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In collaboration with M. Boglione

TMD Fragmentations from thrust dependent processes

Old Dominion University

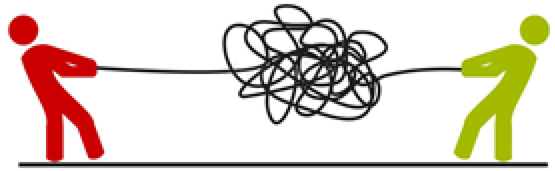
Jefferson Lab

Extraction of TMD Fragmentation Functions

Access to the 3D-dynamics of confinement

Standard TMD factorization →

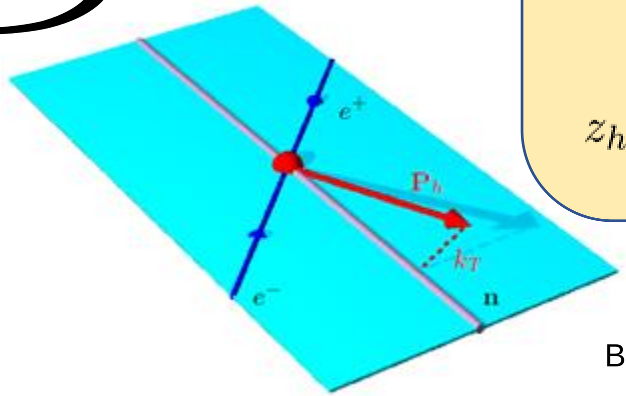
- SIDIS $d\sigma \sim H_{\text{SIDIS}}\, F\, D$
- DIA $d\sigma \sim H_{\text{DIA}}\, D_1\, D_2$



Always two TMDs that have to be extracted *simultaneously*

A process with a single hadron may offer a cleaner access to TMD FFs

$$d\sigma \overset{??}{\propto} D$$



Single-Inclusive Hadroproduction (SIA)

$$e^+e^- \rightarrow h\,X$$

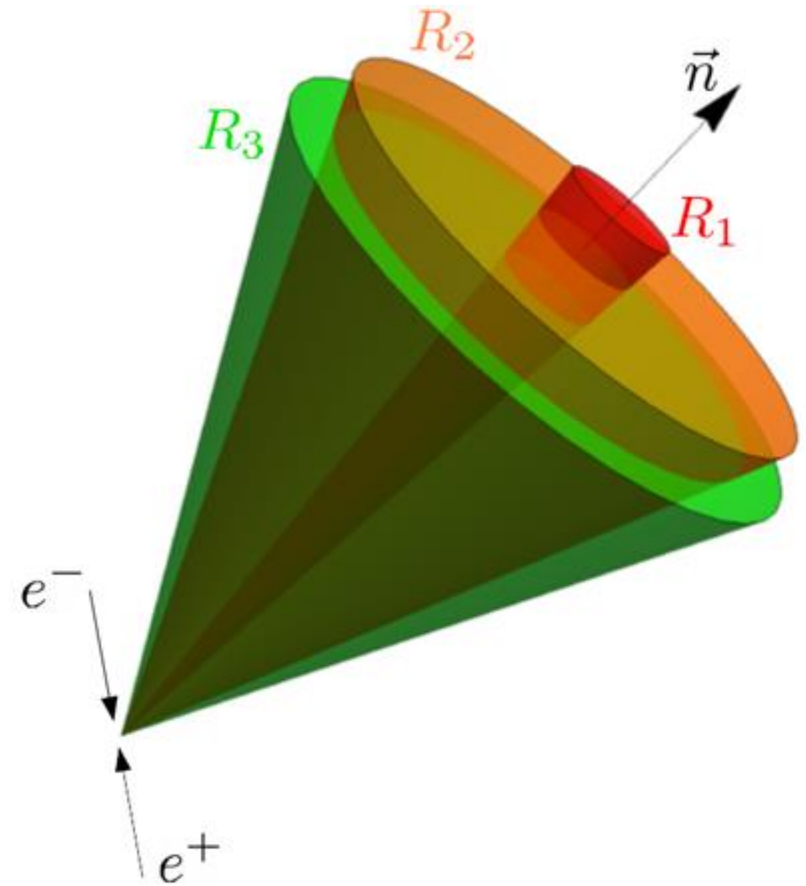
The transverse momentum of the detected hadron is measured w.r.t. the thrust axis

$$z_h = \frac{E}{Q/2}, \quad T = \frac{\sum_i |\vec{P}_{(\text{c.m.}),\,i} \cdot \hat{n}|}{\sum_i |\vec{P}_{(\text{c.m.}),\,i}|}, \quad P_T \text{ w.r.t } \vec{n}$$

BELLE collab., *Phys.Rev.D 99 (2019)*

Three kinematic regions

Depending on where the hadron is located within the jet the underlying kinematics can be remarkably different, resulting in different factorization theorems



The hadron is detected very close to the **axis** of the jet.

- ❑ Extremely small P_T
- ❑ Soft radiation affects significantly the transverse deflection of the hadron from the thrust axis

The hadron is detected in the **central region** of the jet.

- ❑ Most common scenario
- ❑ Majority of experimental data expected to fall into this case

The hadron is detected near the **boundary** of the jet.

- ❑ Moderately small P_T
- ❑ The hadron transverse momentum affects the topology of the final state directly

The three regions are uniquely determined by the specific role of **soft** and **soft-collinear** radiation:

$$u \to 1 - T$$
$$b_T \to P_T/z$$

	soft	soft-collinear	collinear
R_1	TMD-relevant	TMD-relevant	TMD-relevant
R_2	TMD-irrelevant	TMD-relevant	TMD-relevant
R_3	TMD-irrelevant	TMD-irrelevant	TMD-relevant

TMD FF + NP soft

$$d\sigma \sim H J(u) \Sigma(u, b_T)\, D(z, b_T)$$

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- Soft radiation affects significantly the transverse deflection of the hadron from the thrust axis

TMD FF

$$d\sigma \sim H J(u) S(u)\, D(z, b_T)$$

The hadron is detected in the **central region** of the jet.

- Most common scenario
- Majority of experimental data expected to fall into this case

Generalized FJF

$$d\sigma \sim H J(u) S(u)\, G(z, u, b_T)$$

The hadron is detected near the **boundary** of the jet.

- Moderately small P_T
- The hadron transverse momentum affects the topology of the final state directly

Non-global logs

$$d\sigma \sim H J(u) \Sigma(u, b_T)\, D(z, b_T)$$

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TMD FF + NP soft

CSS definition (rearrangement)

Non-perturbative effects that modify the **topology** of the final state

Non-perturbative effects that generate **transverse momentum**

$$d\sigma \sim H J(u) S(u)\, D(z, b_T)$$

The hadron is detected in the **central region** of the jet.

- Most common scenario
- Majority of experimental data expected to fall into this case

TMD FF

Non-global logs

$$d\sigma \sim H J(u) \Sigma'(u, b_T)\, D^{\mathrm{CSS}}(z, b_T)$$

The hadron is detected very close to the **axis** of the jet.

- ❑ Extremely small P_T
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TMD FF + NP soft

Non-perturbative effects that modify the **topology** of the final state

Non-perturbative effects that generate **transverse momentum**

NOT THE SAME TMD FRAGMENTATION FUNCTION!

Is **universality** in danger?

$$d\sigma \sim H J(u) S(u)\, D(z, b_T)$$

The hadron is detected in the **central region** of the jet.

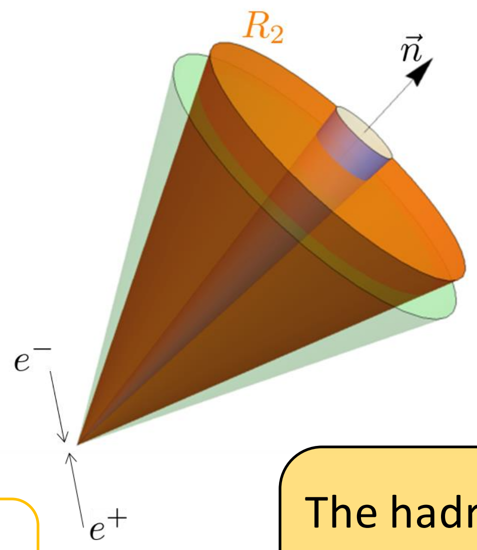
- ❑ Most common scenario
- ❑ Majority of experimental data expected to fall into this case

TMD FF

Standard TMD factorization can be extended beyond the standard processes (DY, SIDIS, DIA) at the cost of including a new, independent, non-perturbative function (the **soft model**).

$$D^{\text{CSS}}(z, b_T) = D(z, b_T)\sqrt{M_S(b_T)}$$

Universality is saved!



$$d\sigma \sim H J(u) S(u)\, D(z, b_T)$$

TMD FF

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Rapidity divergences in the central region

$$\frac{\partial}{\partial y_1} \ldots \mathcal{S}(\tau, y_1, \ldots)\, D(z, b_T, y_1) \neq 0$$

SIA^{thr} has a **double nature**:
- Thrust dependent observable
- TMD observable

The thrust τ *naturally* regularizes the rapidity divergences.
The 2-jet limit $\tau \to 0$ corresponds to removing the regulator and to exposing the rapidity divergences in fixed order calculations.

So the final result depends on a regulator?
Yes, but...

1) The thrust is *measured*.
2) When the regulator is removed the (factorized) cross section vanishes, as showed by resummation.

The rapidity cut-offs $y_{1,2}$ *artificially* regularize the rapidity divergences.
The limits $y_{1,2} \to \pm\infty$ correspond to removing the regulator and to exposing the rapidity divergences in fixed order calculations.

So the final result depends on a regulator?
Yes (in principle), but…

1) The rapidity cut-offs are just mathematical tools.
2) In standard TMD factorization they cancel among themselves before the limit $y_{1,2} \to \pm\infty$ is taken and the final cross section is rapidity cut-offs independent.

Both kind of regularization coexists in SIA$^{\text{thr}}$.
Therefore, it should not be surprising that the two mechanisms intertwine and that thrust and rapidity regulators are strictly related.

This signals a *redundancy* of regulators: one can be expressed in terms of the other.
In particular, the rapidity cut-off y_1 should be a function of thrust, such that when it is removed, also τ is removed. In other words:

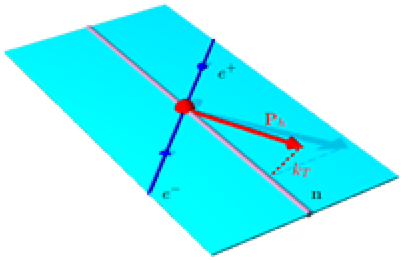
$$\tau \to 0 \Longleftrightarrow y_1 \to +\infty$$

Peculiar and very unique feature of the central region!

Naively from kinematics…

$$y_h \geq -\log\sqrt{\tau} \qquad y_1 \propto -\log\sqrt{\tau}$$

Rapidity of detected hadron



…but also formally:
The double counting due to the overlap between soft and collinear (forward) radiation is cancelled only if the rapidity cut-off is fixed to a function of thrust and transverse momentum

SOFT-COLLINEAR

$$k_T \lesssim c_1/b_T \longleftarrow \text{COLLINEAR}$$

$$\text{SOFT} \longrightarrow k_T \lesssim Q\, e^{y_1}/u_E$$

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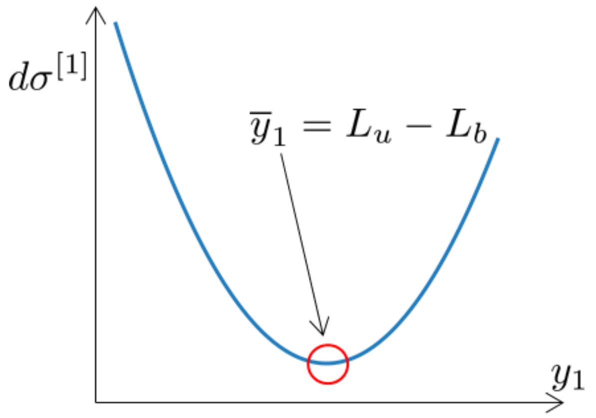
The double counting due to the overlap between soft and collinear (forward) radiation is cancelled only if the rapidity cut-off is fixed to a function of thrust and transverse momentum

SOFT-COLLINEAR

$$k_T \lesssim c_1/b_T$$ ← COLLINEAR

SOFT → $$k_T \lesssim Q\, e^{y_1}/u_E$$

$$u_E = u\, e^{\gamma_E};\; c_1 = 2e^{-\gamma_E}$$
$$L_u = \log u_E$$
$$L_b = \log\left(b_T Q/c_1\right)$$



$$y_1 = L_u - L_b$$

This is also the **minimum** of the factorized cross section as a function of y_1

It is the (unique) solution of the Collins-Soper evolution equation!

$$\frac{\partial}{\partial y_1} d\sigma_{R_2} = 0$$

$$\overline{y}_1 = L_u - L_b^\star \left(1 + \frac{1 - e^{-\frac{2\beta_0}{\gamma_K^{[1]}}\left(g_K - \widetilde{K}^\star\right)}}{\lambda_b^\star} \right)$$

- Large and positive
- Consistent with pert. solution: $\overline{y}_1 = L_u - L_b$ as $b_T \to 0$
- Consistent with kinematics: $\widehat{y}_1 = -\log\sqrt{\tau} + b_T$-logs

...but also formally:
The double counting due to the overlap between soft and collinear (forward) radiation is cancelled only if the rapidity cut-off is fixed to a function of thrust and transverse momentum

SOFT-COLLINEAR

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SOFT → $$k_T \lesssim Q\, e^{y_1}/u_E$$

$$u_E = u\, e^{\gamma_E};\ c_1 = 2e^{-\gamma_E}$$
$$L_u = \log u_E$$
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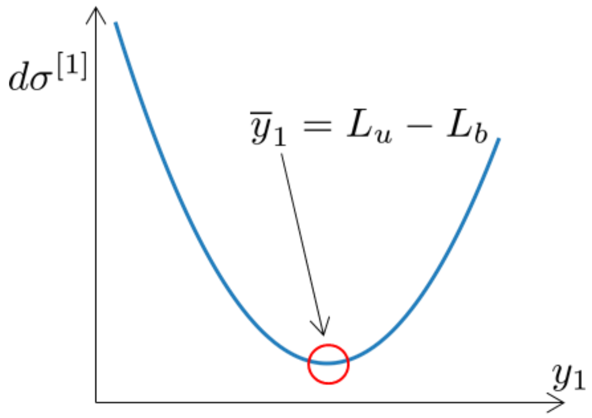

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Factorization errors are affected by the choice of g_K.
HINTS FOR PHENOMENOLOGY:

- Monotonic increasing (unique minimum)
- Constant at large distances
  $g_K(b_T) \to g_0$ (const.)

Factorization theorem in the central region

$$d\sigma_{R_2} \sim H\ J(u)\,\frac{\mathcal{S}(u,\overline{y}_1,y_2)}{\mathcal{Y}_L(u,y_2)}\,\widetilde{D}_{h/j}(z,b_T,\overline{y}_1)$$

$$= H\,J\,\frac{\mathcal{S}}{\mathcal{Y}_L}\bigg|_{\text{ref. scale}} \exp\left\{\int_{\mu_J}^{Q}\frac{d\mu'}{\mu'}\gamma_J + \frac{1}{2}\int_{\mu_S}^{Q}\frac{d\mu'}{\mu'}\gamma_S\right\} \times \widetilde{D}_{h/j}(z,b_T)\Big|_{y_1=0}$$

$$\times \exp\left\{\frac{1}{2}\int_{\mu_S}^{\mu_S e^{\overline{y}_1}}\frac{d\mu'}{\mu'}\left[\widehat{g}-\gamma_K\,\log\left(\frac{\mu'}{\mu_S}\right)\right] - \overline{y}_1\ \widetilde{K}\Big|_{\mu_S}\right\}$$

Genuinely thrust. Exponent is *half* of standard thrust distributionin e+e- annihilation

Genuinely TMD. Reference scales as* in standard TMD factorization

Correlation part. It encodes the correlations between the measured variables

The function g_K does not only appear into the TMD FF!

$$\frac{d\sigma_{R_2}}{dz\,dT\,d^2\vec{P}_T} = -\frac{\sigma_B\,N_C}{1-T}\sum_j e_j^2\left(1+a_S\,H^{[1]}\right)$$

$$\times\int\frac{d^2\vec{b}_T}{(2\pi)^2}e^{i\vec{b}_T\cdot\vec{P}_T/z}\,e^{L_{b^\star}\,n_1+n_2}\,\widetilde{D}^{\text{NLL}}_{h/j}(z,b_T)\Big|_{\substack{\mu=Q\\ y_1=0}}\left(1+a_S\,C_1\right)\frac{e^{Lf_1+f_2+\frac{1}{L}f_3}}{\Gamma\left(1-g_1\right)}\left(g_1+\frac{1}{L}g_2\right)$$

Phenomenology $e^+e^- \to \pi X$

BELLE collaboration
Phys.Rev.D 99 (2019) 11, 112006

T	z												P_T/z max	N
	0.20 – 0.25	0.25 – 0.30	0.30 – 0.35	0.35 – 0.40	0.40 – 0.45	0.45 – 0.50	0.50 – 0.55	0.55 – 0.60	0.60 – 0.65	0.65 – 0.70	0.70 – 0.75	0.75 – 0.80		
0.80 – 0.85													$0.16\,Q$	57
0.85 – 0.90													$0.15\,Q$	60
0.90 – 0.95													$0.14\,Q$	61
0.95 – 1.00													$0.13\,Q$	52

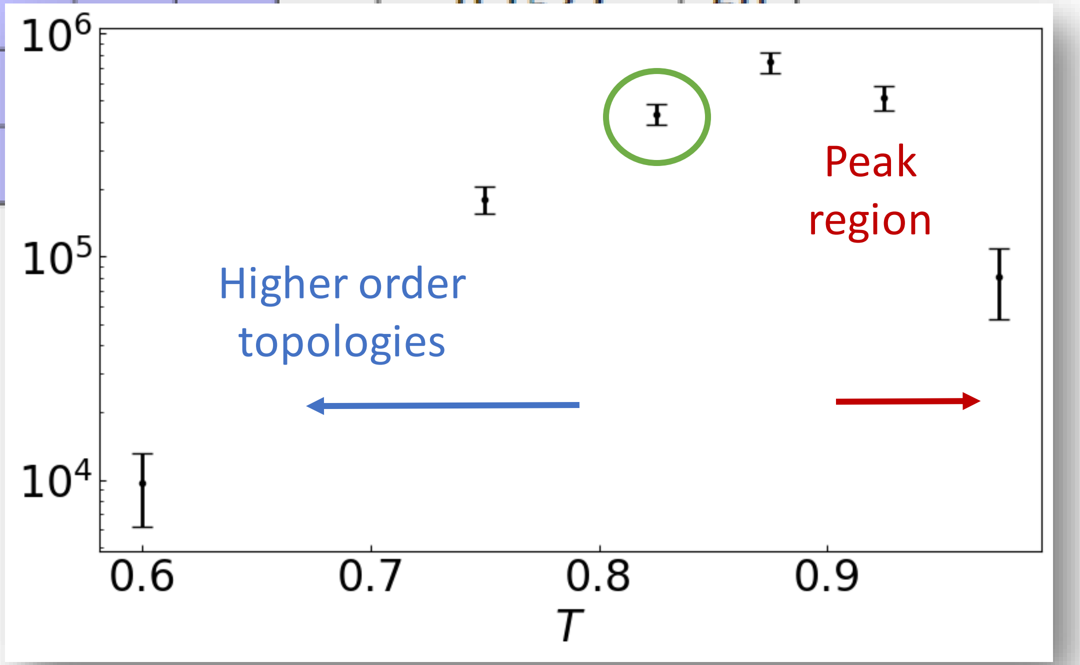
Avoiding Region 1

Avoiding Region 3

230 Data in total

Phenomenology $e^+e^- \to \pi X$

T	z												P_T/z max	N
	$0.20-0.25$	$0.25-0.30$	$0.30-0.35$	$0.35-0.40$	$0.40-0.45$	$0.45-0.50$	$0.50-0.55$	$0.55-0.60$	$0.60-0.65$	$0.65-0.70$	$0.70-0.75$	$0.75-0.80$		
$0.80-0.85$													$0.16\,Q$	57
$0.85-0.90$														
$0.90-0.95$														
$0.95-1.00$														



Subsample of 57 data points

"Pure" TMD extraction

A preliminary fit of the subsample data helps to fix the functional form of the non-perturbative content of the TMD FF

Non-perturbative content of the TMD FF

1. g_K function, describing the long-distance behavior of the Collins-Soper kernel.

- Even function of b_T
- Quadratic behavior at small
- Constant behavior at large

$$g_K \sim g_2 b_T^2 + \ldots \text{ for } b_T \to 0.$$
$$g_K \to g_0 \text{ for } b_T \to \infty.$$

$$g_K^A(b_T) = g_0 \tanh\left(\beta^2 \frac{b_T^2}{b_{MAX}^2}\right),$$
$$g_K^B(b_T) = g_0 \tanh\left(\beta^2 b_T^\star b_T\right).$$

2. M_D model for the (unpolarized) TMD FF, describing its characteristic long-distance behavior.

- Gaussian behavior at small
- Exponential decay at large

$$M_D \sim e^{-cb_T^2} \times \ldots \text{ for } b_T \to 0.$$
$$M_D \sim e^{-db_T} \times \ldots \text{ for } b_T \to \infty.$$

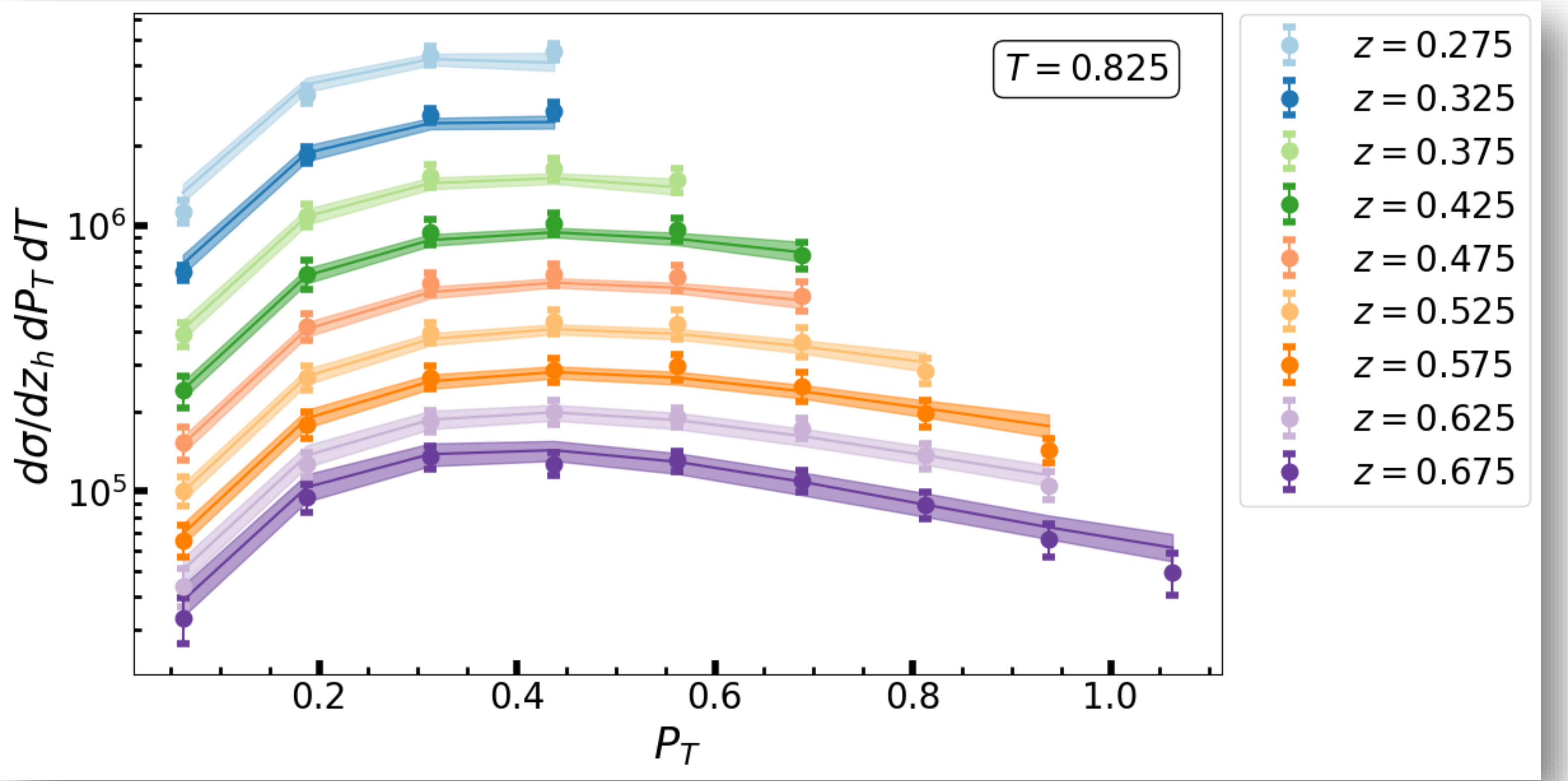
$$\boxed{M_D\left(z_h, P_T, M, p\right) = \frac{\Gamma(p)}{\pi\Gamma(p-1)} M^{2\,(p-1)} \left(M^2 + \frac{P_T^2}{z_h^2}\right)^{-p}}$$

Where $p = p(R, W)$ and $M = M(R, W)$

$$R(z) = 1 - \alpha \frac{f(z)}{f(z_0)} \text{ with } f(z) = z\,(1-z)^{\frac{1-z_0}{z_0}},$$
$$W(z) = \frac{m_\pi}{R(z)}.$$

Preliminary step: TMD FF functional form

χ^2/d.o.f.	0.6183
z_0	$0.5521^{+0.0415}_{-0.0398}$
α	$0.3644^{+0.0250}_{-0.0282}$
g_0	$0.2943^{+0.0329}_{-0.0261}$
β	$4.7100^{+1.9856}_{-1.9856}$



The aim is to test the validity of the functional forms chosen for g_K and M_D

The value of the free parameters should not be taken at face value, especially those associated to g_K

Final fit: inclusion of NP thrust effects

Several recipes available.
We choose the simplest: minimal approach

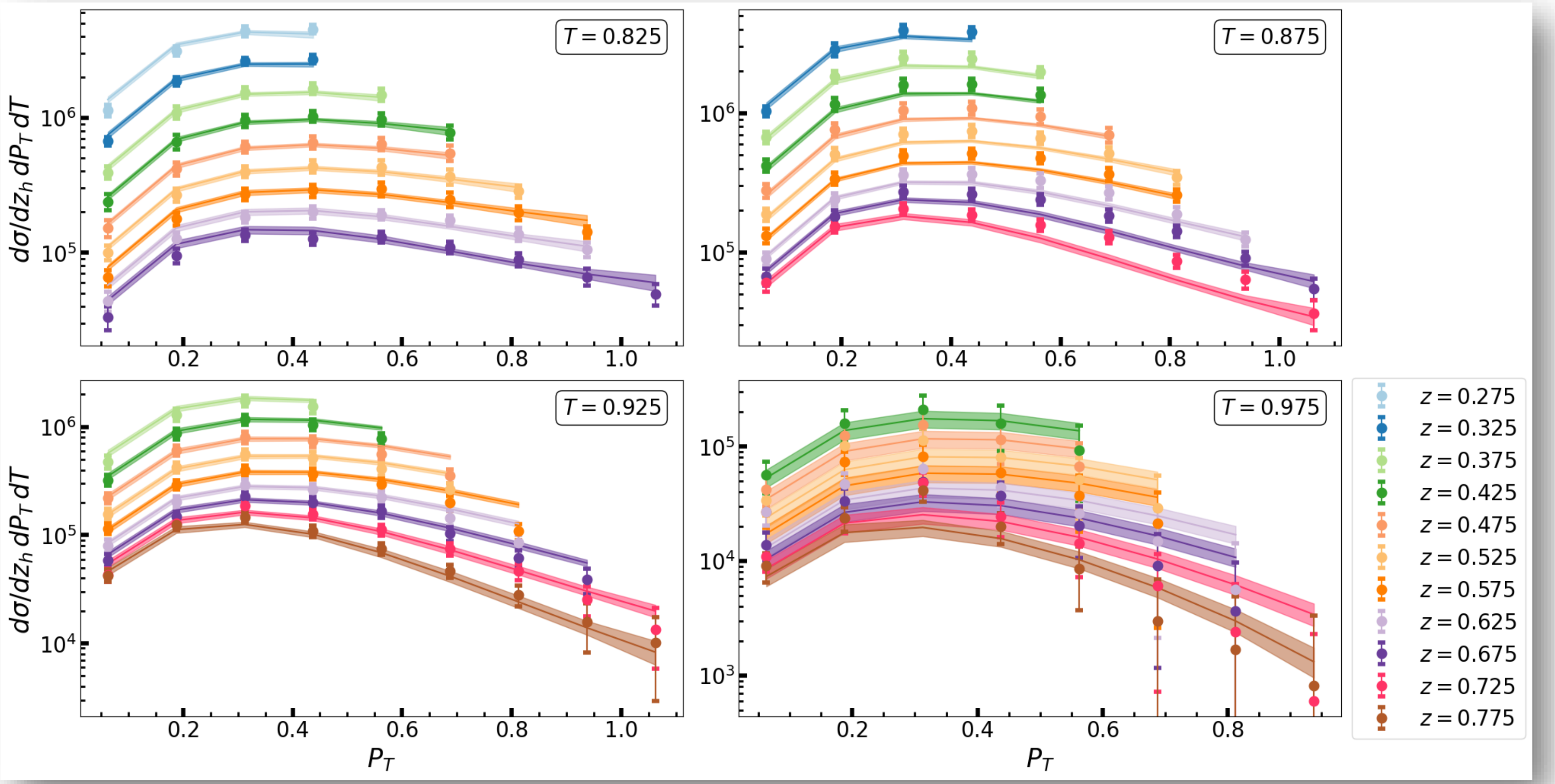
$$\frac{d\sigma}{dz\,dT\,d^2\vec{P}_T} = \left.\frac{d\sigma^{\text{pert.}}}{dz\,dT\,d^2\vec{P}_T}\right|_{T-T_0} f_{\text{NP}}(T)$$

With:
$$f_{NP} = \tanh\left(\rho\,(1-T)\right)^2$$

Model (B)

$$g_K^B(b_T) = g_0\,\tanh\left(\beta^2\, b_T^{\star}\, b_T\right)$$

χ^2/d.o.f.	1.3421
z_0	$0.5334^{+0.0192}_{-0.0189}$
α	$0.3394^{+0.0127}_{-0.0134}$
g_0	$0.1205^{+0.0305}_{-0.0367}$
β	$2.0610^{+2.1042}_{-0.5193}$
T_0	$0.0467^{+0.0117}_{-0.0077}$
ρ	$8.1643^{+0.3053}_{-0.3011}$



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We choose the simplest: minimal approach

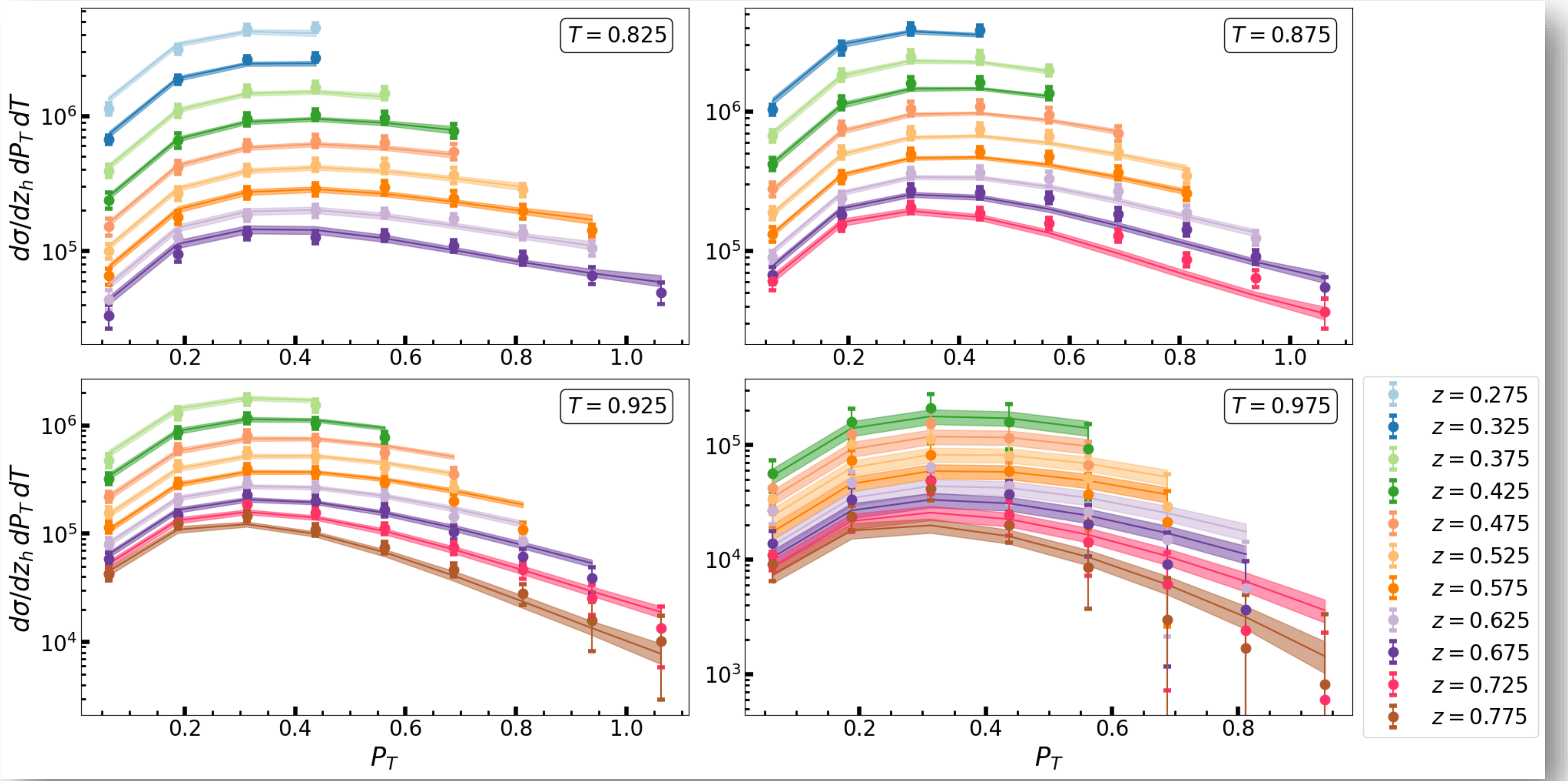
$$\frac{d\sigma}{dz\, dT\, d^2\vec{P}_T} = \frac{d\sigma^{\text{pert.}}}{dz\, dT\, d^2\vec{P}_T}\Big|_{T-T_0} f_{\rm NP}(T)$$

With:
$$f_{NP} = \tanh\left(\rho\,(1-T)\right)^2$$

Model (A)

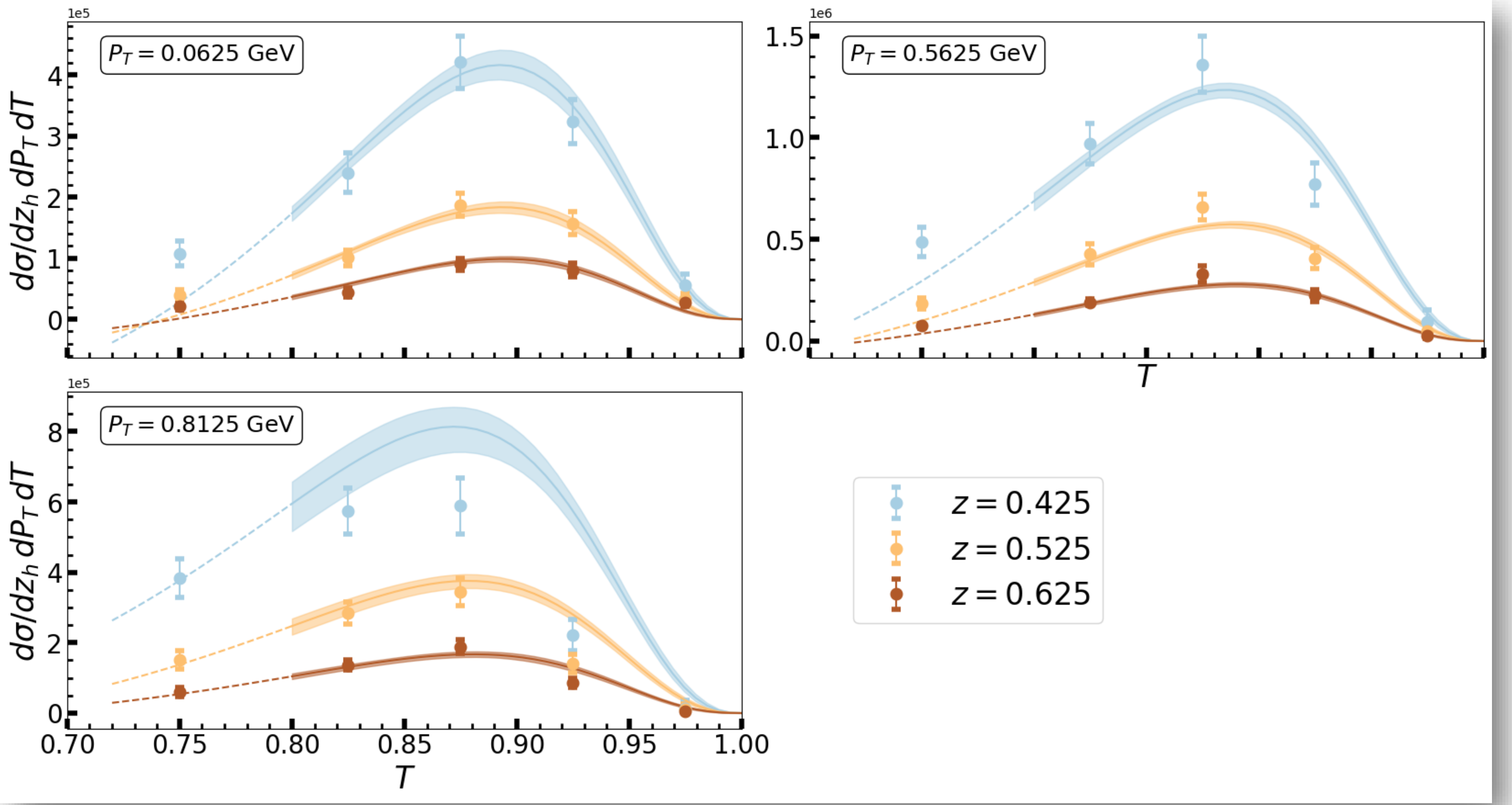
$$g_K^A(b_T) = g_0 \tanh\left(\beta^2 \frac{b_T^2}{b_{MAX}^2}\right)$$

χ^2/d.o.f.	1.0749
z_0	$0.5335^{+0.0194}_{-0.0180}$
α	$0.3403^{+0.0114}_{-0.0122}$
g_0	$0.1044^{+0.0446}_{-0.0742}$
β	$1.6765^{+0.8150}_{-0.8150}$
T_0	$0.0617^{+0.0295}_{-0.0134}$
ρ	$7.7205^{+0.2834}_{-0.2099}$

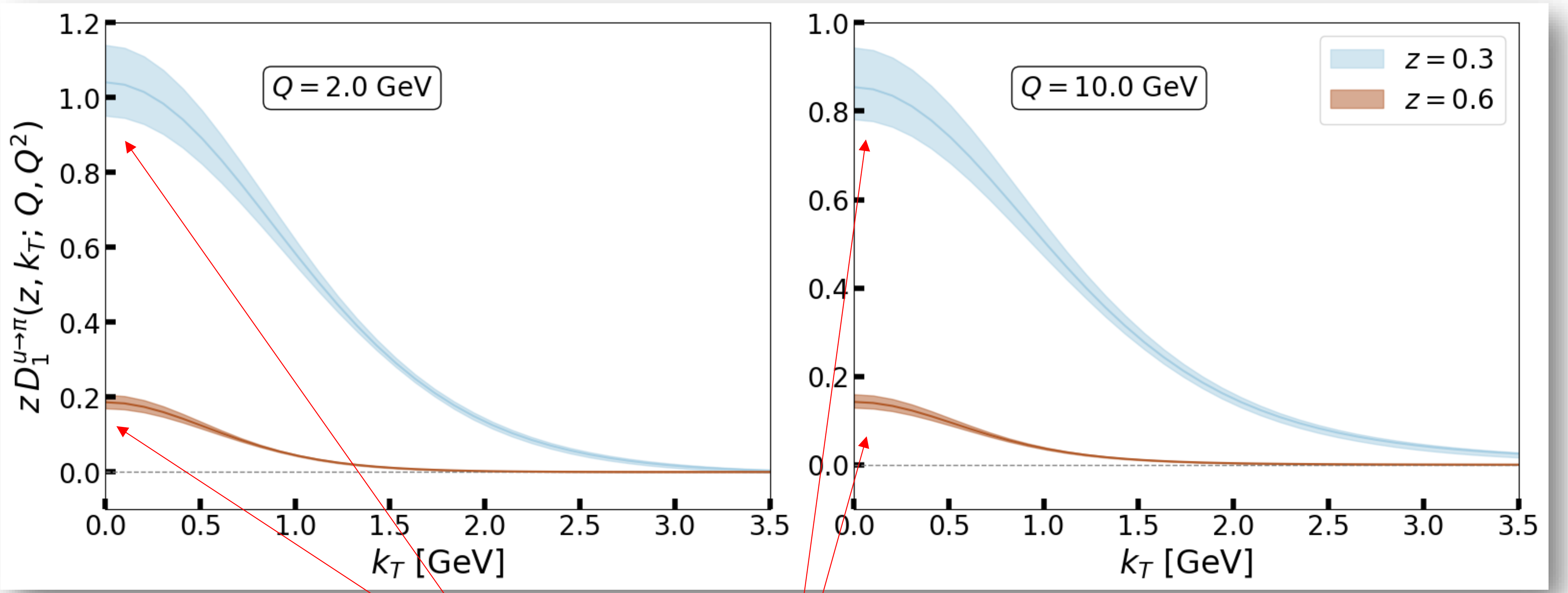


Description of thrust dependence

First time that the thrust dependence is consistently described!



Unpolarized TMD Fragmentation Function



Expected different behavior at low k_T compared to TMD FFs extracted from standard processes (e.g. SIDIS)

Conclusions

- The factorization theorem(s) for SIA with thrust is now **complete** and **consistent**

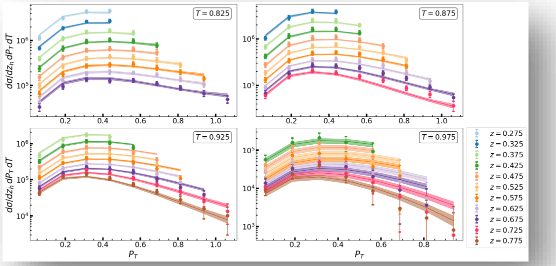
TMDs universality

$$D^{\rm CSS}(z,b_T) = D(z,b_T)\,\sqrt{M_S(b_T)}$$

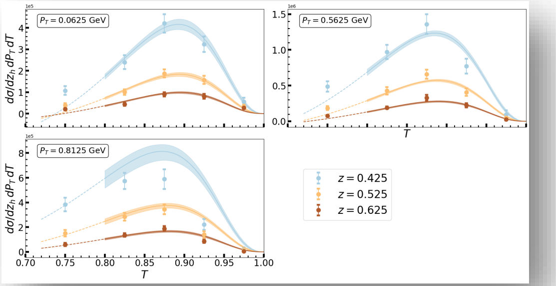
Rapidity divergences (R_2)

$$\overline{y}_1 = L_u - L_{b^\star}\left(1 + \frac{1 - e^{\frac{2\beta_0}{\gamma_K^{[1]}}\left(\widetilde{K}_\star(a_S(\mu_b^\star)) - g_K(b_T)\right)}}{2\beta_0\, a_S(\mu_b^\star) L_{b^\star}}\right)$$

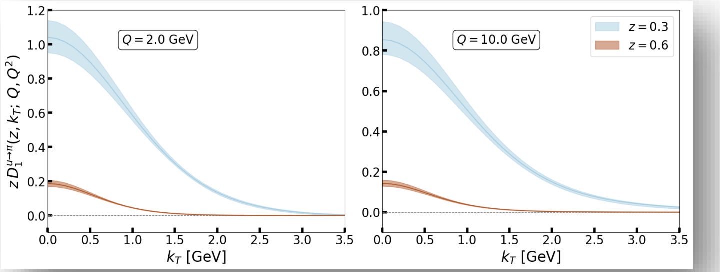
- The BELLE data have finally been described phenomenologically...



...with respect to ALL the measured variables (z, P_T, T)



- Extraction of the unpolarized TMD FF (for charged pions)



Future

- ❑ Refine the inclusion of the non-perturbative thrust effects $$\frac{d\sigma}{dz\,dT\,d^2\vec{P}_T} = \frac{d\sigma^{\text{pert.}}}{dz\,dT\,d^2\vec{P}_T}\bigg|_{T-T_0} f_{\text{NP}}(T)$$

- ❑ Address the matching between the three kinematic regions → Relevant for comparison with SCET

- ❑ Comparison with DIA!! $e^+e^- \to \pi^+\pi^- X$ → Access to the soft model

Thank You!

Back-up

Matching R_2 with R_3 and relation with SCET

	soft	soft-collinear	collinear
R_1	TMD-relevant	TMD-relevant	TMD-relevant
R_2	TMD-irrelevant	TMD-relevant	TMD-relevant
$M_{2,3}$	TMD-irrelevant	TMD-irrelevant	TMD-relevant
R_3	TMD-irrelevant	TMD-irrelevant	TMD-relevant

SCET factorization theorem

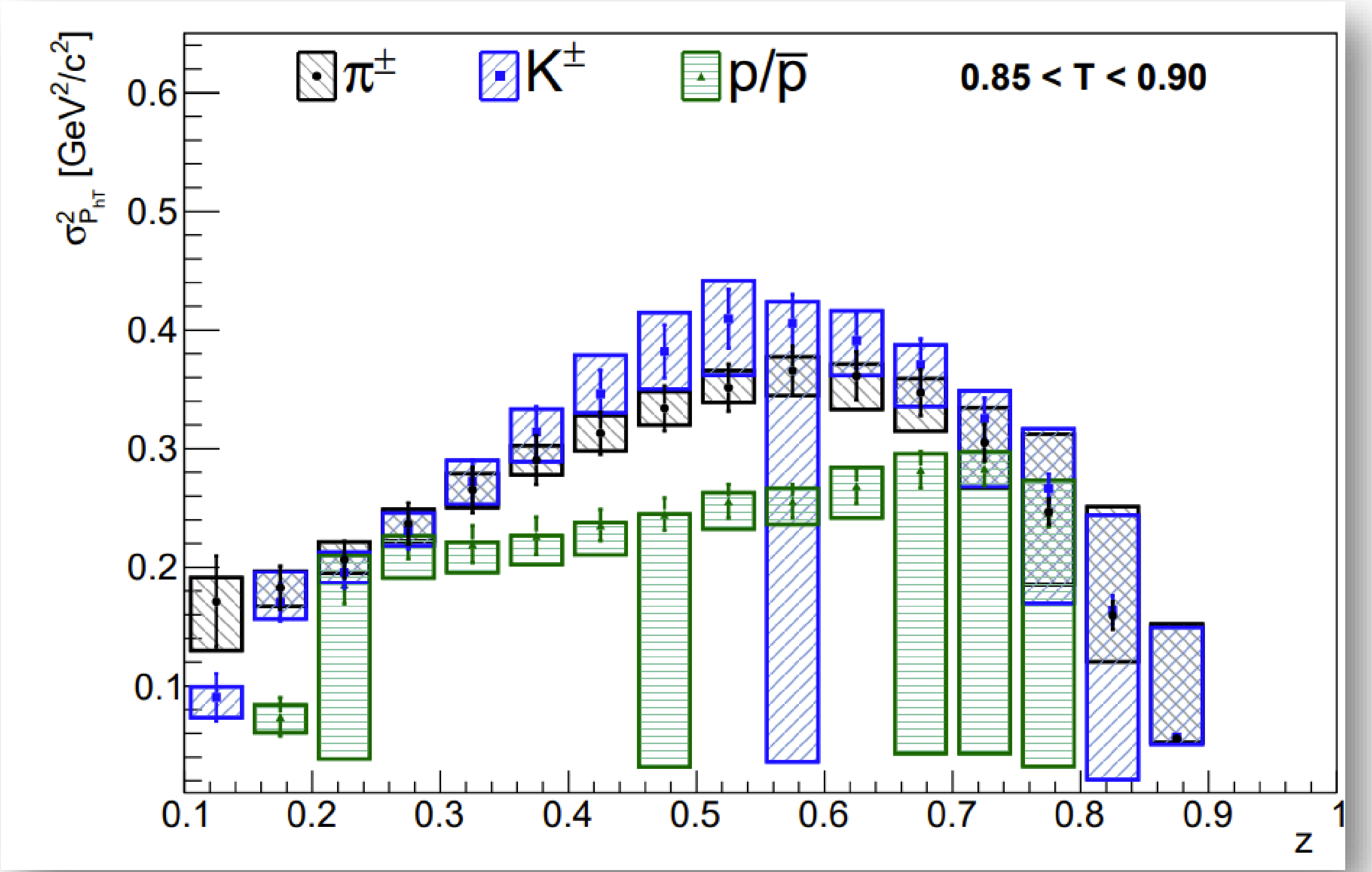
$$d\sigma_{M_{2,3}} \sim H\, J\, \frac{\mathcal{S}}{\mathcal{Y}_L\, \mathcal{Y}_R}\, \mathcal{G}_{h/j}$$

$$\frac{d\sigma_{R_{M_{2,3}}}}{dz\, du\, d^2\vec{b}_T} = \boxed{\mathcal{R}\left(a_S(\mu), \mathcal{L}_b, L_R; u, b_T\right)}\, \frac{d\sigma_{R_2}(y_1)}{dz\, du\, d^2\vec{b}_T}.$$

Non-perturbative!

$$\boxed{d\sigma_{R_3} \xrightarrow{\text{large } b_T} d\sigma_{M_{2,3}} \xleftarrow{\text{small } b_T} d\sigma_{R_2};}$$

Cross sections width and choice of the model



Maximum width in z_0

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$$W(z) = \frac{m_\pi}{R(z)}.$$

$$M_D\,(z_h, P_T, M, p) = \frac{\Gamma(p)}{\pi\Gamma(p-1)} M^{2\,(p-1)} \left(M^2 + \frac{P_T^2}{z_h^2}\right)^{-p}$$