TMD factorisation for diffractive jets in photon-nucleus interactions

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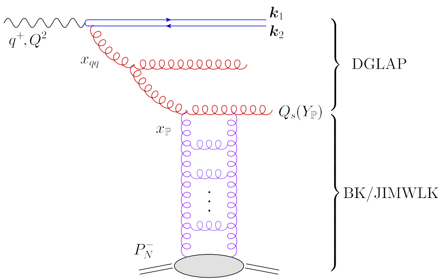
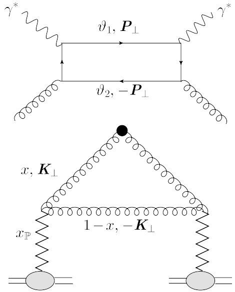


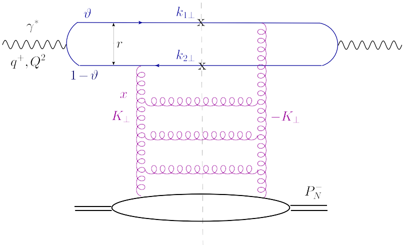


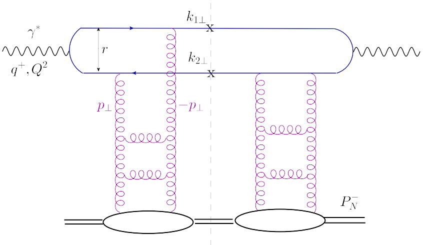
Outline

- Colour Glass Condensate: effective theory for scattering amplitudes in QCD at high energy: small Bjorken $x_{\rm Bj} \leq 10^{-2}$, small Pomeron $x_{\mathbb{P}} \leq 10^{-2}$
  - all-order resummations of non-linear ("higher-twist") corrections associated with high gluon occupation numbers: "gluon saturation"
- Intrinsec semi-hard scale (saturation momentum) which grows with the energy and the nuclear mass number A : $Q_s^2(x, A) \sim A^{1/3}/x^{0.3}$
  - effective infrared cutoff which allows for perturbative calculations
- When several scales $(Q^2,\, P_\perp^2 \gg K_\perp^2 \sim Q_s^2)$ are present: TMD factorisation
  - seminal paper by Dominguez, Marquet, Xiao, Yuan (arXiv:1101.0715)
  - many recent developments for inclusive dijets (talk by Paul Caucal)
- This talk: TMD factorisation also holds for diffractive jets at small $x_{\mathbb{P}}$
- Diffraction is controlled by strong scattering/saturation : first principles calculations of the diffractive TMDs

Inclusive vs. exclusive dijets in γA

- High energy photon-nucleus interactions: DIS, nucleus-nucleus UPCs
- Hard dijets: $P_\perp \equiv |\boldsymbol{k}_1 - \boldsymbol{k}_2|/2 \gg K_\perp \equiv |\boldsymbol{k}_1 + \boldsymbol{k}_2|$ ("correlation limit")





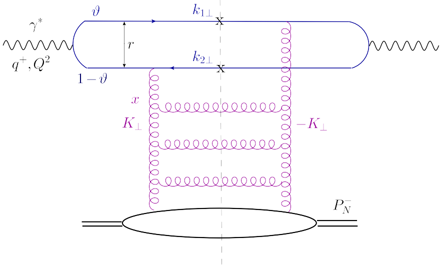
- Small quark-antiquark dipole $r \sim 1/P_\perp \Rightarrow$ weak scattering

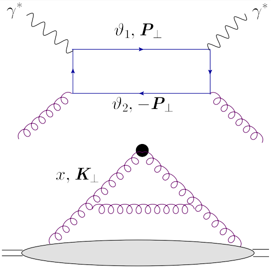
$$T_{q\bar{q}}(r) = 1 - \frac{\mathrm{tr}}{N_c}\langle V(\boldsymbol{x})V^\dagger(\boldsymbol{y})\rangle \simeq \begin{cases} r^2 Q_s^2, & \text{for } rQ_s \ll 1 \text{ (color transparency)} \\ 1, & \text{for } rQ_s \gtrsim 1 \text{ (black disk limit)} \end{cases}$$

- Elastic scattering is stronger suppressed when the scattering is weak

$$\sigma_{inel} \propto 2\mathrm{Im}\, T \longleftrightarrow \sigma_{el} \propto |T|^2$$

TMD factorisation for inclusive dijets *(cf. P. Caucal)*

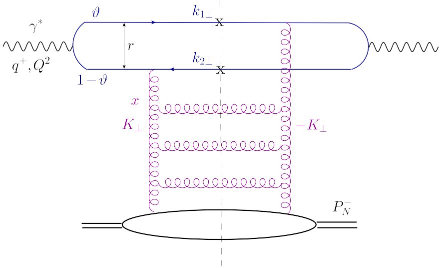


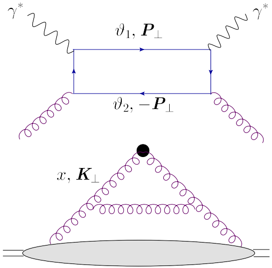


$$\frac{\mathrm{d}\sigma_{\rm incl}^{\gamma A\to q\bar q X}}{\mathrm{d}\vartheta_1 \mathrm{d}\vartheta_2 \mathrm{d}^2\boldsymbol{P}\mathrm{d}^2\boldsymbol{K}} = \underbrace{H(\vartheta_1,\vartheta_2,Q^2,P_\perp^2)}_{\text{hard factor}} \ \underbrace{\frac{\mathrm{d}\,xG_{WW}(x,K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}}}_{\text{WW gluon TMD}}$$

- Hard factor: $q\bar q$ pair formation and its coupling to a gluon from the target
- $P_\perp$ dependence determined by a single hard scattering: leading twist

$$H = \alpha_{\rm em}\alpha_s \left(\sum e_f^2\right)(\vartheta_1^2+\vartheta_2^2)\,\frac{1}{P_\perp^4} \quad \text{for } Q^2 \ll P_\perp^2$$



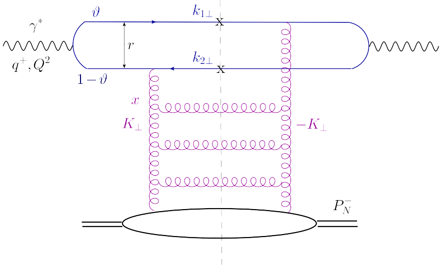


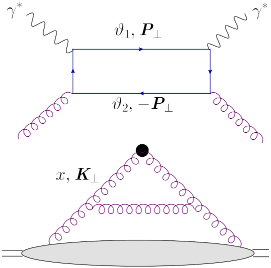
- Weiszäcker-Williams UGD: gluon occupation number in the target

$$\frac{\mathrm{d}xG_{WW}(x, K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}} \simeq \frac{S_\perp(N_c^2-1)}{4\pi^3}\frac{1}{\alpha_s N_c}\begin{cases}\ln\frac{Q_s^2}{K_\perp^2} & \text{for} \quad K_\perp \ll Q_s \\ \frac{Q_s^2}{K_\perp^2} & \text{for} \quad K_\perp \gg Q_s.\end{cases}$$

- xP_N^-: longitudinal momentum transferred from the target $(x \lesssim 10^{-2})$
- $Y = \ln\frac{1}{x}$: rapidity phase-space for high energy evolution: $Q_s \equiv Q_s(Y)$

TMD factorisation for inclusive dijets *(cf. P. Caucal)*





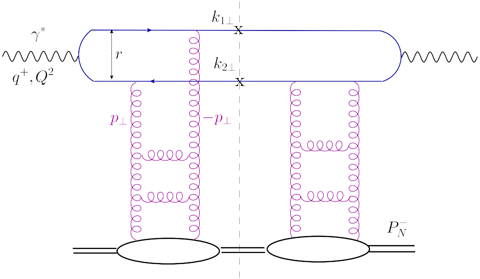
- Weiszäcker-Williams UGD: gluon occupation number in the target

$$\frac{\mathrm{d}xG_{WW}(x,K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}} \simeq \frac{S_\perp(N_c^2-1)}{4\pi^3}\,\frac{1}{\alpha_s N_c}\begin{cases}\ln\frac{Q_s^2}{K_\perp^2} & \text{for} \quad K_\perp \ll Q_s\\ \frac{Q_s^2}{K_\perp^2} & \text{for} \quad K_\perp \gg Q_s.\end{cases}$$

- Saturation (multiple scattering) at $K_\perp \lesssim Q_s$: occupation numbers $\sim 1/\alpha_s$
- Bremsstrahlung tail (single hard scattering) at $K_\perp \gg Q_s$

Exclusive dijets is higher twist

- Colorless exchange: Pomeron $\Rightarrow$ rapidity gap $Y_{\mathbb{P}} = \ln \frac{1}{x_{\mathbb{P}}}$
- Elastic scattering: $\sigma_{\rm el} \propto |T_{q\bar{q}}(r, Y_{\mathbb{P}})|^2$ with $r \sim 1/P_\perp$
- High $P_\perp \gg Q_s(Y_{\mathbb{P}})$: small dipole $\Longrightarrow$ weak scattering



$$\frac{{\rm d}\sigma_{\rm el}^{\gamma A \to q\bar{q}A}}{{\rm d}\vartheta_1 {\rm d}\vartheta_2 {\rm d}^2\boldsymbol{P}} \propto \frac{Q_s^4}{P_\perp^6}$$

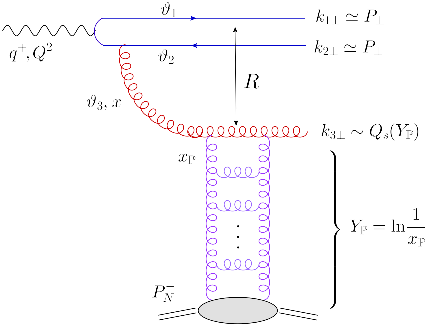
$$Q_s^2 \equiv Q_s^2(A, Y_{\mathbb{P}}) \sim A^{1/3}\, {\rm e}^{0.3 Y_{\mathbb{P}}}$$

- "Higher twist": strongly suppressed at large $P_\perp \gg Q_s$
- Diffraction abhors weak scattering

Diffractive 2+1 jets

(E.I., A.H. Mueller, D.N. Triantafyllopoulos, Phys.Rev.Lett. 128 (2022) 20)

- Can one have diffractive dijets at leading twist ?
- Yes ... provided one allows for strong scattering !
- 2+1 jets: 2 hard ($P_\perp \gg Q_s$) and 1 semi-hard ($K_\perp \sim Q_s$)



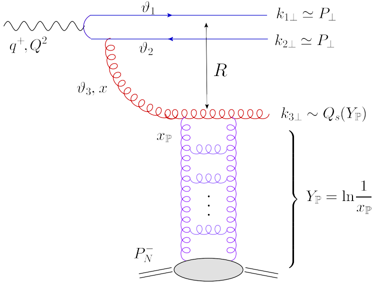
$$R \sim \frac{1}{Q_s} \gg r \sim \frac{1}{P_\perp}$$

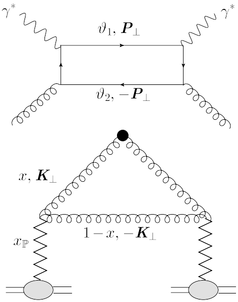
- Effective gluon-gluon dipole
- Strong scattering: $T_{gg}(R, Y_{\mathbb{P}}) \sim 1$
- Leading twist: $\sim Q_s^2/P_\perp^4$
- Semi-inclusive dijet production

- No penalty for scattering, but only for gluon emission by a small $q\bar{q}$ dipole

TMD factorisation for diffractive 2+1 jets

- The third jet is relatively soft: $k_3^+ = \vartheta_3 q^+$ with $\vartheta_3 \sim \frac{Q_s^2}{Q^2} \ll 1$
  - gluon formation time must be small enough to scatter: $\frac{k_3^+}{k_{3\perp}^2} \lesssim \frac{q^+}{Q^2}$
- It controls the hard dijet imbalance: $K_\perp \equiv |\boldsymbol{k}_1 + \boldsymbol{k}_2| = k_{3\perp} \sim Q_s \ll P_\perp$
- It can alternatively be seen as a part of the Pomeron wavefunction

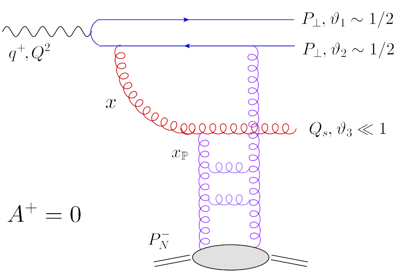


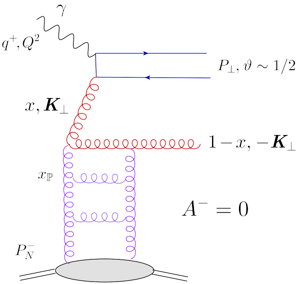


- x: energy fraction of the exchanged gluon with respect to the Pomeron

TMD factorisation for diffractive 2+1 jets (2)

- The strong ordering in both $k_\perp$ and k^+ is essential for factorisation
- The dipole picture holds in the projectile light cone gauge $A^+ = 0$
  - right moving partons couple to the A^- component of the target field

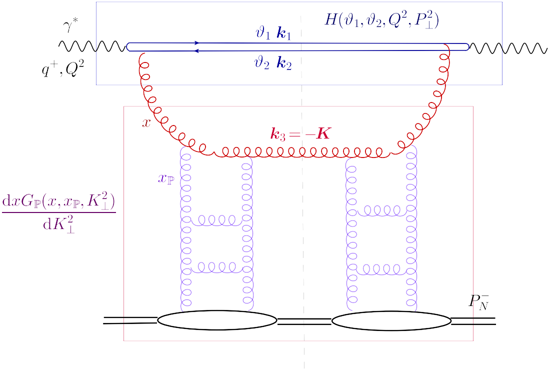




- The TMD picture holds in the target light cone gauge $A^- = 0$
  - only the soft gluon couples to the target field: $v^i A^i$ with $v^i = k^i/k^+$

TMD factorisation for diffractive 2+1 jets (3)

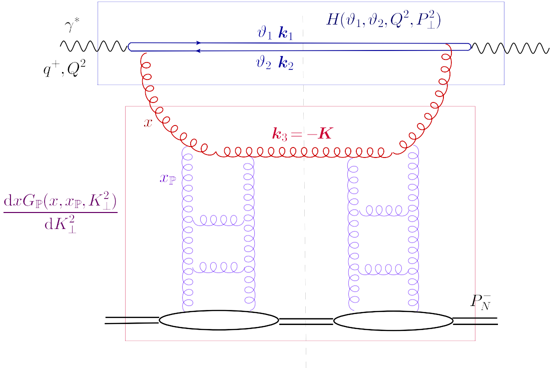
$$\frac{\mathrm{d}\sigma_{2+1}^{\gamma_{T,L}^* A\to q\bar{q}gA}}{\mathrm{d}\vartheta_1\mathrm{d}\vartheta_2\mathrm{d}^2\boldsymbol{P}\mathrm{d}^2\boldsymbol{K}\mathrm{d}Y_{\mathbb{P}}} = H_{T,L}(\vartheta_1,\vartheta_2,Q^2,P_\perp^2)\,\frac{\mathrm{d}xG_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}}$$



- The hard factor: the same as for inclusive dijets (same physics)
- The UGD of the Pomeron: first example of a diffractive TMD

TMD factorisation for diffractive 2+1 jets (3)

$$\frac{d\sigma_{2+1}^{\gamma_{T,L}^* A\to q\bar{q}gA}}{d\vartheta_1 d\vartheta_2 d^2\boldsymbol{P} d^2\boldsymbol{K} dY_{\mathbb{P}}} = H_{T,L}(\vartheta_1,\vartheta_2,Q^2,P_\perp^2)\,\frac{dxG_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2)}{d^2\boldsymbol{K}}$$



- Implicit in early studies of inclusive diffraction
  (Hebecker, Golec-Biernat, Wüsthoff, Hautmann, Soper ... 97-01)
- Operatorial definition clarified by *Hatta, Xiao, and Yuan (2205.08060)*

The Pomeron UGD

$$\frac{\mathrm{d}xG_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}} = \frac{S_\perp(N_c^2-1)}{4\pi^3} \underbrace{\Phi_g(x, x_{\mathbb{P}}, K_\perp^2)}_{\text{occupation number}}$$

- Explicitly computed in terms of the gluon-gluon dipole amplitude $T_{gg}(R, Y_{\mathbb{P}})$

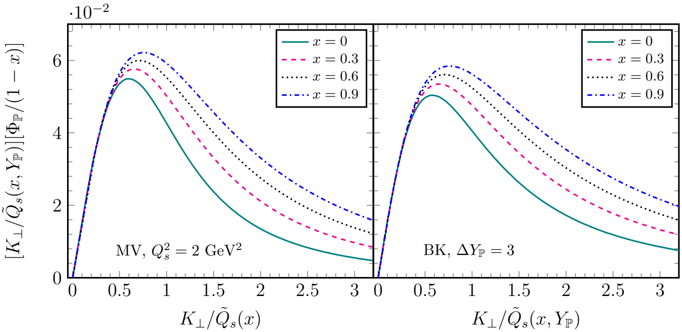
$$\Phi_g(x, x_{\mathbb{P}}, K_\perp^2) \simeq (1-x)\begin{cases} 1, & K_\perp \lesssim \tilde{Q}_s(x) \\ \dfrac{\tilde{Q}_s^4(x)}{K_\perp^4}, & K_\perp \gg \tilde{Q}_s(x) \end{cases}$$

- Valid for small $x_{\mathbb{P}} \lesssim 10^{-2}$ but any $x \leq 1$
  - effective saturation momentum: $\tilde{Q}_s^2(x, Y_{\mathbb{P}}) = (1-x)Q_s^2(Y_{\mathbb{P}})$
- Saturation when $K_\perp \lesssim \tilde{Q}_s(x)$: occupation numbers of order 1
- Very fast decrease $\sim 1/K_\perp^4$ at large gluon momenta $K_\perp \gg \tilde{Q}_s(x)$
  - the bulk of the distribution lies at saturation: $K_\perp \lesssim \tilde{Q}_s(x)$

Numerical results

(E.I., A.H. Mueller, D.N. Triantafyllopoulos, S.-Y. Wei, arXiv:2207.06268)

- Left: McLerran-Venugopalan model. Right: adding high-energy evolution
- Pronounced peak at $K_\perp \simeq \tilde{Q}_s$: diffraction is controlled by saturation



- BK evolution of $T_{gg}(R, Y_\mathbb{P})$: evolution of $\Phi_\mathbb{P}(x, x_\mathbb{P}, K_\perp)$ in $x_\mathbb{P}$ and $K_\perp$
  - increasing $Q_s^2(Y_\mathbb{P})$, but the shape remains the same (geometric scaling)

The gluon diffractive PDF

- By integrating the gluon momentum $K_\perp$: the usual collinear factorisation

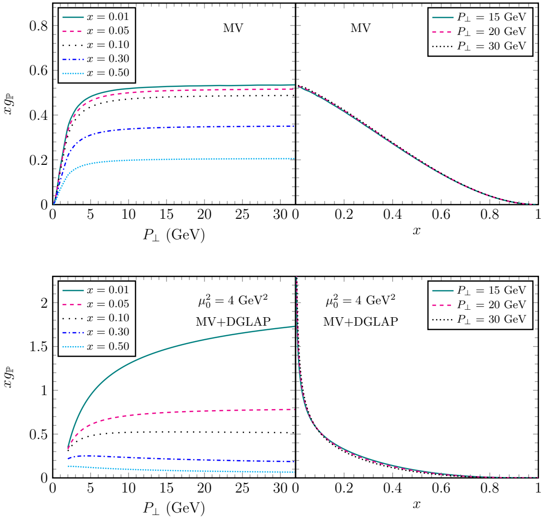
$$\frac{\mathrm{d}\sigma_{2+1}^{\gamma A\to q\bar{q}gA}}{\mathrm{d}\vartheta_1\mathrm{d}\vartheta_2\mathrm{d}^2\boldsymbol{P}\mathrm{d}Y_{\mathbb{P}}} = H(\vartheta_1, \vartheta_2, Q^2, P_\perp^2)\, xG_{\mathbb{P}}(x, x_{\mathbb{P}}, P_\perp^2)$$

- ... but with an explicit result for the gluon diffractive PDF:

$$xG_{\mathbb{P}}(x, x_{\mathbb{P}}, P_\perp^2) \equiv \int^{P_\perp} \mathrm{d}^2\boldsymbol{K}\, \frac{\mathrm{d}xG_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}} \,\propto\, (1-x)^2\, Q_s^2(A, Y_{\mathbb{P}})$$

- The integral is rapidly converging and effectively cut off at $K_\perp \sim \tilde{Q}_s(x)$
- The $(1-x)^2$ vanishing at the end point is a hallmark of saturation
- DGLAP evolution with increasing $P_\perp^2$
- Initial condition for DGLAP determined by saturation (MV+BK)

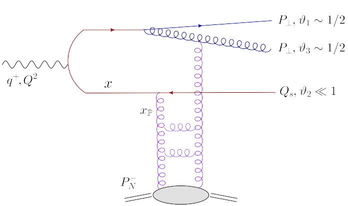
The gluon diffractive PDF: numerical results

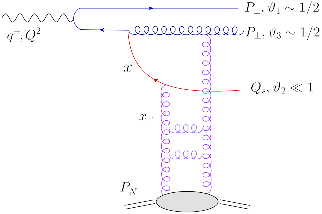


- DGLAP: increase for very small $x \leq 0.01$, slight decrease for $x > 0.05$
- When $x \to 1$, the distribution vanishes even faster

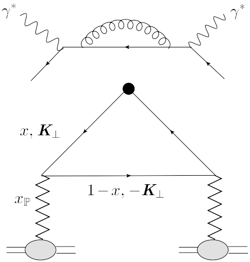
2+1 jets with a hard gluon

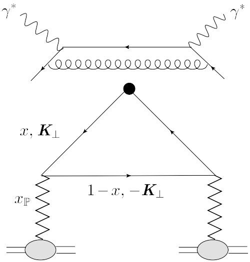
- The third (semi-hard) jet can also be a quark: same-order





- TMD factorisation: quark unintegrated distribution of the Pomeron

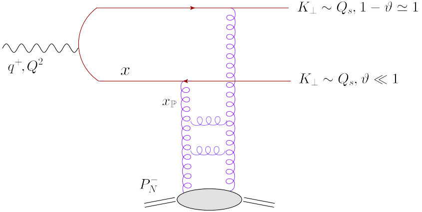


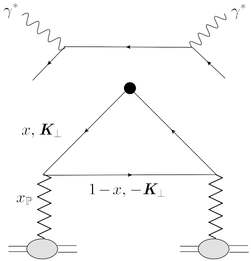


Universality of the quark diffractive TMD

- Diffractive SIDIS in the aligned jet configuration: $\vartheta \ll 1$
- High virtuality: $Q^2 \gg Q_s^2$, but semi-hard transverse momenta:

$$K_\perp^2 \simeq \vartheta(1-\vartheta)Q^2 \sim Q_s^2 \qquad \text{(to have strong scattering)}$$





$$\frac{\mathrm{d}\sigma_{\mathrm{el}}^{\gamma_T^* A \to q\bar{q}A}}{\mathrm{d}^2\boldsymbol{K}\,\mathrm{d}Y_{\mathbb{P}}} = \frac{4\pi^2\alpha_{em}}{Q^2}\,\frac{\mathrm{d}x q_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}}$$

- The hard factor: cross-section for virtual photon absorbtion

The quark diffractive TMD

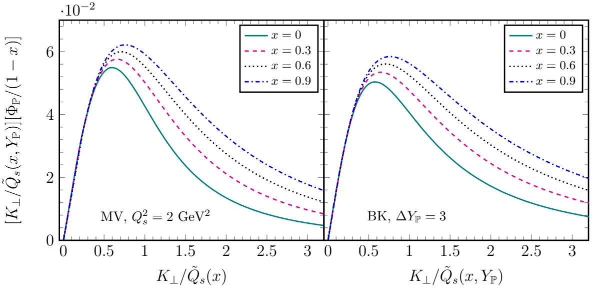
$$\frac{\mathrm{d}xq_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}} = \frac{S_\perp N_c}{4\pi^3} \underbrace{\Phi_q(x, x_{\mathbb{P}}, K_\perp^2)}_{\text{occupation number}}$$

- Related to the quark-antiquark dipole amplitude $T_{q\bar{q}}(R, Y_{\mathbb{P}})$

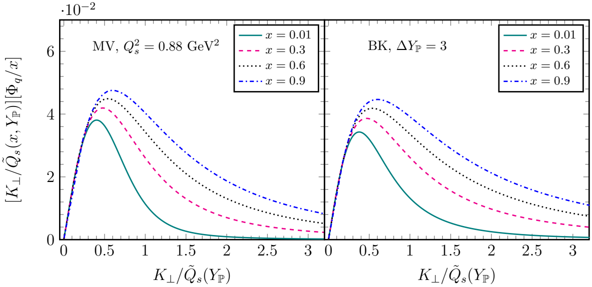
$$\Phi_q(x, x_{\mathbb{P}}, K_\perp^2) \simeq x \begin{cases} 1, & K_\perp \lesssim \tilde{Q}_s(x) \\ \dfrac{\tilde{Q}_s^4(x)}{K_\perp^4}, & K_\perp \gg \tilde{Q}_s(x) \end{cases}$$

- Like for the gluon diffractive TMD, but with $1 - x \to x$
  - gluons dominate at small x, quarks are more important near $x = 1$
- Saturation when $K_\perp \lesssim \tilde{Q}_s(x)$: occupation numbers of order 1
- Once again, the bulk of the distribution lies at saturation: $K_\perp \lesssim \tilde{Q}_s(x)$

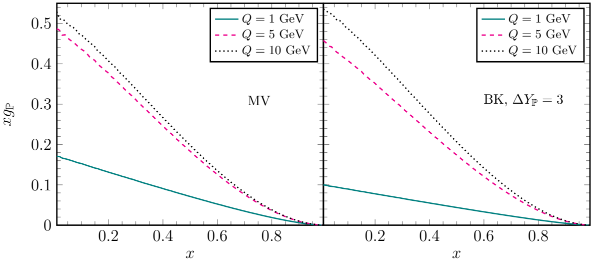
Gluon vs. quark diffractive TMDs



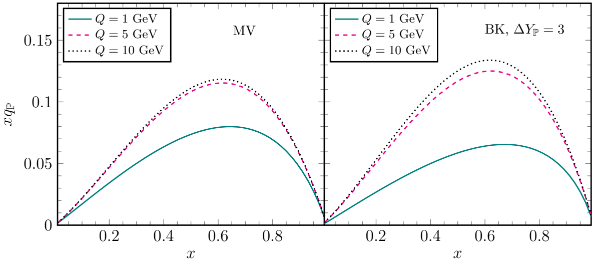
- First line: gluon. Second line: quark



Gluon vs. quark diffractive PDFs



- First line: gluon. Second line: quark



Conclusions

- Diffractive jet production in photon-hadron interactions at high energies admits TMD factorisation
- Demonstrated "by construction": the factorisation emerges from explicit calculations within the dipole picture/CGC effective theory
- For sufficiently small $x_{\mathbb{P}} \lesssim 10^{-2}$ and/or large $A \sim 200$, diffractive TMDs and PDFs can be computed from first principles
- So far, only two diffractive TMDs: unpolarised quark and gluon
- Perhaps more complicated final states requires new diffractive TMDs (?)
- Applications to the phenomenology of DIS at the EIC and of nucleus-nucleus ultra-peripheral collisions at the LHC (e-Print: 2304.12401)