QCD EVOLUTION 2023

The Resolution to the problem of consistent large transverse momentum in TMDs

J. Osvaldo Gonzalez-Hernandez
Università di Torino & INFN Torino

Based on:

JOGH, T.C. Rogers T., N. Sato
Phys.Rev.D 106 (2022) 3, 034002 • e-Print: 2205.05750 [hep-ph]

JOGH, T. Rainaldi, T.C. Rogers
e-Print: 2303.04921 [hep-ph]
Accepted in Phys. Rev. D

Work in progress Jlab/ODU/Torino collaboration:
F. Aslan, M. Boglione, JOGH, T.C. Rogers, T. Rainaldi, A. Simonelli

OUTLINE

- ✳ CSS formula & Potential issues in pheno applications.
- ✳ Constraints on TMD models and HSO approach.
- ✳ Standard treatment vs HSO approach.

"Hadron structure oriented approach"

✳CSS formula & Potential issues in pheno applications.

Take the SIDIS cross section as an example

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\ \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}q_{\mathrm{T}}^2} = \frac{\pi^2 \alpha_{\mathrm{em}}^2 z}{Q^2\, x\, y} \left[F_1\, x\, y^2 + F_2\,(1-y)\right]$$

$$F = F^{\mathrm{TMD}} + \boxed{O\left(m/Q, q_{\mathrm{T}}/Q\right)}\,, \longleftarrow \quad \text{errors}$$

$$F_1^{\mathrm{TMD}} \equiv 2\,z \sum_j |H|_j^2 \left[f_{j/p}, D_{h/j}\right]\,, \qquad F_2^{\mathrm{TMD}} \equiv 4\,z\,x \sum_j |H|_j^2 \left[f_{j/p}, D_{h/j}\right]$$

$$
\begin{aligned}
[f_{j/p}, D_{h/j}] \to \int \frac{\mathrm{d}^2 \boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\, e^{-i\boldsymbol{q}_\mathrm{T}\cdot \boldsymbol{b}_\mathrm{T}} & \boxed{\tilde{f}_{j/p}(x, \boldsymbol{b}_\mathrm{T}; \mu_{Q_0}, \mu_{Q_0}^2)\ \tilde{D}_{h/j}(z, \boldsymbol{b}_\mathrm{T}; \boldsymbol{b}_\mathrm{T}; \mu_{Q_0}, \mu_{Q_0}^2)} \\
\times \exp\left\{ 2\int_{\mu_{Q_0}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[\gamma(\alpha_s(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(\alpha_s(\mu')) \right] + \ln \frac{Q^2}{Q_0^2} \tilde{K}(\boldsymbol{b}_\mathrm{T}; \mu_{Q_0}) \right\} .
\end{aligned}
$$

Operator definitions:
Universality, predictive power, true properties of hadrons.

These definitions imply a behavior at small bT (large kT), calculable in pQCD.

$$
\begin{aligned}
[f_{j/p}, D_{h/j}] \to \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\ e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\ & \boxed{\tilde{f}_{j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},\mu_{Q_0}^2)\ \tilde{D}_{h/j}(z,\boldsymbol{b}_\mathrm{T};\boldsymbol{b}_\mathrm{T};\mu_{Q_0},\mu_{Q_0}^2)} \\
& \times \exp\left\{2\int_{\mu_{Q_0}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right] + \ln\frac{Q^2}{Q_0^2}\tilde{K}(\boldsymbol{b}_\mathrm{T};\mu_{Q_0})\right\}\ .
\end{aligned}
$$

$$
\begin{aligned}
[f_{j/p}, D_{h/j}] \to \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\ e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\ & \tilde{f}_{j/p}(x,\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}^2)\ \tilde{D}_{h/j}(z,\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}^2) \\
& \times \exp\left\{2\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right] + \ln\frac{Q^2}{\mu_{b_*}^2}\tilde{K}(b_*;\mu_{b_*})\right\} \\
& \times \exp\left\{-g_{j/p}(x,b_\mathrm{T}) - g_{h/j}(z,b_\mathrm{T}) - g_K(b_\mathrm{T})\ln\left(\frac{Q^2}{Q_0^2}\right)\right\}\ .
\end{aligned}
$$

Same formula, just reorganized

$$
-g_{j/p}(x,b_\mathrm{T}) \equiv \ln\left(\frac{\tilde{f}_{j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)}{\tilde{f}_{j/p}(x,\boldsymbol{b}_*;\mu_{Q_0},Q_0^2)}\right)\ , \qquad -g_{h/j}(z,b_\mathrm{T}) \equiv \ln\left(\frac{\tilde{D}_{h/j}(z,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)}{\tilde{D}_{h/j}(z,\boldsymbol{b}_*;\mu_{Q_0},Q_0^2)}\right)\ ,
$$

$$
g_K(b_\mathrm{T}) \equiv \tilde{K}(b_*;\mu) - \tilde{K}(b_\mathrm{T};\mu)\ .
$$

Precise definitions for g functions, $b_*(b_T)$ is a transition function bounded by some b_{max}. Note that b_* dependence cancels exactly. It is really unimportant which b_* we choose.

$$[f_{j/p}, D_{h/j}] \to \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\, e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\, \tilde{f}_{j/p}(x,\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}^2)\, \tilde{D}_{h/j}(z,\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}^2)$$
$$\times \exp\left\{2\int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right] + \ln\frac{Q^2}{\mu_{b_*}^2}\tilde{K}(b_*;\mu_{b_*})\right\}$$
$$\times \exp\left\{-g_{j/p}(x,b_\mathrm{T}) - g_{h/j}(z,b_\mathrm{T}) - g_K(b_\mathrm{T})\ln\left(\frac{Q^2}{Q_0^2}\right)\right\}.$$

Same formula, just reorganized

$$-g_{j/p}(x,b_\mathrm{T}) \equiv \ln\left(\frac{\tilde{f}_{j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)}{\tilde{f}_{j/p}(x,\boldsymbol{b}_*;\mu_{Q_0},Q_0^2)}\right), \qquad -g_{h/j}(z,b_\mathrm{T}) \equiv \ln\left(\frac{\tilde{D}_{h/j}(z,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)}{\tilde{D}_{h/j}(z,\boldsymbol{b}_*;\mu_{Q_0},Q_0^2)}\right),$$

$$g_K(b_\mathrm{T}) \equiv \tilde{K}(b_*;\mu) - \tilde{K}(b_\mathrm{T};\mu)\,.$$

$$\boxed{\boldsymbol{b}_*(b_\mathrm{T}) = \frac{\boldsymbol{b}_\mathrm{T}}{\sqrt{1+b_\mathrm{T}^2/b_{\max}^2}}},$$

Precise definitions for g functions, $b_*(b_T)$ is a transition function bounded by some b_{max}. Note that b_* dependence cancels exactly. High sensitivity to b_* or b_{max} signals an issue.

$$
\begin{aligned}
[f_{j/p}, D_{h/j}] \to \int \frac{\mathrm{d}^2 \boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\, e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\; \tilde{f}^{\mathrm{OPE}}_{j/p}(x, \boldsymbol{b}_*; \mu_{b_*}, \mu^2_{b_*})\; \tilde{D}^{\mathrm{OPE}}_{h/j}(z, \boldsymbol{b}_*; \mu_{b_*}, \mu^2_{b_*}) \\
\times \exp\left\{ 2\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right] + \ln\frac{Q^2}{\mu^2_{b_*}}\tilde{K}(b_*;\mu_{b_*})\right\} \\
\times \exp\left\{ -g_{j/p}(x, b_\mathrm{T}) - g_{h/j}(z, b_\mathrm{T}) - g_K(b_\mathrm{T}) \ln\left(\frac{Q^2}{Q_0^2}\right)\right\} \boxed{+ O(b_{\max}\, m)}
\end{aligned}
$$

$\longleftarrow$ errors

Use of OPE introduces errors. Must keep b_{max} reasonably small.

$$
\boxed{\frac{\mathrm{d}}{\mathrm{d}b_{\max}}\left[f_{j/p}, D_{h/j}\right] = O\left(m b_{\max}\right)}
$$

$$
\begin{aligned}
\left[f_{j/p}, D_{h/j}\right] \to \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\; e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\; &\tilde{f}^{\mathrm{OPE}}_{j/p}(x,\boldsymbol{b}_*;\mu_{b_*},\mu^2_{b_*})\; \tilde{D}^{\mathrm{OPE}}_{h/j}(z,\boldsymbol{b}_*;\mu_{b_*},\mu^2_{b_*}) \\
&\times \exp\left\{2\int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right] + \ln\frac{Q^2}{\mu^2_{b_*}}\tilde{K}(b_*;\mu_{b_*})\right\} \\
&\boxed{\times \exp\left\{-g_{j/p}(x,b_\mathrm{T}) - g_{h/j}(z,b_\mathrm{T}) - g_K(b_\mathrm{T})\ln\left(\frac{Q^2}{Q_0^2}\right)\right\}} + O(b_{\max}\, m)
\end{aligned}
$$

Models →

Definitions:
Smooth transition
to small-b_T region
by construction

$$
-g_{h/j}(z,b_\mathrm{T}) \equiv \ln\left(\frac{\tilde{D}_{h/j}(z,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)}{\tilde{D}_{h/j}(z,\boldsymbol{b}_*;\mu_{Q_0},Q_0^2)}\right)
$$

$$
-g_{j/p}(x,b_\mathrm{T}) \equiv \ln\left(\frac{\tilde{f}_{j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)}{\tilde{f}_{j/p}(x,\boldsymbol{b}_*;\mu_{Q_0},Q_0^2)}\right)
$$

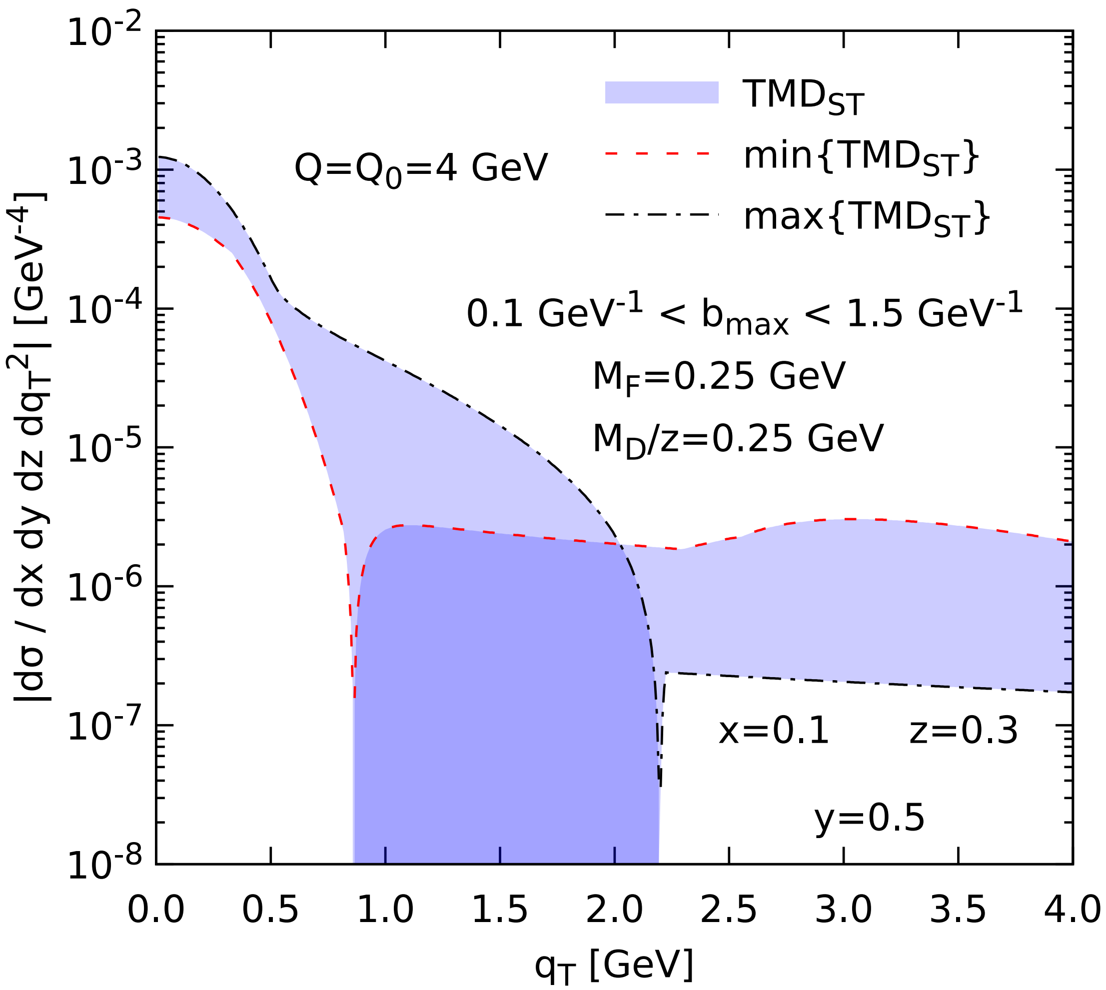
$$
g_K(b_\mathrm{T}) \equiv \tilde{K}(b_*;\mu) - \tilde{K}(b_\mathrm{T};\mu)\,.
$$

Typical choices:
generally unconstrained

$$
g_{h/j}(z,b_\mathrm{T}) = \frac{1}{4\,z^2} M_D^2\, b_\mathrm{T}^2
$$

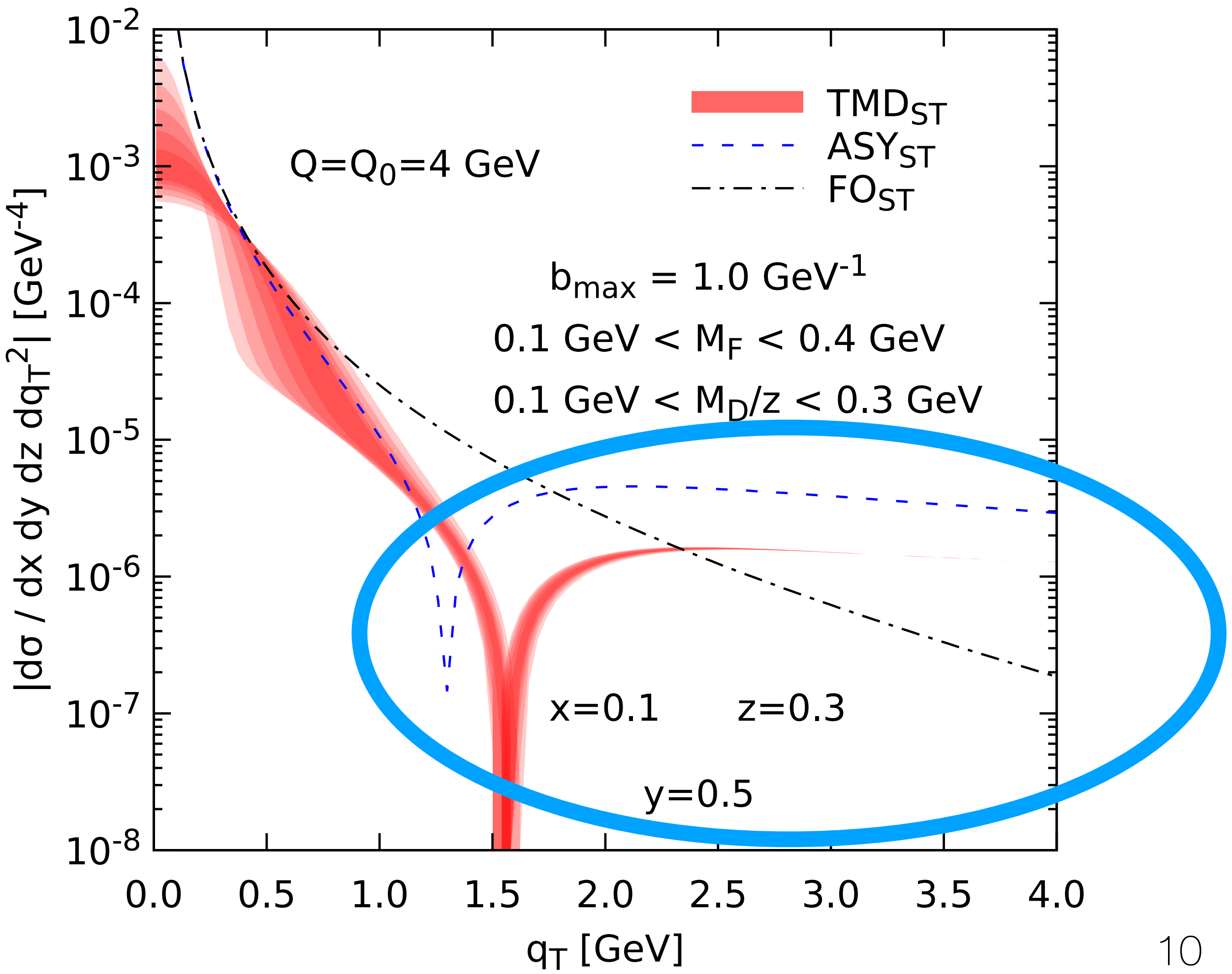
$$
g_{j/p}(x,b_\mathrm{T}) = \frac{1}{4}\, M_F^2\, b_\mathrm{T}^2
$$

$$
g_K(b_\mathrm{T}) = \frac{g_2}{2\,M_K^2}\ln\left(1 + M_K^2 b_\mathrm{T}^2\right)
$$



Issues:

Note the large-q_T (small-b_T) region should be determined by the OPE. **Small mass parameters can't really compensate for this b_{max} dependence.**

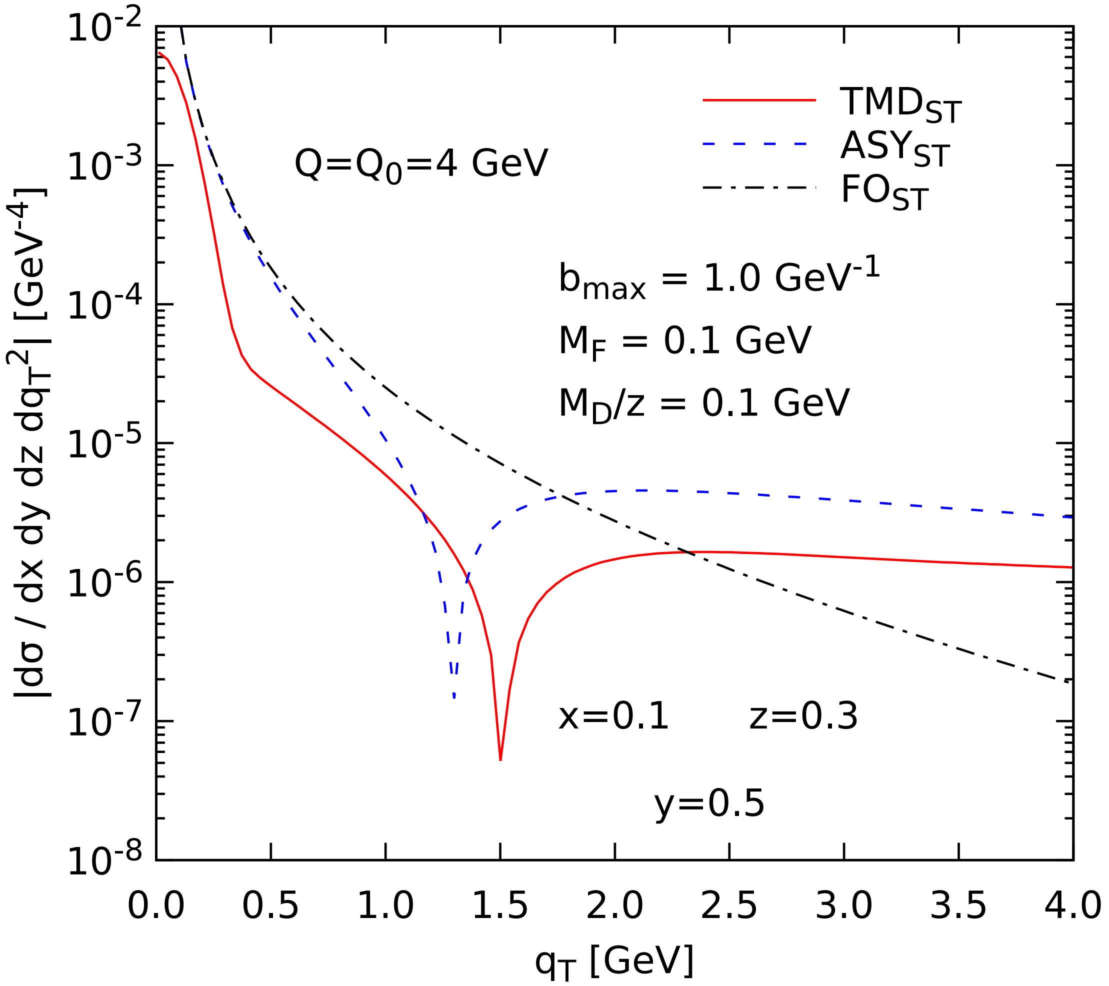


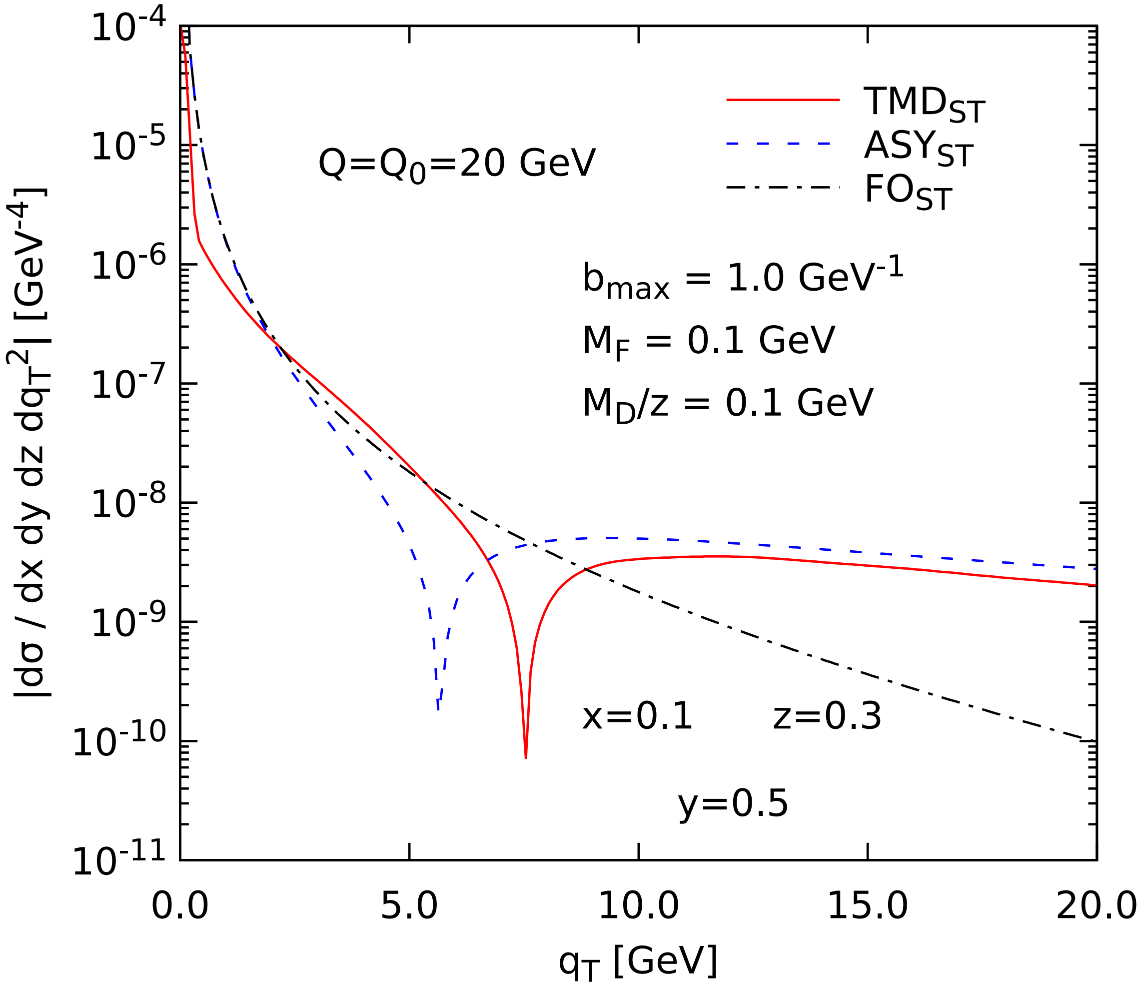
Typical choices: generally unconstrained

$$g_{h/j}(z, b_{\rm T}) = \frac{1}{4\, z^2} M_D^2\, b_{\rm T}^2$$

$$g_{j/p}(x, b_{\rm T}) = \frac{1}{4}\, M_F^2\, b_{\rm T}^2$$

$$g_K(b_{\rm T}) = \frac{g_2}{2\, M_K^2} \ln\left(1 + M_K^2 b_{\rm T}^2\right)$$





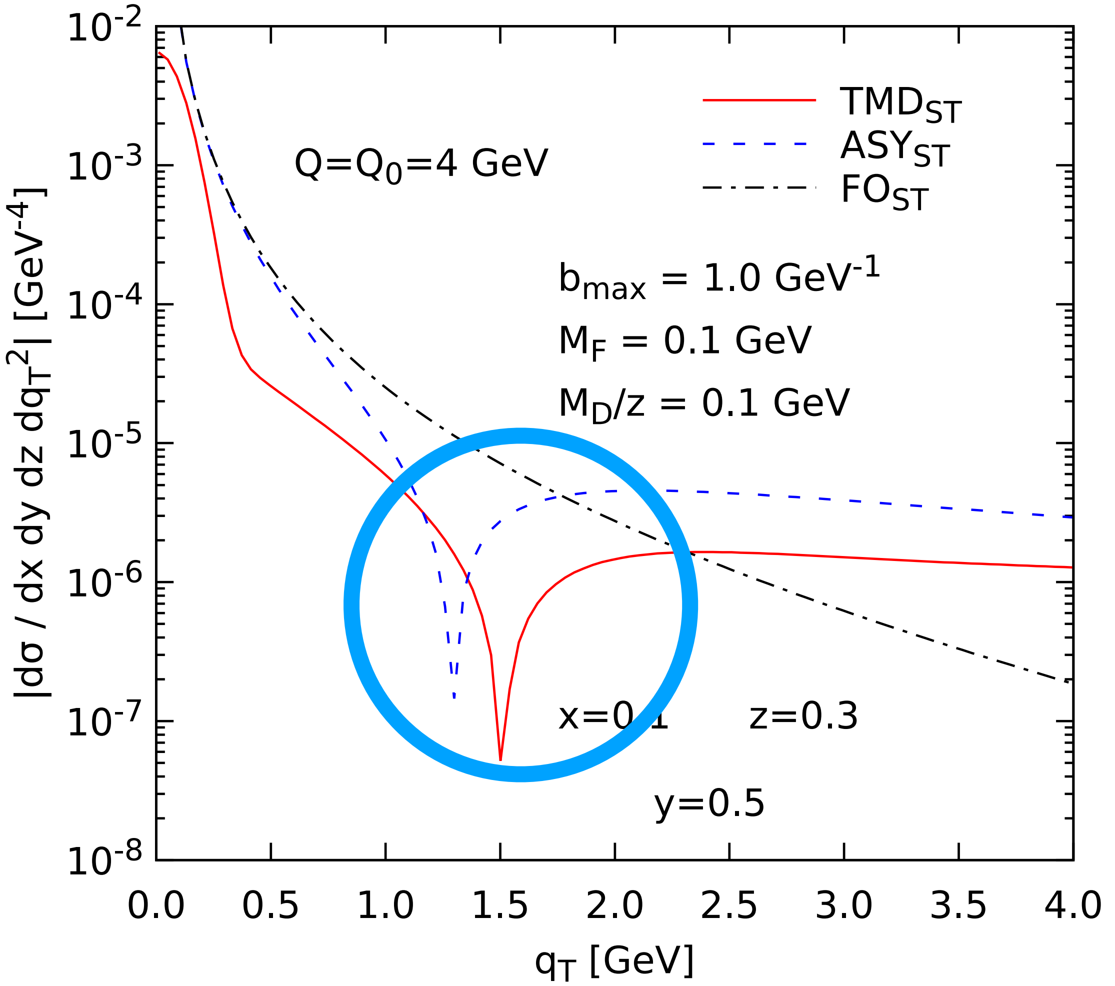
Issues:

Typical choices:
generally unconstrained

$$g_{h/j}(z, b_{\mathrm{T}}) = \frac{1}{4\,z^2} M_D^2\, b_{\mathrm{T}}^2$$

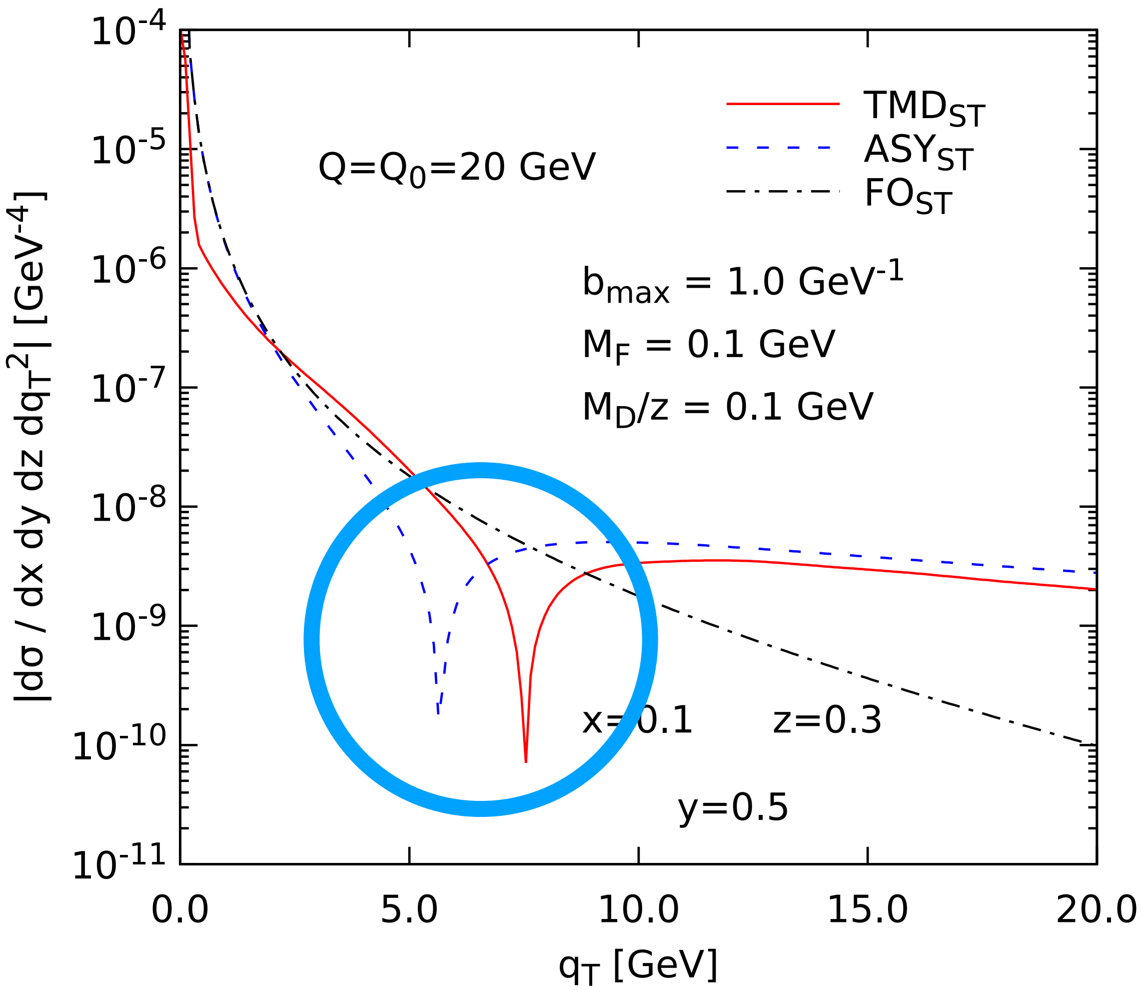
$$g_{j/p}(x, b_{\mathrm{T}}) = \frac{1}{4}\, M_F^2\, b_{\mathrm{T}}^2$$

$$g_K(b_{\mathrm{T}}) = \frac{g_2}{2\,M_K^2} \ln\left(1 + M_K^2 b_{\mathrm{T}}^2\right)$$



Issues:

Asymptotic term does not approximate well the TMD term, even at a scale of **Q_0=20 GeV**



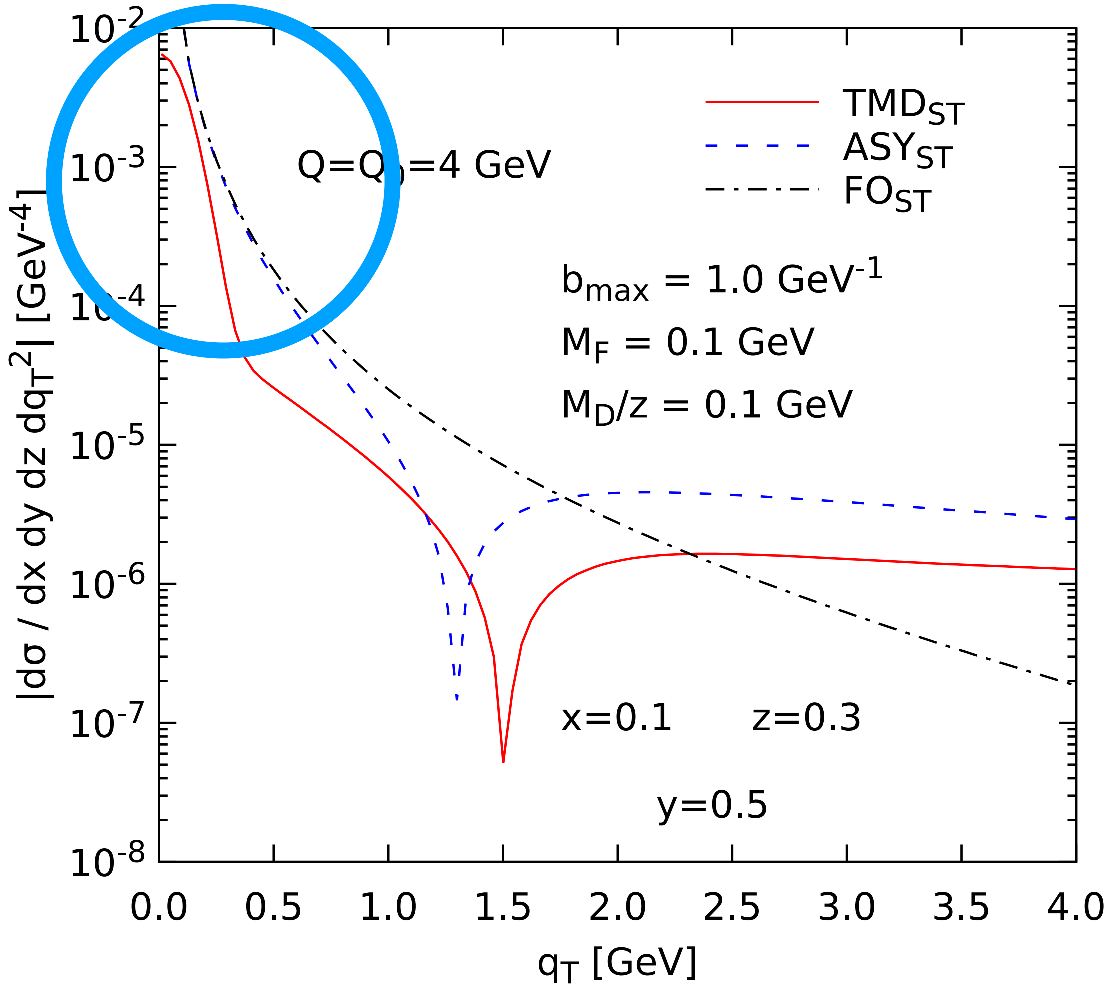
Typical choices:

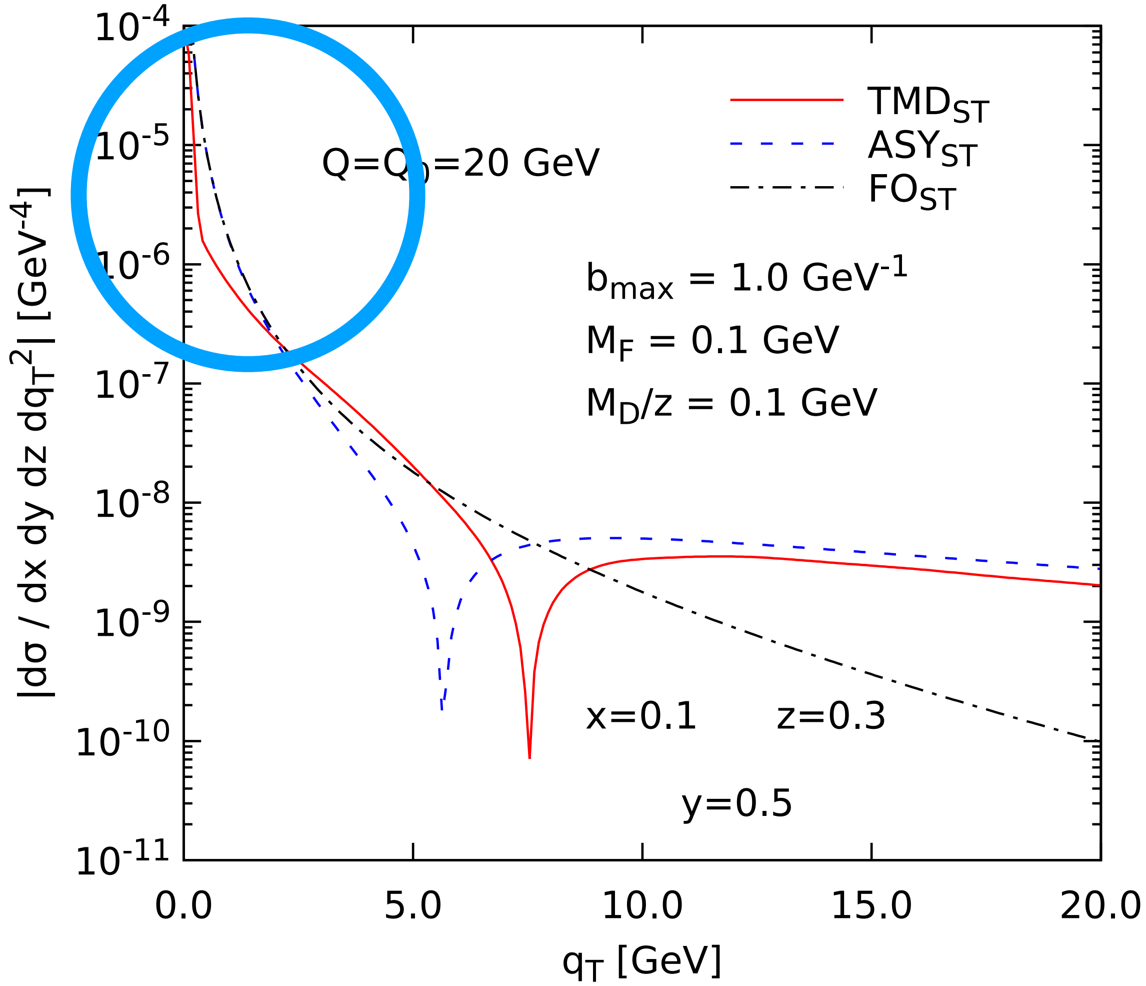
generally unconstrained

$$g_{h/j}(z, b_{\rm T}) = \frac{1}{4\, z^2} M_D^2\, b_{\rm T}^2$$

$$g_{j/p}(x, b_{\rm T}) = \frac{1}{4}\, M_F^2\, b_{\rm T}^2$$

$$g_K(b_{\rm T}) = \frac{g_2}{2\, M_K^2} \ln\left(1 + M_K^2 b_{\rm T}^2\right)$$





Issues:

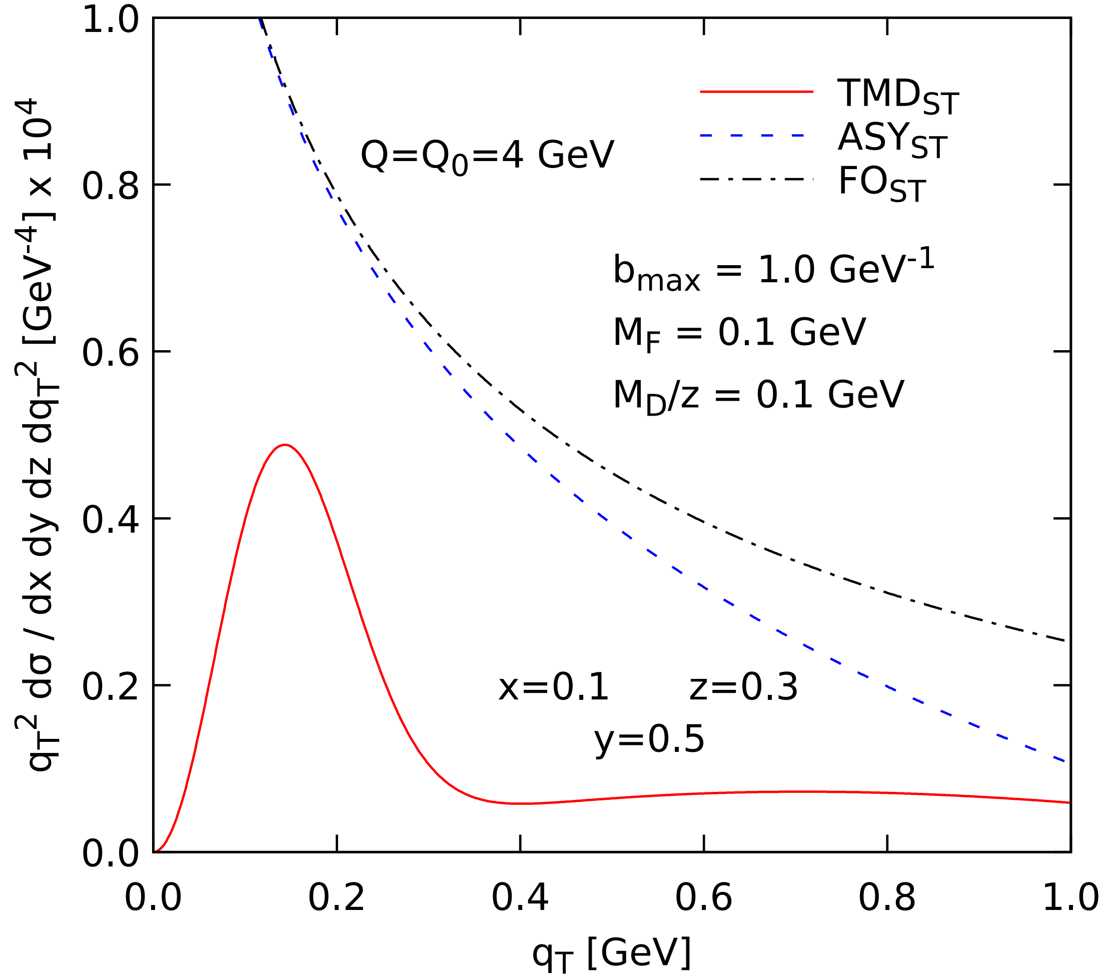
No region of "overlap" between TMD term and FO. This means smooth matching is not possible

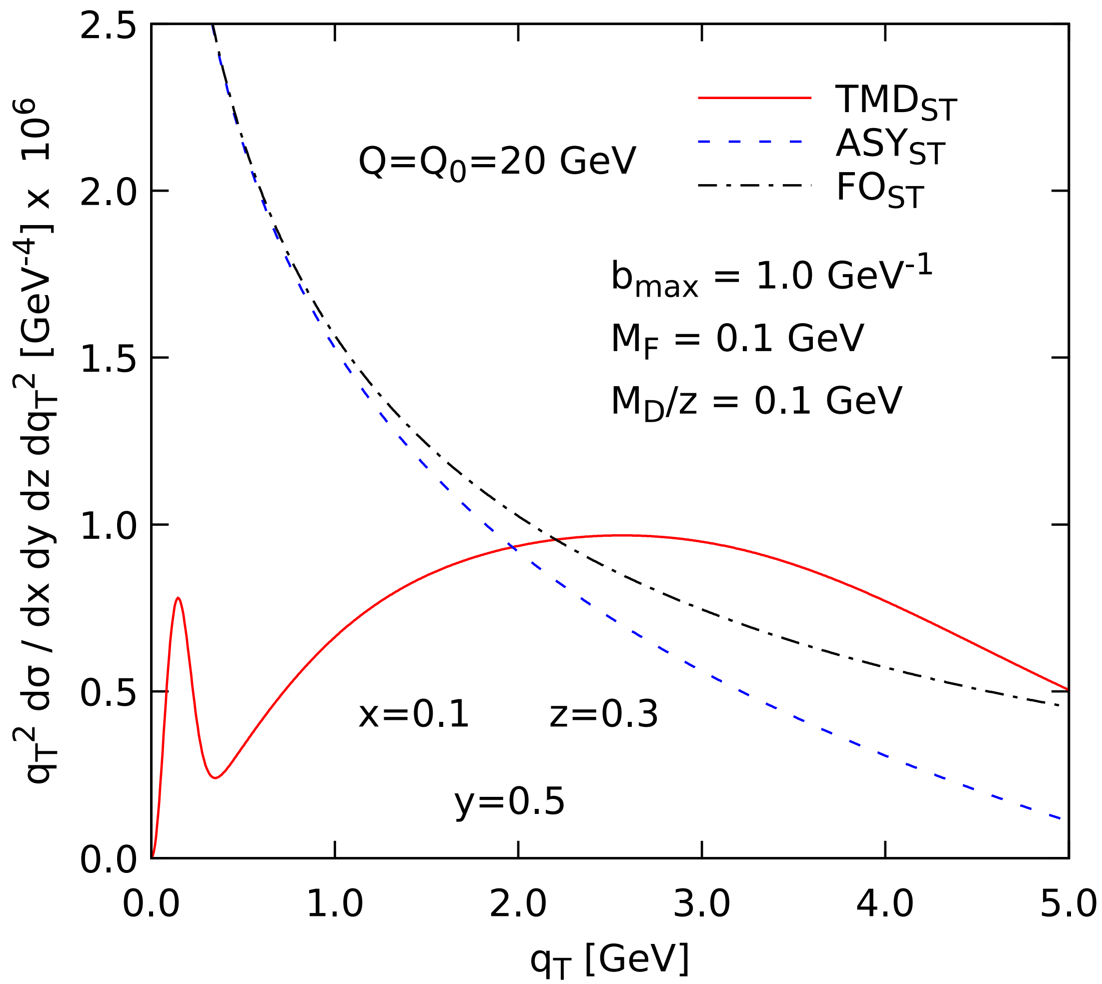
Typical choices:
generally unconstrained

$$g_{h/j}(z, b_T) = \frac{1}{4\,z^2} M_D^2\, b_T^2$$

$$g_{j/p}(x, b_T) = \frac{1}{4}\, M_F^2\, b_T^2$$

$$g_K(b_T) = \frac{g_2}{2\,M_K^2} \ln\left(1 + M_K^2 b_T^2\right)$$





Issues:

No region of "overlap" between TMD term and FO. This means smooth matching is not possible

Typical choices:

generally unconstrained

$$g_{h/j}(z, b_{\rm T}) = \frac{1}{4\,z^2} M_D^2\, b_{\rm T}^2$$

$$g_{j/p}(x, b_{\rm T}) = \frac{1}{4}\, M_F^2\, b_{\rm T}^2$$

$$g_K(b_{\rm T}) = \frac{g_2}{2\,M_K^2} \ln\left(1 + M_K^2 b_{\rm T}^2\right)$$

✳ Constraints on TMD models and HSO approach.

* These issues can be resolved by carefully constraining the TMD models.

* We work in momentum space

* Constraints are ultimately equivalent to those that one **attempts** to implement by means of the OPE (although, as we saw, this is not automatic):

✳These issues can be resolved by carefully constraining the TMD models.

✳We work in momentum space

✳Constraints are ultimately equivalent to those that one **attempts** to implement by means of the OPE (although, as we saw, this is not automatic):

1) pQCD tail

$$f^{\rm pert}_{{\rm inpt},i/p}(x,\boldsymbol{k}_{\rm T};\mu_{Q_0},Q_0^2) = \frac{1}{2\pi}\frac{1}{k_{\rm T}^2}\left[A^f_{i/p}(x;\mu_{Q_0}) + B^f_{i/p}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_{\rm T}^2}\right] + \frac{1}{2\pi}\frac{1}{k_{\rm T}^2}A^{f,g}_{i/p}(x;\mu_{Q_0}),$$

2) Integral relations

$$f^c(x;\mu) \equiv \pi\int_0^{\mu^2} {\rm d}k_{\rm T}^2\, f_{i/p}(x,\boldsymbol{k}_{\rm T};\mu;\zeta)$$

Note collinear function defined with a cutoff in the k_T integral. This retains a parton model interpretation.

NOTE: No b_* prescription

* These issues can be resolved by carefully constraining the TMD models.

* We work in momentum space

* Constraints are ultimately equivalent to those that one **attempts** to implement by means of the OPE (although, as we saw, this is not automatic):

0) Define the **input scale Q_0:**

smallest scale where perturbation theory can be trusted

Model in the HSO approach

$$
\begin{aligned}
f_{\text{inpt},i/p}(x,\boldsymbol{k}_{\text{T}};\mu_{Q_0},Q_0^2) = {} & \frac{1}{2\pi}\,\frac{1}{k_{\text{T}}^2+m_{f_{i,p}}^2}\left[A^f_{i/p}(x;\mu_{Q_0}) + B^f_{i/p}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_{\text{T}}^2+m_{f_{i,p}}^2}\right] \\
& + \frac{1}{2\pi}\,\frac{1}{k_{\text{T}}^2+m_{f_{g,p}}^2}A^{f,g}_{i/p}(x;\mu_{Q_0}) \\
& + C^f_{i/p}\,f_{\text{core},i/p}(x,\boldsymbol{k}_{\text{T}};Q_0^2)\,,
\end{aligned}
$$

Model in the HSO approach

$$f_{\text{inpt},i/p}(x,\boldsymbol{k}_\text{T};\mu_{Q_0},Q_0^2) = \frac{1}{2\pi}\frac{1}{k_\text{T}^2+m_{f_{i,p}}^2}\left[A^f_{i/p}(x;\mu_{Q_0}) + B^f_{i/p}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_\text{T}^2+m_{f_{i,p}}^2}\right] + \frac{1}{2\pi}\frac{1}{k_\text{T}^2+m_{f_{g,p}}^2}A^{f,g}_{i/p}(x;\mu_{Q_0}) + C^f_{i/p}\boxed{f_{\text{core},i/p}(x,\boldsymbol{k}_\text{T};Q_0^2)\,,}$$

Any "core" model here

examples:

$$f^{\text{Gauss}}_{\text{core},i/p}(x,\boldsymbol{k}_\text{T};Q_0^2) = \frac{e^{-k_\text{T}^2/M_\text{F}^2}}{\pi M_\text{F}^2}$$

$$f^{\text{Spect}}_{\text{core},i/p}(x,\boldsymbol{k}_\text{T};Q_0^2) = \frac{6M_{0\text{F}}^6}{\pi\,(2M_\text{F}^2+M_{0\text{F}}^2)}\,\frac{M_\text{F}^2+k_\text{T}^2}{(M_{0\text{F}}^2+k_\text{T}^2)^{\,4}}$$

Model in the HSO approach

$$
\begin{aligned}
f_{\mathrm{inpt},i/p}(x,\boldsymbol{k}_{\mathrm{T}};\mu_{Q_0},Q_0^2) &= \frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2+\boxed{m_{f_{i,p}}^2}}\left[A^f_{i/p}(x;\mu_{Q_0})+B^f_{i/p}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_{\mathrm{T}}^2+\boxed{m_{f_{i,p}}^2}}\right] \\
&+\frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2+\boxed{m_{f_{g,p}}^2}}A^{f,g}_{i/p}(x;\mu_{Q_0}) \\
&+C^f_{i/p}\,f_{\mathrm{core},i/p}(x,\boldsymbol{k}_{\mathrm{T}};Q_0^2)\,,
\end{aligned}
$$

Transition between
small and large k_T

Behaves as the pQCD tail, for large k_T

$$
f^{\mathrm{pert}}_{\mathrm{inpt},i/p}(x,\boldsymbol{k}_{\mathrm{T}};\mu_{Q_0},Q_0^2) = \frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2}\left[A^f_{i/p}(x;\mu_{Q_0})+B^f_{i/p}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_{\mathrm{T}}^2}\right]+\frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2}A^{f,g}_{i/p}(x;\mu_{Q_0})\,,
$$

Model in the HSO approach

$$
\begin{aligned}
f_{\mathrm{inpt},i/p}(x,\boldsymbol{k}_{\mathrm{T}};\mu_{Q_0},Q_0^2) = {} & \frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2+m_{f_{i,p}}^2}\left[A_{i/p}^{f}(x;\mu_{Q_0}) + B_{i/p}^{f}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_{\mathrm{T}}^2+m_{f_{i,p}}^2}\right] \\
& + \frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2+m_{f_{g,p}}^2}A_{i/p}^{f,g}(x;\mu_{Q_0}) \\
& + \boxed{C_{i/p}^{f}}\, f_{\mathrm{core},i/p}(x,\boldsymbol{k}_{\mathrm{T}};Q_0^2)\,,
\end{aligned}
$$

Determined by the integral relation

Integral relation

$$
f^{c}(x;\mu) \equiv \pi\int_0^{\mu^2} \mathrm{d}k_{\mathrm{T}}^2\, f_{i/p}(x,\boldsymbol{k}_{\mathrm{T}};\mu;\zeta)
$$

$$
C_{i/p}^{f} \equiv \frac{1}{N_{i/p}^{f}}\left[f_{i/p}^{c}(x;\mu_{Q_0}) \right.
$$

$$
\left. - A_{i/p}^{f}(x;\mu_{Q_0})\ln\left(\frac{\mu_{Q_0}}{m_{f_{i,p}}}\right) - B_{i/p}^{f}(x;\mu_{Q_0})\ln\left(\frac{\mu_{Q_0}}{m_{f_{i,p}}}\right)\ln\left(\frac{Q_0^2}{\mu_{Q_0}m_{f_{i,p}}}\right) \quad -A_{i/p}^{f,g}(x;\mu_{Q_0})\ln\left(\frac{\mu_{Q_0}}{m_{f_{g,p}}}\right)\right]
$$

Model in the HSO approach

$$
\begin{aligned}
f_{\mathrm{inpt},i/p}(x,\boldsymbol{k}_\mathrm{T};\mu_{Q_0},Q_0^2) = {} & \frac{1}{2\pi}\frac{1}{k_\mathrm{T}^2+m_{f_{i,p}}^2}\left[A_{i/p}^f(x;\mu_{Q_0}) + B_{i/p}^f(x;\mu_{Q_0}) \ln\frac{Q_0^2}{k_\mathrm{T}^2+m_{f_{i,p}}^2}\right] \\
& + \frac{1}{2\pi}\frac{1}{k_\mathrm{T}^2+m_{f_{g,p}}^2} A_{i/p}^{f,g}(x;\mu_{Q_0}) \\
& + \boxed{C_{i/p}^f}\, f_{\mathrm{core},i/p}(x,\boldsymbol{k}_\mathrm{T};Q_0^2)\,,
\end{aligned}
$$

Determined by the integral relation

Integral relation (using $\overline{\mathrm{MS}}$ functions)

$$
f^c(x;\mu) \equiv \pi\int_0^{\mu^2} \mathrm{d}k_\mathrm{T}^2\, f_{i/p}(x,\boldsymbol{k}_\mathrm{T};\mu;\zeta)
$$

$$
\begin{aligned}
C_{i/p}^f \equiv \frac{1}{N_{i/p}^f}\Bigg[& f_{i/p}^{\overline{\mathrm{MS}}}(x;\mu_{Q_0}) + \frac{\alpha_s(\mu_{Q_0})}{2\pi}\left\{\sum_{jj'}\delta_{j'j}[\mathcal{C}_\Delta^{j'/j}\otimes d_{h/j'}](z;\mu_{Q_0}) + [\mathcal{C}_\Delta^{g/j}\otimes d_{h/g}](z;\mu_{Q_0})\right\}\Bigg] \\
& - A_{i/p}^f(x;\mu_{Q_0})\ln\left(\frac{\mu_{Q_0}}{m_{f_{i,p}}}\right) - B_{i/p}^f(x;\mu_{Q_0})\ln\left(\frac{\mu_{Q_0}}{m_{f_{i,p}}}\right)\ln\left(\frac{Q_0^2}{\mu_{Q_0} m_{f_{i,p}}}\right) - A_{i/p}^{f,g}(x;\mu_{Q_0})\ln\left(\frac{\mu_{Q_0}}{m_{f_{g,p}}}\right)\Bigg]
\end{aligned}
$$

Model in the HSO approach

$$f_{\text{inpt},i/p}(x, \boldsymbol{k}_\text{T}; \mu_{Q_0}, Q_0^2) = \frac{1}{2\pi} \frac{1}{k_\text{T}^2 + m_{f_{i,p}}^2} \left[A_{i/p}^f(x; \mu_{Q_0}) + B_{i/p}^f(x; \mu_{Q_0}) \ln \frac{Q_0^2}{k_\text{T}^2 + m_{f_{i,p}}^2} \right]$$
$$+ \frac{1}{2\pi} \frac{1}{k_\text{T}^2 + m_{f_{g,p}}^2} A_{i/p}^{f,g}(x; \mu_{Q_0})$$
$$+ C_{i/p}^f \, f_{\text{core},i/p}(x, \boldsymbol{k}_\text{T}; Q_0^2) \,,$$

In b_T space

$$\tilde{f}_{\text{inpt},i/p}(x, \boldsymbol{b}_\text{T}; \mu_{Q_0}, Q_0^2) = K_0 \left(b_\text{T} m_{f_{i,p}} \right) \left[A_{i/p}^f(x; \mu_{Q_0}) + B_{i/p}^f(x; \mu_{Q_0}) \ln \left(\frac{b_\text{T} Q_0^2 e^{\gamma_E}}{2 m_{f_{i,p}}} \right) \right]$$
$$+ K_0 \left(b_\text{T} m_{f_{g,p}} \right) A_{g/p}^f(x; \mu_{Q_0})$$
$$+ C_{i/p}^f \, \tilde{f}_{\text{core},i/p}(x, \boldsymbol{b}_\text{T}; Q_0^2) \,,$$

from this expression one can recover the OPE

Model in the HSO approach

$$f_{\mathrm{inpt},i/p}(x,\boldsymbol{k}_{\mathrm{T}};\mu_{Q_0},Q_0^2) = \frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2+m_{f_{i,p}}^2}\left[A^f_{i/p}(x;\mu_{Q_0}) + B^f_{i/p}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_{\mathrm{T}}^2+m_{f_{i,p}}^2}\right]$$
$$+\frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2+m_{f_{g,p}}^2}A^{f,g}_{i/p}(x;\mu_{Q_0})$$
$$+C^f_{i/p}\,f_{\mathrm{core},i/p}(x,\boldsymbol{k}_{\mathrm{T}};Q_0^2)\,,$$

In b_T space

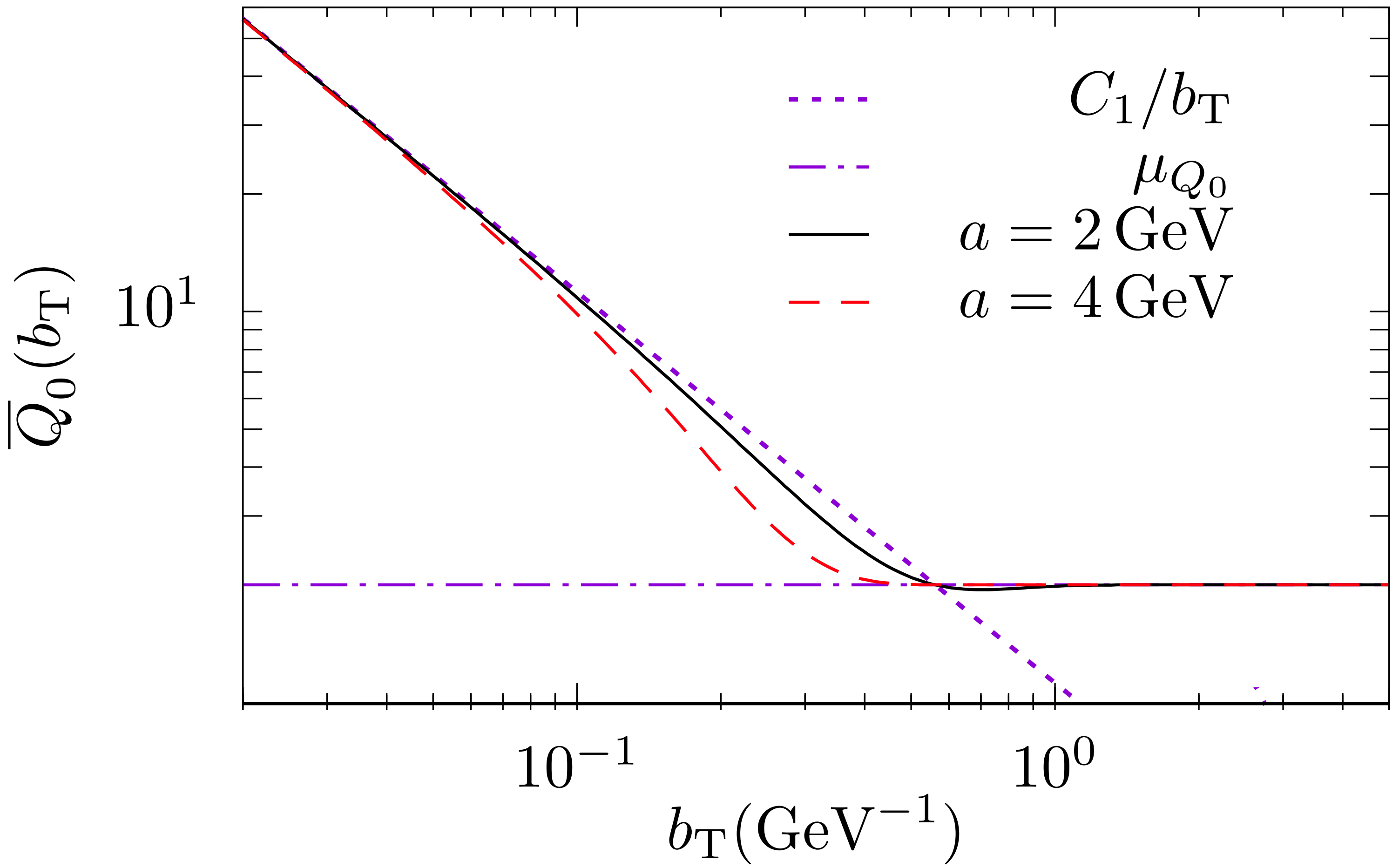
$$\tilde{f}_{\mathrm{inpt},i/p}(x,\boldsymbol{b}_{\mathrm{T}};\mu_{Q_0},Q_0^2) = K_0\left(b_{\mathrm{T}} m_{f_{i,p}}\right)\left[A^f_{i/p}(x;\mu_{Q_0}) + B^f_{i/p}(x;\mu_{Q_0})\ln\left(\frac{b_{\mathrm{T}} Q_0^2 e^{\gamma_E}}{2 m_{f_{i,p}}}\right)\right]$$
$$+K_0\left(b_{\mathrm{T}} m_{f_{g,p}}\right)A^f_{g/p}(x;\mu_{Q_0})$$
$$+C^f_{i/p}\,\tilde{f}_{\mathrm{core},i/p}(x,\boldsymbol{b}_{\mathrm{T}};Q_0^2)\,,$$

Expressions useful for pheno at $Q \approx Q_0$

Scale setting for evolution to large Q

$$\overline{Q}_0(b_{\rm T}) = Q_0 \ {\rm GeV} \left[1 - \left(1 - \frac{C_1}{Q_0 b_{\rm T}}\right) e^{-a^2 \, b_{\rm T}^2}\right]$$

$Q_0 = 2\,{\rm GeV}$



- ✳ goes as $1/b_T$ for small b_T
- ✳ approaches input scale Q_0 at large b_T
- ✳ analogous to b_* in usual treatment

Model in the HSO approach

Need RG improvements for pheno at Q >> Q_0

$$\sim \alpha_s(Q_0)^n \ln^m\left(\frac{q_\mathrm{T}}{Q_0}\right)$$

Wider range of qT available upon evolution to large Q

$$\tilde{f}_{i/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2) = \tilde{f}_{\mathrm{inpt},i/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{\bar{Q}_0},\bar{Q}_0^2)E(\bar{Q}_0/Q_0,b_\mathrm{T})$$

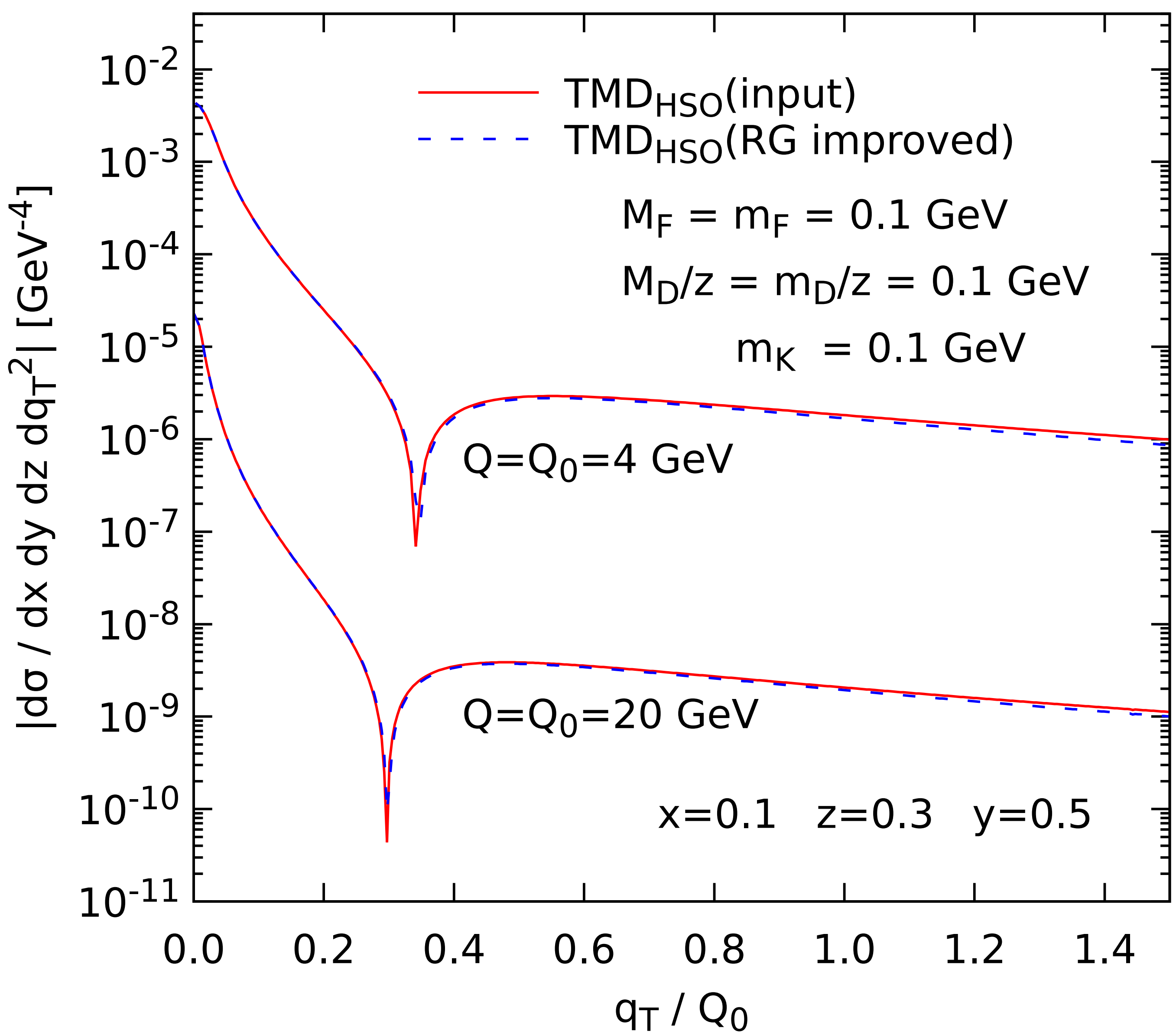
$$\overline{Q}_0(b_\mathrm{T}) = Q_0\ \mathrm{GeV}\left[1-\left(1-\frac{C_1}{Q_0 b_\mathrm{T}}\right)e^{-a^2\, b_\mathrm{T}^2}\right]$$

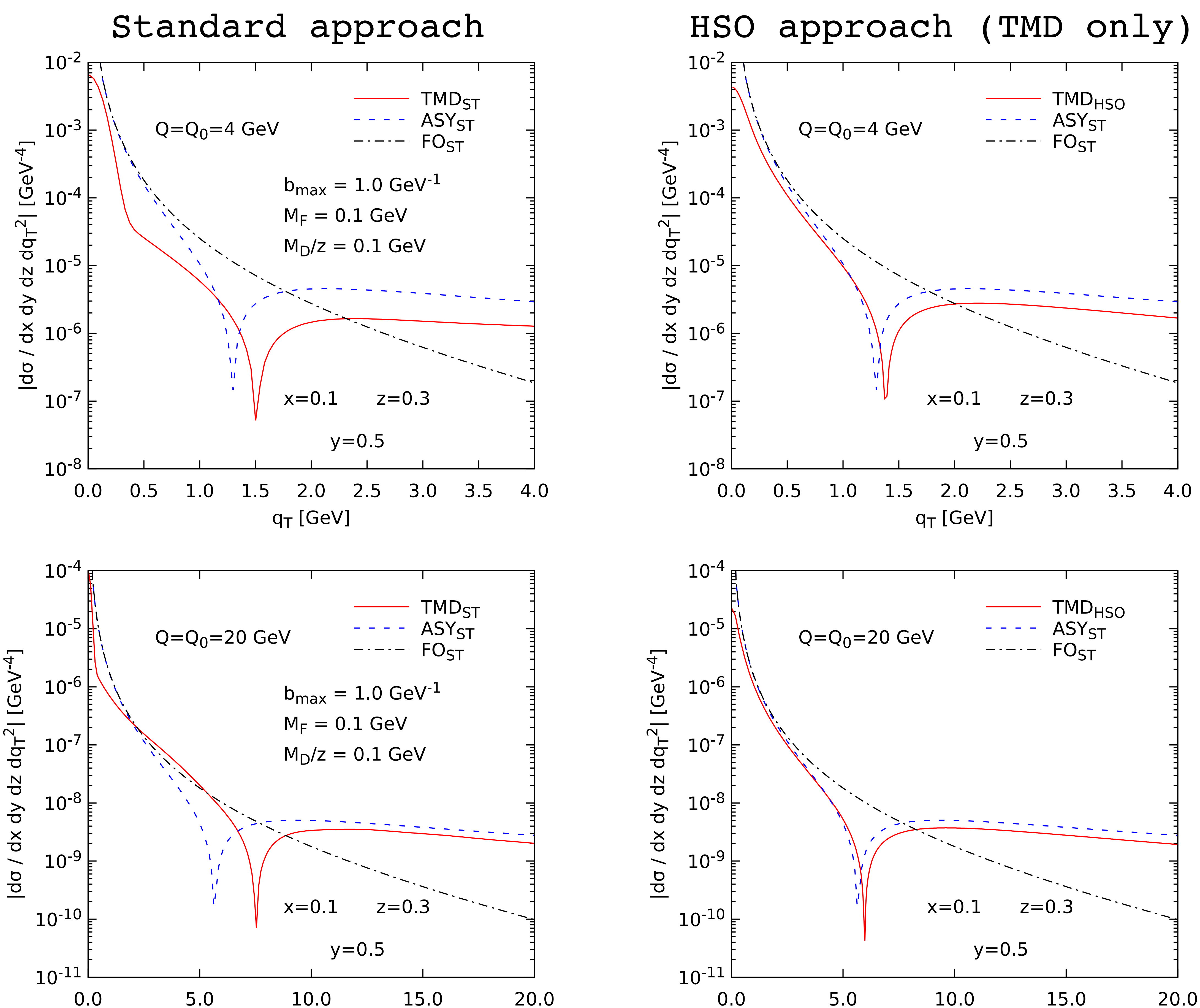
$$E(\bar{Q}_0/Q_0,b_\mathrm{T}) \equiv \exp\left\{\int_{\mu_{\bar{Q}_0}}^{\mu_{Q_0}}\frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1)-\ln\frac{Q_0}{\mu'}\gamma_K(\alpha_s(\mu'))\right]+\ln\frac{Q_0}{\bar{Q}_0}\tilde{K}_\mathrm{inpt}(b_\mathrm{T};\mu_{\bar{Q}_0})\right\}.$$

The usual evolution factor

Scale transformation not really needed for pheno at $Q \approx Q_0$

Work with $Q=Q_0$ for now





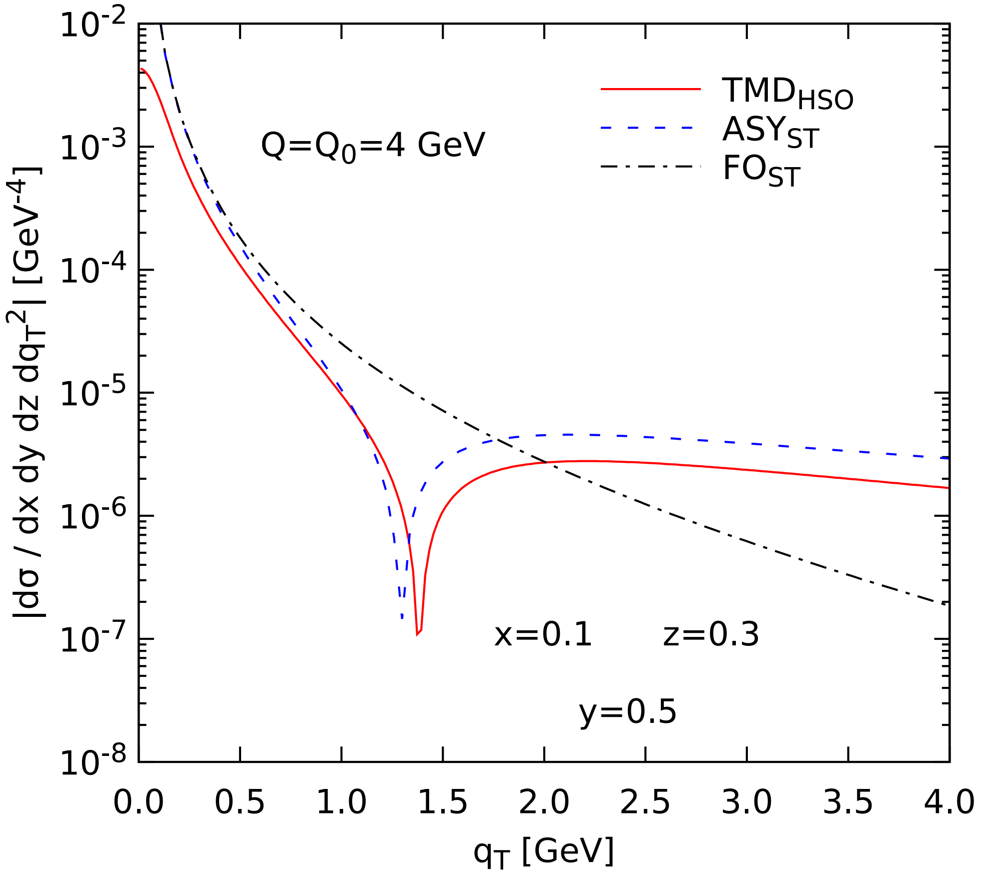
Standard approach
HSO approach (TMD only)
TMD_ST
ASY_ST
FO_ST
TMD_HSO
Q=Q_0=4 GeV
Q=Q_0=20 GeV
b_max = 1.0 GeV^-1
M_F = 0.1 GeV
M_D/z = 0.1 GeV
x=0.1 z=0.3
y=0.5
|dσ / dx dy dz dq_T^2| [GeV^-4]
q_T [GeV]
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
10^-11
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
0.0 5.0 10.0 15.0 20.0

Asymptotic term

The usual asymptotic term

$$\lim_{q_T/Q \to 0} F^{\mathrm{FO}}$$

Still not a good approximation to the TMD term at large q_T



Asymptotic term

The usual asymptotic term

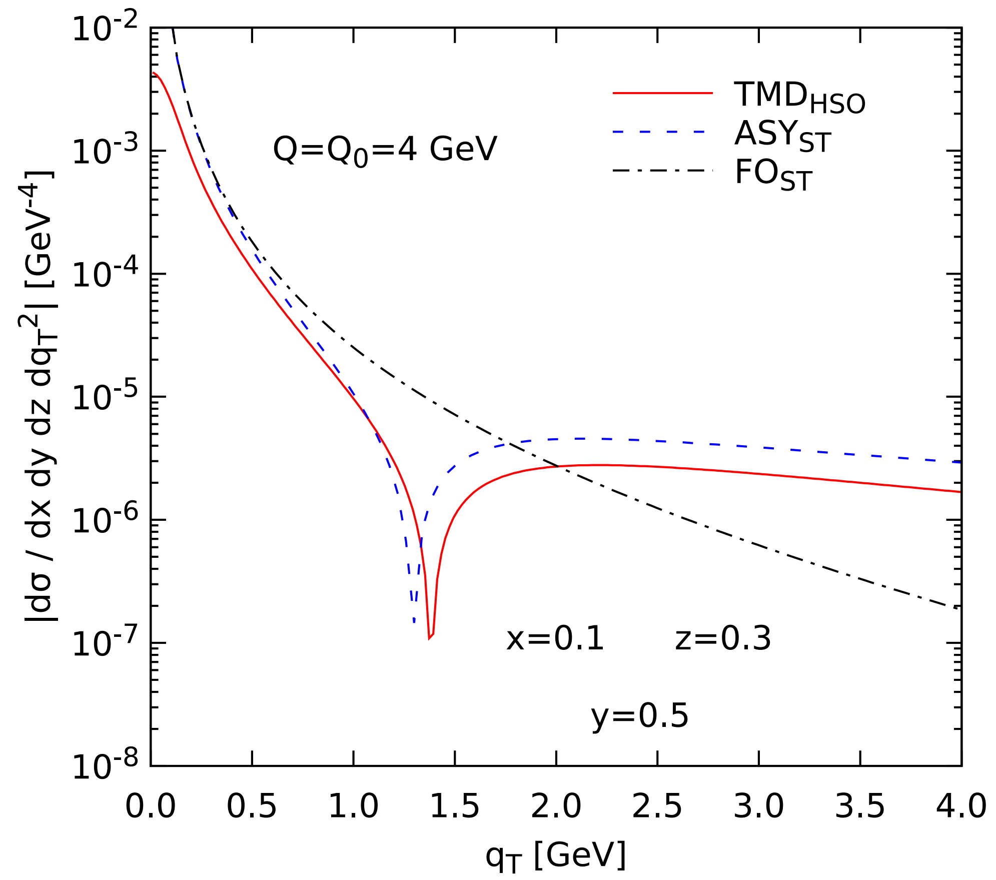
$$\lim_{q_{\mathrm{T}}/Q \to 0} F^{\mathrm{FO}}$$

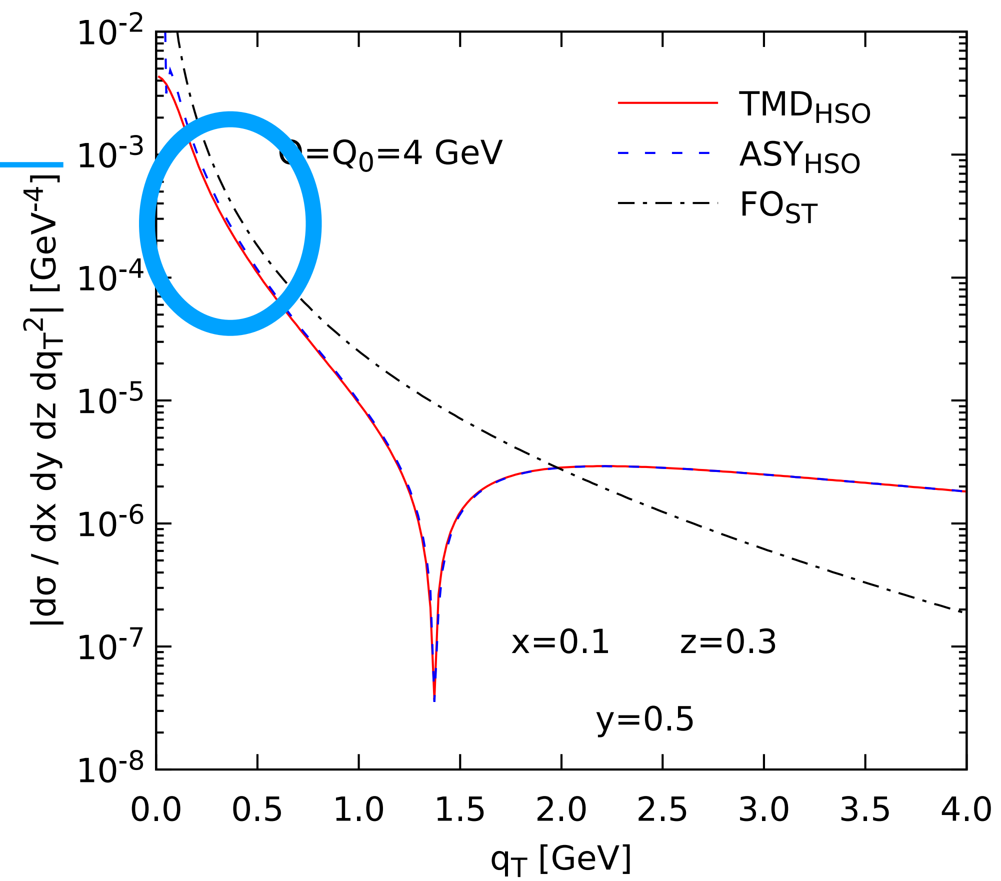
Still not a good approximation to the TMD term at large q_T

We compute instead

$$\lim_{m/q_{\mathrm{T}} \to 0} F^{\mathrm{TMD}}$$

Stays a good approximation to the TMD term at large q_T, from around **this region**





Asymptotic term

The usual asymptotic term

$$\lim_{q_{\mathrm{T}}/Q \to 0} F^{\mathrm{FO}}$$

We compute instead

$$\lim_{m/q_{\mathrm{T}} \to 0} F^{\mathrm{TMD}}$$

$$\left[\lim_{q_{\mathrm{T}}/Q \to 0} F^{\mathrm{FO}}\right]^{O(\alpha_s^n)} - \left[\lim_{m/q_{\mathrm{T}} \to 0} F^{\mathrm{TMD}}\right]^{O(\alpha_s^n)} = O\left(\alpha_s^{n+1}\boxed{, m^2/Q^2)}\right.$$

If using different schemes for collinear functions

Asymptotic term

The usual asymptotic term

$$\lim_{q_{\mathrm{T}}/Q \to 0} F^{\mathrm{FO}}$$

We compute instead

$$\lim_{m/q_{\mathrm{T}} \to 0} F^{\mathrm{TMD}}$$

$$\left[\lim_{q_{\mathrm{T}}/Q \to 0} F^{\mathrm{FO}}\right]^{O(\alpha_s^n)} - \left[\lim_{m/q_{\mathrm{T}} \to 0} F^{\mathrm{TMD}}\right]^{O(\alpha_s^n)} = O\left(\boxed{\alpha_s^{n+1}}, m^2/Q^2\right)$$

From two places (fixing the scheme for collinear functions)

Asymptotic term

The usual asymptotic term

We compute instead

$$\lim_{q_T/Q \to 0} F^{\mathrm{FO}}$$

$$\lim_{m/q_T \to 0} F^{\mathrm{TMD}}$$

$$\left[\lim_{q_T/Q \to 0} F^{\mathrm{FO}} \right]^{O(\alpha_s^n)} - \left[\lim_{m/q_T \to 0} F^{\mathrm{TMD}} \right]^{O(\alpha_s^n)} = O\left(\boxed{\alpha_s^{n+1}}, m^2/Q^2 \right)$$

1) Additional terms in the bracket

$$[f,D] = \boxed{D^{\mathrm{pert}}(z, z\boldsymbol{q}_T; \mu_Q; Q^2) f^c(x;\mu_Q) + \frac{1}{z^2} f^{\mathrm{pert}}(x, -\boldsymbol{q}_T; \mu_Q; Q^2) d^c(z;\mu_Q)}$$

$$+ \int \mathrm{d}^2\boldsymbol{k}_T \, \Big\{ f^{\mathrm{pert}}(x, \boldsymbol{k}_T - \boldsymbol{q}_T/2; \mu_Q; Q^2) D^{\mathrm{pert}}(z, z(\boldsymbol{k}_T + \boldsymbol{q}_T/2); \mu_Q; Q^2)$$

$$- D^{\mathrm{pert}}(z, z\boldsymbol{q}_T; \mu_Q; Q^2) f^{\mathrm{pert}}(x, \boldsymbol{k}_T - \boldsymbol{q}_T/2; \mu_Q; Q^2) \Theta(\mu_Q - |\boldsymbol{k}_T - \boldsymbol{q}_T/2|)$$

$$- D^{\mathrm{pert}}(z, z(\boldsymbol{k}_T + \boldsymbol{q}_T/2); \mu_Q; Q^2) f^{\mathrm{pert}}(x, -\boldsymbol{q}_T; \mu_Q; Q^2) \Theta(\mu_Q - |\boldsymbol{k}_T + \boldsymbol{q}_T/2|) \Big\} + O\left(\frac{m^2}{q_T^2}\right)$$

Asymptotic term

The usual asymptotic term

$$\lim_{q_T/Q \to 0} F^{\mathrm{FO}}$$

We compute instead

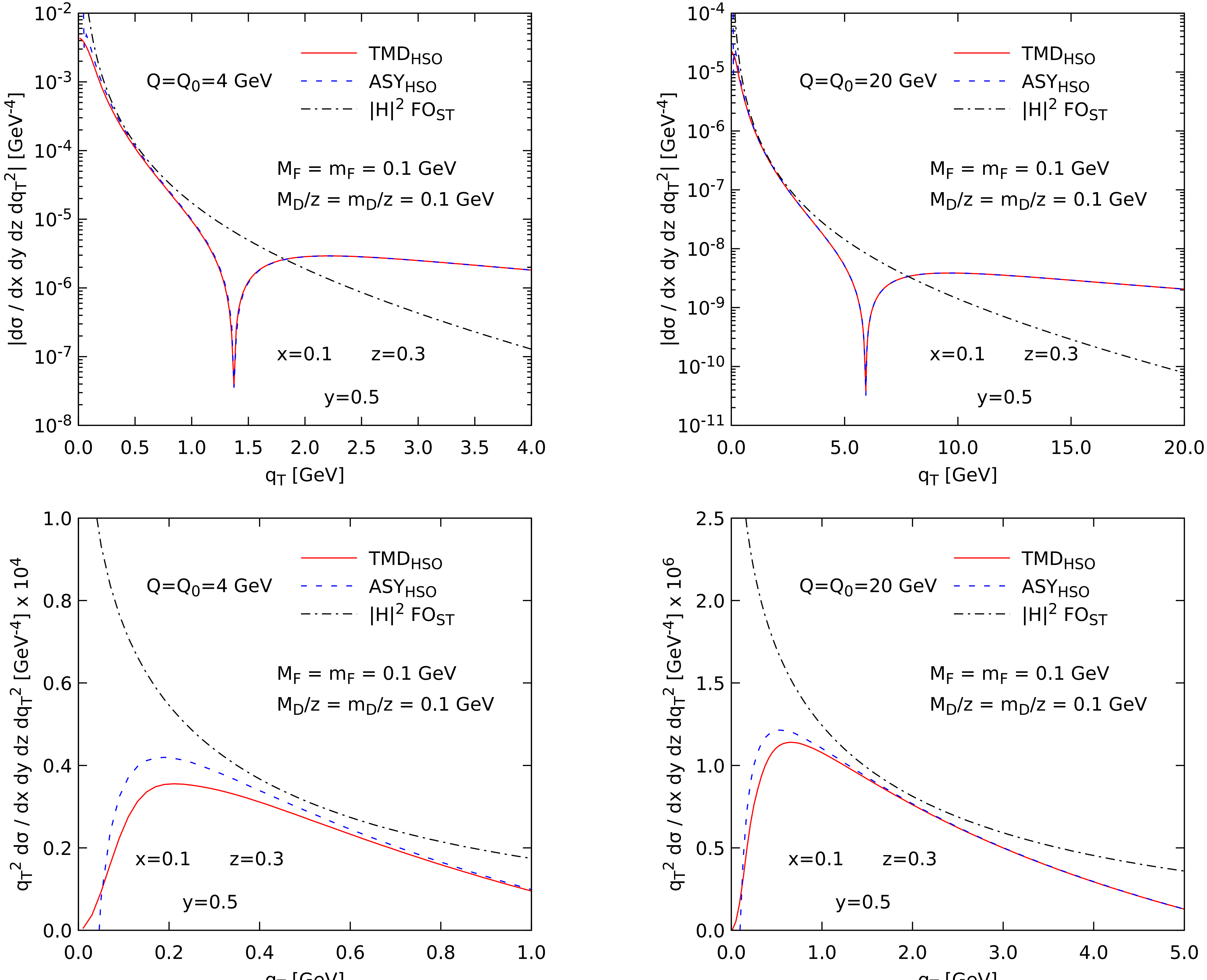
$$\lim_{m/q_T \to 0} F^{\mathrm{TMD}}$$

$$\left[\lim_{q_T/Q \to 0} F^{\mathrm{FO}}\right]^{O(\alpha_s^n)} - \left[\lim_{m/q_T \to 0} F^{\mathrm{TMD}}\right]^{O(\alpha_s^n)} = O\left(\boxed{\alpha_s^{n+1}}, m^2/Q^2\right)$$

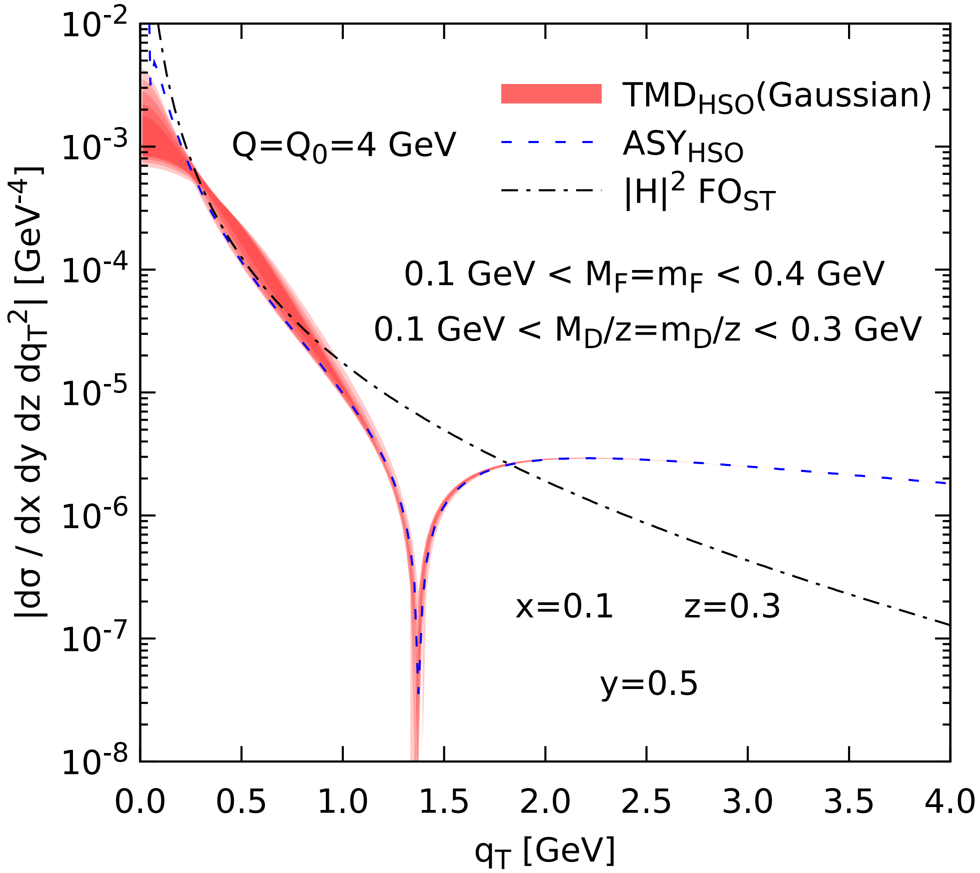
2) Hard coefficient in TMD term

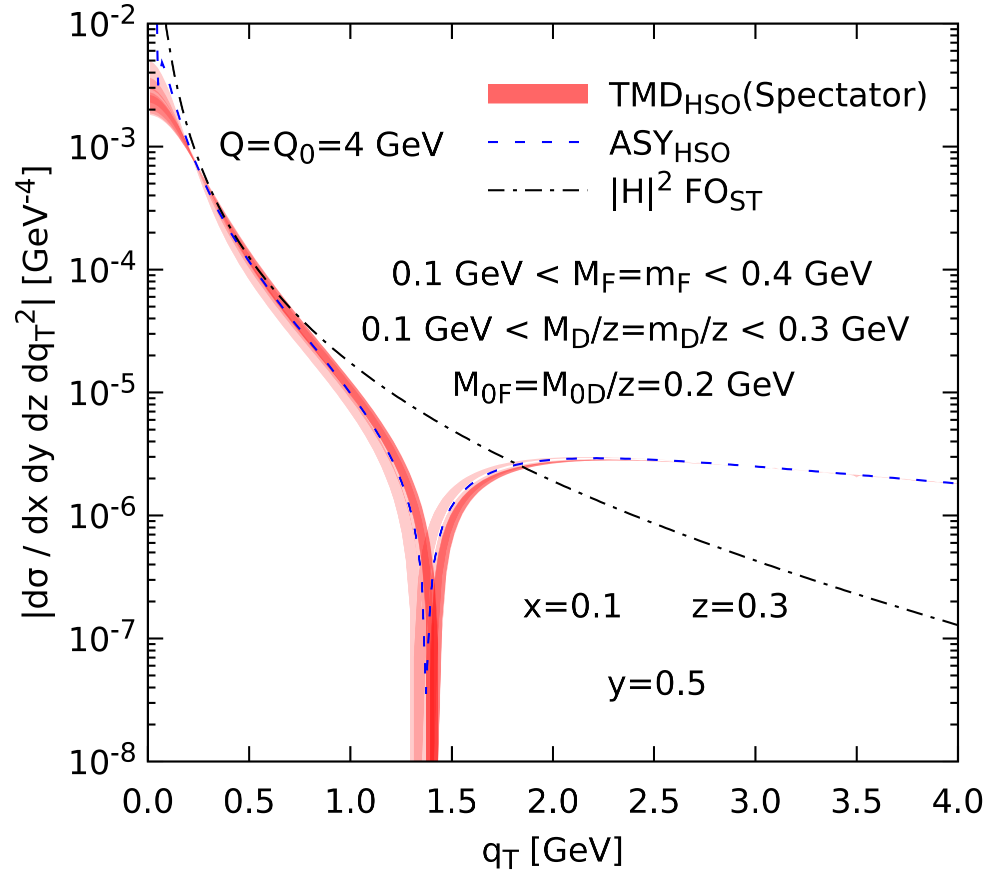
$$F_1^{\mathrm{TMD}} \equiv 2\,z \sum_j \boxed{|H|_j^2} \left[f_{j/p}, D_{h/j}\right] \qquad F_2^{\mathrm{TMD}} \equiv 4\,z\,x \sum_j \boxed{|H|_j^2} \left[f_{j/p}, D_{h/j}\right]$$

HSO approach



HSO approach

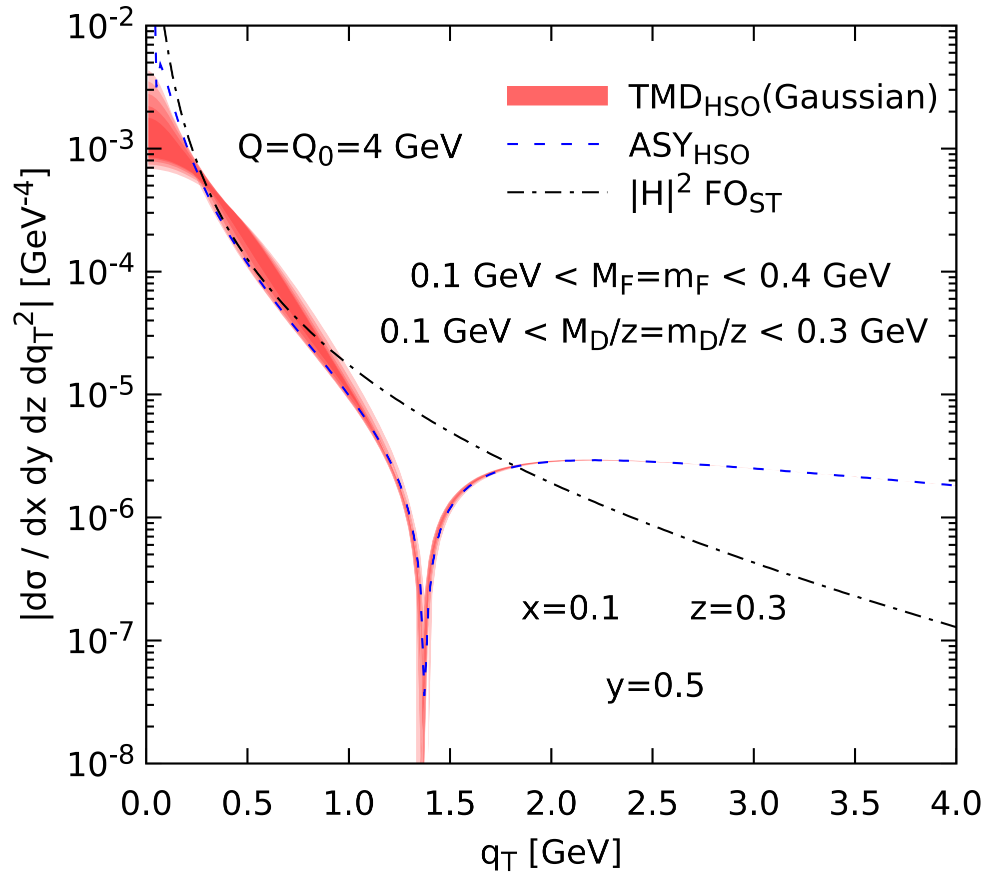


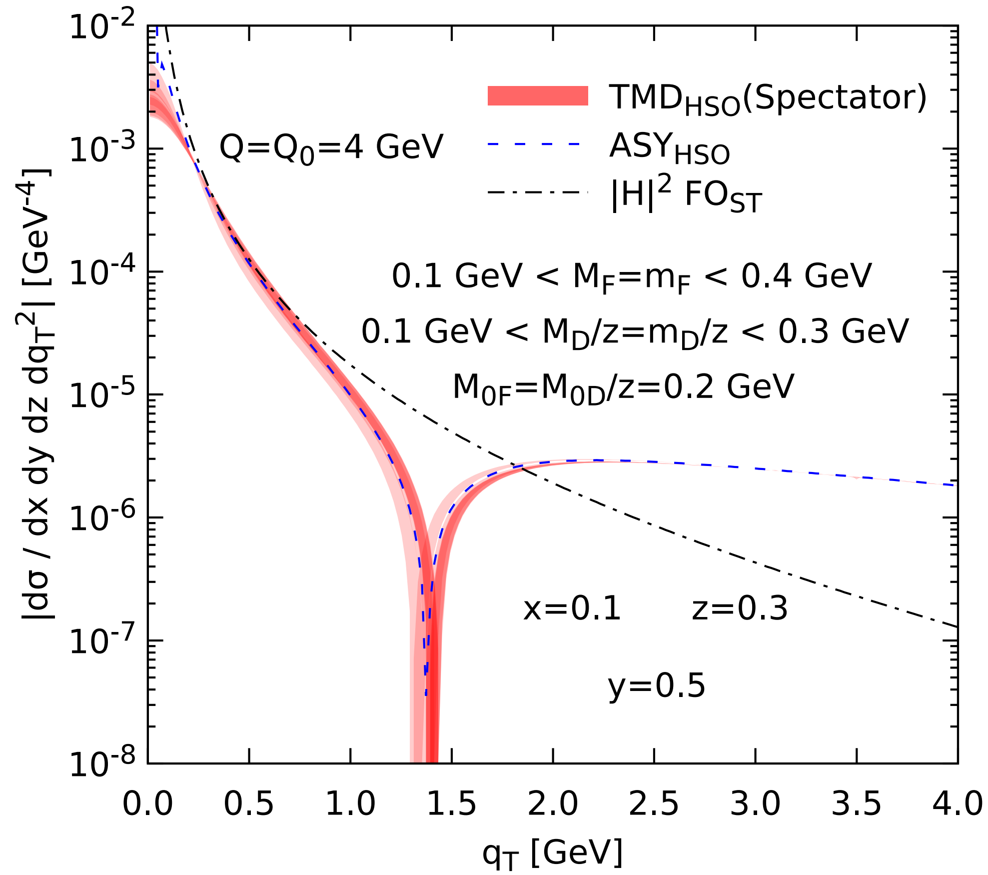


$$f^{\text{Gauss}}_{\text{core},i/p}(x,\boldsymbol{k}_{\text{T}};Q_0^2) = \frac{e^{-k_{\text{T}}^2/M_{\text{F}}^2}}{\pi M_{\text{F}}^2}$$

$$f^{\text{Spect}}_{\text{core},i/p}(x,\boldsymbol{k}_{\text{T}};Q_0^2) = \frac{6M_{0\text{F}}^6}{\pi\left(2M_{\text{F}}^2+M_{0\text{F}}^2\right)}\,\frac{M_{\text{F}}^2+k_{\text{T}}^2}{\left(M_{0\text{F}}^2+k_{\text{T}}^2\right)^4}$$

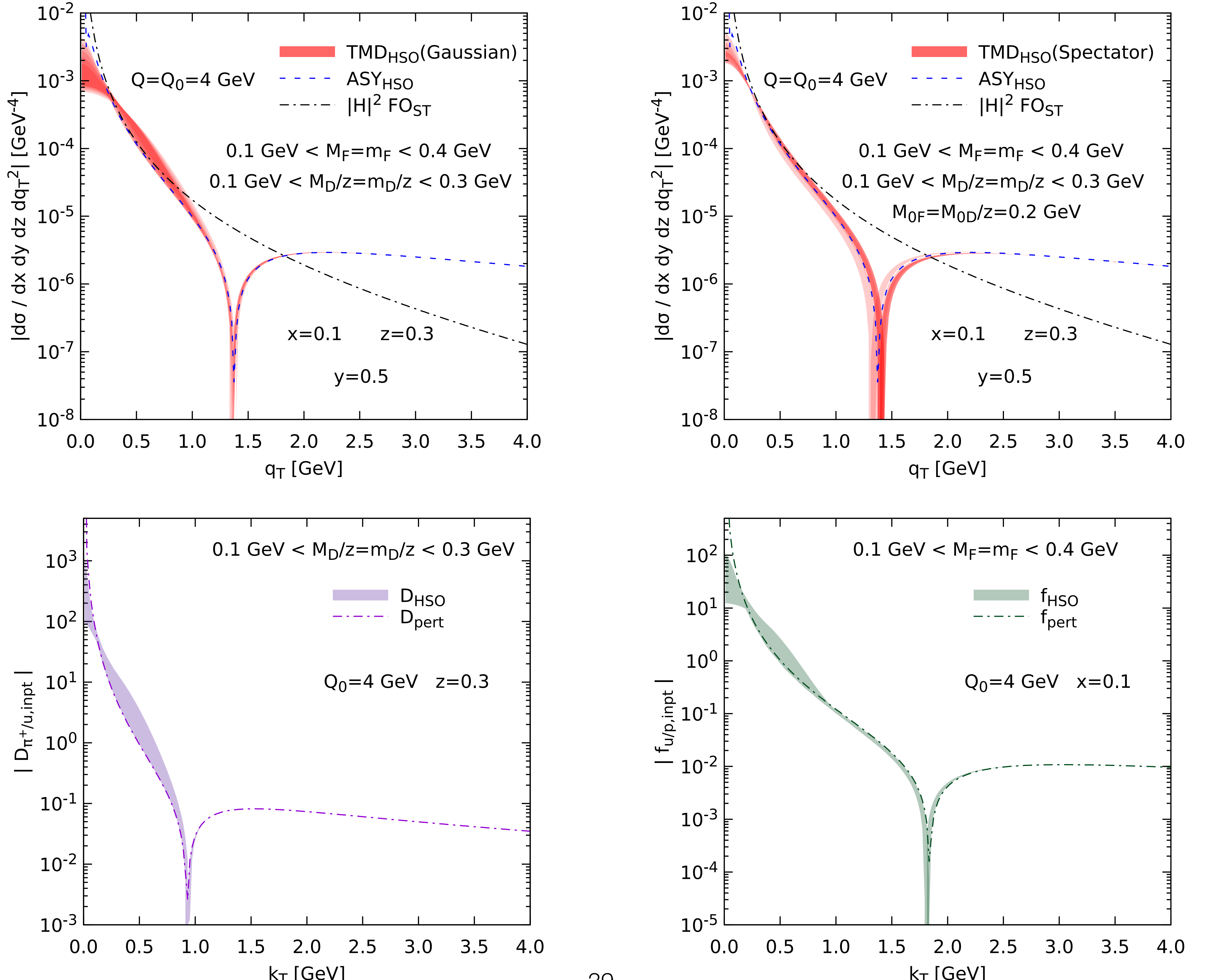
HSO approach





Consistency of the band with the asymptotic term means the models for TMDs have been made consistent with collinear factorization. In the usual approach, this is the **aim** when embedding the OPE.

HSO approach



✳Standard treatment vs HSO approach.

$\mathbf{b_{max}}$ sensitivity

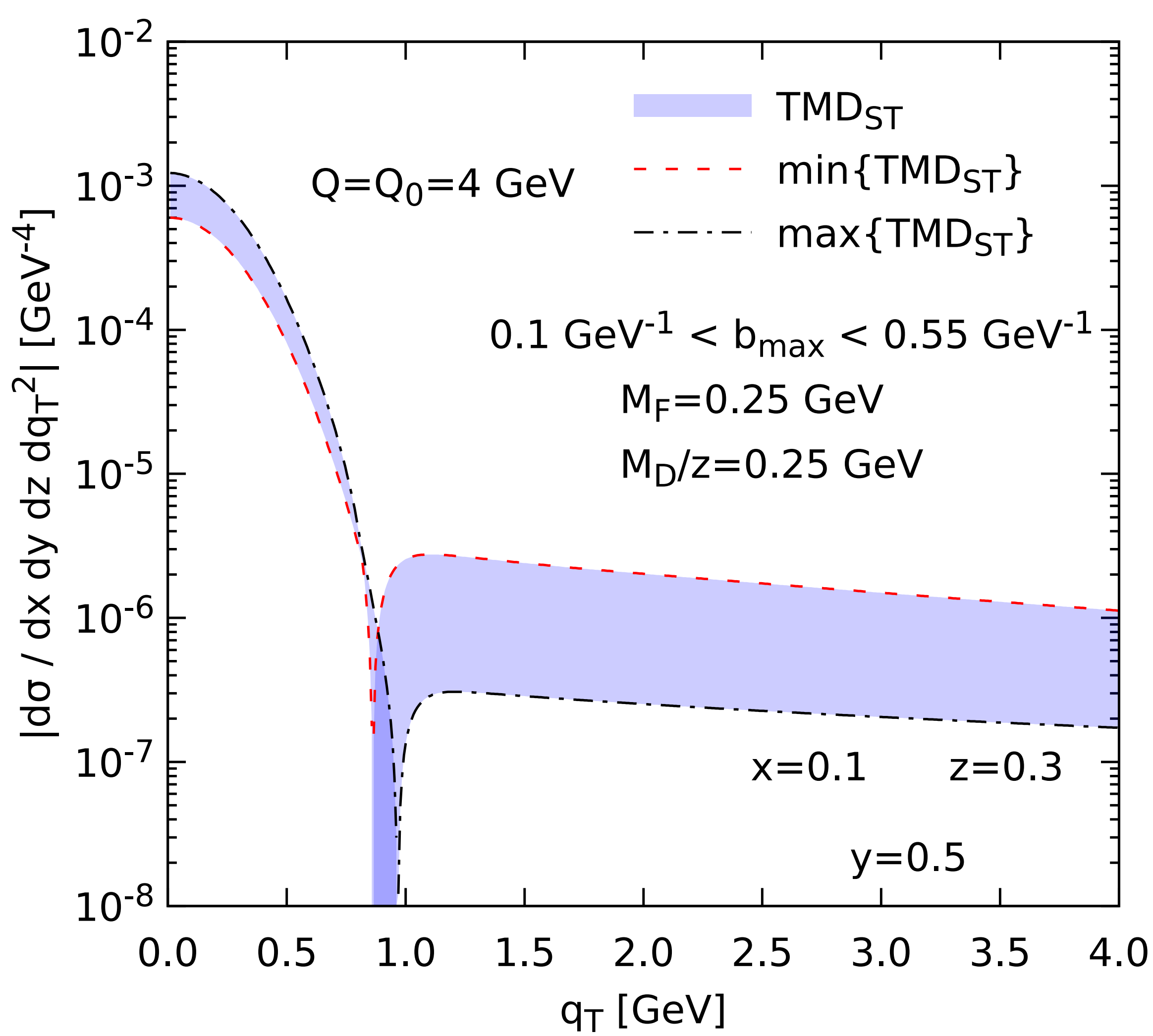
b_* prescription **not used** in HSO. It is instructive though to construct g-functions from HSO approach

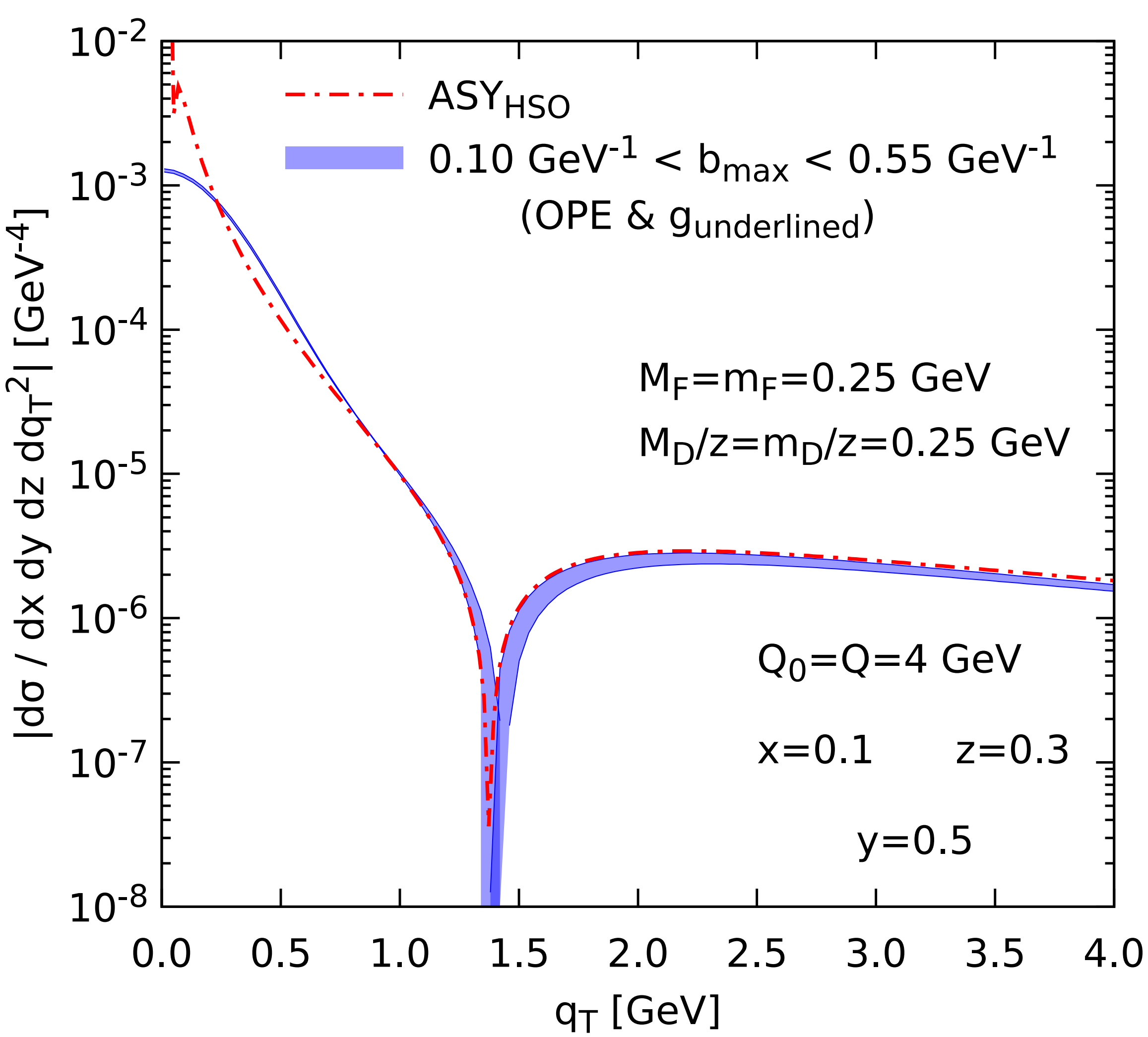
$$-g_{j/p}(x, b_{\mathrm{T}}) \equiv \ln\left(\frac{\tilde{f}_{j/p}(x, \boldsymbol{b}_{\mathrm{T}}; \mu_{Q_0}, Q_0^2)}{\tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_{Q_0}, Q_0^2)}\right), \qquad -g_{h/j}(z, b_{\mathrm{T}}) \equiv \ln\left(\frac{\tilde{D}_{h/j}(z, \boldsymbol{b}_{\mathrm{T}}; \mu_{Q_0}, Q_0^2)}{\tilde{D}_{h/j}(z, \boldsymbol{b}_*; \mu_{Q_0}, Q_0^2)}\right),$$

$$g_K(b_{\mathrm{T}}) \equiv \tilde{K}(b_*; \mu) - \tilde{K}(b_{\mathrm{T}}; \mu) \,.$$

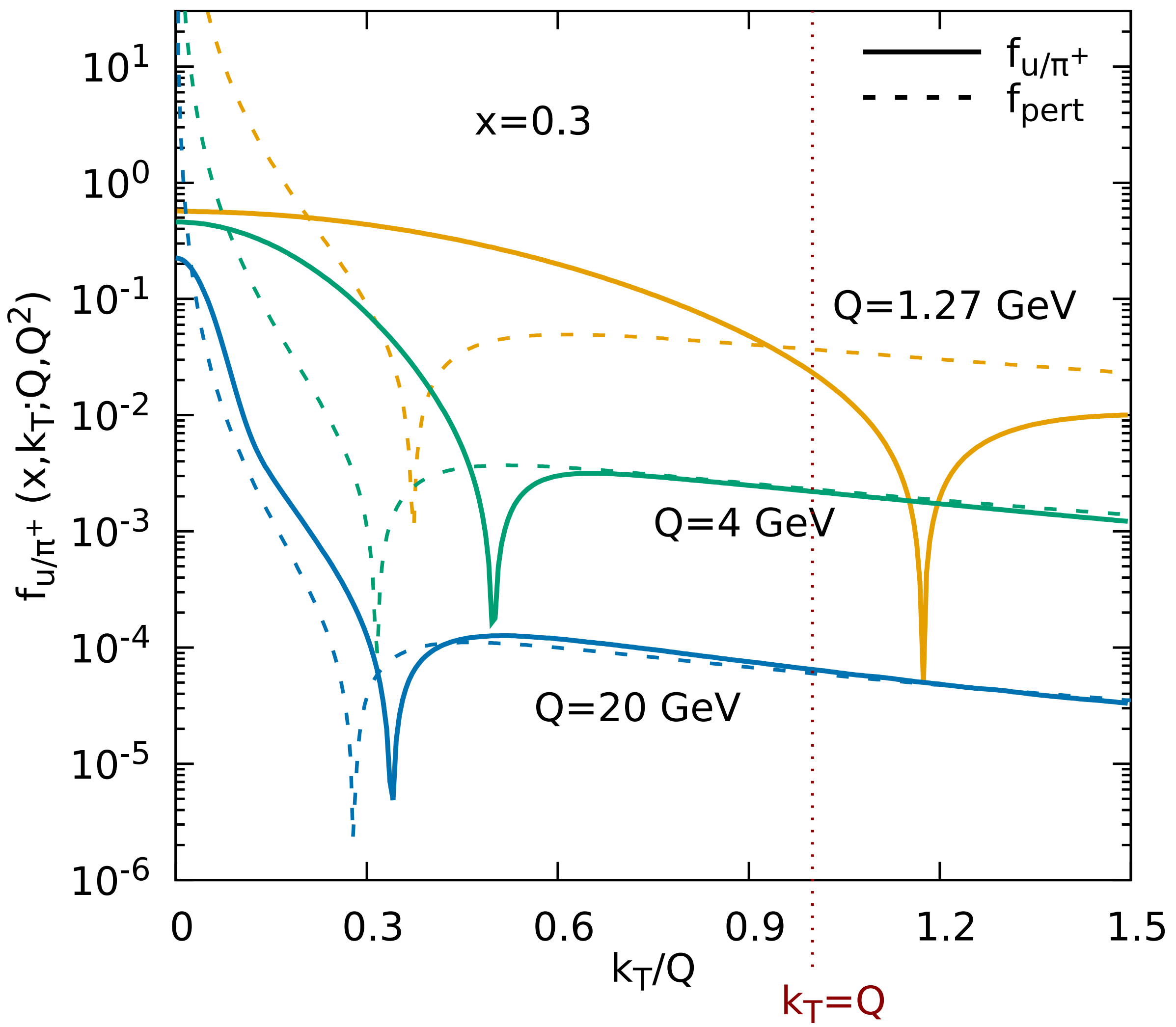
b_{max} sensitivity

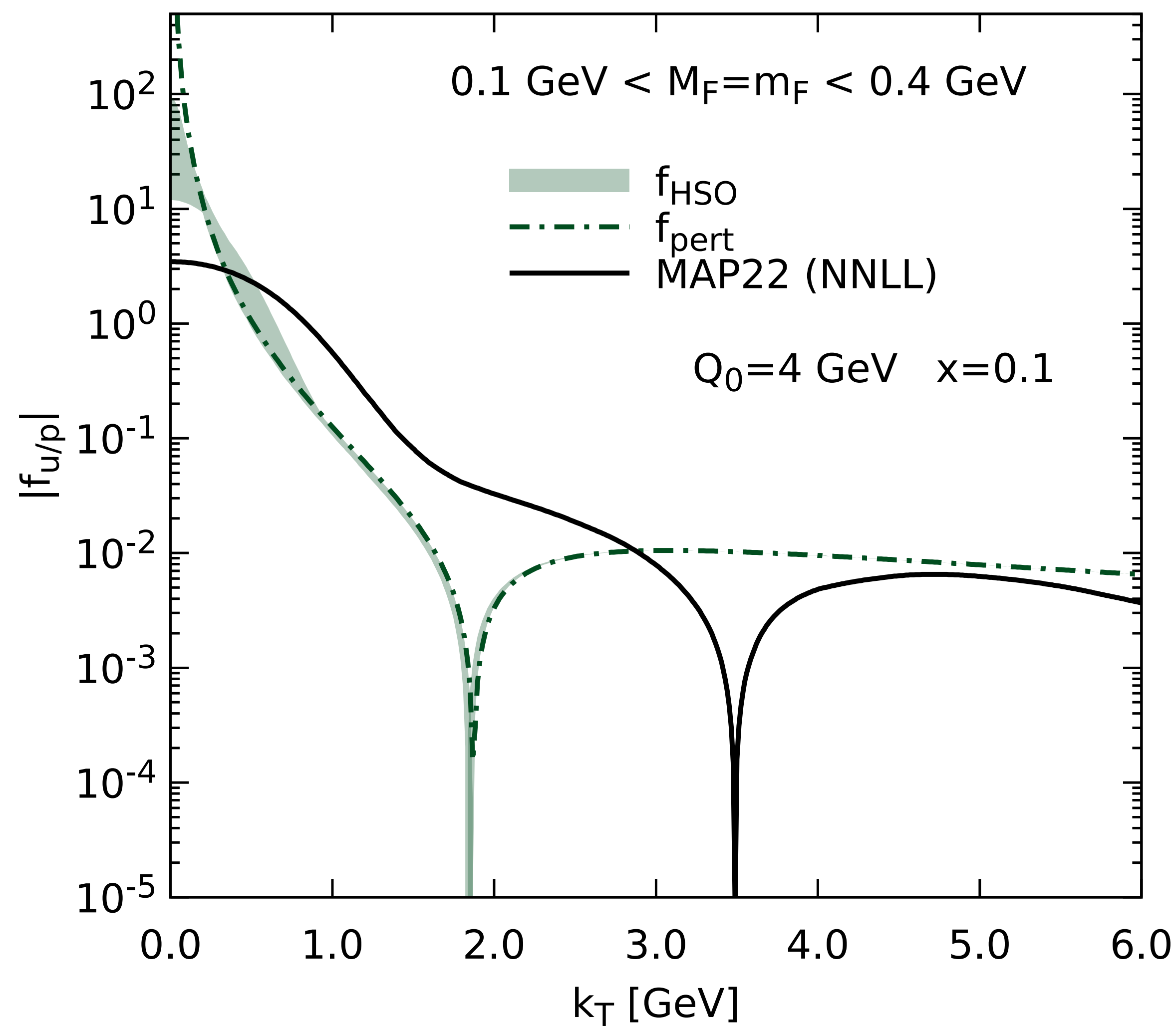
b_* prescription **not used** in HSO. It is instructive though to construct g-functions from HSO approach





Some other comparisons





Final Remarks

Theoretical constraints are important to really assess/study hadronic structure

We propose an approach to treat TMDs in full consistency with collinear factorization.

We call it HSO “Hadron structure oriented” approach. A framework to embed models of nonperturbative behavior into the CSS formalism

No b_* prescription

Effectively, imposes constraints to models, like g-functions.

Pheno applications to come.

Thanks.

Back up slides

Standard approach　　　　With explicit constraints