

# Collinear dynamic beyond DGLAP

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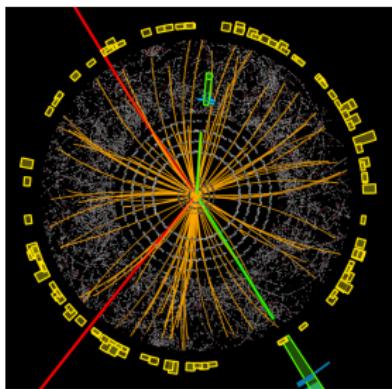
arXiv:2210.10061

University of Amsterdam

QCD Evolution 2023

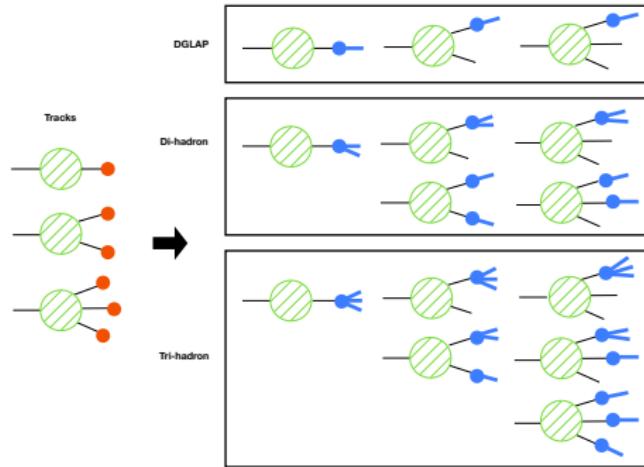


# You will have to wait to the end!



Tracks

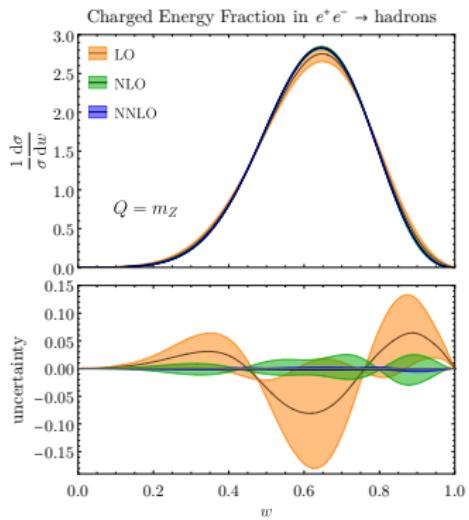
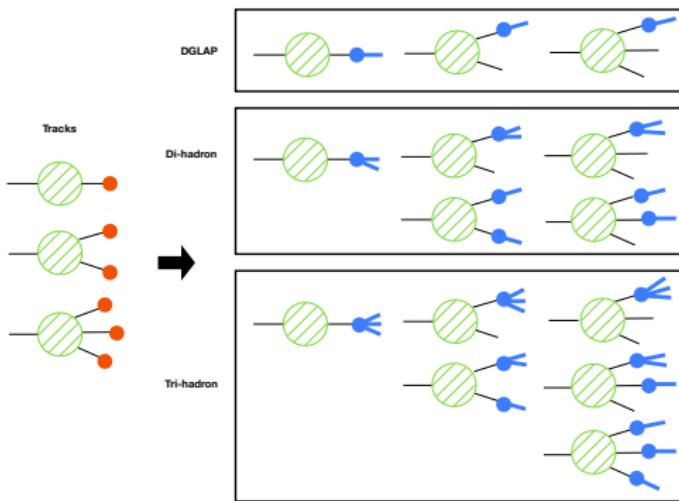
This talk  
⇒



Universal collinear RGE

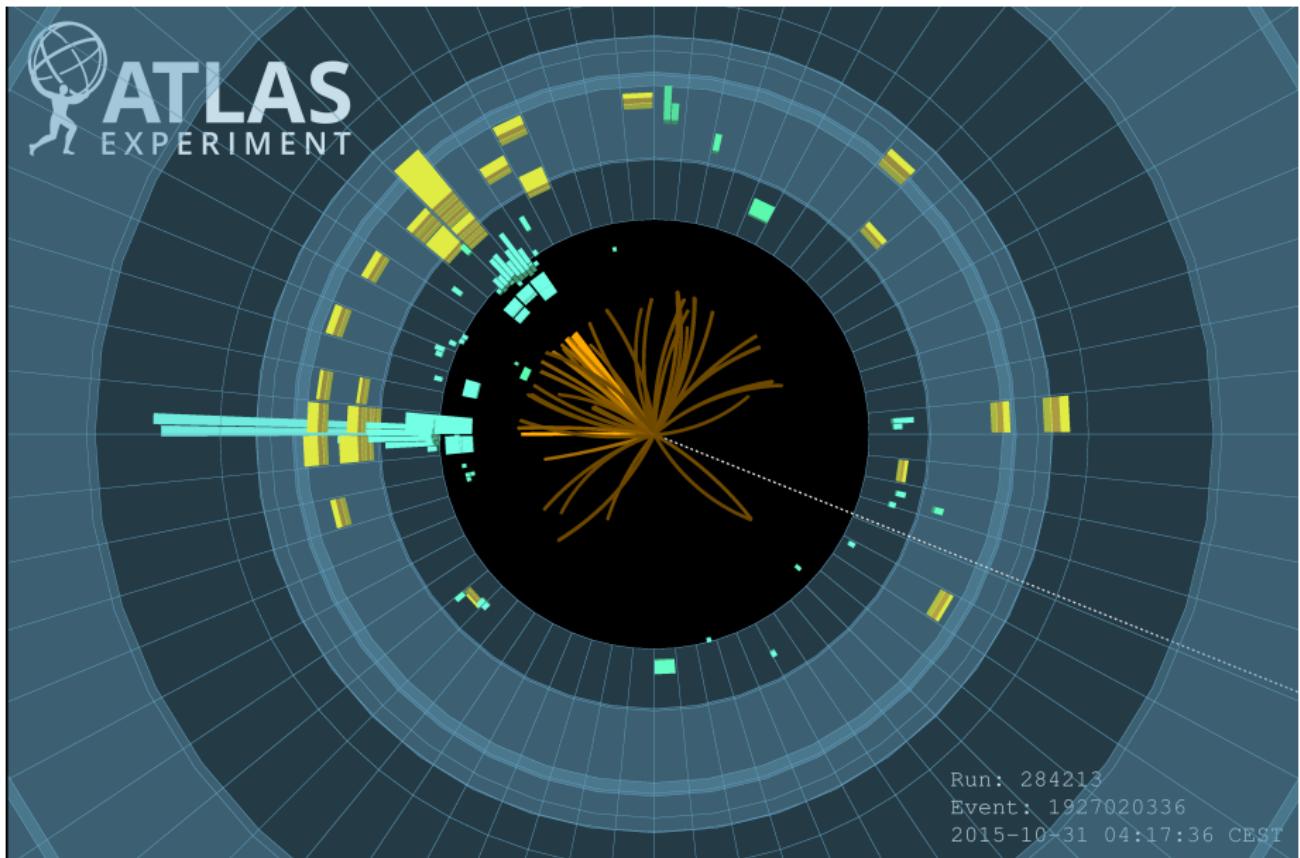
# What can you expect today?

$$\frac{d}{d \ln \mu^2} T_q = a_s \left[ K_{q \rightarrow q} \otimes T_q + K_{q \rightarrow qg} \otimes T_q T_g + K_{q \rightarrow qgg} \otimes T_q T_g T_g \right. \\ \left. + K_{q \rightarrow qq\bar{q}} \otimes T_q T_q T_{\bar{q}} + K_{q \rightarrow qQ\bar{Q}} \otimes T_q T_Q T_{\bar{Q}} + \dots \right]$$

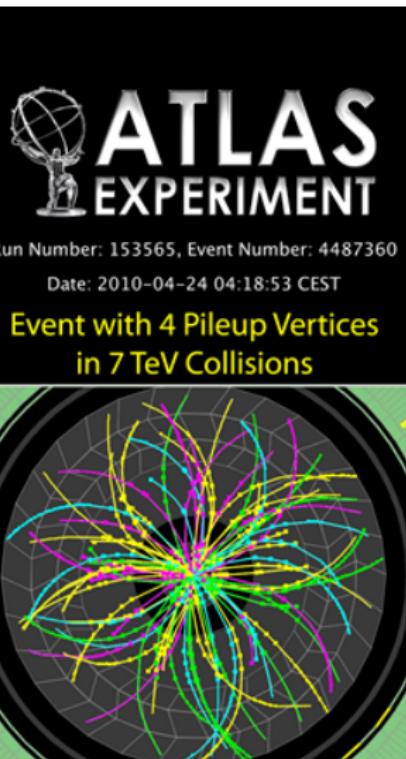
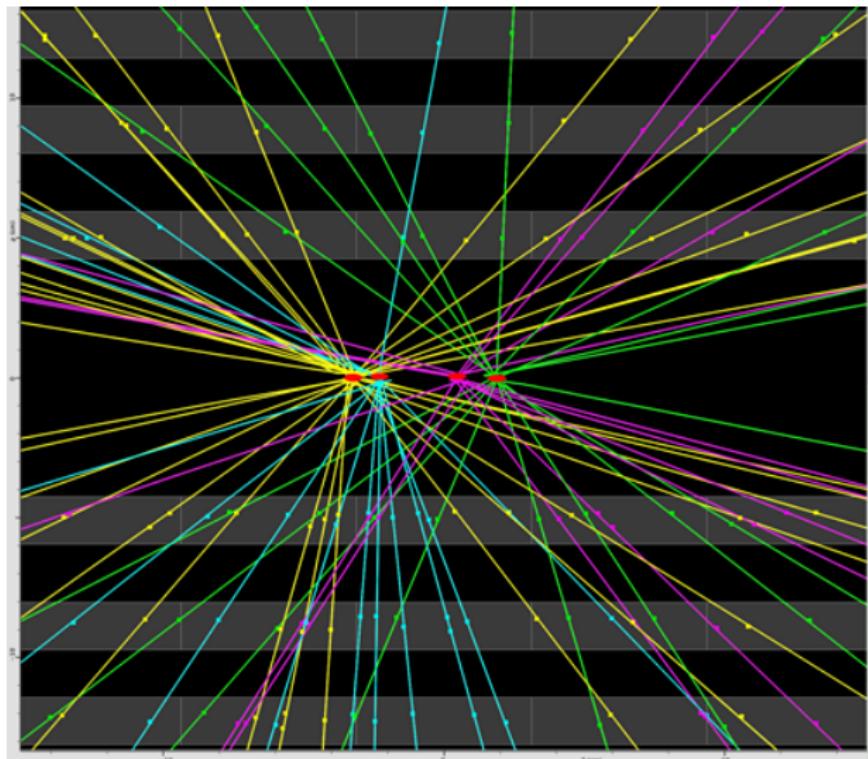


# Tracks

Q: What is there to love about tracks? A: Precision!



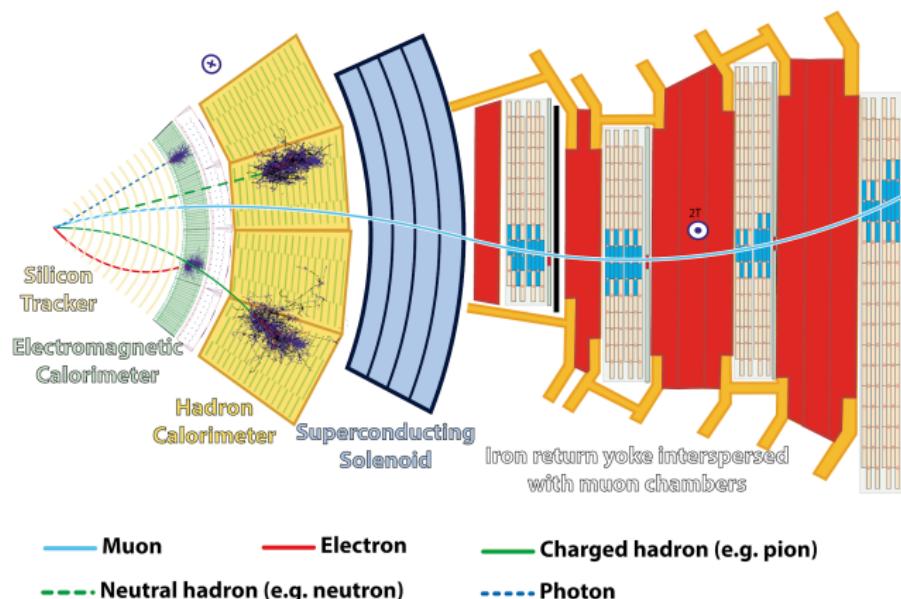
Q: What is there to love about tracks? A: Less mess!



# No more love for calorimeters?

Should we just donate our calorimeters to charity now?

- ▶ Of course not!



When is it better to exclusively measure on tracks?

Observables where precision is watered down by calorimeter related uncertainties

# Not so fast

- Track-based observables are great experimentally!
- But theorists had a bit of catching up to do
- Track-based observables are not safe
  - ▶ Not including all final states  $\Rightarrow$  IRC unsafe



## Track function formalism

IR divergences are absorbed into non-perturbative **track functions**

*Chang, Procura, Thaler, Waalewijn (2013)*

# What is a track function?

Track function definition: English

$T_i(x) \stackrel{\text{LO}}{=} \left( \begin{array}{l} \text{Probability density of finding that the charged} \\ \text{particles fragmenting from a parton } i \text{ carry} \\ \text{a certain fraction } x \text{ of the total momentum} \end{array} \right)$

Track function definition: QFT

$$T_q(x) = \frac{1}{2N_c} \sum_X \delta\left(x - \frac{P_c^-}{P^-}\right) \int dy^+ d^{d-2}y e^{iy^+ P^-/2} \\ \times \text{tr} \left[ \vec{n} \langle 0 | \chi_n(y^+, 0^-, \mathbf{y}) | X \rangle \langle X | \bar{\chi}_n(0) | 0 \rangle \right]$$

- ▶ Like a FF, but measuring the momentum of a group of particles

# Implementation

For a generic cross section with some observable  $e$

$$\frac{d\sigma}{de} = \sum_N \int d\Pi_N \frac{d\sigma_N}{d\Pi_N} \delta[e - \hat{e}(\{p_i\})]$$

Partonic cross section with  $N$  final state partons

Track function formalism (leading order)

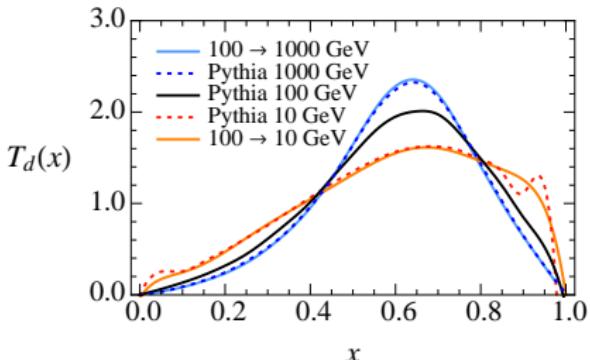
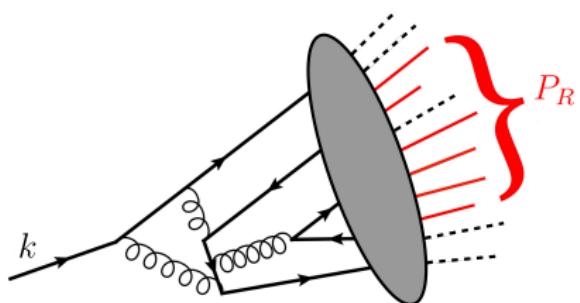
Observe on tracks only  $\Rightarrow$  attach track function to each parton

For the same observable on tracks  $\bar{e}$

$$\frac{d\sigma}{d\bar{e}} = \sum_N \int d\Pi_N \frac{d\bar{\sigma}_N}{d\Pi_N} \int \left( \prod_{i=1}^N dx_i T_i(x_i) \right) \delta[\bar{e} - \hat{e}(\{x_i p_i\})]$$

Chang, Procura, Thaler, Waalewijn (2013)

# Basic properties



Probability density concerning a subset of fragments

- Only has support for  $x \in [0, 1]$

- Normalised to 1

$$\int_0^1 dx T_i(x) = 1$$

- Calculable scale dependence

# Evolution

Why should we care?

NLO evolution for  $T(x) \Rightarrow$  precision for track-based observables

But wait, there is more!

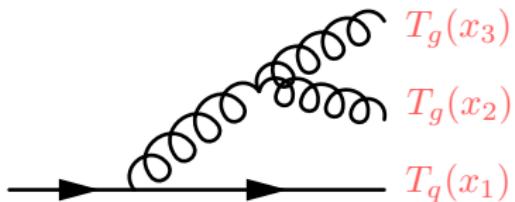
- Predicts structure of IR divergences in perturbative calculations

*For fragmentation at NNLO see: Gehrmann, Schürmann (2022), Gehrmann, Stagnitto (2022)*

- Higher-order collinear corrections to next-gen parton showers

*Dasgupta et al. (2020), Li, Skands (2017), Höche, Prestel (2017), ...*

# Structure of the evolution



There goes linearity

All parton branches contribute  $\Rightarrow$  Non-linear evolution equations

$$\frac{d}{d \ln \mu^2} T_q = a_s \left[ \underbrace{K_{q \rightarrow q} \otimes T_q}_{\text{linear}} + \underbrace{K_{q \rightarrow qg} \otimes T_q T_g}_{\text{quadratic}} + \underbrace{K_{q \rightarrow qgg} \otimes T_q T_g T_g}_{\text{cubic}} + \dots \right]$$

# Beyond LO

At leading order track evolution kernels are just the DGLAP kernels

$$K_{q \rightarrow qg}^{(0)}(z, 1-z) = P_{qq}^{(0)}(z)$$

## Goal

Calculate the evolution kernels beyond LO

Problem: Track functions are scaleless objects

$$T_q(x) = \delta(1-x) + 2a_s C_F \left( \frac{1}{\epsilon_{uv}} - \frac{1}{\epsilon_{ir}} \right) \left[ \frac{1+x^2}{1-x} \right]_+ + \mathcal{O}(a_s^2)$$

The burden of being scaleless

Calculate evolution kernels  $\Rightarrow$  single out UV divergence

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The burden of being scaleless

Calculate evolution kernels  $\Rightarrow$  single out UV divergence

## Indirect calculation

Calculate evolution via another object defined on tracks

- 1 Calculate bare track jet function
- 2 Treat overlapping soft divergences
- 3 Renormalize the track jet function
- 4 Extract evolution kernels from single poles

# Step 1

## 1 Calculate bare track jet function

- Treat overlapping soft divergences
- Renormalize the track jet function
- Extract evolution kernels from single poles

# Step 1: Calculate bare track jet function

Consider the standard invariant mass jet function

$$J_i^{\text{bare}}(s) = \sum_N \sum_{\{i_f\}} \int d\Phi_N^c \frac{1}{S_{\{i_f\}}} \sigma_{i \rightarrow \{i_f\}}(\{z_f\}, \{s_{ff'}\}, s)$$

Now consider the same object, but on tracks

$$\begin{aligned} J_{i,\text{track}}^{\text{bare}}(s, x) &= \sum_N \sum_{\{i_f\}} \int d\Phi_N^c \frac{1}{S_{\{i_f\}}} \sigma_{i \rightarrow \{i_f\}}(\{z_f\}, \{s_{ff'}\}, s) \\ &\quad \times \int dx_1 \cdots dx_N T_{i_1}^{(0)}(x_1) \cdots T_{i_N}^{(0)}(x_N) \delta(x - z_1 x_1 - \cdots - z_N x_N) \end{aligned}$$

## Step 1: Calculate bare track jet function

Consider the standard invariant mass jet function

$$J_i^{\text{bare}}(s) = \sum_N \sum_{\{i_f\}} \int d\Phi_N^c \frac{1}{S_{\{i_f\}}} \sigma_{i \rightarrow \{i_f\}}(\{z_f\}, \{s_{ff'}\}, s)$$

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## Step 2

- 1 Calculate bare track jet function
- 2 Treat overlapping soft divergences
  - Renormalize the track jet function
  - Extract evolution kernels from single poles

## Step 2: Treat overlapping soft divergences

Single momentum fraction  $\Rightarrow$  soft divergences at  $z \rightarrow 0, 1$

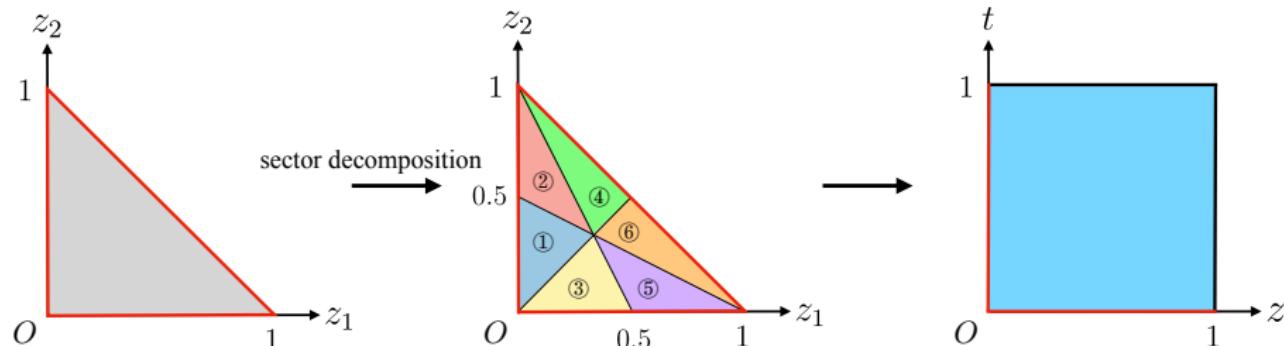
$$\frac{1}{z^{1+\epsilon}} = -\frac{1}{\epsilon} \delta(z) + \left[ \frac{1}{z} \right]_+ + \dots$$

Multiple momentum fractions  $\Rightarrow$  divergent regions overlap

- Partons can become soft either individually or simultaneous

$$\frac{1}{z_1^\epsilon (z_1 + z_2)^{1+\epsilon}}$$

# Sector decomposition



- Divide the phase space up into 6 sectors

- sector 1:  $z_1 < z_2 < z_3, \dots$

- Change variables in each sector

*Binoth, Heinrich (2000)  
Binoth, Heinrich (2004)  
Anastasiou, Melnikov (2004)*

Sector decomposition

Disentangles overlapping divergences

## Step 3

- 1 Calculate bare track jet function
  - 2 Treat overlapping soft divergences
  - 3 Renormalize the track jet function
- 
- Extract evolution kernels from single poles

## Step 3: Renormalize the track jet function

A useful property

Renormalization of  $J$  is not sensitive to the momentum fractions

Renormalization kernels for the two jet functions are the same

$$J_i(s) = \int_0^s ds' Z_{J_i}(s') J_i^{\text{bare}}(s - s')$$
$$J_{i,\text{track}}(s, x) = \int_0^s ds' Z_{J_i}(s') J_{i,\text{track}}^{\text{bare}}(s - s', x)$$

Ritzmann, Waalewijn (2014)  
Becher, Neubert (2006)  
Becher, Bell (2010)

## Step 4

- 1 Calculate bare track jet function
- 2 Treat overlapping soft divergences
- 3 Renormalize the track jet function
- 4 Extract the evolution kernels from the single poles

## Step 4: Extract evolution from the single poles

After renormalization  $J_{\text{track}}$  will still contain (IR) poles in  $\epsilon$

$$J_q^{(2)}(s) = \text{finite}$$

$$J_{q,\text{track}}^{(2)}(s, x) = \text{finite} - \frac{1}{2\epsilon} \delta(s) K_{q \rightarrow qgg}^{(1)} \otimes T_q^{(0)} T_g^{(0)} T_g^{(0)}(x) + \dots$$

### Evolution kernels

Obtain evolution kernels from poles of  $J_{\text{track}}$  that remain after renormalization\*

\*Some one-loop cross terms need to be subtracted first

# Universality

## Claim

The track function evolution kernels are universal

What do we mean with this claim?

- Reduction to DGLAP kernels
- Reduction to the evolution of di-hadron fragmentation functions
- Generalization to  $N$ -hadron fragmentation functions

# Reduction to DGLAP

**Track functions:** All branches  $\Rightarrow$  non-linear evolution

$$\frac{d}{d \ln \mu^2} T_i = \underbrace{K_{i \rightarrow i} \otimes T_i}_{\text{linear}} + \underbrace{K_{i \rightarrow i_1 i_2} \otimes T_{i_1} T_{i_2}}_{\text{quadratic}} + \underbrace{K_{i \rightarrow i_1 i_2 i_3} \otimes T_{i_1} T_{i_2} T_{i_3}}_{\text{cubic}} + \dots$$

**Fragmentation functions:** One branch  $\Rightarrow$  linear evolution

$$\frac{d}{d \ln \mu^2} D_{i \rightarrow h} = P_{ij} \otimes D_{j \rightarrow h}$$

Gribov, Lipatov (1972)

Dokshitzer (1977)

Altarelli, Parisi (1977)

## Universality

The DGLAP kernels can be obtained from the track evolution kernels by integrating over all unobserved branches

# Reduction to DGLAP



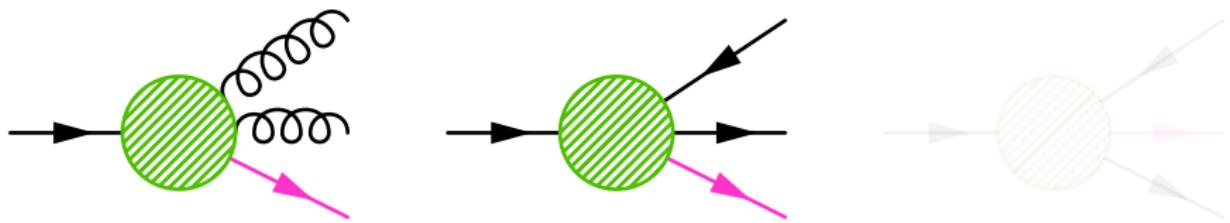
$$\begin{aligned} P_{q\bar{q}}(z) = & K_{q \rightarrow q} \delta(1 - z) + K_{q \rightarrow qg}(z, 1 - z) \\ & + \int dz_1 dz_2 dz_3 \delta(1 - z_1 - z_2 - z_3) K_{q \rightarrow qgg}(z_1, z_2, z_3) \delta(z - z_1) \\ & + \int dz_1 dz_2 dz_3 \delta(1 - z_1 - z_2 - z_3) K_{q \rightarrow q\bar{q}}(z_1, z_2, z_3) \delta(z - z_1) \\ & + \int dz_1 dz_2 dz_3 \delta(1 - z_1 - z_2 - z_3) K_{q \rightarrow q\bar{q}}(z_1, z_2, z_3) \delta(z - z_2) \\ & + \dots \end{aligned}$$

# Reduction to DGLAP



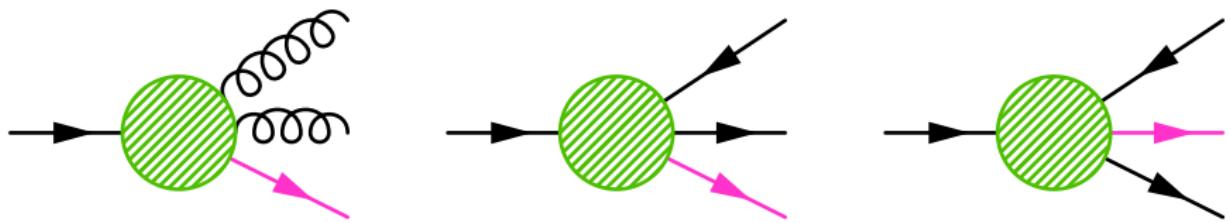
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# Reduction to DGLAP



$$\begin{aligned} P_{q\bar{q}}(z) = & K_{q \rightarrow q} \delta(1 - z) + K_{q \rightarrow qg}(z, 1 - z) \\ & + \int dz_1 dz_2 dz_3 \delta(1 - z_1 - z_2 - z_3) K_{q \rightarrow qgg}(z_1, z_2, z_3) \delta(z - z_1) \\ & + \int dz_1 dz_2 dz_3 \delta(1 - z_1 - z_2 - z_3) K_{q \rightarrow q\bar{q}}(z_1, z_2, z_3) \delta(z - z_1) \\ & + \int dz_1 dz_2 dz_3 \delta(1 - z_1 - z_2 - z_3) K_{q \rightarrow q\bar{q}}(z_1, z_2, z_3) \delta(z - z_2) \\ & + \dots \end{aligned}$$

# Reduction to DGLAP



$$\begin{aligned} P_{qg}(z) = & K_{q \rightarrow q} \delta(1 - z) + K_{q \rightarrow qg}(z, 1 - z) \\ & + \int dz_1 dz_2 dz_3 \delta(1 - z_1 - z_2 - z_3) K_{q \rightarrow qgg}(z_1, z_2, z_3) \delta(z - z_1) \\ & + \int dz_1 dz_2 dz_3 \delta(1 - z_1 - z_2 - z_3) K_{q \rightarrow qq\bar{q}}(z_1, z_2, z_3) \delta(z - z_1) \\ & + \int dz_1 dz_2 dz_3 \delta(1 - z_1 - z_2 - z_3) K_{q \rightarrow q\bar{q}}(z_1, z_2, z_3) \delta(z - z_2) \\ & + \dots \end{aligned}$$

# Going beyond DGLAP

## Evolution of track functions

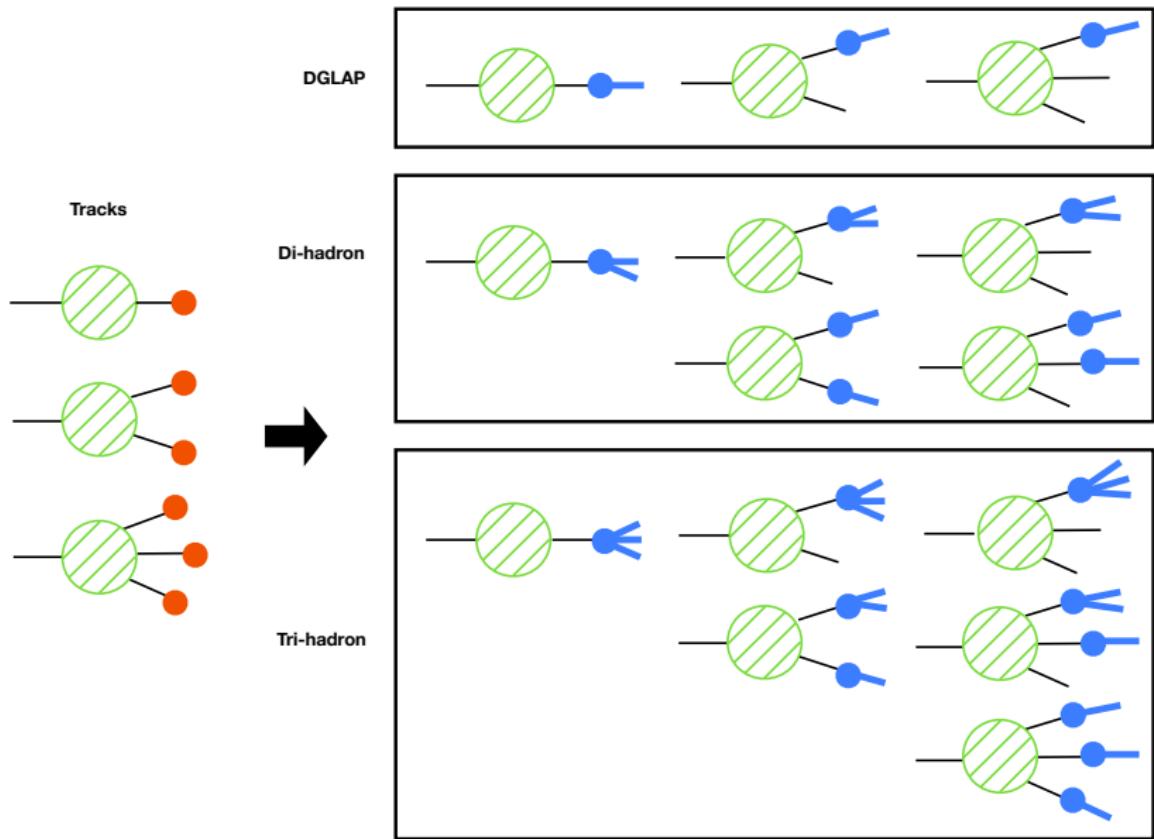
$$\frac{d}{d \ln \mu^2} T_i = \underbrace{K_{i \rightarrow i} \otimes T_i}_{\text{linear}} + \underbrace{K_{i \rightarrow i_1 i_2} \otimes T_{i_1} T_{i_2}}_{\text{quadratic}} + \underbrace{K_{i \rightarrow i_1 i_2 i_3} \otimes T_{i_1} T_{i_2} T_{i_3}}_{\text{cubic}} + \dots$$

## Evolution for di-hadron fragmentation functions

$$\frac{d}{d \ln \mu^2} D_{i \rightarrow h_1 h_2} = \underbrace{??? \otimes D_{j \rightarrow h_1 h_2}}_{\text{linear}} + \underbrace{??? \otimes D_{i_1 \rightarrow h_1} D_{i_2 \rightarrow h_2}}_{\text{quadratic}}$$

*Konishi, Ukawa (1979)*  
*Sukhatme, Lassila (1980)*  
*Sukhatme, Lassila, Orava (1982)*  
*de Florian, Vanni (2004)*  
*Vendramin (1981)*  
*Majumder, Wang (2004)*

# Reduction



# Application

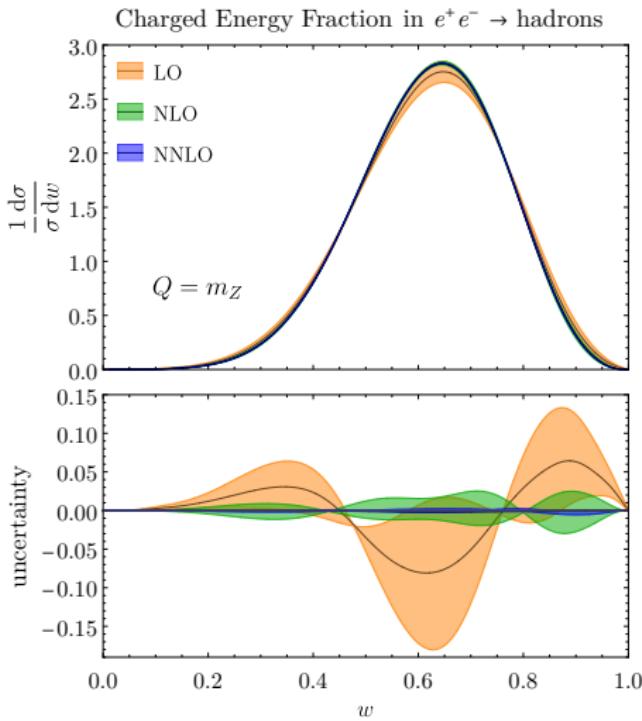
- $w$ : Charged energy fraction

$$\frac{d\sigma_{e^+e^- \rightarrow \text{hadrons}}}{dw}$$

- Convolution of track functions

$$\frac{d\sigma^{\text{LO}}}{dw} \propto \int dx T_q(x) T_{\bar{q}}(w-x)$$

- Requires evolution of  $T_i(x)$
- Significant error reduction



# Energy correlators (work in progress)

What is an energy correlator?

Observable that describes correlations within the energy flow of a jet

Neutral particles are detected via calorimeter cells

- ▶ Low angular resolution limits precision

Energy correlators on tracks

Measuring EECs on tracks allows for higher precision measurements

For all **charged** particles, measure the energies and angles

$$\frac{d\sigma^{[N]}}{dz} = \sum_{\{i\}} \int d\sigma \frac{E_{i_1} \cdots E_{i_N}}{Q^N} \delta\left(z - \frac{1 - \cos \theta_{\max}}{2}\right)$$

Lee, Mečaj, Moult (2022)

Dixon, Moult, Zhu (2019)

Chen, Luo, Moult, Yang, Zhang (2020)

Chen, Moult, Zhang, Zhu (2020)

## Conclusions

# Conclusion

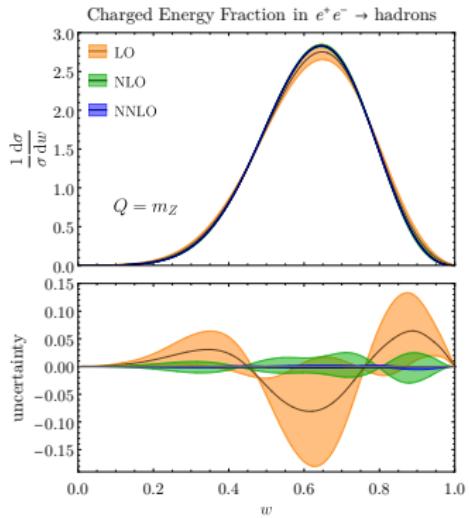
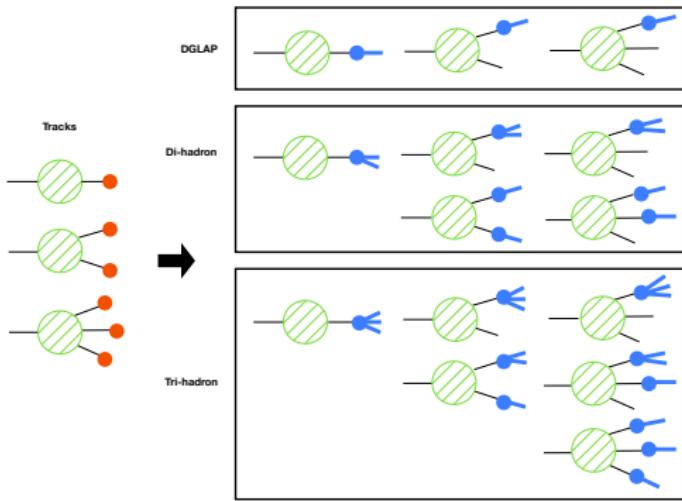
- Track-based observables can allow for greater precision
  - ▶ Track function formalism makes theory predictions possible
- Evolution of track functions is non-linear
  - ▶ Evolution kernels now known at NLO
- Track function evolution kernels are universal
  - ▶ Can be reduced to DGLAP and  $N$ -hadron fragmentation
- Application
  - ▶ Working towards energy correlators on tracks

Main achievement

Universal collinear evolution kernels calculated to NLO

# Thank you for your attention!

$$\frac{d}{d \ln \mu^2} T_q = a_s \left[ K_{q \rightarrow q} \otimes T_q + K_{q \rightarrow qg} \otimes T_q T_g + K_{q \rightarrow qgg} \otimes T_q T_g T_g \right. \\ \left. + K_{q \rightarrow qq\bar{q}} \otimes T_q T_q T_{\bar{q}} + K_{q \rightarrow qQ\bar{Q}} \otimes T_q T_Q T_{\bar{Q}} + \dots \right]$$



Backup slides

# Backup I - Convolution

$$\begin{aligned} & K_{i \rightarrow i_1 i_2 i_3} \otimes T_{i_1} T_{i_2} T_{i_3}(x) \\ &= \int dx_1 dx_2 dx_3 T_{i_1}(x_1) T_{i_2}(x_2) T_{i_3}(x_3) \int dz_1 dz_2 dz_3 \delta(1 - z_1 - z_2 - z_3) \\ & \quad \times \delta(x - z_1 x_1 - z_2 x_2 - z_3 x_3) K_{i \rightarrow i_1 i_2 i_3}(z_1, z_2, z_3) \end{aligned}$$

## Backup II - Sector decomposition

$$\begin{aligned} & K_{i \rightarrow i_1 i_2 i_3} \otimes T_{i_1} T_{i_2} T_{i_3}(x) \\ &= \sum_{n=1}^6 \int dx_1 dx_2 dx_3 T_{i_1}(x_1) T_{i_2}(x_2) T_{i_3}(x_3) \int dz dt \\ & \quad \times \delta\left(\textcolor{red}{x} - {}^n z_1 x_1 - {}^n z_2 x_2 - {}^n z_3 x_3\right) {}^n K_{i \rightarrow i_1 i_2 i_3}(z, t) \end{aligned}$$

where

$${}^n z_i \in \left\{ \frac{zt}{1+z+zt}, \frac{z}{1+z+zt}, \frac{1}{1+z+zt} \right\}$$