

Transverse momentum-dependent parton distributions
for longitudinally polarized nucleons
from Lattice QCD

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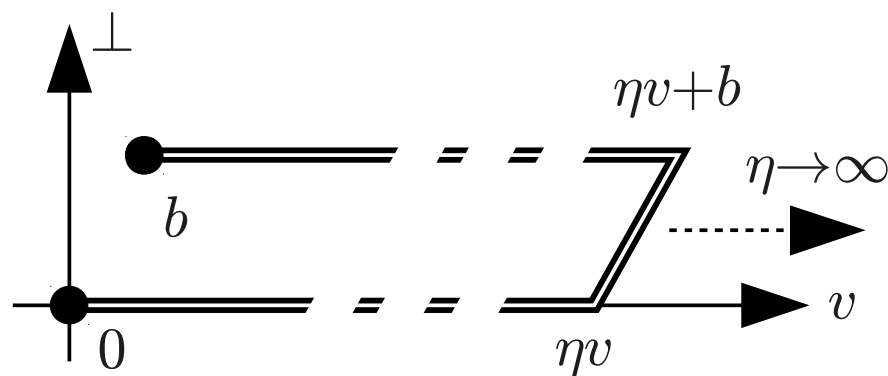
Fundamental TMD correlator

$$\tilde{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \dots) \equiv \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \dots, b] q(b) | P, S \rangle$$

$$\Phi^{[\Gamma]}(x, k_T, P, S, \dots) \equiv \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b \cdot P)}{(2\pi) P^+} \exp(i x (b \cdot P) - i b_T \cdot k_T) \frac{\tilde{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \dots)}{\tilde{\mathcal{S}}(b^2, \dots)} \Big|_{b^+=0}$$

- “Soft factor” $\tilde{\mathcal{S}}$ required to subtract divergences of Wilson line \mathcal{U}
- $\tilde{\mathcal{S}}$ is typically a combination of vacuum expectation values of Wilson line structures
- Here, will consider only ratios in which soft factors cancel

Gauge link structure



Beyond tree level: Rapidity divergences suggest taking staple direction slightly off the light cone. Approach of Aybat, Collins, Qiu, Rogers makes v space-like. Parametrize in terms of Collins-Soper parameter

$$\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$$

Light-like staple for $\hat{\zeta} \rightarrow \infty$. Perturbative evolution equations for large $\hat{\zeta}$.

Decomposition of Φ into TMDs

All leading twist structures:

$$\Phi[\gamma^+] = f_1 - \left[\frac{\epsilon_{ij} k_i S_j}{m_H} f_{1T}^\perp \right] \text{odd}$$

$$\Phi[\gamma^+ \gamma^5] = \Lambda g_1 + \frac{k_T \cdot S_T}{m_H} g_{1T}$$

$$\Phi[i\sigma^{i+} \gamma^5] = S_i h_1 + \frac{(2k_i k_j - k_T^2 \delta_{ij}) S_j}{2m_H^2} h_{1T}^\perp + \frac{\Lambda k_i}{m_H} h_{1L}^\perp + \left[\frac{\epsilon_{ij} k_j}{m_H} h_1^\perp \right] \text{odd}$$

TMD Classification

All leading twist structures:

$q \rightarrow$ H ↓	U	L	T	
U	f_1		h_1^\perp	← Boer-Mulders (T-odd)
L		g_1	h_{1L}^\perp	
T	f_{1T}^\perp	g_{1T}	$h_1 \quad h_{1T}^\perp$	

↑
Sivers (T-odd)

Decomposition of $\tilde{\Phi}$ into amplitudes

$$\tilde{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \hat{\zeta}, \mu) \equiv \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \eta v, \eta v + b, b] q(b) | P, S \rangle$$

Decompose in terms of invariant amplitudes; at leading twist,

$$\begin{aligned} \frac{1}{2P^+} \tilde{\Phi}_{\text{unsubtr.}}^{[\gamma^+]} &= \tilde{A}_{2B} + im_H \epsilon_{ij} b_i S_j \tilde{A}_{12B} \\ \frac{1}{2P^+} \tilde{\Phi}_{\text{unsubtr.}}^{[\gamma^+ \gamma^5]} &= -\Lambda \tilde{A}_{6B} + i[(b \cdot P)\Lambda - m_H(b_T \cdot S_T)] \tilde{A}_{7B} \\ \frac{1}{2P^+} \tilde{\Phi}_{\text{unsubtr.}}^{[i\sigma^{i+} \gamma^5]} &= im_H \epsilon_{ij} b_j \tilde{A}_{4B} - S_i \tilde{A}_{9B} \\ &\quad - im_H \Lambda b_i \tilde{A}_{10B} + m_H[(b \cdot P)\Lambda - m_H(b_T \cdot S_T)] b_i \tilde{A}_{11B} \end{aligned}$$

(Decompositions analogous to work by Metz et al. in momentum space)

Fourier-transformed TMDs

$$\tilde{f}(x, b_T^2, \dots) \equiv \int d^2 k_T \exp(ib_T \cdot k_T) f(x, k_T^2, \dots)$$

$$\tilde{f}^{(n)}(x, b_T^2, \dots) \equiv n! \left(-\frac{2}{m_H^2} \partial_{b_T^2} \right)^n \tilde{f}(x, b_T^2, \dots)$$

In limit $|b_T| \rightarrow 0$, recover k_T -moments:

$$\tilde{f}^{(n)}(x, 0, \dots) \equiv \int d^2 k_T \left(\frac{k_T^2}{2m_H^2} \right)^n f(x, k_T^2, \dots) \equiv f^{(n)}(x)$$

ill-defined for large k_T , so will not attempt to extrapolate to $b_T = 0$, but give results at finite $|b_T|$.

In this study, only consider first x -moments (accessible at $b \cdot P = 0$), rather than scanning range of $b \cdot P$:

$$f^{[1]}(k_T^2, \dots) \equiv \int_{-1}^1 dx f(x, k_T^2, \dots)$$

→ [Bessel-weighted asymmetries](#) (Boer, Gamberg, Musch, Prokudin, JHEP 1110 (2011) 021)

Relation between Fourier-transformed TMDs and invariant amplitudes \tilde{A}_i

Invariant amplitudes directly give selected x -integrated TMDs in Fourier (b_T) space, up to soft factors:

$$\tilde{f}_1^{[1](0)}(b_T^2, \hat{\zeta}, \dots, \eta v \cdot P) = 2\tilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P) / \tilde{S}(b^2, \dots)$$

$$\tilde{g}_1^{[1](0)}(b_T^2, \hat{\zeta}, \dots, \eta v \cdot P) = -2\tilde{A}_{6B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P) / \tilde{S}(b^2, \dots)$$

$$\tilde{h}_{1L}^{\perp1}(b_T^2, \hat{\zeta}, \dots, \eta v \cdot P) = -2\tilde{A}_{10B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P) / \tilde{S}(b^2, \dots)$$

Generalized shifts

Form ratios in which soft factors, (Γ -independent) multiplicative renormalization factors cancel

h_{1L}^\perp worm gear shift:

$$\langle k_x \rangle_{LT} \equiv m_N \frac{\tilde{h}_{1L}^{\perp1}}{\tilde{f}_1^{[1](0)}} = \frac{\int dx \int d^2 k_T k_x \Phi^{[s^j i \sigma^{j+} \gamma^5]}(x, k_T, P, S, \dots)}{\int dx \int d^2 k_T \Phi^{[\gamma^+]}}(x, k_T, P, S, \dots) \Big|_{\Lambda=1, s_T=(1,0)}$$

Average transverse momentum of quarks polarized in the same transverse (“ T ”) direction in a longitudinally (“ L ”) polarized nucleon; normalized to the number of valence quarks. “Dipole moment” in $b_T^2 = 0$ limit, “shift”.

Issue: k_T -moments in this ratio singular; generalize to ratio of Fourier-transformed TMDs at *nonzero* b_T^2 ,

$$\langle k_x \rangle_{LT}(b_T^2, \dots) \equiv m_N \frac{\tilde{h}_{1L}^{\perp1}(b_T^2, \dots)}{\tilde{f}_1^{[1](0)}(b_T^2, \dots)}$$

(remember singular $b_T \rightarrow 0$ limit corresponds to taking k_T -moment). “Generalized shift”.

Generalized shifts from amplitudes

Now, can also express this in terms of invariant amplitudes:

$$\langle k_x \rangle_{LT}(b_T^2, \dots) \equiv m_N \frac{\tilde{h}_{1L}^{\perp1}(b_T^2, \dots)}{\tilde{f}_1^{[1](0)}(b_T^2, \dots)} = -m_N \frac{\tilde{A}_{10B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\tilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Analogously for generalized axial charge (here, no k_T weighting):

$$\Delta\Sigma(b_T^2, \dots) = \frac{\tilde{g}_1^{[1](0)}(b_T^2, \dots)}{\tilde{f}_1^{[1](0)}(b_T^2, \dots)} = -\frac{\tilde{A}_{6B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\tilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Lattice setup

- Evaluate directly $\tilde{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \hat{\zeta}, \mu) \equiv \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \eta v, \eta v + b, b] q(b) | P, S \rangle$
- Euclidean time: Place entire operator at one time slice, i.e., $b, \eta v$ purely spatial
- Since generic b, v space-like, no obstacle to boosting system to such a frame!
- **Parametrization of correlator in terms of \tilde{A}_i invariants** permits direct translation of results back to original frame
- Form desired ratios of \tilde{A}_i invariants
- Extrapolate $\eta \rightarrow \infty, \hat{\zeta} \rightarrow \infty$ numerically.
- Use variety of $P, b, \eta v$; **here $b \perp P, b \perp v$** (lowest x -moment, kinematical choices/constraints)

Ensemble details

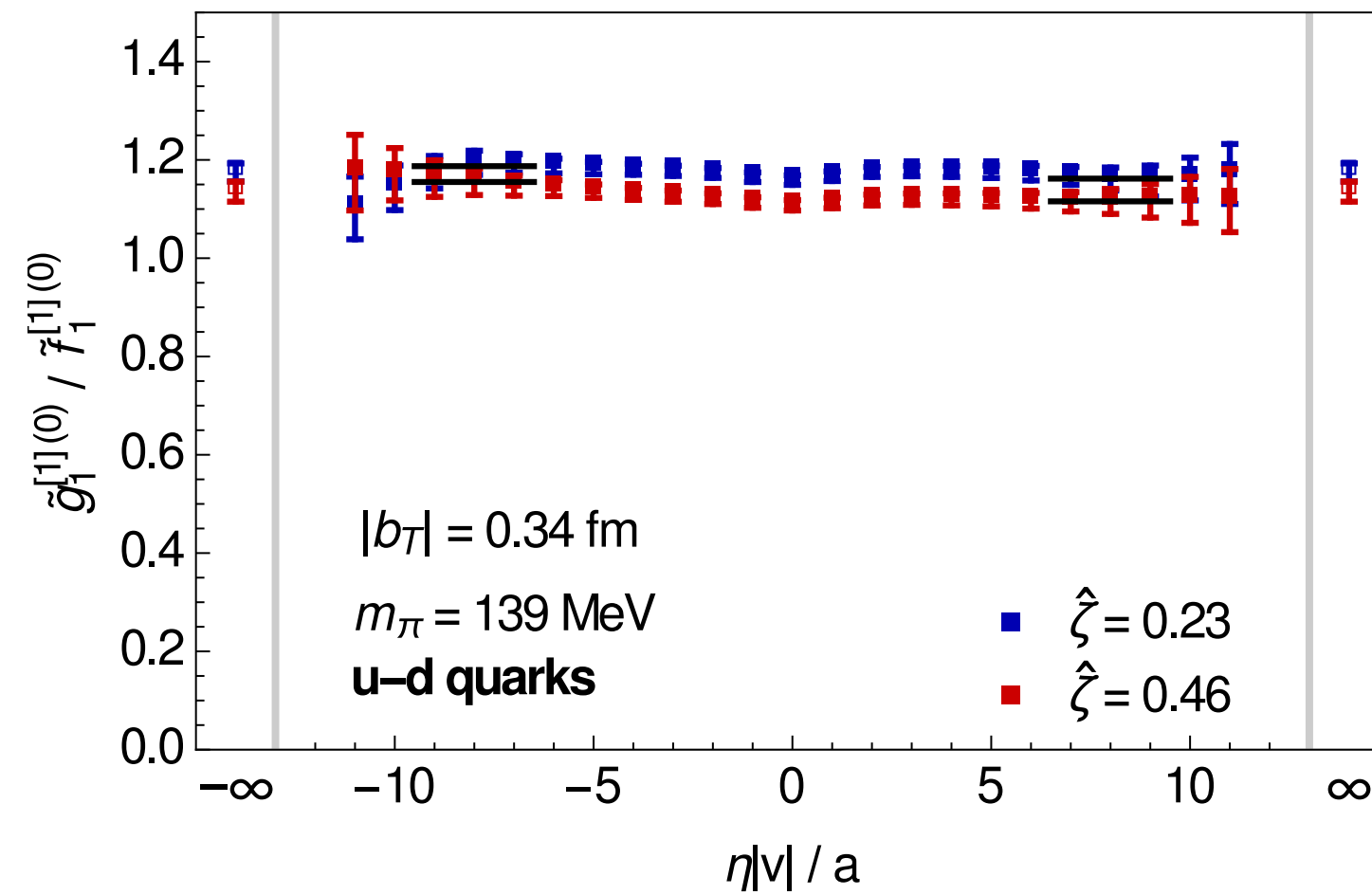
DWF ensemble provided by the RBC/UKQCD collaboration

$L^3 \times T$	$a(\text{fm})$	m_π (MeV)	#conf.	#meas.
$48^3 \times 96$	0.114	139	130	(33280 sloppy + 520 exact) AMA

WARNING: Data obtained at one fairly small source-sink separation, $8a = 0.91$ fm
→ significant systematic uncertainty due to excited state contamination!

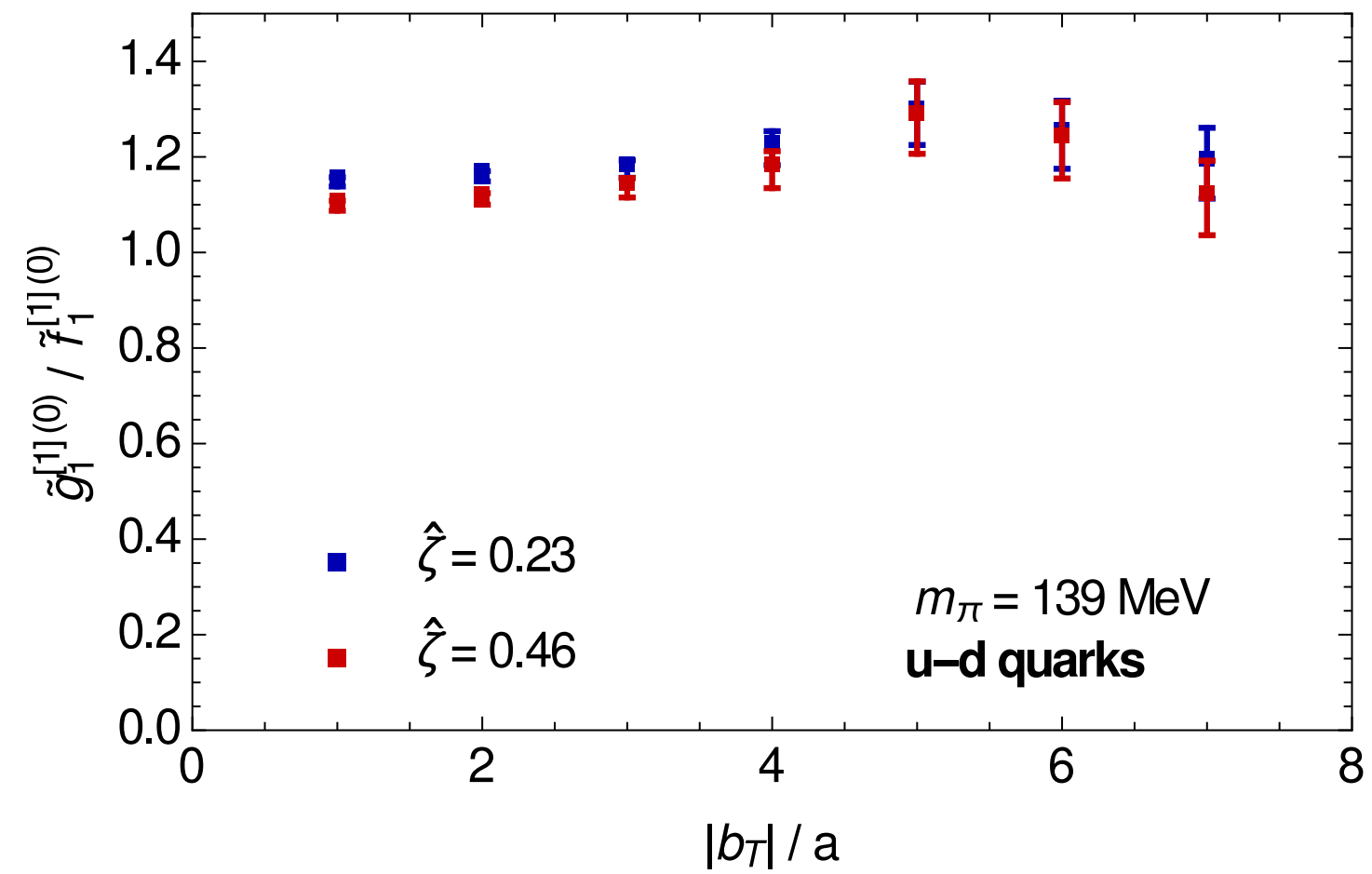
Results: Generalized axial charge

Dependence on staple extent, at $|b_T| = 0.34$ fm



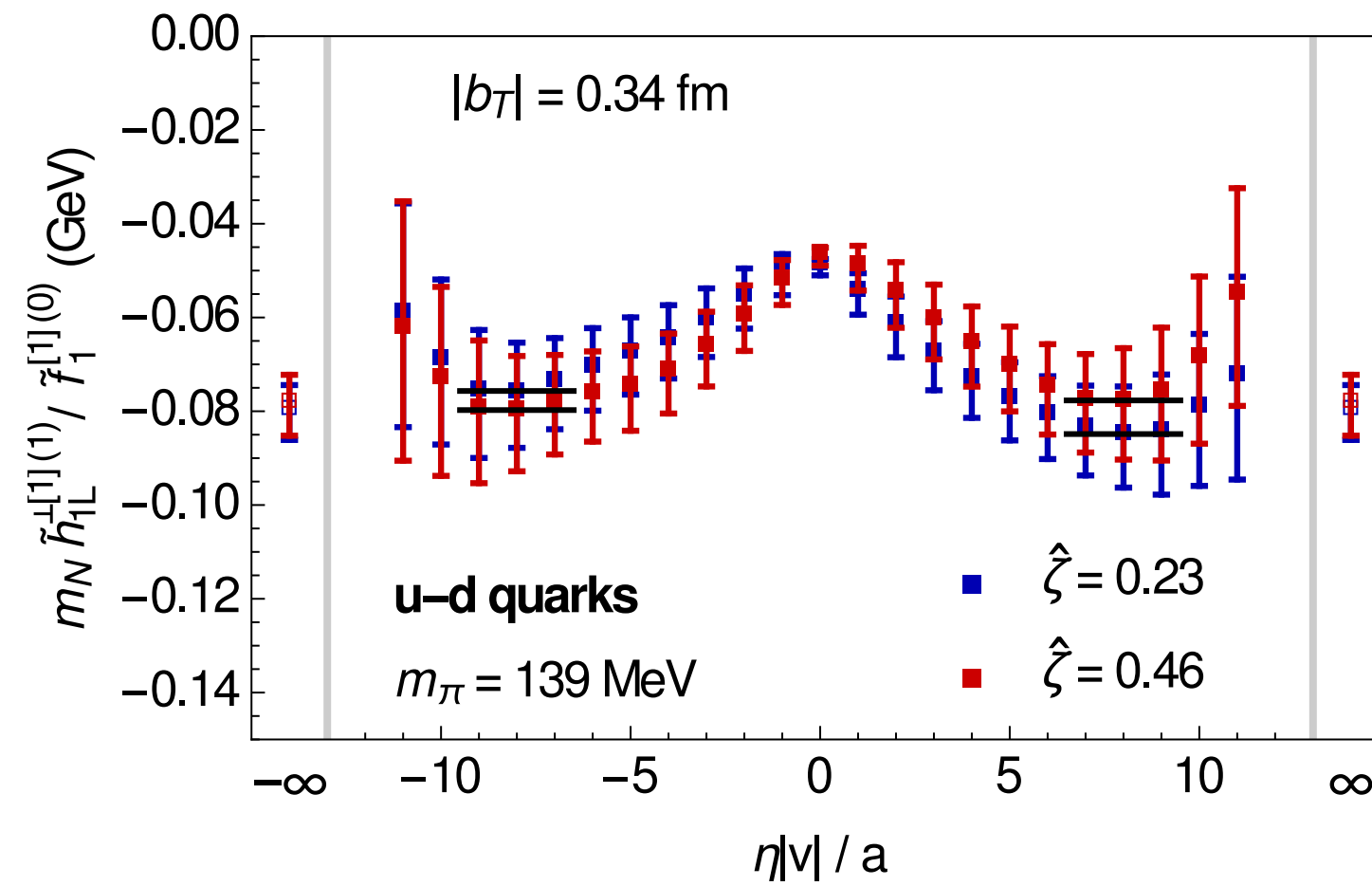
Results: Generalized axial charge

Dependence of SIDIS/DY limit on $|b_T|$



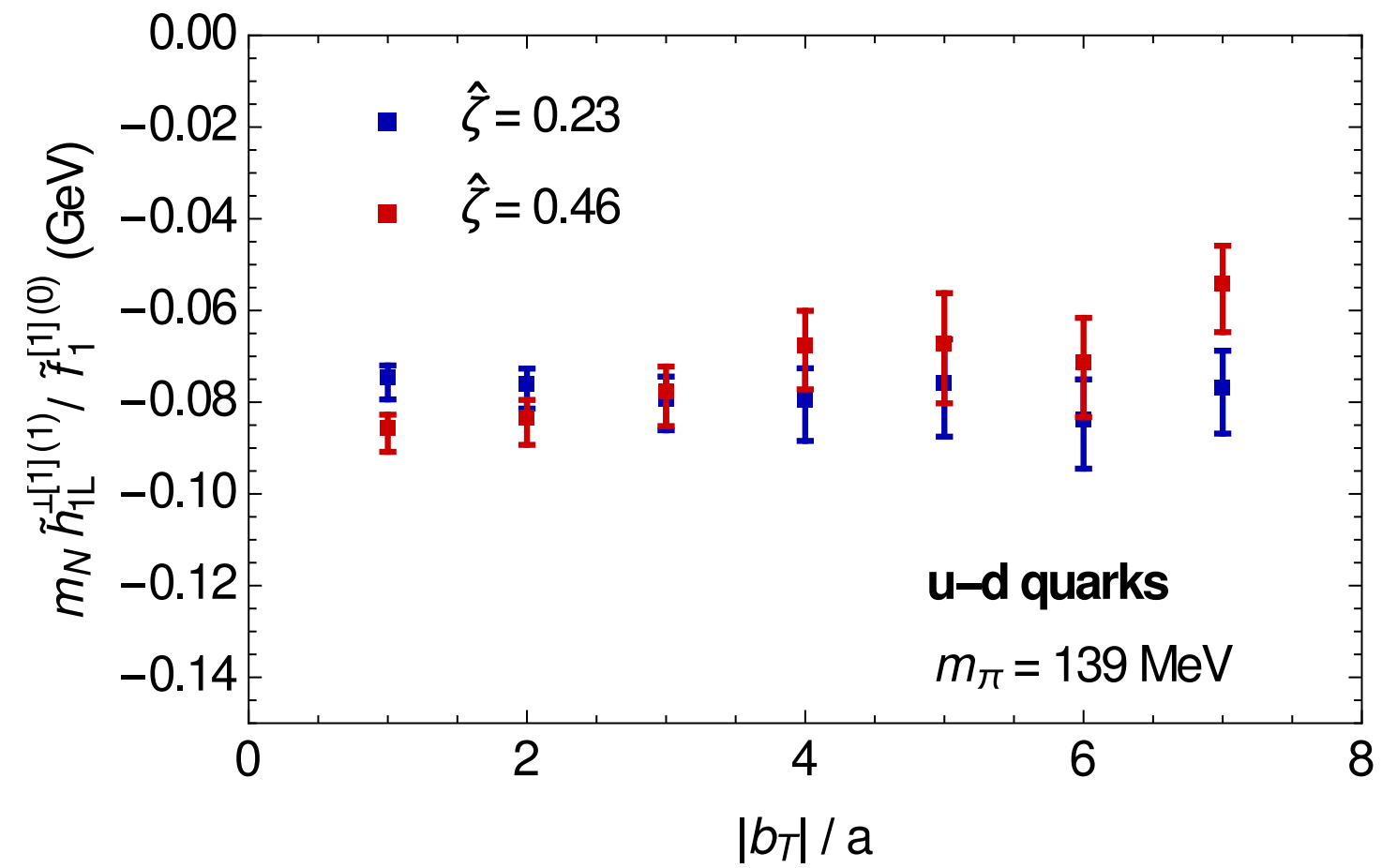
Results: h_{1L}^\perp worm gear shift

Dependence on staple extent, at $|b_T| = 0.34$ fm



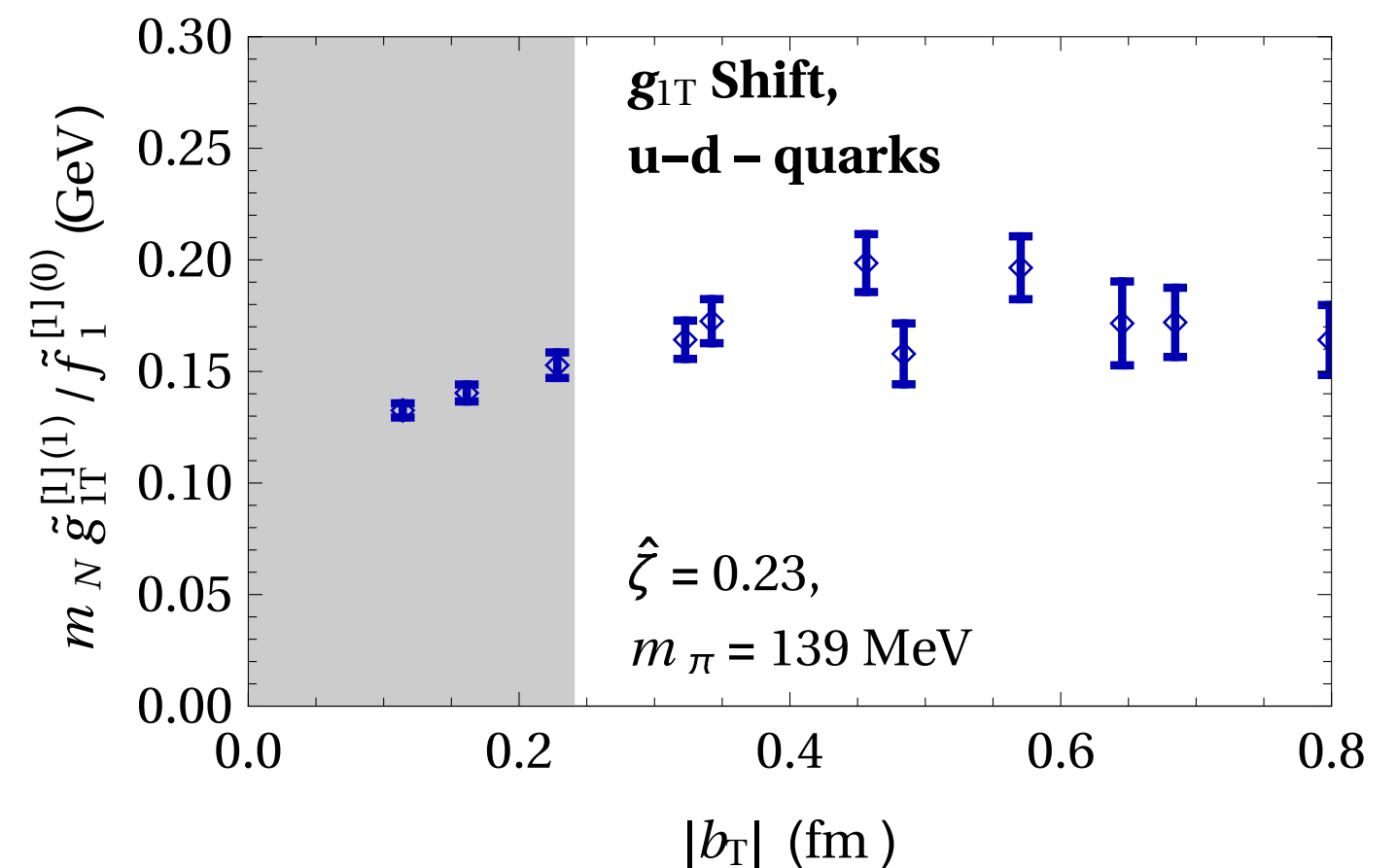
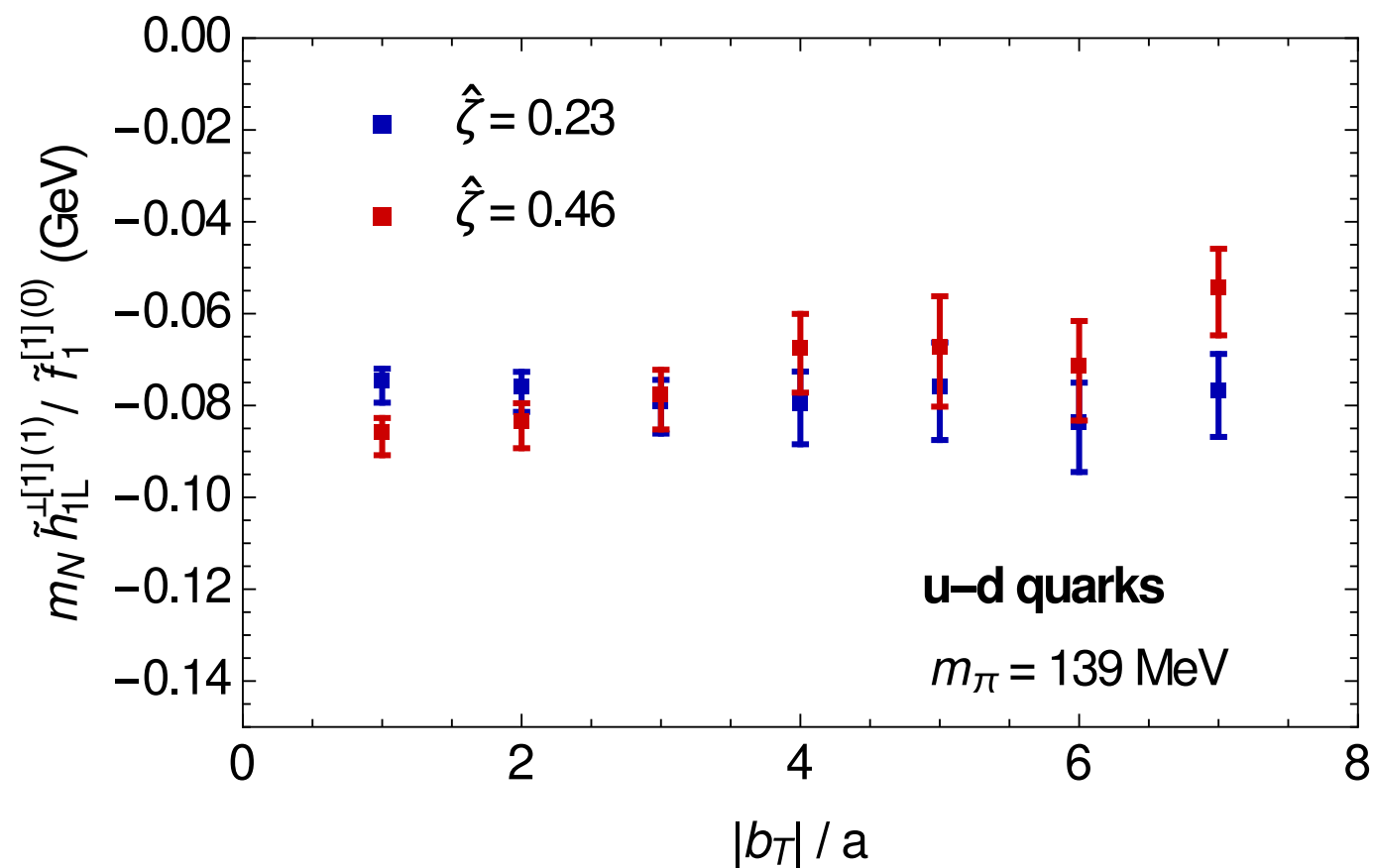
Results: h_{1L}^\perp worm gear shift

Dependence of SIDIS/DY limit on $|b_T|$



Comparison: h_{1L}^\perp vs. g_{1T} worm gear shift

SIDIS/DY limit as a function of $|b_T|$



A wide variety of models predicts h_{1L}^\perp and g_{1T} to have the same magnitude (and opposite sign):

Spectator model, light-front constituent quark model, covariant parton model, bag model, light-front quark-diquark model, light-front version of the chiral quark-soliton model, nonrelativistic quark model ...

Significant QCD effects not captured by quark models!

Conclusions

- Calculations of TMD observables using bilocal quark operators with staple-shaped gauge link structures have reached the physical pion mass.
- To avoid soft factors, multiplicative renormalization constants, considered appropriate ratios of Fourier-transformed TMDs (“shifts”).
- Here, observables in longitudinally polarized nucleons are extracted: generalized axial charge and h_{1L}^\perp worm gear shift.
- h_{1L}^\perp worm gear shift considerably smaller in magnitude compared to its counterpart g_{1T} worm gear shift, exhibiting significant QCD effects not captured by a wide variety of commonly used quark models.