Rethinking running coupling in JIMWLK/BK

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Introduction and motivation

- Small-x evolution: BFKL $\to$ BK $\to$ JIMWLK

Talks by Mehtar-Tani & Boussarie

- JIMWLK allows to evolve arbitrary combination of many Wilson lines without large N_c approximation
- NLO JIMWLK equation was derived $\approx$ 10 years ago

Kovner, Lublinsky & Mulian (2013), Balitsky & Chirilli (2007), Grabovsky (2013); ML & Mulian (2016)

- Large transverse logs in NLO JIMWLK/BK: improvements are necessary

Altinoluk, Armesto, Beuf Hatta, Iancu, Lublinsky, Müller, Stasto, Triantafyllopoulos, Xiao, ...

- The principal part: large logs multiplied by QCD β-function
- Resummation of these logs led to r.c. BK with generalization to r.c. JIMWLK

Balitsky, Kovchegov & Weigert, ...

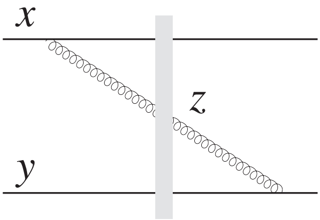
- There has been no r.c. JIMWLK implementation that would explicitly reproduce any specific r. c. prescription consistent with NLO JIMWLK
- All known r. c. prescriptions violate semi-positivity of JIMWLK Hamiltonian

Punchline

- In NLO JIMWLK, not all large logs with QCD β-function belong in running coupling
- Subset of the logs comes from DGLAP evolution of the projectile
- Why misidentification? Integral of DGLAP splitting function $\propto$ QCD β-function
- We identified both types of logs, and provided a scheme for their resummation:
  - DGLAP logs $\rightsquigarrow$ evolution equation for JIMWLK kernel
  - r. c. logs $\rightsquigarrow$ simple scale for the QCD running coupling
- This procedure leads to semi-positive definite JIMWLK Hamiltonian

LO JIMWLK Hamiltonian

- LO JIMWLK Hamiltonian $\partial \mathcal{O}/\partial Y = -\mathcal{H}^{\mathrm{JIMWLK}}\mathcal{O}$



$$\mathcal{H}^{\mathrm{JIMWLK}}_{\mathrm{LO}} = \int_{x,y,z} K_{\mathrm{LO}} \left[J^a_L(x) J^a_L(y) + J^a_R(x) J^a_R(y) - 2 J^a_L(x) S^{ab}(z) J^b_R(y) \right]$$

$$K_{\mathrm{LO}}(x,y,z) = \frac{\alpha_s}{2\pi^2} \frac{(x-z)_i (y-z)_i}{(x-z)^2 (y-z)^2} \equiv \frac{\alpha_s}{2\pi^2} \frac{X \cdot Y}{X^2 Y^2}$$

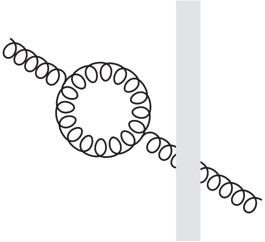
- Eikonal propagation through target

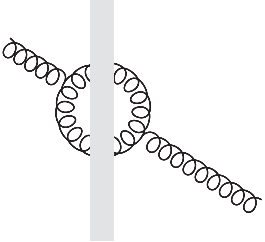
$$S(z) = \mathcal{P} \exp\left(ig \int dz^+ A^-(z_+, z) \right)$$

- Lee derivatives

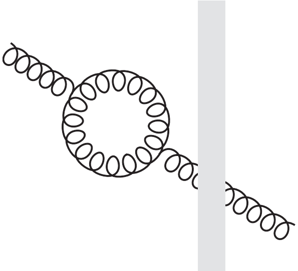
$$J^a_L(x) S(z) = T^a S(x) \delta^{(2)}(x-z) \qquad J^a_R(x) = S^{\dagger ab}(x) J^b_L(x)$$

NLO JIMWLK Hamiltonian: UV divergent contributions





NLO JIMWLK Hamiltonian: *UV* divergent contributions I



$$\int_{x,y,z} K'_{JSJ}\left[J_L^a(x)J_L^a(y)+J_R^a(x)J_R^a(y)-2J_L^a(x)S^{ab}(z)J_R^b(y)\right]$$

$$K'_{JSJ}=K_{\rm LO}\frac{\alpha\beta_0}{4\pi}\left(\ln\left(X^2\mu^2\right)+\ln\left(Y^2\mu^2\right)\right)+\ldots$$

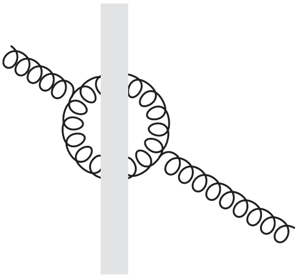
- The structure similar to the leading order
- Proportional to the WW kernel $\frac{X\cdot Y}{X^2Y^2}$
- No reasonable r. c. prescription, as the number of UV logs is twice as many

$$\alpha(X^2)\to\alpha\left(1+\frac{\alpha\beta_0}{4\pi}\ln X^2\mu^2\right)$$

- Forcing r. c. would lead to $\frac{\alpha(X^2)\alpha(Y^2)}{\alpha}$

Kovchegov & Weigert, Balitsky, Albacete & Kovchegov 2007

NLO JIMWLK Hamiltonian: *UV* divergent contributions II



$$\int_{x\,y\,z\,z'} K_{JSSJ} f^{abc} f^{def} J_L^a(x) S^{be}(z) S^{cf}(z') J_R^d(y)$$

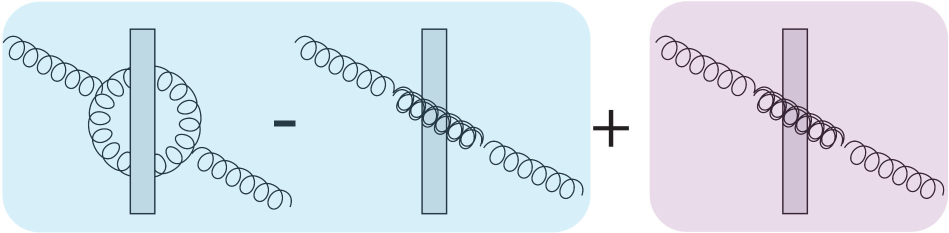
- When $z' \to z$, $f^{abc} f^{def} S^{be}(z) S^{cf}(z') \to N_c S^{ad}(z)$

- In the coincidence limit, integral of JSSJ kernel contains wanted UV singularity

$$N_c \int_{\mathbf{z}'} K_{JSSJ} = \frac{\alpha_s}{2\pi^2} \frac{\alpha_s \beta_0}{4\pi} \left(\frac{1}{X^2} \ln\left(X^2\mu^2\right) + \frac{1}{Y^2} \ln\left(Y^2\mu^2\right) + \frac{(X-Y)^2}{X^2 Y^2} \ln\left(\frac{(X-Y)^2}{X^2 Y^2 \mu^2} \right) \right) + \dots$$

- Strategy is to shift UV divergent “single gluon” scattering part to K_{JSJ}

NLO JIMWLK Hamiltonian: *UV* divergent contributions



K_{JSSJ} K_{JSJ}

- ✓ No UV divergence in K_{JSSJ}
- ✓ Allows for r. c. in K_{JSJ}: cancel an extra $\ln\mu^2$
- ✗ UV-finite pieces, including potentially large logarithms, are not uniquely defined. Dependence on the coordinate of the subtraction point
- ◆ All logarithms multiplying β_0 were attributed to r. c.

This led to Balitsky and Kovchegov-Weigert r. c. prescriptions.

Dressed gluon state

- K'_{JSJ}: production of a bare gluon state from the valence charge
- r. c. in QFT: the matrix element of the interaction Hamiltonian b/w dressed states
- Gluon wave function renormalization at arbitrary scale Q in one loop

$$Z^{1/2}(Q^2) = 1 + \frac{\alpha_s}{8\pi}\beta_0 \ln \frac{Q^2}{\mu^2}$$

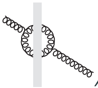
and associated renormalized gluon field

$$A^Q_\mu(x) = Z^{-1/2}(Q^2) A_\mu(x)$$

- LO kernel of JIMWLK Hamiltonian is to be multiplied by $Z^{-1/2}(Q^2)$
- This will lead to the modification of NLO:

$$K'_{JSJ} \to K_{LO} \frac{\alpha_s \beta_0}{4\pi} \left(\ln(X^2\mu^2) + \ln(Y^2\mu^2) - \ln \frac{\mu^2}{Q^2} + \dots \right)$$

DGLAP splitting

- UV divergence of K_{JSSJ} () is to cancel if JIMWLK Hamiltonian is reformulated in terms of dressed gluon amplitude
- At NLO the dressed gluon state contains a two-gluon (and $q-\bar{q}$) component due to gluon splitting; to be included in the definition of the dressed gluon scattering amplitude
- For splitting to two gluons

$$\mathbb{S}_Q^{ab}(z) = S^{ab}(z) + \frac{\alpha_s}{2\pi^2}\int d\xi \underbrace{\frac{1}{\xi_+(1-\xi)_+}\left(\xi^2 + (1-\xi)^2 + \xi^2(1-\xi)^2\right)}_{\sigma(\xi)}$$

$$\times \int_Z^{Q^{-1}} \frac{1}{Z^2}\left(\underbrace{\mathrm{Tr}[T^a S(z+(1-\xi)Z)T^b S^+(z-\xi Z)]}_{D_{ab}(z+(1-\xi)Z,z-\xi Z)} - N_c S^{ab}(z)\right)$$

Last term: $\frac{\alpha\beta_0}{4\pi}\ln\frac{\mu^2}{Q^2}S^{ab}(z)$

- Expressing LO JIMWLK in terms of $\mathbb{S}_Q$ cancels UV divergence of K_{JSSJ} in NLO

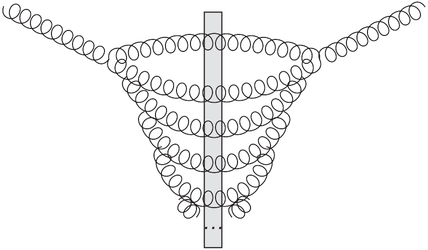
Resolution scale independence and RG

- Promoting to closed equation describing multiple consecutive DGLAP splittings

$$\frac{\partial \mathbb{S}_Q(z)}{\partial \ln Q^2} = -\alpha_s \int_\xi \sigma(\xi)\left(\mathbb{D}_Q - \mathbb{S}_Q(z)\right)$$

- Independence of the introduced scale, Q:

$$\frac{dH}{d\ln Q} = \frac{\partial H}{\partial \ln Q} + \int_u \left[\frac{\delta H}{\delta \mathbb{S}_Q(u)} \frac{\partial \mathbb{S}_Q(u)}{\partial \ln Q}\right] = 0$$



Initial conditions and r. c.

- Initial conditions: at $Q_{\text{in}} = Q_s^P$

$$\mathcal{H}_{\text{in}} = \int K_{\text{in}} \left[\{ \mathbb{S}_{Q_{\text{in}}}(z) - \mathbb{S}_{Q_{\text{in}}}(x) \} \{ \mathbb{S}_{Q_{\text{in}}}(z) - \mathbb{S}_{Q_{\text{in}}}(y) \}^{\dagger} \right]^{ab} J_L^a(x)\, J_L^b(y)$$

- The kernel at this scale is given by

$$K_{\text{in}} = \frac{\alpha_s^{\lambda}(X^2)\, \alpha_s^{\lambda}(Y^2) \alpha_s^{1-2\lambda}(XY)}{2\pi^2} \frac{X \cdot Y}{X^2 Y^2} \left[1 + \text{ small } \text{ NLO } \text{ corrections}\right]$$

and does not contain large logs, as $Q_s^P |X| \sim 1$

λ is not uniquely fixed by NLO; $\lambda = 1/2$ is our preference; $\lambda = 1$ is "triumvirate" form

c.f. G. Chirilli & Y. Kovchegov, 2013

- Evolve up to $Q_f = Q_s^T$

Dilute/BFKL regime

- Initial JIMWLK kernel is convenient to write in the form:

$$\mathcal{H}_{\text{in}} \propto \int_{x,y,z,z_1,z_2} \underbrace{\frac{X\cdot Y}{X^2Y^2}\Big(\underbrace{\delta_{z_1,z_2}}_{\delta(z_1-z_2)} \delta_{z_1,z} + \delta_{x,z_1}\delta_{y,z_2} - \delta_{x,z_1}\delta_{z,z_2} - \delta_{y,z_2}\delta_{z,z_1}\Big)}_{\propto K_{\text{in}}} \left[\mathbb{S}_{Q_0}(z_1)\mathbb{S}^{\dagger}_{Q_0}(z_2)\right]^{ab} J^a_L(x)\, J^b_L(y)$$

- DGLAP evolution leads to smearing of δ-functions

$$\mathcal{H}_{Q} \propto \int_{x,y,z,z_1,z_2} \frac{X\cdot Y}{X^2Y^2}\Big(\underbrace{r_{z_1,z_2}}_{r(z_1-z_2)} r_{z_1,z} + r_{x,z_1}r_{y,z_2} - r_{x,z_1}r_{z,z_2} - r_{y,z_2}r_{z,z_1}\Big) \left[\mathbb{S}_{Q}(z_1)\mathbb{S}^{\dagger}_{Q}(z_2)\right]^{ab} J^a_L(x)\, J^b_L(y)$$

- r function:

$$r(z) = \begin{cases} \delta(z), & \text{for } z > 1/Q_s^P \\ \frac{1}{z^2}\left[\left(\frac{1}{zQ_s^P}\right)^{\frac{\alpha_s\beta_0}{2\pi}} - 1\right], & \text{for } 1/Q_s^P > z > 1/Q_s^T \\ \frac{1}{z^2}\left[\left(\frac{Q_s^T}{Q_s^P}\right)^{\frac{\alpha_s\beta_0}{2\pi}} - 1\right], & \text{for } z < 1/Q_s^T \end{cases}$$

Saturation regime

- Target saturation momentum plays two roles:
  - provides correlation length for Wilson lines
  - provides color neutralization scale: a Wilson line separated from the rest by a distance greater than $1/Q_s$ is vanishingly small

- For evolution in distance range from $1/Q_s^P$ to $1/Q_s^T$, neglect quadratic term in DGLAP evolution $\mathbb{D}_Q - N_c\mathbb{S}_Q(z) \rightarrow -N_c\mathbb{S}_Q(z)$

- The kernel is

$$K_Q = \left[\frac{Q_s^T}{Q_s^P}\right]^{\frac{\alpha_s}{2\pi}b} K_{in}$$

Interpolating equation between two regimes

- Explicit solutions in dilute and saturation regime of DGLAP provided us with Q-dependent kernel for JIMWLK Hamiltonian

- For practical implementation, an interpolating equation is needed

Conclusions

- Not all large logs of NLO JIMWLK multiplying QCD β-function belong to running coupling
- Subset of the logs comes from DGLAP evolution of the projectile
- We identified both types of logs, and provided the scheme for their resummation:
  - DGLAP logs $\rightsquigarrow$ evolution equation for JIMWLK kernel
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