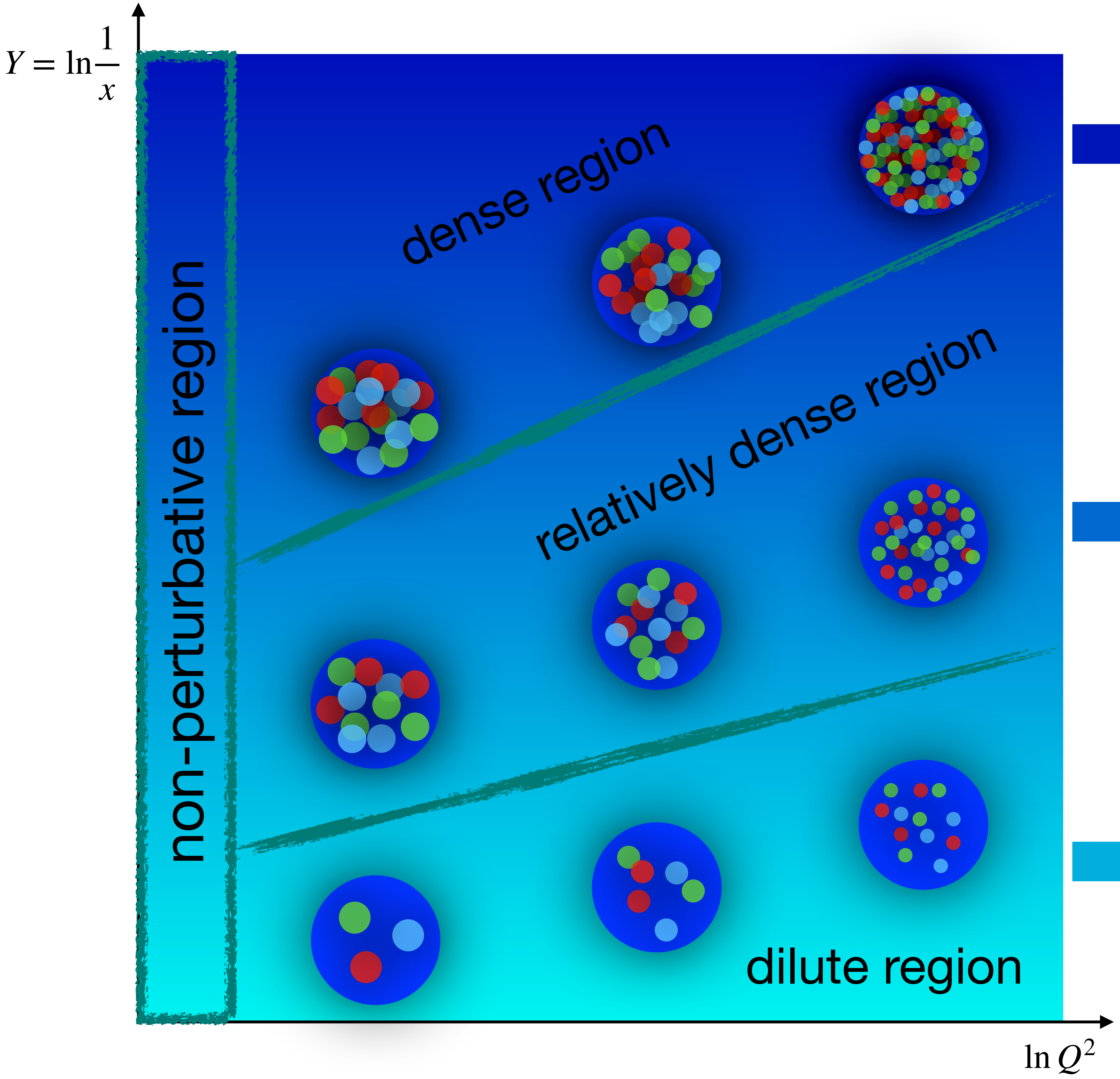
A unified picture for dilute-dense dynamics in QCD medium

Hongxi Xing

Institute of Quantum Matter
South China Normal University

QCD Evolution 2023

QCD “phase diagram” for nuclei from dilute to dense region



Dense region: $x \ll \mathcal{O}(1)$

Probing length $\lambda \sim 1/xp \gg L \sim A^{1/3}$

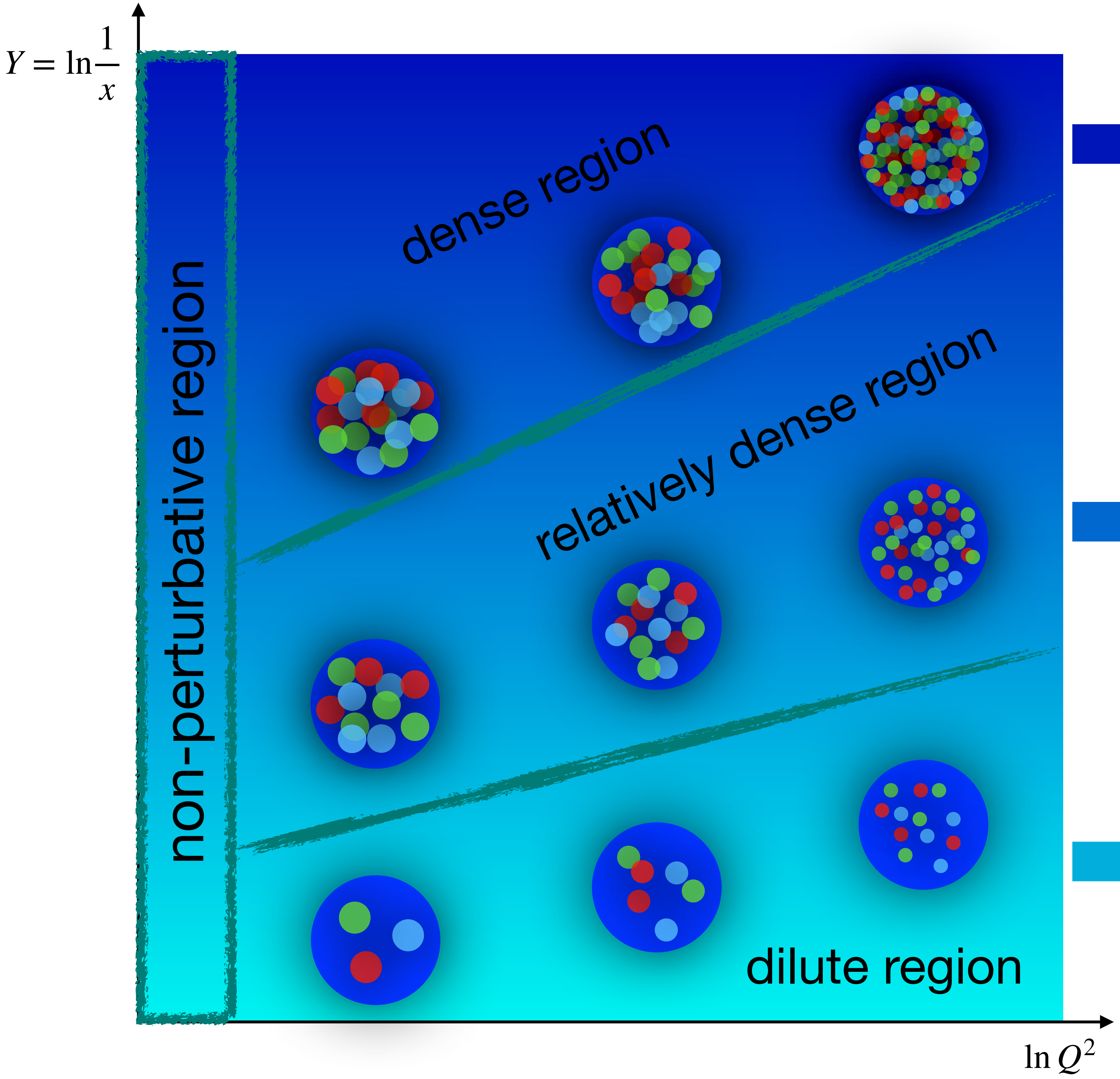
Relatively dense region: $x \lesssim \mathcal{O}(1)$

Probing length $\lambda \sim 1/xp \lesssim L \sim A^{1/3}$

Dilute region: $x \sim \mathcal{O}(1)$

Probing length $\lambda \sim 1/xp \ll L \sim A^{1/3}$

QCD theoretical frameworks from dilute to dense region



Color Glass Condensate (CGC)

Wilson lines, nonlinear BK/JIMWLK evolution

See review: Gelis, Iancu, Venugopalan, 2003

High-twist formalism

Multi-parton correlation, DGLAP-type evolution

Qiu, Stermann, 1991

Kang, Wang, Wang, Xing, 2014

Leading twist collinear factorization

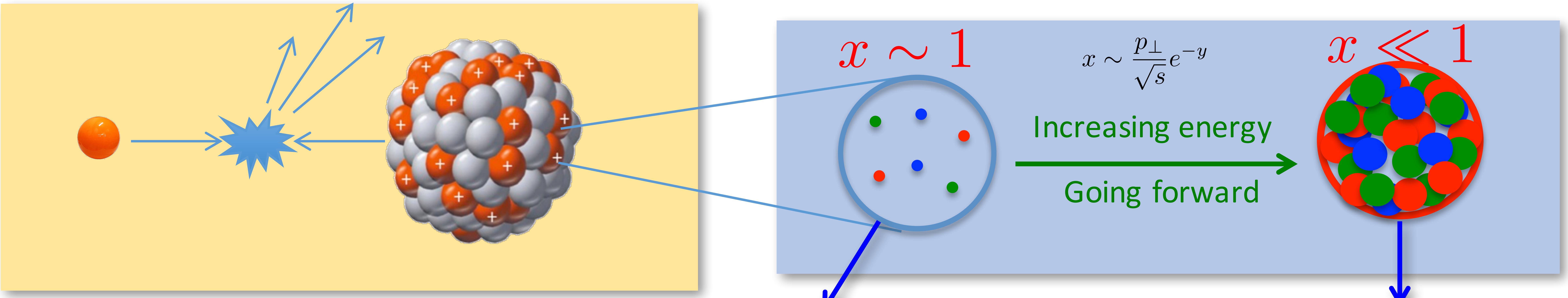
PDF, DGLAP evolution

Collins, Soper, 1981

Scan the phase diagram in proton-nucleus collisions

- Multiple scattering in dilute and dense medium

Probing length: $\lambda \sim \dfrac{1}{xp}$



- A dilute system
- Probes interact independently

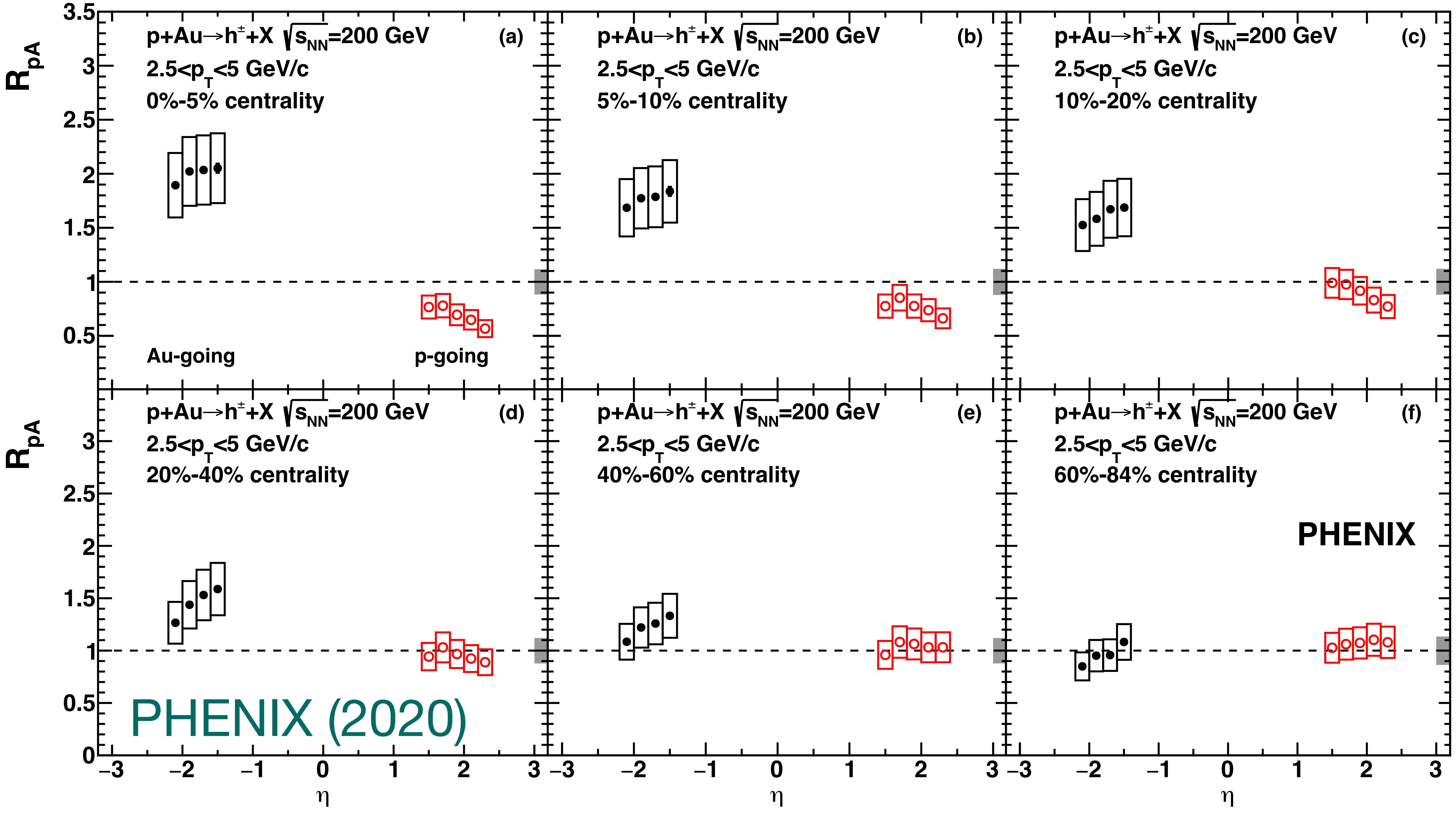
- A dense system
- Probes interact coherently

A ⟵ Backward

Forward ⟶ P

Scan the phase diagram in proton-nucleus collisions

- Experimental phenomena in dilute and dense medium



Nuclear modification factor

$$R_{pA} = \frac{\sigma_{pA}}{\sigma_{pp}}$$

dilute region: enhancement

dense region: suppression

$$x \sim \frac{p_\perp}{\sqrt{s}} e^{-y}$$

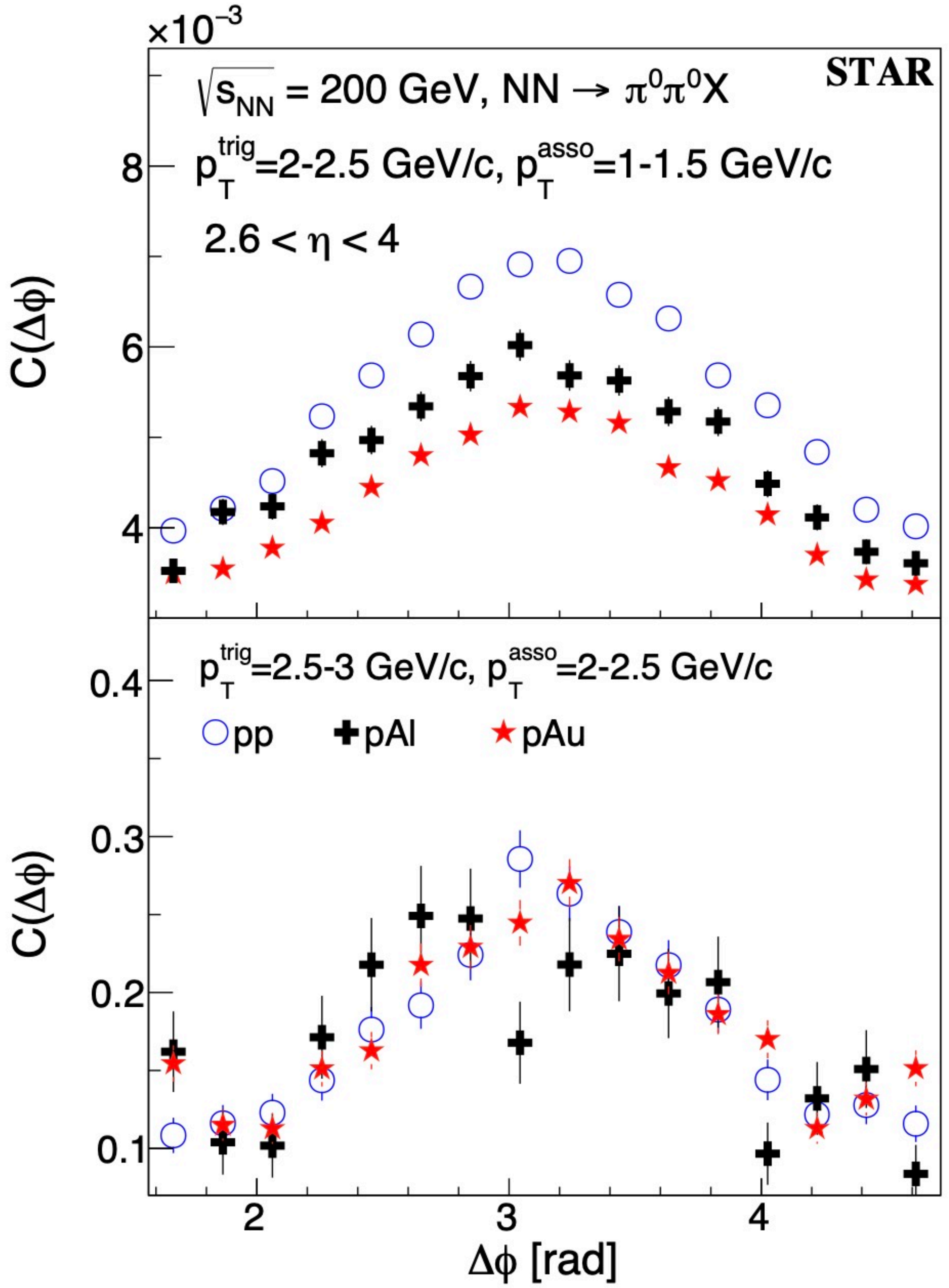
Evidence of CGC?

PHYSICAL REVIEW LETTERS

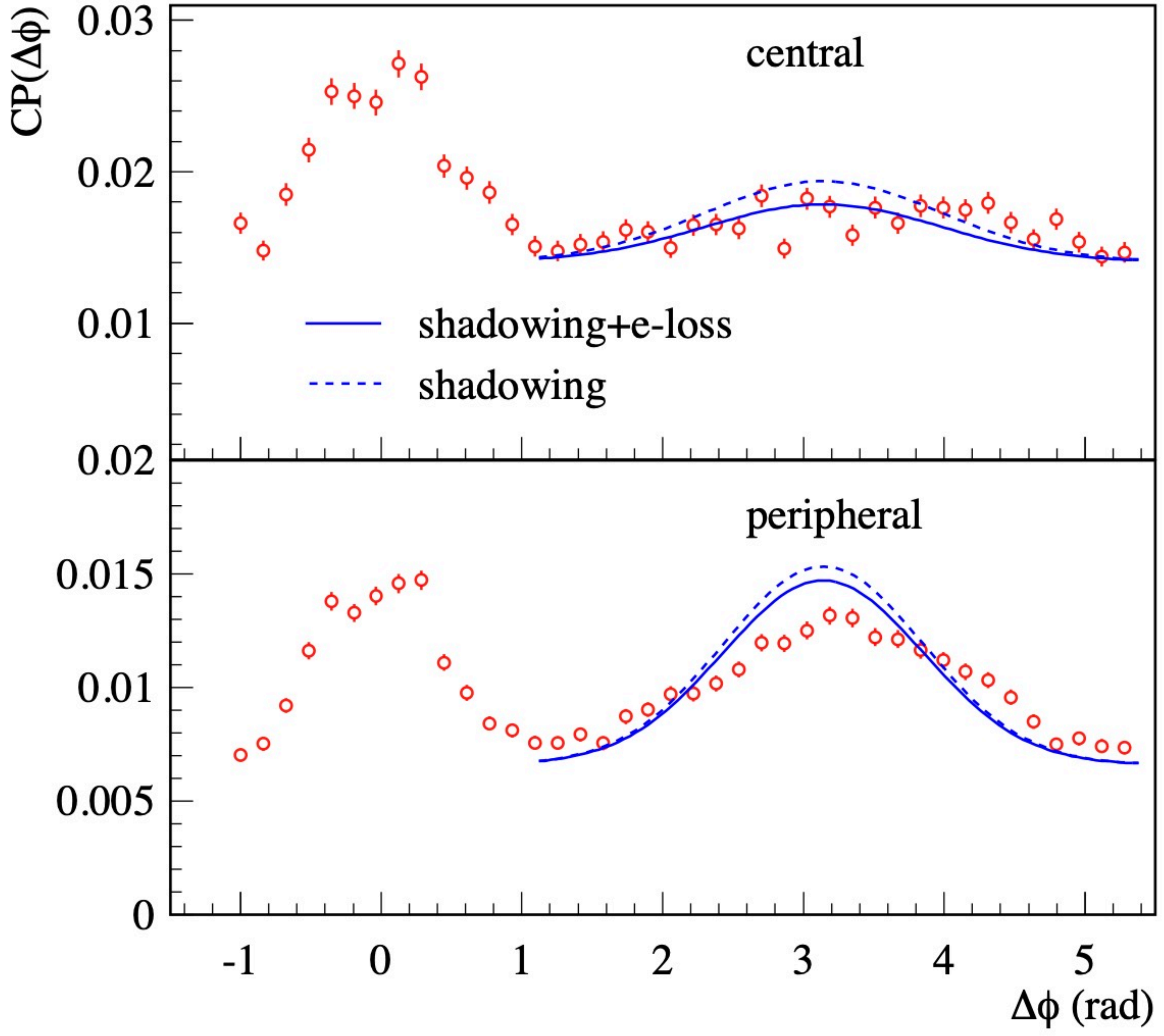
Highlights Recent Accepted Collections Authors Referees Search Press

Evidence for Nonlinear Gluon Effects in QCD and Their Mass Number Dependence at STAR

M. S. Abdallah *et al.* (STAR Collaboration)
Phys. Rev. Lett. **129**, 092501 – Published 22 August 2022



Qiu, Vitev, PRL, 2004
Kang, Vitev, HX, PRD, 2012



- High-twist calculation also explain the data
- Which framework is correct?

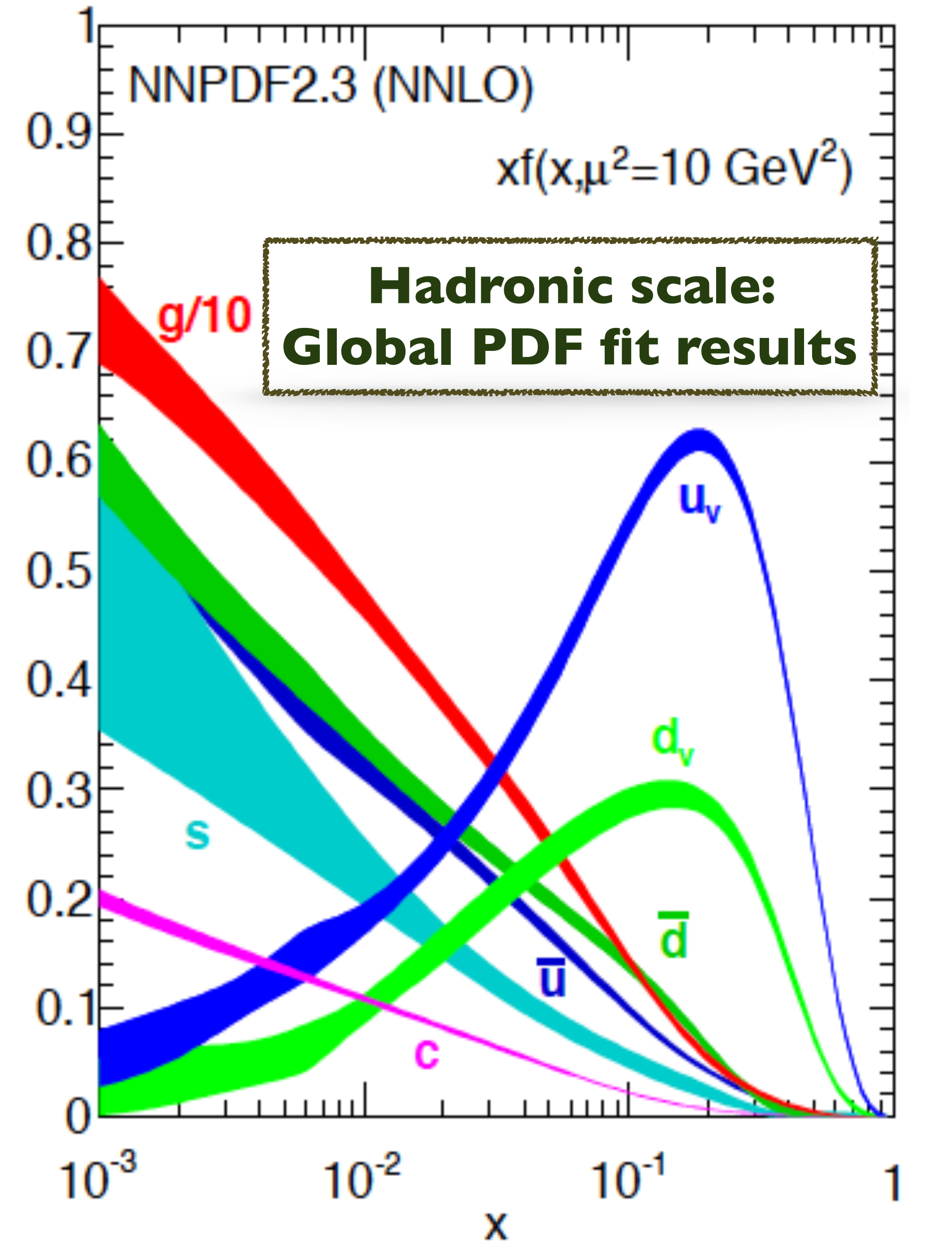
Theoretical framework for multiple scattering expansion

- Generalized factorization theorem

perturbative expansion

$$\sigma^h_{phys} = \left[\alpha_s^0 C_2^{(0)} + \alpha_s^1 C_2^{(1)} + \alpha_s^2 C_2^{(2)} + \ldots\right] \otimes T_2(x)$$
$$+ \frac{1}{Q}\left[\alpha_s^0 C_3^{(0)} + \alpha_s^1 C_3^{(1)} + \alpha_s^2 C_3^{(2)} + \ldots\right] \otimes T_3(x)$$
$$+ \frac{1}{Q^2}\left[\alpha_s^0 C_4^{(0)} + \alpha_s^1 C_4^{(1)} + \alpha_s^2 C_4^{(2)} + \ldots\right] \otimes T_4(x)$$
$$+ \ldots$$

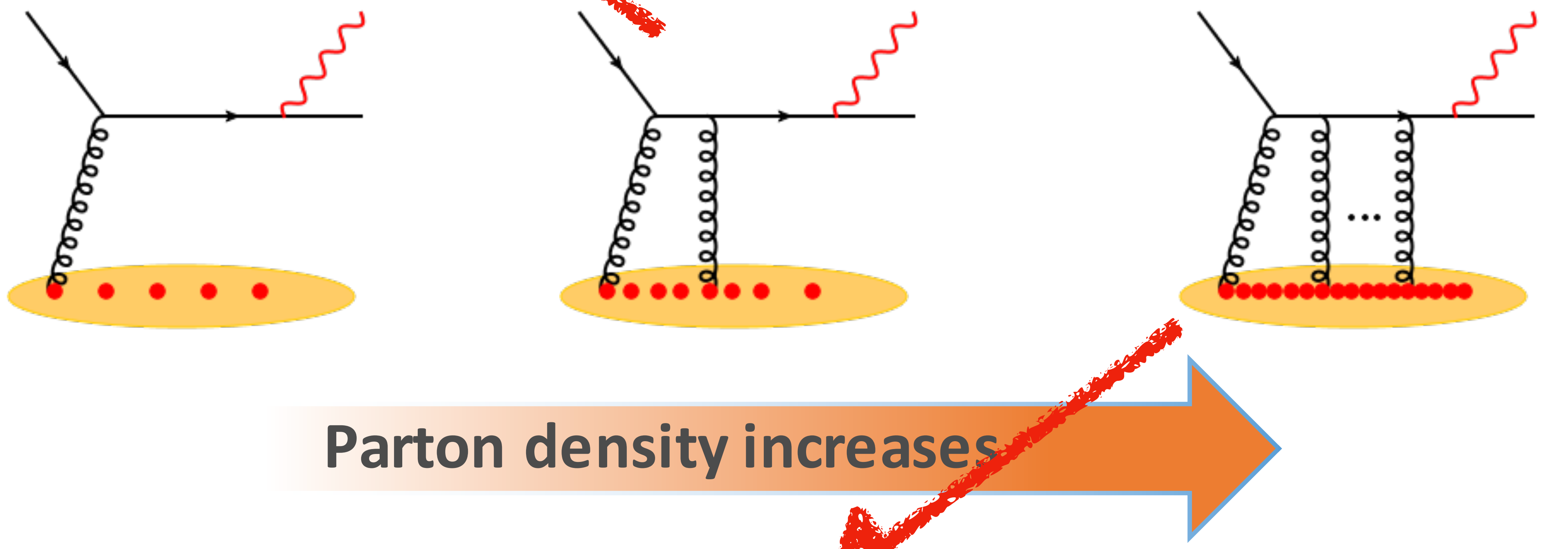
Multiple scattering expansion



twist-4 expansion

- Nuclear enhanced power correction

$$\frac{1}{Q^2} \to \frac{A^{1/3}}{Q^2}$$

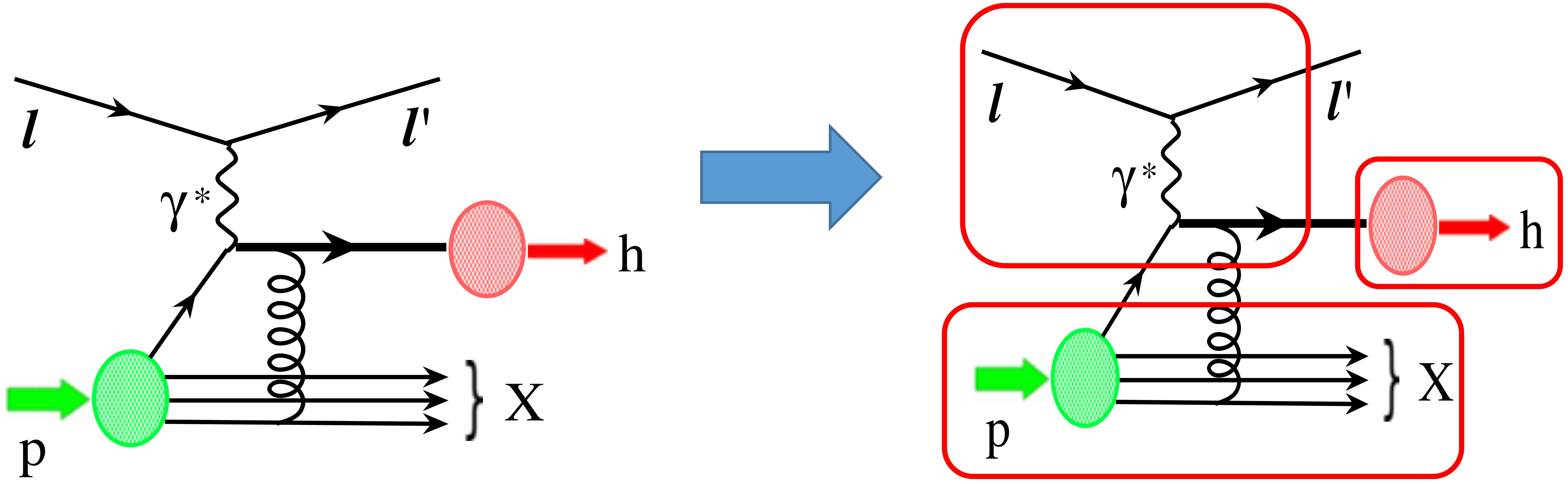


CGC: sum of all multiple scatterings

Incoherent multiple scattering - from dilute to relative dense

- QCD factorization at twist-4

Qiu, Sterman, 1991; Luo, Qiu, Sterman, 1993
Kang, Wang, Wang, **HX,** PRL 2014



$$\frac{d\langle \ell_{hT}^2 \sigma\rangle}{dz_h} \propto \quad D_{q/h}(z,\mu^2)\otimes H^{LO}(x,z)\otimes T_{qg}(x,0,0,\mu^2)$$
$$+\frac{\alpha_s}{2\pi} D_{q/h}(z,\mu^2)\otimes H^{NLO}(x,z,\mu^2)\otimes T_{qg(gg)}(x,0,0,\mu^2)$$

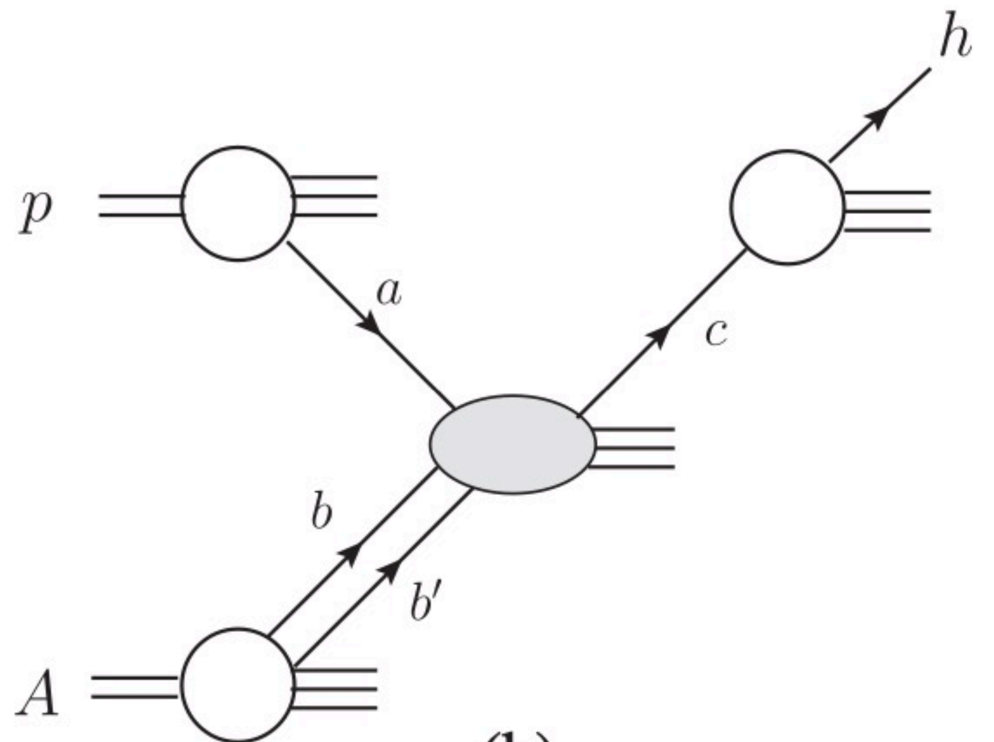
Multiple scattering hard probe and medium properties can be factorized!!!

Incoherent multiple scattering - from dilute to relative dense

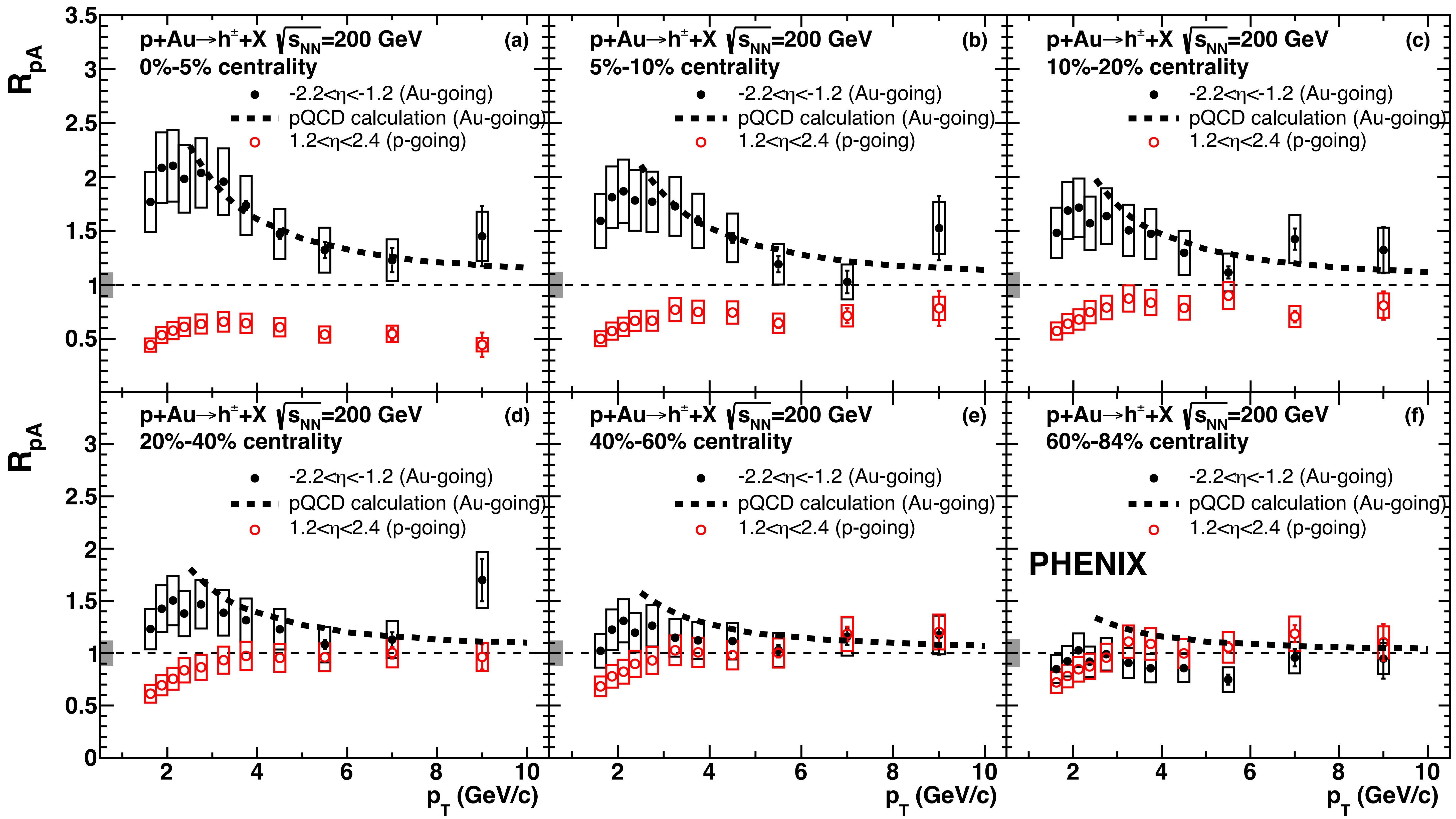
- Enhancement from twist-4 contribution

$$E_h \frac{d\sigma^{(D)}}{d^3P_h} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\right)\frac{\alpha_s^2}{S}\sum_{a,b,c}\int\frac{dz}{z^2}D_{c\to h}(z)\int\frac{dx'}{x'}f_{a/p}(x')\int\frac{dx}{x}\delta(\hat{s}+\hat{t}+\hat{u})$$

$$\times\sum_{i=I,F}\left[x^2\frac{\partial^2 T^{(i)}_{b/A}(x)}{\partial x^2} - x\frac{\partial T^{(i)}_{b/A}(x)}{\partial x} + T^{(i)}_{b/A}(x)\right] c^i H^i_{ab\to cd}(\hat{s},\hat{t},\hat{u})$$



(b)

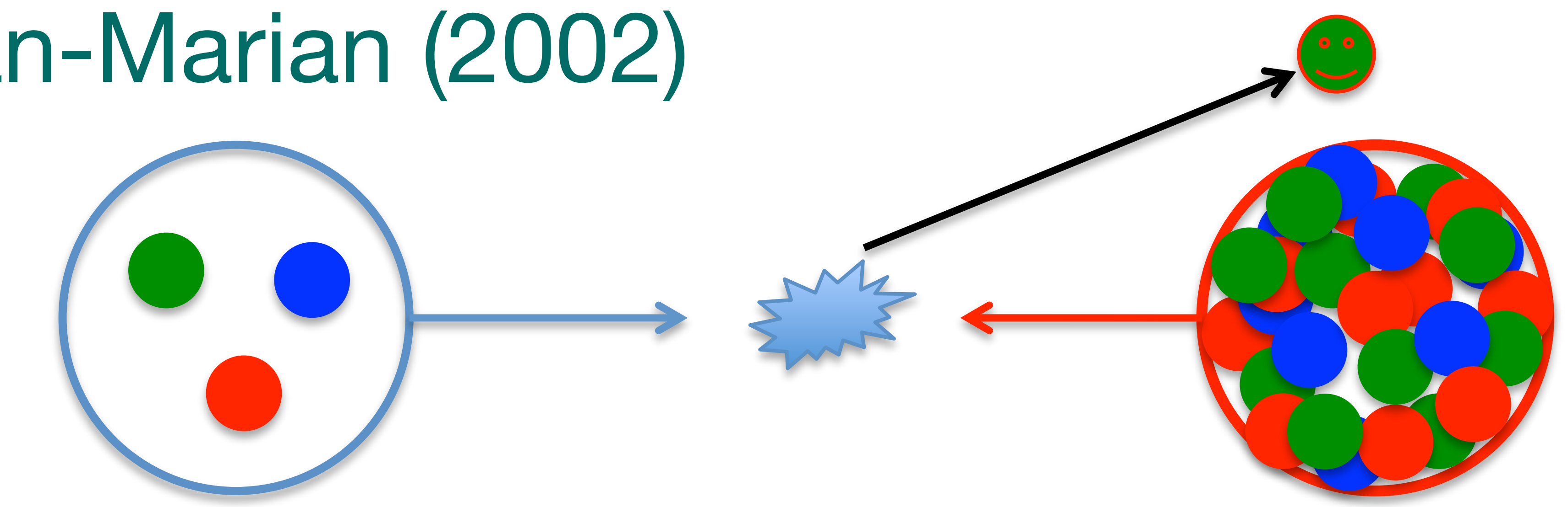


Prediction of nuclear enhancement from incoherent multiple scattering

Kang, Vitev, **HX,** PRD 2014
Li, Kang, **HX**, 2023
PHENIX, PRC, 2020

Coherent multiple scattering - CGC

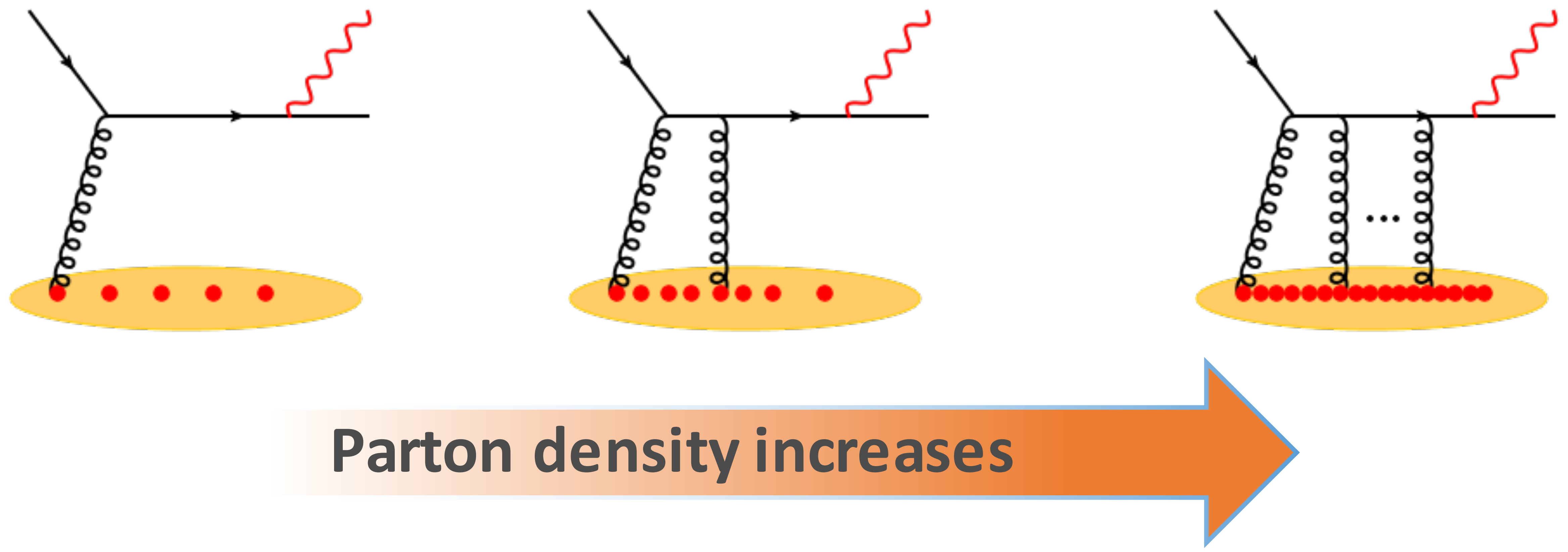
- Hybrid (dilute-dense) factorization   Dumitru, Jalilian-Marian (2002)



$$\sigma \sim x_p f_{q/p}(x_p) \otimes H \otimes \mathcal{F}(x_g, k_\perp) \otimes D_{h/q}(z)$$

$x_p = \frac{p_\perp}{z\sqrt{s}} e^{y}$ → $x_p p_a \gg k_{Ta}$ → Probing valance quark – DGLAP evolution

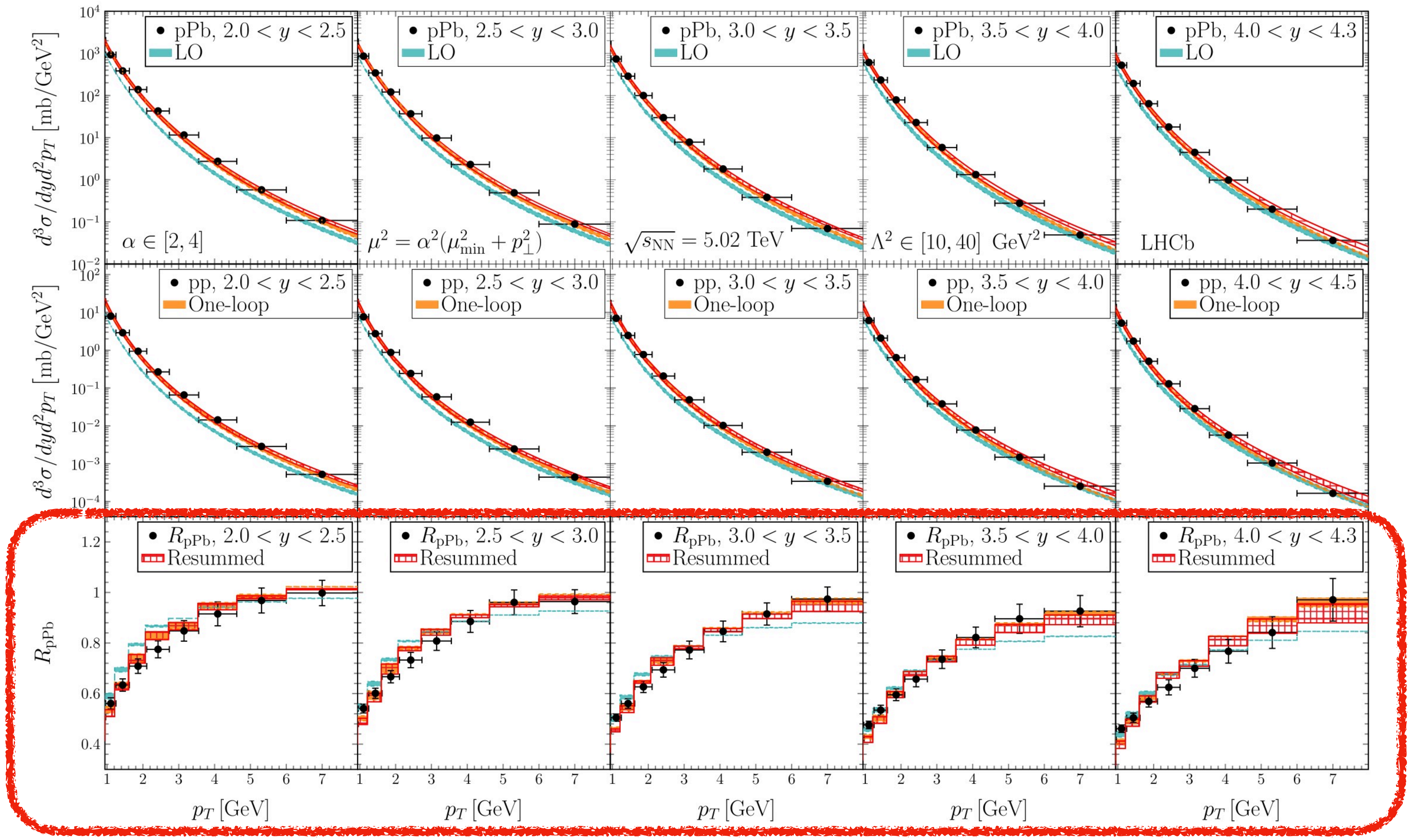
$x_g = \frac{p_\perp}{z\sqrt{s}} e^{-y}$ → $x_g p_b \sim k_{Tb}$ → Probing dense gluon – BK evolution



- All multiple scatterings become equally important, need to be resumed.
- Coherent multiple scattering are encoded in the so-called unintegrated gluon distribution $\mathscr{F}(x_g, k_\perp)$

Coherent multiple scattering - dilute region

- Hybrid (dilute-dense) factorization



Suppression from CGC calculation

Albacete, Marquet, PLB 2010

Dimitri, Jalilian-Marian, PRL 2012

Chirilli, Xiao, Yuan, PRL 2012

Stasto, Xiao, Zaslavsky, PRL 2014

Kang, Vitev, HX, PRL 2014

Iancu, Mueller, Triantafyllopoulos, JHEP 2016

Liu, Kang, Liu, PRD 2020

Shi, Wang, Wei, Xiao, PRL 2022

……

A unified picture of dilute and dense limits

- efforts along this direction

Quark jets scattering from a gluon field: From saturation to high p_t

Jamal Jalilian-Marian
*Department of Natural Sciences, Baruch College, CUNY, 17 Lexington Avenue, New York, New York 10010, USA
and CUNY Graduate Center, 365 Fifth Avenue, New York, New York 10016, USA*

(Received 18 September 2018; published 30 January 2019)

We continue our studies of possible generalization of the color glass condensate effective theory of high energy QCD to include the high p_t (or equivalently large x) QCD dynamics as proposed in [Phys. Rev. D 96, 074020 (2017)]. Here, we consider scattering of a quark from both the small and large x gluon degrees of freedom in a proton or nucleus target and derive the full scattering amplitude by including the interactions between the small and large x gluons of the target. We thus generalize the standard eikonal approximation for parton scattering, which can now be deflected by a large angle (and therefore have large p_t) and also lose a significant fraction of its longitudinal momentum (unlike the eikonal approximation). The corresponding production cross section can thus serve as the starting point toward the derivation of a general evolution equation that would contain the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution equation at large Q^2 and the Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner evolution equation at small x. This amplitude can also be used to construct the quark Feynman propagator, which is the first ingredient needed to generalize the color glass condensate effective theory of high energy QCD to include the high p_t dynamics. We outline how it can be used to compute observables in the large x (high p_t) kinematic region where the standard color glass condensate formalism breaks down.

DOI: 10.1103/PhysRevD.99.014043

Next-to-eikonal corrections in the CGC: gluon production and spin asymmetries in pA collisions

Tolga Altinoluk,[a] Néstor Armesto,[a] Guillaume Beuf,[a] Mauricio Martínez[b] and Carlos A. Salgado[a]
[a] *Departamento de Física de Partículas and IGFAE, Universidade de Santiago de Compostela, E-15706 Santiago de Compostela, Galicia-Spain*
[b] *Department of Physics, The Ohio State University, Columbus, OH 43210, U.S.A.*
E-mail: tolga.altinoluk@usc.es, nestor.armesto@usc.es, guillaume.beuf@usc.es, martinezguerrero.1@osu.edu, carlos.salgado@usc.es

Abstract: We present a new method to systematically include corrections to the eikonal approximation in the background field formalism. Specifically, we calculate the subleading, power-suppressed corrections due to the finite width of the target or the finite energy of the projectile. Such power-suppressed corrections involve Wilson lines decorated by gradients of the background field — thus related to the density - of the target. The method is of generic applicability. As a first example, we study single inclusive gluon production in pA collisions, and various related spin asymmetries, beyond the eikonal accuracy.

Keywords: QCD Phenomenology, Hadronic Colliders

ArXiv ePrint: 1404.2219

Gluon TMD in particle production from low to moderate x

I. Balitsky[a,b] and A. Tarasov[b]
[a] *Department of Physics, Old Dominion University, 4600 Elkhorn Ave, Norfolk, VA 23529, U.S.A.*
[b] *Theory Group, Jefferson Lab, 12000 Jefferson Ave, Newport News, VA 23606, U.S.A.*
E-mail: balitsky@jlab.org, atarasov@jlab.org

Abstract: We study the rapidity evolution of gluon transverse momentum dependent distributions appearing in processes of particle production and show how this evolution changes from small to moderate Bjorken x.

Keywords: Deep Inelastic Scattering (Phenomenology), QCD Phenomenology

ArXiv ePrint: 1603.06548

Gluon-mediated inclusive Deep Inelastic Scattering from Regge to Bjorken kinematics

Renaud Boussarie[a] and Yacine Mehtar-Tani[b]
[a] *Centre de Physique Théorique, École polytechnique, CNRS, I.P. Paris, F-91128 Palaiseau, France*
[b] *Physics Department and RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, U.S.A.*
E-mail: renaud.boussarie@polytechnique.edu, mehtartani@bnl.gov

Abstract: We revisit high energy factorization for gluon mediated inclusive Deep Inelastic Scattering (DIS) for which we propose a new semi-classical approach that accounts systematically for the longitudinal extent of the target in contrast with the shockwave limit. In this framework, based on a partial twist expansion, we derive a factorization formula that involves a new gauge invariant unintegrated gluon distribution which depends explicitly on the Feynman x variable. It is shown that both the Regge and Bjorken limits are recovered in this approach. We reproduce in particular the full one loop inclusive DIS cross-section in the leading twist approximation and the all-twist dipole factorization formula in the strict $x = 0$ limit. Although quantum evolution is not discussed explicitly in this work, we argue that the proper treatment of the x dependence of the gluon distribution encompasses the kinematic constraint that must be imposed on the phase-space of gluon fluctuations in the target to ensure stability of small-x evolution.

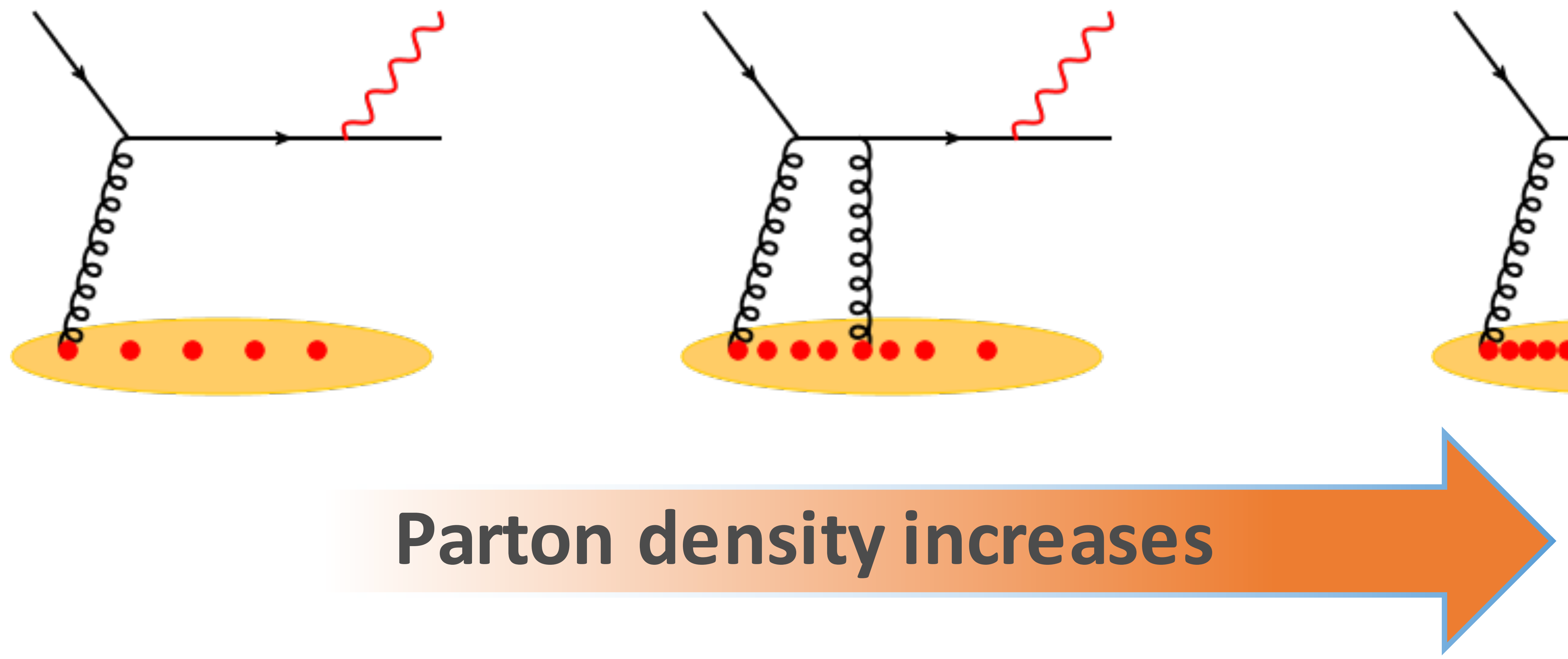
Keywords: Deep Inelastic Scattering or Small-X Physics, Parton Distributions

ArXiv ePrint: 2112.01412

T. Altinoluk, N. Armesto, G. Beuf, M. Martínez, and C. A. Salgado, JHEP **07**, 068 (2014), arXiv:1404.2219 [hep-ph].
T. Altinoluk, N. Armesto, G. Beuf, and A. Moscoso, JHEP **01**, 114 (2016), arXiv:1505.01400 [hep-ph].
T. Altinoluk and A. Dumitru, Phys. Rev. D **94**, 074032 (2016), arXiv:1512.00279 [hep-ph].
P. Agostini, T. Altinoluk, and N. Armesto, Eur. Phys. J. C **79**, 600 (2019), arXiv:1902.04483 [hep-ph].
P. Agostini, T. Altinoluk, and N. Armesto, Eur. Phys. J. C **79**, 790 (2019), arXiv:1907.03668 [hep-ph].
P. Agostini, T. Altinoluk, N. Armesto, F. Dominguez, and J. G. Milhano, Eur. Phys. J. C **82**, 1001 (2022), arXiv:2207.10472 [hep-ph].
T. Altinoluk, G. Beuf, A. Czajka, and A. Tymowska, Phys. Rev. D **104**, 014019 (2021), arXiv:2012.03886 [hep-ph].
T. Altinoluk and G. Beuf, Phys. Rev. D **105**, 074026 (2022), arXiv:2109.01620 [hep-ph].
G. A. Chirilli, JHEP **01**, 118 (2019), arXiv:1807.11435 [hep-ph].
G. A. Chirilli, JHEP **06**, 096 (2021), arXiv:2101.12744 [hep-ph].
I. Balitsky and A. Tarasov, JHEP **10**, 017 (2015), arXiv:1505.02151 [hep-ph].
I. Balitsky and A. Tarasov, JHEP **06**, 164 (2016), arXiv:1603.06548 [hep-ph].
I. Balitsky and A. Tarasov, JHEP **07**, 095 (2017), arXiv:1706.01415 [hep-ph].
I. Balitsky and A. Tarasov, JHEP **05**, 150 (2018), arXiv:1712.09389 [hep-ph].
I. Balitsky and G. A. Chirilli, Phys. Rev. D **100**, 051504 (2019), arXiv:1905.09144 [hep-ph].
R. Boussarie and Y. Mehtar-Tani, Phys. Lett. B **831**, 137125 (2022), arXiv:2006.14569 [hep-ph].
R. Boussarie and Y. Mehtar-Tani, JHEP **07**, 080 (2022), arXiv:2112.01412 [hep-ph].
J. Jalilian-Marian, Phys. Rev. D **96**, 074020 (2017), arXiv:1708.07533 [hep-ph].
J. Jalilian-Marian, Phys. Rev. D **99**, 014043 (2019), arXiv:1809.04625 [hep-ph].
J. Jalilian-Marian, Phys. Rev. D **102**, 014008 (2020), arXiv:1912.08878 [hep-ph].

The relation between CGC and high-twist expansion

- Take direct photon production as an example



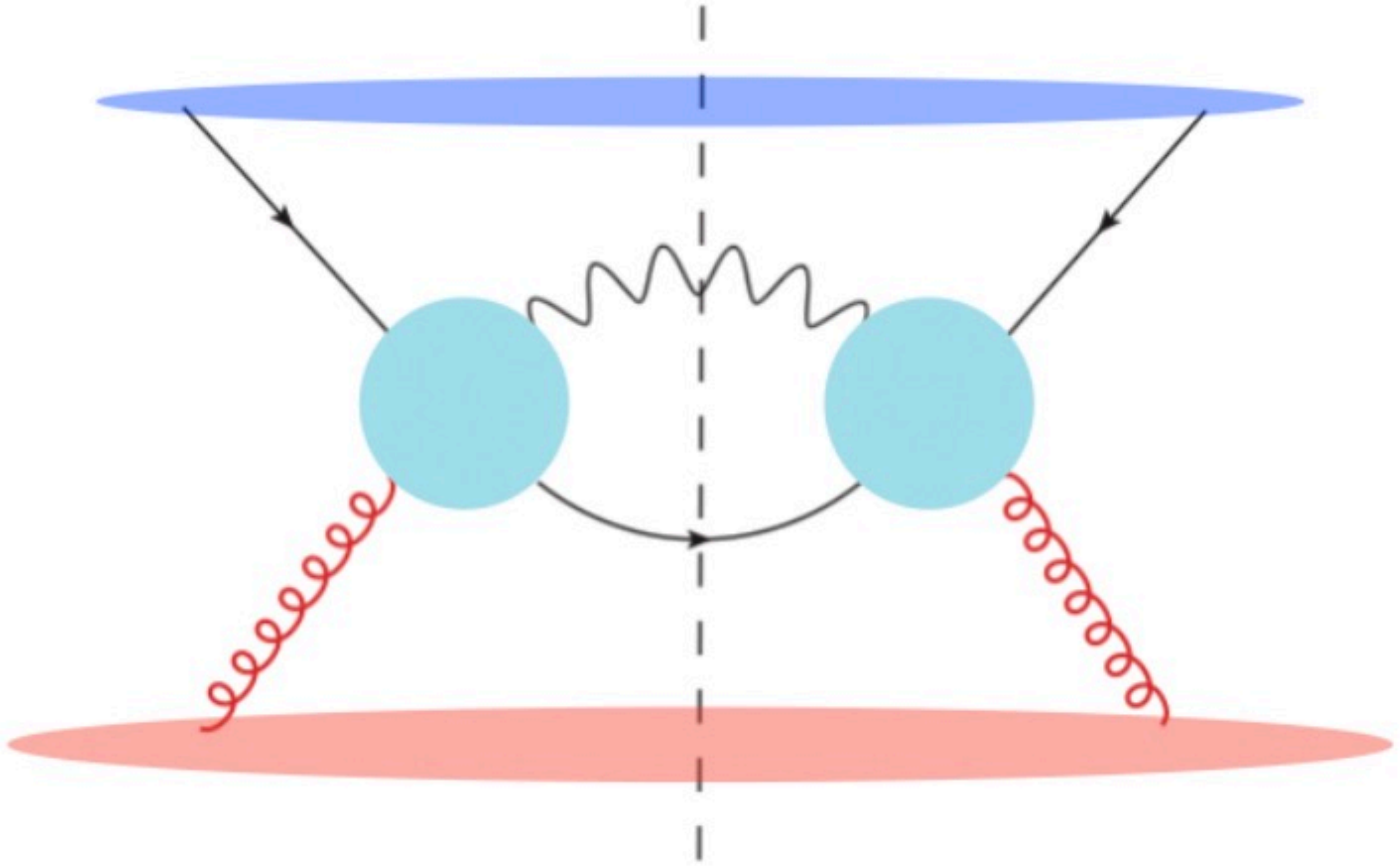
- Higher twist become important at moderate $p_{\gamma\perp}^2$

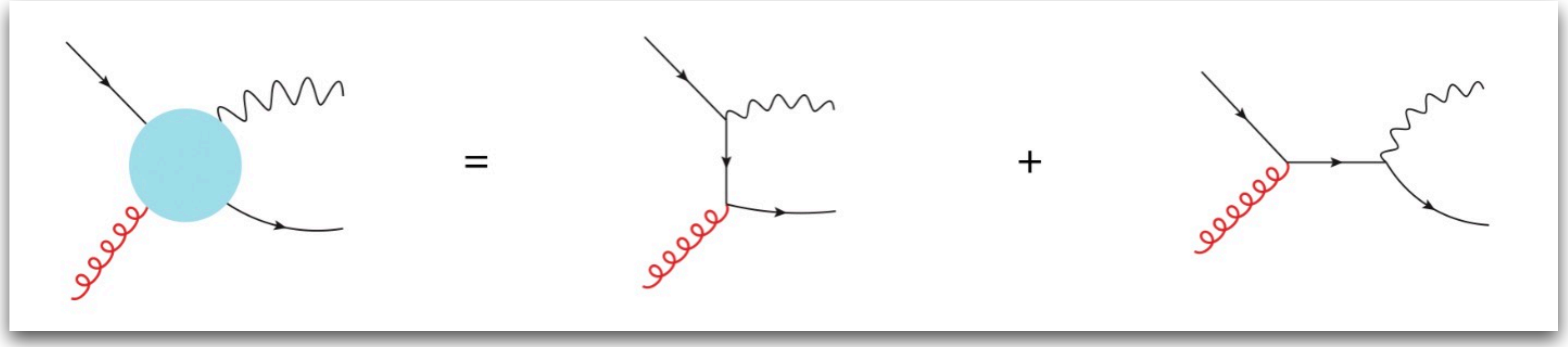
$$\mathrm{d}\sigma \sim \frac{1}{p_{\gamma\perp}^4}\Big[\underbrace{A}_{\substack{\text{leading twist}\\ \text{(twist-2)}}} + \underbrace{B\frac{\langle k_\perp^2\rangle}{p_{\gamma\perp}^2} + C\frac{\langle k_\perp^2\rangle^2}{p_{\gamma\perp}^4}}_{\substack{\text{Higher twist}\\ \text{(twist-4 and twist-6)}}}\dots\Big]$$

$$\langle k_\perp^2\rangle \sim Q_s^2 \propto A^{1/3}x^{-\lambda}$$

Direct photon production in p+A collisions

- Single scattering (q+g channel)





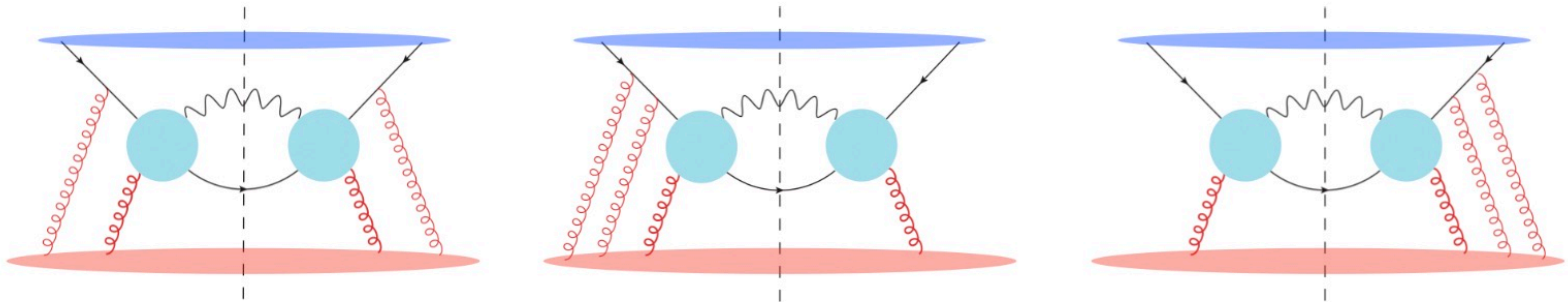
- leading twist collinear factorization

$$E_\gamma \frac{\mathrm{d}\sigma^S_{pA\to\gamma}}{\mathrm{d}^3\boldsymbol{p_\gamma}} = \alpha_{em}\alpha_s \frac{1}{s}\int \frac{\mathrm{d}x_p}{x_p} f(x_p) \int \frac{\mathrm{d}x}{x} f_{g/A}(x)\ H^U_{qg\to q\gamma}(\hat{s},\hat{t},\hat{u})\, \delta(\hat{s}+\hat{t}+\hat{u})$$

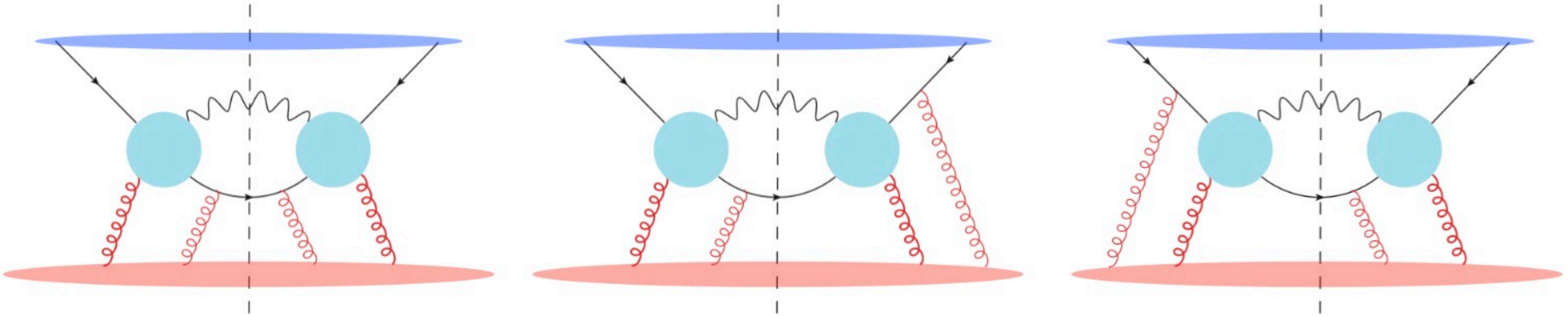
$$f_{g/A}(x) = \frac{1}{xP^+}\int \frac{\mathrm{d}y^-}{2\pi}\ e^{-ixP^+y^-}\ \langle P_A|F^{+\omega}(0^-)F^+{}_\omega(y^-)|P_A\rangle \qquad H^U_{qg\to q\gamma}(\hat{s},\hat{t},\hat{u}) = \frac{1}{2N_c}[-2(\frac{\hat{t}}{\hat{s}}+\frac{\hat{s}}{\hat{t}})]$$

Looking backward - incoherent multiple scattering from high-twist

- Initial state double scattering and single-triple interference



- Final state double scattering and initial-final interference



Looking backward - incoherent multiple scattering from high-twist

- Complete twist-4 contribution

$$E_\gamma \frac{\mathrm{d}\sigma^D_{qA\to\gamma}}{\mathrm{d}^3\boldsymbol{p}_\gamma} = \int \mathrm{d}x_p f_q(x_p) x_b \frac{4\pi^2\alpha_s^2\alpha_e}{N_c^2} \frac{\xi^2 - 2\xi + 2}{\boldsymbol{p}^6_{\gamma\perp}} \left[\cdots\right]_{x_1 = x_b, x_2 = x_3 = 0}$$

$$T(x_1, x_2, x_3) = \int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi}\frac{dy_2^-}{2\pi} e^{ix_1P^+y^-} e^{ix_2P^+(y_1^- - y_2^-)} e^{ix_3P^+y_2^-} \frac{1}{xP^+}\left\langle P_A \middle| F^{+\omega}(0^-) F^{+\kappa}(y_2^-) F^+{}_\kappa(y_1^-) F^+{}_\omega(y^-) \middle| P_A \right\rangle$$

result from initial state rescattering

$[\cdots]$ Cuts / Derivatives	Central	Asymmetric
2nd	$\xi^4[x_b^2 \frac{\partial^2 T^{C,I}}{\partial x_1^2}]$	0
1st	$-3\xi^4[x_b \frac{\partial T^{C,I}}{\partial x_1}] + (1-\xi)\xi^3[x_b \frac{\partial T^{C,I}}{\partial x_2}]$	$(1-\xi)\xi^3[x_b \frac{\partial T^{A,I}}{\partial x_2}]$
0th	$4\xi^4 T^{C,I}$	0

- Positive contribution from incoherent multiple scattering

$$E_\gamma \frac{\mathrm{d}\sigma^D_{pA\to\gamma}}{\mathrm{d}^3\boldsymbol{p}_\gamma} = \frac{4\pi^2\alpha_s^2\alpha_e}{N_c}\frac{1}{s} \int \frac{\mathrm{d}x_p}{x_p} f(x_p) \int \frac{\mathrm{d}x}{x} c^I H^U_{qg\to q\gamma}(\hat{s}, \hat{t}, \hat{u})\, \delta(\hat{s} + \hat{t} + \hat{u})$$

$$c^I = -\frac{1}{\hat{s}} - \frac{1}{\hat{t}}$$

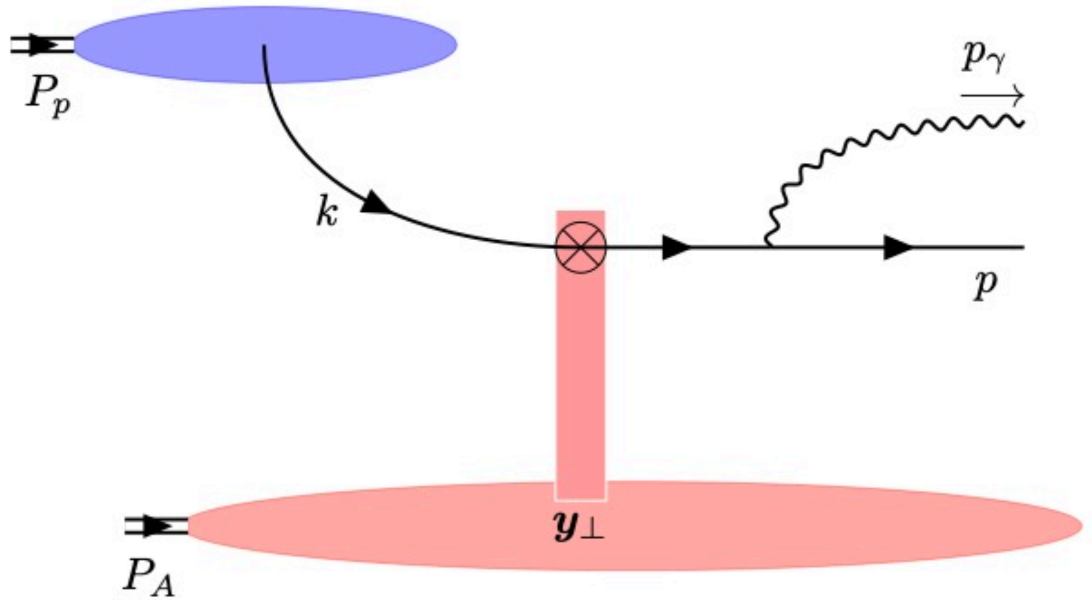
$$\boxed{\left[x^2 \frac{\partial^2 T^I(x)}{\partial x^2} - x\frac{\partial T^I(x)}{\partial x} + x T^I(x)\right]}$$

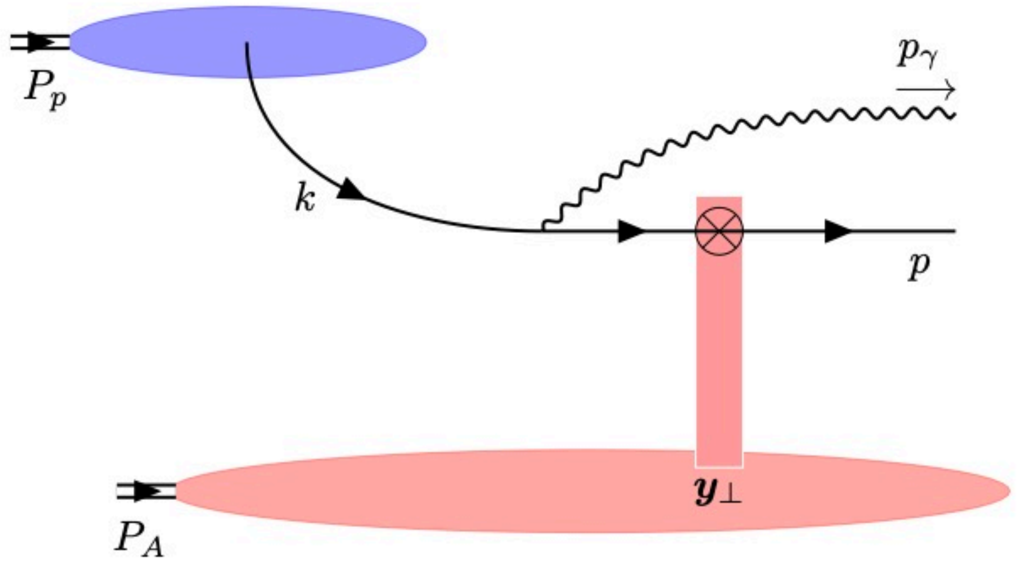
Only initial state rescattering contributes

positive -> nuclear enhancement

Looking forward - coherent multiple scattering from CGC

- Direct photon production with the CGC/saturation framework





  - CGC differential cross section

$$\frac{\mathrm{d}\sigma^{p+A\to\gamma+X}}{\mathrm{d}\eta_\gamma \mathrm{d}^2\boldsymbol{p}_{\gamma\perp}} = \frac{\alpha_{\mathrm{em}} e_f^2}{2\pi^2}\int_{x_{p,min}}^{1} \mathrm{d}x_p f(x_p)\xi^2\left[1+(1-\xi)^2\right]\int \mathrm{d}^2\boldsymbol{l}_\perp \frac{\boldsymbol{l}_\perp^2 F(\bar{x}_A,\boldsymbol{l}_\perp)}{\left(\xi\boldsymbol{l}_\perp-\boldsymbol{p}_{\gamma\perp}\right)^2\boldsymbol{p}_{\gamma\perp}^2}$$

  - Dipole correlator

$$F(x_A,\boldsymbol{l}_\perp) = \int\frac{\mathrm{d}^2\boldsymbol{y}_\perp}{2\pi}\int\frac{\mathrm{d}^2\boldsymbol{y}'_\perp}{2\pi}e^{-i\boldsymbol{l}_\perp\cdot(\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp)}S^{(2)}(x_A;\boldsymbol{y}_\perp,\boldsymbol{y}'_\perp)$$

$$S^{(2)}(x_A;\boldsymbol{y}_\perp,\boldsymbol{y}'_\perp) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[V^\dagger(\boldsymbol{y}'_\perp)V(\boldsymbol{y}_\perp)\right]\right\rangle_{x_A} \qquad V_{ij}(\boldsymbol{y}_\perp) = \mathcal{P}\exp\left(ig\int_{-\infty}^{\infty}\mathrm{d}z^- A^{+,c}(y^-,\boldsymbol{y}_\perp)t_{ij}^c\right)$$

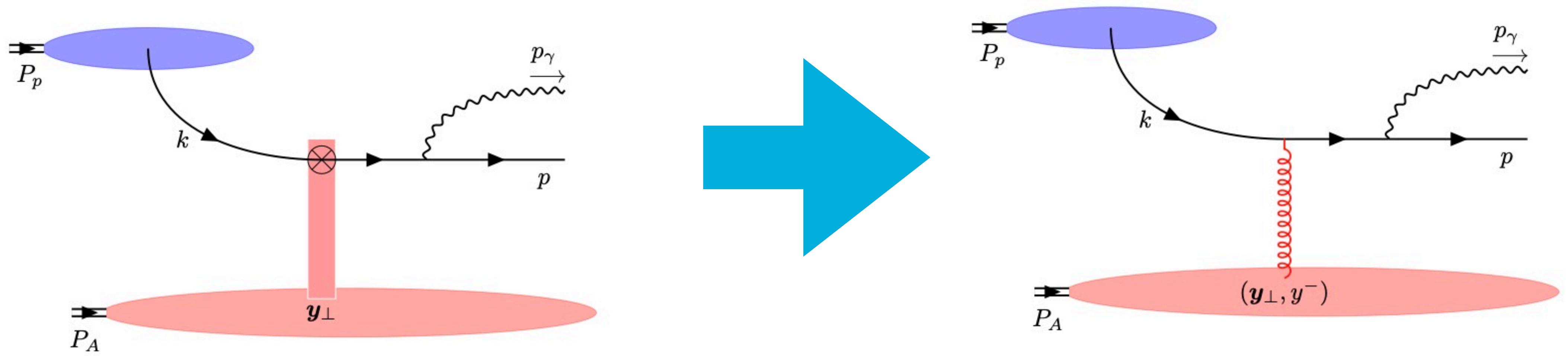
From CGC to leading twist collinear factorization

- Consistency between CGC and single scattering

  - considering large $p_{\gamma\perp}$ to go beyond small-x

$$\frac{1}{(\xi l_\perp - p_{\gamma\perp})^2} \approx \frac{1}{p_{\gamma\perp}^2} + \frac{\xi^2 l_\perp^2}{p_{\gamma\perp}^4} + \dots$$

twist-2 twist-4



  - Twist-2 cross section

$$\frac{\mathrm{d}\sigma^{p+A\to\gamma+X}}{\mathrm{d}\eta_\gamma \mathrm{d}^2\boldsymbol{p}_{\gamma\perp}} = \frac{\alpha_{\mathrm{em}} e_f^2 \alpha_s}{N_c} \int_{x_{p,min}}^{1} \mathrm{d}x_p f(x_p) \frac{\xi^2\left[1+(1-\xi)^2\right]}{\boldsymbol{p}_{\gamma\perp}^4} \bar{x}_A f_{g/A}(\bar{x}_A)\Big|_{\bar{x}_A\to 0}$$

$$\lim_{x\to 0} x f_{g/A}(x) = \frac{N_c}{2\pi^2\alpha_s} \int \mathrm{d}^2\boldsymbol{l}_\perp \boldsymbol{l}_\perp^2 F(x, \boldsymbol{l}_\perp)$$

Baier, Mueller, Schiff, 2004

$$e^{i\bar{x}_A P_A^+ \Delta y} \sim 1 \to \bar{x}_A A^{1/3} \ll 1$$

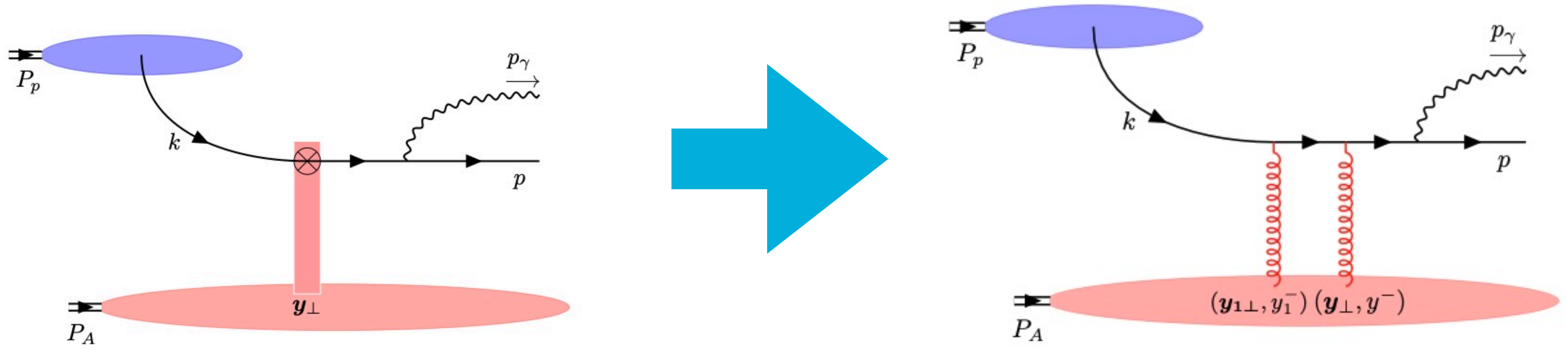
Dropping out the phase in small-x limit

From CGC to twist-4 collinear factorization

- Consistency between CGC and double scattering

  - considering large $p_{\gamma\perp}$ to go beyond small-x

$$\frac{1}{(\xi l_\perp - p_{\gamma\perp})^2} \approx \underbrace{\frac{1}{p_{\gamma\perp}^2}}_{\text{twist-2}} + \underbrace{\frac{\xi^2 l_\perp^2}{p_{\gamma\perp}^4}}_{\text{twist-4}} + \ldots$$



  - Twist-4 cross section

$$\left.\frac{\mathrm{d}\sigma^{p+A\to\gamma+X}}{\mathrm{d}\eta_\gamma \mathrm{d}^2\boldsymbol{p}_{\gamma\perp}}\right|_{\mathrm{NLT}} = \frac{(2\pi)^2\alpha_{\mathrm{em}} e_f^2 \alpha_s^2}{N_c^2} \int_{\frac{p_\gamma^-}{P_p^-}}^{1} \mathrm{d}x_p f(x_p) \frac{\xi^4\left[1+(1-\xi)^2\right]}{\boldsymbol{p}_{\gamma\perp}^6} T_{g/A}(\bar{x}_A, 0, 0)\Big|_{\bar{x}_A\to 0}$$

$$\lim_{x\to 0} T_{g/A}(x,0,0) = \frac{2N_c^2}{(2\pi)^4\alpha_s^2}\int \boldsymbol{l}_\perp^4 \mathrm{d}^2\boldsymbol{l}_\perp F(x, \boldsymbol{l}_\perp)$$

Some terms are missing comparing to twist-4 result with finite x !

A unified picture of dilute and dense limits

- Bringing back the longitudinal "sub-eikonal" phase for single scattering

$$\mathrm{d}\sigma \propto \int \mathrm{d}x_p f(x_p)\ \mathcal{H} \otimes \mathcal{T}$$

Expand the Wilson line:

$$(2\pi)\delta(l^- - l'^-)\gamma^- \int \mathrm{d}^2\boldsymbol{y}_\perp e^{-i(\boldsymbol{l}_\perp - \boldsymbol{l}'_\perp)\cdot \boldsymbol{y}_\perp} \int \mathrm{d}y^- e^{i(l^+ - l'^+)y^-} igA_a^+(y^-, \boldsymbol{y}_\perp)(t^a)_{ij}$$

Collinear expansion:

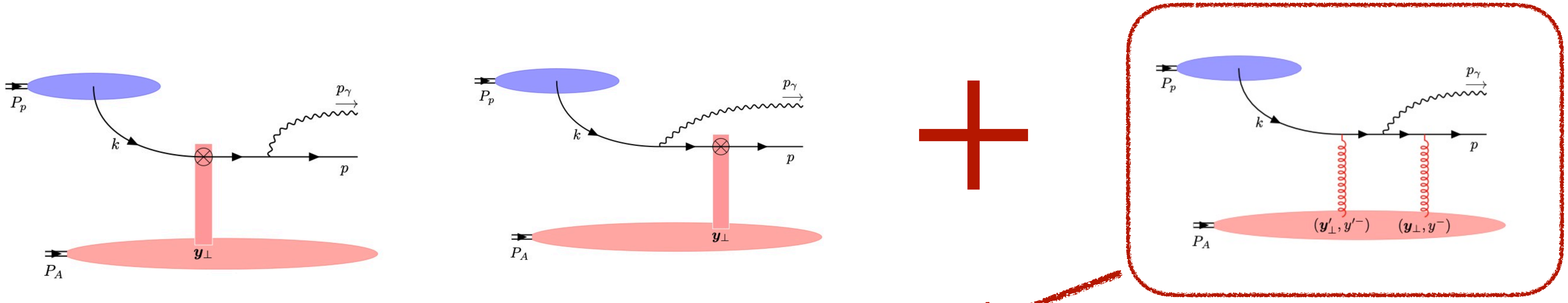
$$\mathcal{H}_2(p_\gamma; y, y') = \frac{8\xi^2\left[1 + (1-\xi)^2\right]}{\boldsymbol{p}_{\gamma\perp}^4} e^{i\bar{x}_A P_A^+(y^- - y'^-)} \frac{\partial^2}{\partial \boldsymbol{y}_\perp \cdot \partial \boldsymbol{y}'_\perp} \int \frac{\mathrm{d}^2\boldsymbol{l}_\perp}{(2\pi)^2} e^{-i\boldsymbol{l}_\perp\cdot(\boldsymbol{y}_\perp - \boldsymbol{y}'_\perp)} + \ldots .$$

$$\frac{\mathrm{d}\sigma^{p+A\to\gamma+X}}{\mathrm{d}\eta_\gamma \mathrm{d}^2\boldsymbol{p}_{\gamma\perp}} = \frac{\alpha_{\mathrm{em}} e_f^2 \alpha_s}{N_c} \int_{x_{p,min}}^1 \mathrm{d}x_p f(x_p) H(\xi, \boldsymbol{p}_{\gamma\perp}) \bar{x}_A f_{g/A}^{(0)}(\bar{x}_A)$$

Matching exactly to leading-twist result beyond small-x limit

A unified picture of dilute and dense limits

- Missing diagram in CGC



$$\text{phases} = e^{\frac{i}{x_p s}\left[\frac{\boldsymbol{p}_\perp^2}{(1-\xi)} + \frac{\boldsymbol{p}_{\gamma\perp}^2}{\xi} - \boldsymbol{l}_\perp^2\right] P_A^+ y'^-} e^{\frac{i}{x_p s}\boldsymbol{l}_\perp^2 P_A^+ y^-} \left\{ 1 - e^{\frac{i}{x_p s}\frac{\left[\boldsymbol{p}_{\gamma\perp} - \xi \boldsymbol{l}_\perp\right]^2}{\xi(1-\xi)} P_A^+ (y^- - y'^-)} \right\}$$

- formation time for photon production: $\tau_{\gamma,\text{form}}^{-1} = \dfrac{1}{x_p s}\dfrac{\left[\boldsymbol{p}_{\gamma\perp} - \xi \boldsymbol{l}_\perp\right]^2}{\xi(1-\xi)} P_A^+$

- LPM effect: $\tau_{\gamma,\text{form}} \gg y^- - y'^-$, coherent double scattering cancels, while this diagrams remains a net incoherent double scattering.

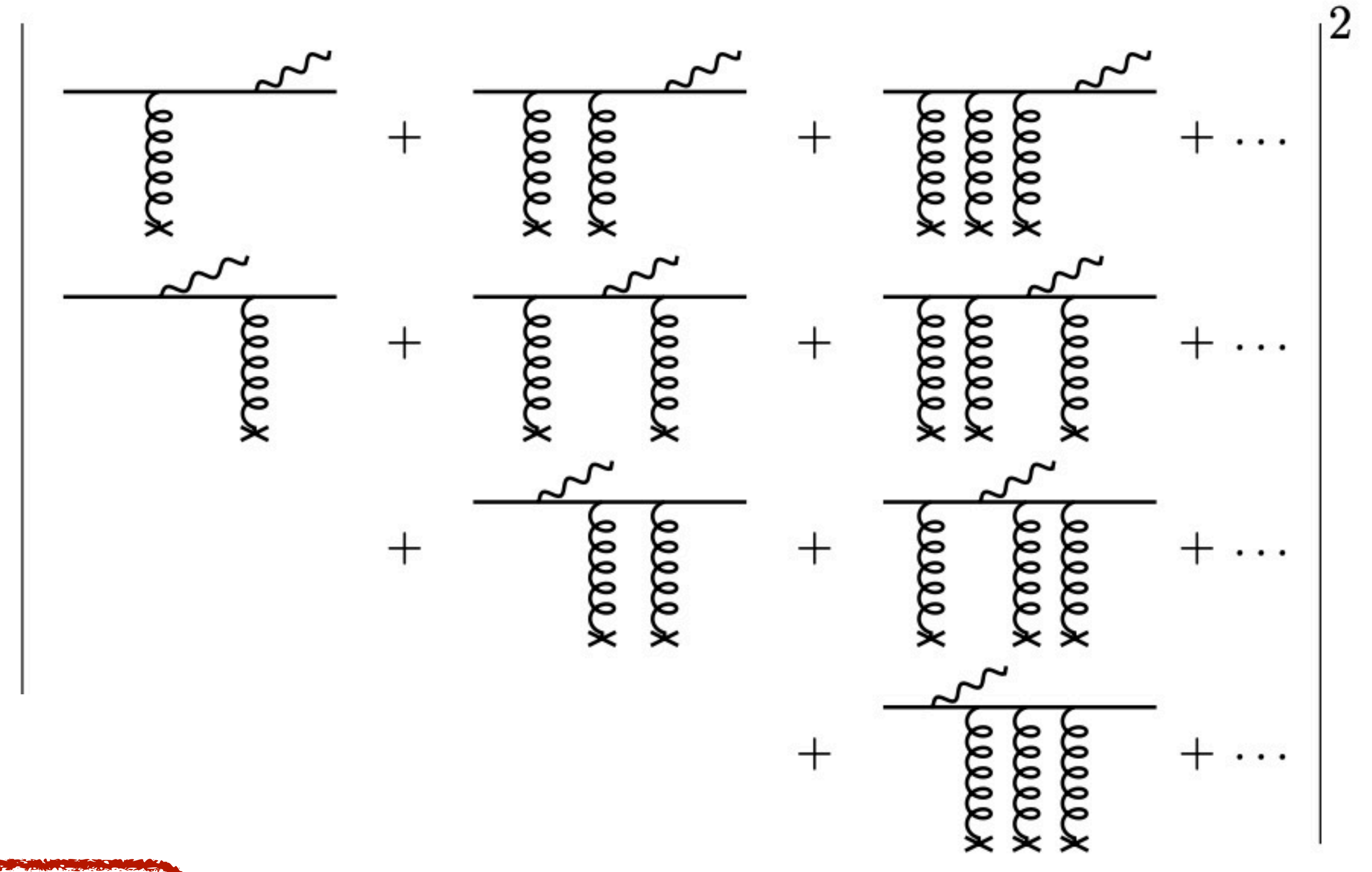
A unified picture of dilute and dense limits

- Consistency between CGC and double scattering

$$\mathrm{d}\sigma \propto \int \mathrm{d}x_p f(x_p)\ \mathcal{H} \otimes \mathcal{T}$$

$$\mathcal{T}(z_1, z_2, z_3, z_4) = \frac{1}{N_c} \left\langle \mathrm{Tr}\left[A^+(z_1^-, \boldsymbol{z_{1\perp}}) A^+(z_2^-, \boldsymbol{z_{2\perp}}) A^+(z_3-, \boldsymbol{z_{3\perp}}) A^+(z_4^-, \boldsymbol{z_{4\perp}}) \right] \right\rangle$$

$$\begin{aligned}
&\mathcal{H}^{coll}_{\mathrm{C,I}}(p_\gamma; y, y', y_1, y_2) \\
&= 8H(\xi, \boldsymbol{p_{\gamma\perp}}) e^{i\bar{x}_A P_A^+(y^- - y'^-)} \frac{\partial \delta^{(2)}(\boldsymbol{y}_\perp - \boldsymbol{y}_{1\perp})}{\partial \boldsymbol{y}_\perp} \cdot \frac{\partial \delta^{(2)}(\boldsymbol{y}'_\perp - \boldsymbol{y}_{2\perp})}{\partial \boldsymbol{y}'_\perp} \times \Bigg[\boxed{\delta^{(2)}(\boldsymbol{y}_{1\perp} - \boldsymbol{y}_{2\perp})} \\
&+ \frac{1}{\boldsymbol{p}^2_{\gamma\perp}} \frac{\partial^2 \delta^{(2)}(\boldsymbol{y}_{1\perp} - \boldsymbol{y}_{2\perp})}{\partial \boldsymbol{y}_{1\perp} \cdot \partial \boldsymbol{y}_{2\perp}} \left[4\xi^2 + \xi(1-\xi)(i\bar{x}_A P_A^+ \Delta y_{12}^-) - 3\xi^2(i\bar{x}_A P_A^+ \Delta y^-) + \xi^2 (i\bar{x}_A P_A^+ \Delta y^-)^2 \right] \Bigg]
\end{aligned}$$



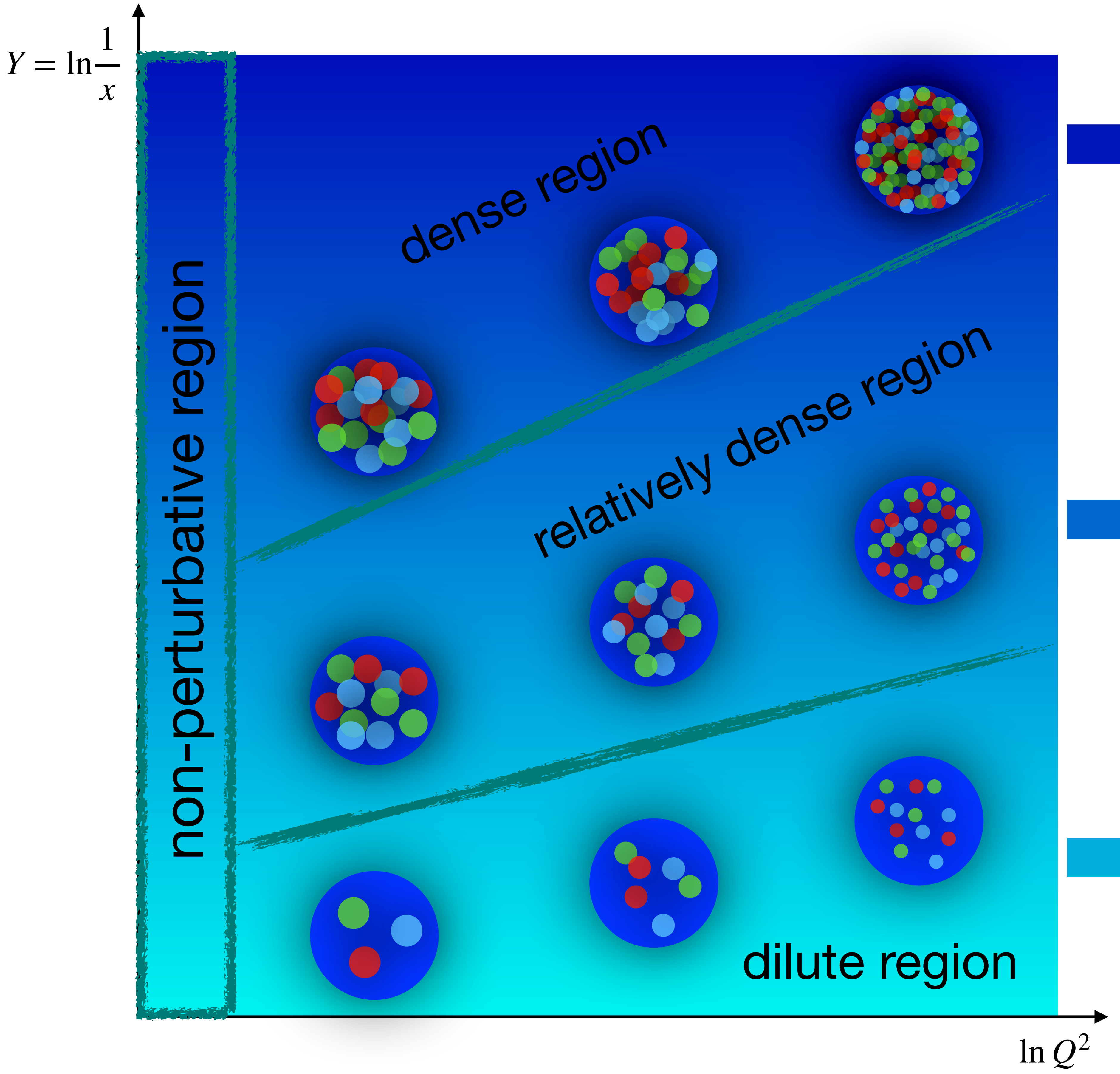
25 diagrams at twist-4

$$\begin{aligned}
\frac{\mathrm{d}\sigma^{p+A\to\gamma+X}_{\mathrm{C,I}}}{\mathrm{d}\eta_\gamma \mathrm{d}^2\boldsymbol{p}_{\gamma\perp}} &= \frac{\alpha_{\mathrm{em}} e_f^2 \alpha_{\mathrm{s}}}{N_c} \int_{x_{\min}}^1 \mathrm{d}x_p f(x_p) H(\xi, \boldsymbol{p_{\gamma\perp}}) \bar{x}_A \boxed{f_{g/A}^{(\mathrm{gauge\ \ link})}(\bar{x}_A)} \\
&+ \frac{(2\pi)^2 \alpha_{\mathrm{em}} e_f^2 \alpha_{\mathrm{s}}^2}{N_c^2 \boldsymbol{p}^2_{\gamma\perp}} \int_{x_{\min}}^1 \mathrm{d}x_p f(x_p) H(\xi, \boldsymbol{p_{\gamma\perp}}) \mathcal{D}_{\mathrm{C,I}}(\xi, \bar{x}_A, x_1, x_2, x_3) T_{\mathrm{C,I}}(x_1, x_2, x_3) \Big|_{\substack{x_1 = \bar{x}_A \\ x_2 = x_3 = 0}}
\end{aligned}$$

Recover the complete result from twist-4 formalism and the gauge link in PDF!

Summary

Yu Fu, Zhong-Bo Kang, Farid Salazar, Xin-Nian Wang, and Hongxi Xing 2023, to appear soon!



Color Glass Condensate (CGC)

Wilson lines, nonlinear BK/

High-twist formalism

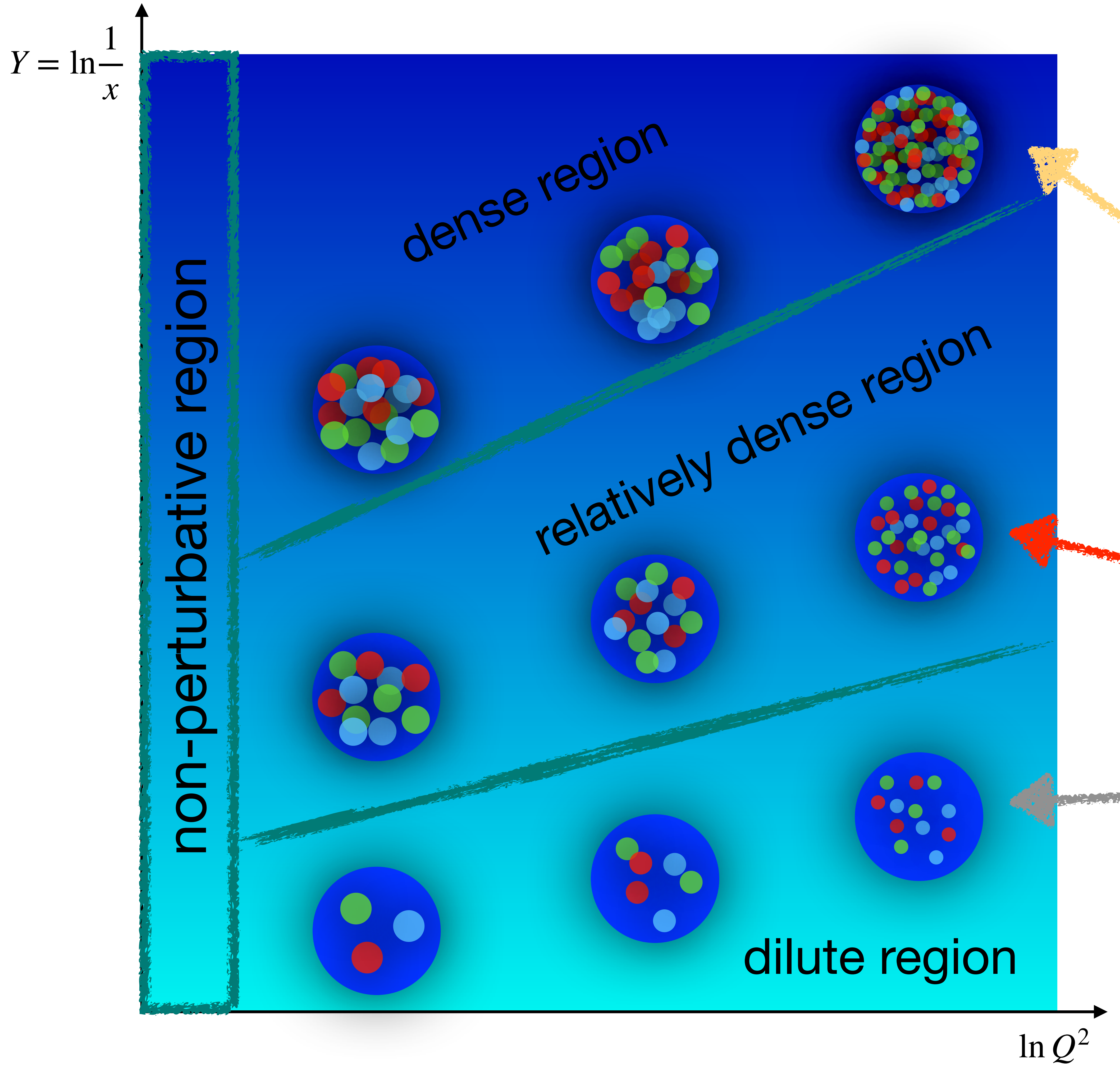
Multi-parton correlation, DGLAP-type evolution

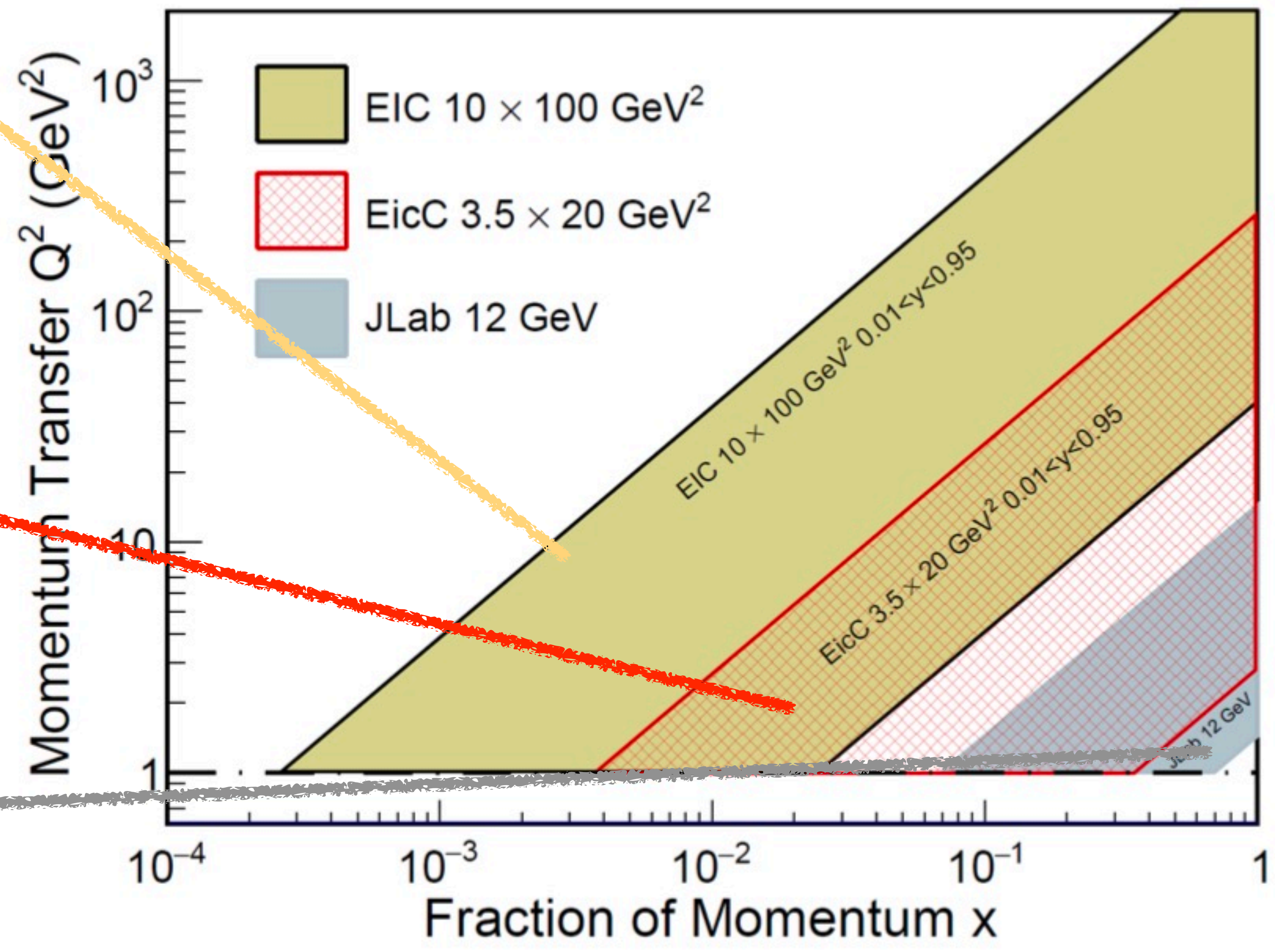
Leading twist collinear factorization

Taking direct photon production in pA collision as an example, we show the consistency between the collinear factorization (dilute) and the extended CGC (dense), and establish a unified picture for dilute-dense dynamics in QCD medium.

Outlook

THANKS!





Mapping out the QCD phase diagram for nuclei with worldwide efforts using a unified theoretical framework!