Weizsäcker-Williams gluon TMD factorization at NLO in the small-x regime

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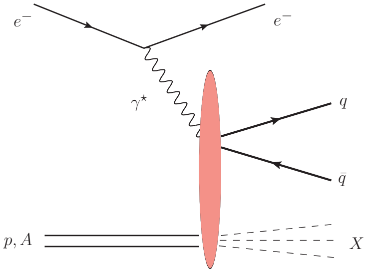
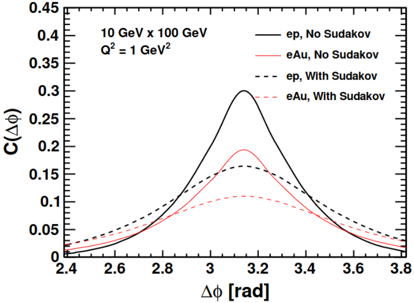
JHEP 2021 (11), 1-108, JHEP 2022 (11), 1-77, arXiv:2304.03304 and work in progress

Back-to-back di-jets in DIS

⇒ probe of the saturated regime of QCD

⇒ access to the Weizsäcker-Williams gluon TMD in the back-to-back limit.

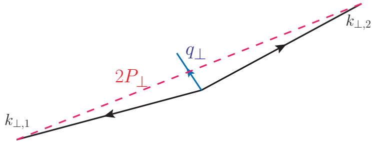




Zheng, Aschenauer, Lee, Xiao, 1403.2413

LO: common language between small-x and TMD communities

- Def: $|\boldsymbol{P}_\perp| = |z_2\boldsymbol{k}_{\perp,1} - z_1\boldsymbol{k}_{\perp,2}| \gg |\boldsymbol{q}_\perp| = |\boldsymbol{k}_{\perp,1} + \boldsymbol{k}_{\perp,2}|$



- LO in photon-gluon fusion channel: TMD factorization Dominguez, Marquet, Xiao, Yuan, 1101.0715

$$\left.\frac{\mathrm{d}\sigma^{\gamma^\star\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp\mathrm{d}^2\boldsymbol{q}_\perp}\right|_{\mathrm{LO}} \propto \mathcal{H}^{ij}(\boldsymbol{P}_\perp)G_Y^{ij}(\boldsymbol{q}_\perp) + \mathcal{O}\left(\frac{q_\perp}{P_\perp}\right) + \mathcal{O}\left(\frac{Q_s}{P_\perp}\right)$$

See also del Castillo, Echevarria, Makris, Scimemi, 2008.07531

- $G_Y(\boldsymbol{q}_\perp)$: WW gluon TMD

$$G^{ij}_{Y=\ln(1/x)}(\boldsymbol{q}_\perp) = 2\int \frac{\mathrm{d}\xi^-\mathrm{d}^2\boldsymbol{\xi}_\perp}{(2\pi)^3P^+} e^{ixP^+\xi^- - iq_\perp\xi_\perp}\left\langle P\left|F^{+i}(\xi^-,\boldsymbol{\xi}_\perp)U_\xi^{[+]\dagger}F^{+j}(0)U_\xi^{[+]}\right|P\right\rangle \quad \text{TMD}$$

$$= \frac{-2}{\alpha_s}\int\frac{\mathrm{d}^2\boldsymbol{b}_\perp\mathrm{d}^2\boldsymbol{b}'_\perp}{(2\pi)^4}e^{-iq_\perp\cdot r_{bb'}}\left\langle \mathrm{Tr}\left[\partial^i V^\dagger(\boldsymbol{b}_\perp)V(\boldsymbol{b}'_\perp)\partial^j V^\dagger(\boldsymbol{b}'_\perp)V(\boldsymbol{b}_\perp)\right]\right\rangle_Y \quad \text{CGC}$$

Conceptual questions relevant for TMD and small-x communities

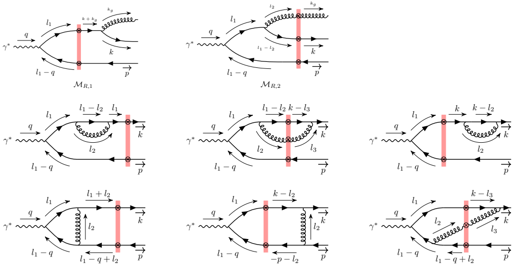
Small x and back-to-back regime

We work in the regime $W^2 \gg Q^2 \sim P_\perp^2 \gg q_\perp^2 \sim Q_s^2$.
Two kinds of large logs: $\ln(W^2/Q^2) \sim \ln(1/x)$ and $\ln(P_\perp/q_\perp)$

- Does TMD factorization hold at NLO in the small x limit?
- Do we recover the same NLO hard factor as in TMD calculations?
  Becher, Schwartz, 0911.0681, del Castillo, Echevarria, Makris, Scimemi, 2111.03703, Zhang, 1709.08970
- Can we isolate Sudakov from small-x logarithms beyond double logarithmic accuracy ?
  At DLA, conjecture from Mueller, Xiao, Yuan, 1308.2993 based on Higgs production in pA: yes!
- What value of $Y = \ln(1/x)$ enters the CGC definition of the WW TMD?
- Can we prove CSS evolution at small x? See Balitsky, Tarasov, arXiv:1505.02151

Outline of the NLO calculation

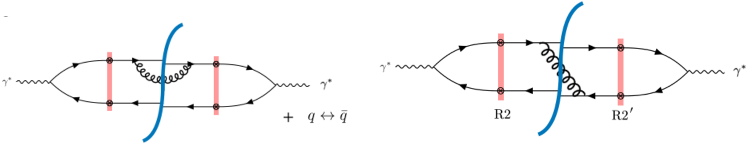
- We have done the full computation for general kinematics in 2108.06347
  Similar calculations & cross-checks in Taels, Altinoluk, Beuf, Marquet, 2204.11650, Iancu, Mulian, 2211.04837, Bergago, Jalilian-Marian, 2301.03117
- In the CGC EFT+ dipole picture of DIS, the diagrams are



- Rapidity divergence $\int_{\Lambda^-}^{k_f^-} \frac{k_g^-}{k_g^-}$ isolated $\Rightarrow$ gives JIMWLK evolution of the LO cross-section.
- Explicit computation of the NLO impact factor.

Back-to-back limit: Sudakov logarithms

- Real diagrams with soft double logarithmic enhancement:



- However: the integration over the soft gluon gives the Sudakov with a positive sign!

$$\mathrm{d}\sigma_{\mathrm{NLO}}^{\gamma^\star \to q\bar{q}+X} \sim \mathcal{H}(\boldsymbol{P}_\perp) \int \mathrm{d}^2\boldsymbol{r}_{bb'}\, e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \left[1 + \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) + \ldots + \alpha_s \ln\left(\frac{k_f^-}{\Lambda^-}\right) \mathcal{K}_{\mathrm{LL}} \otimes \right] G_Y(\boldsymbol{r}_{bb'})$$

- Problem: overlapping phase space between soft gluons and slow gluons included in $\mathcal{K}_{\mathrm{LL}}$.

Solution: kinematic constraint on rapidity evolution of the WW

- Kinematic improvement: impose both k_g^- and k_g^+ ordering (lifetime ordering).
  $\Longrightarrow$ Resum large transverse double logarithms to all orders.
  $\Longrightarrow$ Solve the instability of NLO BFKL or B-JIMWLK evolution.
  Kwiecinski, Martin, Stasto, 9703445, Salam, 9806482, Ciafaloni, Colferai, 9812366, Vera, 0505128, Beuf, 1401.0313, ...
- In practice, add an additional constraint in the LL evolution kernel

$$k_g^+ \geq k_f^+ \equiv \frac{Q_f^2}{2k_f^-} \Longrightarrow k_g^- \leq \frac{\boldsymbol{k}_{g\perp}^2}{Q_f^2} k_f^-$$

.

- With this modification $\mathcal{K}_{\mathrm{LL}} \to \mathcal{K}_{\mathrm{LL,coll}}$, one recovers the expected double logarithm.
  Taels, Altinoluk, Beuf, Marquet, 2204.11650

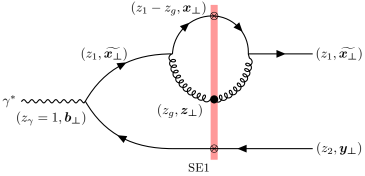
$$\mathrm{d}\sigma_{\mathrm{NLO}}^{\gamma^\star \to q\bar{q}+X} \sim \mathcal{H}(\boldsymbol{P}_\perp) \int \mathrm{d}^2 \boldsymbol{r}_{bb'} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}}$$

$$\times \left[1 - \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) - \frac{\alpha_s}{\pi} s_L \ln\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) + \alpha_s \mathcal{K}_{\mathrm{LL,coll}} \otimes\right] G_{\mathrm{WW}}(\boldsymbol{r}_{bb'}) + \mathcal{O}(\alpha_s)$$

- But: the single log coefficient looks weird, $s_L = -C_F \ln(z_1 z_2 R^2) - N_c \ln\left(\frac{z_f P_\perp^2}{z_1 z_2 c_0^2 Q_f^2}\right)$

NLO coefficient function from virtual graphs

- Back-to-back limit of virtual graphs are very challenging! Need to find a judicious expansion in coordinate space.



$$u_\perp = \widetilde{x_\perp} - y_\perp \sim r_\perp = z_\perp - x_\perp \ll b_\perp = z_1 \widetilde{x_\perp} + z_2 y_\perp$$

- In the end, the leading power contribution can be extracted and computed fully analytically!

$$\mathcal{H}^{\lambda=L,ij}_{\mathrm{NLO,se}} = \mathcal{H}^{ij} \int_0^{z_1} \frac{\mathrm{d}z_g}{z_g} \left(1 - \frac{z_g}{z_1} + \frac{z_g^2}{2z_1^2}\right) \left[-1 + \ln\left(\frac{1+\chi^2}{\chi^2}\right) - \ln\left(\frac{z_g}{z_2(z_1 - z_g)}\right)\right] - \int_0^{z_f} \frac{\mathrm{d}z_g}{z_g} \text{,,slow''}$$

- Byproduct: cancellation of $z_g \to 0$ singularity demands kinematic constraint **and**

$$\frac{k_f^+}{P^+} = \frac{1}{ec_0^2} \frac{M_{q\bar{q}}^2 + Q^2}{W^2 + Q^2}$$

Analytic results for NLO coefficient functions

- Gathering all diagrams together:

$$\mathcal{O}(\alpha_s) = \mathcal{H}^{ij} \times G^{ij}(\boldsymbol{q}_\perp) \times \left[\frac{\alpha_s N_c}{2\pi} f_1^{\lambda=L} + \frac{\alpha_s}{2\pi N_c} f_2^{\lambda=L}\right]$$

$$\begin{aligned} f_1^{\lambda=L}(\chi = Q/M_{q\bar{q}}, z_1, R) = {} & 7 - \frac{3\pi^2}{2} - \frac{3}{2}\ln\left(\frac{z_1 z_2 R^2}{\chi^2}\right) - \ln(z_1)\ln(z_2) + 2\ln\left(\frac{(1+\chi^2) z_f}{z_1 z_2}\right) \\ & - \ln(1+\chi^2)\ln\left(\frac{1+\chi^2}{z_1 z_2}\right) + \left\{ \mathrm{Li}_2\left(\frac{z_2 - z_1\chi^2}{z_2(1+\chi^2)}\right) - \frac{1}{4(z_2 - z_1\chi^2)} \right. \\ & \left. + \frac{(1+\chi^2)(z_2(2z_2 - z_1) + z_1(2z_1 - z_2)\chi^2)}{4(z_2 - z_1\chi^2)^2} \ln\left(\frac{z_2(1+\chi^2)}{\chi^2}\right) + (1 \leftrightarrow 2) \right\} \end{aligned}$$

- Similar expression for subleading $1/N_c$ term f_2.
- Two remaining issues:
  - A $Y_f = \ln(k_f^-/q^-)$ dependence remains in the NLO coefficient function.
  - Single Sudakov log $s_L = -C_F \ln(z_1 z_2 R^2) + N_c(-1 + \ln(1 + Q^2/M_{q\bar{q}}^2))$ does not match previous results from collinear calculation. Hatta, Xiao, Yuan, Zhou, 2106.05307

Matching the NLO coefficient function with target rapidity evolution

- Our kinematically constrained evolution equation: (using gaussian approximation)

$$\frac{\partial S_Y(\boldsymbol{r}_{bb'})}{\partial Y} = \bar{\alpha}_s \int \frac{\mathrm{d}^2 \boldsymbol{z}_\perp}{2\pi} \Theta\left(-Y - \ln(r_<^2 \mu_\perp^2)\right) \frac{\boldsymbol{r}_{bb'}^2}{\boldsymbol{r}_{zb}^2 \boldsymbol{r}_{zb'}^2} \left[S_Y(\boldsymbol{r}_{zb}) S_Y(\boldsymbol{r}_{zb'}) - S_Y(\boldsymbol{r}_{bb'})\right]$$

$(\mu_\perp \sim P_\perp ,\; r_<^2 = \min(\boldsymbol{r}_{zb}^2, \boldsymbol{r}_{zb'}^2))$

- Change of variable $\eta = Y + \ln(r_<^2 Q^2) - \ln(x_{\mathrm{Bj}}/x_0)$:

$$\frac{\partial S_\eta(\boldsymbol{r}_{bb'})}{\partial \eta} = \bar{\alpha}_s \int \frac{\mathrm{d}^2 \boldsymbol{z}_\perp}{2\pi} \Theta\left(\eta - \delta_{bb'z}\right) \frac{\boldsymbol{r}_{bb'}^2}{\boldsymbol{r}_{zb}^2 \boldsymbol{r}_{zb'}^2} \left[S_{\eta-\delta_{zb}}(\boldsymbol{r}_{zb}) S_{\eta-\delta_{zb'}}(\boldsymbol{r}_{zb'}) - S_\eta(\boldsymbol{r}_{bb'})\right]$$

- Recover result by Ducloué, Iancu, Mueller, Soyez, Triantafyllopoulos, 1902.06637+ NLO matching relation for the coefficient function (for $\eta = \eta_f = \ln(x_0/x_f)$)

$$Y_f = \underbrace{-\ln\left(\frac{\boldsymbol{r}_{bb'}^2 \boldsymbol{P}_\perp^2}{c_0^2}\right)}_{\text{cancel the -1 in } s_L!} + 1 - \ln\left(\frac{1 + Q^2/M_{q\bar{q}}^2}{z_1 z_2}\right)$$

Final TMD factorized result

$$\left\langle \mathrm{d}\sigma_{\mathrm{LO}}^{(0),\lambda} + \alpha_s \mathrm{d}\sigma_{\mathrm{NLO}}^{(0),\lambda} \right\rangle_{\eta_f} = \frac{1}{2}\mathcal{H}_{\mathrm{LO}}^{\lambda,ii} \int \frac{\mathrm{d}^2 \boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \hat{G}^0(x_f, \boldsymbol{r}_{bb'})$$

$$\times \left\{ 1 + \frac{\alpha_s}{\pi} \left[-\frac{N_c}{4} \ln^2 \left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2} \right) - s_L \ln \left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2} \right) + \beta_0 \ln \left(\frac{\mu_R^2 \boldsymbol{r}_{bb'}^2}{c_0^2} \right) \right.\right.$$

$$\left.\left. + \frac{N_c}{2} f_1^\lambda(Q/M_{q\bar{q}}, z_1, R) + \frac{1}{2N_c} f_2^\lambda(Q/M_{q\bar{q}}, z_1, R) \right] \right\}$$

$$+ \frac{\alpha_s}{2\pi}\mathcal{H}_{\mathrm{LO}}^{\lambda,ii} \int \frac{\mathrm{d}^2 \boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \hat{h}^0(x_f, \boldsymbol{r}_{bb'}) \left\{ \frac{N_c}{2} \left[1 + \ln(R^2) \right] - \frac{1}{2N_c} \ln(z_1 z_2 R^2) \right\}$$

- $x_f = x_{\mathrm{Bj}}(M_{q\bar{q}}^2 + Q^2)/(e c_0^2 Q^2)$ dependence of TMD given by k-c. k_g^+ ordered non-linear evolution. Saturation corrections $\mathcal{O}(Q_s/q_\perp)$ fully included in this dependence!
- First line should be exponentiated (?) to resum large double and single Sudakov logs.
- $s_L = -C_F \ln(z_1 z_2 R^2) + N_c \ln(1 + Q^2/M_{q\bar{q}}^2) \Rightarrow$ agreement with collinear calculations.

  Hatta, Xiao, Yuan, Zhou, 2106.05307
- Last line: dependence on linearly polarized WW, due to real soft gluon radiation.

Summary

- First proof of WW gluon TMD factorization at NLO at small x: non trivial because of "all twist" $Q_s/q_\perp$ power corrections.

- TMD factorization and isolation of Sudakov logs demand kinematic constraint + target rapidity small x evolution.

- First calculation of Sudakov single log for this process at small x, agreement with collinear calculations.

- We postulate exponentiation of Sudakov logs à la CSS, a rigorous proof will require to go beyond our one-loop computation

- Analytic calculation of NLO hard factors (transverse and longitudinal) $\rightarrow$ same as in TMD calculations?

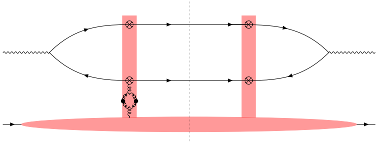
- Very fast numerical evaluation $\Rightarrow$ numerical study included kinematically constrained BK equation to be published soon!

Back-up slide

The single log proportional to β_0

- At NLO, quantum correction to the classical field: $\boldsymbol{A}^i_\perp = \boldsymbol{A}^{i,(0)}_\perp + \underbrace{\boldsymbol{A}^{i,(1)}_\perp}_{\mathcal{O}(\alpha_s)}$

  Gelis, Venugopalan, 0601209



- We have (see Ayala, Jalilian-Marian, McLerran, Venugopalan, 9508302)

$$\boldsymbol{A}^{i,(1)}_\perp = \frac{\alpha_s N_c}{\pi}\beta_0[1/\varepsilon + \mathrm{finite}]\boldsymbol{A}^{i,(0)}_\perp \tag{1}$$

- UV divergence removed by renormalization $\Rightarrow$ renormalization scale dependence of the WW gluon TMD: See also Zhou, 1807.00506

$$\frac{\partial \hat{G}_Y(\boldsymbol{r}_{bb'},\mu)}{\partial \ln(\mu)} = \alpha_s\beta_0 \times \hat{G}_Y(\boldsymbol{r}_{bb'},\mu)\,. \tag{2}$$