

Progresses on the TMD shape function in SIDIS

Talk @ QCD evolution 2023

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In collaboration with:
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Outline

- TMD factorization involving a **shape function** for the quarkonium
- Extraction of the TMD shape function via a matching procedure
 - Relevance of the hard amplitude **pole structure**
- Process dependence of the TMD shape function
- Conclusions and outcome



Quarkonia & gluon TMDs

Processes involving Quarkonia are **sensitive to gluons**

hadron collisions

- $p + p \rightarrow \eta_Q + X$

- $p + p \rightarrow \chi_Q + X$

- $p + p \rightarrow J/\psi + J/\psi + X$

- $p + p \rightarrow J/\psi + X ?$

ep collisions

- $e + p \rightarrow e' + J/\psi + X$

- $e + p \rightarrow e' + J/\psi + jet + X$

and more...



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***ep* collisions**

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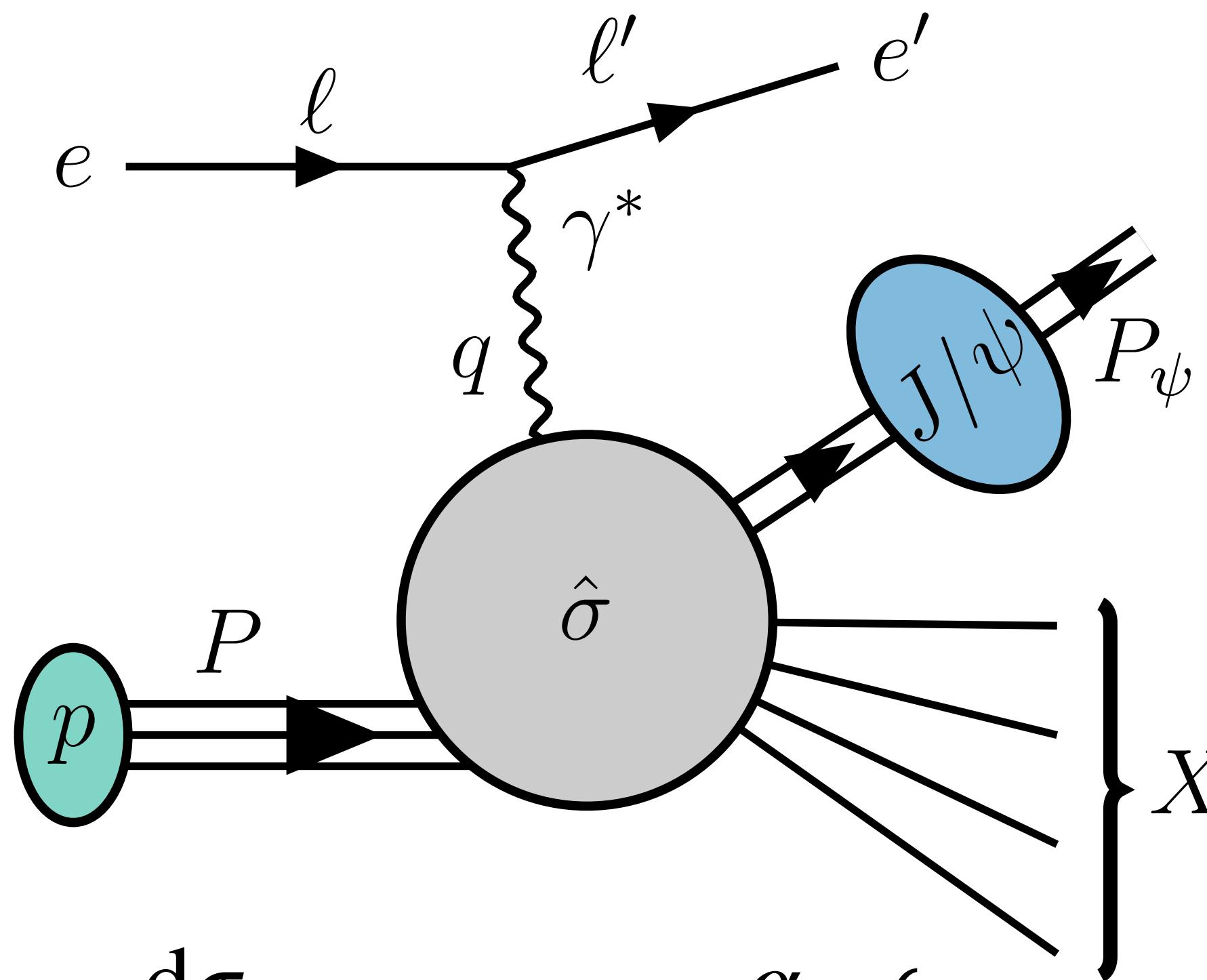
- $e + p \rightarrow e' + J/\psi + jet + X$

and more...



Theoretical framework

$$e(\ell) + p(P) \rightarrow e'(\ell') + \gamma^*(q) + p(P) \rightarrow e'(\ell') + J/\psi(P_\psi) + X$$



SIDIS variables

$$q^2 = -Q^2, S \approx 2P \cdot \ell$$

$$x_B = \frac{Q^2}{2 \cdot q}, y = \frac{P \cdot q}{P \cdot \ell}, z = \frac{P \cdot P_\psi}{P \cdot q}$$

Phase spaces

$$\frac{d^3\ell'}{2E'} = 2\pi yS dx_B dy$$

$$\frac{d^3P_\psi}{2E_\psi} = \frac{dz}{z} d^2P_{\psi\perp} d\phi_\psi$$

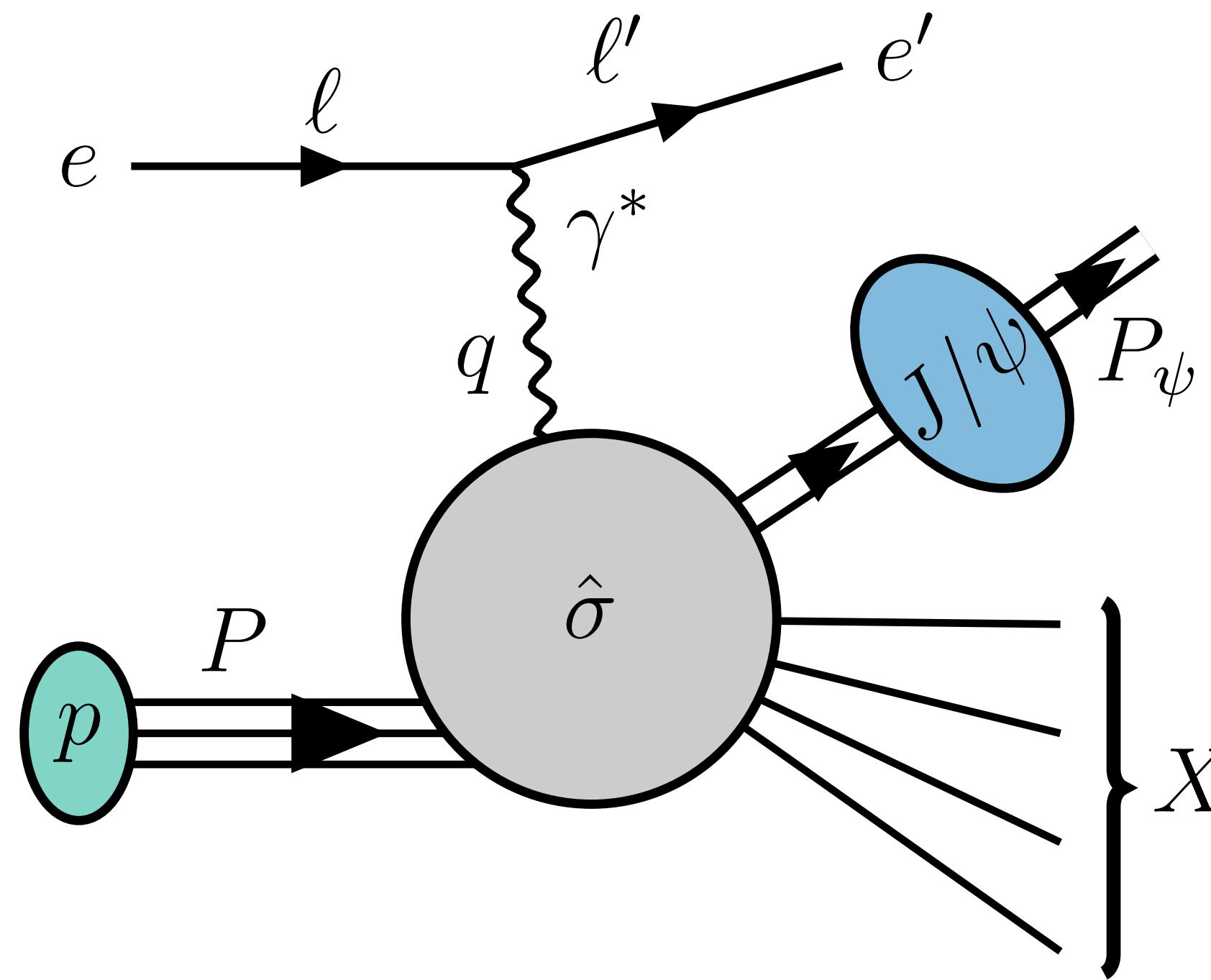
[Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP 02 \(2007\)](#)

[Sun, Zhang, EJPC 77 \(2017\)](#)

$$\begin{aligned} \frac{d\sigma}{dx_B dy dz dP_{\psi\perp}^2 d\phi_\psi} &= \frac{\alpha}{yQ^2} \left\{ [1 + (1-y)^2] F_{UUT} + 4(1-y) F_{UUL} \right. \\ &\quad \left. + (2-y)\sqrt{1-y} \cos\phi_\psi F_{UU}^{\cos\phi_\psi} + 4(1-y) \cos 2\phi_\psi F_{UU}^{\cos 2\phi_\psi} \right\} \end{aligned}$$



The TMD shape function



“light-hadron” SIDIS

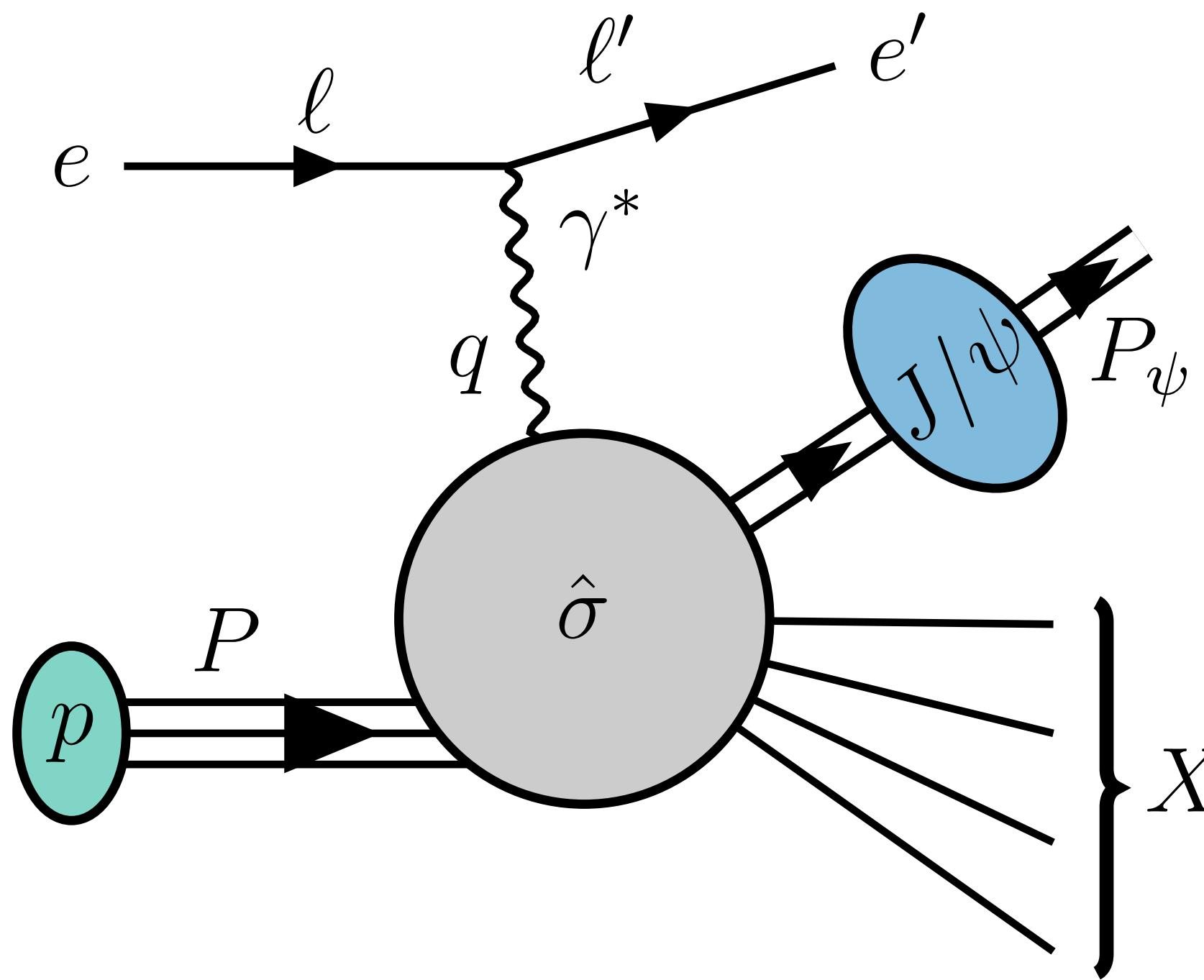
$$\sigma^{ep \rightarrow e' h X} = \hat{\sigma}^{[a]}(\mu_H) \otimes f_p(\hat{x}; \mu_H) \otimes D_{a \rightarrow h}(\hat{z}; \mu_H)$$

[Bodwin, Braaten, Lepage, PRD 51 \(1997\)](#)

“Quarkonium” SIDIS (adopting NRQCD)

$$\sigma^{ep \rightarrow e' J/\psi X} = \hat{\sigma}^{[n]}(\mu_H) \otimes f_p(\hat{x}; \mu_H) \otimes \langle \mathcal{O}_\psi[n] \rangle \delta(\hat{z} - z)$$

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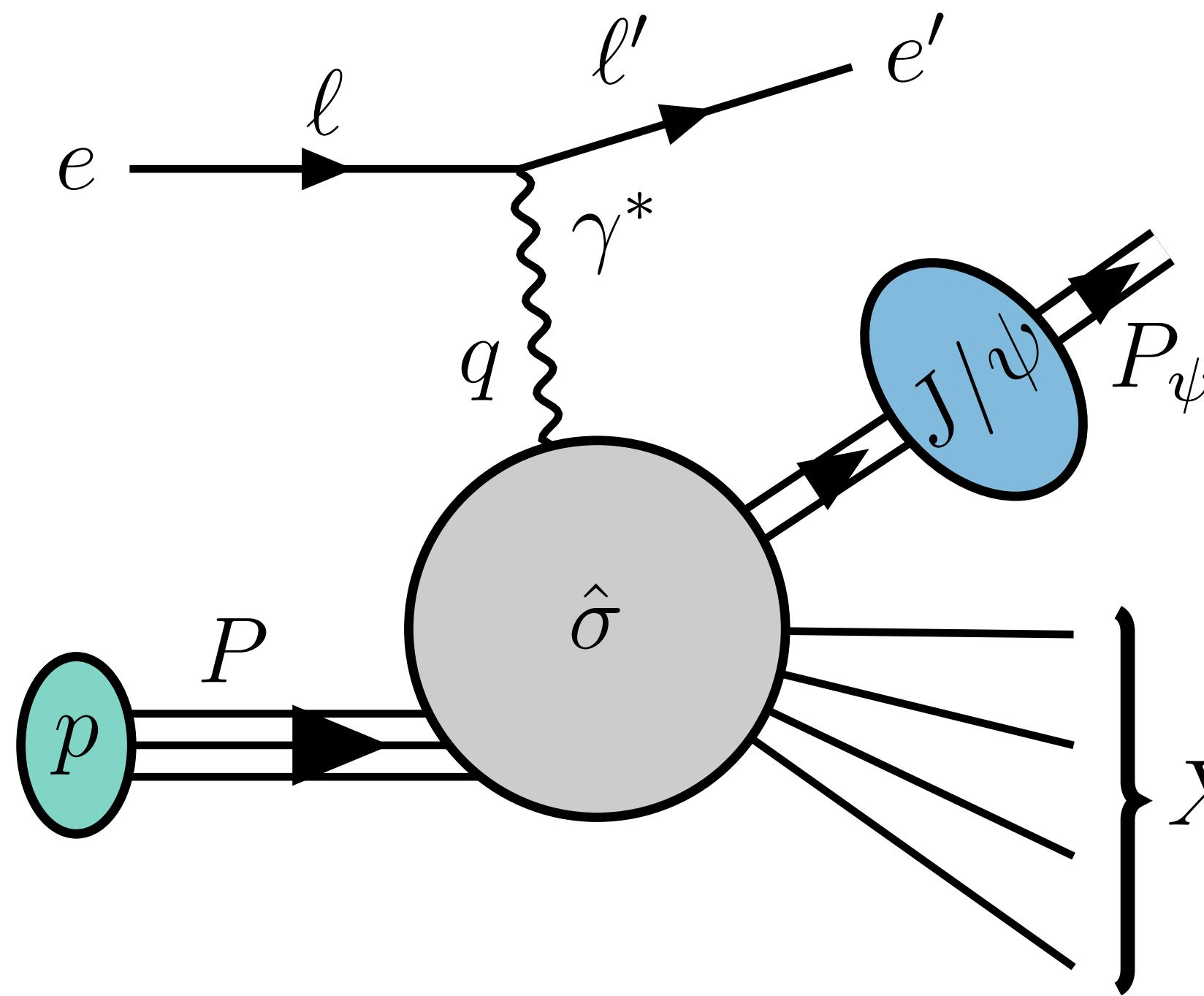
As for $D_{a \rightarrow h}(\hat{z}) \rightarrow D_{a \rightarrow h}(\hat{z}, k_T)$, we have $\langle \mathcal{O}_\psi[n] \rangle \delta(\hat{z} - z) \rightarrow \Delta^{[n]}(\hat{z}, k_T)$

[Echevarría, JHEP 144 \(2019\)](#)

[Fleming, Markis, Mehen, JHEP 112 \(2020\)](#)



The TMD shape function



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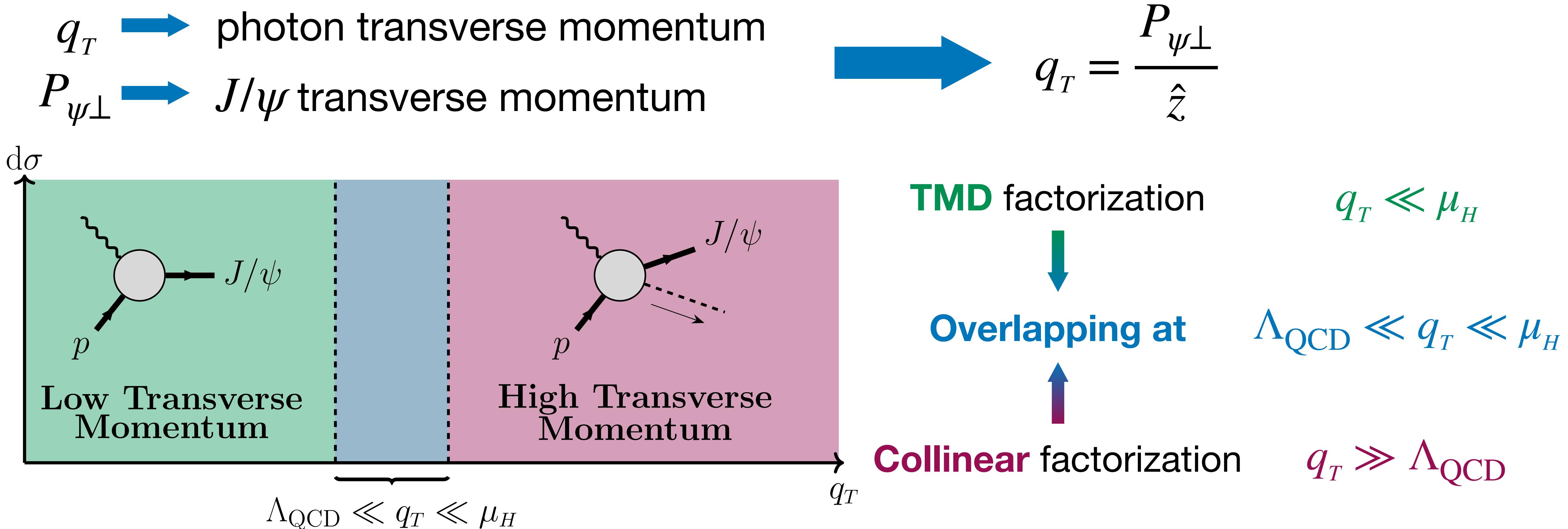
$\Delta^{[n]}$ encodes hadronization
plus
exchange of soft gluons

[Echevarría, JHEP 144 \(2019\)](#)

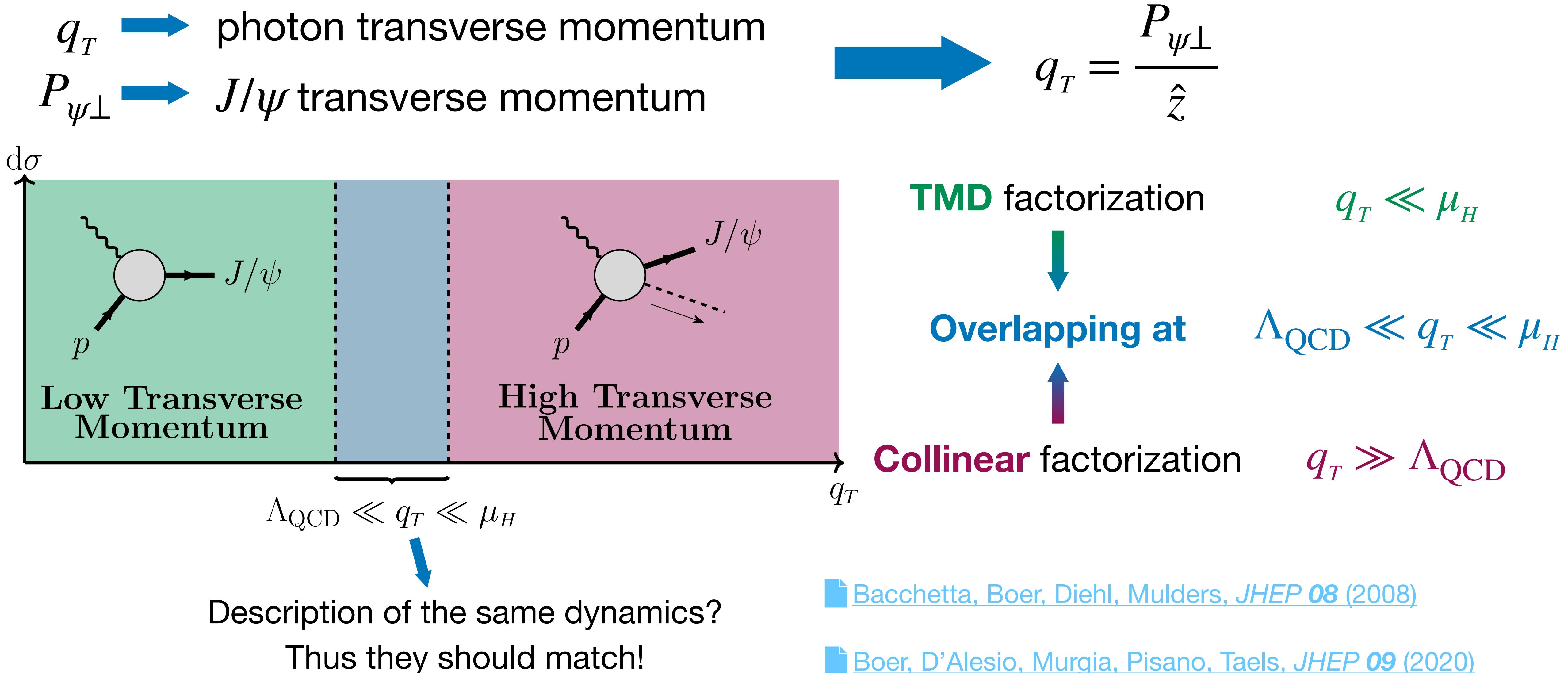
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Matching procedure



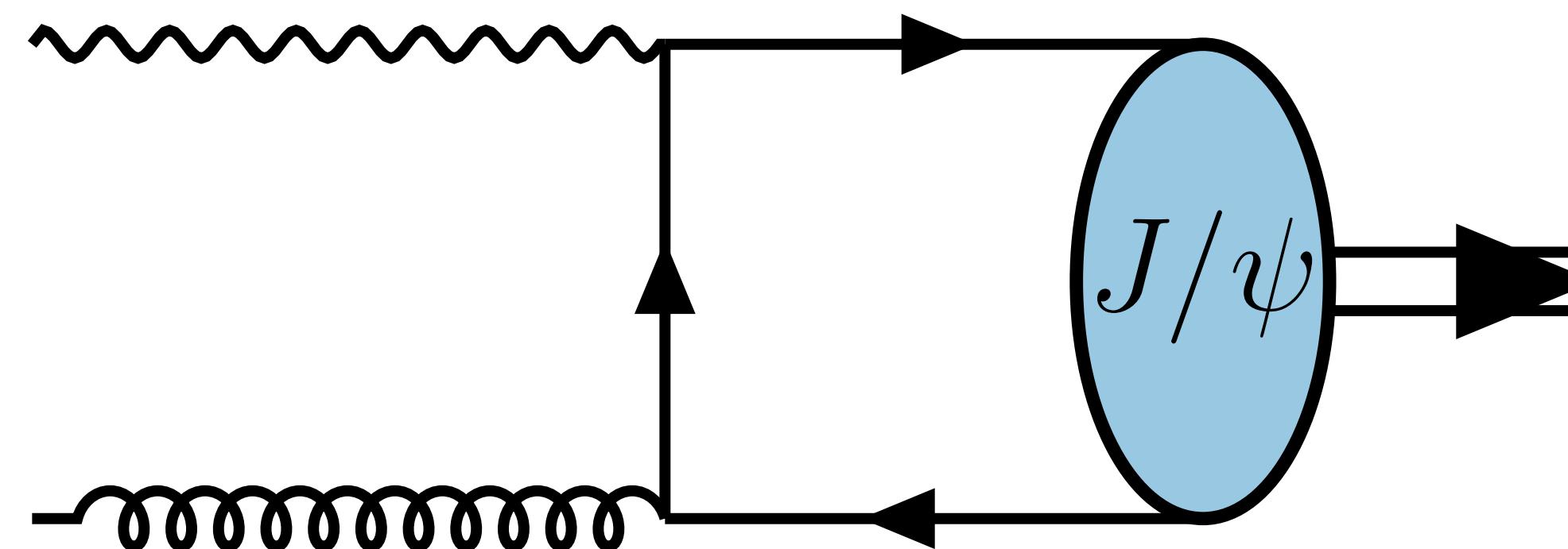
Matching procedure



Structure function at small- q_T (TMD region)

J/ψ production at the lowest α_s -order: $\gamma^* + g \rightarrow c\bar{c}[n]$

 [Bacchetta, Boer, Pisano, Taels, EPJC 80 \(2020\)](#)



Kinematics fixes most of the variables:

- $\hat{x} = x$ (where $x = x_B \frac{M_\psi^2 + Q^2}{Q^2}$)
- $\hat{z} = 1$
- $p_{at} = q_T$

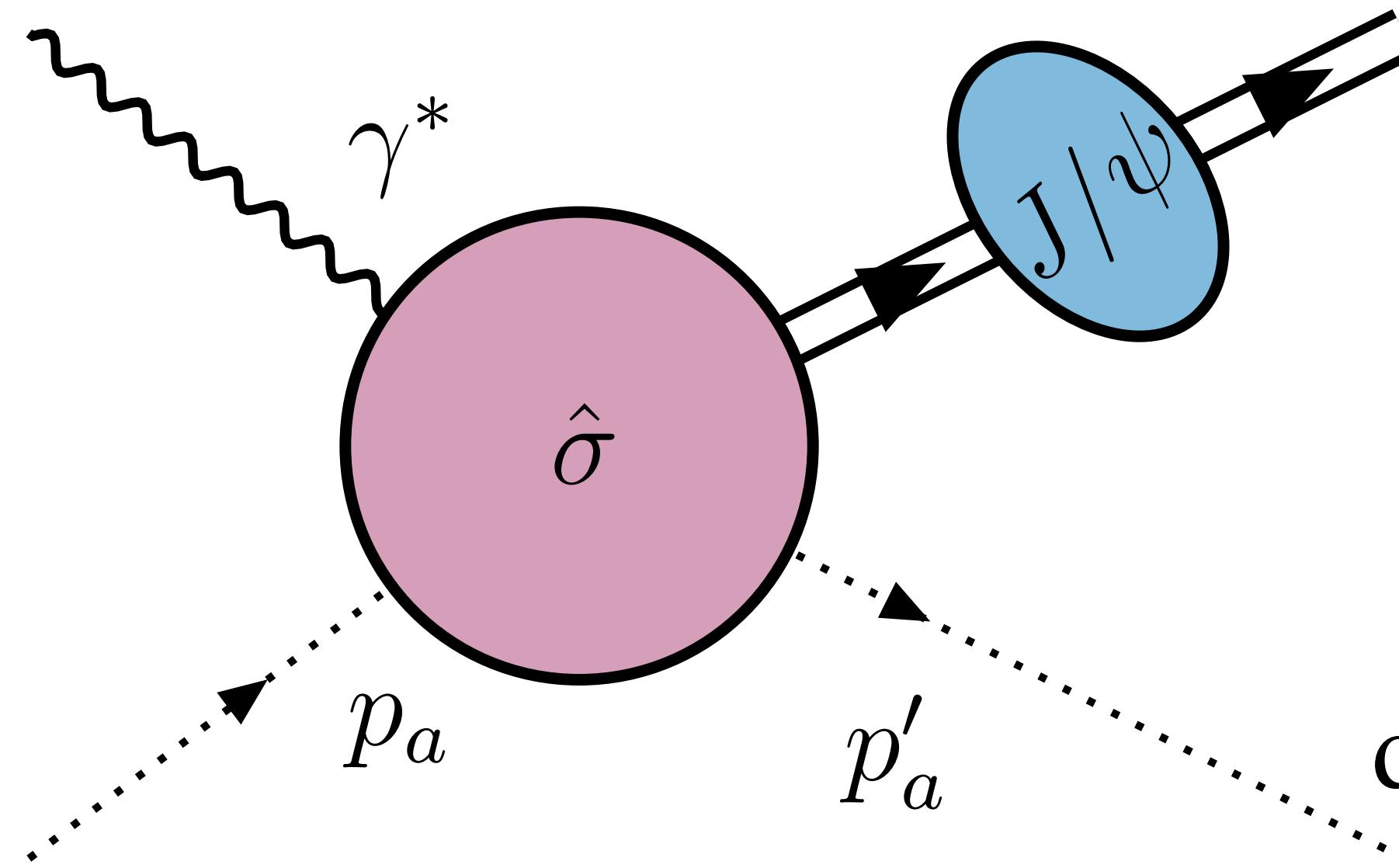
$$d\sigma|_{\text{TMD}} = \frac{\alpha}{yQ^2} \left\{ [1 + (1 - y)^2] \mathcal{F}_{UUT} + 4(1 - y) (\mathcal{F}_{UUL} + \cos 2\phi \mathcal{F}_{UU}^{\cos 2\phi}) \right\}$$

Involves the convolutions:
$$\left\{ \begin{array}{l} \mathcal{C}[f_1^g \Delta^{[n]}](x, q_T) \\ \mathcal{C}[w h_1^{\perp g} \Delta_h^{[n]}](x, q_T) \end{array} \right.$$



Structure function at high- q_T (collinear region)

J/ψ production at the lowest α_s -order: $\gamma^* + a \rightarrow c\bar{c}[n] + a$ $(a = q, \bar{q}, g)$



$$d\sigma^{ep \rightarrow e' J/\psi X} = d\hat{\sigma}^a[n](\mu_H) \otimes f_p^a(\hat{x}; \mu_H) \otimes \langle \mathcal{O}_\psi[n] \rangle \delta(\hat{z} - z)$$

Lepton tensor from

[Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP 02 \(2007\)](#)

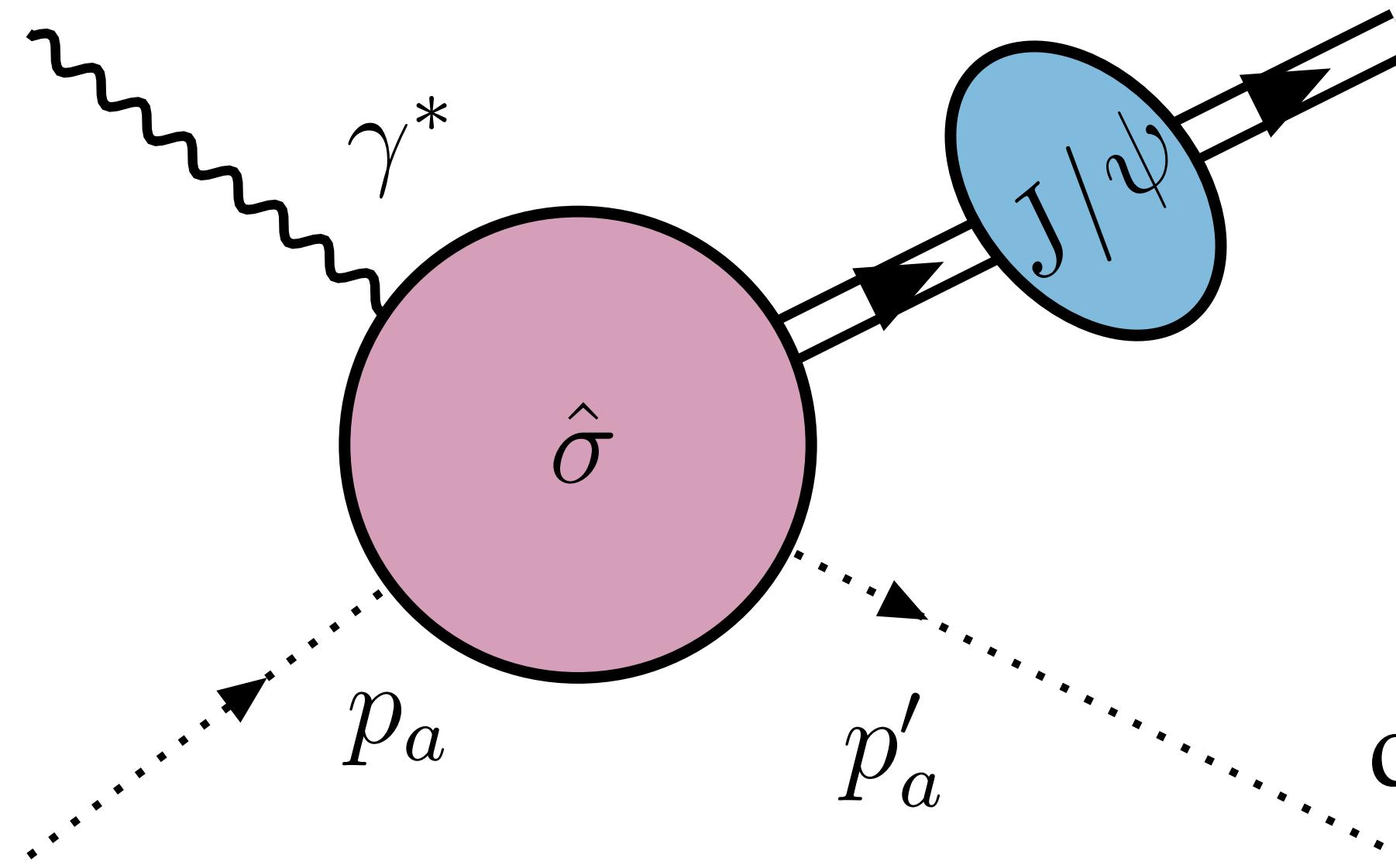
$$d\hat{\sigma}^a[n] \propto \int \frac{d\hat{x}}{\hat{x}} \frac{d\hat{z}}{\hat{z}} \frac{L^{\mu\nu}}{Q^4} H_\mu^{a[n]} H_\nu^{*a[n]} \delta(\hat{x}', \hat{z})$$

$$\hat{x}' = \frac{x_B}{\hat{x}} \frac{M_\psi^2 + Q^2}{Q^2}$$



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$$\hat{x}' = \frac{x_B}{\hat{x}} \frac{M_\psi^2 + Q^2}{Q^2}$$

$$\delta(\hat{x}', \hat{z}) = \delta\left(\frac{(1 - \hat{x}')(1 - \hat{z})}{\hat{x}'\hat{z}} + \frac{1 - \hat{z}}{\hat{z}} \frac{\hat{z} - \hat{x}' M_\psi^2}{\hat{x}'\hat{z}} \frac{M_\psi^2}{Q^2} + \frac{q_T^2}{Q^2}\right) \quad \longrightarrow$$

for $M_\psi \ll Q$ in agreement with

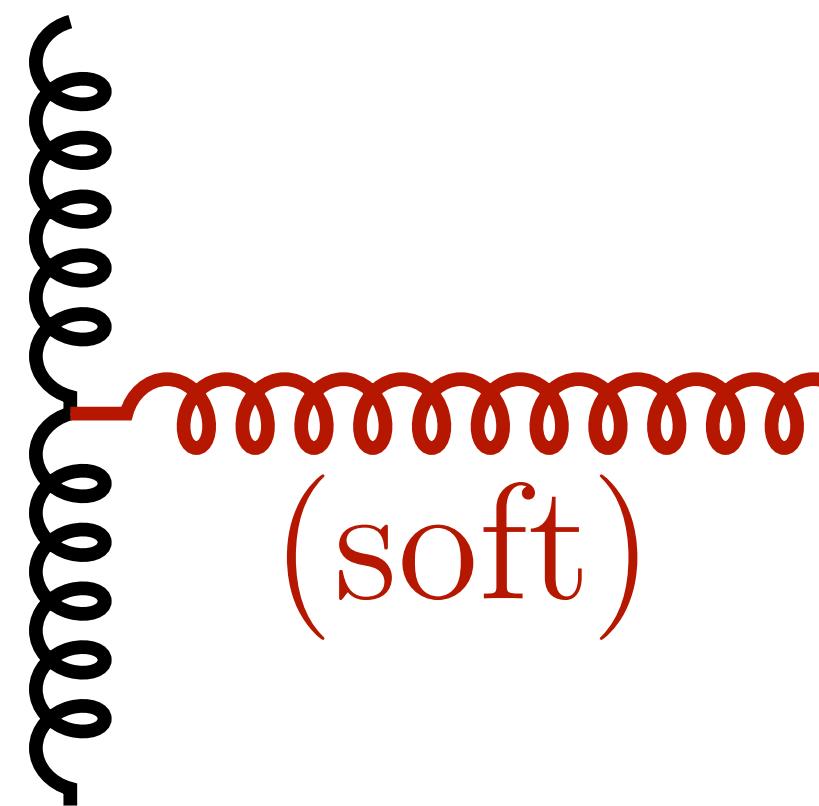
[Meng, Olness, Soper JHEP 11 \(2019\)](#)



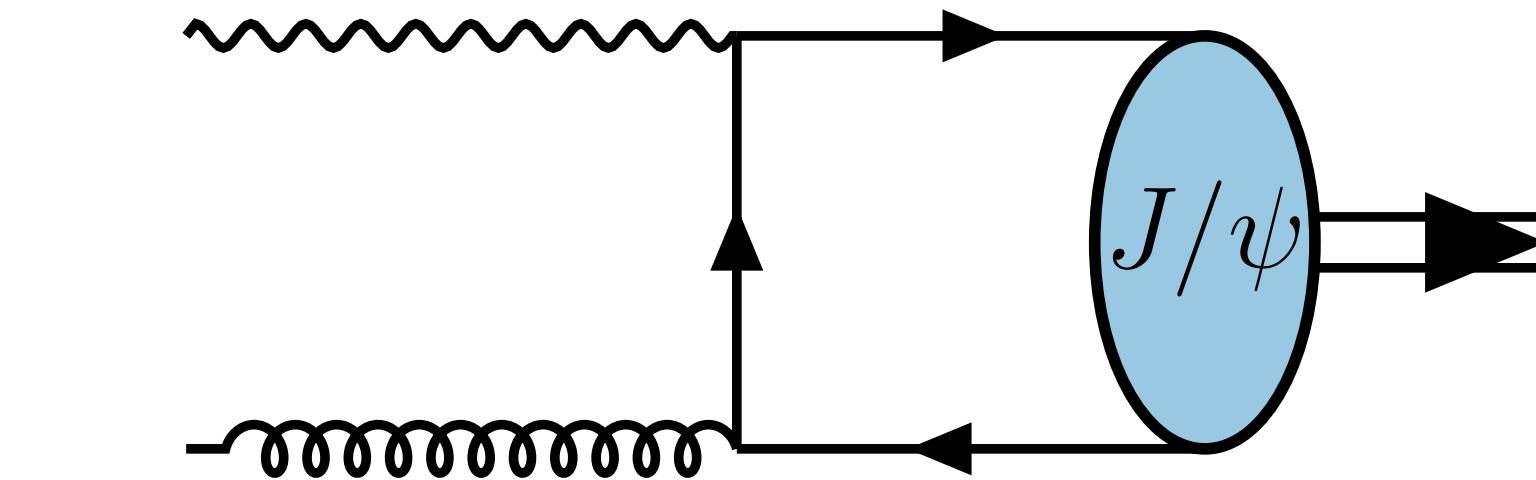
Schematic small- q_T limit valid at $\Lambda_{\text{QCD}} \ll q_T \ll \mu_H$

$$\Lambda_{\text{QCD}} \ll q_T \ll \mu_H$$

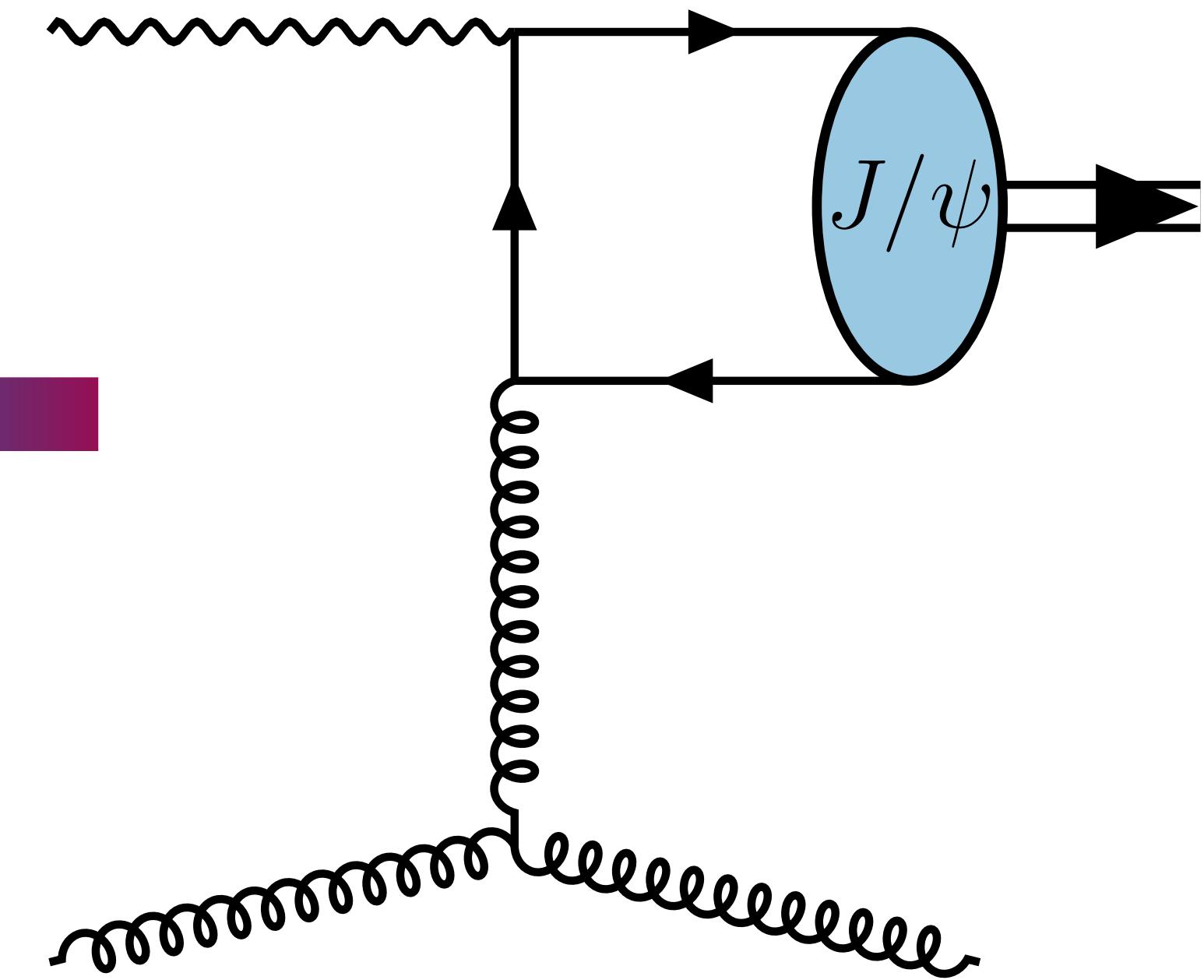
$$q_T \gg \Lambda_{\text{QCD}}$$



q_T divergent behavior
(resummed)



same partonic interaction
appearing in the TMD region



Limit is obtained by expanding $\delta(\hat{x}', \hat{z})$ at small- q_T

Structure functions' pole structure

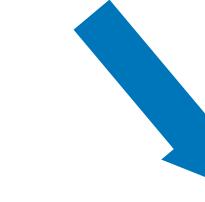
 Boer, D'Alesio, Murgia, Pisano, Taels, *JHEP 09 (2020)*

$$\delta(\hat{x}', \hat{z}) \sim \frac{\hat{x}'}{(1 - \hat{x})_+} \delta(1 - \hat{z}) + \log \frac{M_\psi^2 + Q^2}{q_T^2} \delta(1 - \hat{x}') \delta(1 - \hat{z})$$

Limit based on
a continuous test function

 Boer, Bor, LM, Pisano, Yuan, *2304.09473 (2023)*

$$F_{UU}(\hat{x}', \hat{z}) = F_{UU}^{(0)}(\hat{x}', \hat{z}) + \sum_{k=1} \left(\frac{1 - \hat{z}}{1 - \hat{x}'} \right)^k F_{UU}^{(k)}(\hat{x}', \hat{z})$$


(general notation)  Continuous functions of \hat{x}' and \hat{z}



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Has an impact on the double delta

Continuous functions of \hat{x}' and \hat{z}



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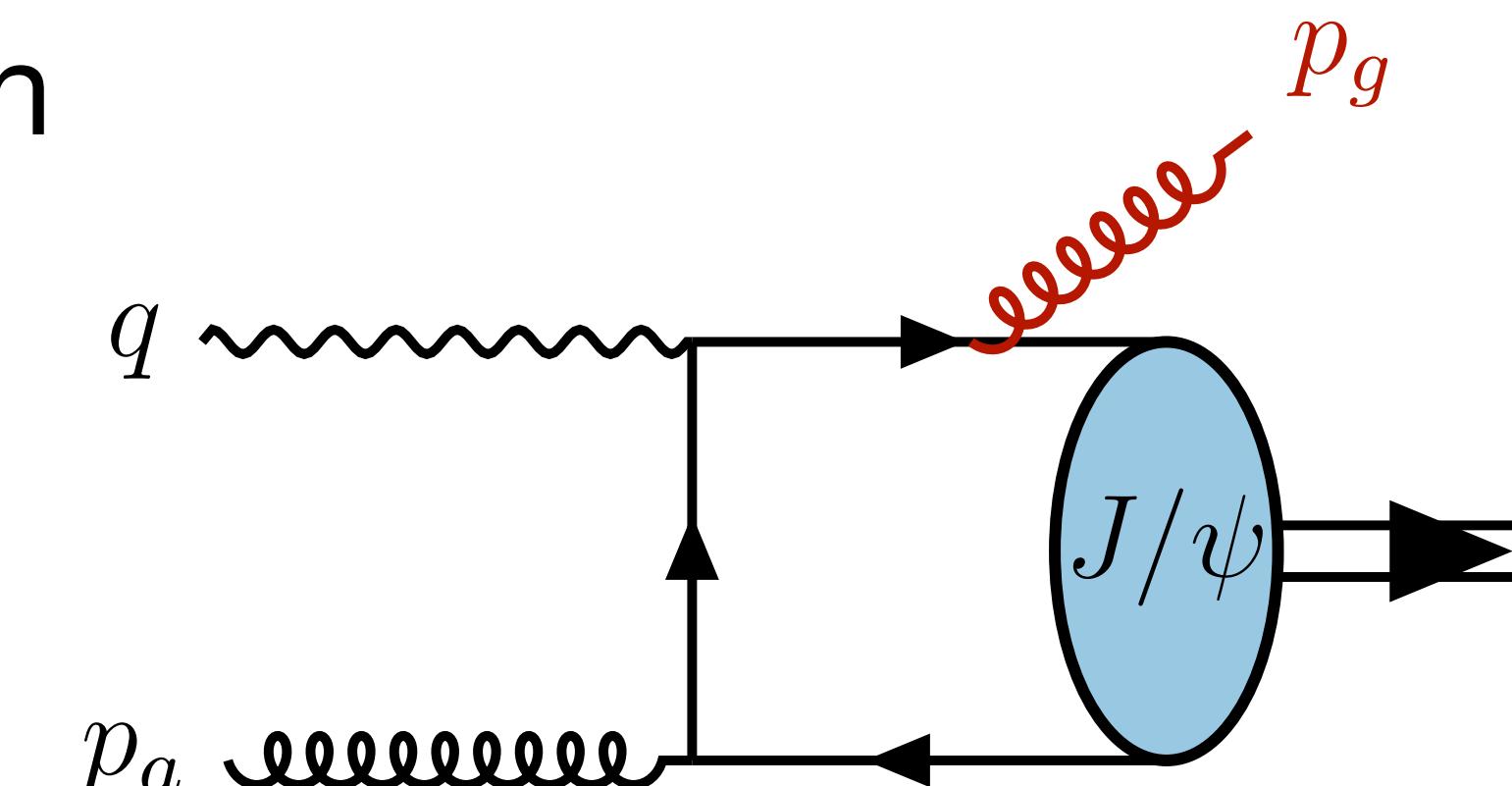
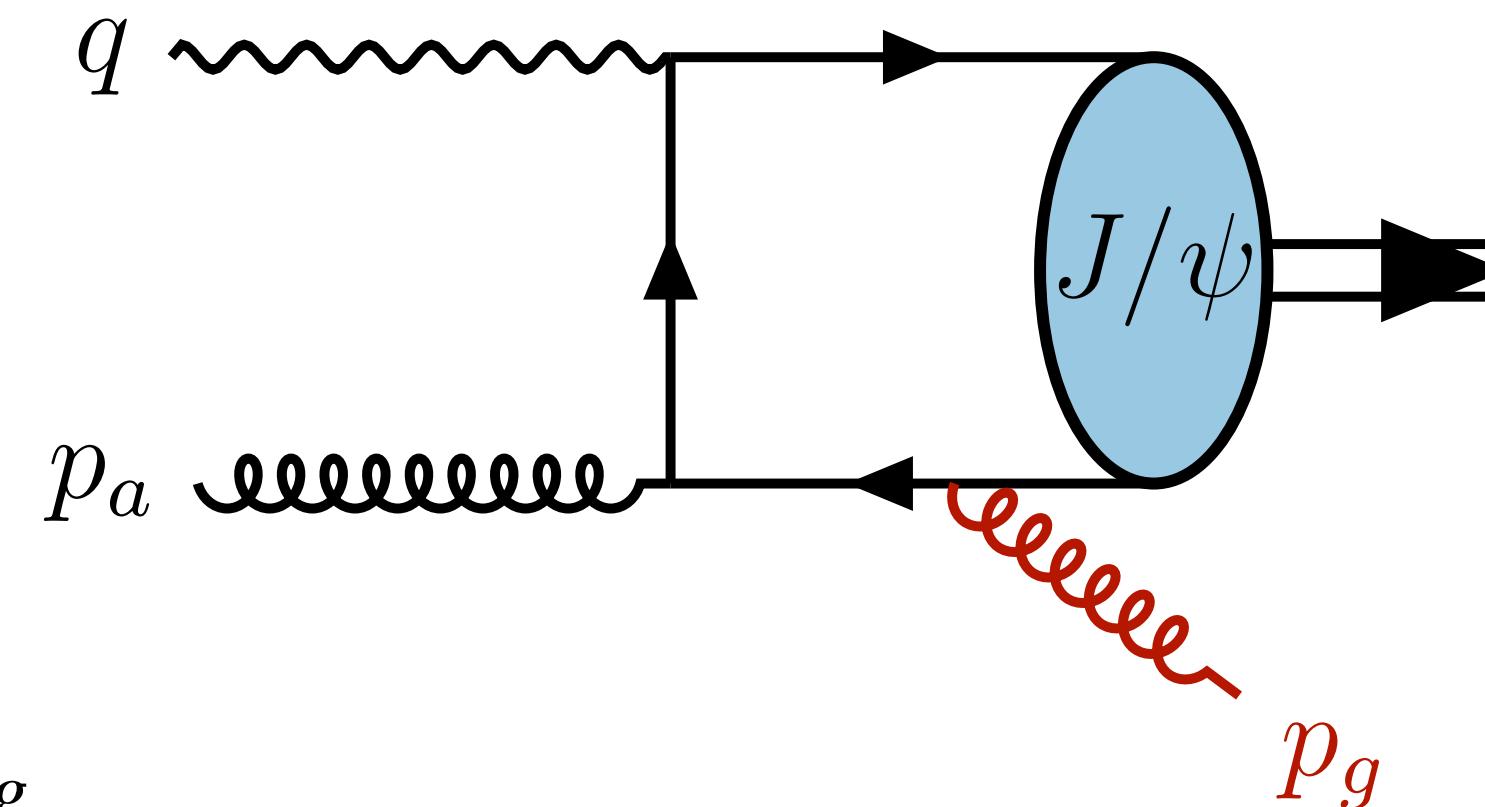
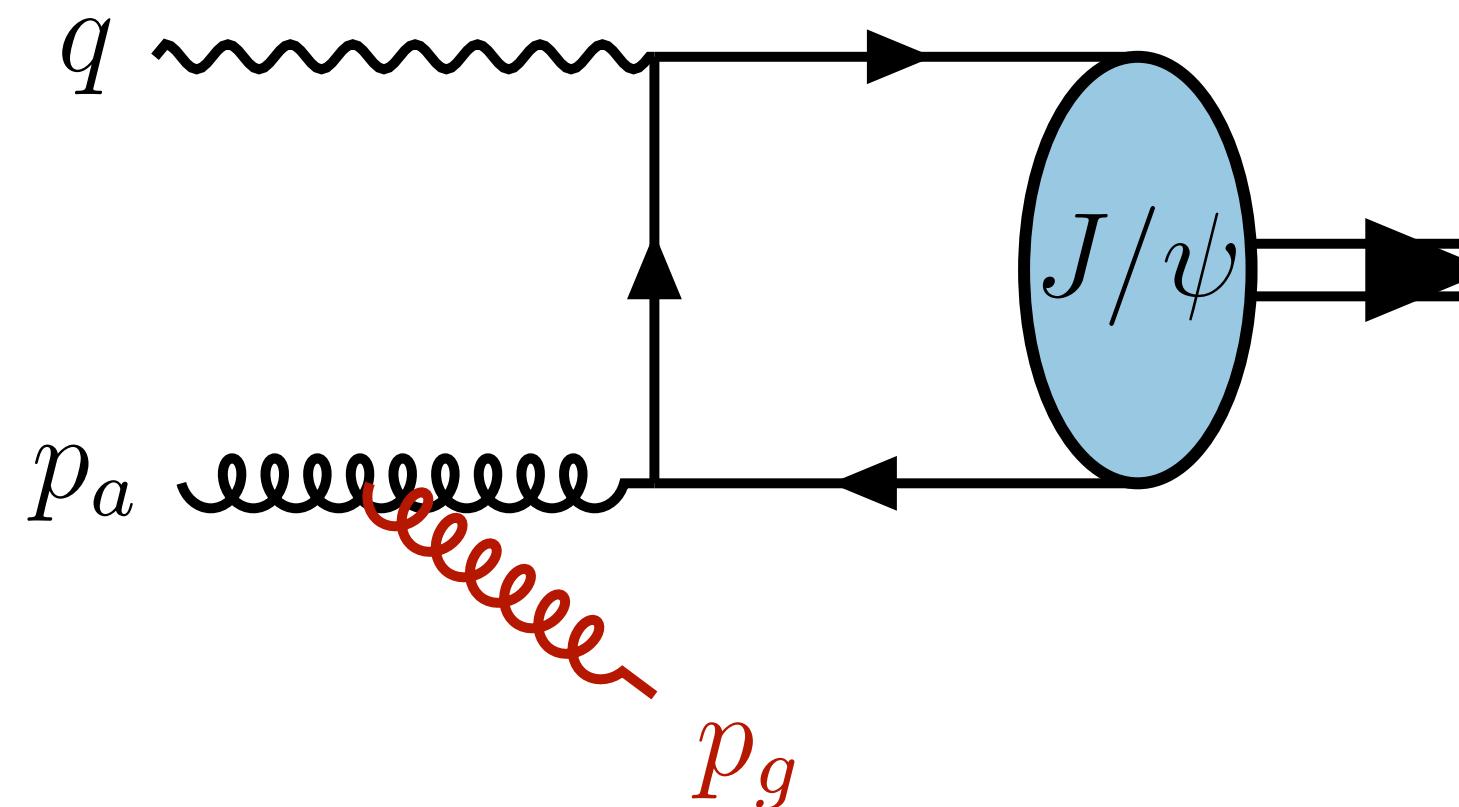
Relevant for $\gamma^* g$
in $F_{UUT}^{(k)}$ and $F_{UUL}^{(k)}$
with $k = 1, 2$

$$\log \frac{M_\psi^2 + Q^2}{q_T^2} \rightarrow \frac{1}{2} \left(\log \frac{M_\psi^2 + Q^2}{q_T^2} - 1 - \log \frac{M_\psi^2}{M_\psi^2 + Q^2} \right)$$



Eikonal method

Same term is found by considering the soft gluon emission



$$d\sigma_1 \propto \int_{\frac{-p_{g\perp}^2}{M_\psi^2 + Q^2}}^1 \frac{dx_g}{x_g} \left[2S_g(p_a, P_\psi) + S_g(P_\psi, P_\psi) \right]$$

$$x_g = \frac{p_a \cdot p_g}{p_a \cdot q}$$

from momentum conservation
and $2p_g^+ p_g^- = -p_{g\perp}^2 = -P_{\psi\perp}^2$

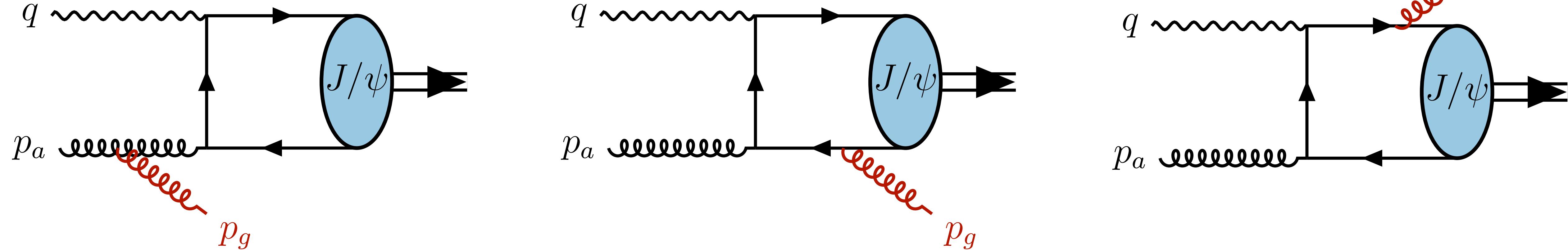
$$\propto \frac{1}{2} \left(\log \frac{M_\psi^2 + Q^2}{q_T^2} - 1 - \log \frac{M_\psi^2}{M_\psi^2 + Q^2} \right)$$

$$S_g(v_1, v_2) = \frac{v_1 \cdot v_2}{(v_1 \cdot p_g)(v_2 \cdot p_g)}$$



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Relation to quark-pair Fragmentation Function?

[Kang, Ma, Qiu, Sterman, PRD 90 \(2014\) & PRD 91 \(2015\)](#)

[Ma, Qiu, Sterman, Zhang, PRL 113 \(2014\)](#)



TMD shape function perturbative tail

Comparison at $\Lambda_{\text{QCD}} \ll q_T \ll \mu_H$ obtained evolving TMDs according to

■ [Echevarria, Kasemets, Mulders, Pisano, JHEP 07 \(2015\)](#)

■ [Sun, Xiao, Yuan, PRD 84 \(2011\)](#)

$$\mathcal{F}_{UU}^{\cos 2\phi}|_{\text{TMD}} = F_{UU}^{\cos 2\phi}|_{\text{coll}} \quad \rightarrow \quad \Delta_{h,\psi}^{[n]} = \delta^{(2)}(k_T^2) \langle \mathcal{O}_\psi[n] \rangle \delta(1-z)$$

■ [Boer, Bor, LM, Pisano, Yuan, 2304.09473 \(2023\)](#)

$$\left. \begin{array}{l} \mathcal{F}_{UUT}|_{\text{TMD}} \neq F_{UUT}|_{\text{coll}} \\ \mathcal{F}_{UUL}|_{\text{TMD}} \neq F_{UUL}|_{\text{coll}} \end{array} \right\} \quad \rightarrow \quad \Delta_\psi^{[n]} = -\frac{\alpha_s}{2\pi^2 k_T^2} C_A \left(1 + \log \frac{M_\psi^2}{M_\psi^2 + Q^2} \right) \langle \mathcal{O}_\psi[n] \rangle \delta(1-z)$$

Up to the precision considered, bulk of the expression given by **CO waves**

$1S_0^{(8)}$ $3P_J^{(8)}$



TMD shape function in other processes?

Previous results are obtained for $\mu_H \equiv \sqrt{M_\psi^2 + Q^2}$

■ [Boer, Bor, LM, Pisano, Yuan, 2304.09473 \(2023\)](#)

In general we get $\Delta_{ep}^{[n]}(\mu_H) = \frac{1}{2\pi} \left[1 + \frac{\alpha_s}{2\pi} C_A \left(1 + \log \frac{M_\psi^2 \mu_H^2}{(M_\psi^2 + Q^2)^2} \right) \log \frac{\mu_H^2}{\mu_b^2} \right] \langle \mathcal{O}[n] \rangle \delta(1 - z)$
(in b_T -space)



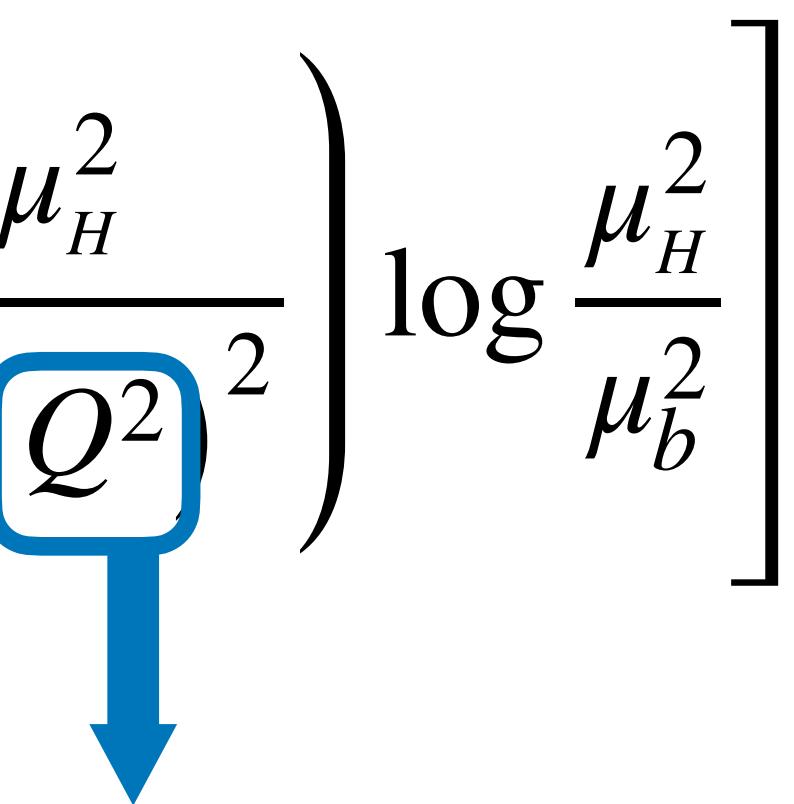
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(in b_T -space)



It is process related!

TMD shape function in other processes?

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In general we get (in b_T -space) $\Delta_{ep}^{[n]}(\mu_H) = \frac{1}{2\pi} \left[1 + \frac{\alpha_s}{2\pi} C_A \left(1 + \log \frac{M_\psi^2 \mu_H^2}{(M_\psi^2 + Q^2)^2} \right) \log \frac{\mu_H^2}{\mu_b^2} \right] \langle \mathcal{O}[n] \rangle \delta(1 - z)$

split up: $\Delta_{ep}^{[n]}(\mu_H) = \Delta_\psi^{[n]}(\mu_H) \times S_{ep}(\mu_H)$

$$\Delta_\psi^{[n]}(\mu_H) = \frac{1}{2\pi} \left[1 + \frac{\alpha_s}{2\pi} C_A \left(1 + \log \frac{M_\psi^2}{\mu_H^2} \right) \log \frac{\mu_H^2}{\mu_b^2} \right] \langle \mathcal{O}[n] \rangle \delta(1 - z)$$

→ **Universal**

e.g. $S_{pp}(\mu_H)$

$$1 + \frac{\alpha_s}{2\pi} C_A \left(3 \log \frac{\mu_H^2}{M_\psi^2} \right) \log \frac{\mu_H^2}{\mu_b^2}$$

$$S_{ep}(\mu_H) = 1 + \frac{\alpha_s}{2\pi} C_A \left(2 \log \frac{\mu_H^2}{M_\psi^2 + Q^2} \right) \log \frac{\mu_H^2}{\mu_b^2}$$

→ **Process dependent**



Summary and outlook

- Factorization involves the presence of TMD shape functions
- We present a matching procedure to extract the TMDShF perturbative tail
- TMD shape functions present universal and process-dependent components

- Perturbative tail at higher order → Relevant for $\Delta_h^{[n]}$
- Non-perturbative dependence
- Extraction of the TMDShF universal component
- The advent of the EIC may shed a light on the role of the TMDShF and its properties

