DOT de`

Topics:

- Overlap between small-x and TMD resummations, rapidity divergences in both cases
- Problem of collinear/running coupling logarithms in BFKL
- BFKL beyond NLL approximation in $\mathcal{N} = 4$ SYM and QCD
- Threshold resummation
- From small to moderate x and back

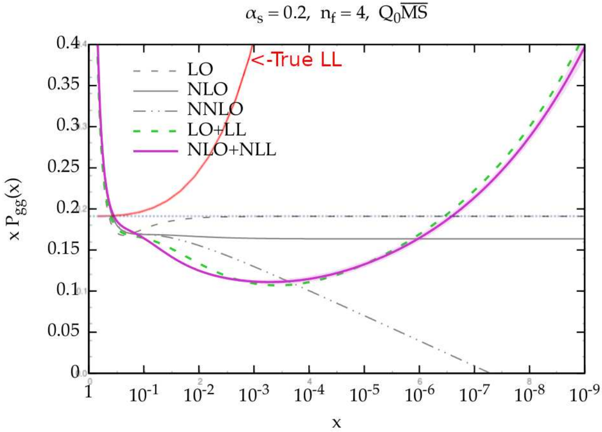
Thank you for your attention!

Backup: DGLAP P_{gg} at small z

LO: $P_{gg}(z) = \frac{2C_A}{z} + \ldots \Leftrightarrow \gamma_N = \frac{\hat{\alpha}_s}{N}$

Plot from hep-ph/1607.02153 with my curve (in red) for the **strict LLA:**

$\frac{\hat{\alpha}_s}{N}\chi_{LO}(\gamma_{gg}(N)) = 1 \Rightarrow \gamma_{gg}(N) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{N^6} + \ldots$



The "LO+LL" and "NLO+NLL" curves represent a form of matching between DGLAP and BFKL expansions, in a scheme by Altarelli, Ball and Forte which is more complicated than the **strict LL or NLL approximation**.

Effect of anomalous dimension beyond LO

Effect of taking **full LLA** for $\gamma_{gg}(N) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{N^6} + \ldots$ together with NLO PDF.