

# Factorizing Lepton Radiation in SIDIS

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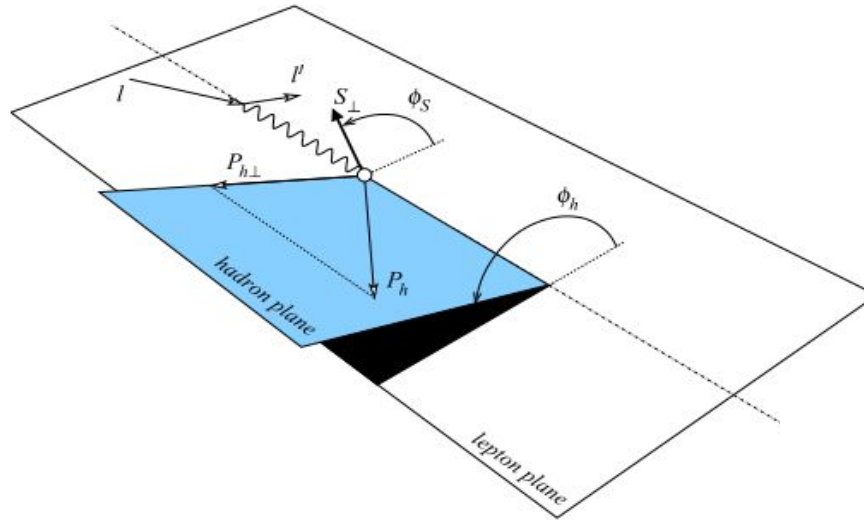
QCD Evolution, 22 May 2023

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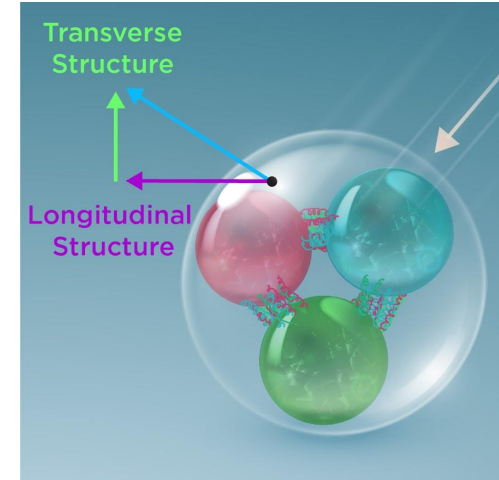
# Semi-Inclusive DIS: Frontier of Hadron Structure

EIC<sup>2</sup>

Jefferson Lab

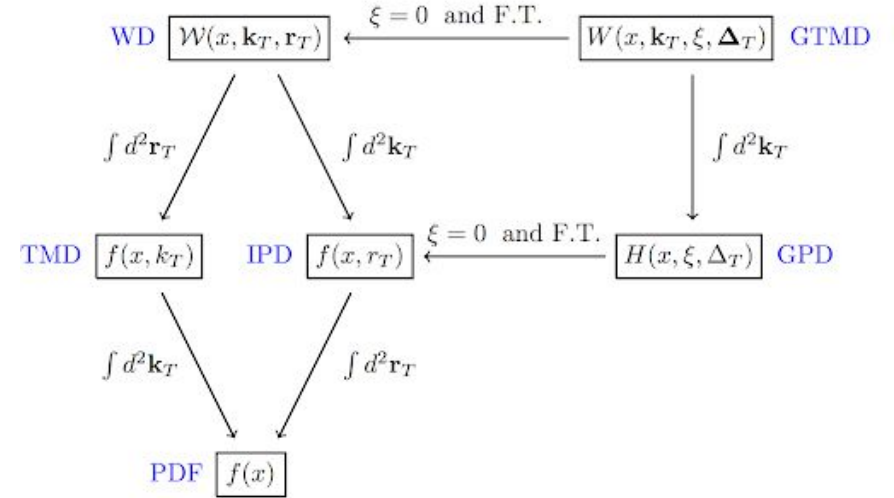


- Observation of two particles allows consideration of two scales of observables:  $P_{h\perp}$  and  $Q^2$

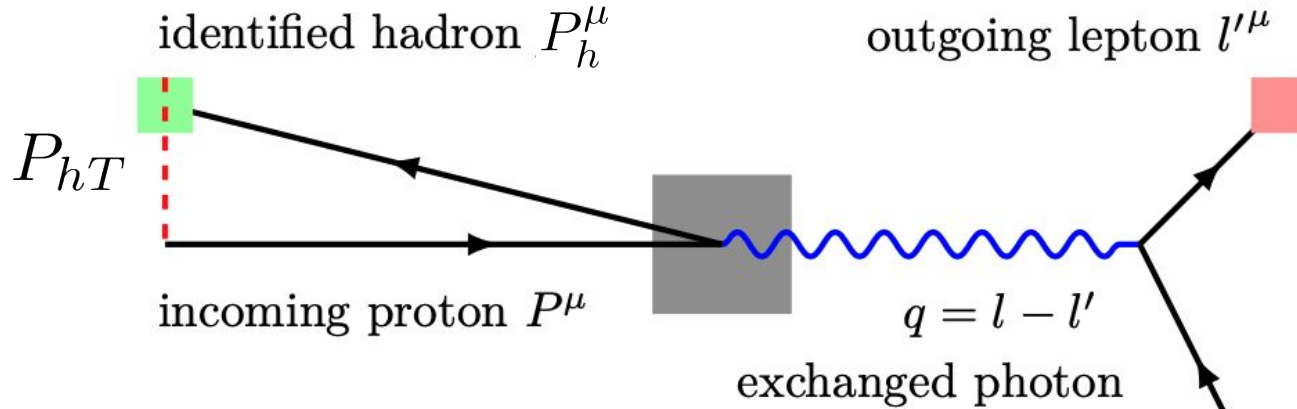


# SIDIS Cross Section

$$\begin{aligned}
 \frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} = & \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} \right. \\
 & + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} \\
 & + S_{\parallel} \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 & + S_{\parallel} \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 & + |S_{\perp}| \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \\
 & + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\
 & + \left. \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] \\
 & + |S_{\perp}| \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} \right. \\
 & + \left. \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \left. \right\}, \quad (
 \end{aligned}$$

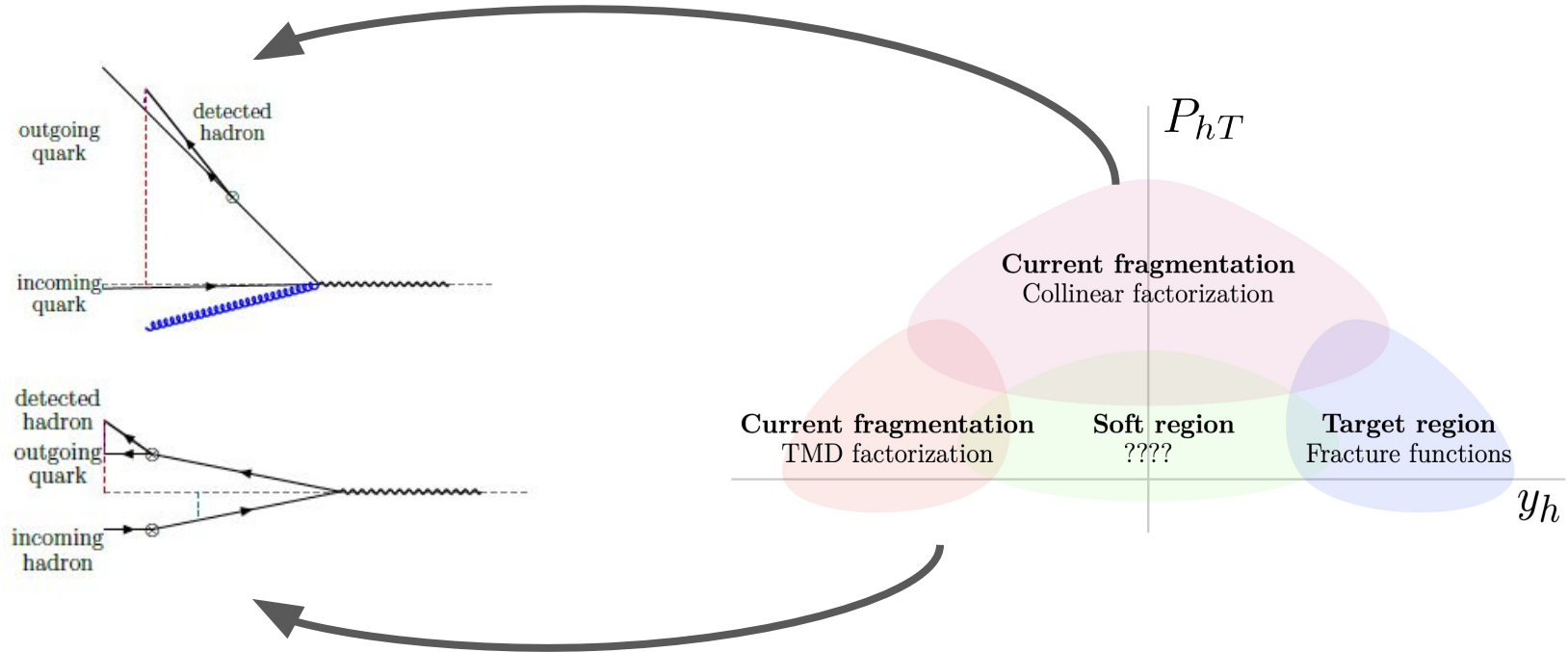


# Breit Frame

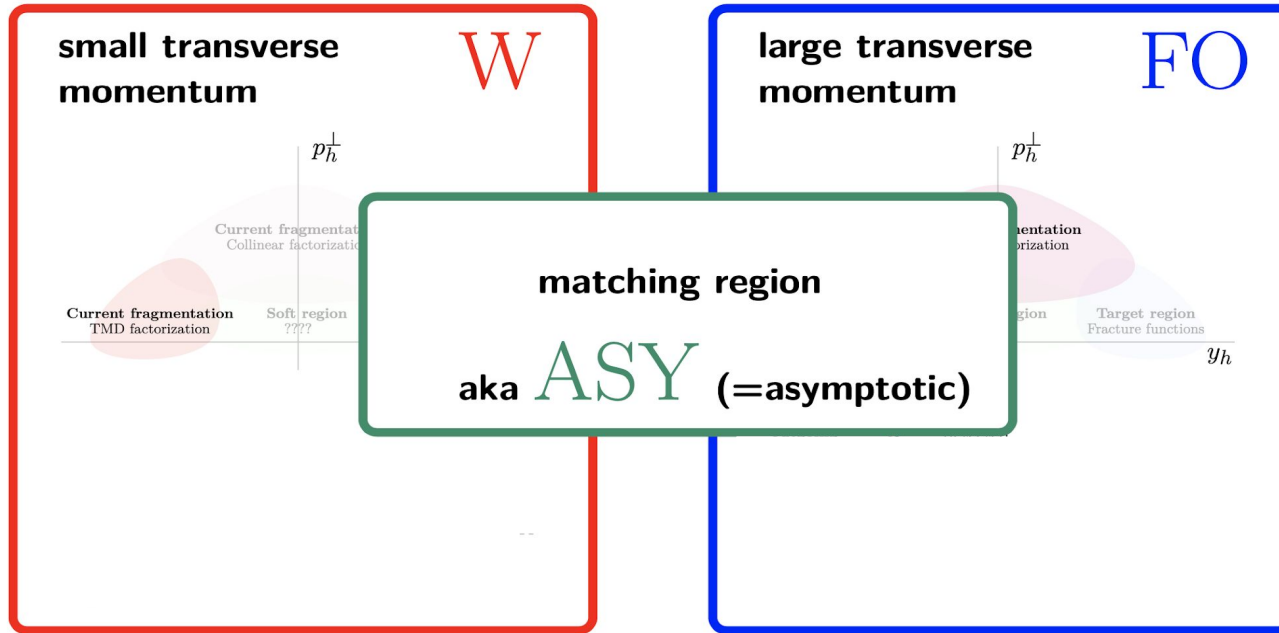


$$y_h = \frac{1}{2} \log \left( \frac{P_{hT}}{P_{h\parallel}} \right)$$

# Separation of Kinematic Regions



# Standard Approach



# Scale Separation of Cross Section

$$\frac{d\sigma}{dx dQ^2 dz dP_{hT}} = \textcolor{red}{W} + \textcolor{blue}{FO} - \textcolor{teal}{ASY} + \mathcal{O}(\Lambda_{QCD}^2/Q^2)$$

where

$$\sim \textcolor{red}{W} \quad \text{for } q_T \ll Q$$

$$P_{hT} = q_T z_h$$

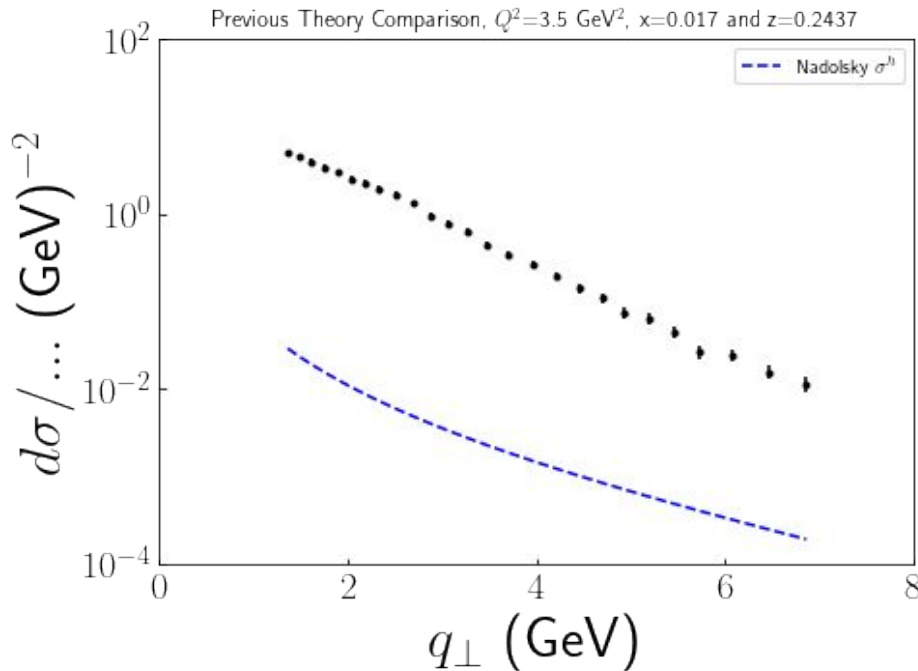
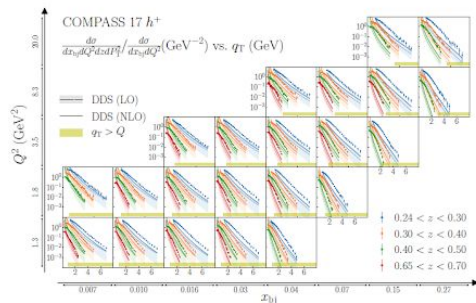
$$\sim \textcolor{blue}{FO} \quad \text{for } q_T \sim Q$$

- W term has resummed logarithms
- Asymptotic term perturbatively expands W term to fixed order
- Covers matching region between W and fixed order and removes non-dominant contributions



# Puzzles from Standard Approach

- Large discrepancy with data (example from COMPASS 2017 run, Aghasyan et al., 2018)
- Several attempted solutions
  - Next-to-leading order corrections (Wang et al., 2019)
  - Power Corrections (Liu and Qiu, 2020)



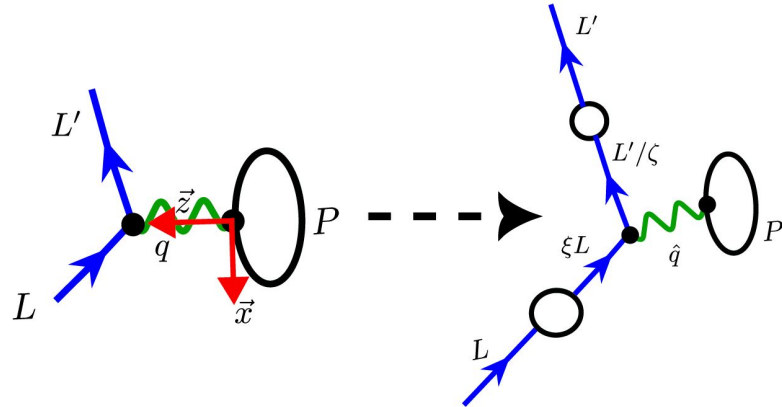
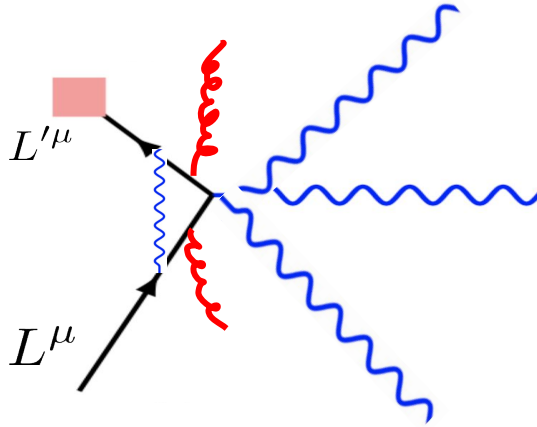
Gonzalez-Hernandez et al., 2018





# QED Effects

- Radiation of photon from initial state changes the direction of the exchanged photon
- Unobserved lepton radiation introduces ambiguity, as the radiation changes the unobserved photon's momentum (Liu et al., 2021)
- Hybrid factorization method to address these issues



# Hybrid Factorization Approach

- Leading collinear radiative corrections resummed into lepton distribution functions and lepton fragmentation functions
- Can still extract PDFs and TMDs from scattering cross sections

$$E_{P_h} E_{L'} \frac{d^6 \sigma^{LA \rightarrow L' h X}}{d^3 \vec{P}_h d^3 \vec{L}'} \approx \sum_{i,j} \int \frac{d\zeta}{\zeta^2} \frac{d\xi}{\xi} D_{e/j}(\zeta) f_{i/e}(\xi) \\ \times \left[ E_{k'} E_{P_h} \frac{d^6 \sigma^{kA \rightarrow k' h X}}{d^3 \vec{P}_h d^3 \vec{k}'} \right]_{k=\xi L, k'=L'/\zeta}$$

- Standard approach still valid in the virtual photon-hadron frame



# Explicit vs Structure Function Expansion

- Explicit Perturbative Expansion

$$E_{k'} E_{P_h} \frac{d^6 \sigma^{kA \rightarrow k' hX}}{d^3 \vec{P}_h d^3 \vec{k}'} \approx \frac{1}{2S} \sum_{n,m} \int \frac{dz}{z^2} \frac{dx}{x} \tilde{D}_{h/n}(z) \tilde{f}_{m/h}(x) \hat{H}^{im \rightarrow jn}(k, k', p, p') \Big|_{p=xP, p'=P_h/z}$$

$$\hat{H}^{im \rightarrow jn}(k, k', p, p') = \sum_{s,r} \hat{H}_{im \rightarrow jn}^{(s,r)}(k, k', p, p') \alpha^s \alpha_S^r + \mathcal{O} \left( \frac{\Lambda_{QCD}^2}{Q^2}, \frac{m_e^2}{Q^2} \right)$$

- Structure Function Expansion

$$E_{k'} E_{P_h} \frac{d^6 \sigma^{kA \rightarrow k' hX}}{d^3 \vec{P}_h d^3 \vec{k}'} = \left( \frac{4\hat{x}_B}{\hat{Q}^2} \sqrt{\hat{z}_h^2 - (\hat{\gamma} \hat{P}_{h\perp} / \hat{Q})^2} \right) \frac{\hat{x}_B}{x_B \xi \zeta} \\ \times \left[ \frac{\alpha^2}{\hat{x}_B \hat{y} \hat{Q}^2} \frac{\hat{y}}{2(1-\hat{\epsilon})} \left( 1 + \frac{\hat{\gamma}^2}{2\hat{x}_B} \right) \sum_n \hat{w}_n F_n^h \left( \hat{x}_B, \hat{Q}^2, \hat{z}_h, \hat{P}_{h\perp}^2 \right) \right]$$



# Kinematic Variables Appearing in Full SIDIS Cross Section

$$x_B = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z = \frac{P \cdot P_h}{P \cdot q}, \quad \gamma = \frac{2Mx_B}{Q}$$

$$\varepsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}$$

$$\begin{aligned} \cos(\phi_h) &= -\frac{l_\mu P_{h\nu} g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, & \cos(\phi_S) &= -\frac{l_\mu S_\nu g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}}, \\ \sin(\phi_h) &= -\frac{l_\mu P_{h\nu} \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, & \sin(\phi_S) &= -\frac{l_\mu S_\nu \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}} \end{aligned}$$

$$S^\mu = S_\parallel \frac{P^\mu - q^\mu M^2/(P \cdot q)}{M\sqrt{1+\gamma^2}} + S_\perp^\mu$$

$$l_\perp^\mu = g_\perp^{\mu\nu} l_\nu, \quad P_{h\perp}^\mu = g_\perp^{\mu\nu} P_{h\nu}$$

$$S_\parallel = \frac{S \cdot q}{P \cdot q} \frac{M}{\sqrt{1+\gamma^2}}, \quad S_\perp^\mu = g_\perp^{\mu\nu} S_\nu$$

$$g_\perp^{\mu\nu} = g_{\mu\nu} - \frac{q^\mu P^\nu + q^\nu P^\mu}{P \cdot q(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2} \left( \frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2} \right)$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} \frac{P_\rho q_\sigma}{P \cdot q \sqrt{1+\gamma^2}}$$



# Kinematic Variables with QED effects Highlighted

$$x_B = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z = \frac{P \cdot P_h}{P \cdot q}, \quad \gamma = \frac{2Mx_B}{Q}$$

$$\cos(\phi_h) = \frac{l_\mu P_{h\nu} g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_h^2}}, \quad \cos(\phi_S) = \frac{l_\mu S_\nu g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}},$$

$$\sin(\phi_h) = \frac{l_\mu P_{h\nu} \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_h^2}}, \quad \sin(\phi_S) = \frac{l_\mu S_\nu \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}}$$

$$l_\perp^\mu = g_\perp^{\mu\nu} l_\nu, \quad P_{h\perp}^\mu = g_\perp^{\mu\nu} P_{h\nu}$$

$$g_\perp^{\mu\nu} = g_{\mu\nu} - \frac{q^\mu P^\nu + q^\nu P^\mu}{P \cdot q (1 + \gamma^2)} + \frac{\gamma^2}{1 + \gamma^2} \left( \frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2} \right)$$

$$\varepsilon = \frac{1 - y - \frac{1}{4} \gamma^2 y^2}{1 - y + \frac{1}{2} y^2 + \frac{1}{4} \gamma^2 y^2}$$

$$S^\mu = S_\parallel \frac{P^\mu - q^\mu M^2 / (P \cdot q)}{M \sqrt{1 + \gamma^2}} + S_\perp^\mu$$

$$S_\parallel = \frac{S \cdot q}{P \cdot q \sqrt{1 + \gamma^2}}, \quad S_\perp^\mu = g_\perp^{\mu\nu} S_\nu$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} \frac{P_\rho q_\sigma}{P \cdot q \sqrt{1 + \gamma^2}}$$



# QED Effects on Relevant Kinematic Variables

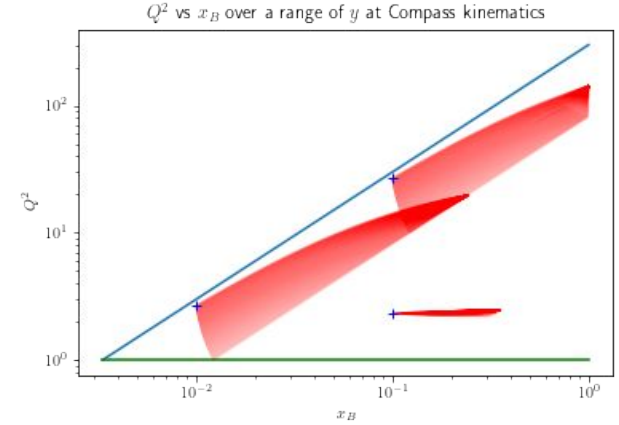
- Explicit dependence of major kinematic variables on radiation parameters  $\xi, \zeta$

$$\hat{x}_B = \frac{\xi y x_B}{\xi \zeta - 1 + y}$$

$$\hat{Q}^2 = \frac{\xi}{\zeta} Q^2$$

$$\hat{z}_h = \frac{\zeta y z_h}{\xi \zeta - 1 + y}$$

$$\hat{y} = \frac{\hat{Q}^2}{\xi S \hat{x}_B}$$

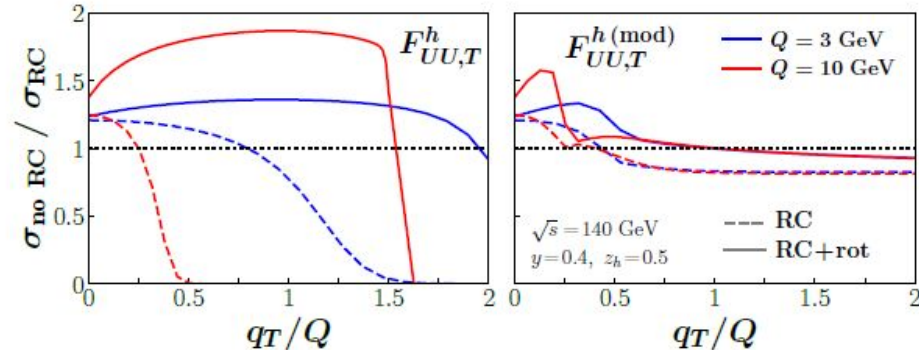


$$\hat{P}_{h\perp}^2(\xi, \zeta, \phi_h, y, z_h, Q, P_{h\perp}) = \frac{-2 \cos \phi_h Q^5 z_h (\zeta \xi - 1) P_{h\perp} \sqrt{1 - y} (\zeta \xi + y - 1) + Q^4 P_{h\perp}^2 (\zeta \xi + y - 1)^2 - Q^6 (y - 1) z_h^2 (\zeta \xi - 1)^2}{Q^4 (\zeta \xi + y - 1)^2}$$

# Previous Work on QED Effects

- Liu et al., 2020 and 2021 established new factorization approach
- Showed effects of radiative corrections to Gaussian approximation of W term with artificial fixed order tail

$$F_{UU,\text{Gaussian}}^h(x_B, y, z_h, Q^2, q_\perp) = \sum_q e_q^2 f_{q/h}(x_B, Q^2) D_{h'/q}(z_h, Q^2) \frac{e^{-(z_h q_\perp)^2 / \langle (z_h q_\perp)^2 \rangle}}{\pi \langle (z_h q_\perp)^2 \rangle}$$

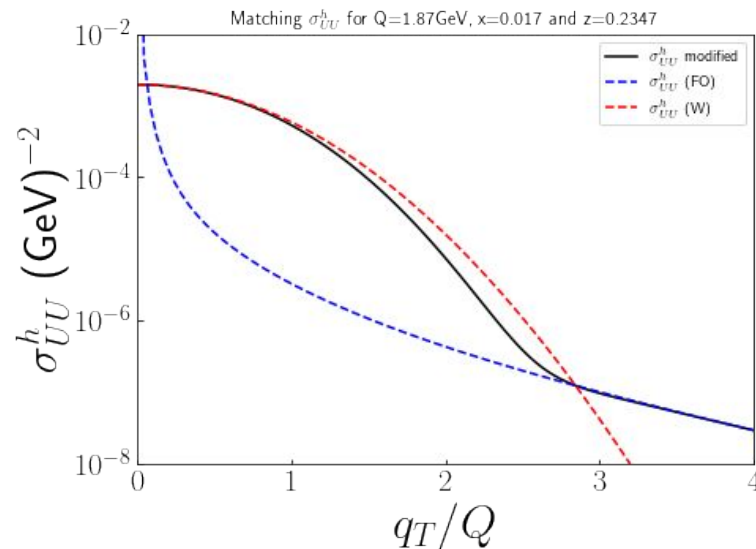


# Setup

- Fixed order calculation comes from Nadolsky et al., 1999 (at finite  $q_T \sim Q$ )
- To match W to FO, toy scheme used in this work (and in Liu et al., 2021):

$$\sigma_{UU,\text{Modified}}^h = \textcolor{red}{W}R + (1 - R)\textcolor{blue}{FO}$$

$$R = e^{-A(q_T/Q)^B}$$

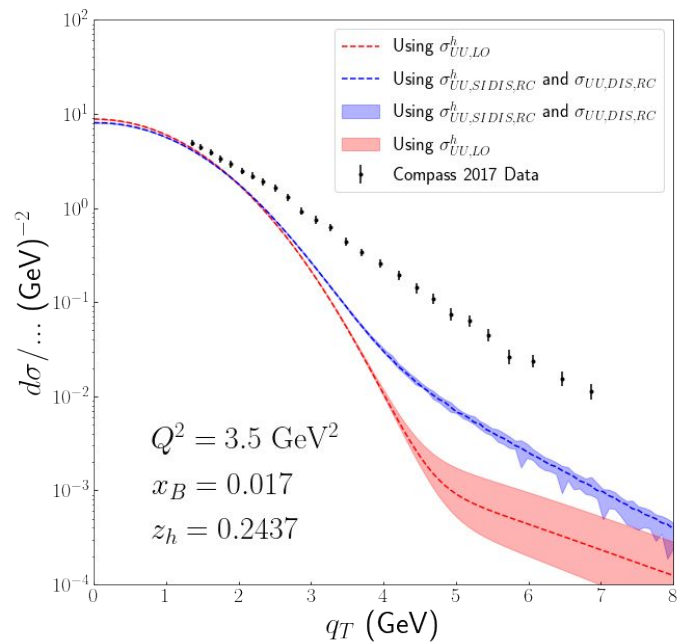


Before QED corrections

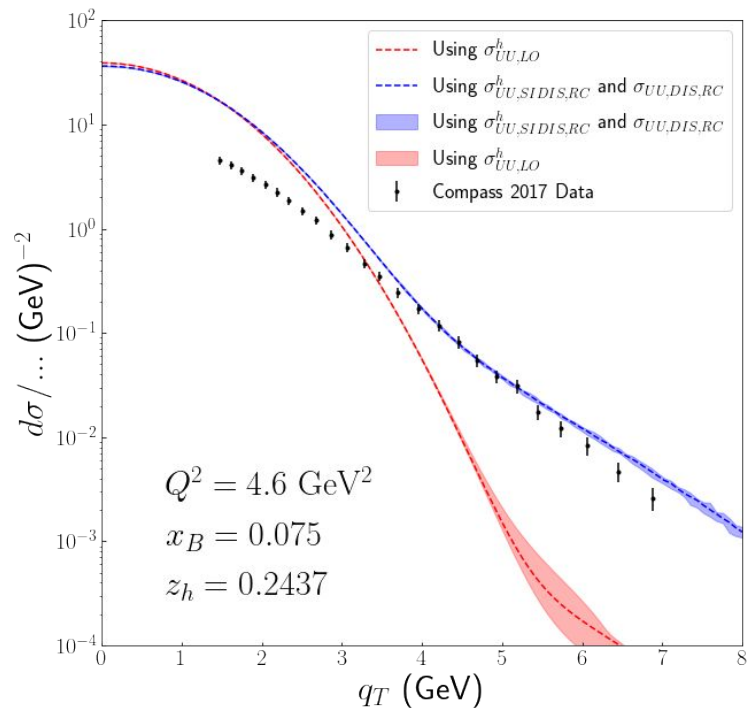




# Results



$$\frac{d\sigma}{dx_B dQ^2 dz_h dP_{hT}^2} / \frac{d\sigma}{dx_B dQ^2}$$



$$P_{hT} = q_T z_h$$



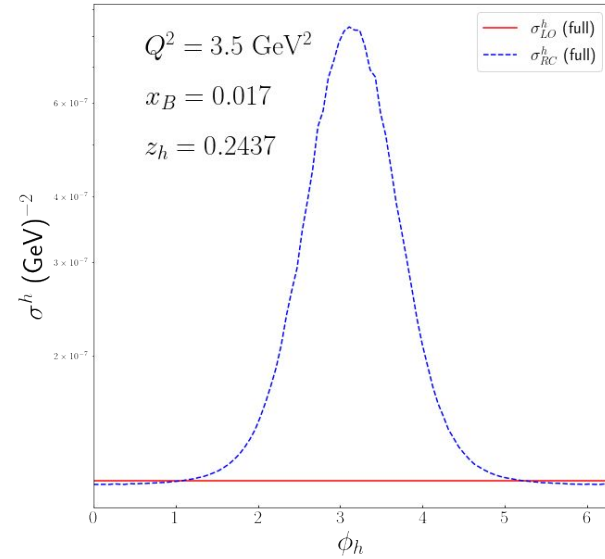
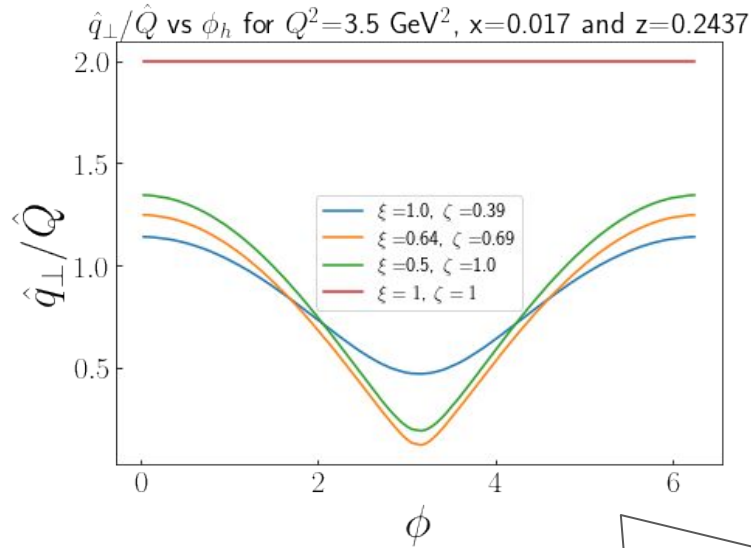
# Role of Angular Effects on $\hat{P}_{hT}$

- Standard approach assumes separable angular dependence
- Lepton radiation introduces internal angular dependence
- Increases weight of back-to-back region

$$\frac{d\sigma}{dx_B dy dz_h dQ^2 dP_{\perp}^2} \propto F_{UU,T} + F_{UU,T} + \cos \phi_h F_{UU}^{\cos \phi_h} + \cos(2\phi_h) F_{UU}^{\cos 2\phi_h}$$

$$\hat{P}_{h\perp}^2(\xi, \zeta, \phi_h, y, z_h, Q, P_{h\perp}) \propto \frac{\cos \phi_h (\zeta \xi - 1) P_{h\perp}}{(\zeta \xi + y - 1)} + P_{h\perp}^2$$

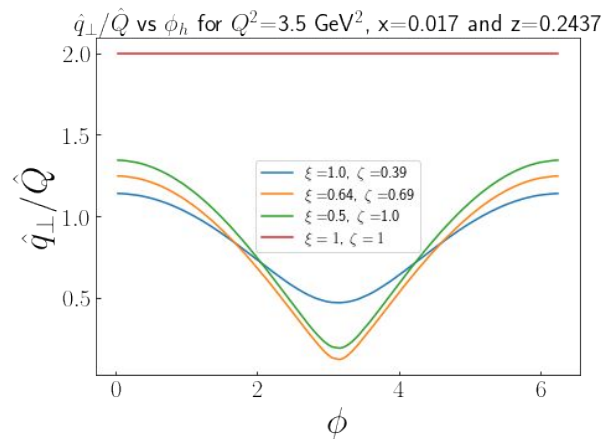
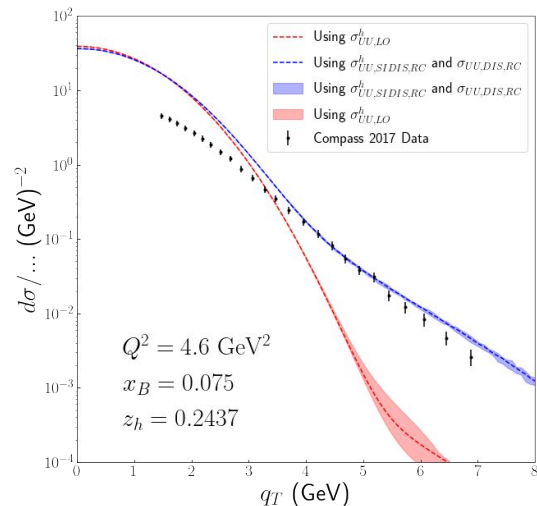
# Importance of Angular Effects



Angular modulation on internal transverse momentum induced by QED effects!

# Conclusion

- Joint QED + QCD factorization scheme for SIDIS process showed significant effects on cross section
- Full transverse momentum spectrum required to address QED effects
- Potential resolution to discrepancies between previous theory and COMPASS data



# Acknowledgements

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# Backup Slides



# Cross Section Expression

$$\begin{aligned}
 E_{L'} E_{P'} \frac{d\sigma}{d^3\vec{L}' d^3\vec{P}'} &= \sum_q \int_{\zeta_{\min}, \xi_{\min}, z_{\min}, 0}^1 \frac{d\zeta}{\zeta^2} \frac{d\xi}{\xi} \frac{dz}{z^2} \frac{dx}{x} \tilde{D}_{L/e}(\zeta) \tilde{f}_{e/L}(\xi) \\
 &\times \tilde{D}_{h/q}(z) \tilde{f}_{q/h}(x) \left( \frac{1}{4(2\pi)^6} \right) \frac{-8e^4 e_q^2 g^2 \zeta}{3\xi Q^2 S x z (\zeta \xi S + U') (S' + \zeta \xi U)} \\
 &(\xi^2 (2Q^4 z^2 + 2\zeta Q^2 U z + \zeta^2 (S^2 x^2 z^2 + U^2)) + 2\xi S' (Q^2 z + \zeta (U - U'))) \\
 &- 2\xi U' (Q^2 z (\xi \zeta - x z) + \zeta^2 \xi U) + (S')^2 + (U')^2 (2\zeta^2 \xi^2 + x^2 z^2)) \\
 &\times \frac{(2\pi)}{(1/\zeta)U' + \xi S + (1/z)T'} \delta \left( x - \frac{(\xi/\zeta)Q^2 - (1/z\zeta)S' + (\xi/z)U}{(1/\zeta)U' + \xi S + (1/z)T'} \right)
 \end{aligned}$$





# Transformation Between Lab Frame and Virtual Breit Frame

- Traditionally Breit frame is photon-hadron frame
- Lepton radiation makes frame determination ambiguous
- All historical factorization formula defined in photon hadron frame
- Introduce **virtual** photon-hadron frame which is determined by a given pair of  $\xi, \zeta$  under one-photon exchange approximation

$$x^\mu = (x^+, x^-, \vec{x}_\perp = (x^1, x^2)) \quad \tilde{x}^\sigma = R_\nu^\sigma \Lambda_\mu^\nu x^\mu$$

$$y \Rightarrow \tilde{y} = y \quad \theta_{\text{No Radiation}} = \arctan \left( \frac{-E' \sin \theta_{L,L'}}{E - E' \cos \theta_{L,L'}} \right)$$

$$\begin{pmatrix} x \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde{x} \\ \tilde{z} \end{pmatrix} = R(\theta(\xi, \zeta)) \begin{pmatrix} x \\ z \end{pmatrix} \quad \theta_{\text{With Radiation}} = \arctan \left( \frac{-E' \sin \theta_{L,L'}}{(\xi\zeta)E - E' \cos \theta_{L,L'}} \right)$$

$$\phi = \tanh^{-1} \left( -\sqrt{\frac{S}{4m^2 + S}} \right)$$



# Helicity Expansion

- Similar to Ji et al., 2006

$$Z^\mu = -\frac{\hat{q}^\mu}{\hat{Q}}$$

$$T^\mu = \left(\frac{1}{\hat{Q}}\right) (\hat{q}^\mu + 2\hat{x}_B P^\mu)$$

$$X^\mu = \left(\frac{1}{\vec{\hat{q}}_T}\right) \left(\frac{(P')^\mu}{z_B} - \hat{q}^\mu - \left(1 + \frac{\vec{\hat{q}}_T^2}{\hat{Q}^2}\right) \hat{x}_B P^\mu\right)$$

$$Y^\mu = \epsilon^{\mu\nu\alpha\beta} Z_\nu T_\alpha X_\beta$$



# Perturbative Coefficients in Helicity Basis

- Working with helicity-based hadronic structure functions (similar to lepton case in Liu et al., 2021)

$$\hat{W}^{\mu\nu} = \underbrace{\frac{1}{2} \left( \hat{X}^\mu \hat{X}^\nu + \hat{Y}^\mu \hat{Y}^\nu \right)}_{\text{Transverse}} \hat{H}_{TT} + \underbrace{\left( \hat{T}^\mu \hat{T}^\nu \hat{H}_L + \left( \hat{T}^\mu \hat{X}^\nu + \hat{X}^\mu \hat{T}^\nu \right) \hat{H}_\Delta + \dots \right)}_{\text{other, } \mathcal{O}(\alpha_S^2)}$$

- Fixed order projection onto partonic states  $H_{TT} = \frac{1}{2} (X^\mu X^\nu + Y^\mu Y^\nu) W_{\mu\nu}$
- Separate into real and virtual terms  $W_{\mu\nu} = R_{\mu\nu} + V_{\mu\nu}$
- Separate based on momentum scale

$$H_{TT}(\vec{\hat{q}}_T) = W_{TT}(\text{small } \vec{\hat{q}}_T) + Y_{TT}(\text{large } \vec{\hat{q}}_T)$$

$$Y_{TT} = R_{TT} - R_{TT}^{\text{asympt}}$$

# CSS-like Formalism

- Similar process as in CSS formalism (Collins et al., 1985)

$$\tilde{W}_{TT}(\vec{b}_T, Q) = \int d^2\vec{q}_T e^{i\vec{q}_T \cdot \vec{b}_T} W_{TT}(\vec{q}_T, Q) = e^{-S} [C_f \otimes f] \otimes [C_D \otimes D]$$

- Expanding in powers of  $\alpha_s$

$$\tilde{W}_{TT} = C_f^{(1)} C_D^{(0)} S^{(0)} + C_f^{(0)} C_D^{(1)} S^{(0)} + C_f^{(0)} C_D^{(0)} S^{(1)}$$

$$S = 1 - \frac{\alpha}{\pi} \left[ \frac{1}{2} A^{(1)} \ln^2 \frac{\nu_Q^2}{\mu_b^2} + B^{(1)} \ln \frac{\nu_Q^2}{\mu_b^2} \right]$$



# NLO Perturbative Coefficients

$$\tilde{W}_{TT} = C_f^{(1)} C_D^{(0)} S^{(0)} + C_f^{(0)} C_D^{(1)} S^{(0)} + C_f^{(0)} C_D^{(0)} S^{(1)}$$

- Comparing derivation with above:

$$A^{(1)} = 1$$

$$B^{(1)} = -\frac{3}{2}$$

$$C_f^{(1)}(\lambda) = \frac{1}{2\lambda}(1 - 2\lambda) - \frac{1}{\lambda} \left( \frac{\lambda^2 + 1}{1 - \lambda} \right)_+ \ln \frac{\mu_{\bar{M}S}}{\mu_b} - \delta(1 - \lambda)$$

$$C_D^{(1)}(\eta) = \frac{1}{2\eta}(1 - 2\eta) - \frac{1}{\eta} \left( \frac{\eta^2 + 1}{1 - \eta} \right)_+ \ln \frac{\mu_{\bar{M}S}}{\mu_b} - \delta(1 - \eta)$$



# Full W term

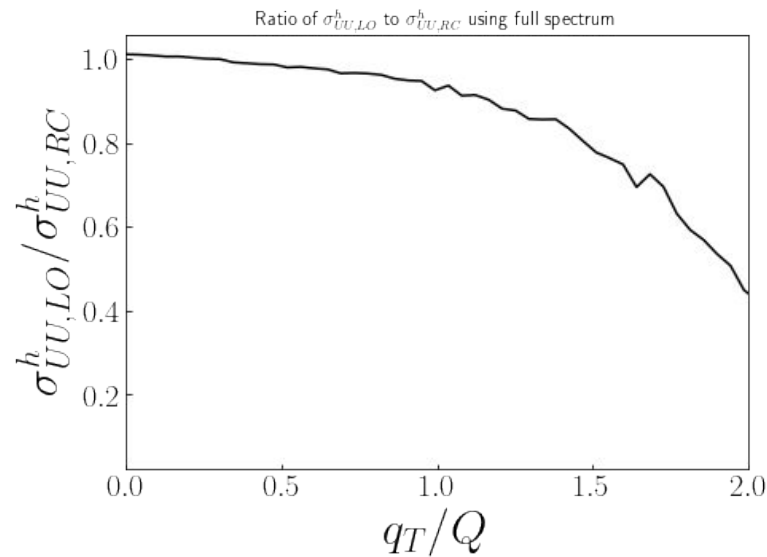
$$\begin{aligned}
 \tilde{W}_{TT} = & \frac{4e_Q^2}{3} \left( \frac{\alpha_S}{\pi} \right) \left[ \delta(1-\lambda)\delta(1-\eta) \left( -\frac{1}{2} \left( \ln^2 \frac{\nu_Q^2}{\mu_b^2} - 3 \ln \frac{\nu_Q^2}{\mu_b^2} \right) \right) \right. \\
 & - \ln \frac{\mu_{\bar{M}S}}{\mu_b} \left( \frac{1}{\lambda} \left( \frac{\lambda^2 + 1}{1-\lambda} \right)_+ \delta(1-\eta) + \frac{1}{\eta} \left( \frac{\eta^2 + 1}{1-\eta} \right)_+ \delta(1-\lambda) \right) \\
 & + \frac{1}{2} \left( \left( \frac{1-2\lambda}{\lambda} \right) \delta(1-\eta) + \left( \frac{1-2\eta}{\eta} \right) \delta(1-\lambda) \right) - 2\delta(1-\lambda)\delta(1-\eta) \\
 & \left. - \frac{1}{2\epsilon} \left( \frac{1}{\lambda} \left( \frac{\lambda^2 + 1}{1-\lambda} \right)_+ \delta(1-\eta) + \frac{1}{\eta} \left( \frac{\eta^2 + 1}{1-\eta} \right)_+ \delta(1-\lambda) + \delta(1-\lambda)\delta(1-\eta) \right) \right]
 \end{aligned}$$

# Matching to Fixed Order Region

- Fixed order calculation comes from Nadolsky et al., 1999 (at finite  $q_T \sim Q$ )
- To match W to FO, toy scheme used in this work (and in Liu et al., 2021):

$$\sigma_{UU,\text{Modified}}^h \approx \sigma_{UU,W}^h R(q_T/Q) + (1 - R(q_T/Q))\sigma_{UU,\text{FO}}^h$$

$$R = e^{-A(q_T/Q)^B}$$



# Breaking of Azimuthal Symmetry

- Standard approach assumes azimuthal symmetry
- Lepton radiation introduces a preferred direction of photon and therefore outgoing hadron
- Breaking of azimuthal symmetry increases weight of back-to-back region

