# Longitudinal Double-Spin Asymmetry at Small x in Polarized Proton-Proton Collisions

Ming Li
The Ohio State University

M. Li, arXiv: 2304.12842

Work in progress with Daniel Adamiak and Yuri Kovchegov

# Outline

- Introduction and Motivation
- Small x Effective Hamiltonian
- Longitudinal double-spin asymmetry for soft gluon production
- Summary

# Origin of Nucleon Spin

**Jaffe-Manohar spin sum rule for proton**

*The RHIC Spin Collaboration (2015)*

$$S_q + L_q + S_G + L_G = \frac{1}{2}$$

**Quark Spin**

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx\, \Delta\Sigma(x, Q^2)$$

$S_q(Q^2 = 10\mathrm{GeV}^2) \approx [0.15,\ 0.20]$

$x \in [0.001, 0.7]$

**Gluon Spin**

$$S_G(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

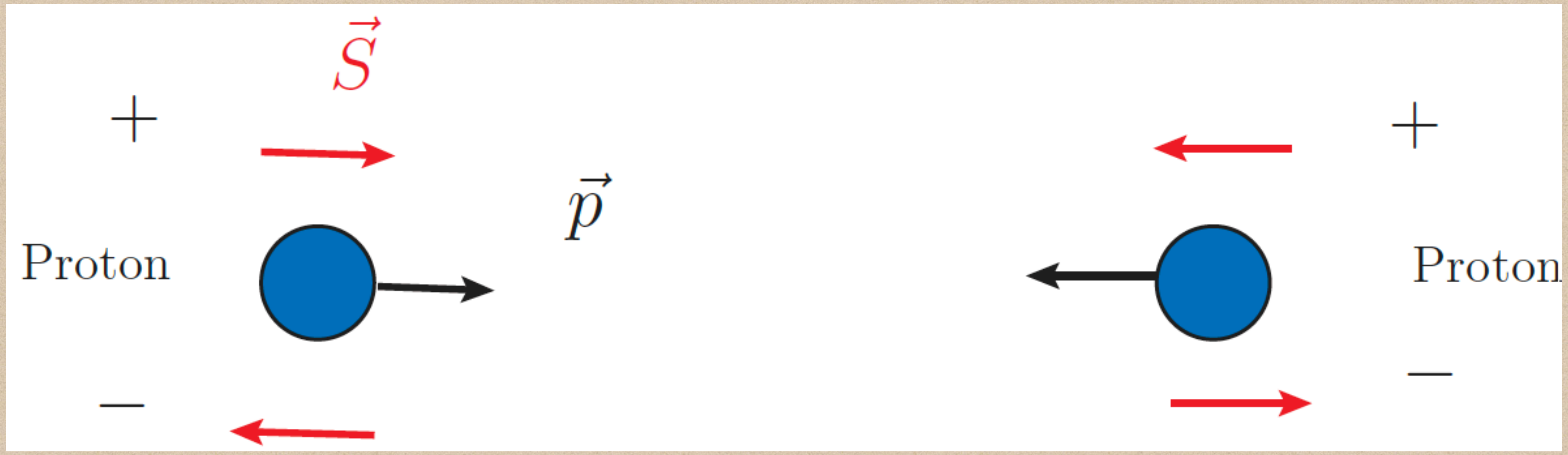
$S_G(Q^2 = 10\mathrm{GeV}^2) \approx [0.13,\ 0.26]$

$x \in [0.05, 0.7]$

**Missing spin of the proton maybe in Quark and Gluon Orbital Angular Momentum $L_q$ and $L_G$ and/or smaller values of $x$**

# Longitudinal Double-Spin Asymmetry

**How to measure quark and gluon intrinsic spin inside a proton?**




$$A_{\rm LL} \equiv \frac{d\Delta\sigma}{d\sigma} \equiv \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$

RHIC has measured $A_{\rm LL}$ for the productions of jets, dijets, $\pi^0$, $\pi^{\pm}$, direct photons…

*RHIC Spin Collaboration, arXiv: 2302.00605*

# Longitudinal Double-Spin Asymmetry

**Longitudinal double-spin asymmetry is related to parton helicity distribution.**

$$A_{\rm LL} \equiv \frac{d\Delta\sigma}{d\sigma} \equiv \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$

**Collinear Factorization (also parity invariance)**

*Babcock, Monsay and Sivers (1979),*
*de Florian, Sassot, Stratmann and Vogelsang (2008)(2014)*

$$d\Delta\sigma = \sum_{ab} \int dx_a \int dx_b \, \Delta f_a(x_a, Q^2)\, \Delta f_b(x_b, Q^2)\, d\Delta\hat{\sigma}_{ab}(x_a, x_b, p_T, \alpha_s(Q^2), p_T/Q)$$

(Anti) quark and gluon helicity distribution $\Delta f_j(x, Q^2) \equiv f_j^+(x, Q^2) - f_j^-(x, Q^2)$

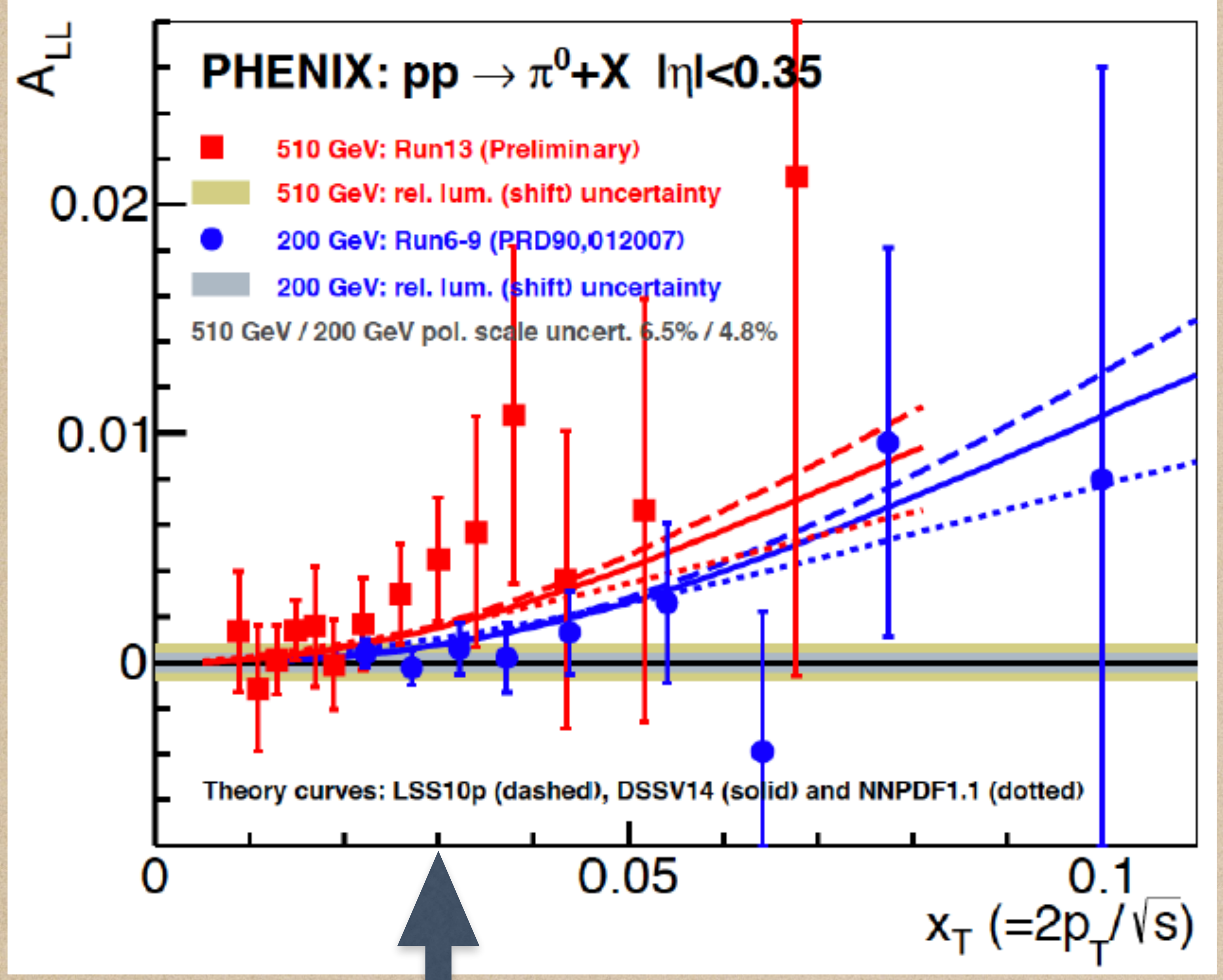
Partonic level double-spin asymmetry $d\Delta\hat{\sigma} = d\hat{\sigma}^{++} - d\hat{\sigma}^{+-}$

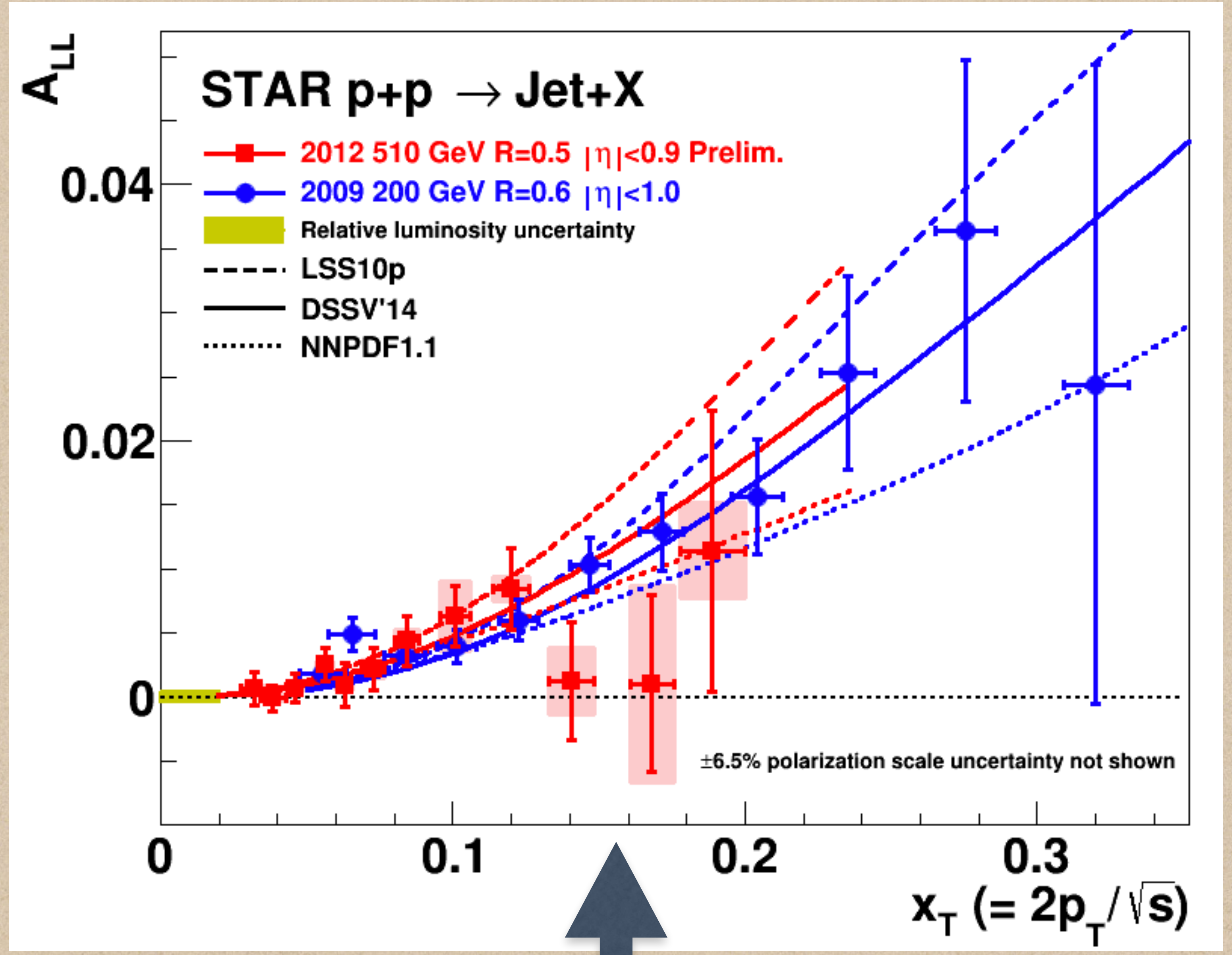
# Longitudinal Double-Spin Asymmetry at small x

**Collinear Factorization (applicable for large transverse momentum)**

$$d\Delta\sigma = \sum_{ab} \int dx_a \int dx_b \, \Delta f_a(x_a, Q^2) \, \Delta f_b(x_b, Q^2) \, d\Delta\hat{\sigma}_{ab}(x_a, x_b, p_T, \alpha_s(Q^2), p_T/Q)$$

*RHIC Spin Collaboration (2015)*




*Small x gluon saturation regime*

$\mathbf{p}_T \sim Q_s$

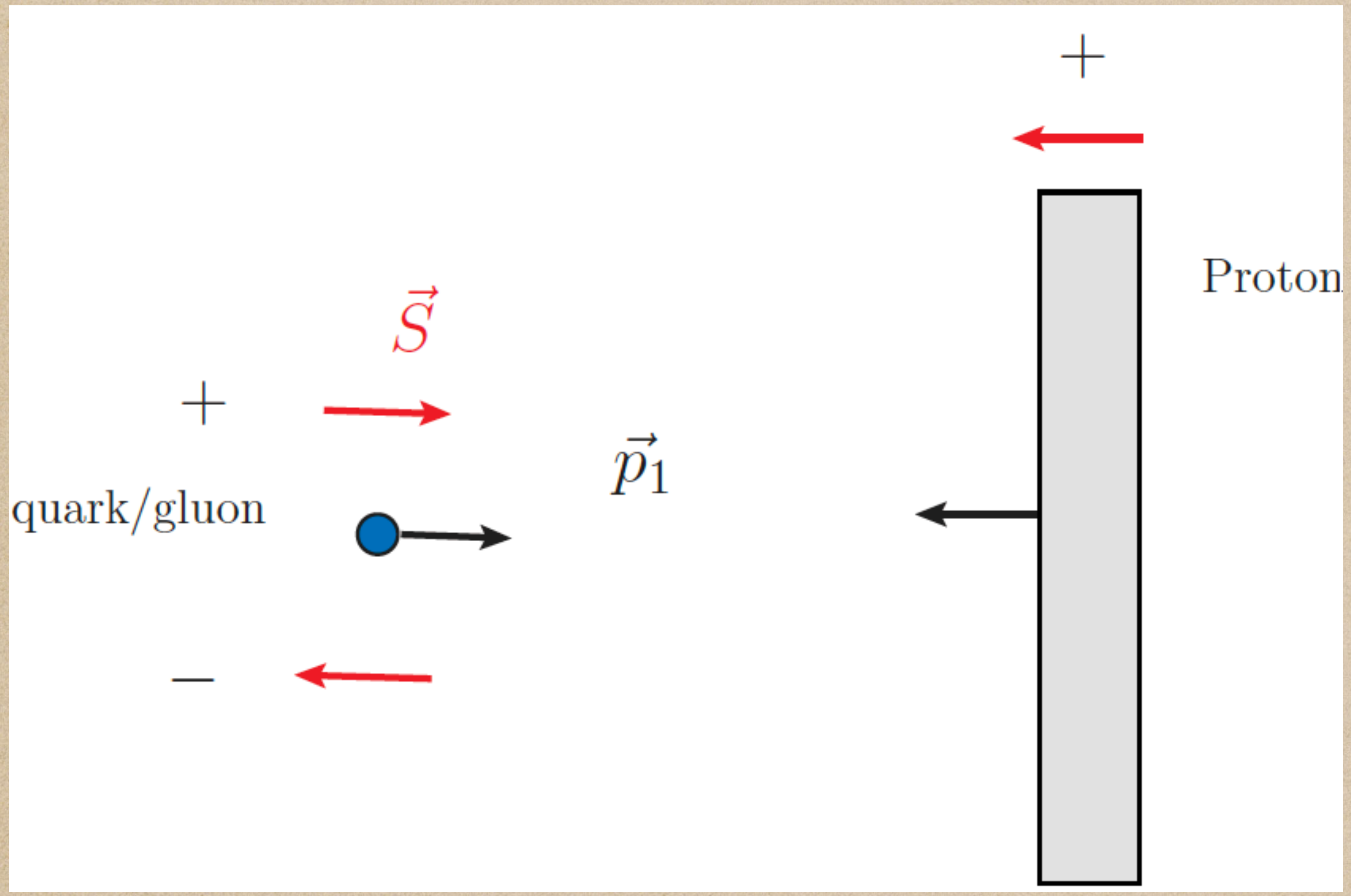
***Low transverse momentum region, sensitive to small x gluons, collinear factorization probably breaks down***

***Large transverse momentum region, collinear factorization successful, but not sensitive to small x gluons.***

**We need transverse momentum dependent framework to describe $A_{LL}$ in low transverse momentum region**

# Double-spin asymmetry at small x: gluon production at mid-rapidity

Treating projectile proton and target proton on equal footing at small x is challenging.



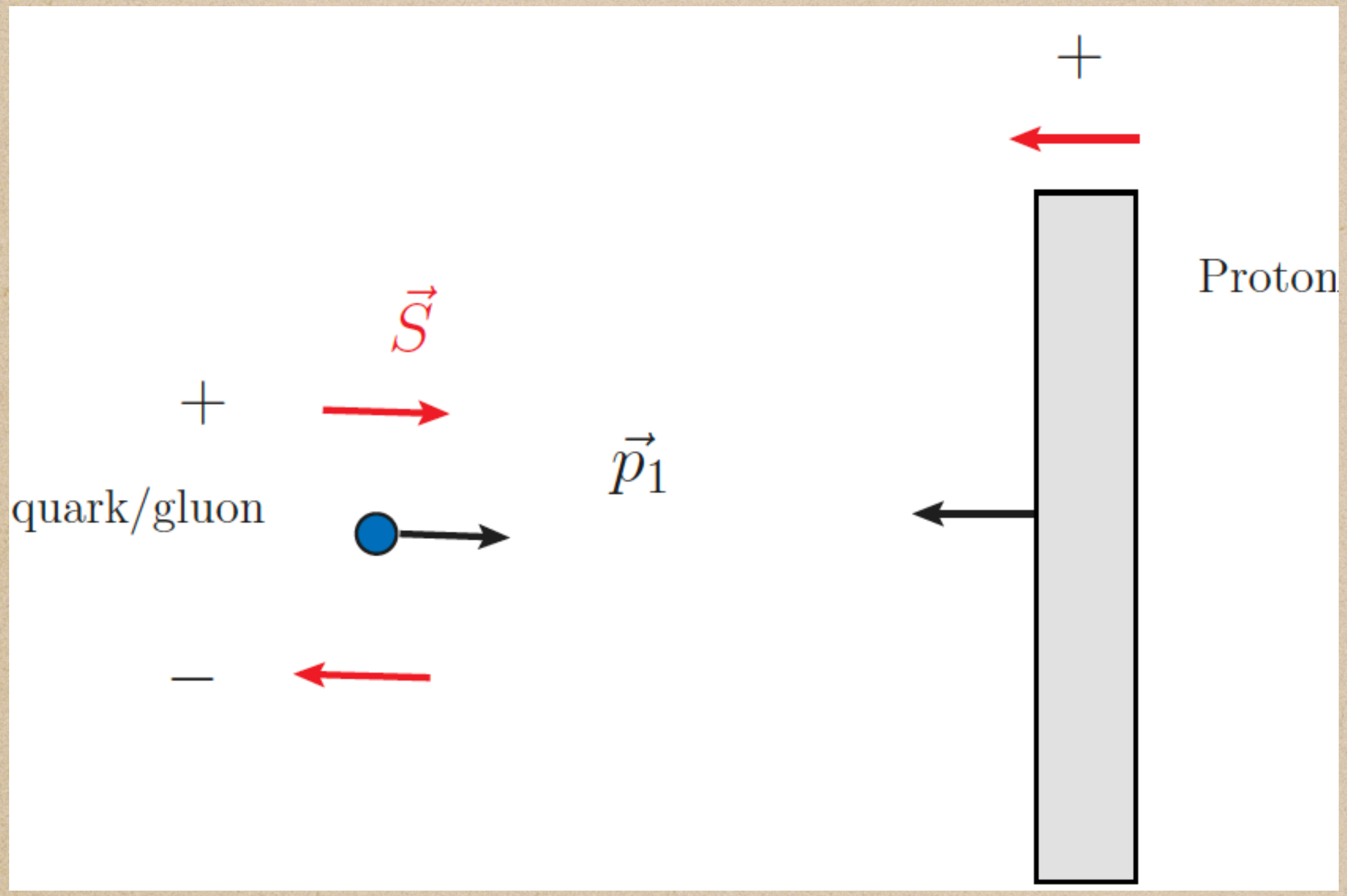

$$\text{quark/gluon} + \text{proton} \longrightarrow \text{gluon} + \text{X}$$

$$A_{\mathrm{LL}}^{g} \equiv \frac{d\Delta\sigma}{d^2\mathbf{p}dy} = \frac{d\sigma^+}{d^2\mathbf{p}dy} - \frac{d\sigma^-}{d^2\mathbf{p}dy}$$

Goal: $A_{\mathrm{LL}}^{g}$ for Gluon production at mid-rapidity

Convention: proton travels along negative-z direction.

# The Formalism: Small-x Effective Hamiltonian




The Shockwave Picture of High energy Scatterings: proton is treated as background gluon and quark fields

S-matrix element for highly boosted states *Bjorken, Kogut and Soper (1971)*

$$S_{\rm fi} = \langle \phi_{\rm f} | \, e^{i\omega \hat{K}^3} \mathscr{P}\exp\left\{ -i \int_{-\infty}^{+\infty} dz^+ V_{\rm I}(z^+) \right\} e^{-i\omega \hat{K}^3} \, | \phi_{\rm i} \rangle$$

$$= \langle \phi_{\rm f} | \, \mathscr{P}\exp\left\{ -i \int_{-\infty}^{+\infty} dz^+ e^{i\omega \hat{K}^3} V_{\rm I}(z^+) e^{-i\omega \hat{K}^3} \right\} | \phi_{\rm i} \rangle \, .$$

S-matrix element of boosted interaction for unboosted states

**Light-cone Hamiltonian in the background fields** → **Boosting the background fields, expanding in powers of $\xi = e^{-\omega}$** → **Small-x effective Hamiltonian up to linear order in $\xi$**

# The Formalism: Small-x Effective Hamiltonian

**Light-Cone Gauge** $A^+ = 0$.

**Expansion in** $\xi$ **is equivalent to expansion in** $x$: $\xi = e^{-|Y_P - Y_T|} \sim x e^{-m_N/Q}$

Order $\xi^0$: $V_{(0)} = a_b^- J_b^+ = a_b^- \left( g\bar{\Psi}\gamma^+ t^b \Psi - ig[A^i, F^{+i}]^b \right)$

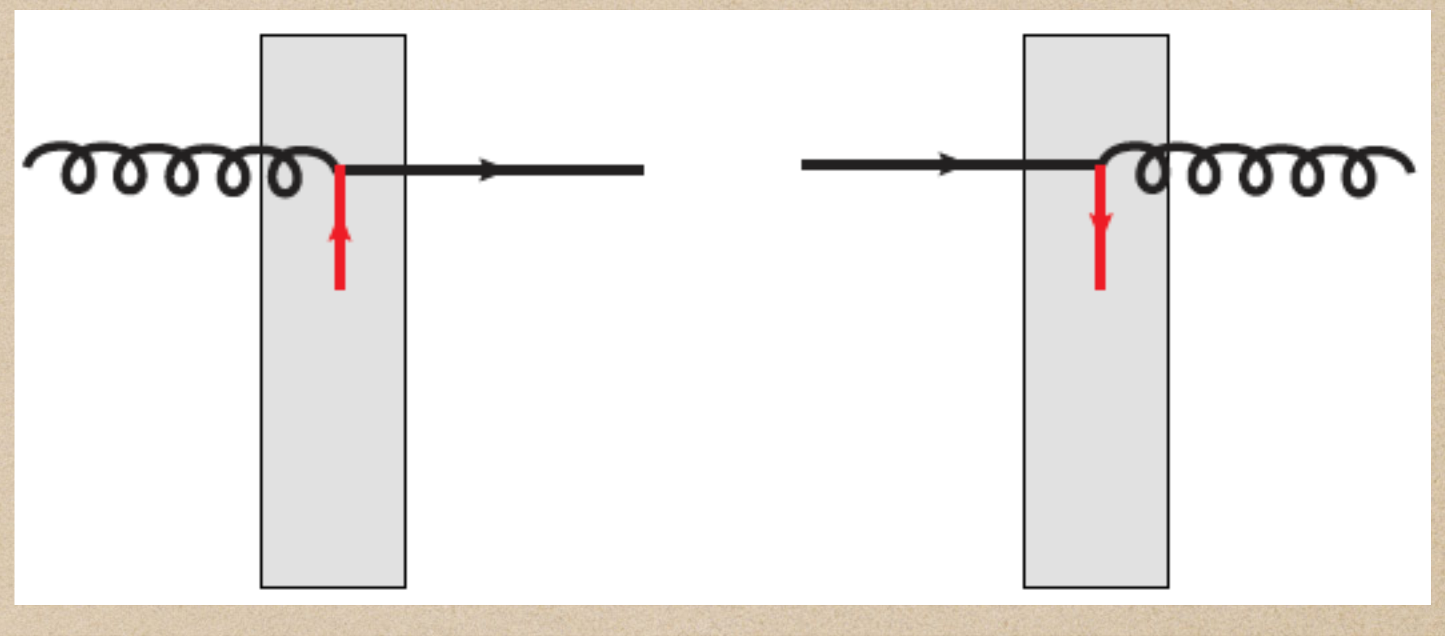
Order $\xi^{\frac{1}{2}}$: $V_{(\frac{1}{2})} = g\bar{\Psi}_G \gamma^i A_i \psi_B + g\bar{\psi}_B \gamma^i A_i \Psi_G$

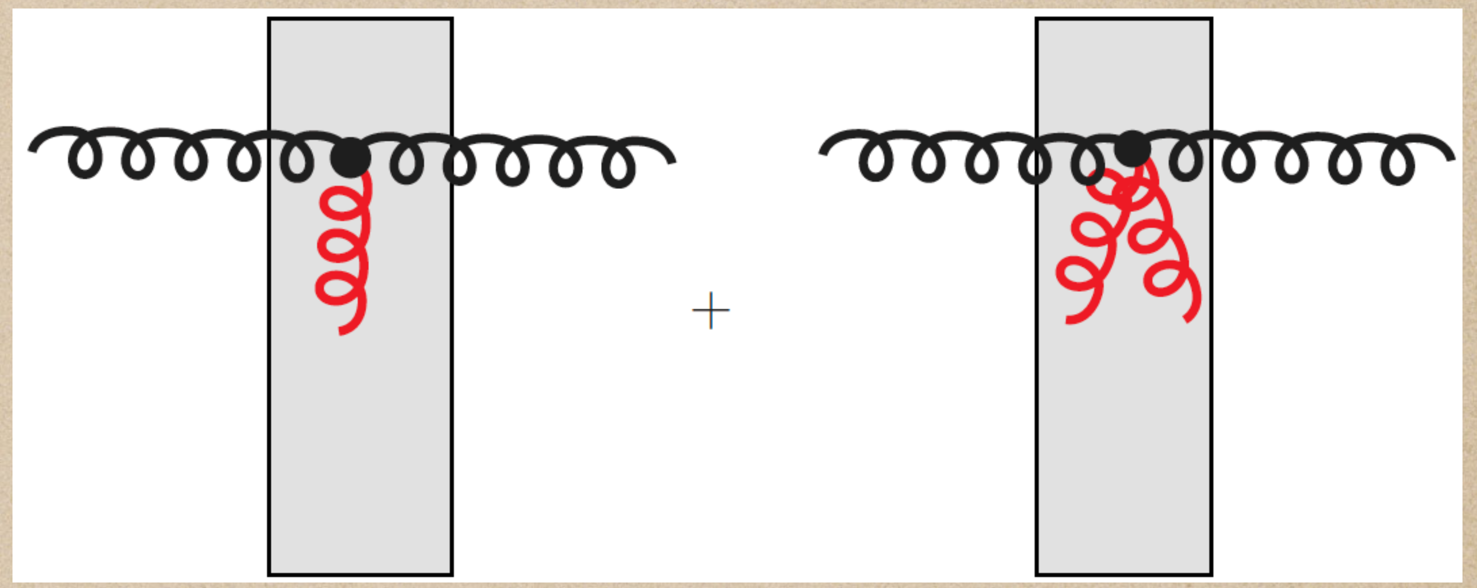
Order $\xi^1$:

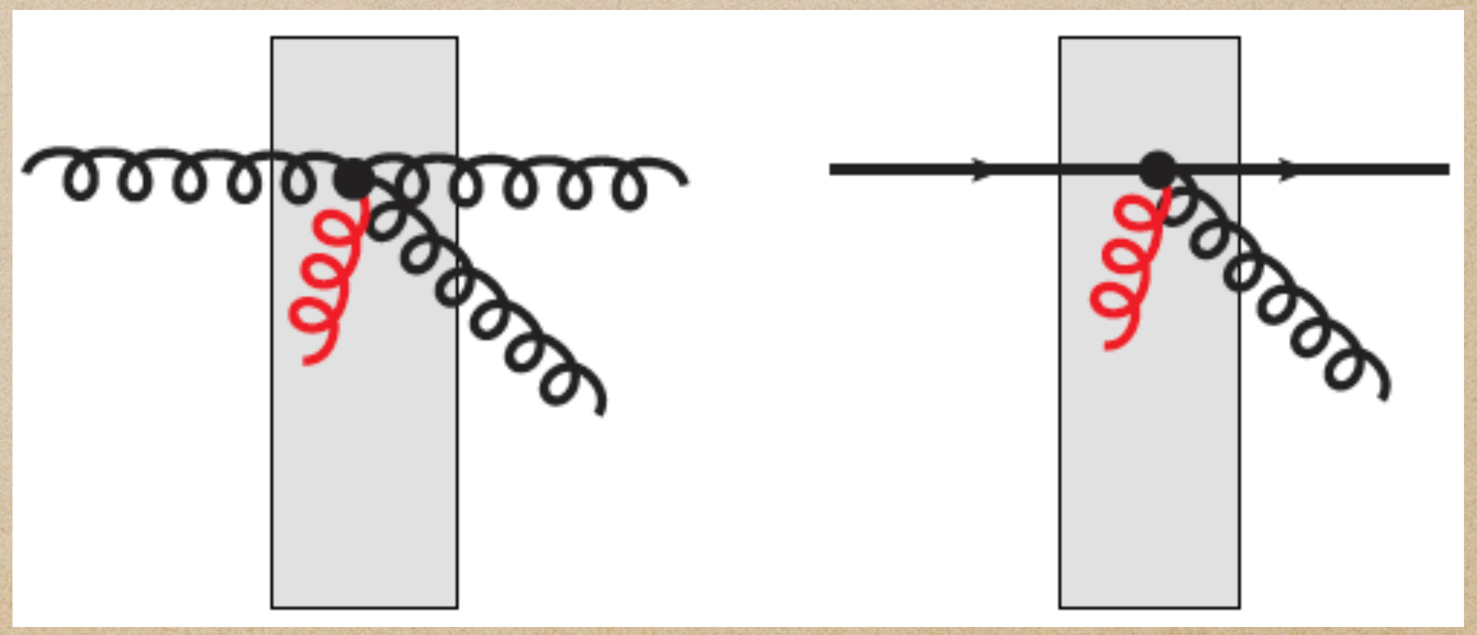
$$V_{(1)} = -\frac{1}{2} A_a^i \left( (\mathscr{D}_l \mathscr{D}^l)^{ab} g_{ij} + 2ig(f_{ij})^{ab} \right) A_b^j + \frac{i}{\sqrt{2}} \Psi_G^\dagger \left( g f_{ji} S^{ij} - \mathscr{D}_l \mathscr{D}^l \right) \frac{1}{\partial_-} \Psi_G$$
$$+ ig \left[ A_i, A_j \right]_b (\mathscr{D}^i A^j)_b + (\mathscr{D}_i A^i)_b \frac{1}{\partial_-} \left( -ig \left[ \partial_- A^j, A_j \right]^b + \sqrt{2} g \Psi_G^\dagger t^b \Psi_G \right)$$
$$+ \frac{1}{\sqrt{2}} g \Psi_G^\dagger A_j \gamma^j \gamma^i \mathscr{D}_i \frac{1}{\partial_-} \Psi_G + h.c.$$

$\Psi = \Psi_G + \Psi_B$ $\qquad \mathscr{D}^i = \partial^i + ig[a^i, .]$

**Background fields:** $a^-, a^i, \psi_B$ $\qquad$ **Quantum fields:** $\Psi_G, A^i$



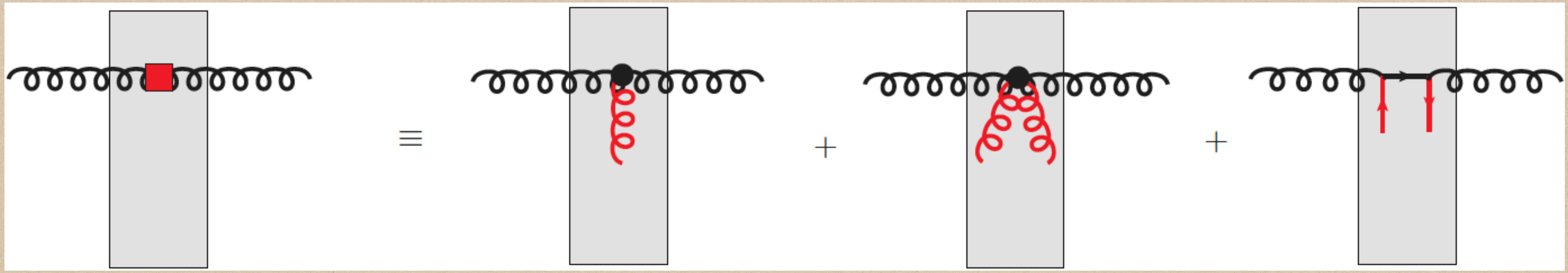


# The Formalism: Polarized Wilson Lines

**Single gluon scattering amplitude up to sub-eikonal order**

$$M^{g\to g} = (2\pi) 2k^+ \delta(k^+ - k'^+)\delta_{\lambda'\lambda}\Big[\delta(\mathbf{x}-\mathbf{y})U_{\mathbf{x}} + \xi\lambda\delta(\mathbf{x}-\mathbf{y})U_{\mathbf{x}}^{\text{pol}[1]}(k^+) + \xi U_{\mathbf{y},\mathbf{x}}^{\text{pol}[2]}(k^+)\Big]^{c'c}$$




$$U_{\mathbf{x}}^{\mathrm{G}[1]} = -\frac{2ig}{2k^+}\int_{-\infty}^{+\infty} dw^+ U_{\mathbf{x}}(+\infty, w^+) f_{12}(w^+, \mathbf{x})\, U_{\mathbf{x}}(w^+, -\infty)$$

$$U_{\mathbf{x}}^{\mathrm{q}[1]} = -\frac{g^2}{2k^+}\int_{-\infty}^{+\infty} dw_2^+ \int_{-\infty}^{w_2^+} dw_1^+\, U_{\mathbf{x}}^{c'h'}(+\infty, w_2^+)\bar{\psi}^{\alpha}_{B,n'}(w_2^+, \mathbf{x})\left[t^{h'}V_{\mathbf{x}}(w_2^+, w_1^+)t^h\right]^{n'n}\left[\frac{\gamma^-\gamma^5}{2}\right]^{\alpha\beta}\psi^{\beta}_{B,n}(w_1^+, \mathbf{x})\, U_{\mathbf{x}}^{hc}(w_1^+, -\infty) + c.c.$$

$$U_{\mathbf{y},\mathbf{x}}^{\mathrm{G}[2]} = \frac{i}{2k^+}\int_{-\infty}^{+\infty} dw^+\, U_{\mathbf{y}}^{c'a}(+\infty, w^+)\int_{\mathbf{z}} \delta(\mathbf{y}-\mathbf{z})\left[\overleftarrow{\mathscr{D}}_l \overrightarrow{\mathscr{D}}^l\right]^{ab}(w^+, \mathbf{z})\delta(\mathbf{z}-\mathbf{x})\, U_{\mathbf{x}}^{bc}(w^+, -\infty),$$

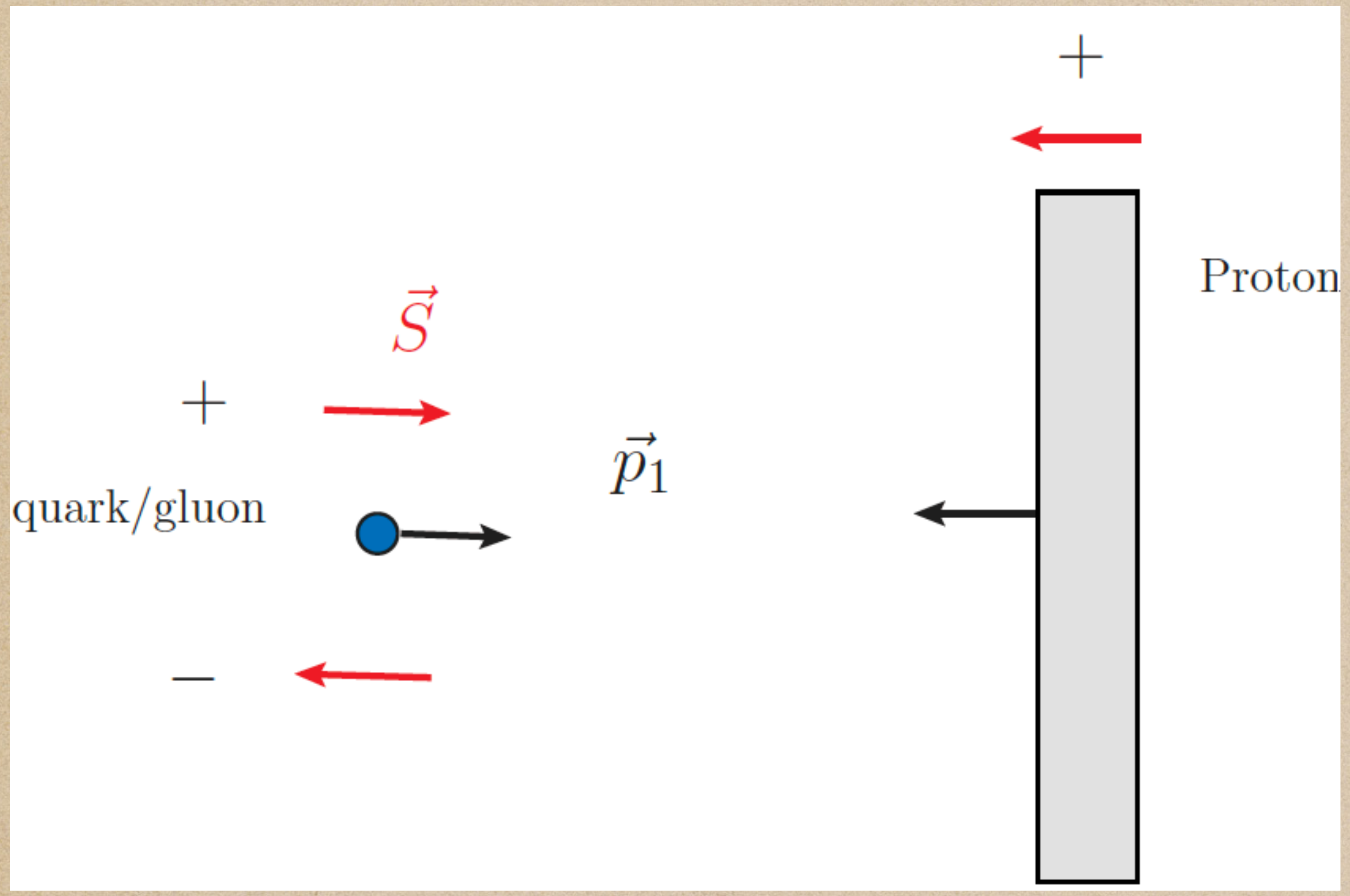
$$U_{\mathbf{x}}^{\mathrm{q}[2]} = -g^2\frac{1}{2k^+}\int_{-\infty}^{+\infty} dw_2^+ \int_{-\infty}^{w_2^+} dw_1^+ U_{\mathbf{x}}^{c'h'}(+\infty, w_2^+)\bar{\psi}^{\alpha}_{B,n'}(w_2^+, \mathbf{x})\left[t^{h'}V_{\mathbf{x}}(w_2^+, w_1^+)t^h\right]^{n'n}\left[\frac{\gamma^-}{2}\right]^{\alpha\beta}\psi^{\beta}_{B,n}(w_1^+, \mathbf{x})U_{\mathbf{x}}^{hc}(w_1^+, -\infty) + c.c.$$

# $A_{LL}^{g}$ at small x: gluon production at mid-rapidity

**Large-$x$ Gluon initiated processes:** $g \to g + g, \quad g \to q + g, \quad g \to \bar{q} + g\,.$

**Large-$x$ Quark (antiquark) initiated processes:** $q \to q + g, \quad q \to g + g;$

$\bar{q} \to \bar{q} + g, \quad \bar{q} \to g + g\,.$




The talk focuses on large-$x$ gluon initiated processes.

$$M = M_{(0)}^{g\to gg} + \xi^{\frac{1}{2}} M_{(\frac{1}{2})}^{g\to qg} + \xi M_{(1)}^{g\to gg} + \ldots$$

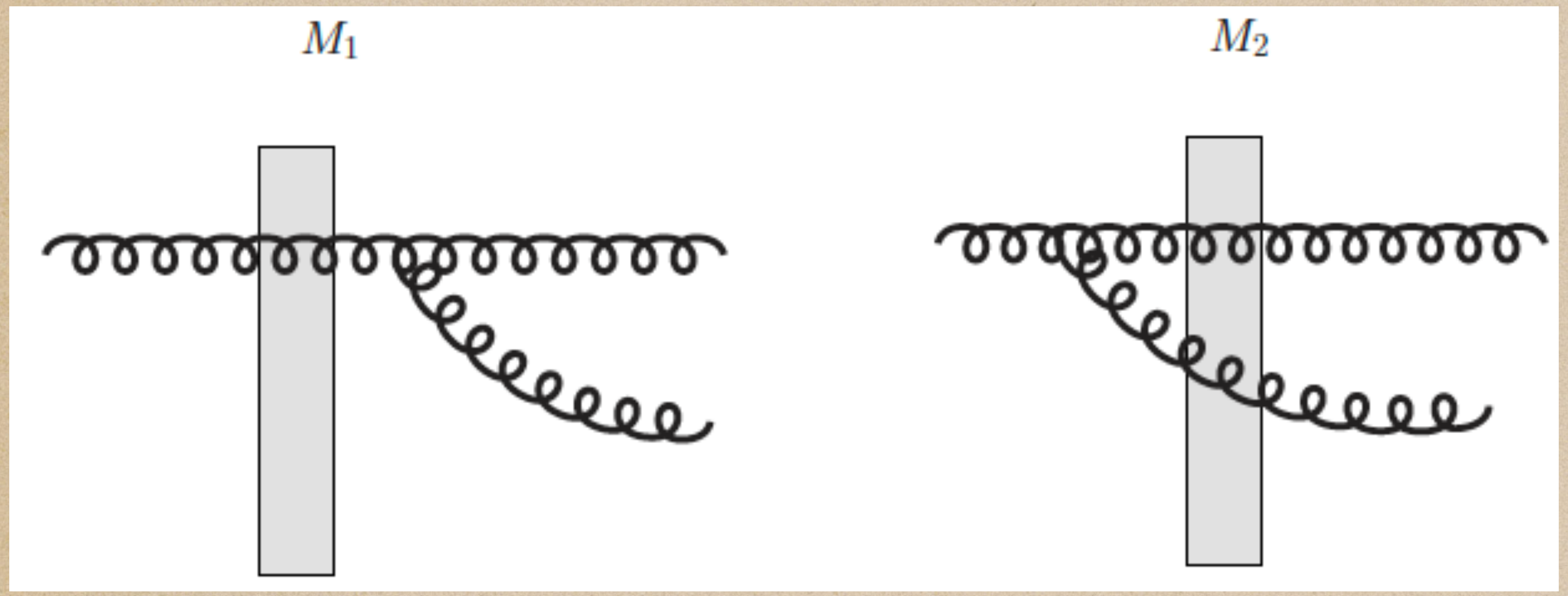
$$|M|^2 = |M_{(0)}^{g\to gg}|^2 + \xi\Big( |M_{(\frac{1}{2})}^{g\to qg}|^2 + M_{(1)}^{g\to gg}(M_{(0)}^{g\to gg})^* + c.c.\Big) + \ldots$$

**Isolate the part that is proportional to the polarization of incoming gluon.**

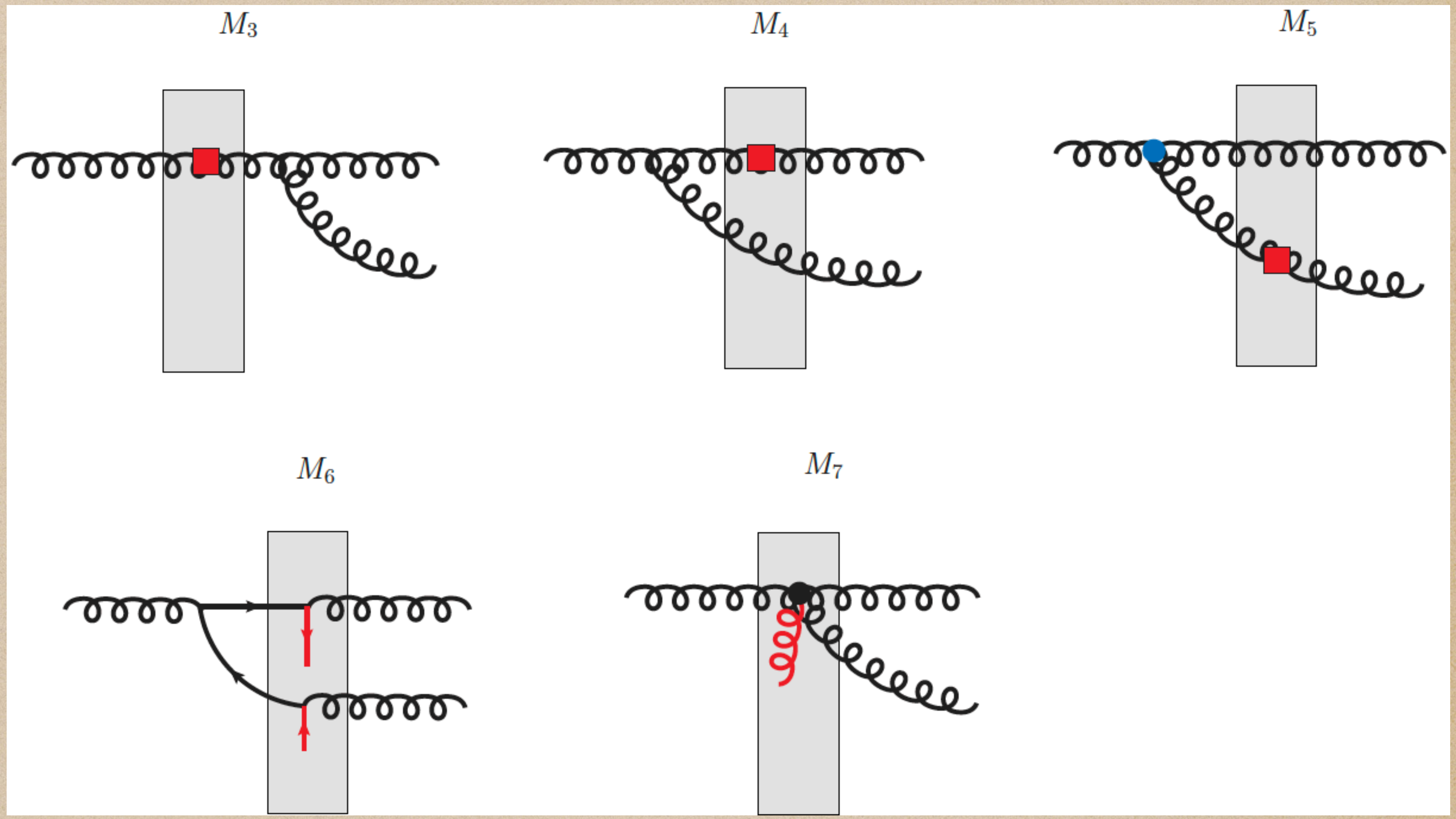
# $A^g_{LL}$ at small x: gluon production at mid-rapidity

**The channel** $g \to g + g$

**Eikonal Order** $\xi^0$



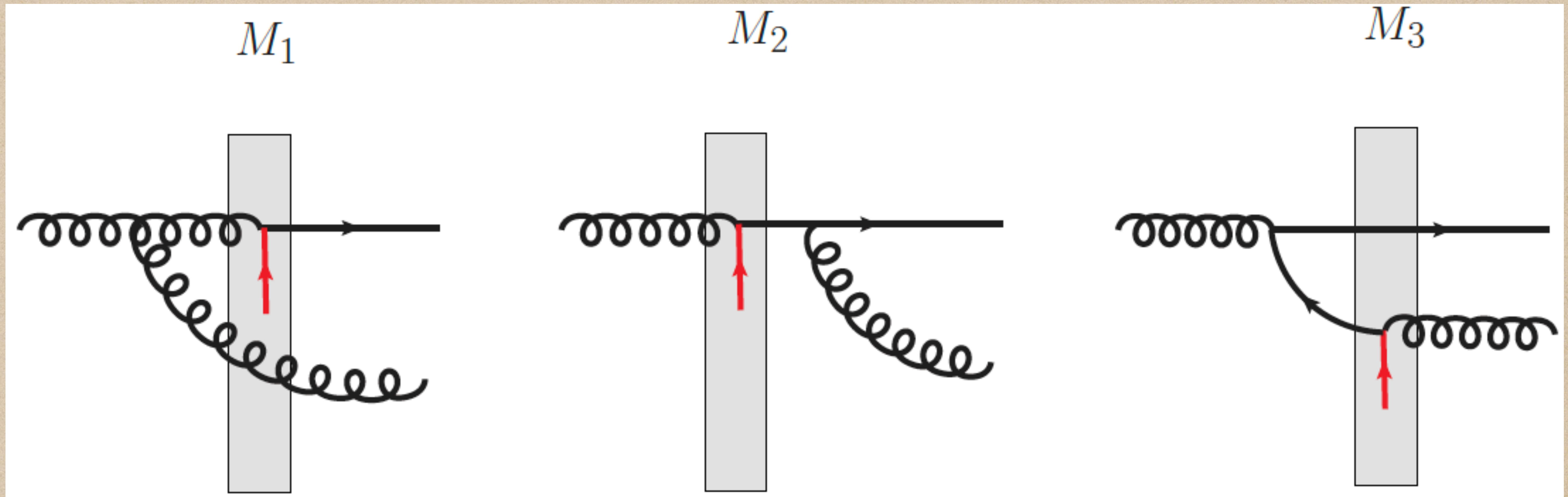

**Sub-eikonal Order** $\xi^1$



# $A_{LL}^{g}$ at small x: gluon production at mid-rapidity

**The channels** $g \to q + g, \quad g \to \bar{q} + g$

**Sub-eikonal Order** $\xi^{\frac{1}{2}}$



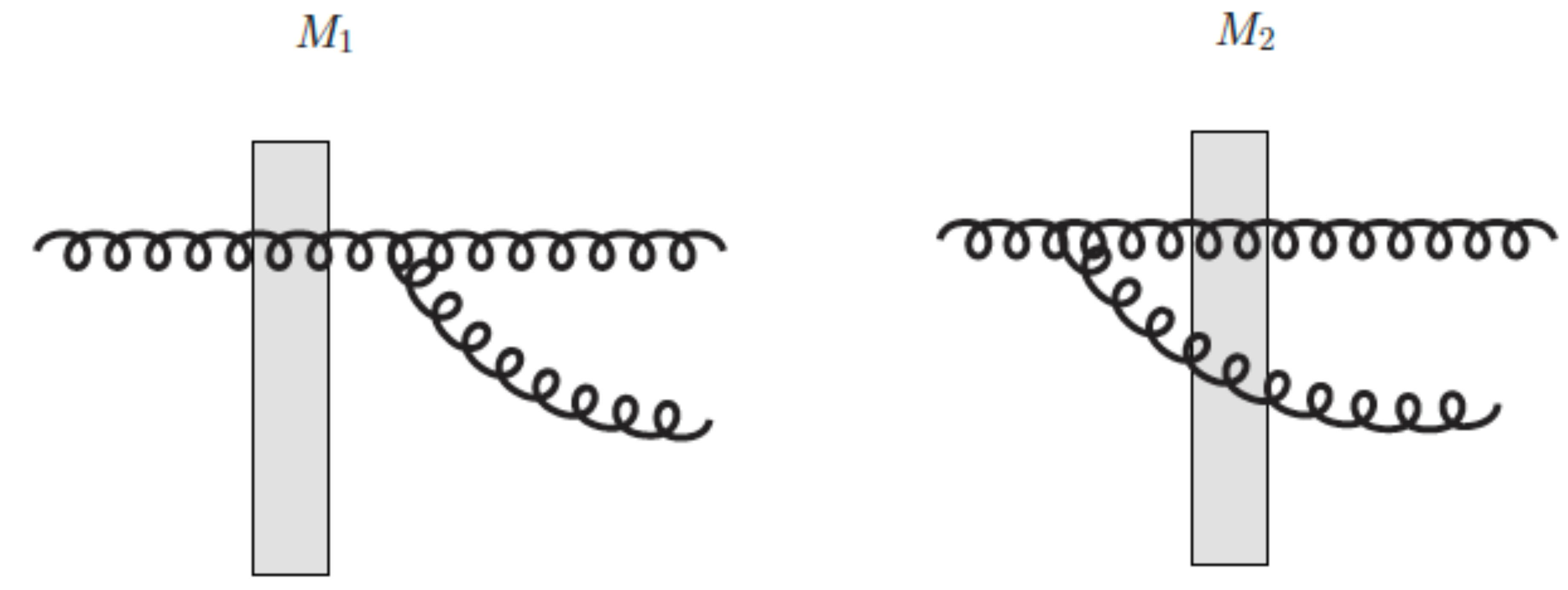

$$|M|^2 = |M_{(0)}^{g\to gg}|^2 + \xi\Big(\,|M_{(\frac{1}{2})}^{g\to qg}|^2 + M_{(1)}^{g\to gg}(M_{(0)}^{g\to gg})^* + c.c.\Big) + \ldots$$

**Various cancellations and combinations between these two channels**

# Review: Eikonal Order Gluon Production

**Eikonal order gluon production at mid-rapidity**

$$\frac{d\sigma}{d^2\mathbf{p}_1 dy} = \frac{\alpha_s C_A}{\pi^2}\int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_1'} e^{-i\mathbf{p}_1\cdot\mathbf{x}_{11'}}\frac{1}{(2\pi)^2}\frac{\mathbf{x}_{10}\cdot\mathbf{x}_{1'0}}{|\mathbf{x}_{10}|^2|\mathbf{x}_{1'0}|^2}\Big[D(\mathbf{x}_0,\mathbf{x}_0) - D(\mathbf{x}_0,\mathbf{x}_1) - D(\mathbf{x}_0,\mathbf{x}_1') + D(\mathbf{x}_1,\mathbf{x}_1')\Big]$$

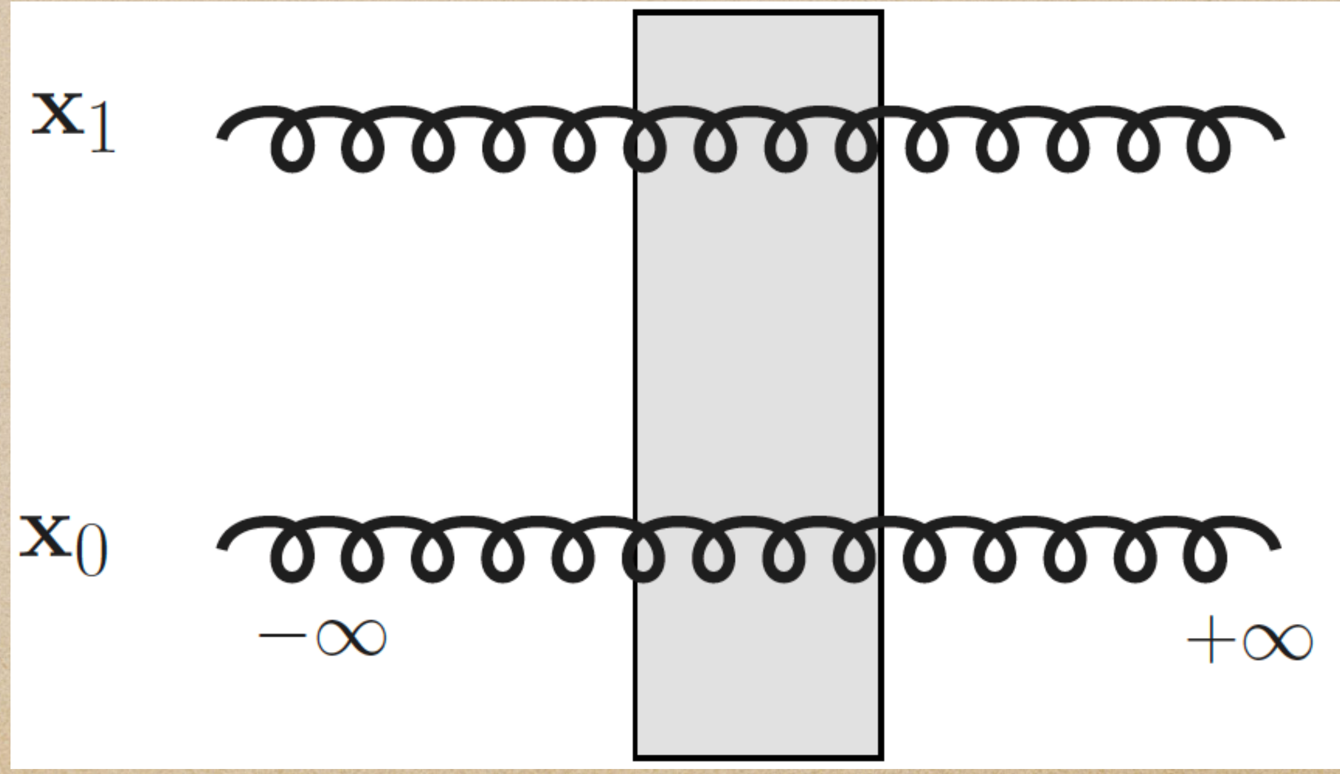



**Gluon Dipole Correlator:**

$$D(\mathbf{x}_0,\mathbf{x}_1) = \frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[U_{\mathbf{x}_0}^\dagger U_{\mathbf{x}_1}\right]\right\rangle$$

**Related to the dipole gluon TMD in small x limit**

$$D(x,\mathbf{k}^2) = \frac{g^2}{4(N_c^2-1)}\frac{xG(x,\mathbf{k}^2)}{\mathbf{k}^2}$$

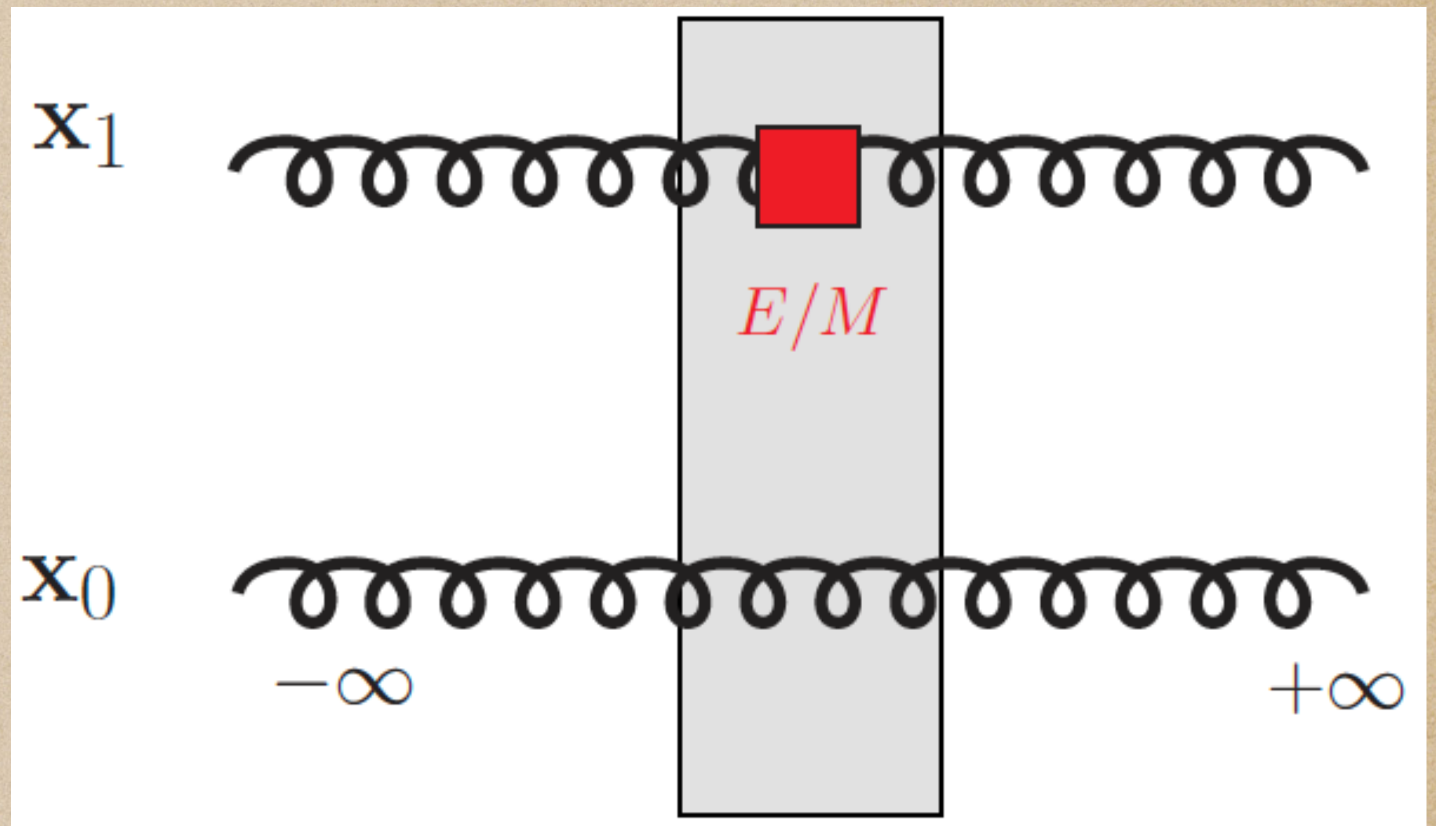


# Polarized Wilson Line Correlators

**Background gluon field polarized Wilson line correlators.**

$$\Delta D^{g}_{M}(p^+;\mathbf{x}_0,\mathbf{x}_1)=\frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[U^{\dagger}_{\mathbf{x}_0}U^{G[1]}_{\mathbf{x}_1}(p^+)\right]\right\rangle$$

$$\Delta D^{g,j}_{\mathrm{E}}(p^+;\mathbf{x}_0,\mathbf{x}_1)=\frac{1}{2(N_c^2-1)}\left\langle \mathrm{Tr}\left[U^{\dagger}_{\mathbf{x}_0}U^{j,G[2]}_{\mathbf{x}_1}(p^+)-U^{j,G[2]\dagger}_{\mathbf{x}_0}(p^+)U_{\mathbf{x}_1}\right]\right\rangle$$

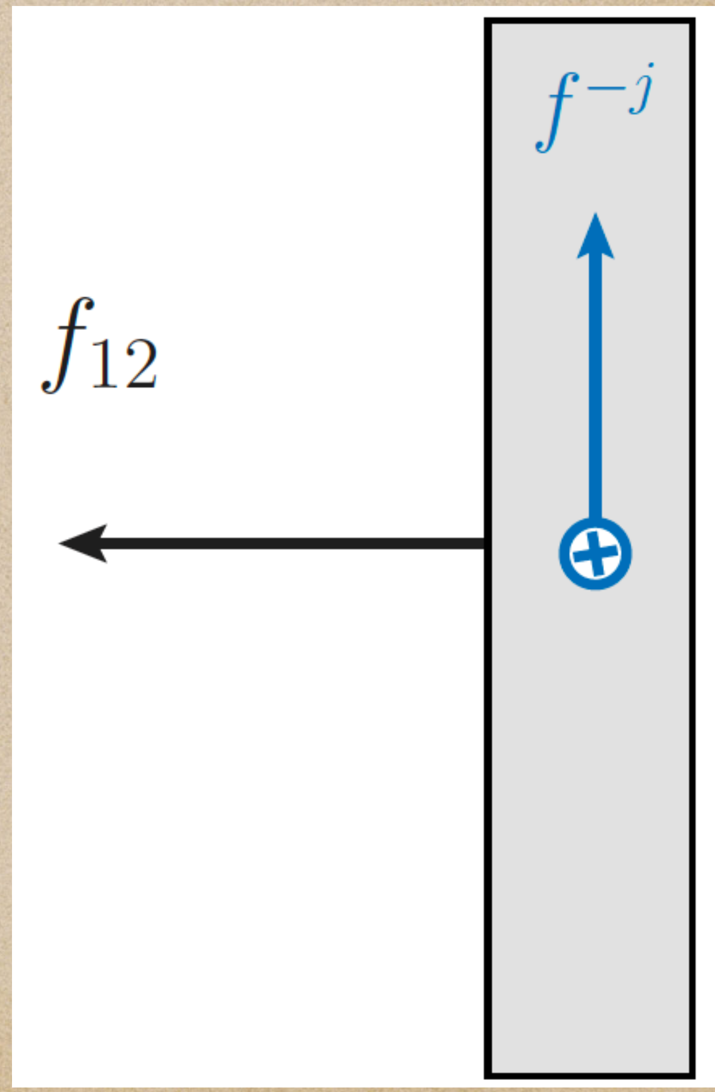



**Magnetically Polarized Wilson Line**

$$U^{G[1]}_{\mathbf{x}}(p^+)=-\frac{2ig}{2p^+}\int_{-\infty}^{+\infty}dw^+U_{\mathbf{x}}(+\infty,w^+)f_{12}(w^+,\mathbf{x})\,U_{\mathbf{x}}(w^+,-\infty),$$

**Electrically Polarized Wilson Line**

$$U^{j,G[2]}_{\mathbf{x}}(p^+)=\frac{ig}{2p^+}\int_{-\infty}^{+\infty}dw^+\,w^+\,U_{\mathbf{x}}(+\infty,w^+)f^{-j}(w^+,\mathbf{x})\,U_{\mathbf{x}}(w^+,-\infty)\,.$$




**No contributions from transverse magnetic field $f^{+j}$ and longitudinal electric field $f^{+-}$.**

High order in eikonality

Longitudinal momentum exchange

# Gluon Helicity TMDs at Small x

In the small x limit: *Cougoulic, Kovchegov, Tarasov, and Tawabutr (2022)*

$$\Delta D_E^g(x, \mathbf{k}^2) = \frac{g^2}{2(N_c^2-1)} g_{1L}^g(x, \mathbf{k}^2)$$

**Gluon Helicity TMD**

$$\Delta D_M^g(x, \mathbf{k}^2) = \frac{g^2}{4(N_c^2-1)} x\Delta H_{3L}^\perp(x, \mathbf{k}^2)$$

**Twist-3 Gluon TMD**

Gluon correlation functions: *Mulders and Rodrigues (2001)*

$$\Gamma^{\mu\nu;\rho\sigma}(k, P, S) = \int d^4\mathbf{x}\, e^{ik\cdot x} \left\langle P, S \,|\, \mathrm{Tr}\left[F^{\mu\nu}(0)\mathcal{U}^{[+]}(0,x)F^{\rho\sigma}(x)\mathcal{U}^{[-]}(x,0)\right] |\, P, S \right\rangle$$

$$\rightarrow \quad g_{1L}^g(x, \mathbf{k}^2) \subset \Gamma^{-i;-j}(k, P, S), \quad x\Delta H_{3L}^\perp(x, \mathbf{k}^2) \subset \Gamma^{ij;l-}(k, P, S)$$

$$\int_{\mathbf{k}} x\Delta H_{3L}^\perp(x, \mathbf{k}^2) = 0$$

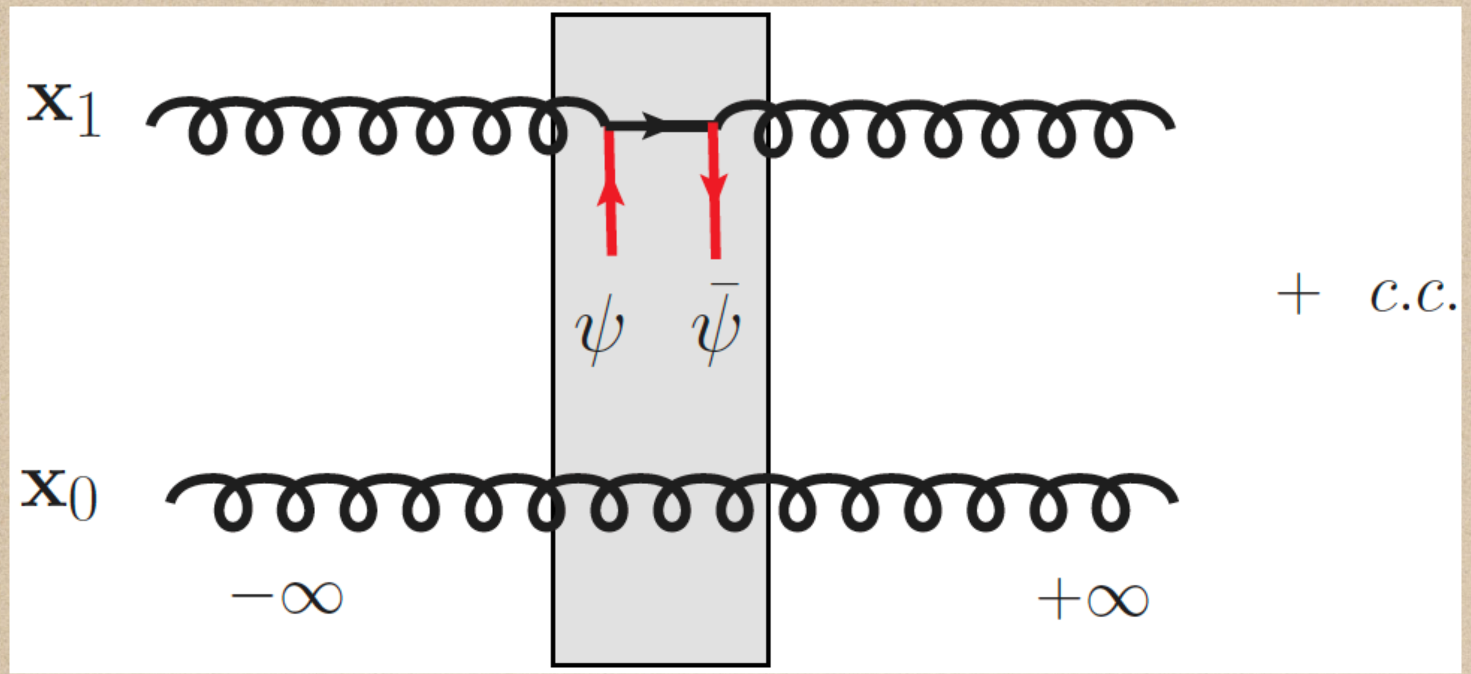
**Twist-3 gluon TMD does not contribute to gluon helicity PDF**

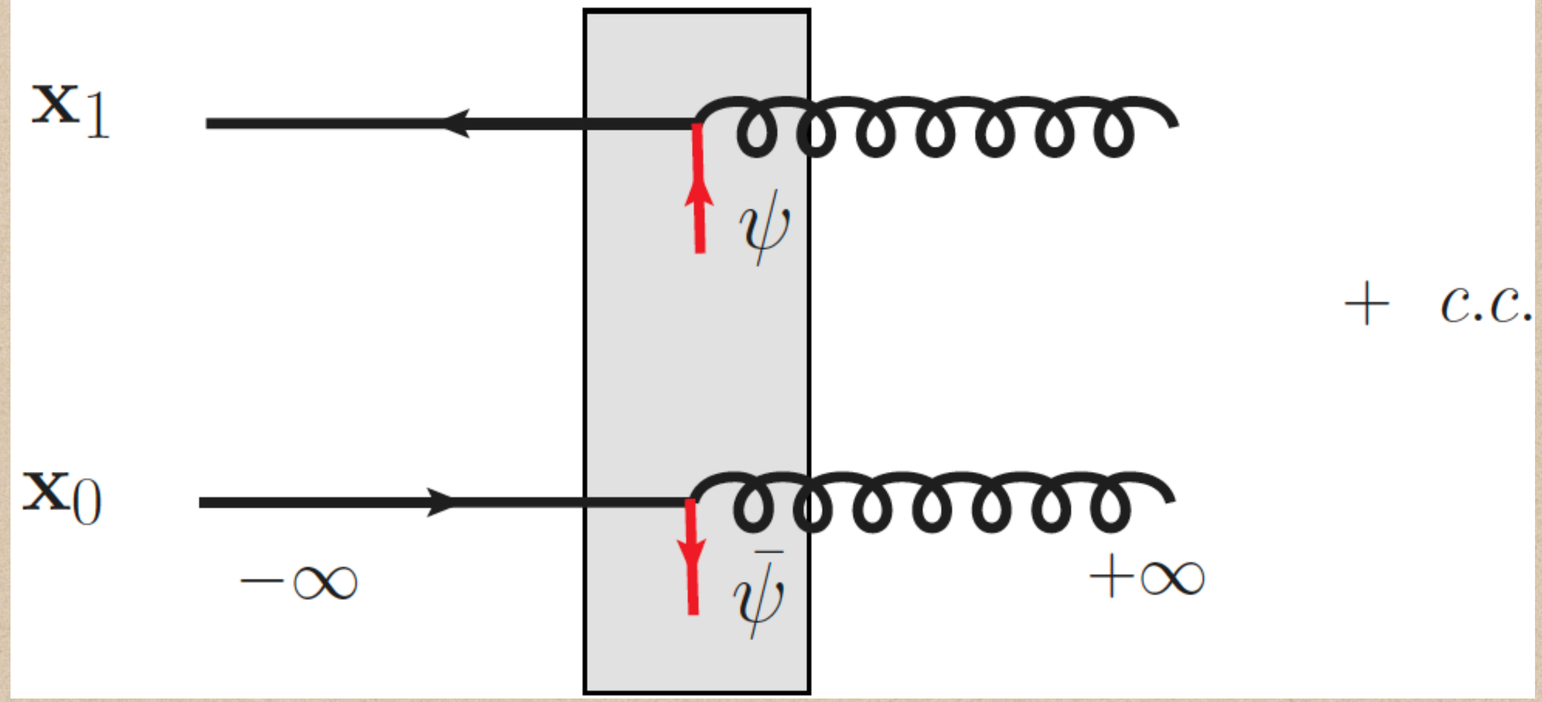
# Polarized Wilson Line Correlators and Quark Helicity TMD at small x

**Background quark field polarized Wilson line correlators.**

$$\Delta D^q_{[1]}(p^+;\mathbf{x}_0,\mathbf{x}_1) = \frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[U^\dagger_{\mathbf{x}_0} U^{q[1]}_{\mathbf{x}_1}(p^+)\right]\right\rangle$$

$$\Delta D^q_{[2]}(p^+;\mathbf{x}_0,\mathbf{x}_1) = \frac{1}{2(N_c^2-1)}\frac{g^2}{2p^+}\int_{-\infty}^{+\infty} dw^+ dw'^+\, U^{ce}_{\mathbf{x}_0}(+\infty,w'^+)\,\bar\psi_B(w'^+,\mathbf{x}_0)$$
$$\times\left[t^e V_{\mathbf{x}_0}(w'^+,-\infty)V^\dagger_{\mathbf{x}_1}(w^+,-\infty)t^d\right]\left[\frac{\gamma^-\gamma^5}{2}\right]\psi_B(w^+,\mathbf{x}_1)\,U^{cd}_{\mathbf{x}_1}(+\infty,w^+) + c.c.$$




**Definition of Quark Helicity TMD**

$$g^q_{1L}(x,\mathbf{k}^2) = \frac{1}{2V^+}\sum_{S_L} S_L \int dz^+ d^2\mathbf{z}\, dy^+ d^2\mathbf{y}\, e^{ixP^-(z^+-y^+)} e^{-i\mathbf{k}\cdot(\mathbf{z}-\mathbf{y})}\langle P,S_L|\bar\Psi(y)\mathcal{U}[y,z]\frac{\gamma^-\gamma^5}{2}\Psi(z)|P,S_L\rangle$$

**In the small $x$ limit:**

$$\Delta D^q_{[2]}(k^+;\mathbf{k}^2) = \frac{g^2}{4(N_c^2-1)}\left[g^q_{1L}(x,\mathbf{k}^2) + g^{\bar q}_{1L}(x,\mathbf{k}^2)\right]$$

**Gauge link:**

$$\mathcal{U}_{\mathbf{x}_0,\mathbf{x}_1} = V_{\mathbf{x}_0}(w'^+,+\infty)t^c V_{\mathbf{x}_0} V^\dagger_{\mathbf{x}_1} t^c V_{\mathbf{x}_1}(+\infty,w^+) = \frac{1}{2}\mathcal{U}^{[+]}_{\mathbf{x}_0,\mathbf{x}_1}\mathrm{tr}[V_{\mathbf{x}_0}V^\dagger_{\mathbf{x}_1}] - \frac{1}{2N_c}\mathcal{U}^{[-]}_{\mathbf{x}_0,\mathbf{x}_1}$$

# $A^{g}_{\rm LL}$ at small x : Results

**<u>In Momentum Space</u>**

**Sub-Eikonal order:**

$$\frac{d\sigma_\lambda}{d^2\mathbf{p}_1 dy} = \lambda\frac{\alpha_s C_A}{\pi^2}\int_{\mathbf{k}}\left[\frac{2}{\mathbf{p}_1^2} - \frac{6\mathbf{p}_1\cdot(\mathbf{p}_1-\mathbf{k})}{\mathbf{p}_1^2|\mathbf{p}_1-\mathbf{k}|^2} + \frac{4}{|\mathbf{p}_1-\mathbf{k}|^2}\right]\Big(\Delta D^g_{\rm M}(p^+;\mathbf{k}^2) + \Delta D^q_{[1]}(p^+;\mathbf{k}^2)\Big)$$
$$+\lambda\frac{\alpha_s C_F}{\pi^2}\int_{\mathbf{k}}\left[\frac{1}{(\mathbf{p}_1-\mathbf{k})^2} - \frac{2\mathbf{p}_1\cdot(\mathbf{p}_1-\mathbf{k})}{\mathbf{p}_1^2|\mathbf{p}_1-\mathbf{k}|^2} + \frac{2}{\mathbf{p}_1^2}\right]\Delta D^q_{[2]}(p^+;\mathbf{k}^2)$$
$$+\lambda\frac{\alpha_s C_A}{\pi^2}\int_{\mathbf{k}}\left[\frac{2(\mathbf{p}_1\times\mathbf{k})^2}{(\mathbf{p}_1-\mathbf{k})^2\mathbf{k}^2\mathbf{p}_1^2} - \frac{(\mathbf{p}_1-\mathbf{k})\cdot\mathbf{k}}{|\mathbf{p}_1-\mathbf{k}|^2\mathbf{k}^2}\right]\Delta D^g_{\rm E}(p^+;\mathbf{k}^2)\,.$$

**Eikonal order:**

$$\frac{d\sigma_0}{d^2\mathbf{p}_1 dy} = \frac{\alpha_s C_A}{\pi^2}\int_{\mathbf{k}}\left[\frac{1}{\mathbf{p}_1^2} - \frac{2\mathbf{p}_1\cdot(\mathbf{p}_1-\mathbf{k})}{\mathbf{p}_1^2|\mathbf{p}_1-\mathbf{k}|^2} + \frac{1}{|\mathbf{p}_1-\mathbf{k}|^2}\right]D(\mathbf{k}^2)\,.$$

**Longitudinal double-spin asymmetry for gluon production at mid-rapidity:**

$$A^g_{\rm LL} = \frac{\dfrac{d\sigma_{\lambda=+1}}{d^2\mathbf{p}_1 dy} - \dfrac{d\sigma_{\lambda=-1}}{d^2\mathbf{p}_1 dy}}{2\dfrac{d\sigma_0}{d^2\mathbf{p}_1 dy}}$$

# $A_{\rm LL}^{g}$ at Small x in the Limit $\mathbf{p}_1 \to +\infty$

**Large transverse momentum limit**

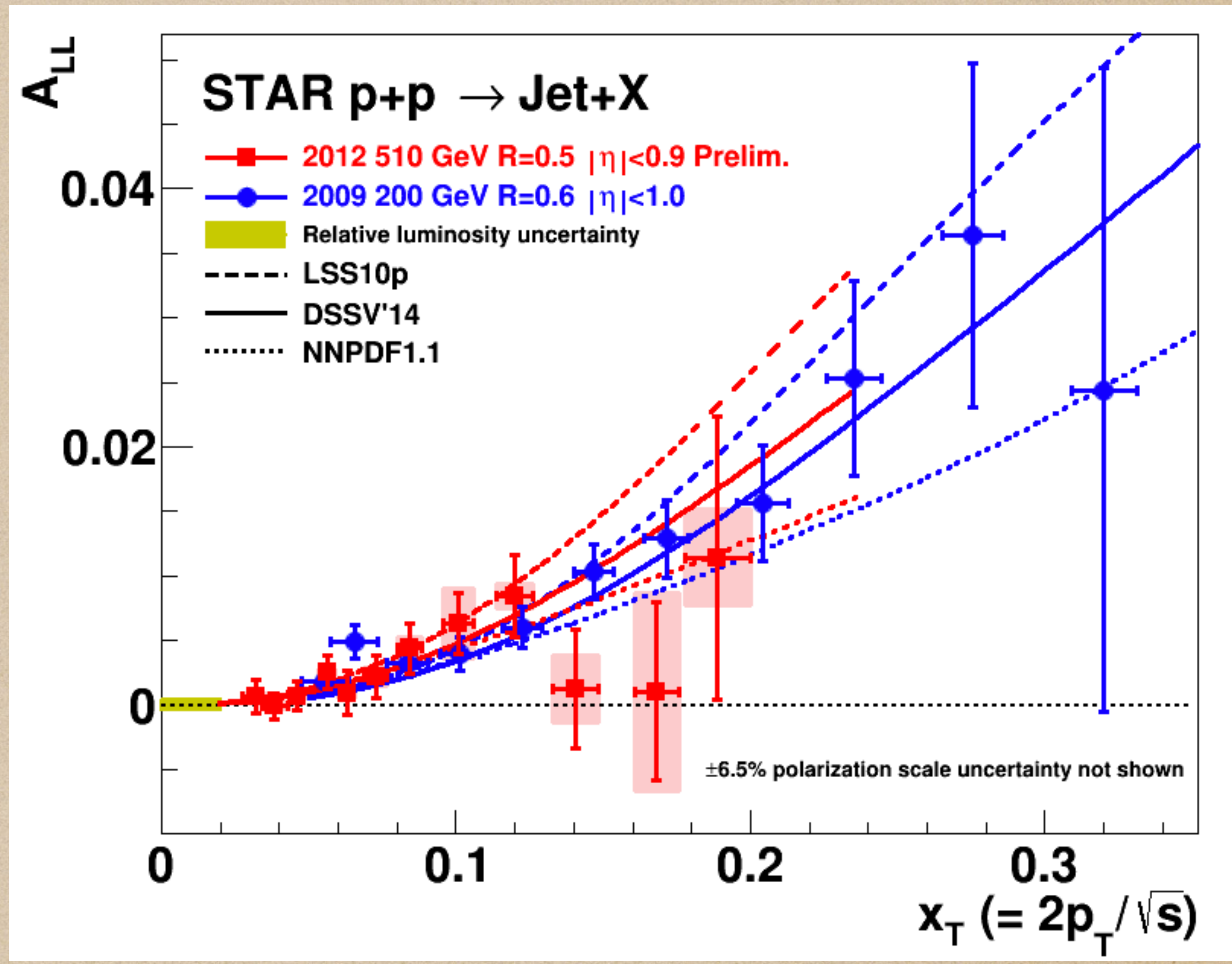
$$\frac{d\sigma_\lambda}{d\mathbf{p}_1 dy}\bigg|_{\mathbf{p}_1\to+\infty} = \lambda \frac{\alpha_s C_A}{\pi^2}\frac{1}{\mathbf{p}_1^2}\Delta D^g_{\rm E}(x,Q^2) + \lambda \frac{\alpha_s C_F}{\pi^2}\frac{1}{\mathbf{p}_1^2}\Delta D^q_{[2]}(x,Q^2) + \mathcal{O}(\frac{1}{\mathbf{p}_1^4})$$

Reproducing collinear factorization.

*Babcock, Monsay and Sivers (1979),*

$$\frac{d\sigma_0}{d^2\mathbf{p}_1 dy}\bigg|_{\mathbf{p}_1\to+\infty} = \frac{\alpha_s C_A}{\pi^2}\frac{1}{\mathbf{p}_1^4}\int_{\mathbf{k}} \mathbf{k}^2 D(\mathbf{k}^2) + \mathcal{O}(\frac{1}{\mathbf{p}_1^6})$$

$$A_{\rm LL}^{g}\bigg|_{\mathbf{p}_1\to+\infty} = \frac{2g^g_{1L}(x,Q^2) + \frac{C_F}{C_A}[g^q_{1L}(x,Q^2) + g^{\bar{q}}_{1L}(x,Q^2)]}{xG(x,Q^2)}\mathbf{p}_1^2 + \mathcal{O}(|\mathbf{p}_1|^0)$$




**Parabolic behavior in the large transverse momentum limit with the coefficient determined by the ratio of gluon and quark helicity PDFs over gluon PDF.**

# Conclusion

- **We developed an effective Hamiltonian approach to study small-x physics beyond Eikonal approximation with a focus on spin-related processes.**
- **For the first time, we calculated $A_{LL}^{g}$ at small x for gluon production at mid-rapidity in a transverse momentum dependent framework. We found four polarized Wilson line correlators which are related to the small x limit of quark and gluon helicity TMDs.**
- **At the large momentum limit, the transverse momentum dependence is parabolic with the coefficient determined by quark and gluon helicity PDFs and unpolarized gluon PDF.**
- **At the low momentum limit, all four polarized dipole correlators contribute comparably and transverse momentum dependent framework beyond the collinear factorization formalism has to be used to extract gluon helicity distribution at small x.**

# Backup: $A_{LL}^{g}$ at small x

**In Coordinate Space**

**Sub-Eikonal order:**

$$\frac{d\sigma_\lambda}{d^2\mathbf{p}_1 dy} = \lambda \frac{\alpha_s C_A}{\pi^2} \int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_1'} e^{-i\mathbf{p}_1\cdot\mathbf{x}_{11'}} \frac{1}{(2\pi)^2} \frac{\mathbf{x}_{1'0}\cdot\mathbf{x}_{10}}{|\mathbf{x}_{1'0}|^2|\mathbf{x}_{10}|^2} \Big[ 2\Delta D^{gq}_{\mathrm{M}[1]}(p^+;\mathbf{x}_0,\mathbf{x}_0) - 2\Delta D^{gq}_{\mathrm{M}[1]}(p^+;\mathbf{x}_0,\mathbf{x}_1) - 2\Delta D^{gq}_{\mathrm{M}[1]}(p^+;\mathbf{x}_0,\mathbf{x}_1')$$

$$-\Delta D^{gq}_{\mathrm{M}[1]}(p^+;\mathbf{x}_1,\mathbf{x}_0) - \Delta D^{gq}_{\mathrm{M}[1]}(p^+;\mathbf{x}_1',\mathbf{x}_0) + 2\Delta D^{gq}_{\mathrm{M}[1]}(p^+;\mathbf{x}_1',\mathbf{x}_1) + 2\Delta D^{gq}_{\mathrm{M}[1]}(p^+;\mathbf{x}_1,\mathbf{x}_1')\Big]$$

$$+\lambda \frac{\alpha_s C_F}{\pi^2} \int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_1'} e^{-i\mathbf{p}_1\cdot\mathbf{x}_{11'}} \frac{1}{(2\pi)^2} \frac{\mathbf{x}_{10}\cdot\mathbf{x}_{1'0}}{|\mathbf{x}_{10}|^2|\mathbf{x}_{1'0}|^2} \Big[ \Delta D^{q}_{[2]}(p^+;\mathbf{x}_1',\mathbf{x}_1) - \Delta D^{q}_{[2]}(p^+;\mathbf{x}_0,\mathbf{x}_1) - \Delta D^{q}_{[2]}(p^+;\mathbf{x}_0,\mathbf{x}_1') + 2\Delta D^{q}_{[2]}(p^+;\mathbf{x}_0,\mathbf{x}_0)\Big]$$

$$+\lambda \frac{\alpha_s C_A}{\pi^2} \left(\delta^{kj} + \frac{2\mathbf{p}_1^k \mathbf{p}_1^j}{\mathbf{p}_1^2}\right) \int_{\mathbf{x}_0,\mathbf{x}_1} e^{-i\mathbf{p}_1\cdot\mathbf{x}_{10}} \frac{1}{2\pi} \frac{\epsilon^{kl}\mathbf{x}_{10}^l}{|\mathbf{x}_{10}|^2} \left[4\Delta D^{g,j}_{\mathrm{E}}(p^+;\mathbf{x}_0,\mathbf{x}_1)\right].$$

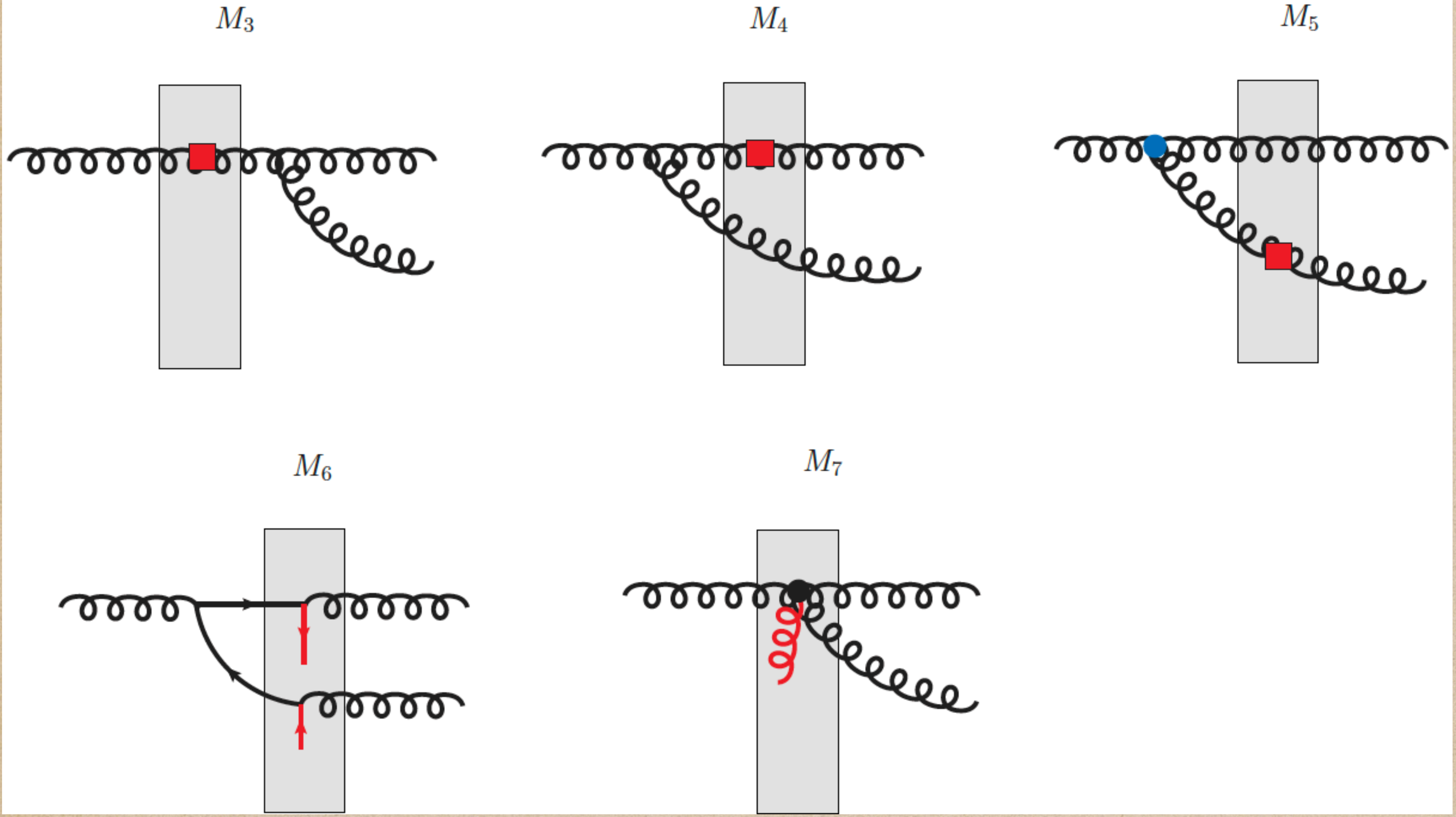
**Eikonal order:**

$$\frac{d\sigma_0}{d^2\mathbf{p}_1 dy} = \frac{\alpha_s C_A}{\pi^2} \int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_1'} e^{-i\mathbf{p}_1\cdot\mathbf{x}_{11'}} \frac{1}{(2\pi)^2} \frac{\mathbf{x}_{10}\cdot\mathbf{x}_{1'0}}{|\mathbf{x}_{10}|^2|\mathbf{x}_{1'0}|^2} \Big[ D(\mathbf{x}_0,\mathbf{x}_0) - D(\mathbf{x}_0,\mathbf{x}_1) - D(\mathbf{x}_0,\mathbf{x}_1') + D(\mathbf{x}_1,\mathbf{x}_1')\Big]$$

# Backup: Gluon Radiation Inside the Shockwave

**The channel $g \to g + g$**

**Sub-eikonal Order $\xi^1$**

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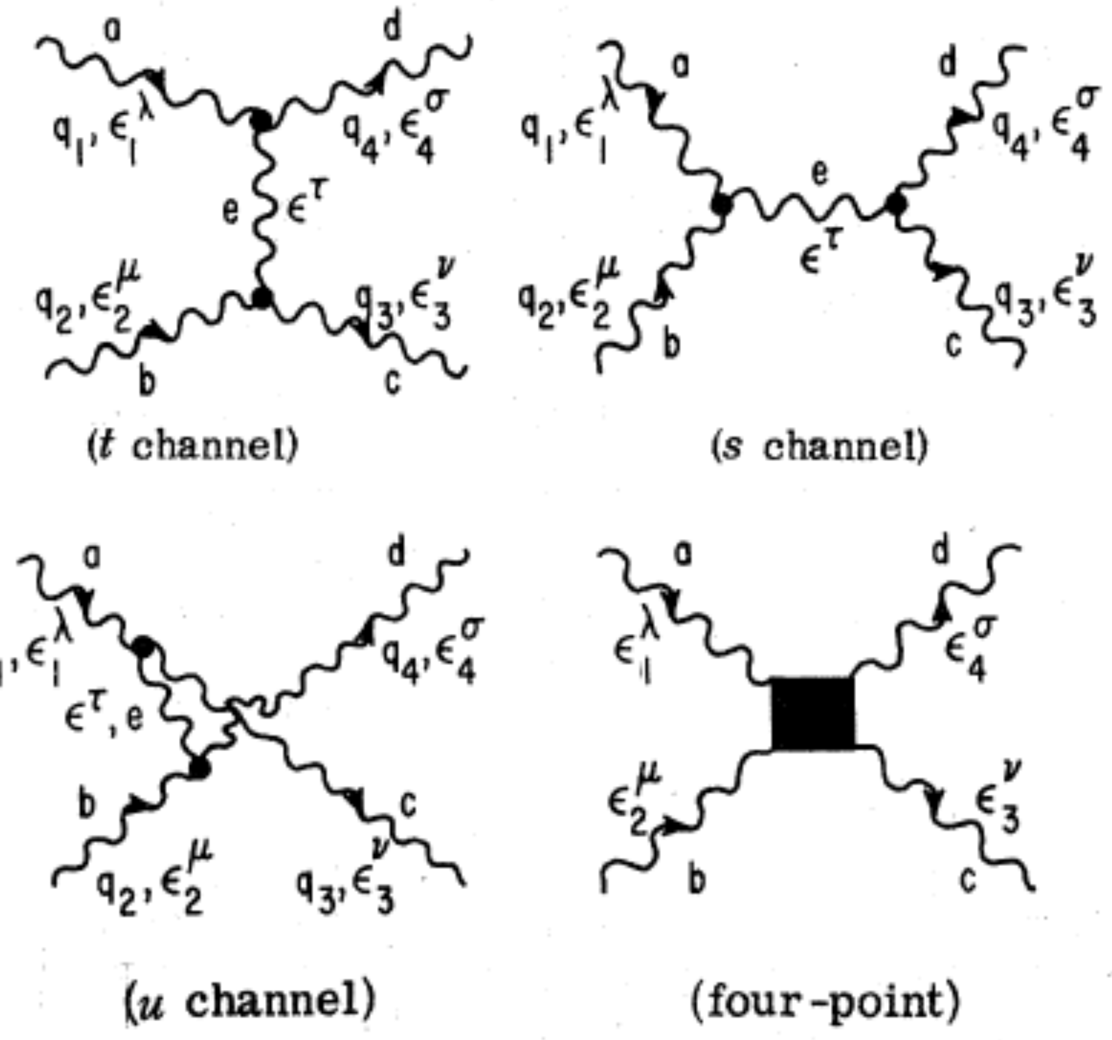
**Quantum-chromodynamic predictions for inclusive spin-spin asymmetries at large transverse momentum**

John Babcock,* Evelyn Monsay, and Dennis Sivers
*Argonne National Laboratory, High Energy Physics Division, Argonne, Illinois 60439*
(Received 20 October 1978)

We discuss predictions for the asymmetry $A_{LL}$ in the inclusive production at large $p_T$ of charged and neutral pions by longitudinally polarized protons. We work in the framework of a hard-scattering model based on perturbative quantum chromodynamics. Various assumptions for the distribution of the proton's spin among its constituents—quarks, ocean antiquarks, and gluons—and the effects of scaling violations on the parton distributions are considered.

F. The process $VV \to VV$

The diagrams for gluon-gluon scattering are



# Backup: Corresponding Partonic Processes

**The channels** $g \rightarrow q + g, \quad g \rightarrow \bar{q} + g$

*Babcock, Monsay and Sivers (1979),*

**Sub-eikonal Order** $\xi^{\frac{1}{2}}$