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Recent advances in the Parton Branching approach for transverse momentum dependent distributions

QCD Evolution Workshop 2023

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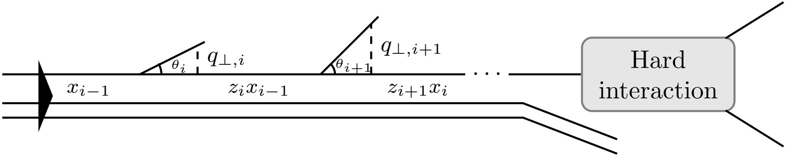
Introduction

- Monte Carlo (MC) generators crucial for HEP predictions
- MC developments to reach high precision in HL LHC, LHeC, FCC, EIC...
- Some observables need transverse momentum degrees of freedom taken into account
  $\rightarrow$ Transverse momentum dependent (TMD) factorization
- There exist analytical TMD factorization approaches e.g. Collins-Soper-Sterman (CSS)
- Parton Branching (PB) approach provides TMD parton distributions that can be used in general purpose MC generators
  $\rightarrow$ Applicable to a wide range of inclusive and exclusive observables

What is PB?

- Provides evolution equations for TMDs and collinear PDFs
- Uses Angular Ordering to associate evolution scale with rescaled transverse momentum of emitted partons

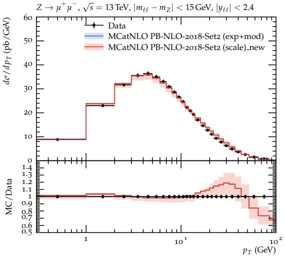


- Equations can be solved with MC methods
- Contains information of transverse momentum from the whole evolution chain

Achievements of PB

- 2 fitted TMDs available in TMDlib/TMDlib2[arXiv:2103.09741]:
  [Bermudez Martinez, Connor, Hautmann, Jung, Lelek, Radescu, Zlebcik, Phys.Rev.D 99 (2019), 074008, arXiv:1804.11152]
  - PB-NLO-HERAI+II-2018-set1 ($\alpha_s(\mu)$)
  - PB-NLO-HERAI+II-2018-set2 ($\alpha_s(q_\perp)$)

  Collinear distributions fitted to DIS at HERA, initial $k_\perp$ not yet



- Good description of DY-$p_\perp$ spectrum at high and low energies
  [Bermudez Martinez et al., EJPC 80 (2020), 598, arXiv:2001.06488]

- For exclusive observables: CASCADE3 has inital state TMD parton shower consistent with PB evolution [S. Baranov et al., Eur.Phys.J.C 81 (2021) 5, 425]

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Where to go from here?

Goal of TMD effects through MC:
Connect different regimes with one approach

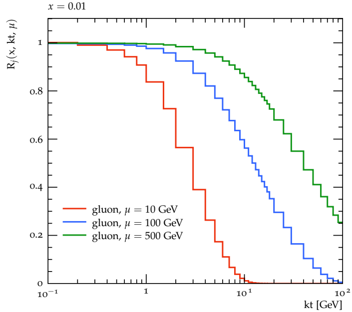
Many different studies, I will present 3:

- PB is an exclusive formulation $\rightarrow$ can address associated jet structure of DY-$p_{\perp}$
  Z+jets results through new "TMD multi-jet merging"
- Instead of only colour-neutral probes (DY di-leptons), investigations of colour-charged probes (e.g. dijets in back-to-back region, or Z+jet) $\rightarrow$ Possible factorization breaking
  Boson-jet and jet-jet $\Delta\phi$ azimuthal correlations to probe such breaking effects
- Can PB approach be extended to the small-x region? $\rightarrow$ interplay small-$q_{\perp}$ and small-x
  $k_{\perp}$-dependence in splittings

TMD broadening

- TMD effects in jets was until recently ([Bermudez Martinez, Hautmann, Mangano, Phys.Lett.B 822 (2021) 136700]) unexplored
- TMD effects at jet production might be important due to TMD broadening

$$a_j(x, k_\perp^2, \mu^2) = \int \frac{d^2 k'_\perp}{\pi} \mathcal{A}_j(x, k'^2_\perp, \mu^2)\Theta(k'^2_\perp - k_\perp^2)$$



$$R_j(x, k_\perp^2, \mu^2) = \frac{a_j(x, k_\perp^2, \mu^2)}{a_j(x, 0, \mu^2)}$$

- $k_\perp$-tail contribution comparable to hard ME emissions at LHC energies

Why TMD multijet merging?

- Connecting regime of low-$p_\perp$ and high $p_\perp$
- Due to TMD broadening TMD effects can be non-neglegible at high $p_\perp$
- New merging procedure needed to be developed to account for double counting between ME and TMD shower

  [Bermudez Martinez, Hautmann, Mangano, Phys.Lett.B 822 (2021) 136700]

  [Bermudez Martinez, Hautmann, Mangano, JHEP09(2022)060]

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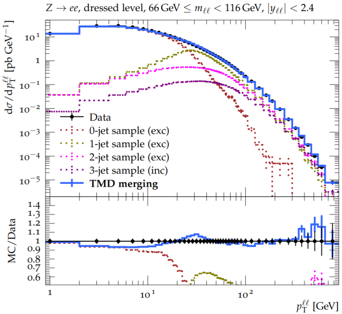
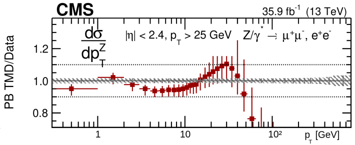
Compared to standard MLM multijet merging:

- Reduced systematic uncertainties
- Improvement description of higher-order emissions beyond the maximum parton multiplicity

TMD multijet merging results



Large $p_\perp$ without merging:



- Good description of Z-boson $p_\perp$ in whole $p_\perp$ spectrum

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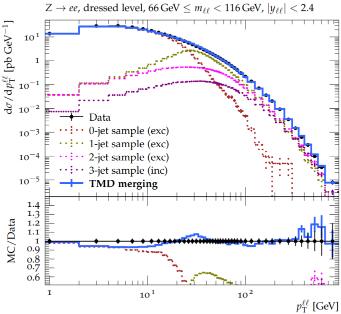
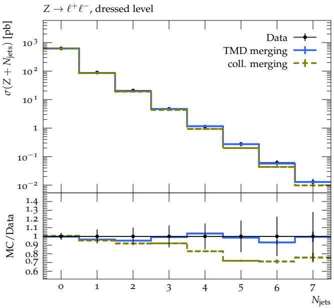
TMD multijet merging results





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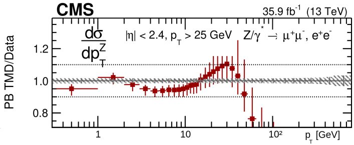
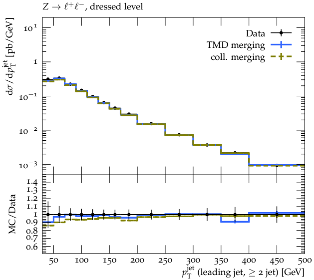
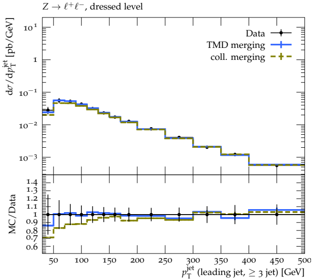
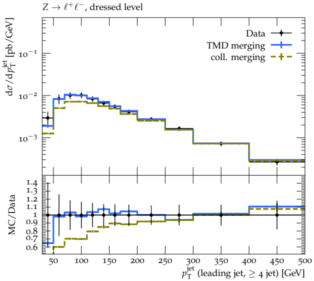


- Good description of Z-boson $p_\perp$ in whole $p_\perp$ spectrum
- Jet multiplicity in Z + jets production well described, also for multiplicities larger than the maximum number of jets in ME

Leading Jet $p_\perp$







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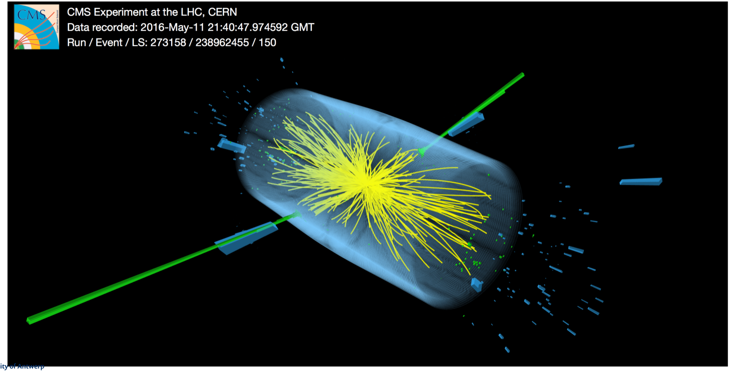
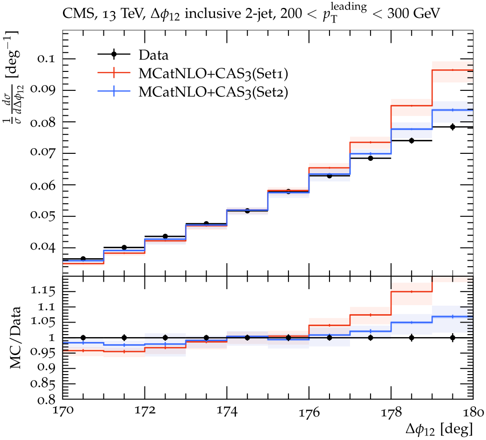
Azimuthal correlations

- Description of high $p_\perp$-jets are an important test of QCD
- LO/NLO + PS predictions of $\Delta\phi_{12} \rightarrow$ large deviations from data (larger than experimental uncertainties)
- Potential factorization breaking due to soft-gluon effects
- Studies with PB performed in: [H. Yang et al., Eur.Phys.J.C 82 (2022) 8, 755]



α_s in dijet azimuthal correlations

- Predictions obtained with MC@NLO+HERWIG6 subtraction terms+PS from CASCADE3
- Scale+TMD uncertainties



- Sensitivity to low $k_\perp$ in back-to-back region $\rightarrow$ Significant difference between predictions with set1 ($\alpha_s(\mu)$) and set2 ($\alpha_s(q_\perp)$)
- Set2 provides better description

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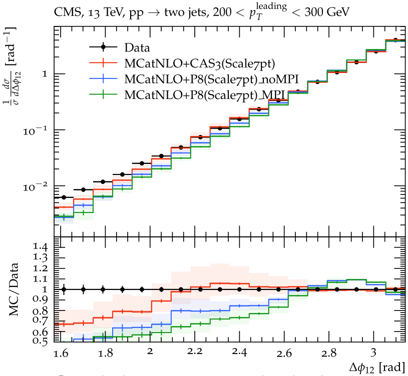
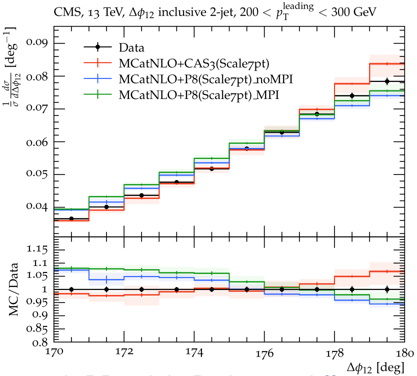
Description of $\Delta\phi_{12}$

Wide range of $\Delta\phi_{12}$



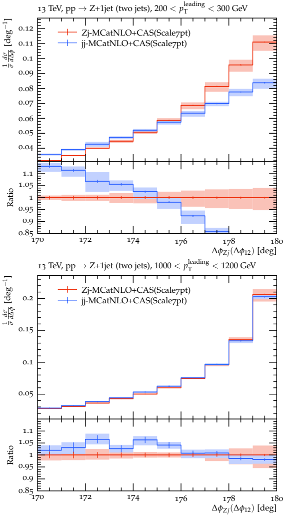
Back-to-back region



- Good description in high $\Delta\phi_{12}$ regions with PB, while Pythia8 is different from measurement in all regions
- Low $\Delta\phi_{12}$ regions not well described due to missing higher-order hard emissions. With TMD merging better description of this region (Presentation at workshop REF2021 by A. Bermudez Martinez)

Description of $\Delta\phi_{12}$ for Z+j and j+j

[van Kampen et al. arXiv:2209.13945v1]



- j+j stronger correlated than Z+j for low leading $p_\perp$ jets
- Different breaking patterns can be expected for strong and weak azimuthal correlations → interesting probe to study both Z+j and j+j

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Including TMD splitting functions

[Hautmann, Hentschinski, Keersmaekers, Kusina, Kutak, Lelek,Phys.Let.B 833 (2022), 137276, arXiv:2205.15873]

Investigation of TMD effects at level of partonic splittings in PB

- We use TMD Splitting functions defined through high-energy factorization [arXiv:hep-ph/9405388]
- We extend the PB approach , using "unitarity", to introduce TMD splitting kernels and new TMD Sudakov form factors
- First step toward a full generator that extends PB approach to the small-x

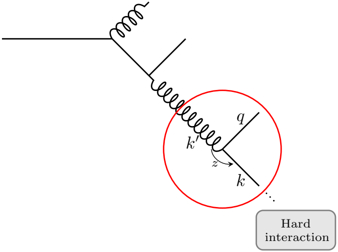
TMD splitting functions

$$P_{qg}\left(\alpha_s, z, k'_\perp, \tilde{q}_\perp\right) = \frac{\alpha_s T_F}{2\pi} \frac{\tilde{q}_\perp^2 z(1-z)}{(\tilde{q}_\perp^2 + z(1-z)k'^2_\perp)^2} \times$$

[arXiv:hep-ph/9405388]

$$\left[\frac{\tilde{q}_\perp^2}{z(1-z)} + 4(1-2z)\tilde{q}_\perp \cdot k'_\perp - 4\frac{(\tilde{q}_\perp \cdot k'_\perp)^2}{k'^2_\perp} + 4z(1-z)k'^2_\perp\right]$$

$$\tilde{q}_\perp = k_\perp - zk'_\perp$$

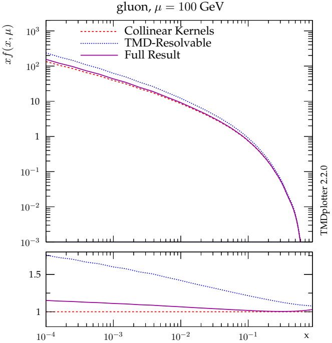


For $k'^2_\perp \ll k^2_\perp$ after angular averaging:
DGLAP splitting function

For $k'^2_\perp \sim \mathcal{O}(k^2_\perp)$:
Series expansion $(k'^2_\perp/\tilde{q}^2_\perp)^n$
Resummation $\ln(1/z)$

Other partonic channels studied in [1511.08439, 1607.01507, 1711.04587]
The splitting functions are positive definite and interpolate consistently between the collinear limit and the high-energy limit

Numerical results with TMD P



Take fixed starting distribution at scale μ_0.

Compare evolved integrated TMDs:
Purple curve: Full result
Red dashed curve: Evolution with collinear kernels

Significant differences especially for low x, not washed out after integration over $k_\perp$

- Differences between red and purple due to dynamical effects from TMD splitting functions

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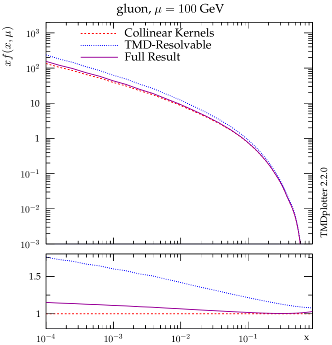

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Blue dotted curve: Model with TMD splitting functions only in resolvable emissions

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- Large differences between Full result and TMD-Resolvable due to violation of momentum conservation in TMD-Resolvable

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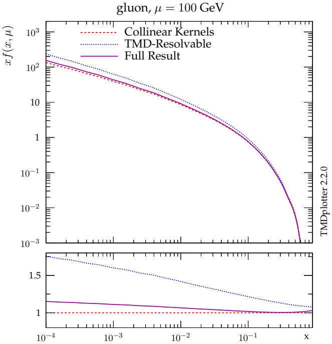

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From TMDs: Effects in whole $k_\perp$ region

Momentum conservation check

	Full Result		
μ^2 (GeV^2)	$\alpha_s(\mu^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, dyn. z_M
3	1.000	1.000	1.000
10	0.999	0.999	0.999
10^2	0.997	0.996	0.997
10^3	0.994	0.992	0.994
10^4	0.991	0.987	0.991
10^5	0.984	0.978	0.983

	TMD-Resolvable		
μ^2 (GeV^2)	$\alpha_s(\mu^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, dyn. z_M
3	1.029	1.038	1.000
10	1.087	1.139	1.007
10^2	1.156	1.304	1.045
10^3	1.195	1.413	1.091
10^4	1.219	1.478	1.129
10^5	1.229	1.507	1.148

	Collinear Kernels		
μ^2 (GeV^2)	$\alpha_s(\mu^2)$ fix. z_M	$\alpha_s(q_\perp^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, dyn. z_M
3	1.000	1.000	1.000
10	0.999	0.999	0.999
10^2	0.997	0.997	0.997
10^3	0.995	0.993	0.995
10^4	0.992	0.989	0.992
10^5	0.986	0.981	0.984

In table:
$\sum_a \int_{x_0}^1 dx \int dk_\perp^2 \, \tilde{\mathcal{A}}_a(x, k_\perp^2, \mu^2)$
$x_0 = 10^{-5}$

Studied for different scales of α_s, soft gluon resolution scales z_M

As expected: Our full result and the result with collinear kernels conserve momentum. When we use TMD splitting function only in resolvable branchings, there is violation of momentum conservation.

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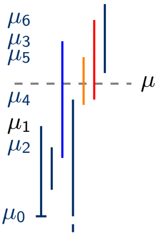
Next step: CCFM phase space?

- CCFM equations contain the the full AO phase space, consistent with both small-x and large-x
- It is however an approach for gluons and non-trivial to extend to all flavours



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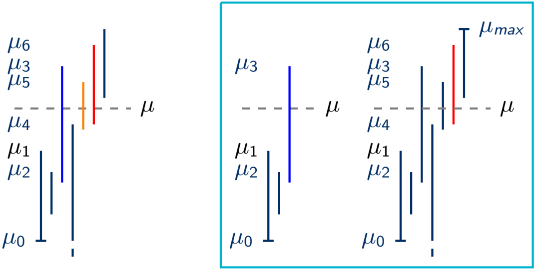
- It is however an approach for gluons and non-trivial to extend to all flavours
- TMD splitting functions might take over role of Non-Sudakov: resummation of small-x

Work in progress: New model with full AO phase space, based on momentum conservation and without Non-Sudakov form factors



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Conclusions

- PB has been proven in the past to work well on inclusive DY and DIS
- Studies are starting to take full advantage of a TMD factorization based MC:
  - With TMD multijet merging one can study jet structure and combine TMD effects with hard ME jet production
  - Colour-charged probes to factorization breaking such as azimuthal correlations are investigated
  - First efforts to extend PB to small-x have been achieved through inclusion of TMD splitting functions

Thank you!

Back-up

PB Evolution Equations

Method to obtain transverse momentum dependent PDFs (TMDs) $\tilde{\mathcal{A}}_a(x, k_\perp^2, \mu^2)$ and collinear PDFs $\tilde{f}_a(x, \mu^2) = \int dk_\perp^2 \tilde{\mathcal{A}}_a(x, k_\perp^2, \mu^2)$:

[Hautmann, Jung, Lelek, Radescu, Zlebcik, JHEP 01 (2018) 070, 1708.03279]

$$\tilde{\mathcal{A}}_a(x, k_\perp^2, \mu^2) = \Delta_a(\mu^2)\tilde{\mathcal{A}}_a(x, k_\perp^2, \mu_0^2) + \sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2}\Theta(\mu^2 - \mu'^2)\Theta(\mu'^2 - \mu_0^2)\frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \times$$

$$\times \int_x^{z_M} dz P_{ab}^R(z, \alpha_s)\tilde{\mathcal{A}}_b(\frac{x}{z}, (k_\perp + (1-z)\mu'_\perp)^2, \mu'^2)$$

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- $P_{ab}^R(z)$: (real emission part of) DGLAP splitting functions: Probability that a branching will happen

  b: incoming parton, a: outgoing parton, z momentum fraction of parton a to b

  $P_{ab}^R(z,\alpha_s) = \sum_1^\infty P_{ab}^{(n)}(z){\alpha_s}^n$

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- Sudakov form factor:

  $\Delta_a(\mu^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\ z\ P_{ba}^R(z, \alpha_s)\right)$

  Interpretation: probability of an evolution without any resolvable branchings

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  Interpretation: probability of an evolution without any resolvable branchings
- z_M: Soft-gluon resolution scale, separates resolvable/non-resolvable branchings
  Fixed (μ-independent) $z_M \approx 1$ $\leftrightarrow$ dynamical (μ-dependent) z_M

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  $$\Delta_a(\mu^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\ z\ P_{ba}^R(z, \alpha_s)\right)$$

  Interpretation: probability of an evolution without any resolvable branchings
- z_M: Soft-gluon resolution scale, separates resolvable/non-resolvable branchings

  Fixed (μ-independent) $z_M \approx 1 \leftrightarrow$ dynamical (μ-dependent) z_M
- Angular Ordering (AO): evolution scale $\mu' = \frac{q_\perp}{1-z}$, $q_\perp$ transverse momentum emitted parton

PB Evolution Equations

Method to obtain transverse momentum dependent PDFs (TMDs) $\tilde{\mathcal{A}}_a(x, k_\perp^2, \mu^2)$ and collinear PDFs $\tilde{f}_a(x, \mu^2) = \int dk_\perp^2 \, \tilde{\mathcal{A}}_a(x, k_\perp^2, \mu^2)$:

[Hautmann, Jung, Lelek, Radescu, Zlebcik, JHEP 01 (2018) 070, 1708.03279]

$$\tilde{\mathcal{A}}_a(x, k_\perp^2, \mu^2) = \Delta_a(\mu^2)\tilde{\mathcal{A}}_a(x, k_\perp^2, \mu_0^2) + \sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2}\Theta(\mu^2 - \mu'^2)\Theta(\mu'^2 - \mu_0^2)\frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \times$$
$$\times \int_x^{z_M} dz P_{ab}^R(z, \alpha_s)\tilde{\mathcal{A}}_b(\frac{x}{z}, (k_\perp + (1-z)\mu'_\perp)^2, \mu'^2)$$

- $P_{ab}^R(z)$: (real emission part of) DGLAP splitting functions: Probability that a branching will happen

  b: incoming parton, a: outgoing parton, z momentum fraction of parton a to b

  $P_{ab}^R(z, \alpha_s) = \sum_1^\infty P_{ab}^{(n)}(z){\alpha_s}^n$
- Sudakov form factor:

  $\Delta_a(\mu^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz \; z \; P_{ba}^R(z, \alpha_s)\right)$

  Interpretation: probability of an evolution without any resolvable branchings
- z_M: Soft-gluon resolution scale, separates resolvable/non-resolvable branchings
  Fixed (μ-independent) $z_M \approx 1$ $\leftrightarrow$ dynamical (μ-dependent) z_M
- Angular Ordering (AO): evolution scale $\mu' = \frac{q_\perp}{1-z}$, $q_\perp$ transverse momentum emitted parton
- $\alpha_s(\mu) \leftrightarrow \alpha_s(q_\perp)$

Other studies with PB

- Studies on the dynamical resolution scale and comparison between single versus multiple emission methods [Hautmann, Keersmaekers, Lelek, van Kampen, Nucl.Phys.B 949 (2019) 114795]
- Including EW corrections in evolution equations and determination of photon TMD
  [Jung, Taheri Monfared, Wening, Phys.Let.B 817 (2021), 136299, arXiv:2102.01494]
- Studies on the 4 and 5 Flavor Variable Number Scheme and Z + heavy flavour events, [Jung, Taheri Monfared, arXiv:2106.09791], [Baranov, Bermudez Martinez, Jung, Lipatov, Malyshev, Taheri Monfared, EPJC 82 (2022), 157]
- Extraction of the CS Kernel from PB predictions [Bermudez Martinez,Vladimirov, Phys.Rev.D 106 (2022) 9, L091501, arXiv:2206.01105]
- Studies on intrinsic $k_\perp$ (to be published soon)
- Studies on Sudakov resummation in PB, including studies on non-perturbative Sudakov form factor and studies on NNLL resummation
- Ongoing/planned projects with fits: Fits with dynamical resolution scale, LO fits, global fits, fits of intrinsic $k_\perp$,...

Unitarity

TMD evolution equations: $\tilde{\mathcal{A}}$: Momentum weighted TMD

$$\tilde{\mathcal{A}}_a(x, k_\perp^2, \mu^2) = \tilde{\mathcal{A}}_a(x, k_\perp^2, \mu_0^2) - \int \frac{d^2\mu'_\perp}{\pi \mu'^2_\perp} F_a(\mu'^2_\perp, k_\perp^2) \tilde{\mathcal{A}}_a(x, k_\perp^2, \mu'^2_\perp) \Theta(\mu'^2_\perp - \mu_0^2) \Theta(\mu^2 - \mu'^2_\perp) +$$

$$+ \sum_b \int \frac{d^2\mu'_\perp}{\pi \mu'^2_\perp} \int_x^{z_M} dz \tilde{P}^R_{ab}(z, k_\perp + (1-z)\mu'_\perp, \mu'_\perp) \tilde{\mathcal{A}}_b\left(\frac{x}{z}, (k_\perp + (1-z)\mu'_\perp)^2, \mu'^2\right) \Theta(\mu'^2_\perp - \mu_0^2) \Theta(\mu^2 - \mu'^2_\perp)$$

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$$\tilde{\mathcal{A}}_a(x,k_\perp^2,\mu^2) = \tilde{\mathcal{A}}_a(x,k_\perp^2,\mu_0^2) - \int \frac{d^2\mu'_\perp}{\pi\mu'^2_\perp} F_a(\mu'^2_\perp,k_\perp^2)\tilde{\mathcal{A}}_a(x,k_\perp^2,\mu'^2_\perp)\Theta(\mu'^2_\perp-\mu_0^2)\Theta(\mu^2-\mu'^2_\perp) +$$

$$+\sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2_\perp}\int_x^{z_M} dz\tilde{P}^R_{ab}(z,k_\perp+(1-z)\mu'_\perp,\mu'_\perp)\tilde{\mathcal{A}}_b\left(\frac{x}{z},(k_\perp+(1-z)\mu'_\perp)^2,\mu'^2\right)\Theta(\mu'^2_\perp-\mu_0^2)\Theta(\mu^2-\mu'^2_\perp)$$

- Real emissions

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$$+\sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2_\perp} \int_x^{z_M} dz \tilde{P}^R_{ab}(z, k_\perp + (1-z)\mu'_\perp, \mu'_\perp)\tilde{\mathcal{A}}_b\left(\frac{x}{z}, (k_\perp + (1-z)\mu'_\perp)^2, \mu'^2\right)\Theta(\mu'^2_\perp - \mu_0^2)\Theta(\mu^2 - \mu'^2_\perp)$$

- Real emissions
- Virtual/Non-resolvable emissions

Unitarity

TMD evolution equations: $\tilde{\mathcal{A}}$: Momentum weighted TMD

$$\tilde{\mathcal{A}}_a(x,k_\perp^2,\mu^2) = \tilde{\mathcal{A}}_a(x,k_\perp^2,\mu_0^2) - \int \frac{d^2\mu'_\perp}{\pi\mu'^2_\perp} F_a(\mu'^2_\perp,k_\perp^2)\tilde{\mathcal{A}}_a(x,k_\perp^2,\mu'^2_\perp)\Theta(\mu'^2_\perp-\mu_0^2)\Theta(\mu^2-\mu'^2_\perp)+$$

$$+\sum_b\int\frac{d^2\mu'_\perp}{\pi\mu'^2_\perp}\int_x^{z_M} dz\tilde{P}^R_{ab}(z,k_\perp+(1-z)\mu'_\perp,\mu'_\perp)\tilde{\mathcal{A}}_b\left(\frac{x}{z},(k_\perp+(1-z)\mu'_\perp)^2,\mu'^2\right)\Theta(\mu'^2_\perp-\mu_0^2)\Theta(\mu^2-\mu'^2_\perp)$$

- Real emissions
- Virtual/Non-resolvable emissions
  $\Rightarrow$ Fix with momentum conservation:

$$0=\sum_a\int_0^1 dx\int dk_\perp^2\tilde{\mathcal{A}}_a(x,k_\perp^2,\mu^2)-\sum_a\int_0^1 dx\int dk_\perp^2\tilde{\mathcal{A}}_a(x,k_\perp^2,\mu_0^2).$$

$$\Rightarrow$$

$$F_a(\mu'^2,k_\perp^2)=\sum_b\int_0^{z_M} dz\; z\bar{P}^R_{ba}(z,k_\perp^2,\mu'^2).$$

$\bar{P}^R_{ba}(z,k_\perp^2,\mu'^2)$: Angular averaged TMD splitting functions

Unitarity (2)

Introduce TMD Sudakov form factors:

$$\Delta_a(\mu^2, k_\perp^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\, z\bar{P}^R_{ba}(z, k_\perp^2, \mu'^2)\right)$$

Rewrite the evolution equation:

$$\tilde{\mathcal{A}}_a\left(x, k_\perp^2, \mu^2\right) = \Delta_a\left(\mu^2, k_\perp^2\right) \tilde{\mathcal{A}}_a\left(x, k_\perp^2, \mu_0^2\right) + \sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2_\perp} \Theta(\mu'^2_\perp - \mu_0^2)\Theta(\mu^2 - \mu'^2_\perp)$$

$$\times \int_x^{z_M} \mathrm{d}z\, \frac{\Delta_a\left(\mu^2, k_\perp^2\right)}{\Delta_a\left(\mu'^2_\perp, k_\perp^2\right)} \tilde{P}^R_{ab}\left(z, k_\perp + (1-z)\mu'_\perp, \mu'_\perp\right) \tilde{\mathcal{A}}_b\left(\frac{x}{z}, (k_\perp + (1-z)\mu'_\perp)^2, \mu'^2_\perp\right)$$

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Introduce TMD Sudakov form factors:

$$\Delta_a(\mu^2, k_\perp^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\, z\bar{P}^R_{ba}(z, k_\perp^2, \mu'^2) \right)$$

Rewrite the evolution equation:

$$\tilde{\mathcal{A}}_a\left(x, k_\perp^2, \mu^2\right) = \Delta_a\left(\mu^2, k_\perp^2\right) \tilde{\mathcal{A}}_a\left(x, k_\perp^2, \mu_0^2\right) + \sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2_\perp} \Theta(\mu'^2_\perp - \mu_0^2)\Theta(\mu^2 - \mu'^2_\perp)$$

$$\times \int_x^{z_M} \mathrm{d}z \frac{\Delta_a\left(\mu^2, k_\perp^2\right)}{\Delta_a\left(\mu'^2_\perp, k_\perp^2\right)} \tilde{P}^R_{ab}\left(z, k_\perp + (1-z)\mu'_\perp, \mu'_\perp\right) \tilde{\mathcal{A}}_b\left(\frac{x}{z}, (k_\perp + (1-z)\mu'_\perp)^2, \mu'^2_\perp\right)$$

Equation has similar structure to other Parton Branching equations [arXiv:1704.01757, arXiv:1708.03279] $\rightarrow$ similar MC

Except for scale generation according to TMD Sudakov form factor: VETO algorithm [arXiv:hep-ph/0603175]