



TMDs from double J/ψ production

QCD Evolution Workshop 2023

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① Introduction

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③ Gluon TMDs and di- J/ψ production

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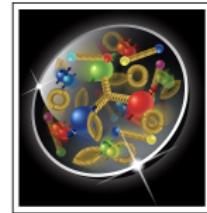
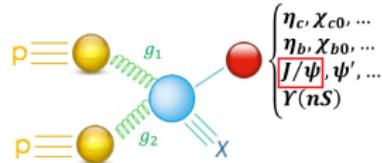
⑤ Conclusions

General introduction

Inclusive production of J/ψ pairs in pp collisions (gluon fusion)



Azimuthal modulations of the cross section for inclusive production of quarkonium pairs in hadronic collisions



- understanding the internal structure of nucleons
- gluon dynamics poorly known

Results → future measurements at LHCb

- Transverse Momentum Dependent PDFs (TMDs)

TMDs → quark and gluon ones

TMDs → 3D structure of the nucleon

Correlations between k_T and the polarisation of the nucleon/parton

2 components ▷ collinear (x)

▷ transversal (\vec{k}_\perp) → generate q_T (final-state)

Quark TMDs extracted from data

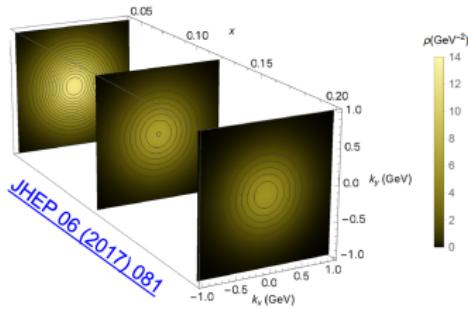
- ↪ SIDIS, DY processes
- ↪ Precision era!

Gluon TMDs → lack of data

- ↪ Extremely poorly know!
- ↪ How to measure them?

Inclusive quarkonium production

A. Bacchetta et al. (JHEP 08 (2008) 023)



Quark				
	U	L	T	
Nucleon	U	f_1		h_1^\perp
	L		g_{1L}	h_{1L}^\perp
	T	f_{1T}^\perp	g_{1T}	h_1 h_{1T}^\perp

Experimental point of view:

- quarkonium production observed in different experiments
- J/ψ : easy to produce and detect
 - ↪ plenty of experimental data

Theoretical point of view:

- Not clear how to treat quarkonium production
- 3 common models → Colour Singlet Model (CSM)
 - Colour Octet Mechanism (COM)
 - Colour Evaporation Model (CEM)
- not complete agreement with experimental data
- however for J/ψ -pair production: **CSM** best description

Why di- J/ψ production?

- **Single J/ψ production:** a lot of data at low p_T ✓
 - ↪ but gluon in the final state → presence of soft gluons (non-perturbative) between Initial State Interactions (ISIs) and Final State Interactions (FSIs) can be problematic
 - ↪ no TMD factorisation ✗
 - **Single η_c production:** no gluon in the final state ✓
 - ↪ but no data at low p_T ✗
 - **Double J/ψ production:**
 - ▷ data at low $p_T^{\psi\psi}$ ✓
 - ▷ no gluon in the final state ✓
 - ↪ gluon fusion: ISI can be encapsulated in the TMDs
 - ↪ consider CSM: no FSIs
- Safe to assume TMD factorisation

F. Scarpa *et al*, Eur. Phys. J.C. 80 no.2, (2020) 87

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TMD factorisation

Study of gluon TMDs → TMD factorisation ($q_T \ll Q$)

General factorised cross section

↪ partonic scattering amplitude (*perturbative*)

↪ k_T -dependent correlators (*non-perturbative*)

$$d\sigma = \int dx_1 dx_2 d^2 \vec{k}_{T1} d^2 \vec{k}_{T2} \delta^{(2)}(\vec{k}_{T1} + \vec{k}_{T2} - \vec{q}_T) \\ \times \Phi_g^{\mu\nu}(x_1, \vec{k}_{T1}) \Phi_g^{\rho\sigma}(x_2, \vec{k}_{T2}) \left[\hat{\mathcal{M}}_{\mu\rho} \hat{\mathcal{M}}_{\nu\sigma}^* \right]_{\substack{k_1=x_1 P_1 \\ k_2=x_2 P_2}} + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

- In order to stay in TMD regime: $q_T \leq Q/2$

Gluon TMDs

2 independent collinear partonic distributions:

- $f_1^g(x)$ *unpolarised*
- $g_1^g(x)$ *circular*

Unpolarised protons → 2 TMDs:

- f_1^g : unpolarised gluon TMD
- $h_1^{\perp g}$: linearly polarised gluon TMD

		Gluon		
		U	C	L
Nucleon	U	f_1		h_1^\perp
	L		g_{1L}	h_{1L}^\perp
	T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

Gluon TMDs and correlators

TMD correlator parametrisation
for an unpolarised proton

▷ unpolarised:

$$f_1^g \rightarrow$$

▷ linearly polarised:

$$h_1^{\perp g} \rightarrow$$

Gluon		
Nucleon	U	C
U	f_1	h_1^\perp
L		g_{1L}
T	f_{1T}^\perp	g_{1T} , h_1^\perp, h_{1T}^\perp

$$\begin{aligned}\Phi_g^{\mu\nu}(x, \vec{k}_T) = & -\frac{1}{2x} \left[g_T^{\mu\nu} f_1^g(x, \vec{k}_T^2) \right. \\ & \left. - \left(\frac{k_T^\mu k_T^\nu}{M_H^2} + g_T^{\mu\nu} \frac{\vec{k}_T^2}{2M_H^2} \right) h_1^{\perp g}(x, \vec{k}_T^2) \right]\end{aligned}$$

→ Second term goes to 0 if $k_T = 0$

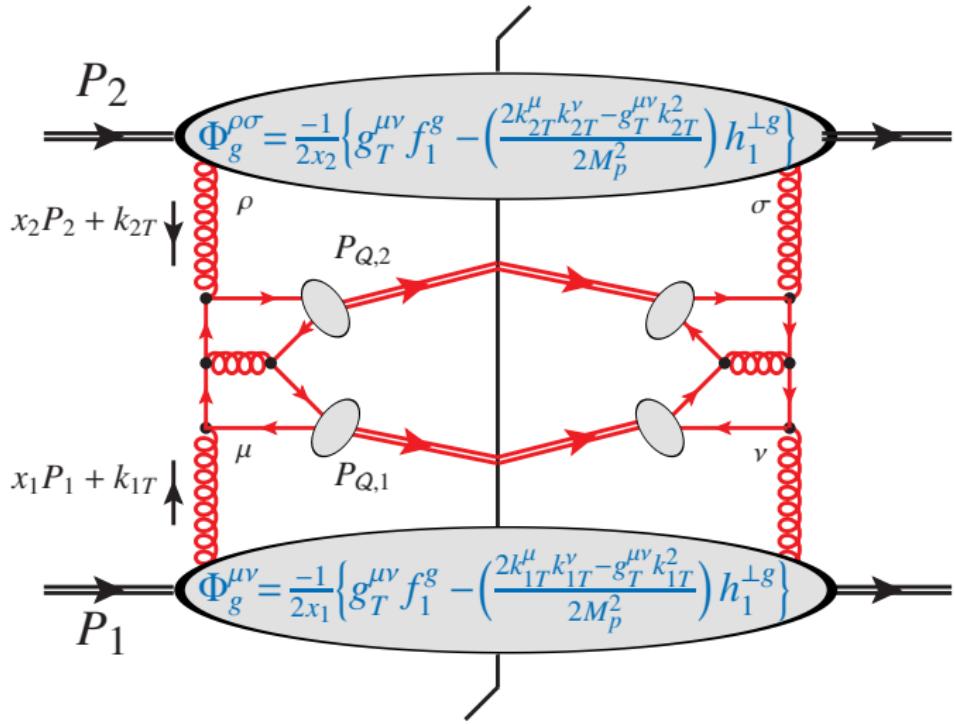
P.J. Mulders and J. Rodrigues (Phys.Rev.D 63 (2001) 094021)

The general formula for the cross section of gluon fusion is:

$$\begin{aligned} d\sigma^{gg} \propto & F_1 \times \mathcal{C}[f_1^g f_1^g] \\ & + F_2 \times \mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}] \\ & + (F_3 \times \mathcal{C}[w_3 f_1^g h_1^{\perp g}] + F'_3 \times \mathcal{C}[w'_3 h_1^{\perp g} f_1^g]) \cos(2\Phi_{CS}) \\ & + (F_4 \times \mathcal{C}[w_4 h_1^{\perp g} h_1^{\perp g}]) \cos(4\Phi_{CS}) \end{aligned}$$

- first two members: azimuthally independent
- third member: $\cos(2\Phi_{CS})$ -modulation
- fourth member: $\cos(4\Phi_{CS})$ -modulation

LO Feynman diagram for $p(P_1) + p(P_2) \rightarrow Q(P_{Q,1}) + Q(P_{Q,1}) + X$



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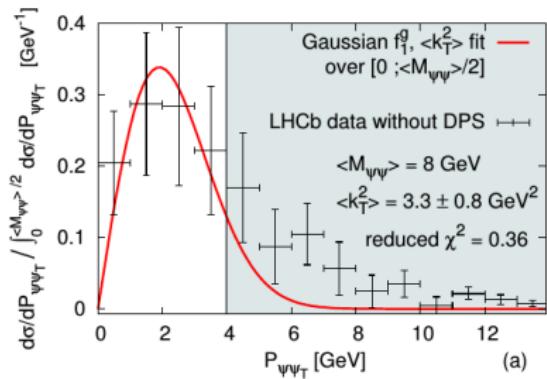
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First "fit" of f_1^g

JPL, C. Pisano, F. Scarpa, M. Schlegel, PLB 784(2018)217

- f_1^g modelled as a Gaussian in \vec{k}_T : $f_1^g(x, \vec{k}_T^2) = \frac{g(x)}{\pi \langle \vec{k}_T^2 \rangle} \exp\left(\frac{-\vec{k}_T^2}{\langle \vec{k}_T^2 \rangle}\right)$
where $g(x)$ is the usual collinear PDF
- First experimental determination [with a pure colorless final state] of $\langle \vec{k}_T^2 \rangle$
by fitting $\mathcal{C}[f_1^g f_1^g]$ over the normalised LHCb $d\sigma/dP_{\psi\psi_T}$ spectrum at 13 TeV
from which we have subtracted the DPS yield determined by LHCb



- Integration over $\phi \Rightarrow \cos(n\phi)$ -terms cancel out
- $F_2 \ll F_1 \Rightarrow$ only $\mathcal{C}[f_1^g f_1^g]$ contributes to the cross-section
- No evolution so far: $\langle \vec{k}_T^2 \rangle \sim 3 \text{ GeV}^2$ accounts both for non-perturbative and perturbative broadenings at a scale close to $M_{\psi\psi} \sim 8 \text{ GeV}$
- Disentangling such (non-)perturbative effects requires data at different scales

Introduction Evolution (1)

- Implementing evolution is more easily done in impact parameter space (b_T), where convolutions become simple products:

$$d\sigma_{UU}^{gg} \propto \int d^2 b_T e^{-i b_T \cdot q_T} \hat{W}(b_T, Q) + \mathcal{O}(q_T^2/Q^2)$$

$$\hat{W}(b_T, Q) = \hat{f}(x_1, b_T; \zeta_f, \mu) \hat{g}(x_2, b_T; \zeta_g, \mu) \mathcal{H}(Q; \mu).$$

- The convolutions are rewritten by Fourier transforming:

$$\begin{aligned} \mathcal{C}[w f g](x_1, x_2, \vec{q}_T) &= \int d^2 \vec{k}_{T1} \int d^2 \vec{k}_{T2} \delta^{(2)}(\vec{k}_{T1} + \vec{k}_{T2} - \vec{q}_T) \\ &\quad \times w_{n,m}(\vec{k}_{T1}, \vec{k}_{T2}) f(x_1, \vec{k}_{T1}) g(x_2, \vec{k}_{T2}) \\ &\Rightarrow \int_0^\infty \frac{db_T}{2\pi} b_T^n J_m(b_T q_T) \hat{f}(x_1, b_T) \hat{g}(x_2, b_T) \end{aligned}$$

Introduction Evolution (2)

$$\begin{aligned}\mathcal{C}[w f g](x_1, x_2, \vec{q}_T; Q) &= \int_0^\infty \frac{db_T}{2\pi} b_T^n J_m(b_T q_T) \\ &\times e^{-S_A(b_T; Q^2, Q)} \hat{f}(x_1, b_T; \mu_b^2, \mu_b) \hat{g}(x_2, b_T; \mu_b^2, \mu_b)\end{aligned}$$

- S_A contains $\ln Q b_T$
- Expressions (based on pQCD) are valid when:
 $b_0/Q \leq b_T \leq b_{T,\max}$
- At lower limit $\mu_b = b_0/b_T$ becomes larger than Q , i.e. evolution should stop ($S_A = 0$)
- At upper limit perturbation theory starts to fail, which is not exactly known. Common to take $b_{T,\max} = 0.5 \text{ GeV}^{-1}$ or $b_{T,\max} = 1.5 \text{ GeV}^{-1}$.
- This effectively boils down to a different resummation:
 $\mu_b(b_T)/Q \rightarrow \mu_b(b_T^*)/Q$

Introduction Evolution (3)

- We need to add a component that takes over as $b_T > b_{T,\max}$:

$$\hat{W}(b_T, Q) \equiv \hat{W}(b_T^*, Q) e^{-S_{NP}(b_T, Q)}$$

- There are different parameterizations for S_{NP} in the literature, but typically it is chosen to be a Gaussian:

$$S_{NP}(b_T; Q) = A \ln \frac{Q}{Q_{NP}} b_T^2 \quad \text{with} \quad Q_{NP} = 1 \text{ GeV}$$

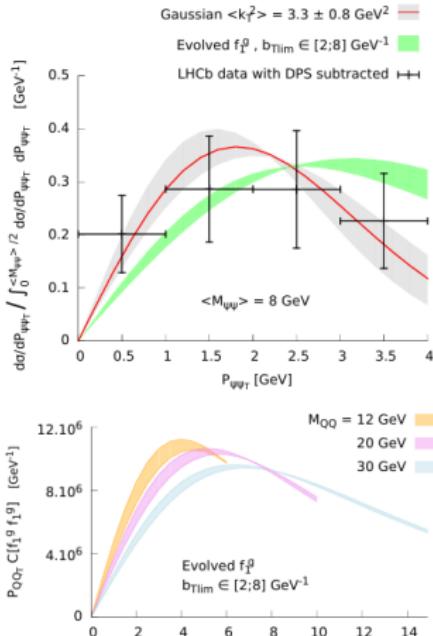
- We obtain the following expression for the convolutions:

$$\begin{aligned} \mathcal{C}[w f g](x_1, x_2, \vec{q}_T; Q) &= \int_0^\infty \frac{db_T}{2\pi} b_T^n J_m(b_T q_T) e^{-S_A(b_T^*; Q^2, Q)} e^{-S_{NP}(b_T; Q)} \\ &\times \hat{f}(x_1, b_T^*; \mu_b^2, \mu_b) \hat{g}(x_2, b_T^*; \mu_b^2, \mu_b) \end{aligned}$$

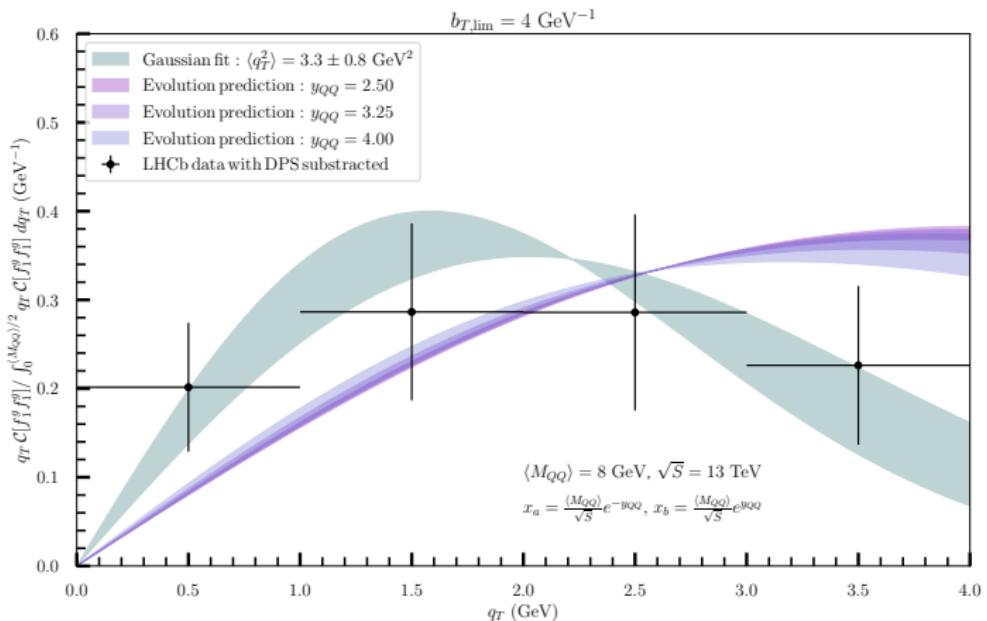
Switching on TMD evolution

F. Scarpa, D. Boer, M.G. Echevarria, JPL, C. Pisano, M. Schlegel, EPJC (2020) 80:87

- With a fit we obtained $\langle k_T^2 \rangle \sim 3 \text{ GeV}^2$
- Let us compare such a value with what a proper NLL evolution up to the scale $M_{\psi\psi} \sim 8 \text{ GeV}$ would give
- Evolution effects are measurable
- So far, no x dependence information

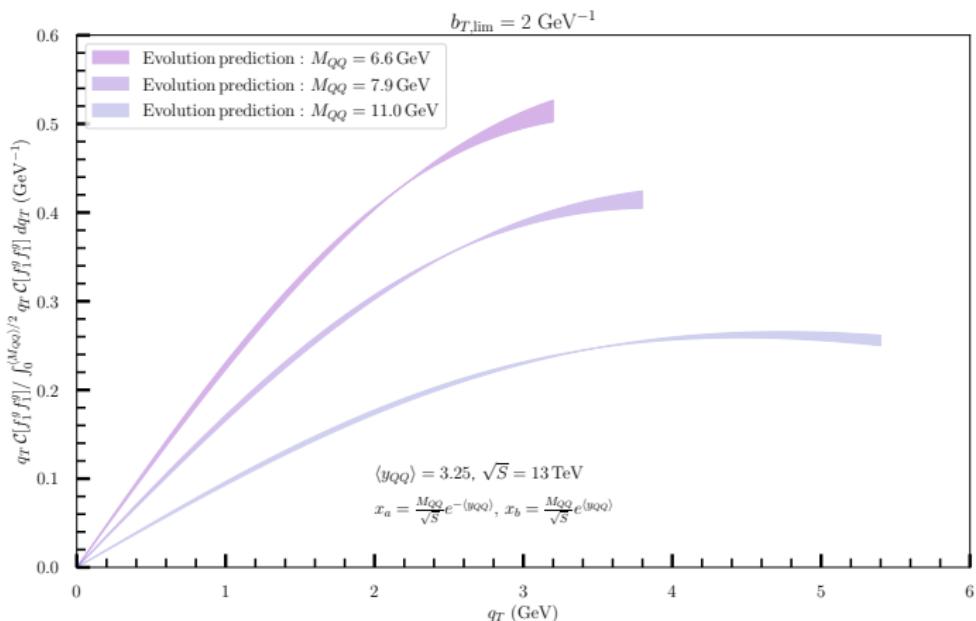


Switching on TMD evolution: preliminary update



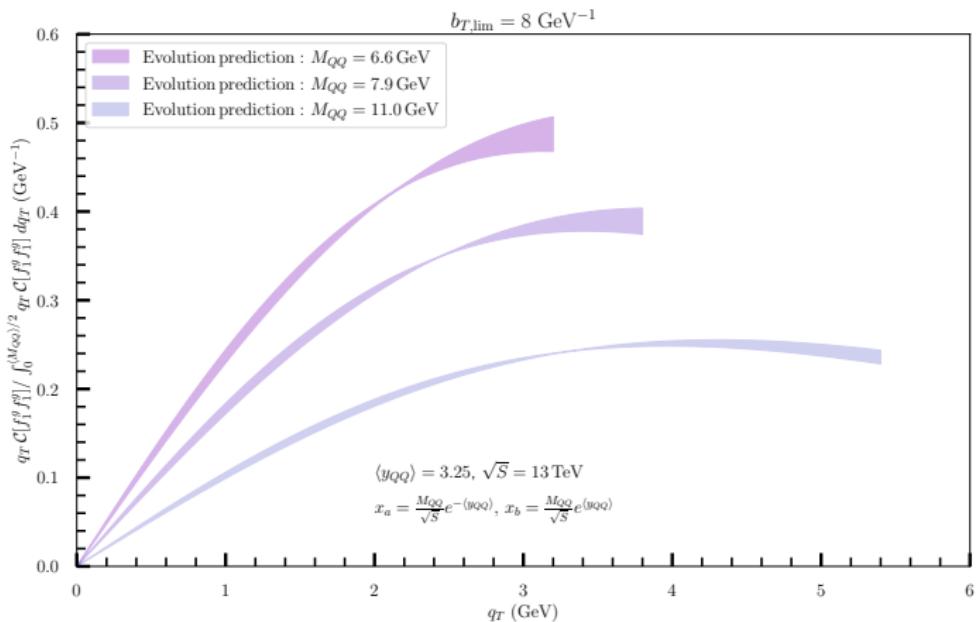
Switching on TMD evolution: preliminary update

LHCb data expected in a few weeks!



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Hadronic cross section

The general formula for the cross section of gluon fusion is:

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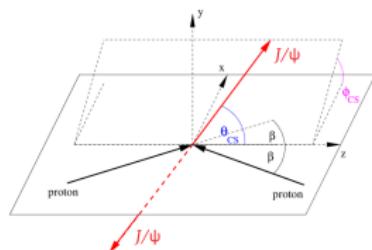
- first two members: azimuthally independent
- third member: $\cos(2\Phi_{CS})$ -modulation
- fourth member: $\cos(4\Phi_{CS})$ -modulation

Computation of azimuthal modulations (average)

The corresponding expressions for $\cos(2\phi_{CS})$ and $\cos(4\phi_{CS})$:

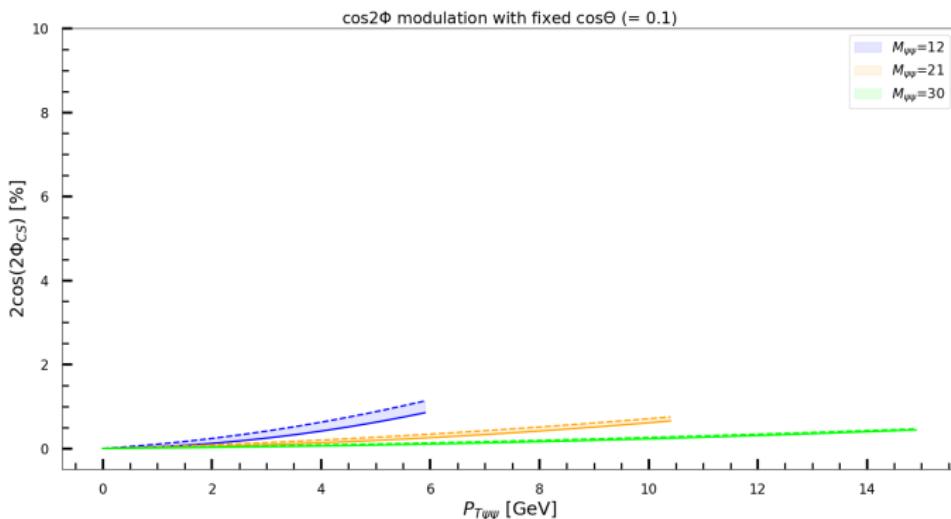
$$\langle \cos(2\phi_{CS}) \rangle = \frac{1}{2} \frac{F_3 \mathcal{C}[w_3 f_1^g h_1^{\perp g}] + F'_3 \mathcal{C}[w'_3 h_1^{\perp g} f_1^g]}{F_1 \mathcal{C}[f_1^g f_1^g] + F_2 \mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}]}$$

$$\langle \cos(4\phi_{CS}) \rangle = \frac{1}{2} \frac{F_4 \mathcal{C}[w_4 h_1^{\perp g} h_1^{\perp g}]}{F_1 \mathcal{C}[f_1^g f_1^g] + F_2 \mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}]}$$

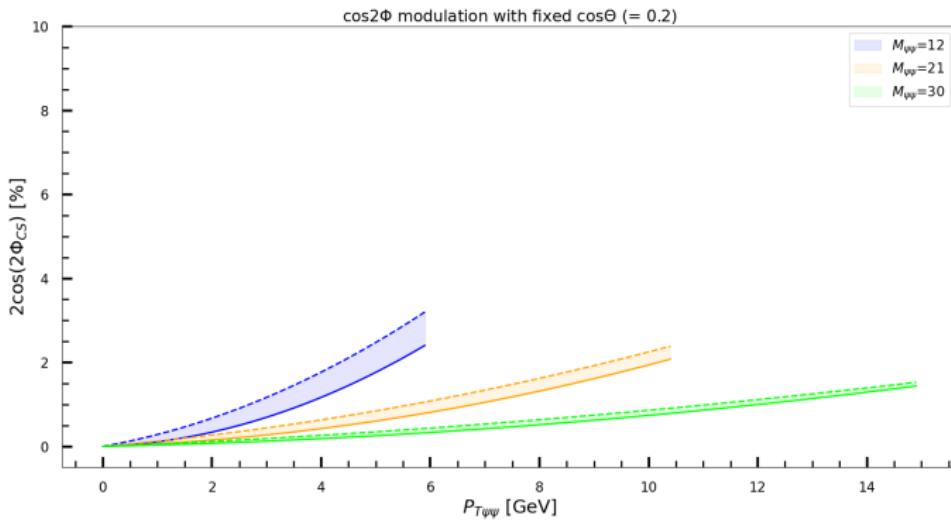


- The hard-scattering coefficients (F_1, F_2, F_3, F'_3, F_4) give the explicit dependence on $M_{\psi\psi}$ and θ_{CS}
- Modulations due to $h_1^{\perp g}$
- Set scale $Q^2 = M_{\psi\psi}^2$
- TMD evolution applied within the convolutions

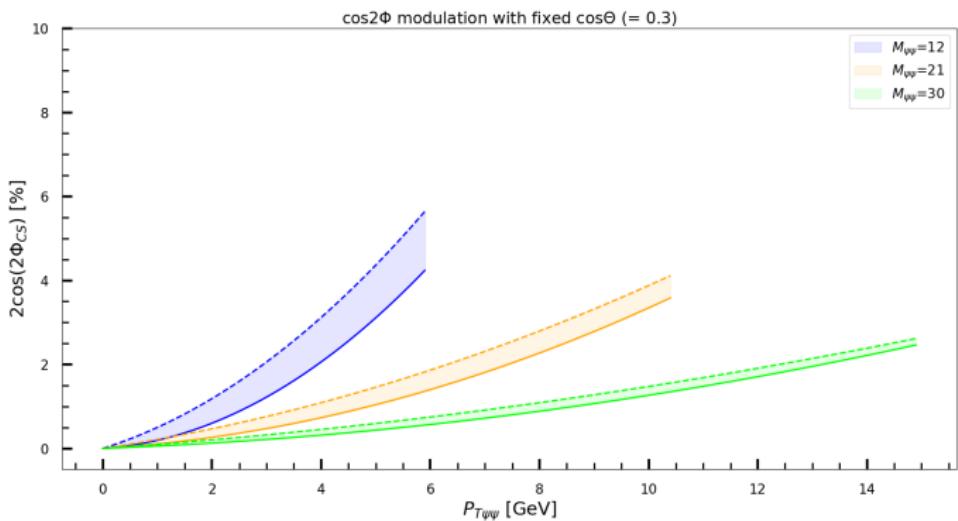
Preliminary results: $2\cos(2\phi)$ at fixed value of $\cos\theta$



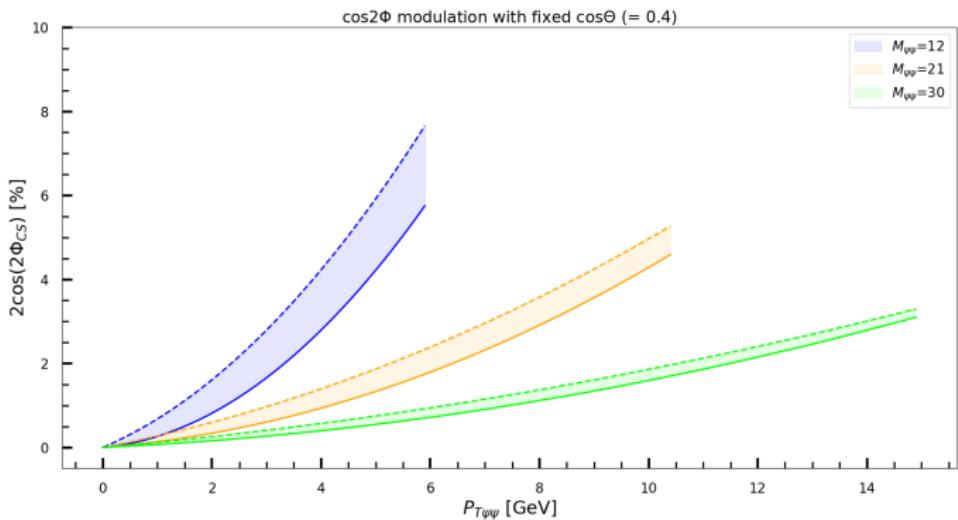
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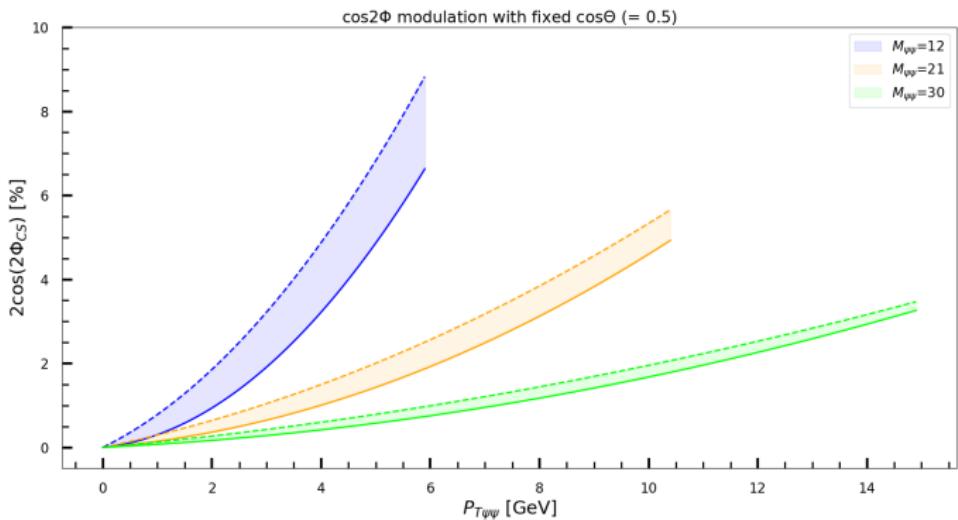
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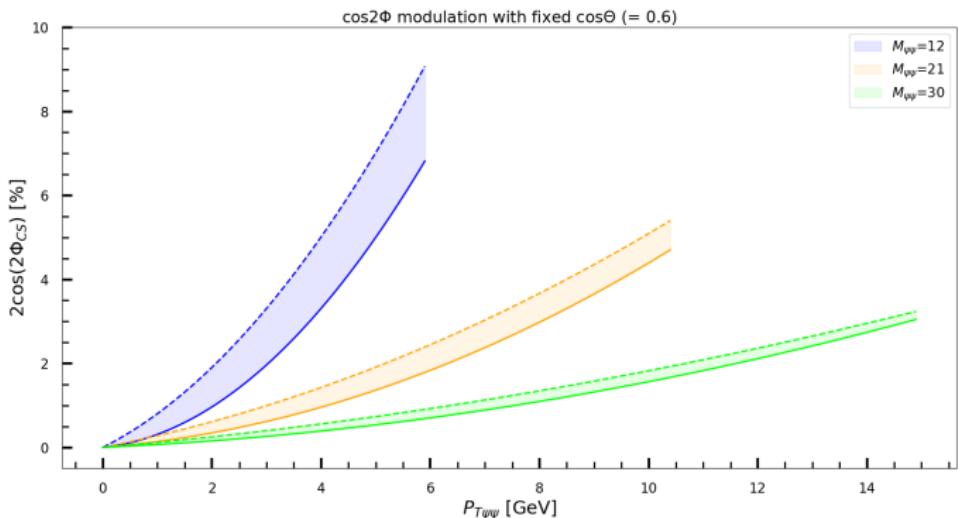
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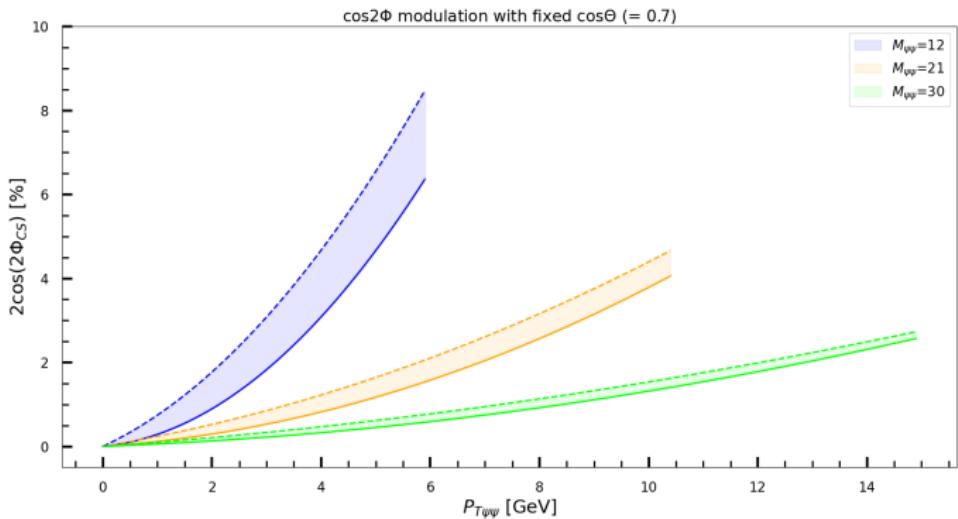
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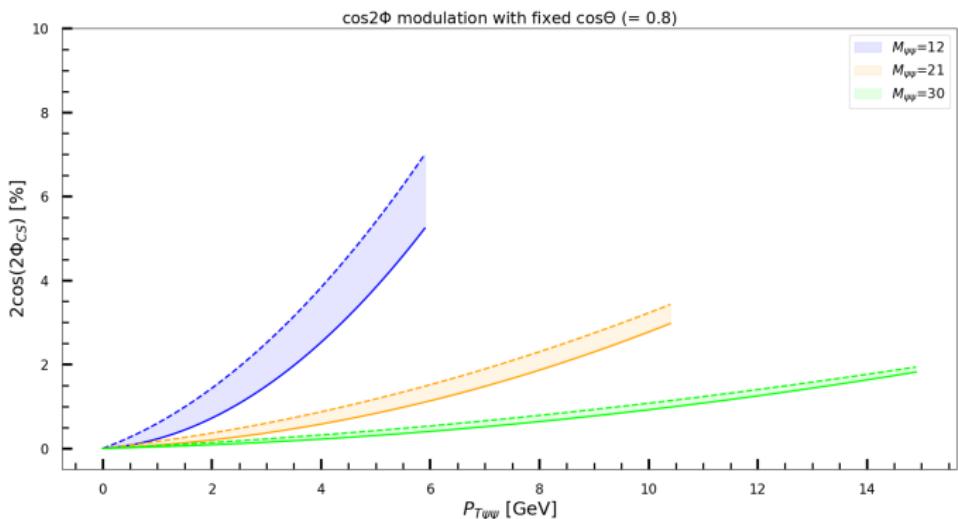
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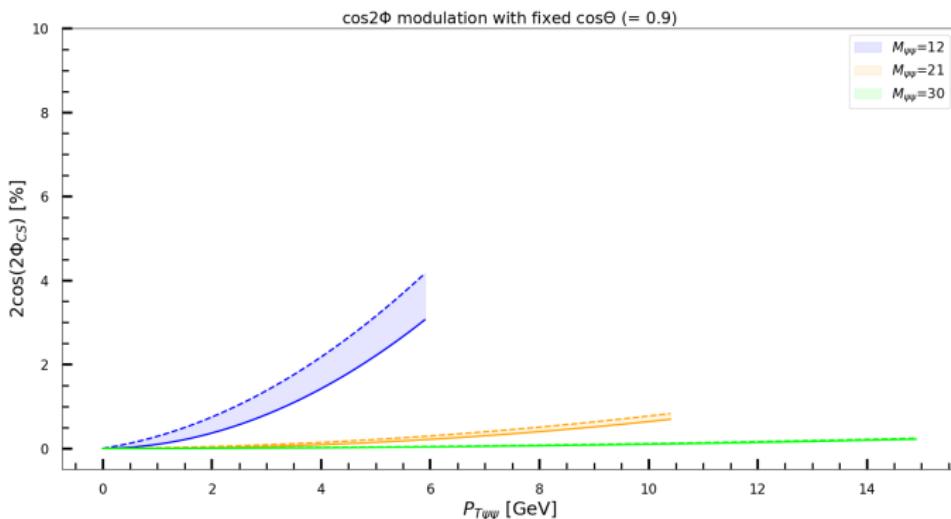
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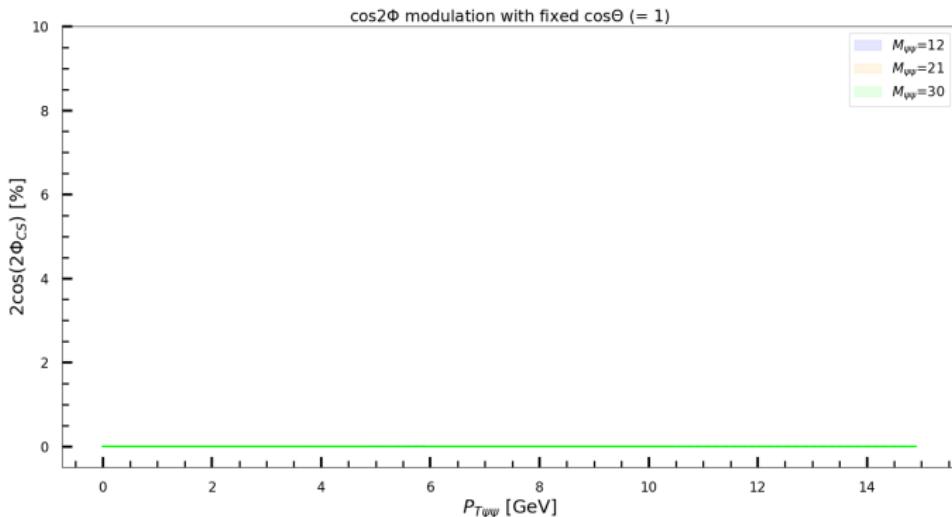
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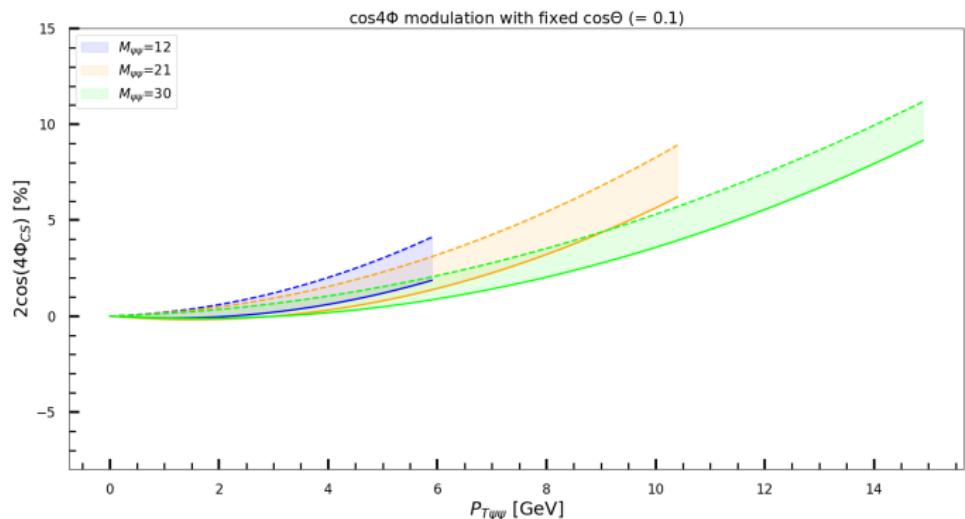
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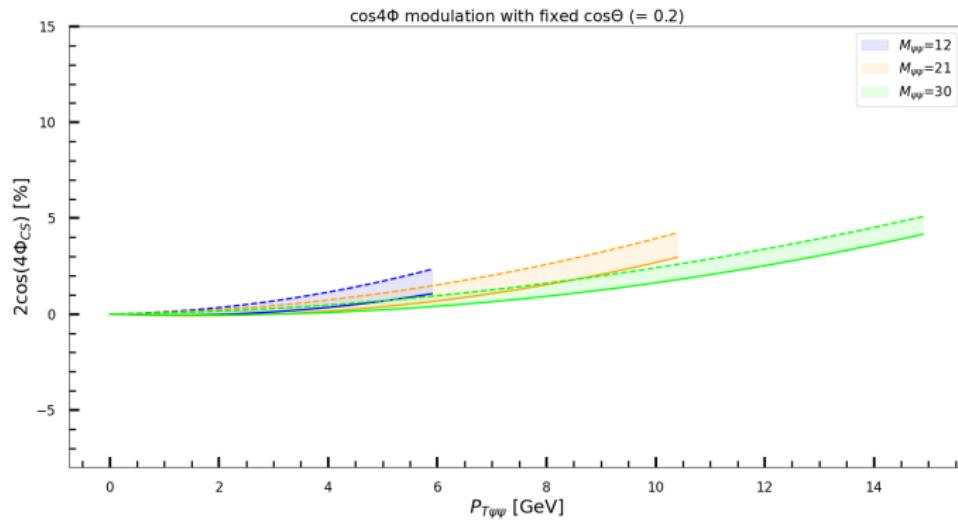
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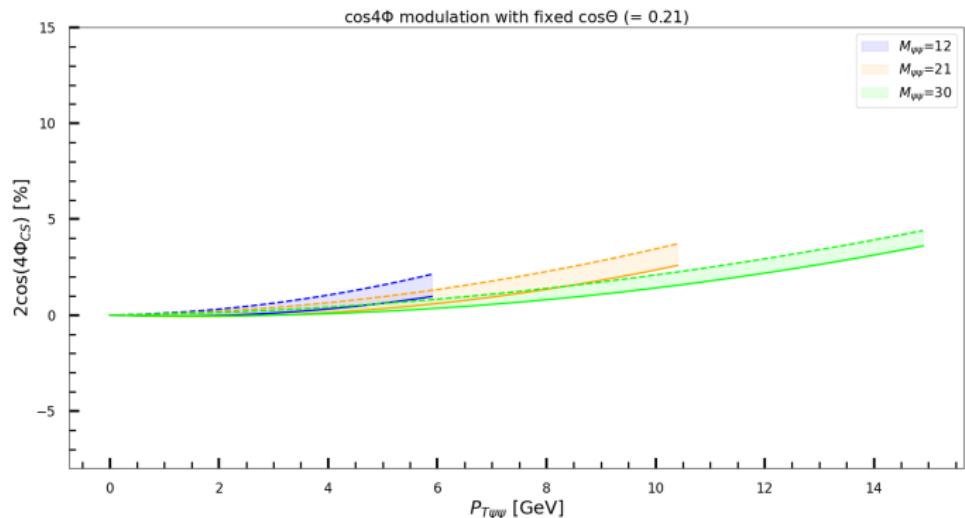
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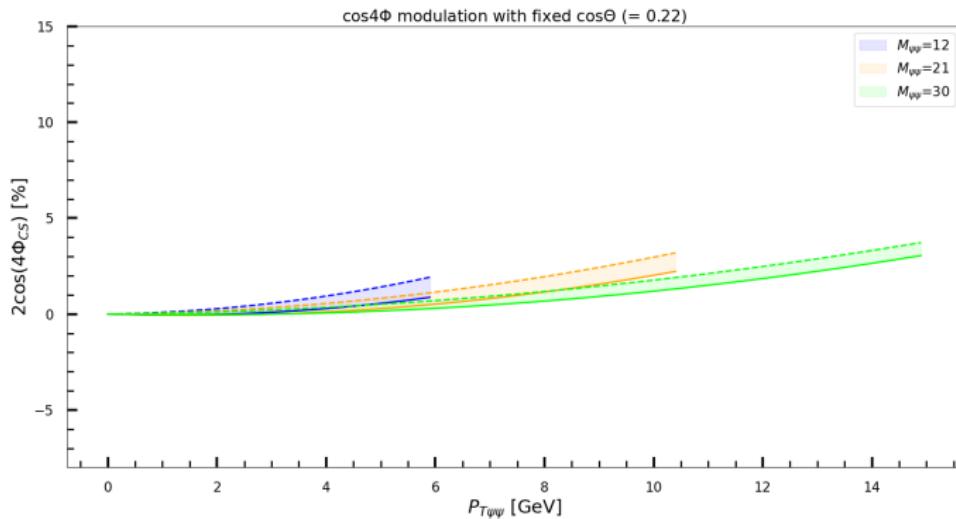
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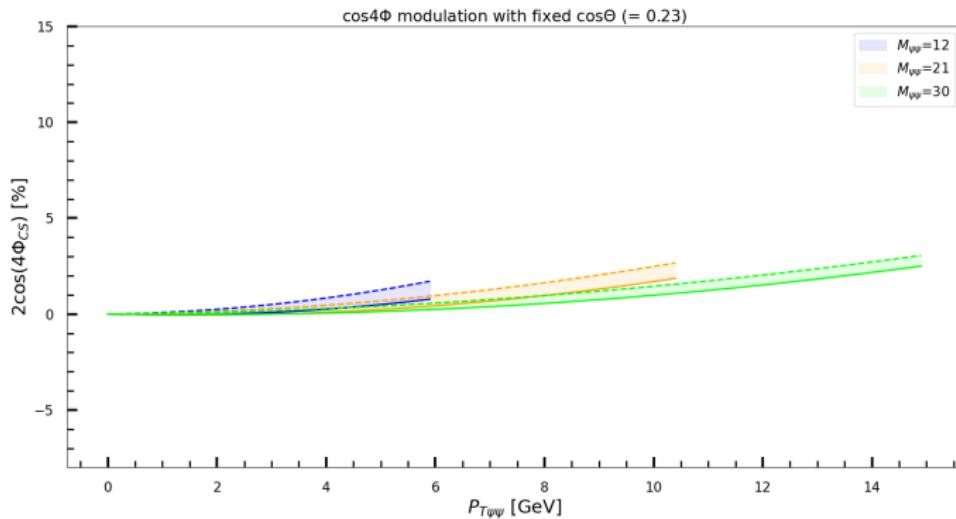
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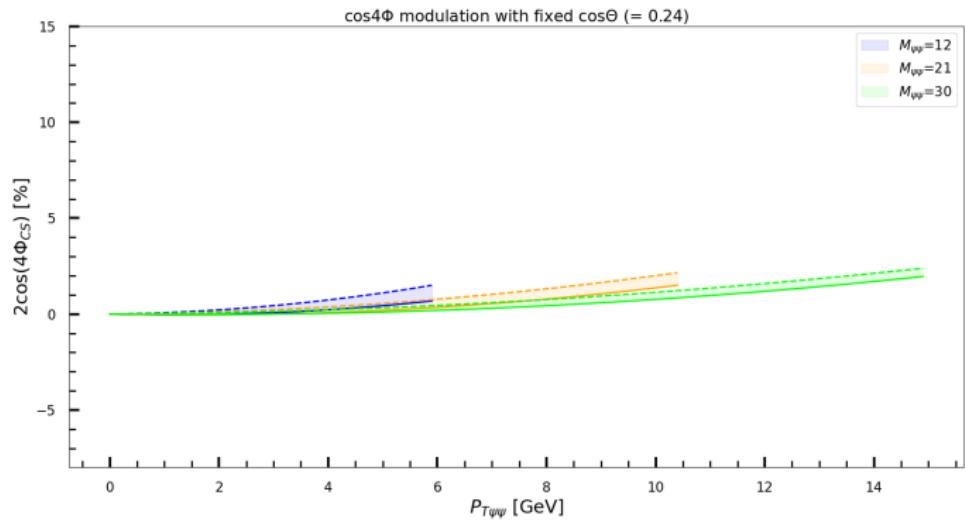
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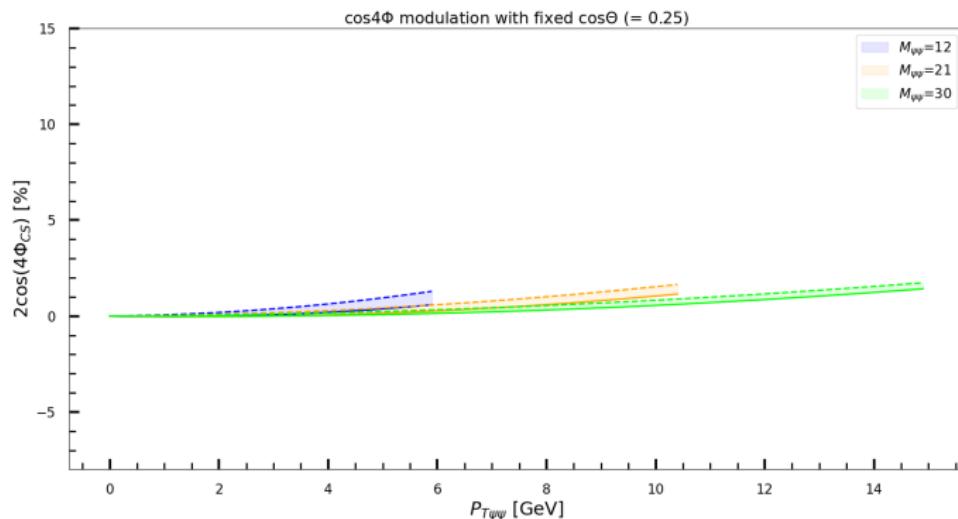
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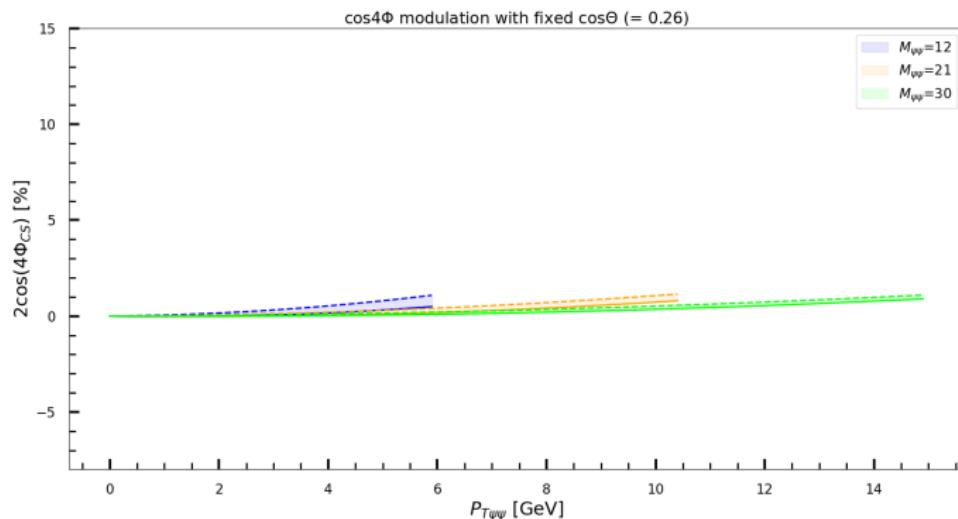
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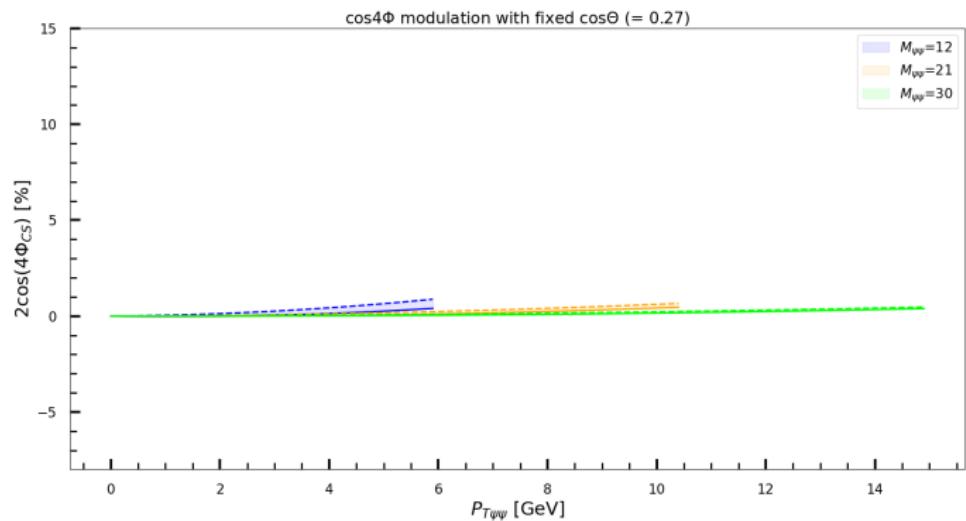
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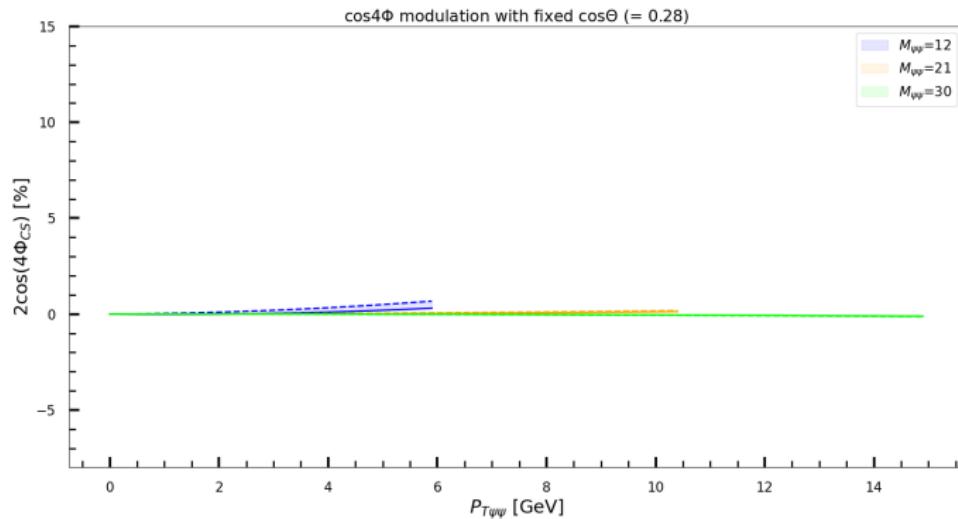
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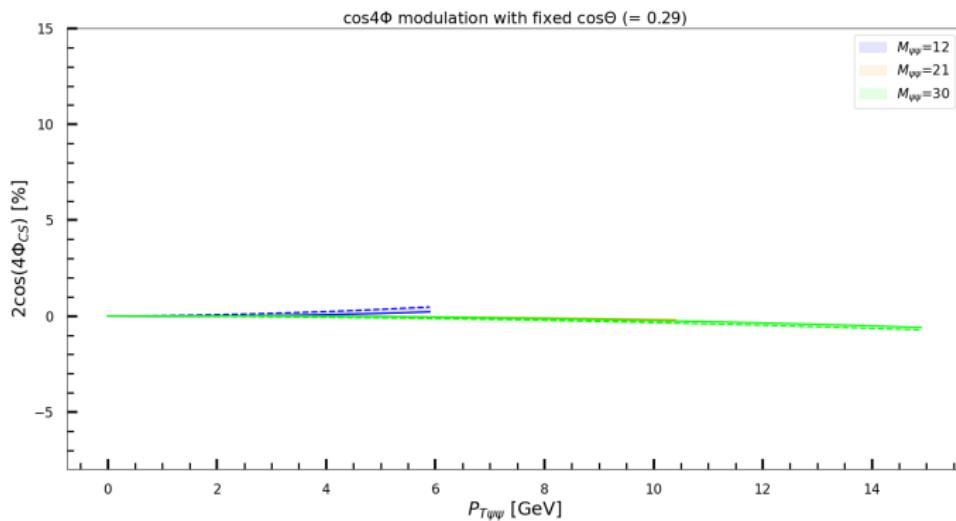
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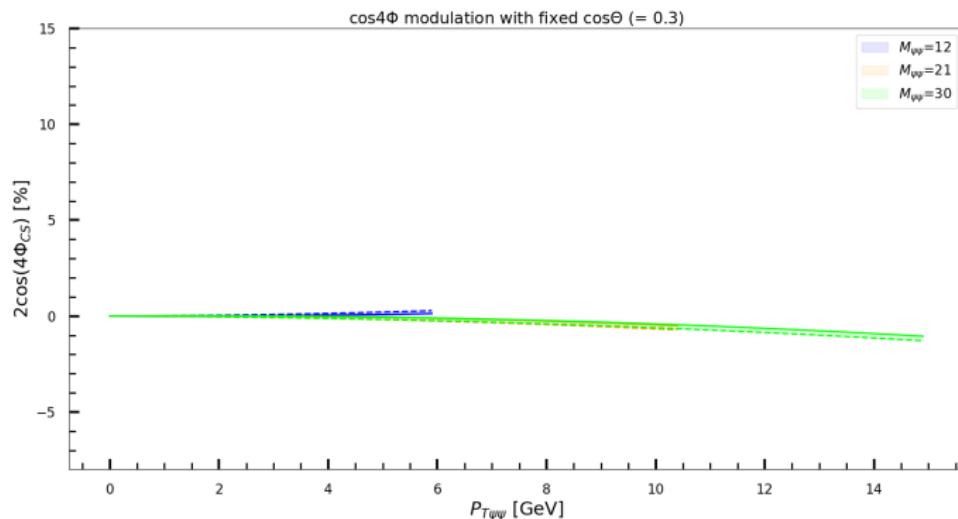
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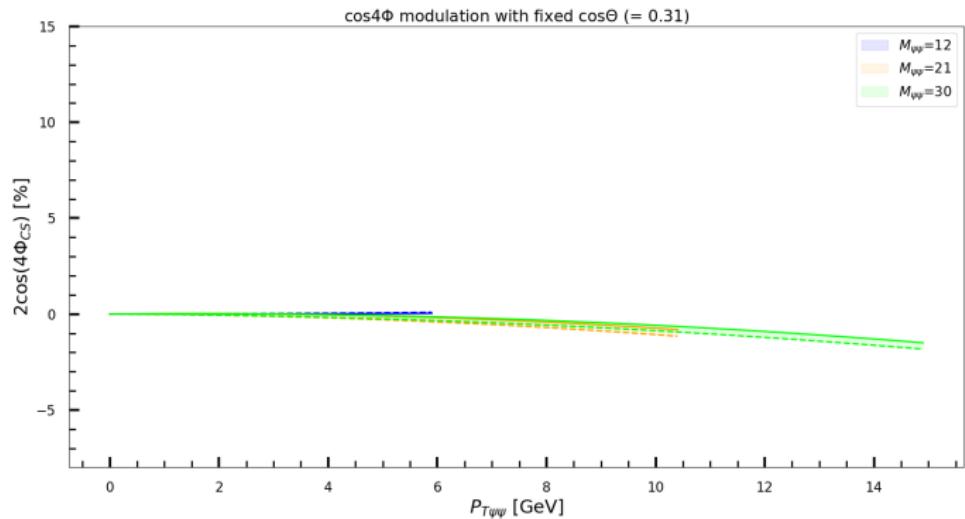
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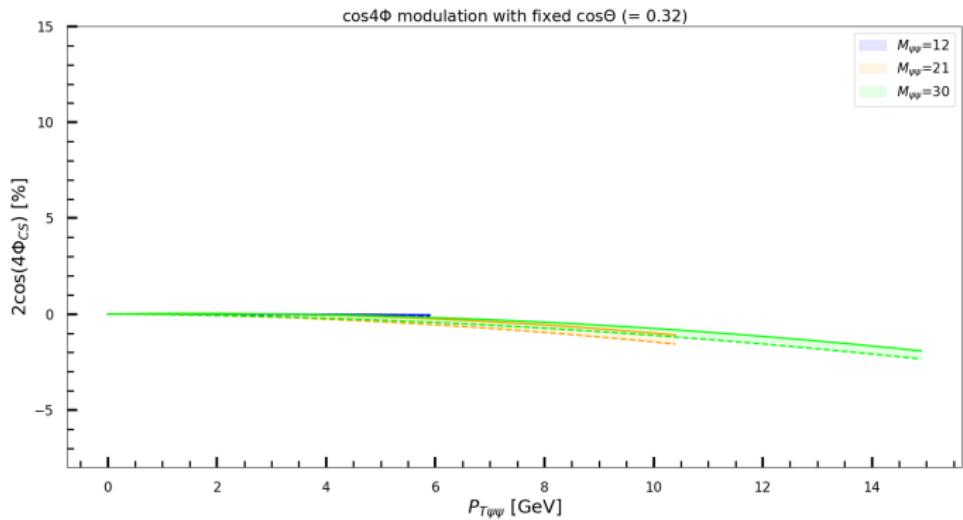
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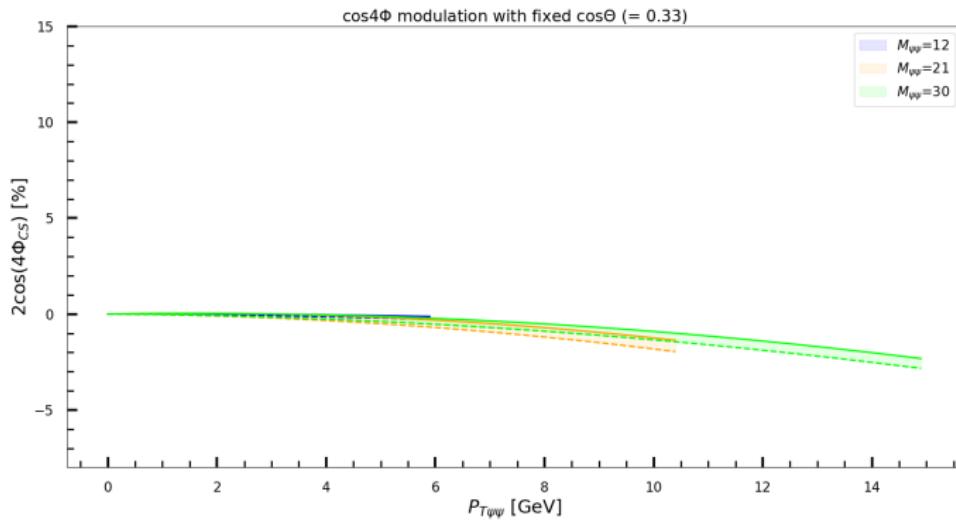
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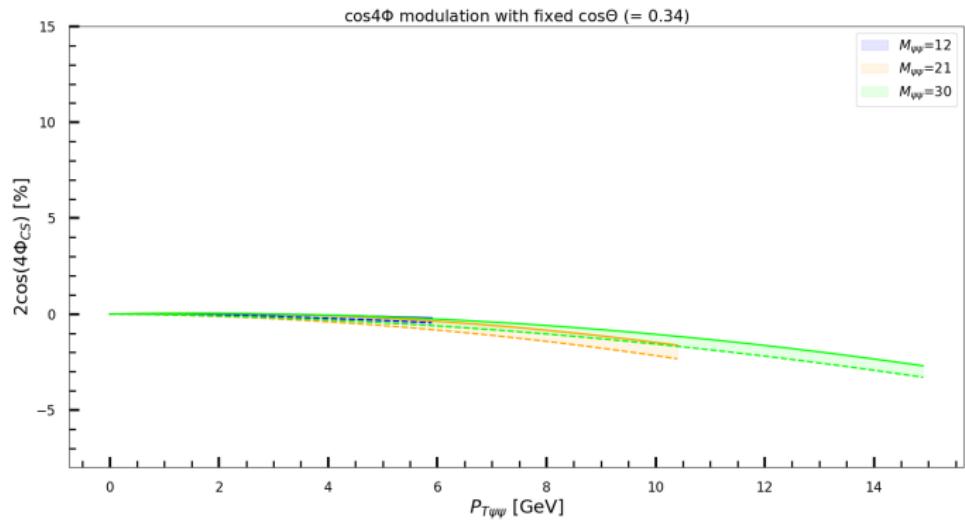
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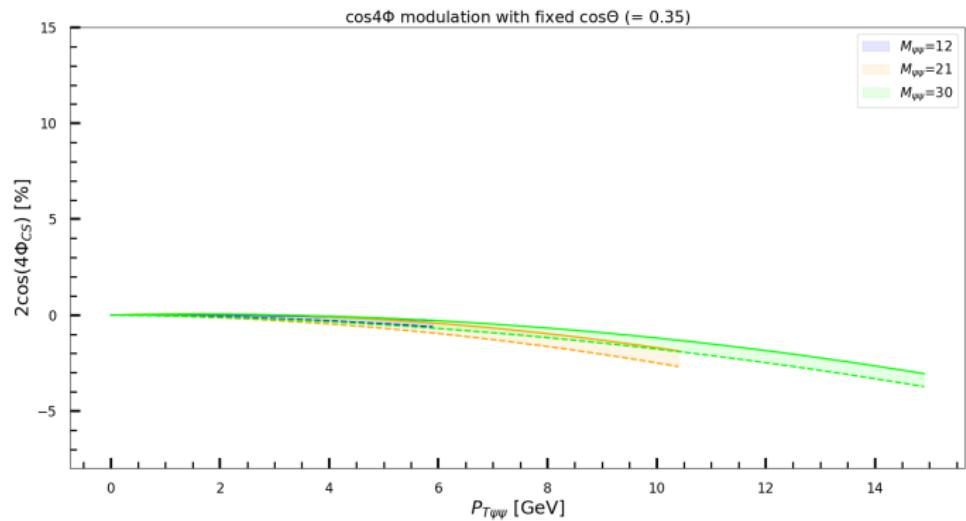
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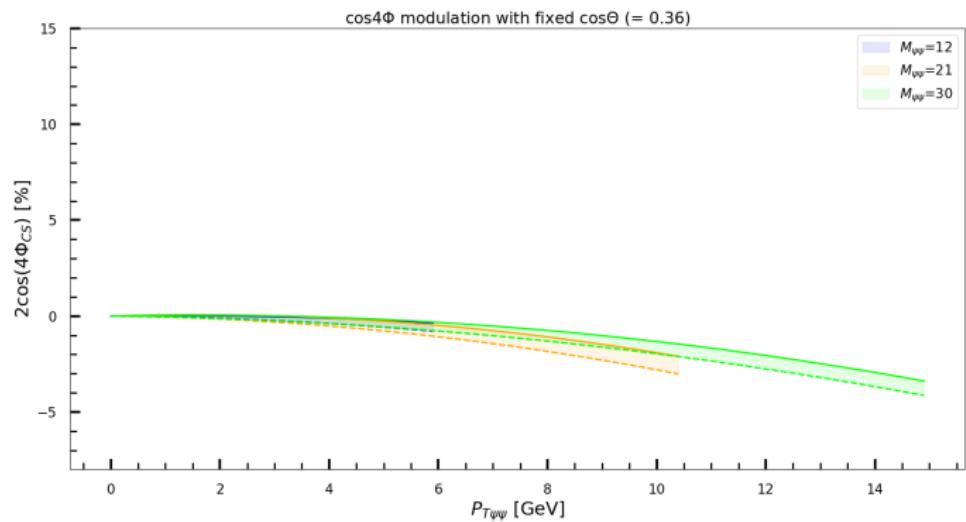
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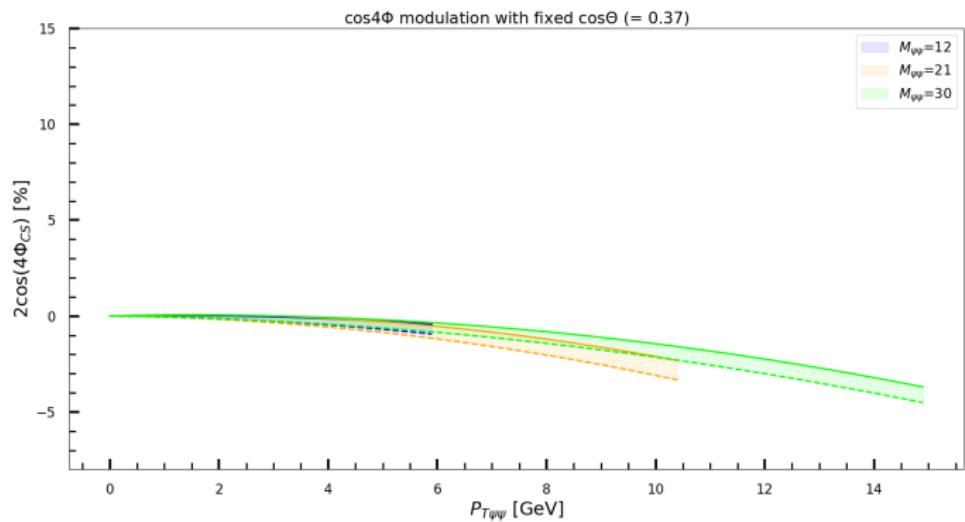
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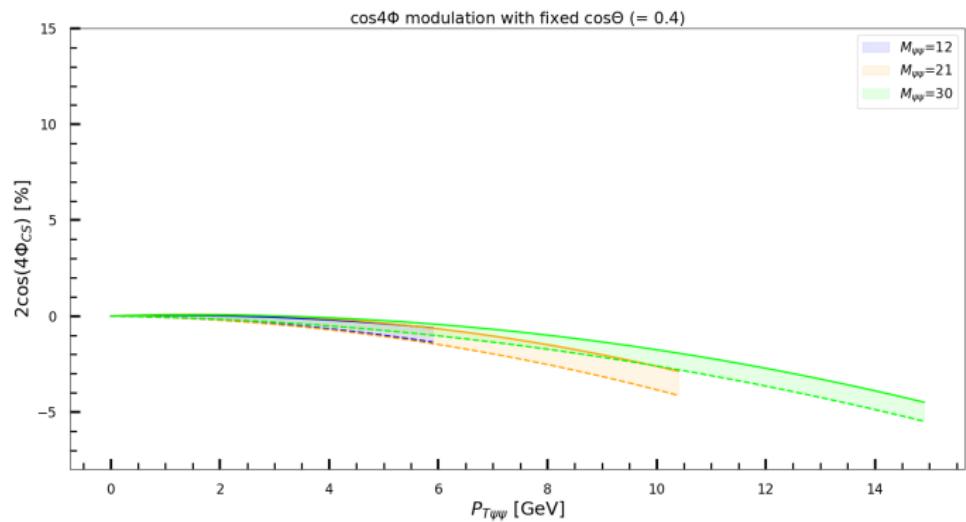
Preliminary results: $2\cos(4\phi)$ at fixed value of $\cos\theta$



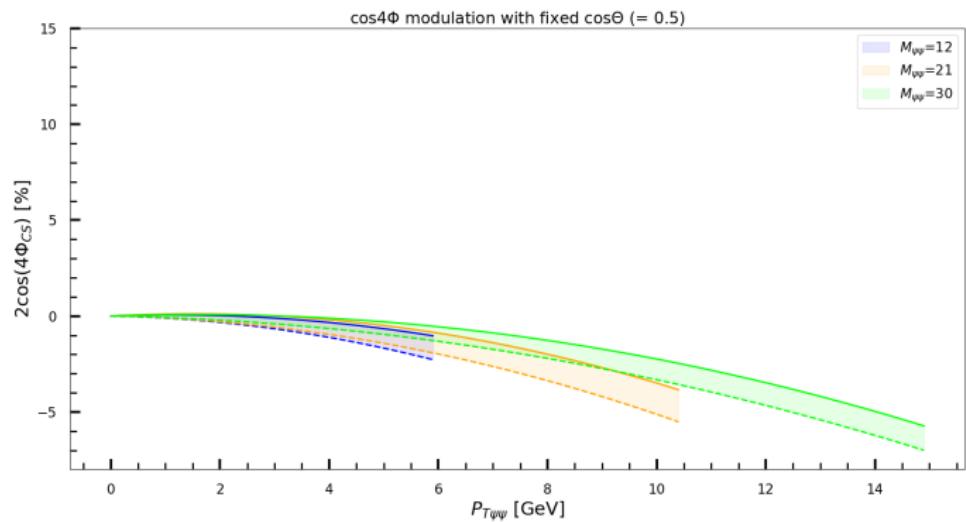
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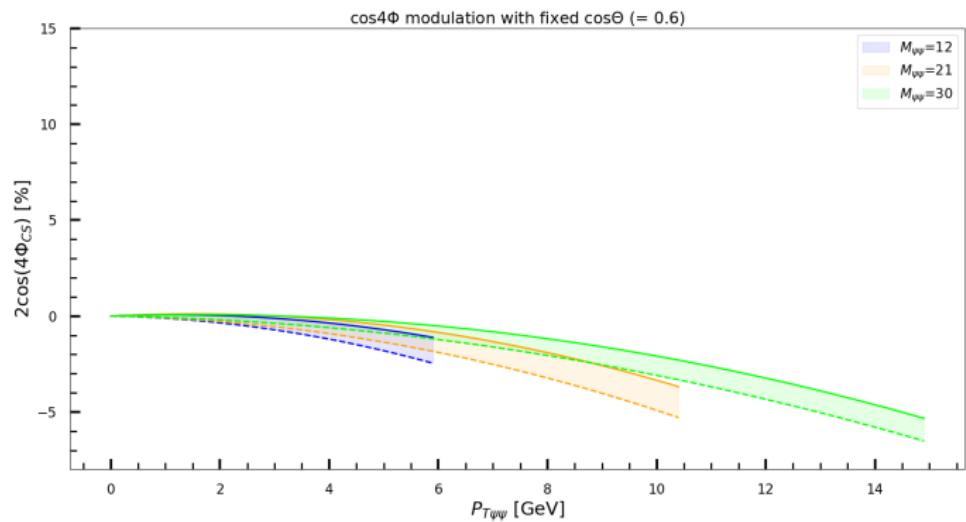
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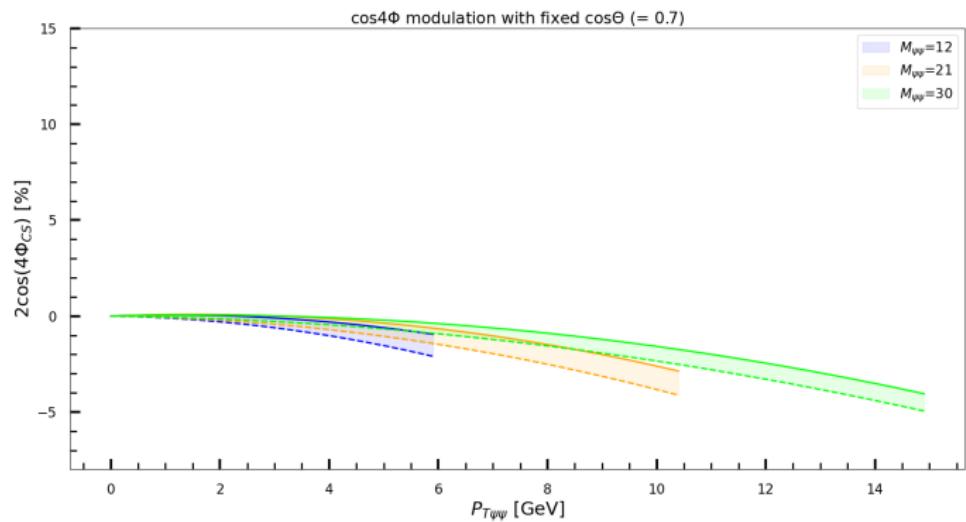
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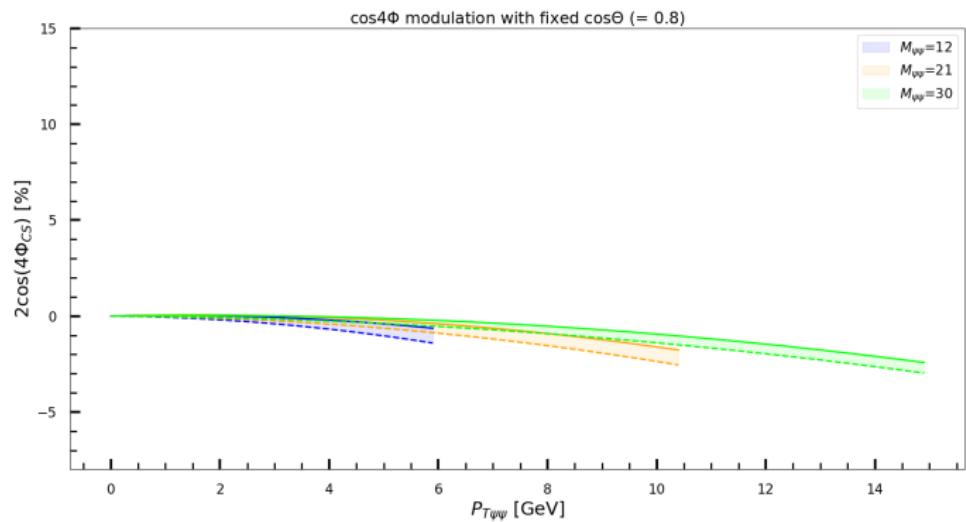
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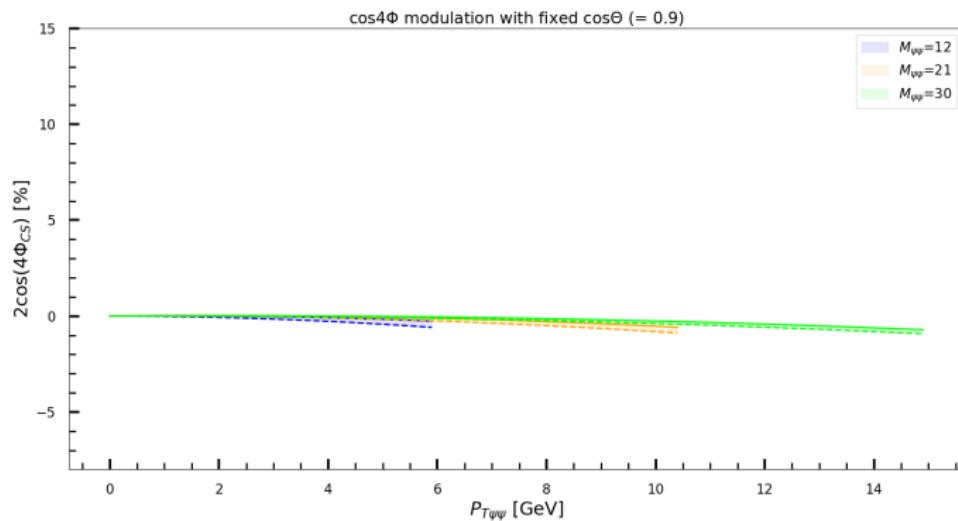
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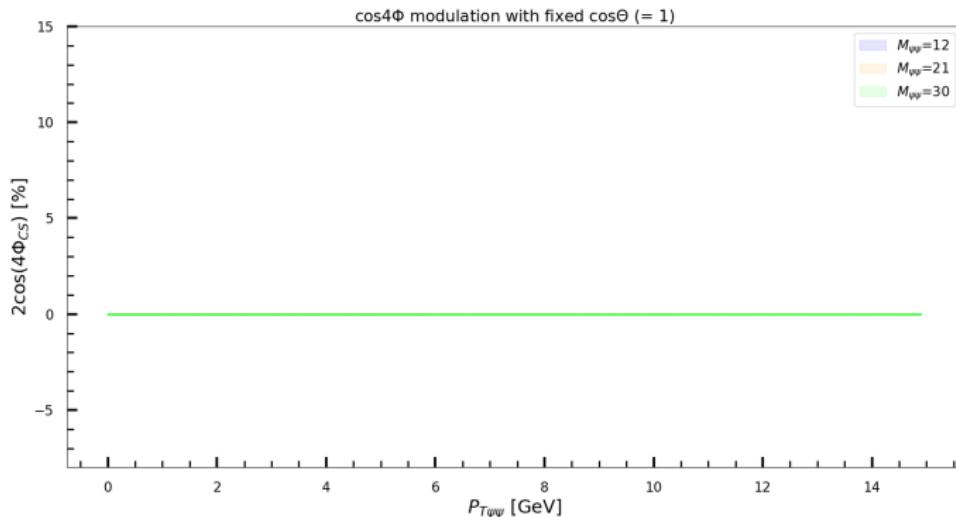
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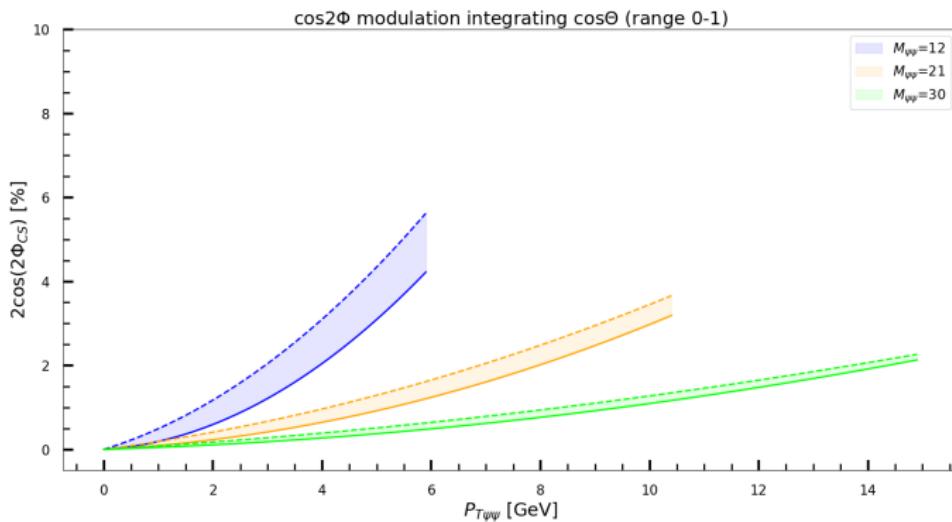
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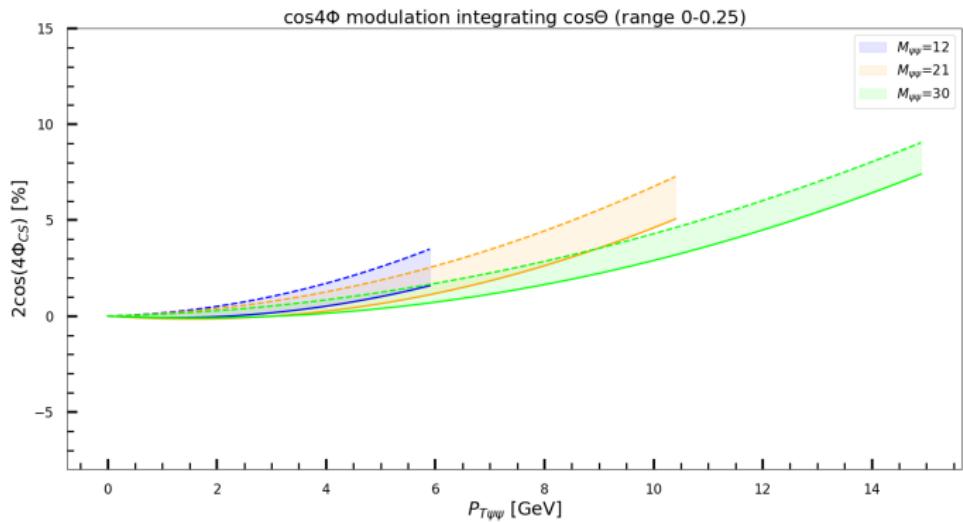
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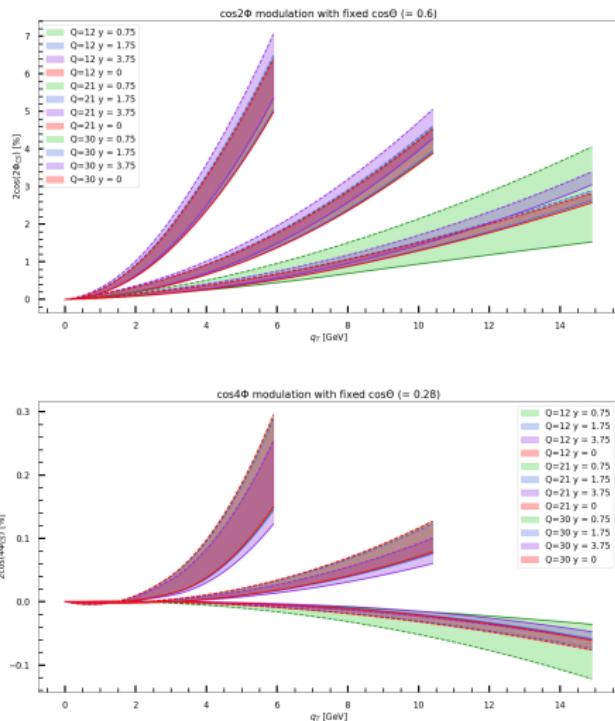
Preliminary results: $2\cos(2\phi)$ with integrated $\cos\theta$



Preliminary results: $2\cos(4\phi)$ with integrated $\cos\theta$



Preliminary results: rapidity dependence?



1 Introduction

2 Gluon TMDs

3 Gluon TMDs and di- J/ψ production

4 Azimuthal modulations

5 Conclusions

- Quarkonium production is a great tool for many purposes
 ↪ exploration of nucleon structure through gluon TMDs
- Double J/ψ production gives the possibility to investigate gluon TMD induced effects
- New LHCb data (@13TeV) coming soon
 - Data driven subtraction of DPS
 - First multidifferential measurements (q_T , Q, y)
 - First constraints on the azimuthal modulations!
- **FUTURE** Studies can be made considering polarised protons (FT LHCb) → access to more gluon TMDs
- Di- J/ψ production: most promising → gluon Sivers function

Backup slides

The Sudakov Factor and Scales

- The solution of the evolution equations results in:

$$\hat{f}_1^g(x_1, b_T; \zeta, \mu) = e^{-\frac{1}{2}S_A(b_T; \zeta, \mu)} \hat{f}_1^g(x_1, b_T; \mu_b^2, \mu_b)$$

$$\hat{h}_1^{\perp g}(x_1, b_T; \zeta, \mu) = e^{-\frac{1}{2}S_A(b_T; \zeta, \mu)} \hat{h}_1^{\perp g}(x_1, b_T; \mu_b^2, \mu_b)$$

- $\mu \sim Q$ avoids large logarithms in \mathcal{H}
- TMDs should be evaluated at their natural scale:
 $\sqrt{\zeta_0} \sim \mu_0 \ll \sqrt{\zeta} \sim \mu$
- \Rightarrow take $\sqrt{\zeta_0} \sim \mu_0 \sim \mu_b \equiv b_0/b_T$ (with $b_0 = 2e^{-\gamma_E}$), in order to minimize both logarithms of μb_T and ζb_T^2 in S_A , and then evolved up to $\sqrt{\zeta} \sim \mu \sim Q$

Perturbative tails

- The large transverse momentum perturbative tail of the TMDs can be written as:

$$\hat{f}_1^g(x, b_T; \mu_b^2, \mu_b) = f_{g/P}(x; \mu_b) + \mathcal{O}(\alpha_s) + \mathcal{O}(b_T \Lambda_{\text{QCD}})$$

$$\begin{aligned}\hat{h}_1^{\perp g}(x, b_T; \mu_b^2, \mu_b) = & -\frac{\alpha_s(\mu_b)}{\pi} \int_x^1 \frac{dx'}{x'} \left(\frac{x'}{x} - 1 \right) \left\{ C_A f_{g/P}(x'; \mu_b) + \right. \\ & \left. C_F \sum_{i=q, \bar{q}} f_{i/P}(x'; \mu_b) \right\} + \mathcal{O}(\alpha_s^2) + \mathcal{O}(b_T \Lambda_{\text{QCD}})\end{aligned}$$

P. Sun et al. (Phys. Rev. D 84 (2011) 094005)

b_T -Domains

- To ensure $b_0/Q \leq b_T$ we take:

$$b_c(b_T) = \sqrt{b_T^2 + \left(\frac{b_0}{Q}\right)^2}$$

- For $b_T \leq b_{T,\max}$:

$$b_T^*(b_c(b_T)) = \frac{b_c(b_T)}{\sqrt{1 + \left(\frac{b_c(b_T)}{b_{T,\max}}\right)^2}}$$

J. Collins et al. (Phys.Rev.D 94 (2016) 3, 034014)

The Non-perturbative Sudakov Factor

$$S_{NP}(b_T; Q) = A \ln \frac{Q}{Q_{NP}} b_T^2 \quad \text{with} \quad Q_{NP} = 1 \text{ GeV}$$

$b_{T,\text{lim}}$ (GeV $^{-1}$)	r (fm $\sim 1/(0.2 \text{ GeV})$)	A (GeV 2)
2	0.2	0.64
4	0.4	0.16
8	0.8	0.04

Table 1: Values of the parameter A for $b_{T,\text{lim}}$ and r determined at $Q = 12 \text{ GeV}$. A is defined at which $\exp(-S_{NP})$ becomes negligible ($\sim 10^{-3}$). To estimate the uncertainty associated with the S_{NP} we vary $b_{T,\text{lim}}$ spanning roughly from $b_{T,\text{max}} = 1.5 \text{ GeV}^{-1}$ to the charge radius of the proton. r is the range over which the interactions occur from the centre of the proton.

Hard scattering coefficients

$$\begin{aligned} F_1 &= \frac{\mathcal{N}}{\mathcal{D} M_{\Psi\Psi}^2} \sum_{n=0}^6 f_{1,n} (\cos \theta_{CS})^{2n} & F_2 &= \frac{2^4 3 M_{\Psi\Psi}^2 \mathcal{N}}{\mathcal{D} M_{\Psi\Psi}^4} \sum_{n=0}^4 f_{2,n} (\cos \theta_{CS})^{2n} \\ F'_3 &= F_3 = \frac{-2^3 (1 - \alpha^2) \mathcal{N}}{\mathcal{D} M_{\Psi\Psi}^2} \sum_{n=0}^5 f_{3,n} (\cos \theta_{CS})^{2n} \\ F_4 &= \frac{(1 - \alpha^2)^2 \mathcal{N}}{\mathcal{D} M_{\Psi\Psi}^2} \sum_{n=0}^6 f_{4,n} (\cos \theta_{CS})^{2n} \end{aligned} \tag{1}$$

with: $\alpha = \frac{2M_\Psi}{M_{\Psi\Psi}}$, $\mathcal{N} = 2^{11} 3^{-4} (N_c^2 - 1)^{-2} \pi^2 \alpha_s^4 |R_\Psi(0)|^4$,

$\mathcal{D} = M_{\Psi\Psi}^4 (1 - (1 - \alpha^2) \cos \theta_{CS}^2)^4$ and $R_\Psi(0)$ is the J/ψ radial wave function at the origin and $N_c = 3$.