



# Generalized Parton Distributions from Lattice QCD: New Developments

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May 2023, QCD Evolution, Orsay, France

no introduction needed

answers to many ‘big questions’ – origin of proton’s mass and spin – are intimately connected to its GPDs

very difficult to access full  $(x, \xi, t)$  of GPDs from ongoing and upcoming experiments

need complimentary theoretical knowledge  
on  $(x, \xi, t)$  dependence from (lattice) QCD

this talk: new developments (past ~6 months)

# new developments

a novel Lorentz invariant formalism for lattice QCD calculations of GPD

fast: ~10 times faster access to t-dependence of GPD

accurate: reduces frame-dependent power corrections

Shohini Bhattacharya (BNL) *et al.*, [Phys. Rev. D 106, 1, 114512 \(2022\)](#)

## Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Unpolarized Quarks

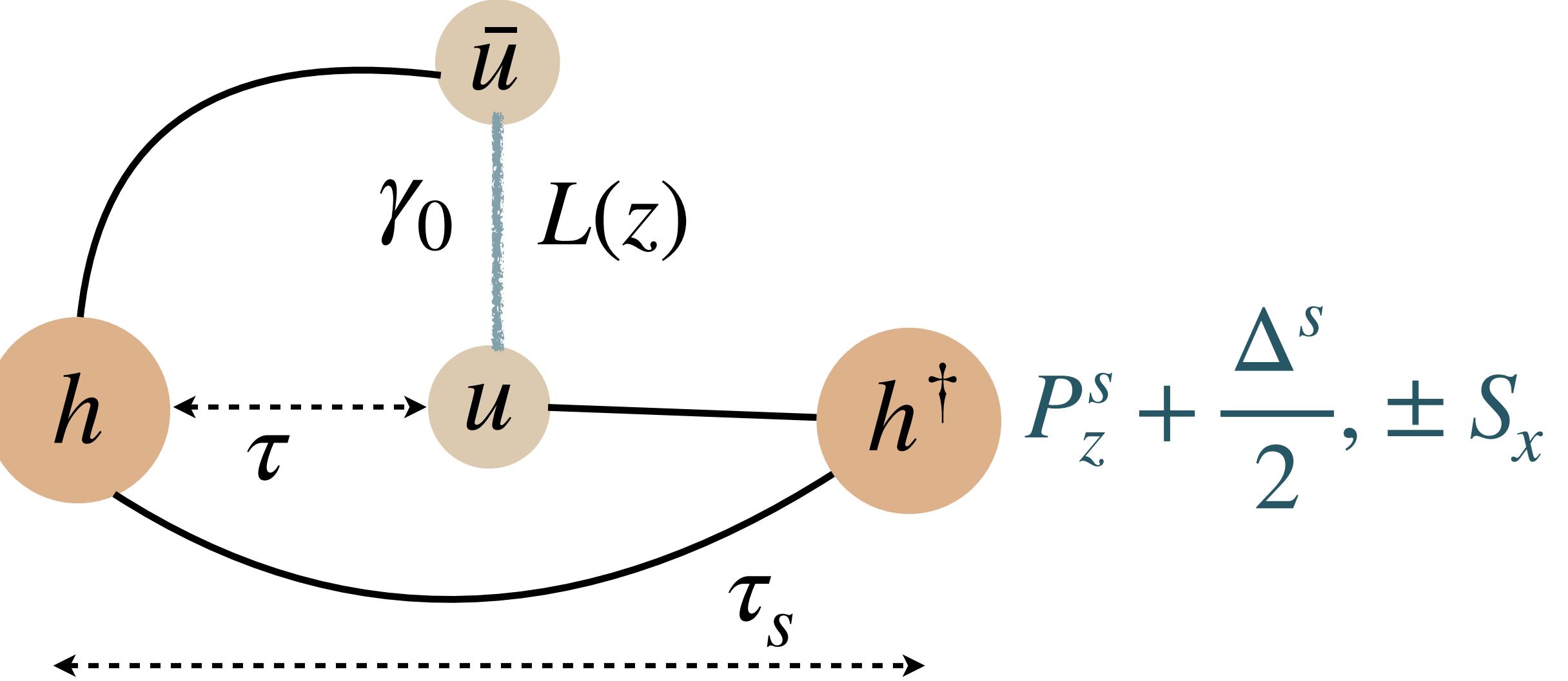
Shohini Bhattacharya,<sup>1,\*</sup> Krzysztof Cichy,<sup>2</sup> Martha Constantinou,<sup>3,†</sup> Jack Dodson,<sup>3</sup> Xiang Gao,<sup>4</sup> Andreas Metz,<sup>3</sup> Swagato Mukherjee,<sup>1</sup> Aurora Scapellato,<sup>3</sup> Fernanda Steffens,<sup>5</sup> and Yong Zhao<sup>4</sup>



our life before

symmetric  
momenta transfer

$$P_z^s - \frac{\Delta^s}{2}, \pm S_x$$



$\mathcal{H}_0^s, \mathcal{E}_0^s$ : pseudo-/quasi-GPD

+ pQCD matching

+  $z^2 \rightarrow 0, P_z \rightarrow \infty$

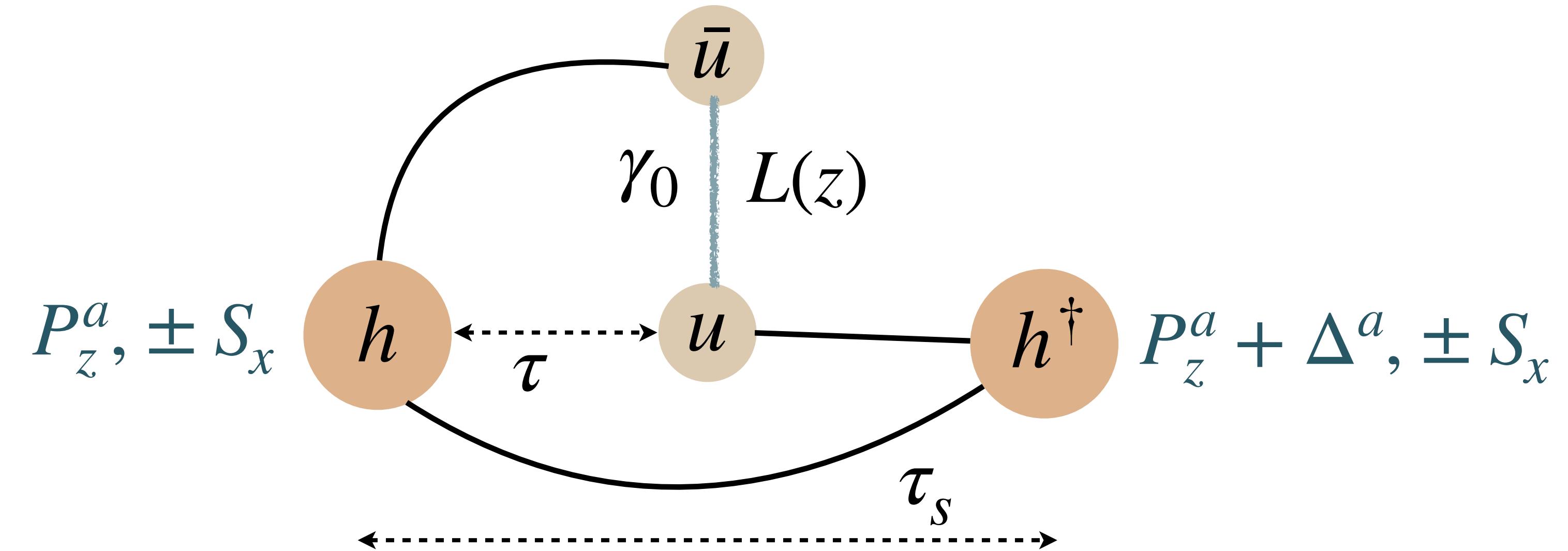
light-cone GPD:  $H, E$

$$F_0^s = \bar{u} \left[ \gamma_0 \mathcal{H}_0^s + \frac{i\sigma^{0\mu} \Delta_\mu^s}{2m} \mathcal{E}_0^s \right] u$$

- need a separate calculation for each  $\Delta^2 = -t$

life we wanted

asymmetric  
momenta transfer



- multiple  $t$  within a single calculation
- each calculation is  $2 \times$  faster than symmetric frame
  - ~ 10 time faster access to t dependence of GPD

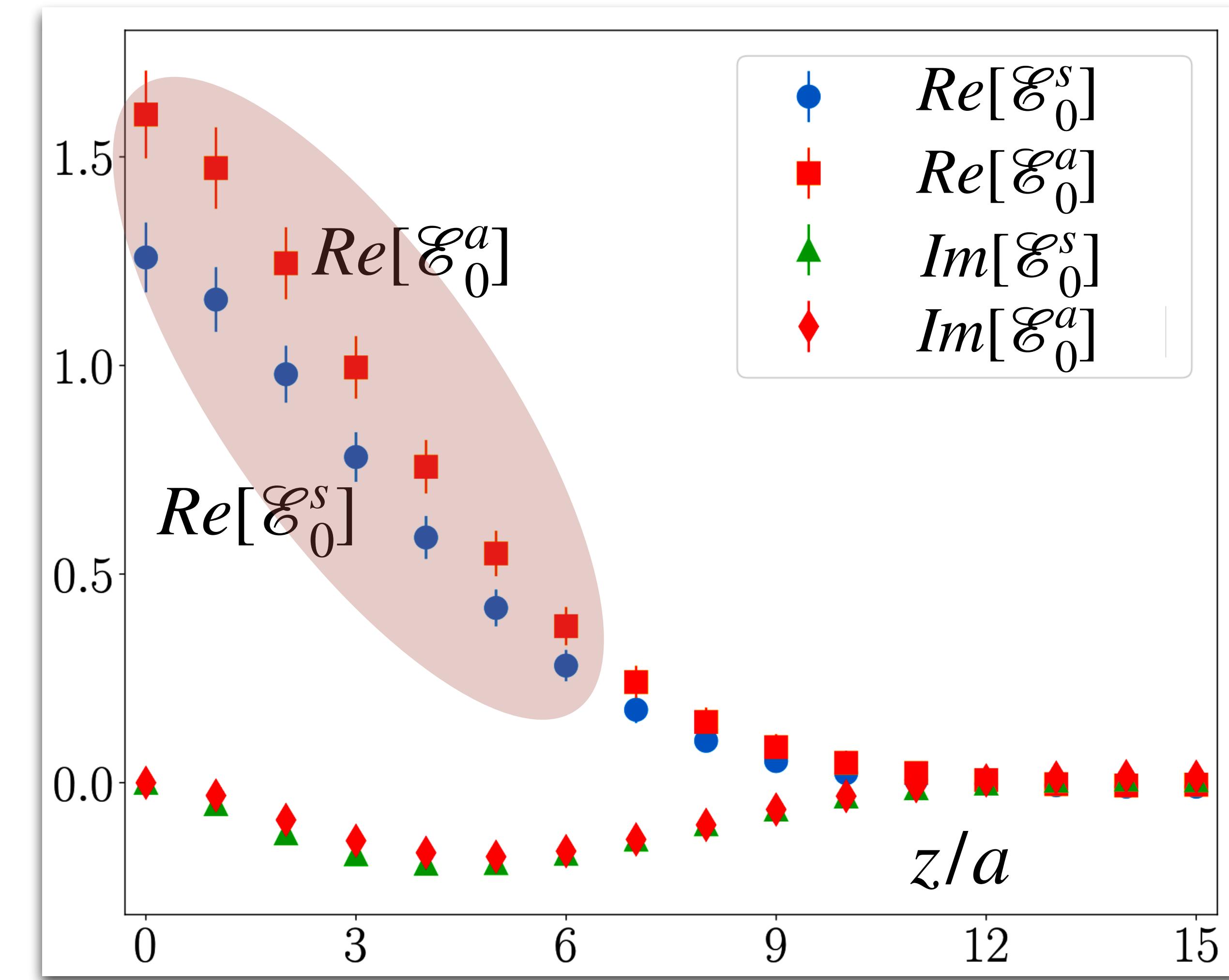
our naive life

frame-dependent  
power corrections

$P_z = 1.25 \text{ GeV}$

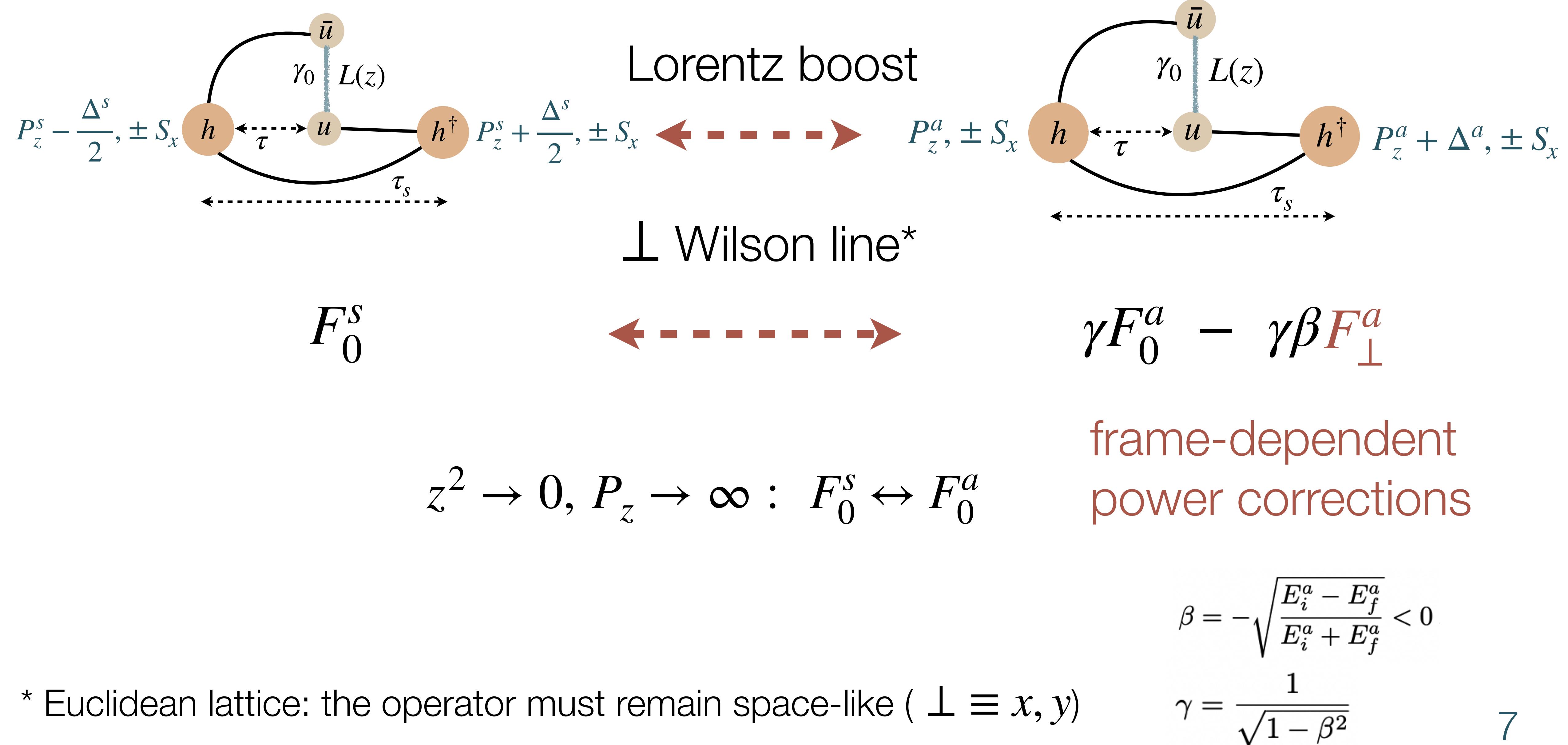
$t = -0.67 \text{ GeV}^2$

$\xi = 0$



$m_\pi = 260 \text{ MeV}$ ,  $a = 0.093 \text{ fm}$ ,  $32^3 \times 64$ ,  $N_f = 2 + 1 + 1$  twisted mass fermions

# putting on our thinking caps



# Lorentz invariant formalism

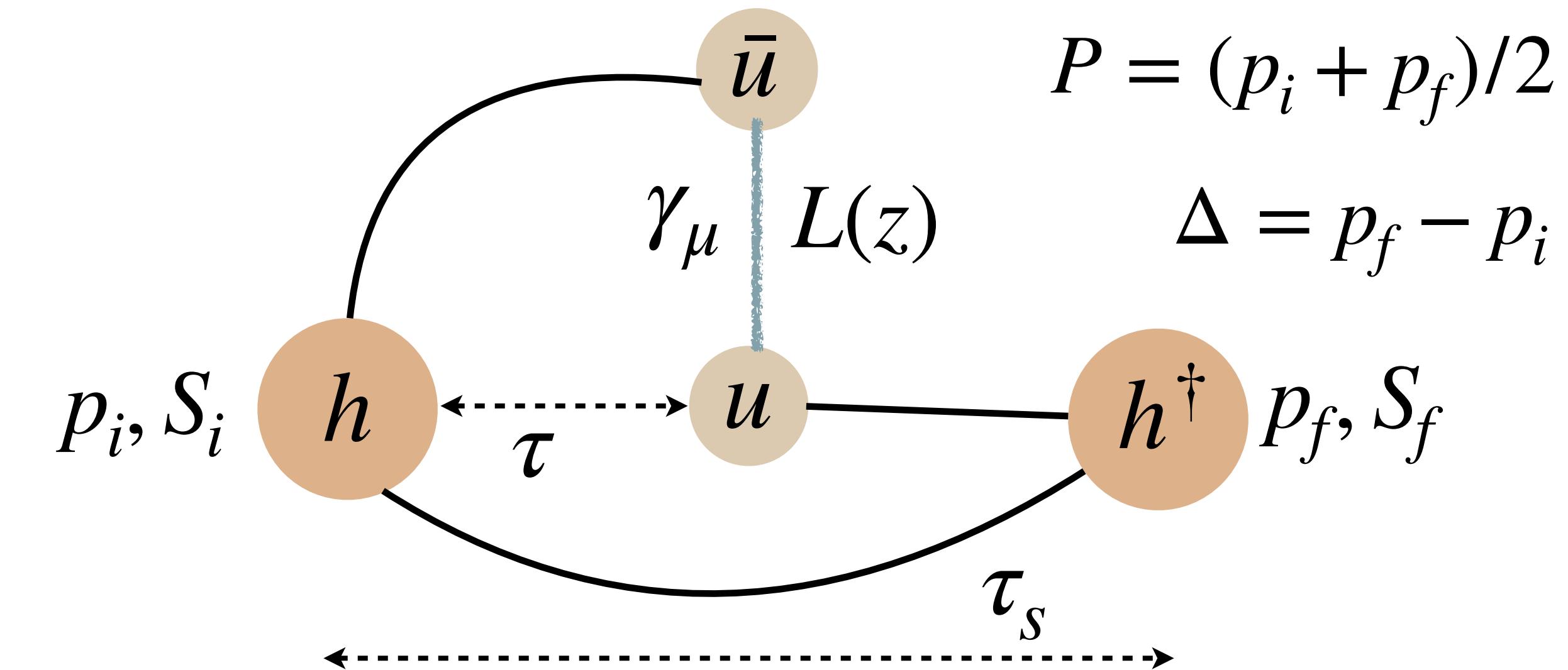
Lorentz covariant  
parameterization:

$$F^\mu(z, P, \Delta)$$

$$\begin{aligned} F^\mu(z, P, \Delta) = \bar{u}(p_f, \lambda') \left[ \frac{P^\mu}{m} A_1 + m z^\mu A_2 + \frac{\Delta^\mu}{m} A_3 + i m \sigma^{\mu z} A_4 + \frac{i \sigma^{\mu \Delta}}{m} A_5 \right. \\ \left. + \frac{P^\mu i \sigma^{z \Delta}}{m} A_6 + m z^\mu i \sigma^{z \Delta} A_7 + \frac{\Delta^\mu i \sigma^{z \Delta}}{m} A_8 \right] u(p_i, \lambda) \end{aligned}$$

8 Lorentz invariant amplitudes:  $A_i(z \cdot P, z \cdot \Delta, \Delta^2, z^2)$

extract  $A_i$  in any frame by combining  $F^\mu$  with varying  $S_i, S_f, \mu$



$$P = (p_i + p_f)/2$$

$$\Delta = p_f - p_i$$

$$h^\dagger p_f, S_f$$

from  $A_i$  to GPD: Lorentz invariant mapping

$$F^+ = \bar{u} \left[ \gamma^+ \mathcal{H}_{LI} + \frac{i\sigma^{+\mu} \Delta_\mu}{2m} \mathcal{E}_{LI} \right] u$$

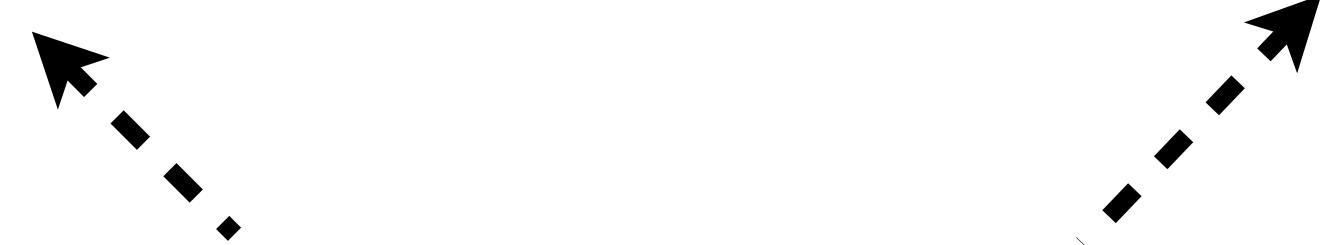
$$\mathcal{H}_{LI} = A_1 + \left( \frac{\Delta \cdot z}{P \cdot z} \right) A_3$$

$$\mathcal{E}_{LI} = -A_1 - \left( \frac{\Delta \cdot z}{\Delta \cdot z} \right) A_3 + 2A_5 + 2(P \cdot z)A_6 + 2(\Delta \cdot z)A_8$$

frame-dependent mapping:

$$\mathcal{H}_0^{s/a} = \sum h_i^{s/a} A_i$$

$$\mathcal{E}_0^{s/a} = \sum e_i^{s/a} A_i$$



frame-dependent kinematic factors

from  $A_i$  to GPD: Lorentz invariant mapping

$$F^+ = \bar{u} \left[ \gamma^+ \mathcal{H}_{LI} + \frac{i\sigma^{+\mu} \Delta_\mu}{2m} \mathcal{E}_{LI} \right] u$$

$$\mathcal{H}_{LI} = A_1 + \left( \frac{\Delta \cdot z}{P \cdot z} \right) A_3$$

~~frame-dependent power corrections~~

$$\mathcal{E}_{LI} = -A_1 - \left( \frac{\Delta \cdot z}{\Delta \cdot z} \right) A_3 + 2A_5 + 2(P \cdot z)A_6 + 2(\Delta \cdot z)A_8$$

$$z^2 \rightarrow 0, P_z \rightarrow \infty :$$

$$\mathcal{H}_0^{s/a} \rightarrow \mathcal{H}_{LI}$$

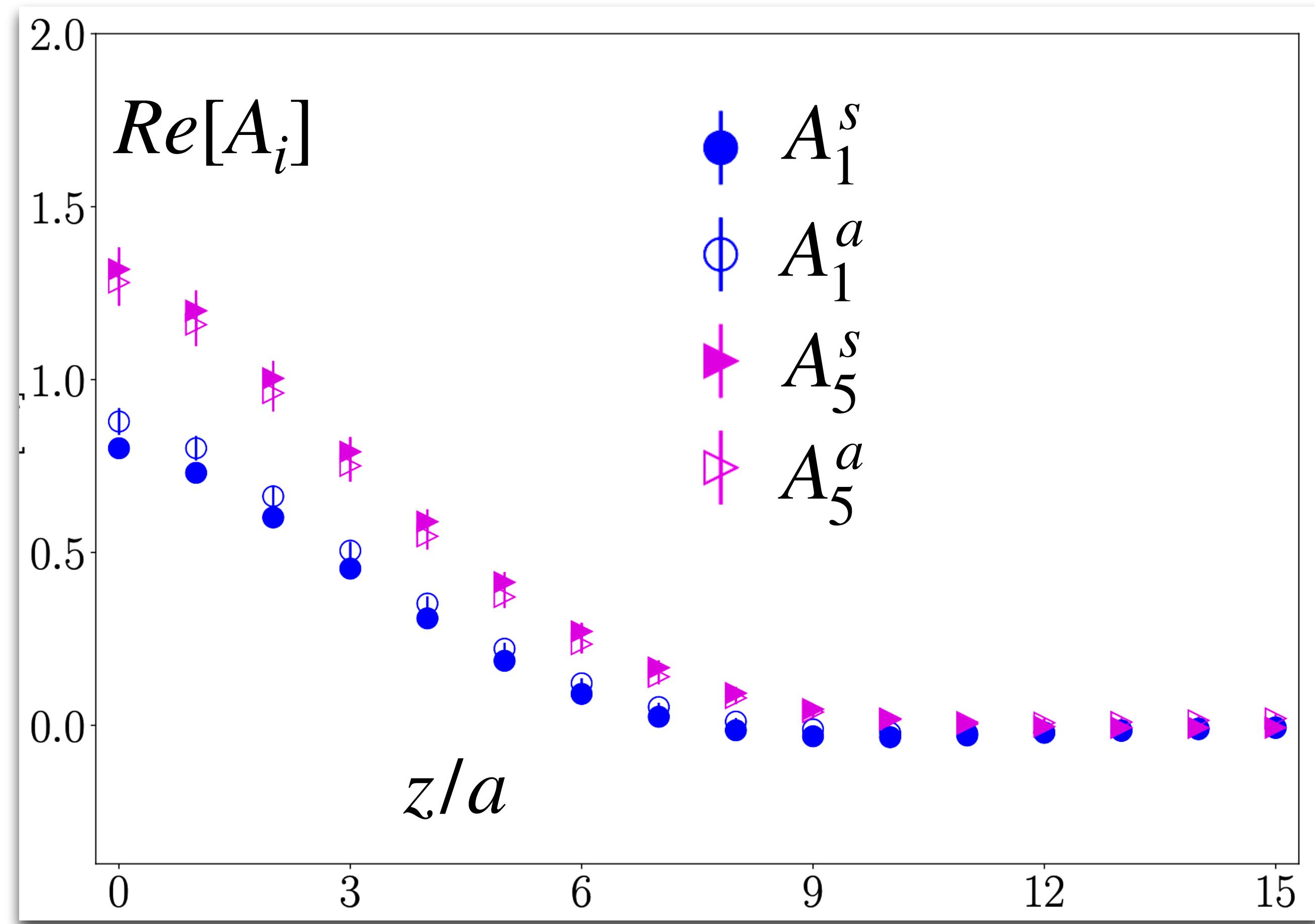
$$\mathcal{E}_0^{s/a} \rightarrow \mathcal{E}_{LI}$$

$A_i$  are frame independent

$P_z = 1.25 \text{ GeV}$

$t = -0.67 \text{ GeV}^2$

$\xi = 0$



filled symbols:  
symmetric frame

unfilled symbols:  
asymmetric frame

# moments of proton GPD: t dependence

$$\int_{-1}^1 x^n H^q(x, \xi = 0, t) dx = A_{n+1,0}^q(t)$$

$$\int_{-1}^1 x^n E^q(x, \xi = 0, t) dx = B_{n+1,0}^q(t)$$

short-distance expansions of  $\mathcal{H}$  and  $\mathcal{E}$  in  $z^2$

$$M_H(z, P, \Delta) = \frac{\mathcal{H}(z, P, \Delta)}{\mathcal{H}(z, P = 0, \Delta = 0)} = \sum_{n=0} \frac{(-izP_z)^n}{n!} \frac{C_n^{\overline{MS}}(\mu^2 z^2)}{C_0^{\overline{MS}}(\mu^2 z^2)} A_{n+1,0}(t) + \mathcal{O}(\Lambda_{QCD}^2 z^2)$$

$C_n^{\overline{MS}}(\mu^2 z^2)$  up to NNLO+RGE

$\mu = 2$  GeV

Xiang Gao (ANL) et al, [2305.11117](#)

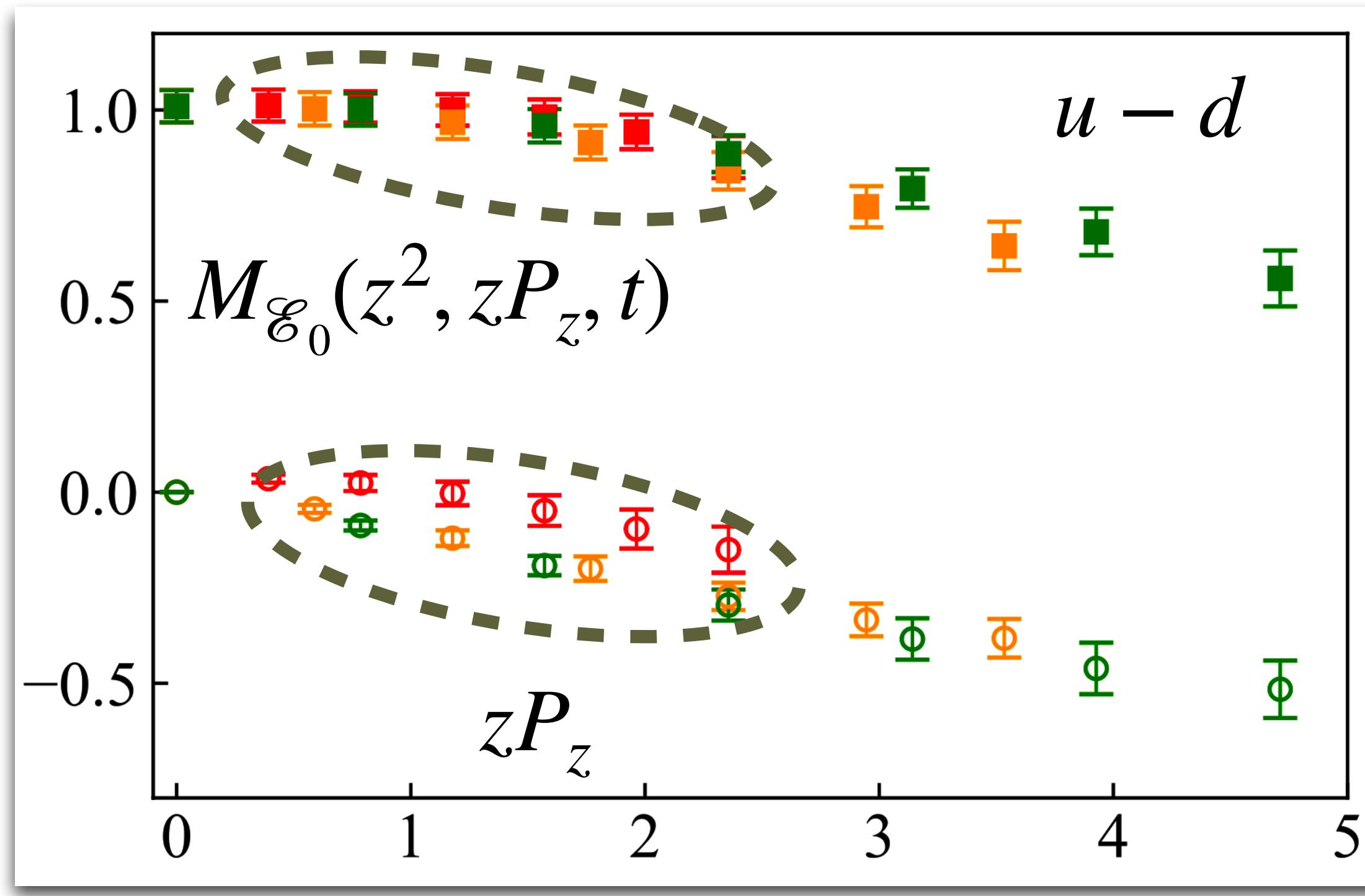
Moments of proton GPDs from the OPE of nonlocal quark bilinears up to NNLO

Shohini Bhattacharya,<sup>1</sup> Krzysztof Cichy,<sup>2</sup> Martha Constantinou,<sup>3</sup> Xiang Gao,<sup>4,\*</sup> Andreas Metz,<sup>3</sup> Joshua Miller,<sup>3</sup> Swagato Mukherjee,<sup>5</sup> Peter Petreczky,<sup>5</sup> Fernanda Steffens,<sup>6</sup> and Yong Zhao<sup>4</sup>



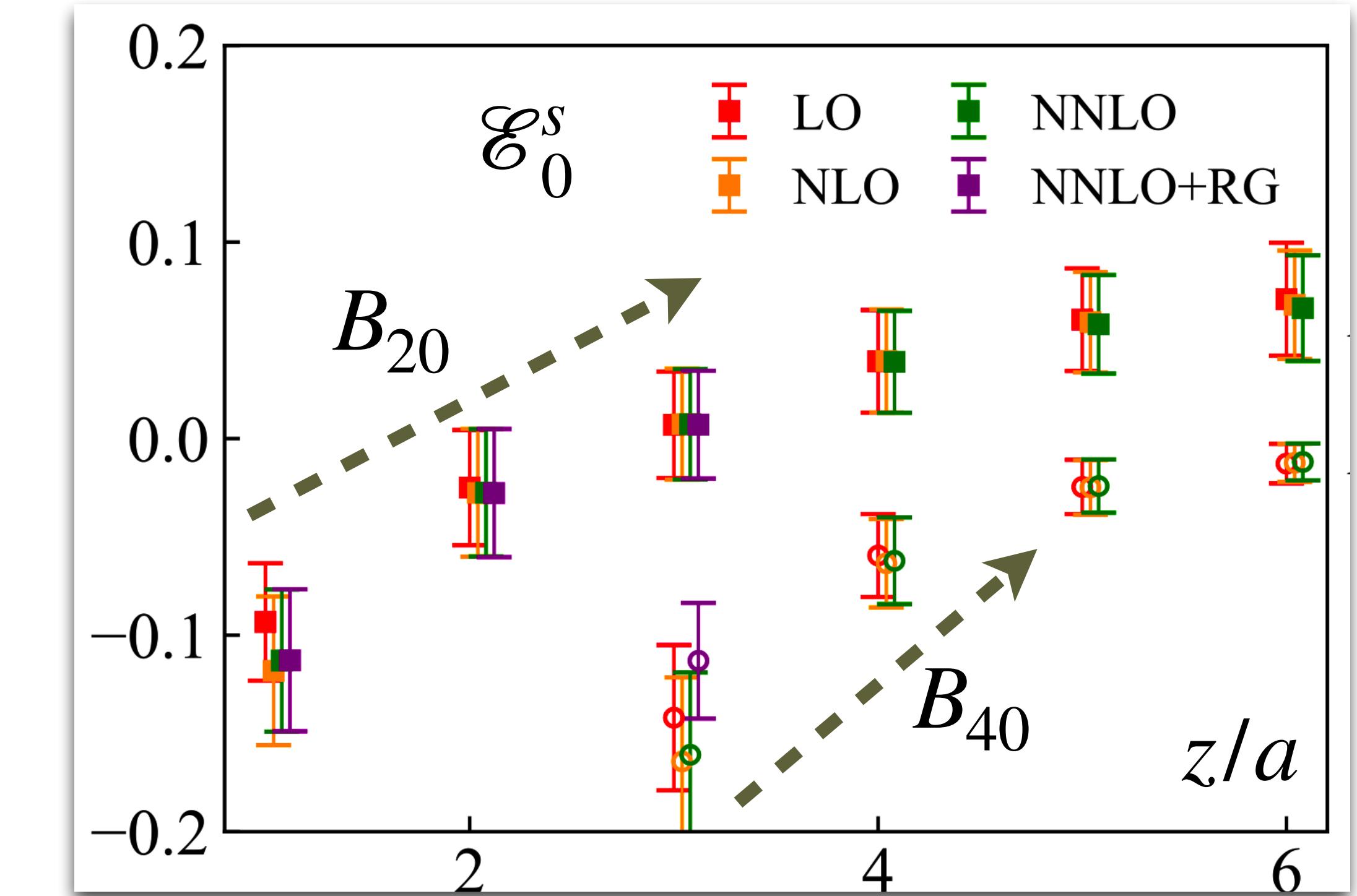
old vs. new

large power corrections for traditional definitions



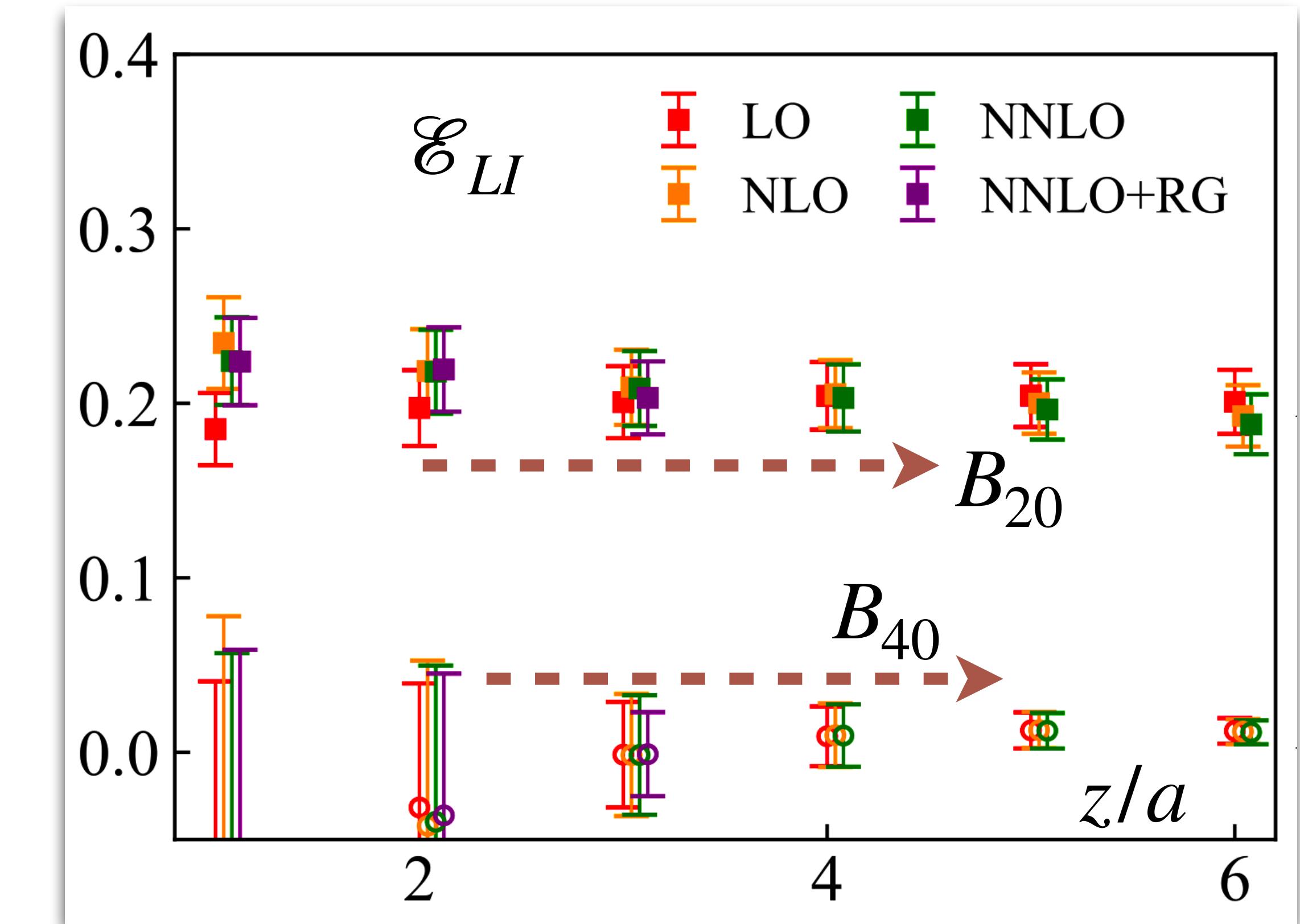
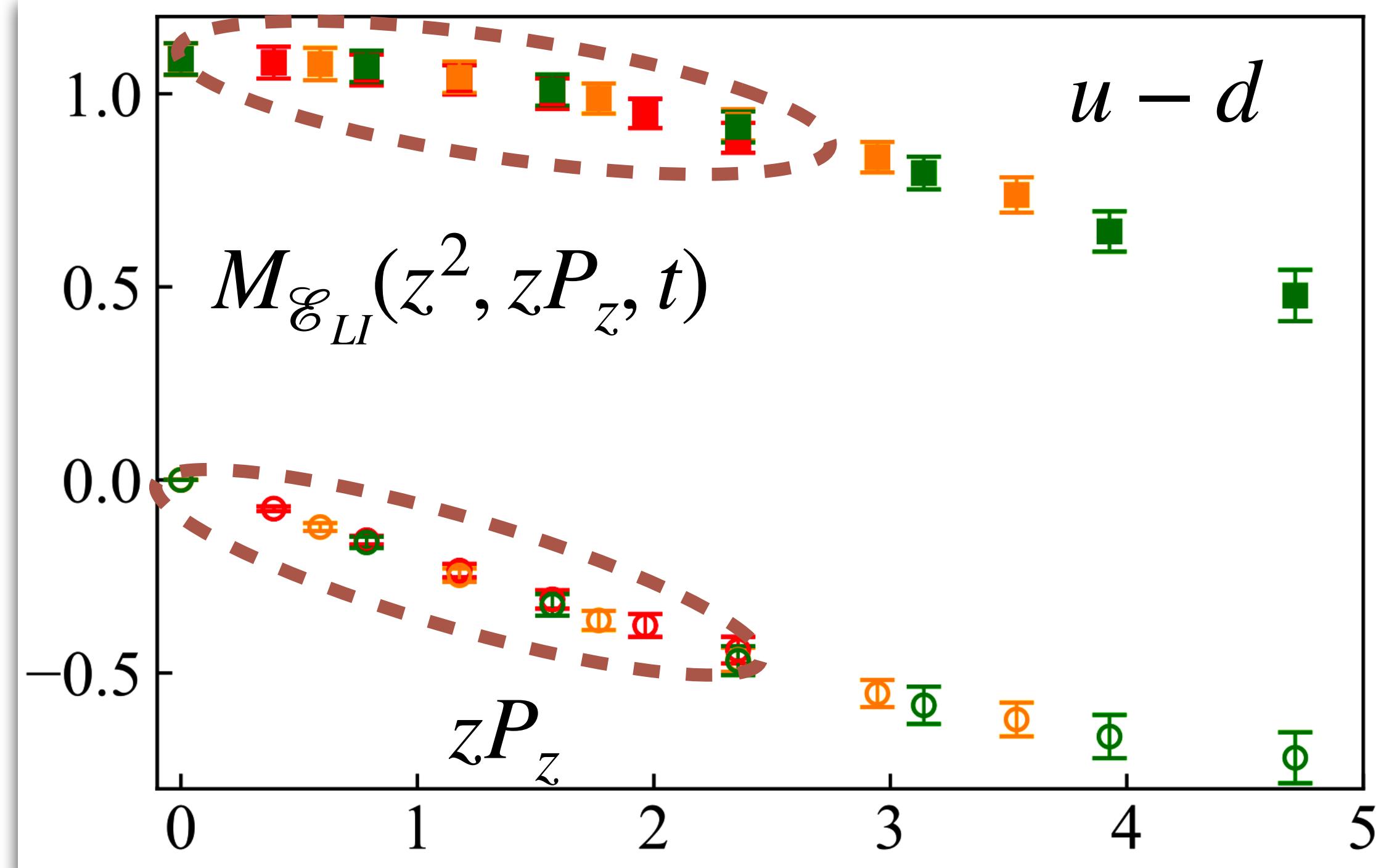
no scaling with  $zP_z$

$$P_z = 0.83, 1.25, 1.67 \text{ GeV}$$



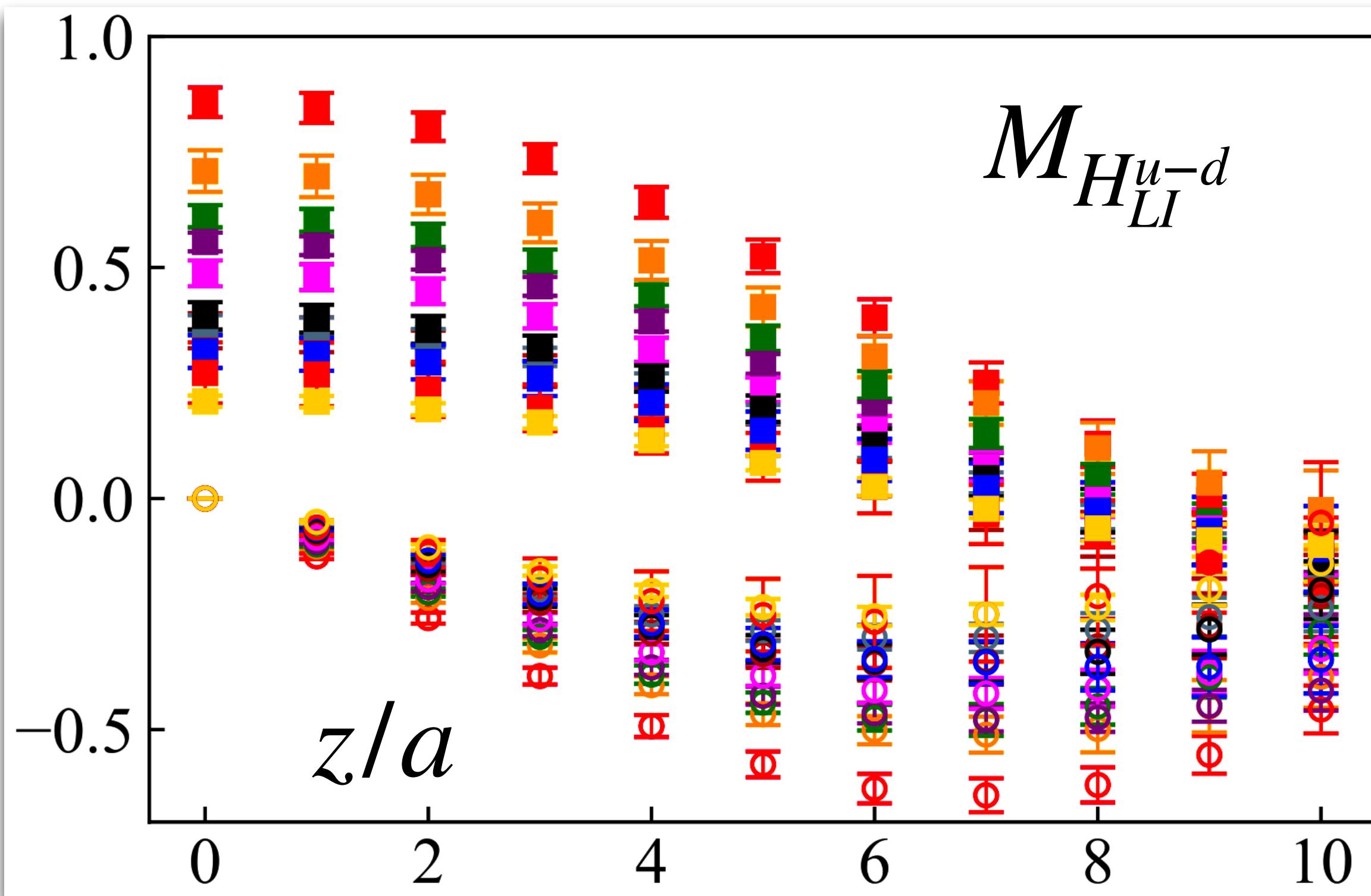
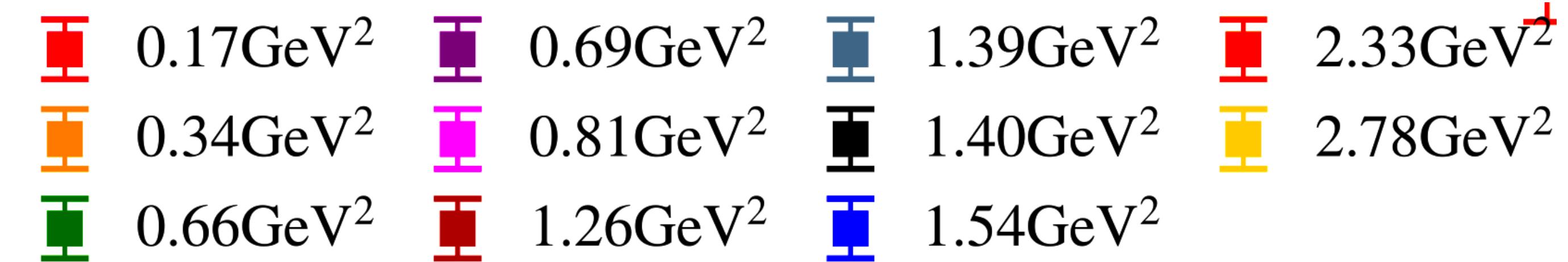
not constant in  $z$

negligible power corrections, stable moments



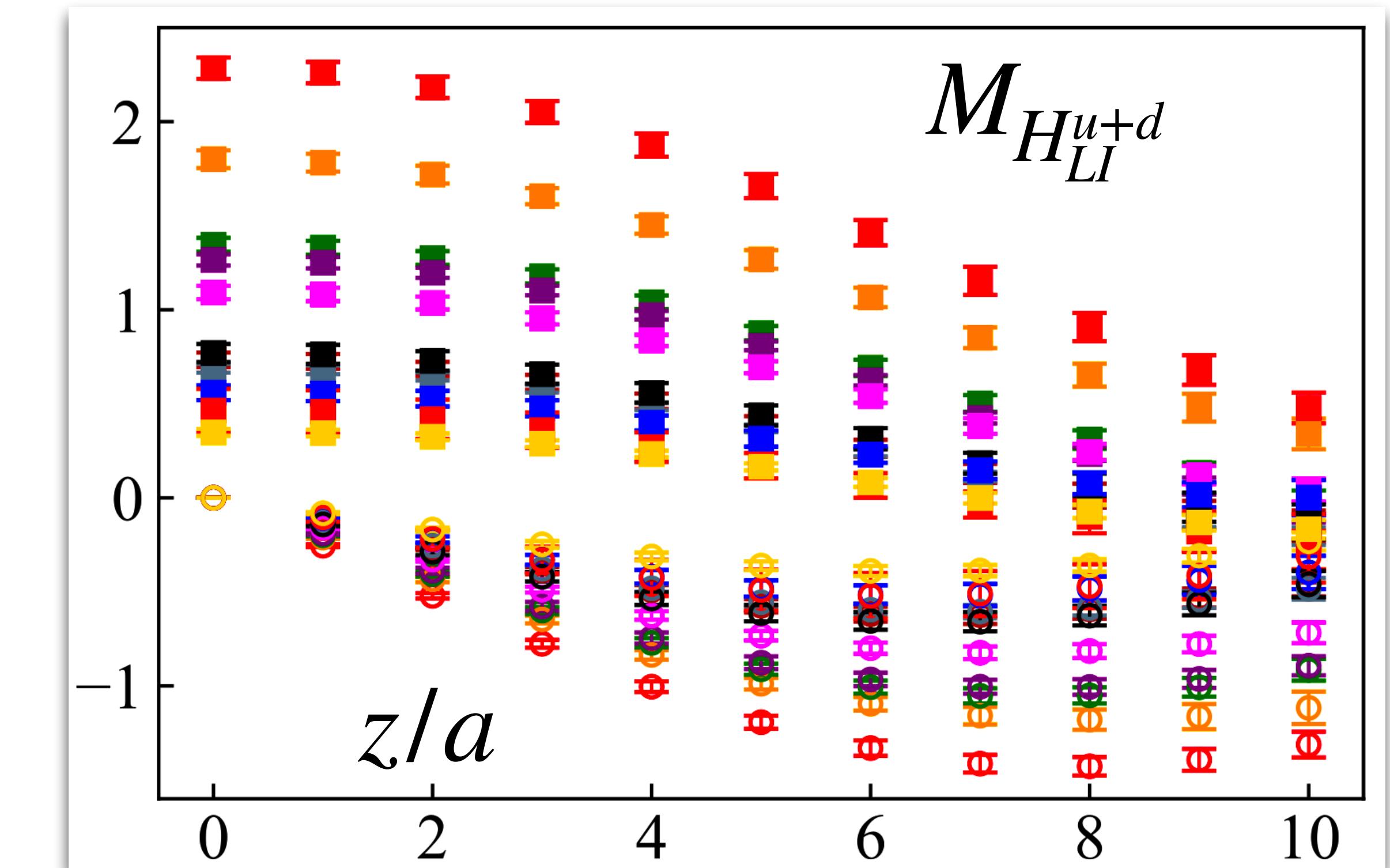
unleashing its full power

$t =$



filled symbols: real part

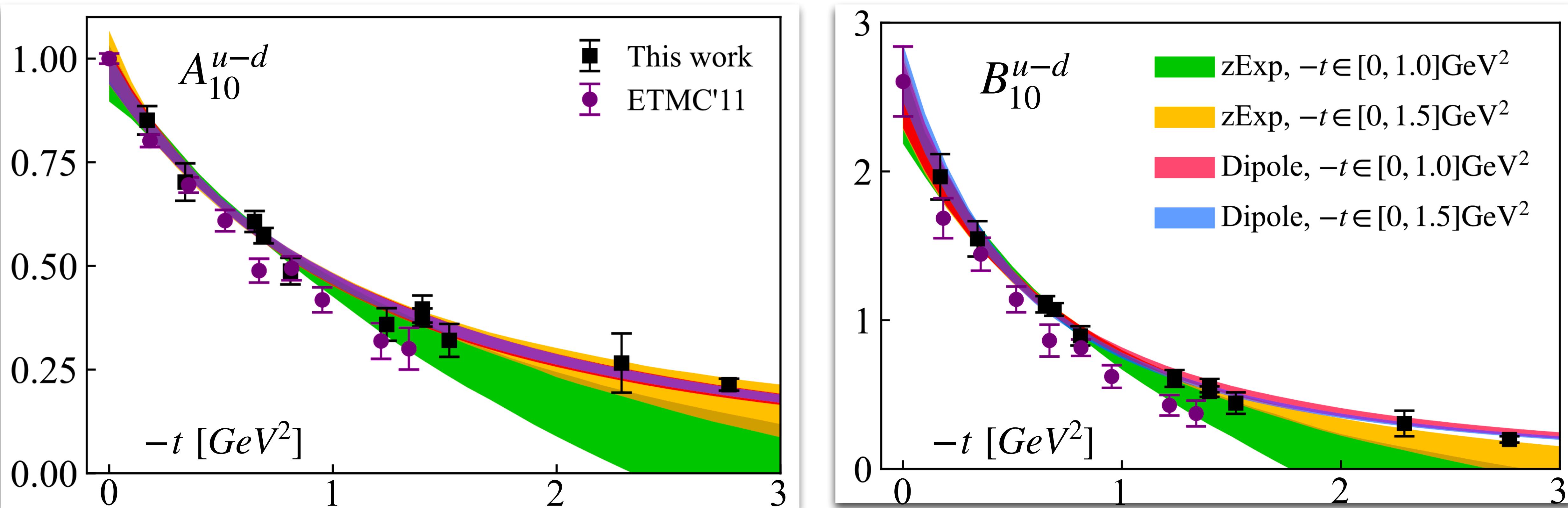
unfilled symbols: imaginary part



disconnected diagrams neglected

$P_z = 1.25 \text{ GeV}$

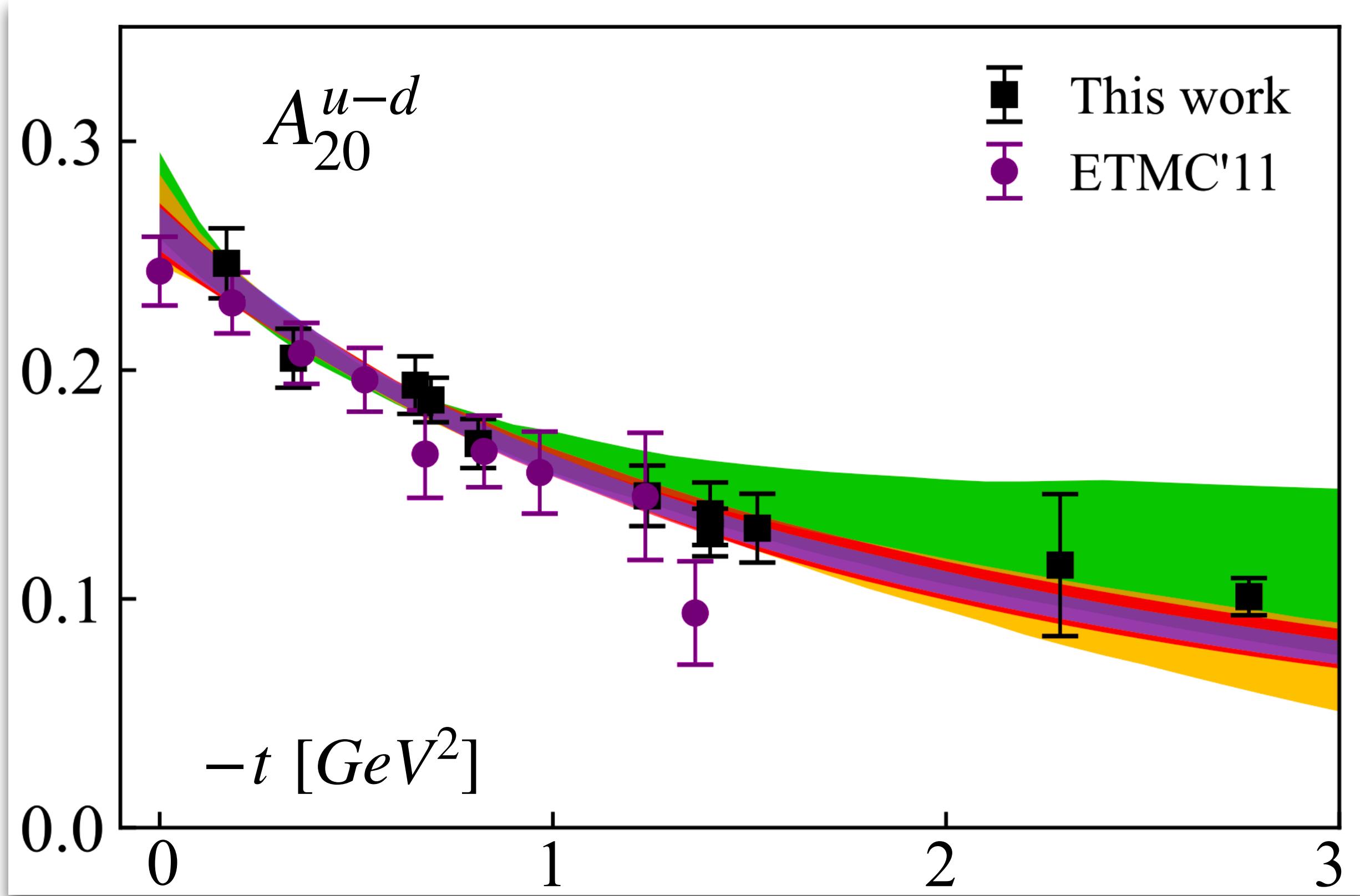
good agreement with traditional lattice QCD calculations  
of GPD moments using local operators, when available



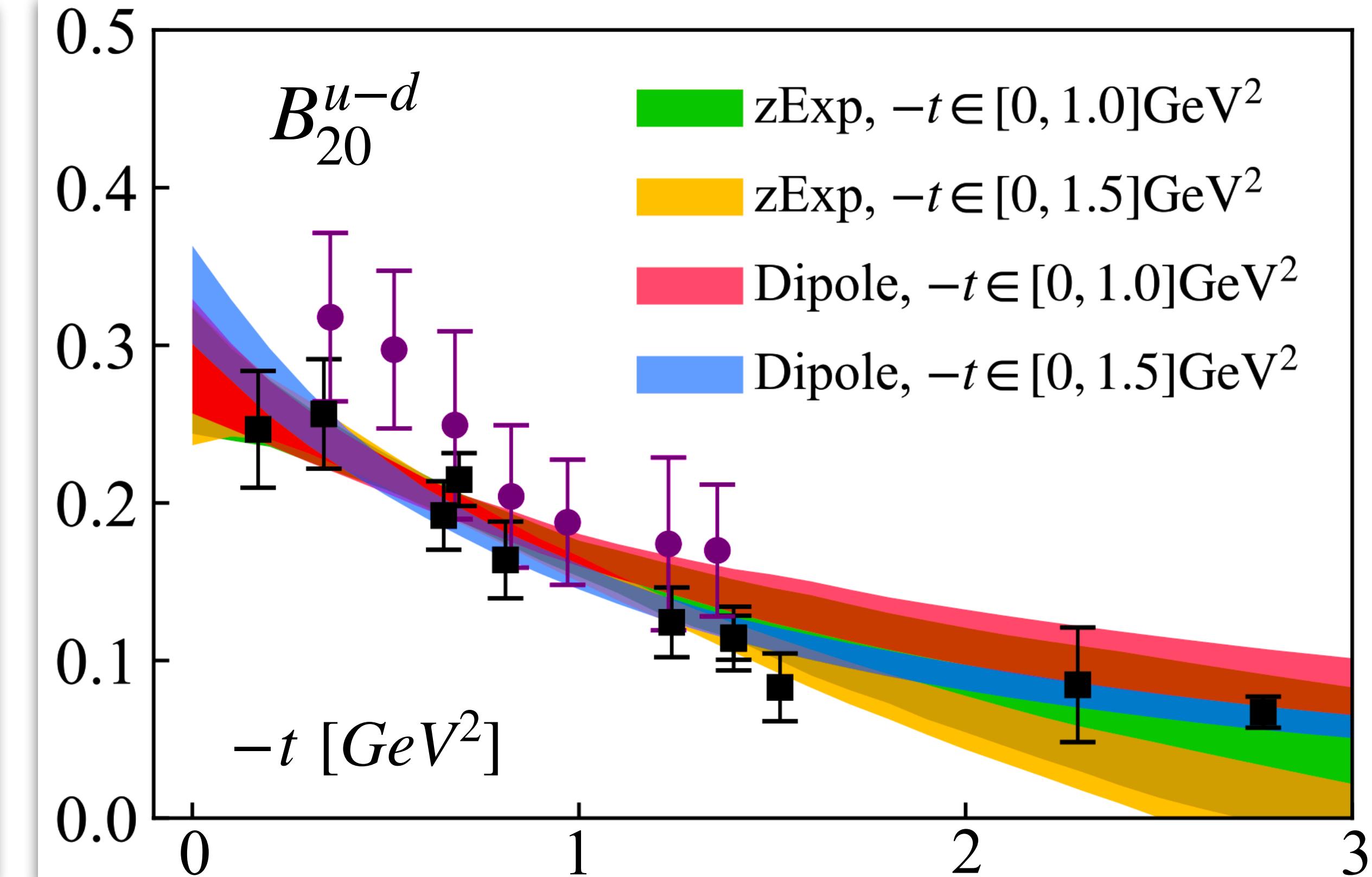
black squares: OPE of nonlocal quark bilinear

$\mu = 2 \text{ GeV}$

purple circles: local operator



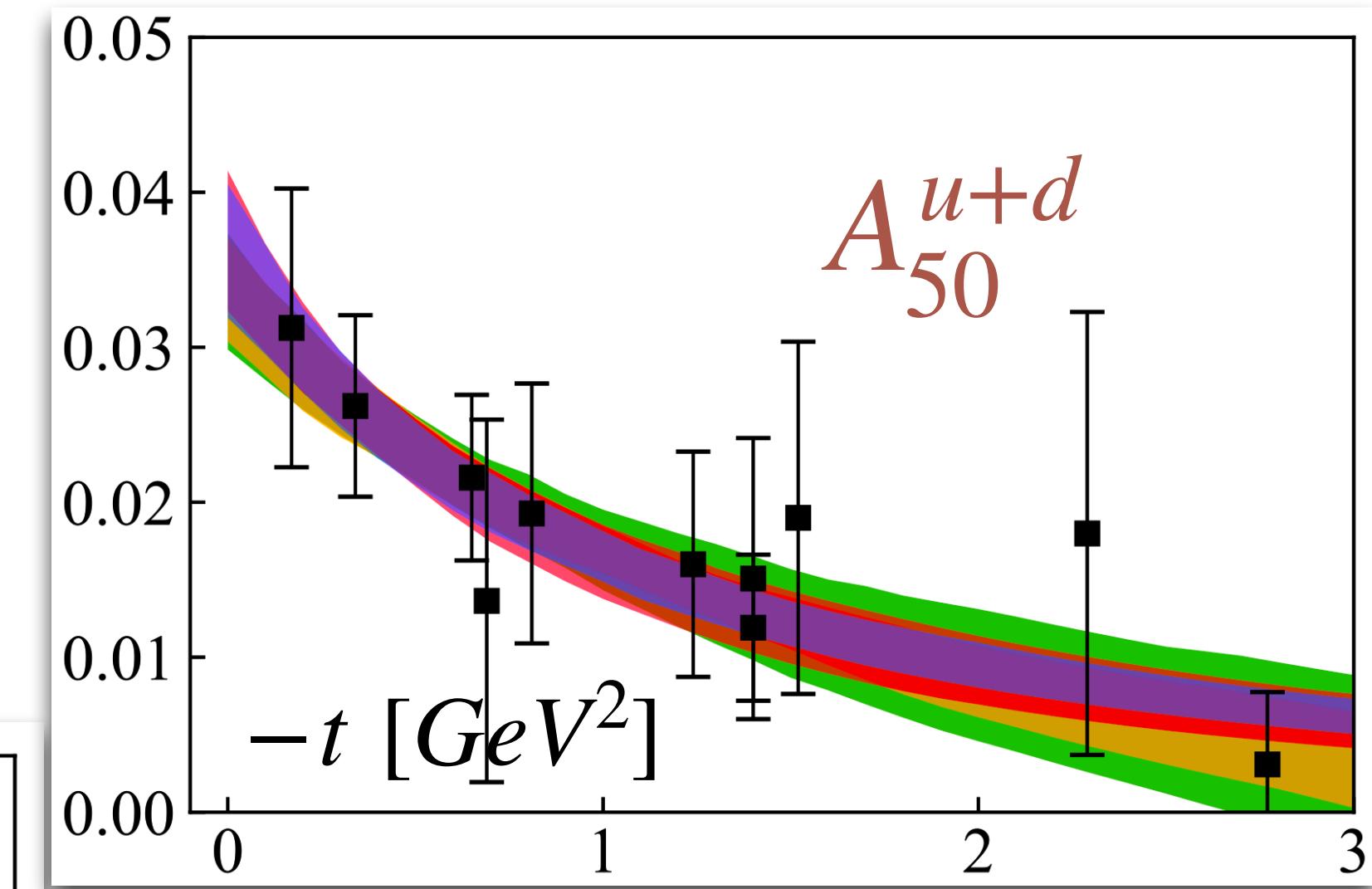
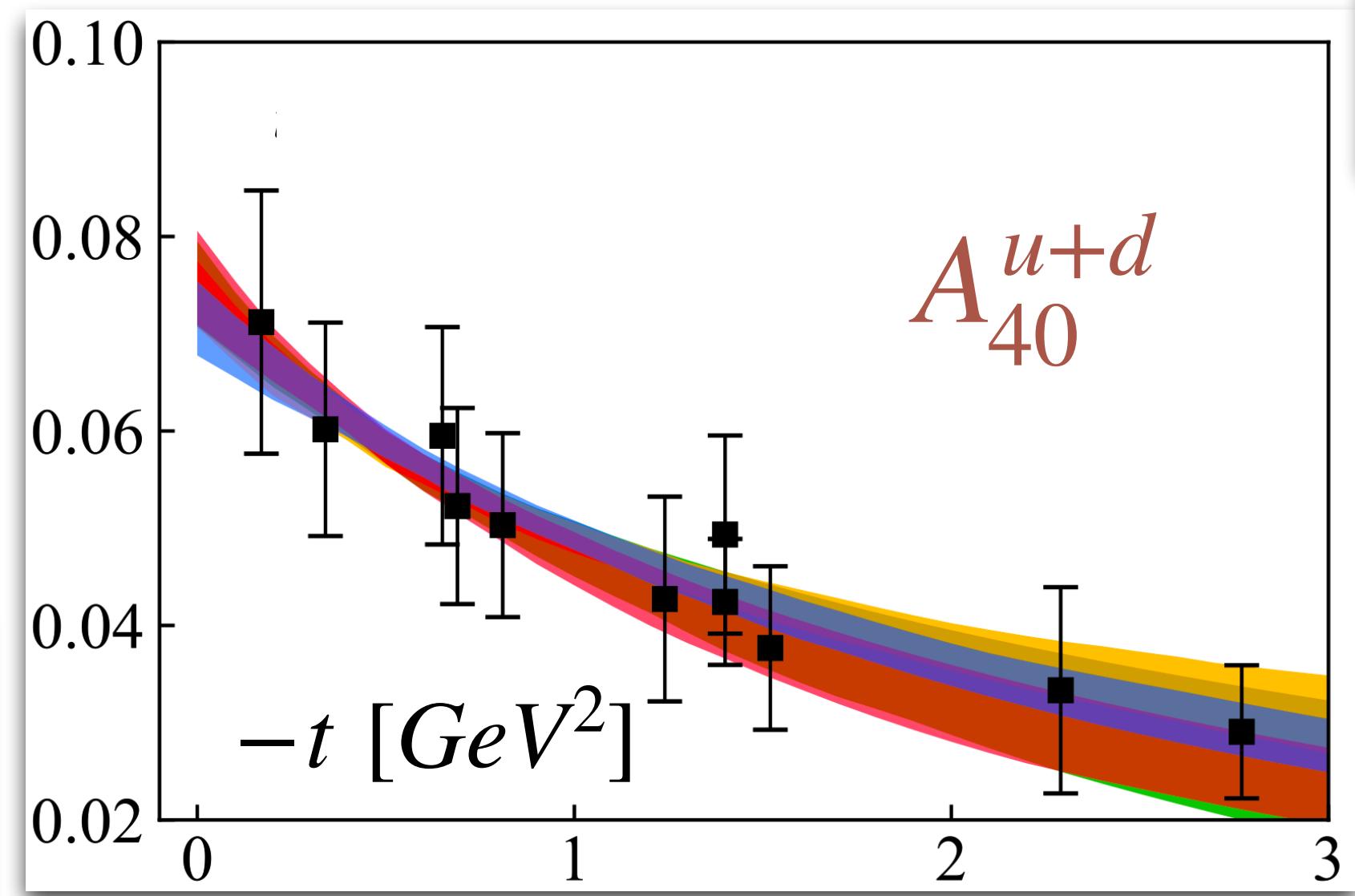
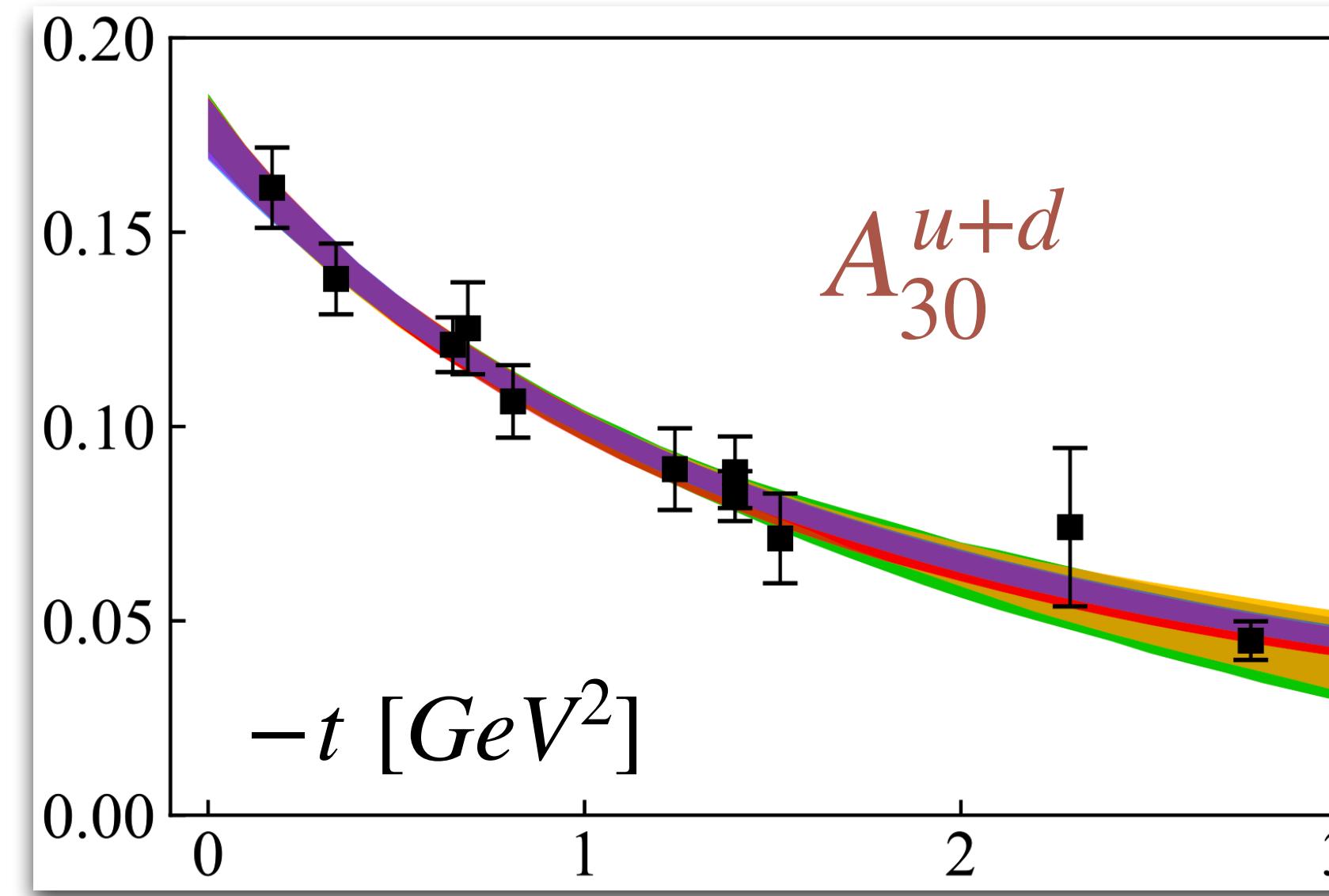
black squares: OPE of nonlocal quark bilinear



purple circles: local operator

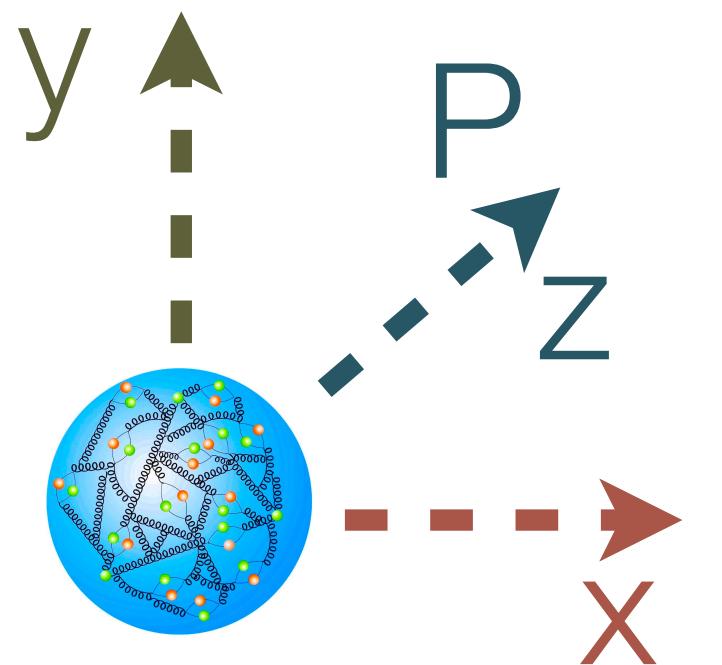
we got more ...

black squares: OPE of nonlocal quark bilinear



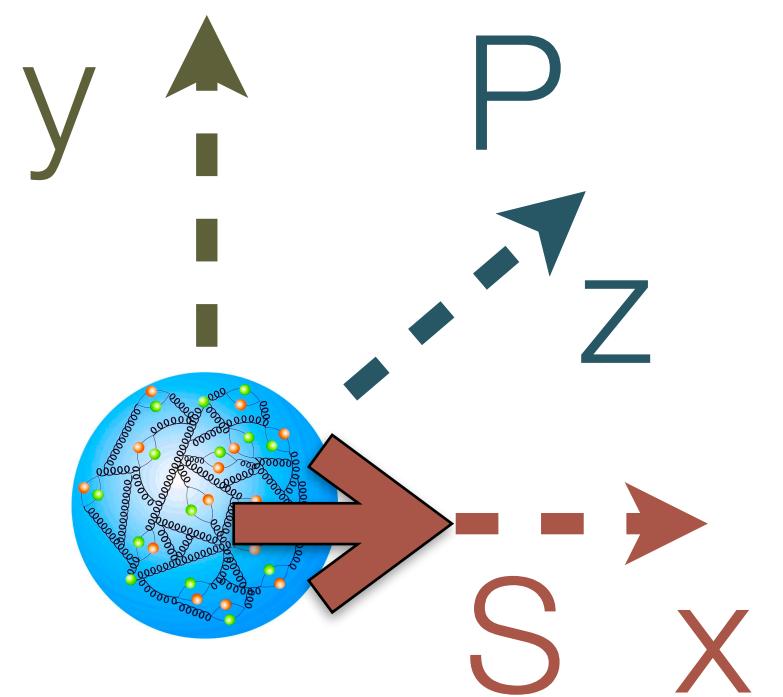
... and spatial imaging

$$\rho_{n+1}(\vec{b}_\perp) = \int \frac{d^2 \vec{\Delta}_\perp}{(2\pi)^2} A_{n+1,0}(-\vec{\Delta}_\perp^2) e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp}$$



$$q(x, \vec{b}_\perp) = \int \frac{d^2 \vec{\Delta}_\perp}{(2\pi)^2} H(x, -\vec{\Delta}_\perp^2) e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp}$$

$$\rho_{n+1}^T(\vec{b}_\perp) = \int \frac{d^2 \vec{\Delta}_\perp}{(2\pi)^2} \left[ A_{n+1,0}(-\vec{\Delta}_\perp^2) + \frac{i\Delta_y}{2m_n} B_{n+1,0}(-\vec{\Delta}_\perp^2) \right] e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp}$$

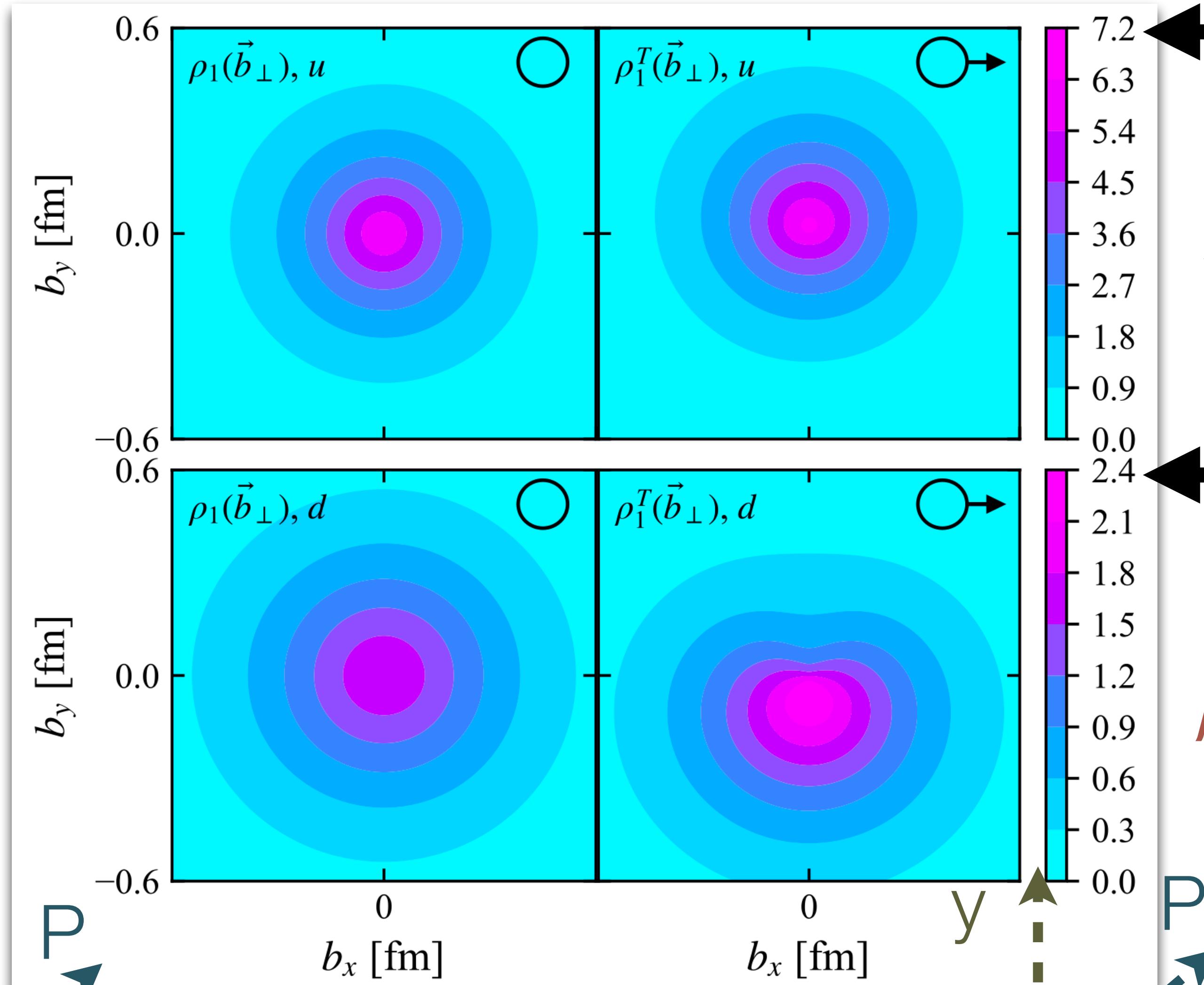
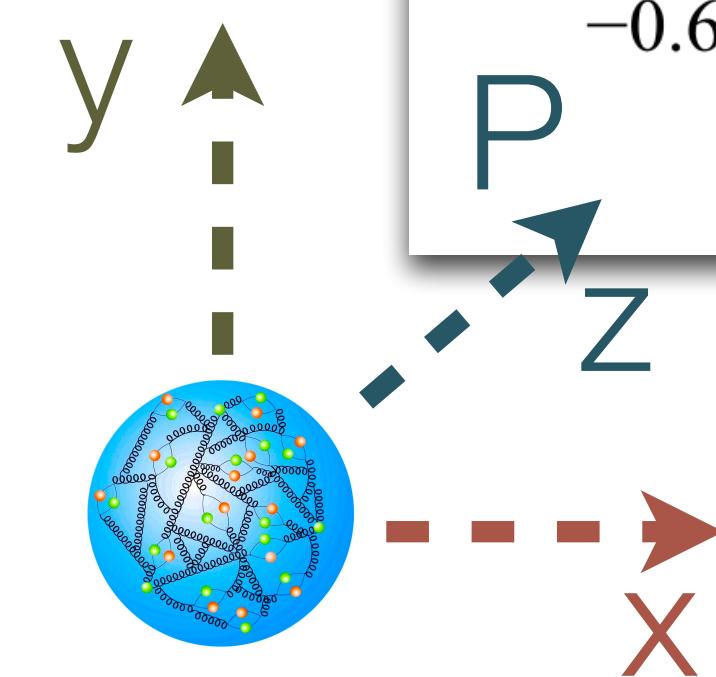


$$q^T(x, \vec{b}_\perp) = \int \frac{d^2 \vec{\Delta}_\perp}{(2\pi)^2} \left[ H(x, -\vec{\Delta}_\perp^2) + \frac{i\Delta_y}{2m_n} E(x, -\vec{\Delta}_\perp^2) \right] e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp}$$

# charge distribution

$\rho_1(\vec{b}_\perp), u$

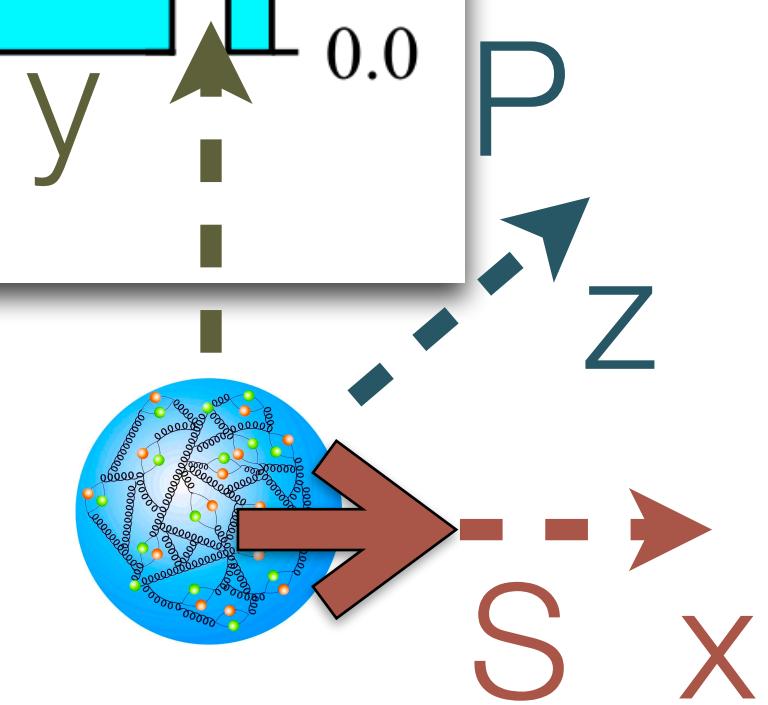
$\rho_1(\vec{b}_\perp), d$



$\sim \times 3$

$\rho_1^T(\vec{b}_\perp), u$

$\rho_1^T(\vec{b}_\perp), d$

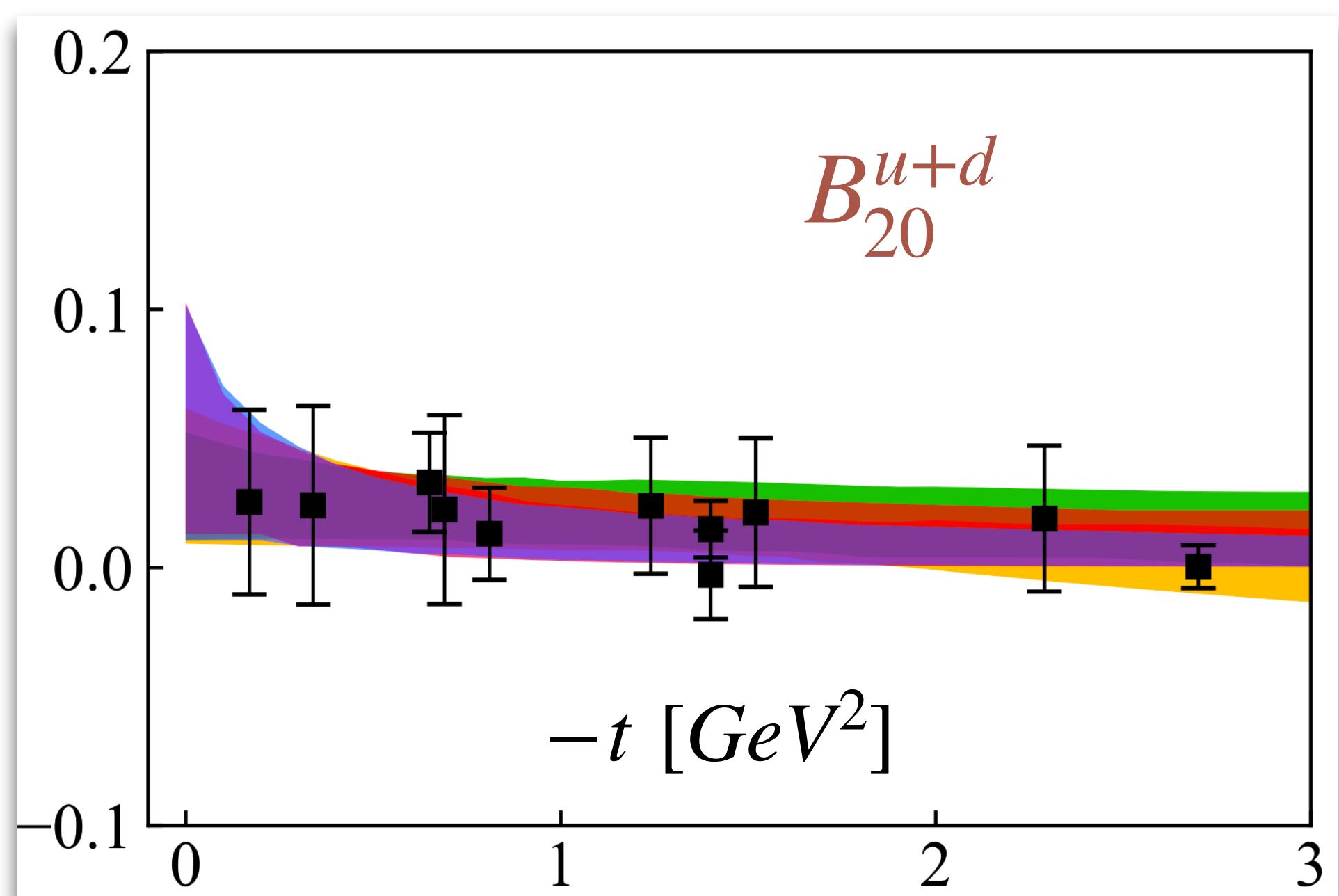
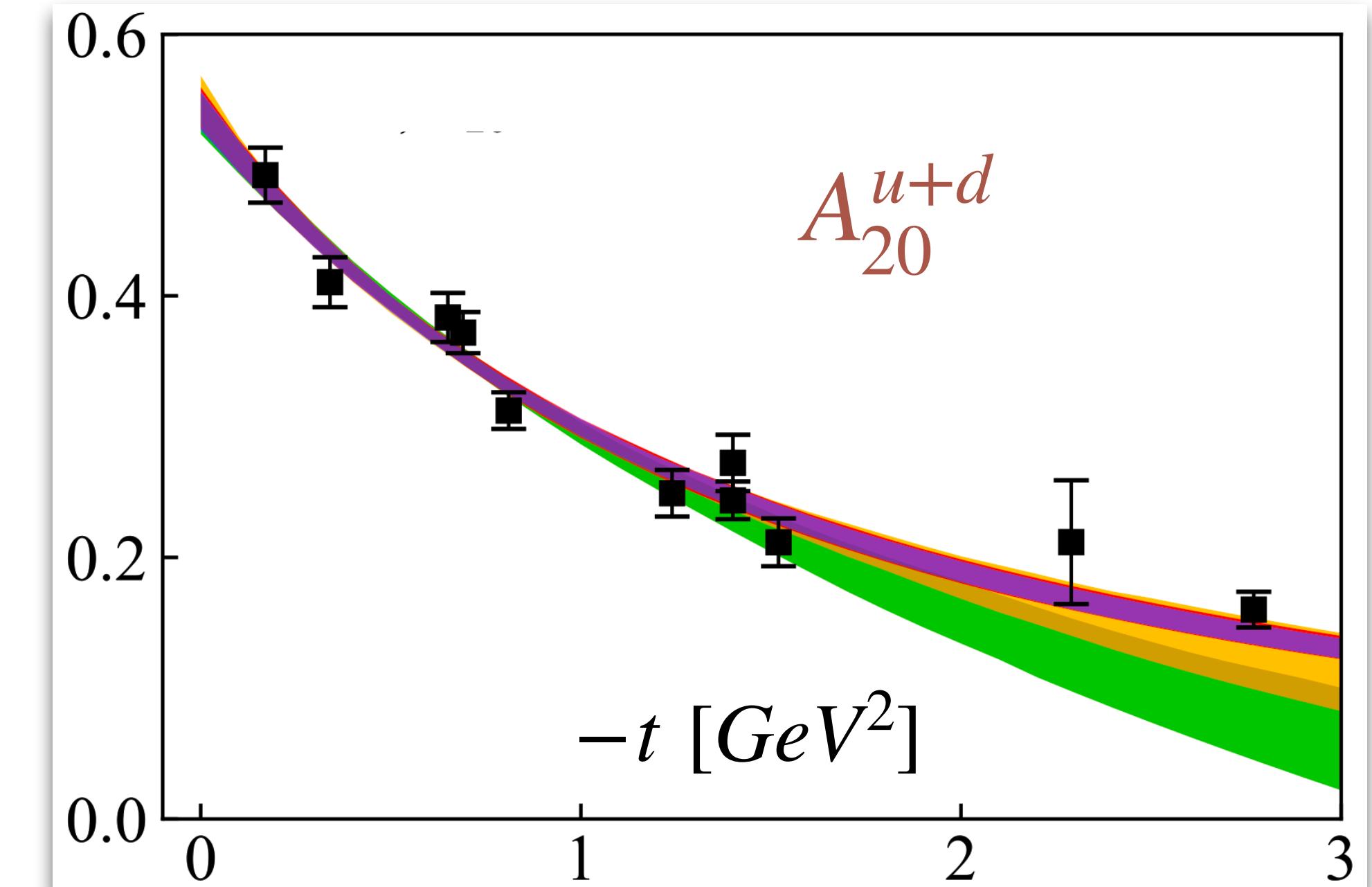


# quark total angular momentum contribution to proton spin

Ji sum rule: 
$$J^q = \frac{1}{2} \left[ A_{20}^q(0) + B_{20}^q(0) \right]$$

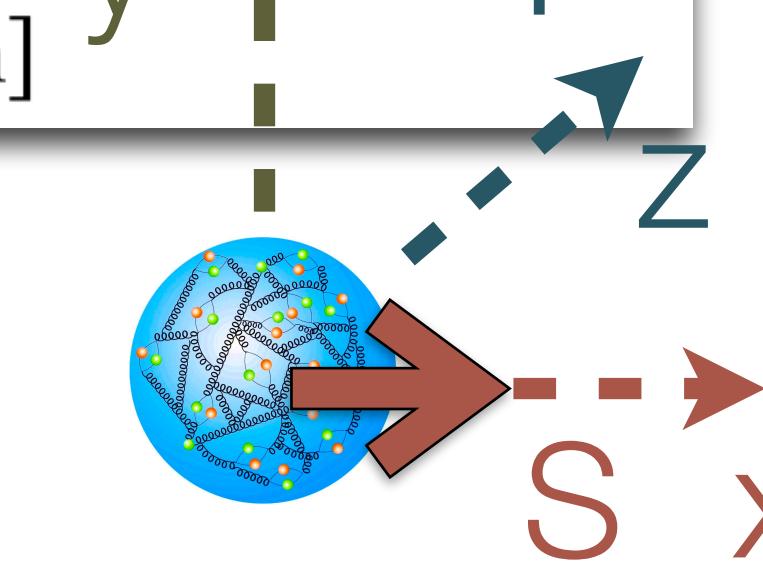
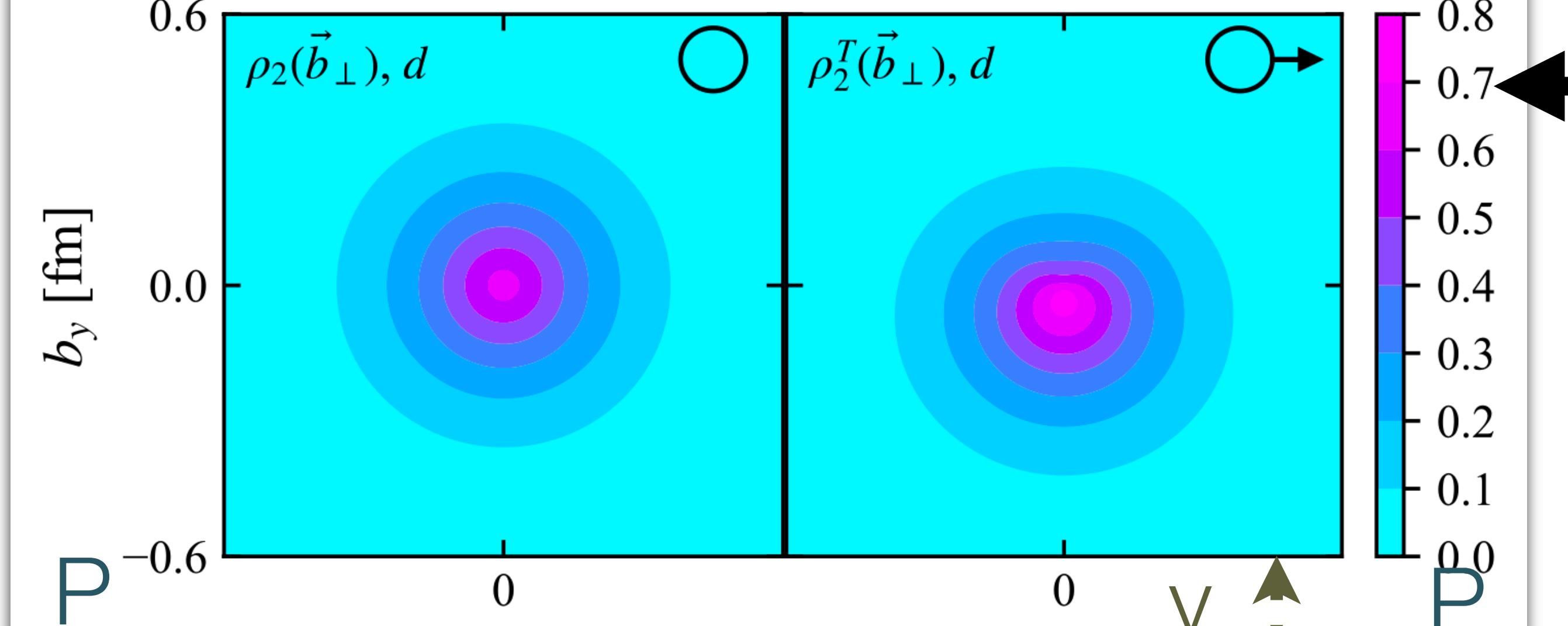
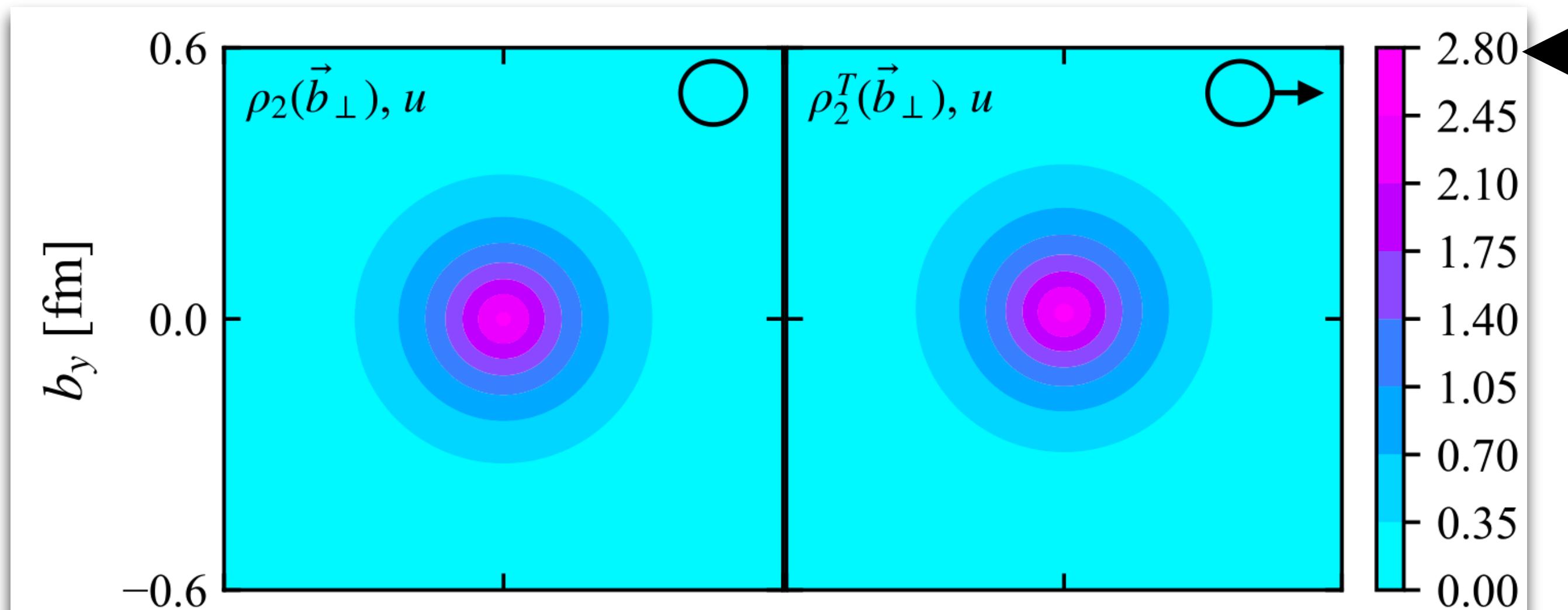
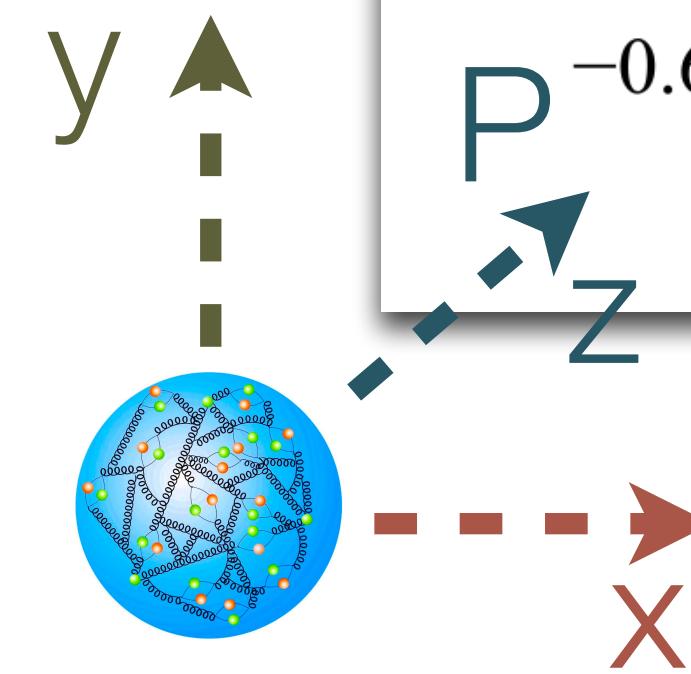
$$J^{u-d} = 0.281(21)(11)$$

$$J^{u+d} = 0.296(22)(33)$$



$\rho_2(\vec{b}_\perp), u$

$\rho_2(\vec{b}_\perp), d$



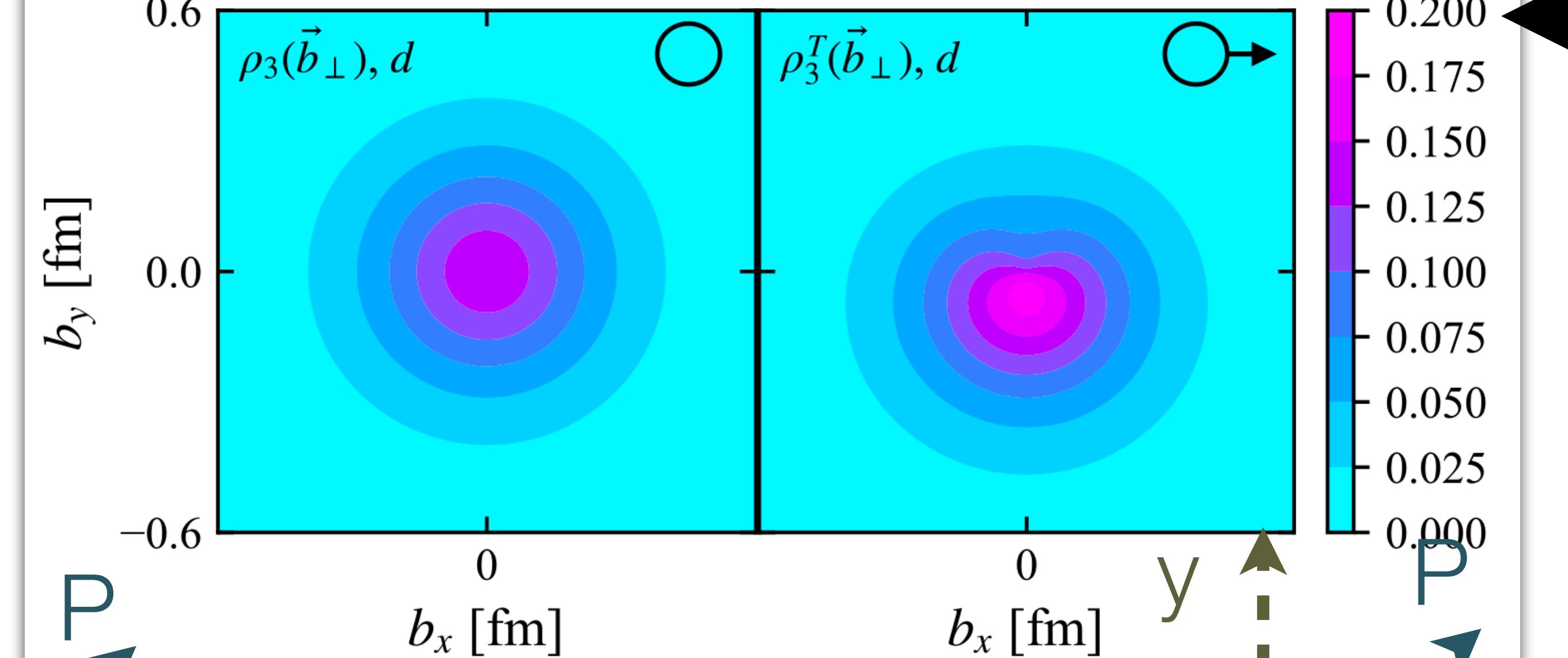
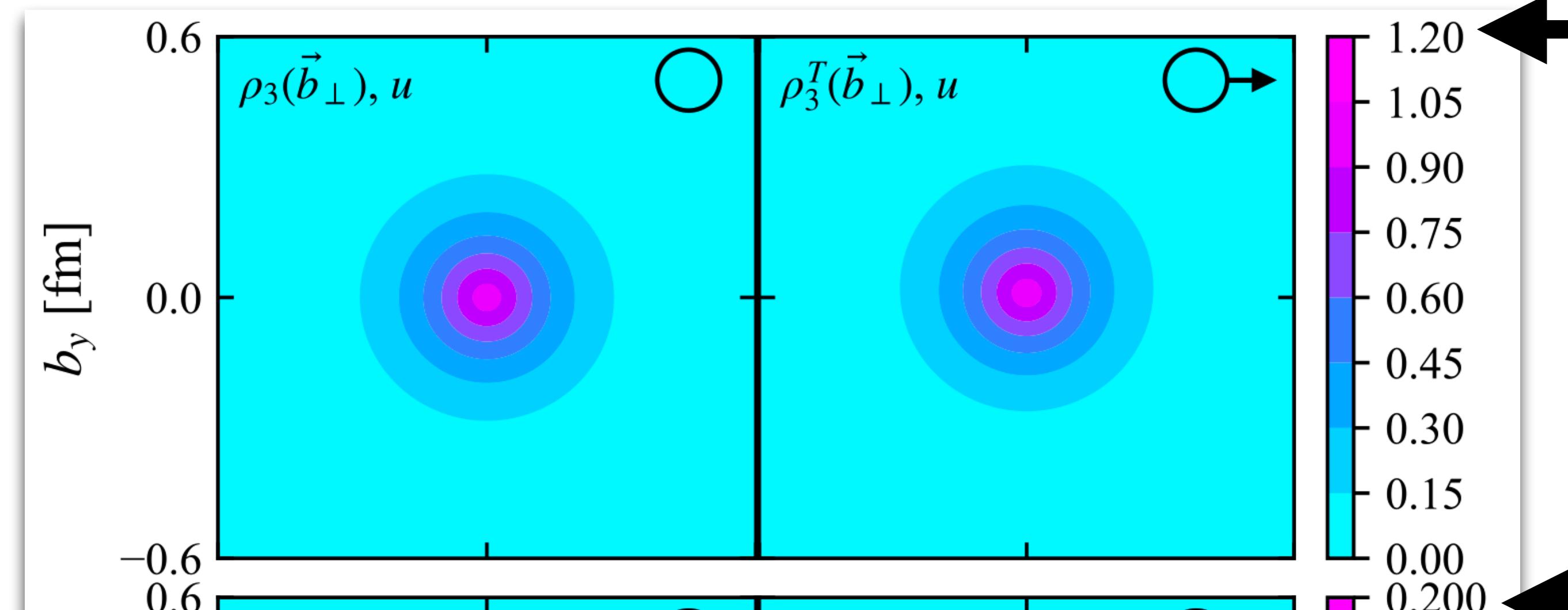
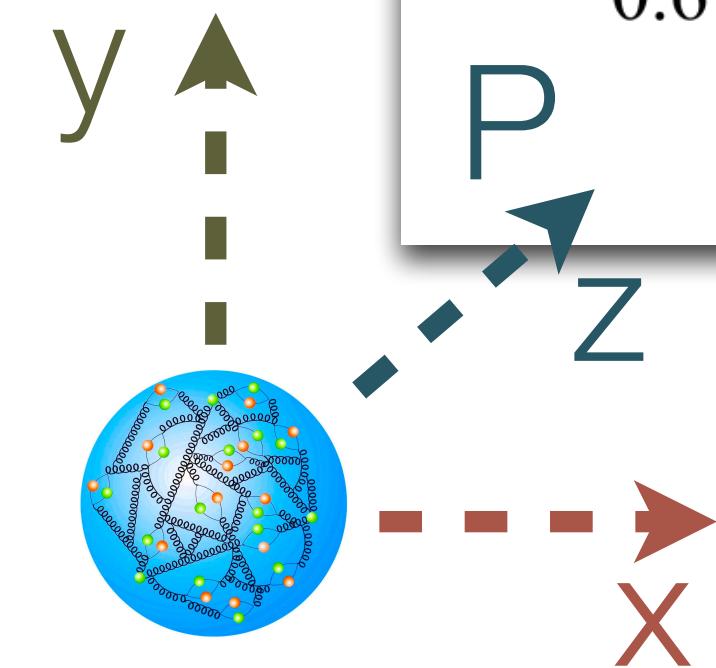
$\rho_2^T(\vec{b}_\perp), u$

$\rho_2^T(\vec{b}_\perp), d$

the 3<sup>rd</sup> ...

$\rho_3(\vec{b}_\perp), u$

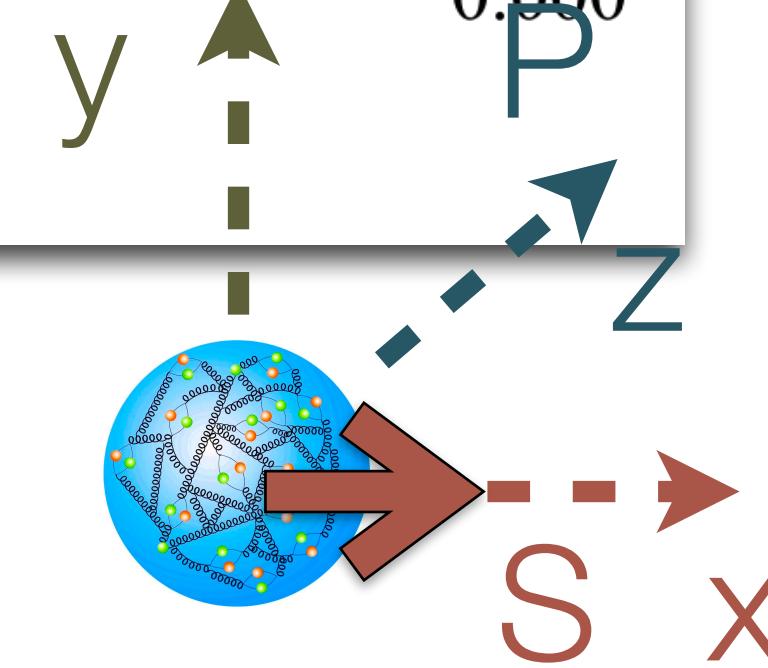
$\rho_3(\vec{b}_\perp), d$



$\sim \times 6$

$\rho_3^T(\vec{b}_\perp), u$

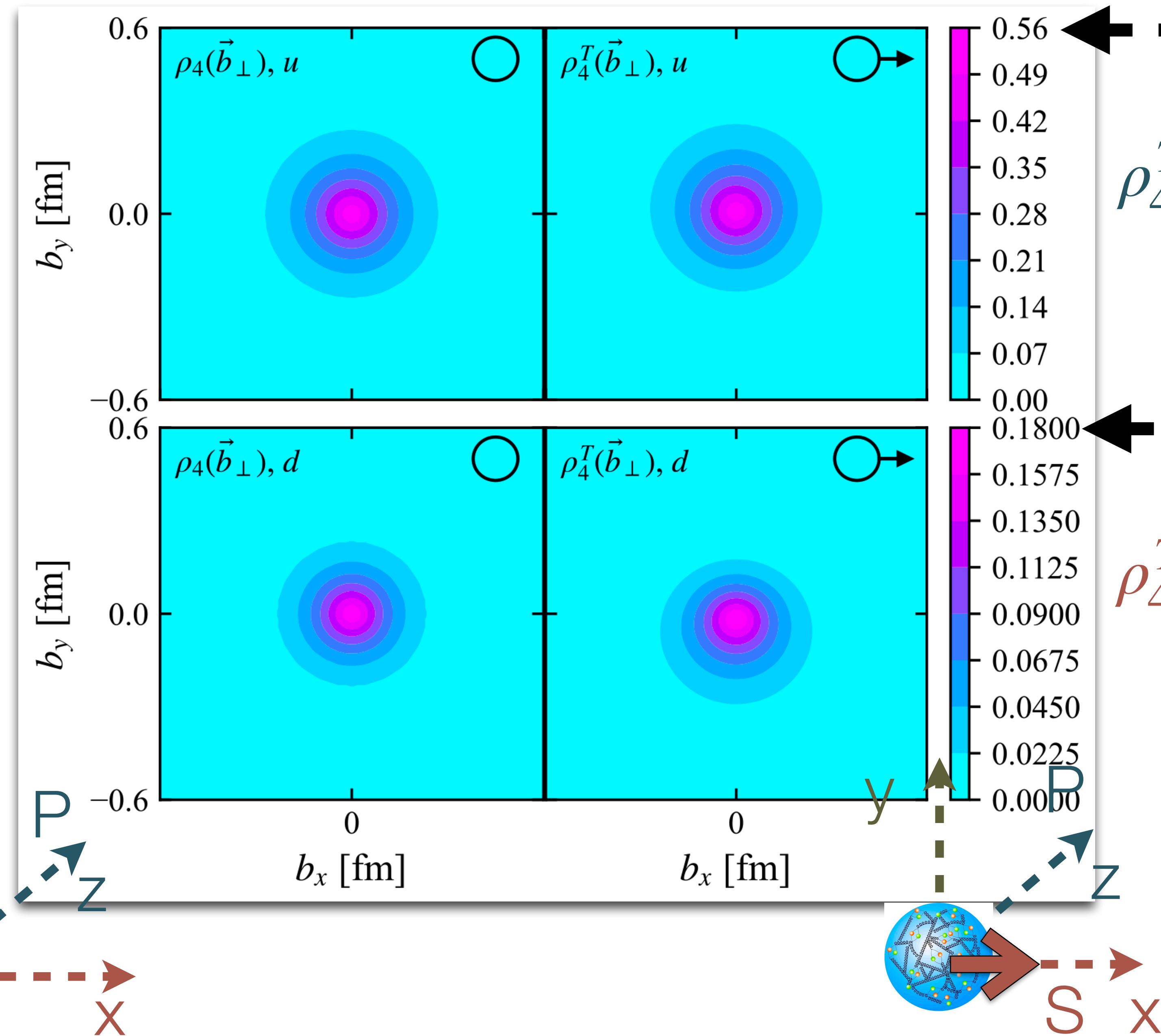
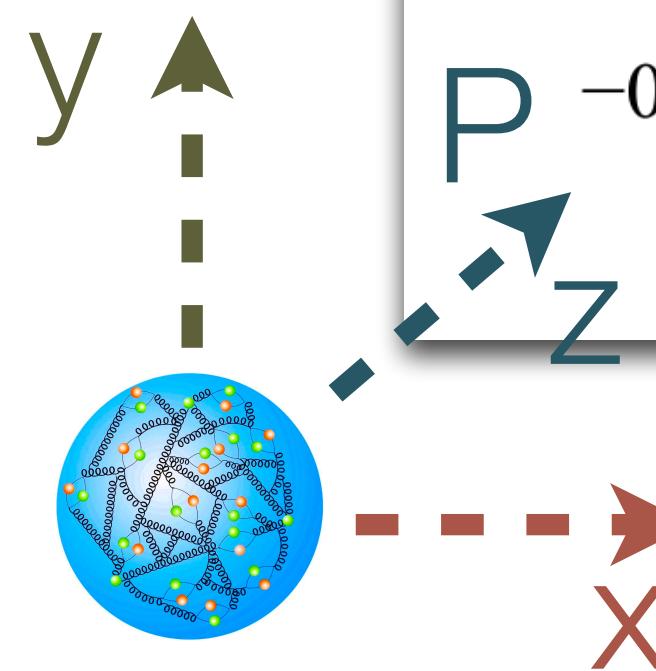
$\rho_3^T(\vec{b}_\perp), d$



... and the 4<sup>th</sup>

$\rho_4(\vec{b}_\perp), u$

$\rho_4(\vec{b}_\perp), d$



past those moments ...

x dependence via quasi-PDF

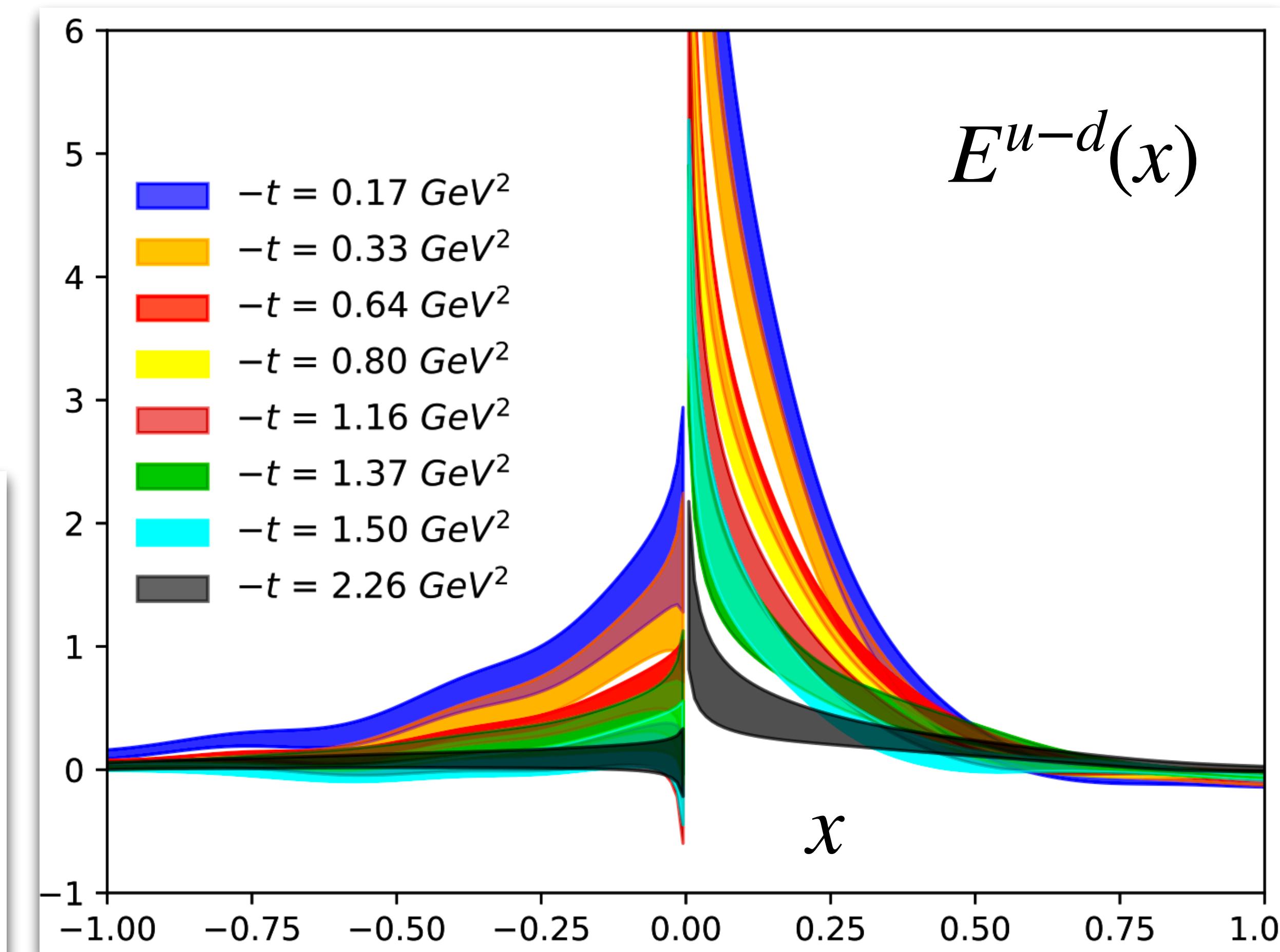
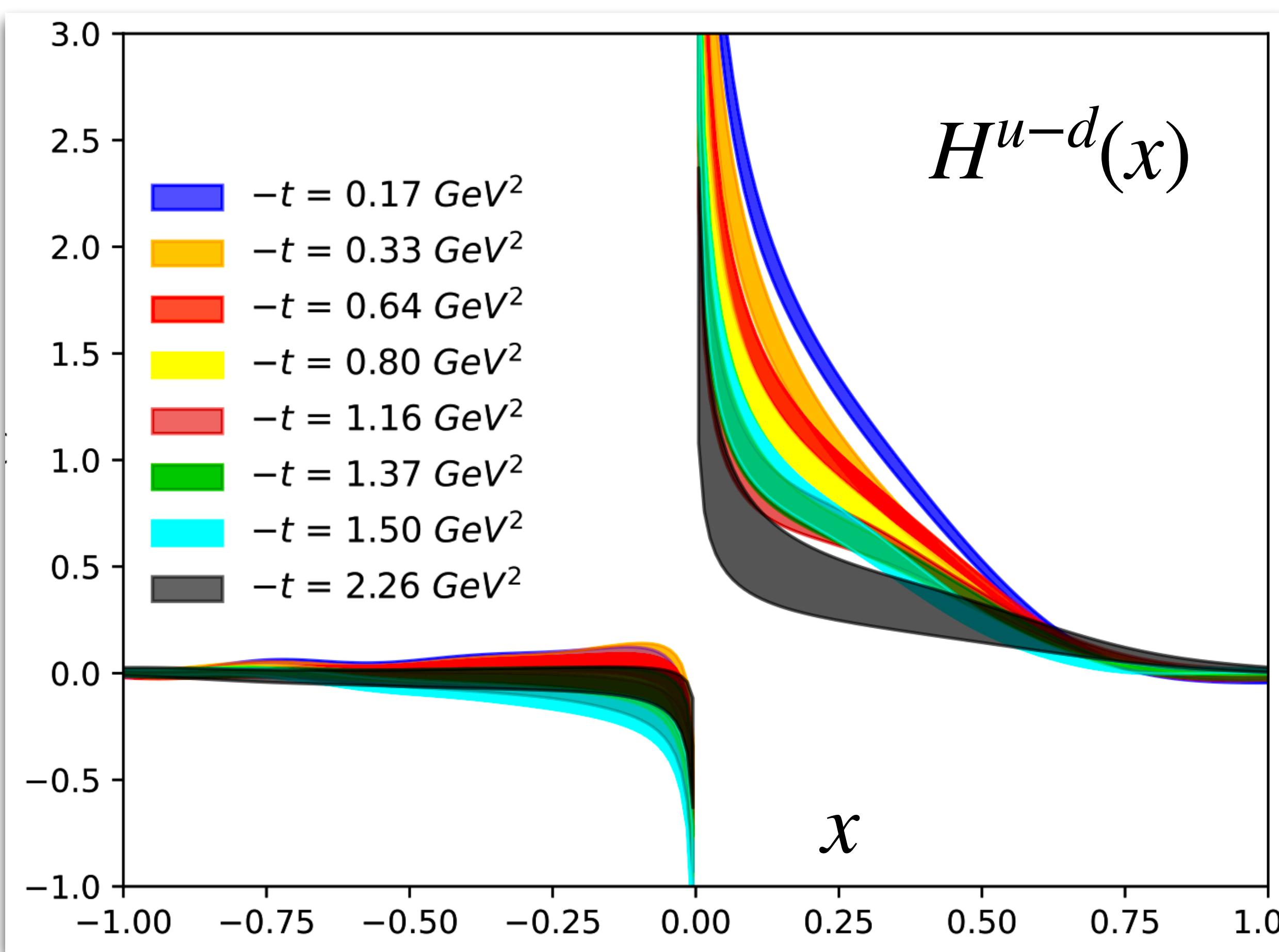
$$H(x) \qquad E(x)$$

Joshua Miller (Temple U) et al, [2304.14970](#)

Generalized Parton Distributions from Lattice QCD\*

KRZYSZTOF CICHY<sup>a</sup>, SHOHINI BHATTACHARYA<sup>b</sup>, MARTHA  
CONSTANTINO<sup>c</sup>, JACK DODSON<sup>c</sup>, XIANG GAO<sup>d</sup>, ANDREAS METZ<sup>c</sup>,  
JOSHUA MILLER<sup>c</sup>, SWAGATO MUKHERJEE<sup>e</sup>, AURORA SCAPELLATO<sup>c</sup>,  
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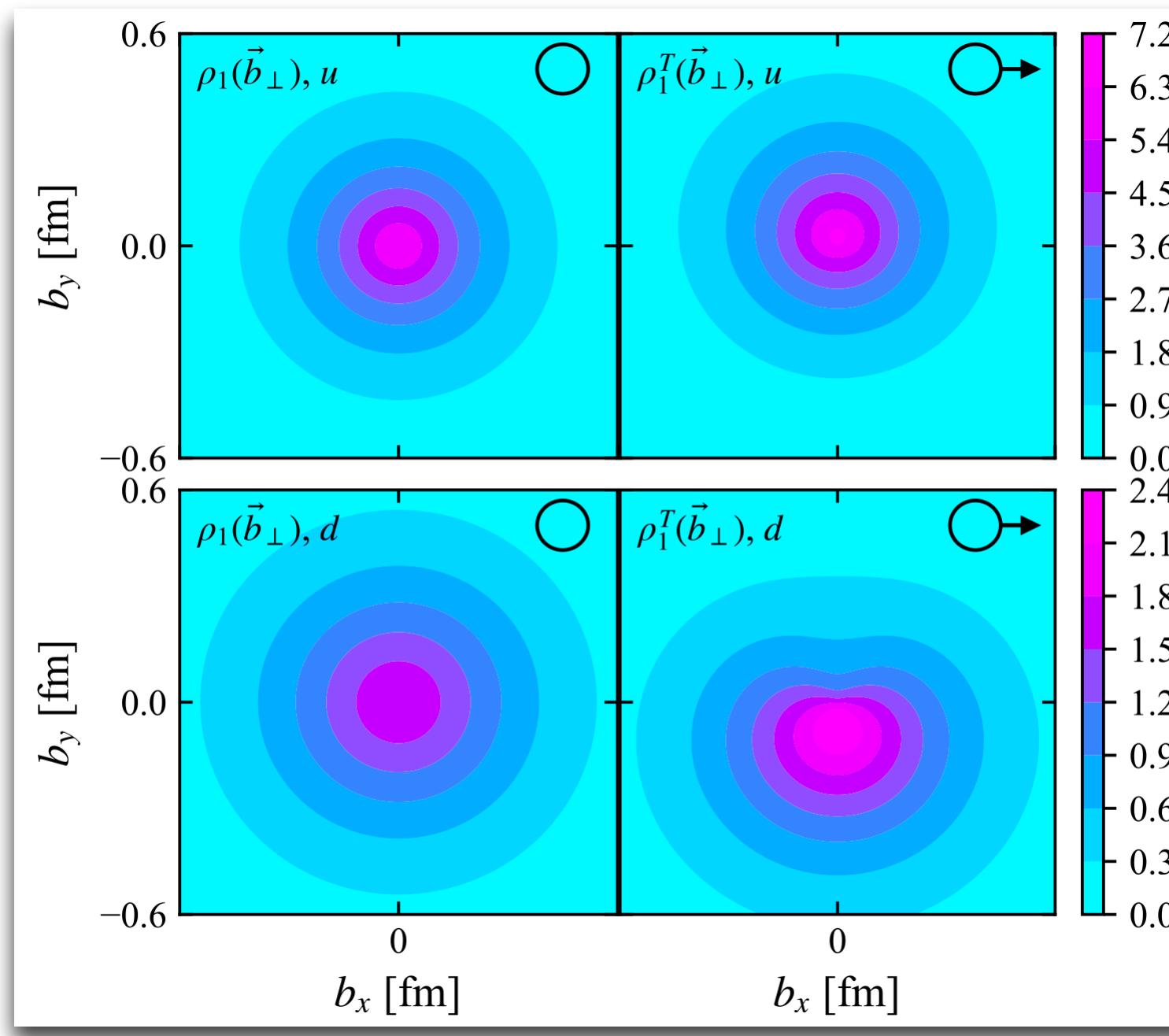
$\mu = 2 \text{ GeV}$

# summary

a new Lorentz invariant formalism for lattice QCD calculations of GPD

t dependence of GPD: faster and more accurate

t dependence of proton's quark GPD     $H(x, \xi = 0, t)$      $E(x, \xi = 0, t)$



$$J^{u+d} \simeq 0.3$$

