

TMD factorization beyond the leading power

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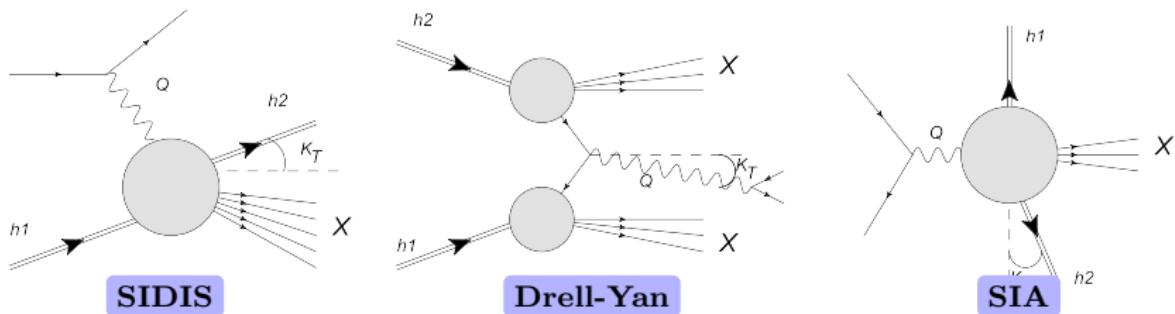


QCD Evolution Workshop 2023

Transverse momentum dependent factorization

$$\frac{d\sigma}{dq_T} \simeq \sigma_0 \int \frac{d^2 b}{(2\pi)^2} e^{-i(bq_T)} |C_V(Q)|^2 F_1(x_1, b; Q, Q^2) F_2(x_2, b; Q, Q^2)$$

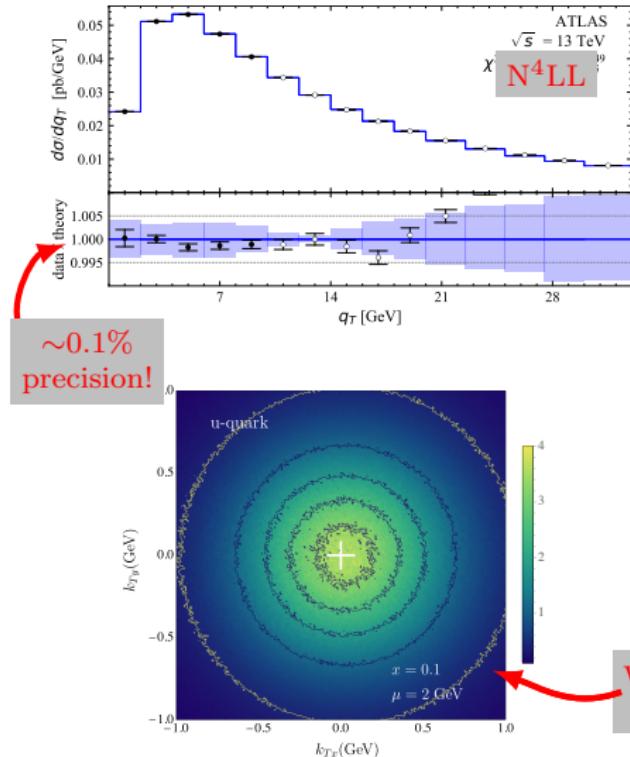
LP term is studied VERY WELL!



$q^2 = \pm Q^2$ momentum of hard probe
 q_T^μ transverse component

$$\left\{ \begin{array}{l} Q^2 \gg \text{anything} \\ q_T^2 \text{ fixed} \end{array} \right.$$



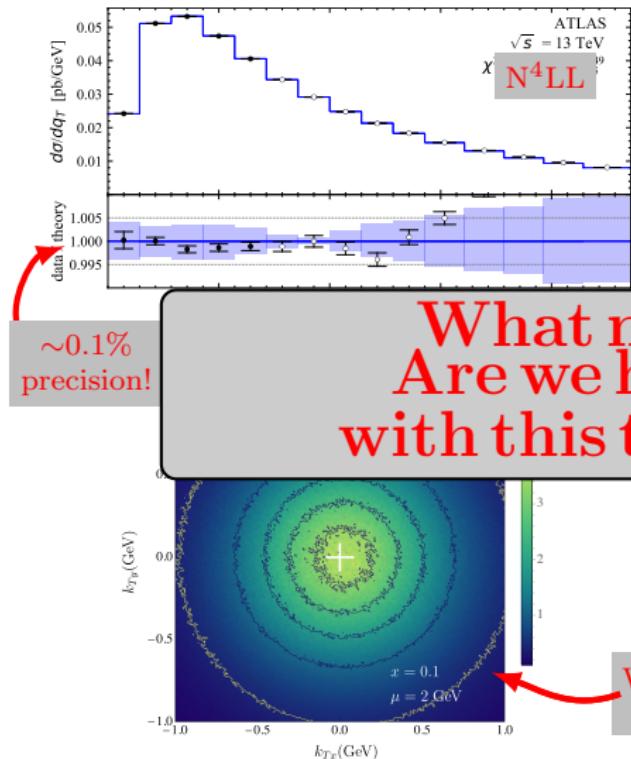


LP is very well studied

- ▶ Factorization theorem proven
- ▶ Multiple phenomenological tests
- ▶ Perturbative precision
 - ▶ Hard function: N⁴LO
 - ▶ Evolution: N³LO
 - ▶ OPE for components
 - ▶ CS kernel N⁴LO
 - ▶ f_1 TMDPDF N³LO
 - ▶ d_1 TMDFF N³LO
 - ▶ ...

N⁴LL for unpolarized
see talk by Pia Zurita
tomorrow





LP is very well studied

- ▶ Factorization theorem proven
 - ▶ Multiple phenomenological tests
 - ▶ Perturbative precision
 - ▶ Hard function: $N^4\text{LO}$

N⁴LL for unpolarized see talk by Pia Zurita tomorrow

With N³LO
evolution



POWER CORRECTIONS

$$\begin{aligned} \frac{d\sigma}{dq_T} &\simeq \sigma_0 \int \frac{d^2 b}{(2\pi)^2} e^{-i(b q_T)} \left\{ |C_V(Q)|^2 F_1(x_1, b; Q, Q^2) F_2(x_2, b; Q, Q^2) \right. && \leftarrow \text{LP} \\ &+ \left(\frac{q_T}{Q}; \frac{k_T}{Q}; \frac{M}{Q} \right) [C_2(Q) \otimes F_3(x, b; Q, Q^2) F_4(x, b; Q, Q^2)](x_1, x_2) && \leftarrow \text{NLP} \\ &+ \left(\frac{q_T^2}{Q^2}; \frac{k_T q_T}{Q^2}; \dots \right) [C_3(Q) \otimes F_5(x, b; Q, Q^2) F_6(x, b; Q, Q^2)](x_1, x_2) && \leftarrow \text{NNLP} \\ &+ \dots \end{aligned}$$

see talk by
S.Rodini



POWER CORRECTIONS

$$\begin{aligned}
 \frac{d\sigma}{dq_T} &\simeq \sigma_0 \int \frac{d^2 b}{(2\pi)^2} e^{-i(b q_T)} \left\{ |C_V(Q)|^2 F_1(x_1, b; Q, Q^2) F_2(x_2, b; Q, Q^2) \right. && \xleftarrow{\text{LP}} \\
 &+ \left(\frac{q_T}{Q}; \frac{k_T}{Q}; \frac{M}{Q} \right) [C_2(Q) \otimes F_3(x, b; Q, Q^2) F_4(x, b; Q, Q^2)](x_1, x_2) && \xleftarrow{\text{NLP}} \\
 &+ \left(\frac{q_T^2}{Q^2}; \frac{k_T q_T}{Q^2}; \dots \right) [C_3(Q) \otimes F_5(x, b; Q, Q^2) F_6(x, b; Q, Q^2)](x_1, x_2) && \xleftarrow{\text{NNLP}} \\
 &+ \dots
 \end{aligned}$$

see talk by
 S.Rodini

LP = 8 TMDs

Leading Twist TMDs

○ : Nucleon Spin ● : Quark Spin

		Quark polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	$f_{1U} = \bullet$			$h_{1U} = \langle \bullet \rangle - \langle \bullet \rangle$ Box-Müller
L		$g_{1L} = \langle \bullet \rangle - \langle \bullet \rangle$ Helicity		$h_{1L} = \langle \bullet \rangle - \langle \bullet \rangle$
T	$f_{1T} = \langle \bullet \rangle - \langle \bullet \rangle$ Shear	$g_{1T} = \langle \bullet \rangle - \langle \bullet \rangle$	$h_{1T} = \langle \bullet \rangle - \langle \bullet \rangle$	$h_{1T} = \langle \bullet \rangle - \langle \bullet \rangle$ Transversity



NLP += 16(32) TMDs

	U	L	$T_{J=0}$	$T_{J=1}$	$T_{J=2}$
U	f_{\bullet}^\perp	g_{\bullet}^\perp		h_{\bullet}	h_{\bullet}^\perp
L	$f_{\bullet L}^\perp$	$g_{\bullet L}^\perp$	$h_{\bullet L}$		$h_{\bullet L}^\perp$
T	$f_{\bullet T}, f_{\bullet T}^\perp$	$g_{\bullet T}, g_{\bullet T}^\perp$	$h_{\bullet T}^{D\perp}$	$h_{\bullet T}^{A\perp}$	$h_{\bullet T}^{S\perp}, h_{\bullet T}^{T\perp}$



Four types of power corrections in TMD factorization

①

$\frac{\Lambda_{\text{QCD}}}{Q}$ corrections \Leftrightarrow genuine power corrections

Higher twist TMD distributions

$$\int d^2 b e^{-i(qb)_T} \int_{-1+x_2}^1 \frac{dy}{y} F_1(x_1, b; Q, Q^2) \Phi_{\oplus}(-x - y, y, x, b; Q, Q^2)$$

- ▶ **New NP content**
- ▶ Dynamics with x , and q_T unknown



Four types of power corrections in TMD factorization

(2) $\frac{k_T}{Q}$ corrections \Leftrightarrow kinematic power corrections (KPC)

Derivatives of TMDs with respect to b^μ \Leftrightarrow tot.derivatives of semi-compact operators

$$\frac{i}{Q} \int d^2 b e^{-i(qb)_T} F_1(x_1, b) \frac{\partial}{\partial b_\mu} F_2(x_2, b)$$

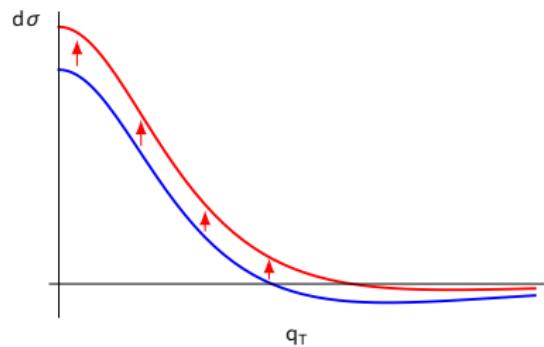
$$\int d^2 \mathbf{k}_1 d^2 \mathbf{k}_2 \delta^{(2)}(\mathbf{q}_T - \mathbf{k}_1 - \mathbf{k}_2) \frac{\mathbf{k}_2^\mu}{Q} F_1(x_1, \mathbf{k}_1) F_2(x_2, \mathbf{k}_2)$$



Four types of power corrections in TMD factorization

(2) $\frac{k_T}{Q}$ corrections \Leftrightarrow kinematic power corrections (KPC)

- ▶ Restore gauge invariance
- ▶ Restore (partially) frame invariance
- ▶ Present at $q_T = 0$
- ▶ **Do not require new NP functions**



Four types of power corrections in TMD factorization

③

Target mass corrections

- ▶ $\sim \frac{M}{Q}$ from exponents (not from kinematics)
- ▶ Requires new definition of TMD distributions
- ▶ Not ever studied (do my best knowladge)
- ▶ $M = 0$ for today



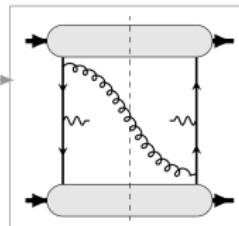
Four types of power corrections in TMD factorization

④

$$\frac{q_T}{Q} \text{ corrections} \Leftrightarrow \frac{1}{bQ} \text{ corrections}$$

Several sources

$$\lim_{b \rightarrow 0} F_{\text{tw-N}}(x, b) = \frac{\alpha_s}{b^{N-1}} F_{\text{tw-2}}(x, b)$$



$$\int d^2b e^{-i(qb)_T} \frac{1}{b^2 Q^2} F_1(x_1, b) F_2(x_2, b)$$

$$\int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \delta^{(2)}(\mathbf{q}_T - \mathbf{k}_1 - \mathbf{k}_2) \frac{\mathbf{q}_T^2}{Q^2} F_1(x_1, \mathbf{k}_1) F_2(x_2, \mathbf{k}_2)$$

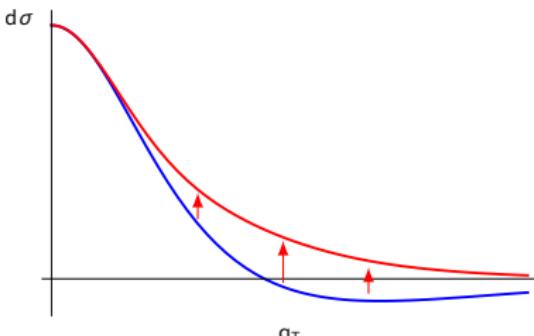
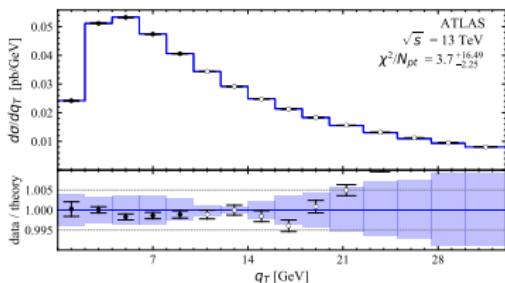


Four types of power corrections in TMD factorization

④

$$\frac{q_T}{Q} \text{ corrections} \Leftrightarrow \frac{1}{bQ} \text{ corrections}$$

- ▶ Provide matching to fixed order
- ▶ **Do not require new NP functions**



Comment: In twist-2 resummation k_T/Q , q_T/Q , Λ/Q are all $\sim 1/bQ$



This picture implies that TMD distributions can be sorted by “irreducible” NP-content
It is **critically** important to determine “twist”

Dynamical twist TMDs (**bad**)

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} f^\perp \\ g^\perp \end{pmatrix} = \left(\frac{\Gamma}{2} \mathbf{L}_\zeta + 2C_F a_s + .. \right) \begin{pmatrix} f^\perp \\ g^\perp \end{pmatrix} + (-2C_F a_s + ..) \begin{pmatrix} f_1 \\ 0 \end{pmatrix} + \begin{pmatrix} \mathbb{P} & \Theta \\ -\Theta & \mathbb{P} \end{pmatrix} \otimes \begin{pmatrix} \Phi_\oplus \\ \Phi_\ominus \end{pmatrix}$$

- ▶ Twist-3 TMDs mix with twist-2 TMDs \Rightarrow no clear identification.

(geometrical) TMD-twist (**better**)

[AV,Moos,Scimemi,21]

- ▶ TMD distributions with different TMD-twists do not mix
- ▶ see next talk(s)



$$W^{\mu\nu} = \frac{1}{N_c} \int \frac{d^2 b}{(2\pi)^2} e^{-i(q_T b)} \left\{ \right.$$

$$D \sim \frac{\partial}{\partial b^\mu}$$

$$\Phi_2 \times \Phi_2$$

$$\begin{aligned}
& + \frac{1}{Q} \left(D \Phi_2 \times \Phi_2 + \Phi_2 \times \Phi_3 \right) \\
& + \frac{1}{Q^2} \left(D^2 \Phi_2 \times \Phi_2 + D \Phi_2 \times \Phi_3 + \Phi_3 \times \Phi_3 + \Phi_2 \times \Phi_4 + \frac{\Phi_2 \times \Phi_2}{b^2} \right) \\
& + \frac{1}{Q^3} \left(D^3 \Phi_2 \times \Phi_2 + \dots + \Phi_3 \times \Phi_4 + \dots + \frac{D \Phi_2 \times \Phi_2}{b^2} \right) \\
& + \frac{1}{Q^4} \left(D^4 \Phi_2 \times \Phi_2 + \dots + \Phi_2 \times \Phi_5 + \dots + \frac{D \Phi_2 \times \Phi_2}{b^2} + \frac{\Phi_2 \times \Phi_2}{b^4} \right) \\
& + \dots \Big\}^{\mu\nu},
\end{aligned}$$



$$W^{\mu\nu} = \frac{1}{N_c} \int \frac{d^2 b}{(2\pi)^2} e^{-i(q_T b)} \left\{$$

$$D \sim \frac{\partial}{\partial b^\mu}$$

Φ_N is TMD of twist-N

$\Phi_2 \times \Phi_2$

LP ✓

$$+ \frac{1}{Q} \left(D \Phi_2 \times \Phi_2 + \Phi_2 \times \Phi_3 \right) \quad \text{NLP ✓}$$

$$+ \frac{1}{Q^2} \left(D^2 \Phi_2 \times \Phi_2 + D \Phi_2 \times \Phi_3 + \Phi_3 \times \Phi_3 + \dots \right)$$

$$+ \frac{1}{Q^3} \left(D^3 \Phi_2 \times \Phi_2 + \dots + \Phi_3 \times \Phi_4 + \dots + \frac{D \Phi_2 \times \Phi_2}{b^2} \right)$$

$$+ \frac{1}{Q^4} \left(D^4 \Phi_2 \times \Phi_2 + \dots + \Phi_2 \times \Phi_5 + \dots + \frac{D \Phi_2 \times \Phi_2}{b^2} + \frac{\Phi_2 \times \Phi_2}{b^4} \right)$$

$$+ \dots \}^{\mu\nu},$$

$$\Phi_2 \times \Phi_4 + \frac{\Phi_2 \times \Phi_2}{b^2}$$

$$\dots + \frac{D \Phi_2 \times \Phi_2}{b^2}$$

$$\dots + \frac{D \Phi_2 \times \Phi_2}{b^2} + \frac{\Phi_2 \times \Phi_2}{b^4}$$



$$W^{\mu\nu} = \frac{1}{N_c} \int \frac{d^2 b}{(2\pi)^2} e^{-i(q_T b)} \left\{$$

$\Phi_2 \times \Phi_2$

$$+ \frac{1}{Q} \left(D \Phi_2 \times \Phi_2 + \Phi_2 \times \Phi_3 \right)$$

$$+ \frac{1}{Q^2} \left(D^2 \Phi_2 \times \Phi_2 + D \Phi_2 \times \Phi_3 + \Phi_3 \times \Phi_3 + \Phi_2 \times \Phi_4 + \frac{\Phi_2 \times \Phi_2}{b^2} \right)$$

$$+ \frac{1}{Q^3} \left(D^3 \Phi_2 \times \Phi_2 + \dots + \Phi_3 \times \Phi_4 + \dots + \frac{D \Phi_2 \times \Phi_2}{b^2} \right)$$

$$+ \frac{1}{Q^4} \left(D^4 \Phi_2 \times \Phi_2 + \dots + \Phi_2 \times \Phi_5 + \dots + \frac{D \Phi_2 \times \Phi_2}{b^2} + \frac{\Phi_2 \times \Phi_2}{b^4} \right)$$

$$+ \dots \left. \right\}^{\mu\nu},$$

resummed KPC
this talk

- ▶ Resporation of gauge and Lorenz invariance
- ▶ Non-vanishing $q_T = 0$, and larger than $\frac{\Lambda}{Q}$
- ▶ Alike LP expression, but with a different convolution integral



$$W^{\mu\nu} = \frac{1}{N_c} \int \frac{d^2 b}{(2\pi)^2} e^{-i(q_T b)} \left\{$$

- ▶ Same NP content as at LP
- ▶ Looks conceptually possible

$$\begin{aligned}
 & \Phi_2 \times \Phi_2 \\
 + \frac{1}{Q} & \left(D \Phi_2 \times \Phi_2 + \Phi_2 \times \Phi_3 \right) + \frac{b}{b^2} \Phi_2 \times \Phi_2 \\
 + \frac{1}{Q^2} & \left(D^2 \Phi_2 \times \Phi_2 + D \Phi_2 \times \Phi_3 + \Phi_3 \times \Phi_3 + \Phi_2 \times \Phi_4 + \frac{\Phi_2 \times \Phi_2}{b^2} \right) \\
 + \frac{1}{Q^3} & \left(D^3 \Phi_2 \times \Phi_2 + \dots + \Phi_3 \times \Phi_4 + \dots + \frac{D \Phi_2 \times \Phi_2}{b^2} \right) + \frac{b}{b^4} \Phi_2 \times \Phi_2 \\
 + \frac{1}{Q^4} & \left(D^4 \Phi_2 \times \Phi_2 + \dots + \Phi_2 \times \Phi_5 + \dots + \frac{D \Phi_2 \times \Phi_2}{b^2} + \frac{\Phi_2 \times \Phi_2}{b^4} \right) \\
 + \dots & \left. \right\}^{\mu\nu},
 \end{aligned}$$

large- q_T tale
possible?



Restoration of Gauge-invariance & frame-independence

LP: $W_{\text{LP}}^{\mu\nu} \propto \int \frac{d^2 b}{(2\pi)^2} e^{-i(q_T b)} C_0 \left(\frac{2q_+ q_-}{\mu^2} \right) \left(-g_T^{\mu\nu} \tilde{f}_1(x_1, b^2) \tilde{f}_1(x_2, b^2) + \dots \right),$

$q_\mu W_{\text{LP}}^{\mu\nu} \simeq q_T^\nu$
EM-gauge invariance = charge conservation = transversality of $W^{\mu\nu}$
brocken (up to NLP)

$x_1 = \frac{q^+}{P_1^+}, \quad x_2 = \frac{q^-}{P_2^-}$
vectors n, \bar{n} are not “well-defined” $+ \mathcal{O}(q_T^2/Q^2)$
reparametrization invariance = frame invariance
brocken (up to N²LP)



Restoration of Gauge-invariance & frame-independence

$$\textbf{LP:} \quad W_{\text{LP}}^{\mu\nu} \propto \int \frac{d^2 b}{(2\pi)^2} e^{-i(q_T b)} C_0 \left(\frac{2q_+ q_-}{\mu^2} \right) \left(-g_T^{\mu\nu} \tilde{f}_1(x_1, b^2) \tilde{f}_1(x_2, b^2) + \dots \right),$$

$$\textbf{NLP:} \quad W_{\text{NLP}}^{\mu\nu} \propto \int \frac{d^2 b}{(2\pi)^2} e^{-i(q_T b)} C_0 \left(\frac{2q_+ q_-}{\mu^2} \right) \left(-\frac{i\bar{n}^\mu}{q^-} \tilde{f}_1 D^\nu \tilde{f}_1 - \frac{in^\mu}{q^+} D^\nu \tilde{f}_1 \tilde{f}_1 + \dots \right),$$

$$q_\mu (W_{\text{LP}}^{\mu\nu} + W_{\text{NLP}}^{\mu\nu}) \simeq q_T^\nu \frac{k_T}{q^\pm}$$

EM-gauge invariance **restored** (up to N²LP)



Restoration of Gauge-invariance & frame-independence

LP: $W_{\text{LP}}^{\mu\nu} \propto \int \frac{d^2 b}{(2\pi)^2} e^{-i(q_T b)} C_0 \left(\frac{2q_+ q_-}{\mu^2} \right) \left(-g_T^{\mu\nu} \tilde{f}_1(x_1, b^2) \tilde{f}_1(x_2, b^2) + \dots \right),$

NLP: $W_{\text{NLP}}^{\mu\nu} \propto \int \frac{d^2 b}{(2\pi)^2} e^{-i(q_T b)} C_0 \left(\frac{2q_+ q_-}{\mu^2} \right) \left(-\frac{i\bar{n}^\mu}{q^-} \tilde{f}_1 D^\nu \tilde{f}_1 - \frac{in^\mu}{q^+} D^\nu \tilde{f}_1 \tilde{f}_1 + \dots \right),$

N²LP: $W_{\text{N}^2\text{LP}}^{\mu\nu} \propto \int \frac{d^2 b}{(2\pi)^2} e^{-i(q_T b)} C_0 \left(\frac{2q_+ q_-}{\mu^2} \right) \left(\frac{g^{\mu\nu} D_\alpha \tilde{f}_1 D^\alpha \tilde{f}_1 - D^\mu \tilde{f}_1 D^\nu \tilde{f}_1 - D^\nu \tilde{f}_1 D^\mu \tilde{f}_1}{q^+ q^-} + \frac{n^\mu n^\nu}{(q^+)^2} D^2 \tilde{f}_1 \tilde{f}_1 + \frac{\bar{n}^\mu \bar{n}^\nu}{(q^-)^2} \tilde{f}_1 D^2 \tilde{f}_1 \right.$

$$\left. + \frac{g_T^{\mu\nu}}{2q^+ q^-} \left(\tilde{f}'_1 D^2 \tilde{f}_1 + D^2 \tilde{f}_1 \tilde{f}'_1 \right) + \dots \right)$$

$$q_\mu (W_{\text{LP}}^{\mu\nu} + W_{\text{NLP}}^{\mu\nu} + W_{\text{N}^2\text{LP}}^{\mu\nu}) \simeq q_T^\nu \frac{k_T^2}{q^+ q^-}$$

EM-gauge invariance restored (up to N³LP)



Restoration of Gauge-invariance & frame-independence

LP: $W_{\text{LP}}^{\mu\nu} \propto \int \frac{d^2 b}{(2\pi)^2} e^{-i(q_T b)} C_0\left(\frac{2q_+ q_-}{\mu^2}\right) \left(-g_T^{\mu\nu} \tilde{f}_1(x_1, b^2) \tilde{f}_1(x_2, b^2) + \dots \right),$

NLP: $W_{\text{NLP}}^{\mu\nu} \propto \int \frac{d^2 b}{(2\pi)^2} e^{-i(q_T b)} C_0\left(\frac{2q_+ q_-}{\mu^2}\right) \left(-\frac{i\bar{n}^\mu}{q^-} \tilde{f}_1 D^\nu \tilde{f}_1 - \frac{in^\mu}{q^+} D^\nu \tilde{f}_1 \tilde{f}_1 + \dots \right),$

N²LP: $W_{\text{N}^2\text{LP}}^{\mu\nu} \propto \int \frac{d^2 b}{(2\pi)^2} e^{-i(q_T b)} C_0\left(\frac{2q_+ q_-}{\mu^2}\right) \left(\frac{g^{\mu\nu} D_\alpha \tilde{f}_1 D^\alpha \tilde{f}_1 - D^\mu \tilde{f}_1 D^\nu \tilde{f}_1 - D^\nu \tilde{f}_1 D^\mu \tilde{f}_1}{q^+ q^-} + \frac{n^\mu n^\nu}{(q^+)^2} D^2 \tilde{f}_1 \tilde{f}_1 + \frac{\bar{n}^\mu \bar{n}^\nu}{(q^-)^2} \tilde{f}_1 D^2 \tilde{f}_1 \right. \\ \left. + \frac{g_T^{\mu\nu}}{2q^+ q^-} \left(\tilde{f}'_1 D^2 \tilde{f}_1 + D^2 \tilde{f}_1 \tilde{f}'_1 \right) + \dots \right)$

$$f' = \frac{x\partial}{\partial x} f$$

N²LP compensation of n -reparametrization



Restoration of Gauge-invariance & frame-independence

LP: $W_{\text{LP}}^{\mu\nu} \propto \int \frac{d^2 b}{(2\pi)^2} e^{-i(q_T b)} C_0\left(\frac{2q_+ q_-}{\mu^2}\right) \left(-g_T^{\mu\nu} \tilde{f}_1(x_1, b^2) \tilde{f}_1(x_2, b^2) + \dots\right),$

NLP: $W_{\text{NLP}}^{\mu\nu} \propto \int \frac{d^2 b}{(2\pi)^2} e^{-i(q_T b)} C_0\left(\frac{2q_+ q_-}{\mu^2}\right) \left(-\frac{i\bar{n}^\mu}{q^-} \tilde{f}_1 D^\nu \tilde{f}_1 - \frac{in^\mu}{q^+} D^\nu \tilde{f}_1 \tilde{f}_1 + \dots\right),$

N²LP: $W_{\text{N}^2\text{LP}}^{\mu\nu} \propto \int \frac{d^2 b}{(2\pi)^2} e^{-i(q_T b)} C_0\left(\frac{2q_+ q_-}{\mu^2}\right) \left(\frac{g^{\mu\nu} D_\alpha \tilde{f}_1 D^\alpha \tilde{f}_1 - D^\mu \tilde{f}_1 D^\nu \tilde{f}_1 - D^\nu \tilde{f}_1 D^\mu \tilde{f}_1}{q^+ q^-} + \frac{n^\mu n^\nu}{(q^+)^2} D^2 \tilde{f}_1 \tilde{f}_1 + \frac{\bar{n}^\mu \bar{n}^\nu}{(q^-)^2} \tilde{f}_1 D^2 \tilde{f}_1 \right.$

$\left. + \frac{g_T^{\mu\nu}}{2q^+ q^-} (\tilde{f}'_1 D^2 \tilde{f}_1 + D^2 \tilde{f}_1 \tilde{f}'_1) + \dots \right) + \frac{q_T^2}{2q^+ q^-} C'_0 W_{\text{LP}}^{\mu\nu}$

New term at NLO!
complete $q^+ q^- \rightarrow Q^2$ in coeff.function



and so on...

the computation can be done up to any power n

$$\begin{aligned} W^{\mu\nu} = & \frac{p_+ p_-}{N_c} \int d^2 b \int dx d\bar{x} e^{-i(qb)} \delta(q^+ - xp^+) \delta(q^- - \bar{x}p^-) \sum_{a,b} \sum_{n,m=0}^{\infty} \frac{(\partial_{\bar{x}})^n (\partial_x)^m}{n! m!} \frac{(-1)^{n+m}}{(2p_+ p_-)^{n+m}} \frac{1}{x^n \bar{x}^m} \left\{ \right. \\ & - g_T^{\mu\nu} \partial_T^{2m} f_1 \partial_T^{2m} f_1 - i(n^\nu g_T^{\mu\alpha} + n^\mu g_T^{\nu\alpha}) \frac{\partial_\alpha}{xp_+} \partial_T^{2n} f_1 \partial_T^{2m} f_1 - i(\bar{n}^\nu g_T^{\mu\alpha} + \bar{n}^\mu g_T^{\nu\alpha}) \partial_T^{2n} f_1 \frac{\partial_\alpha}{\bar{x}p_-} \partial_T^{2m} f_1 \\ & + \left(g_T^{\alpha\beta} g_T^{\mu\nu} - g_T^{\mu\alpha} g_T^{\nu\beta} - g_T^{\mu\beta} g_T^{\nu\alpha} \right) \frac{\partial_\alpha}{xp_+} \partial_T^{2n} f_1 \frac{\partial_\beta}{\bar{x}p_-} \partial_T^{2m} f_1 \\ & + g_T^{\alpha\beta} (\bar{n}^\mu n^\nu + n^\mu \bar{n}^\nu) \frac{\partial_\alpha}{xp_+} \partial_T^{2n} f_1 \frac{\partial_\beta}{\bar{x}p_-} \partial_T^{2m} f_1 + \left(\bar{n}^\mu \bar{n}^\nu \partial_T^{2n} \frac{\partial_T^{2m+2}}{\bar{x}^2 p_-^2} f_1 + n^\mu n^\nu \partial_T^{2n} \frac{\partial_T^{2m+2}}{x^2 p_+^2} f_1 \partial_T^{2m} f_1 \right) \\ & - \frac{i}{2} (\bar{n}^\mu g_T^{\nu\alpha} + \bar{n}^\nu g_T^{\mu\alpha}) \frac{\partial_\alpha}{xp_+} \partial_T^{2n} f_1 \frac{\partial_T^{2m+2}}{\bar{x}^2 p_-^2} f_1 - \frac{i}{2} (n^\mu g_T^{\nu\alpha} + n^\nu g_T^{\mu\alpha}) \frac{\partial_T^{2n+2}}{x^2 p_+^2} f_1 \frac{\partial_\alpha}{\bar{x}p_-} \partial_T^{2m} f_1 \\ & \left. - \frac{g_T^{\mu\nu}}{4} \frac{\partial_T^{2n+2}}{x^2 p_+^2} f_1 \frac{\partial_T^{2m+2}}{\bar{x}^2 p_-^2} f_1 \right\}. \end{aligned}$$

[AV, in preparation]

Important

- ▶ Restoration of gauge invariance ✓
- ▶ Restoration of re-parametrization invariance ✓
 - ▶ still some redundant dependence on definition of TMD
- ▶ Same coeff. function at all powers
- ▶ Argument of coeff.function restores to Q^2 (checked at NLO ✓)
- ▶ At $\zeta = \bar{\zeta} = Q^2$ KPC can be resummed

Resummation of KPCs can be formulated on the level of individual TMD distributions

Extracting twist-2 contribution from bi-quark operator with any y

$$\bar{q}(y)\gamma^\mu q(0) = [\bar{q}(y)\gamma^\mu q(0)]_{\text{tw}2} + y[\bar{q}(y)\gamma^\mu q(0)]_{\text{tw}3} + y^2[\bar{q}(y)\gamma^\mu q(0)]_{\text{tw}4} + \dots$$

$$\frac{\partial}{\partial y^\mu} [\bar{q}(y)\gamma^\mu q(0)]_{\text{tw}2} = 0, \quad \frac{\partial^2}{\partial y^\nu \partial y_\nu} [\bar{q}(y)\gamma^\mu q(0)]_{\text{tw}2} = 0$$

Same as for collinear distributions
[Balitsky & Braun, 88]

Solution is momentum space (unpolarized)

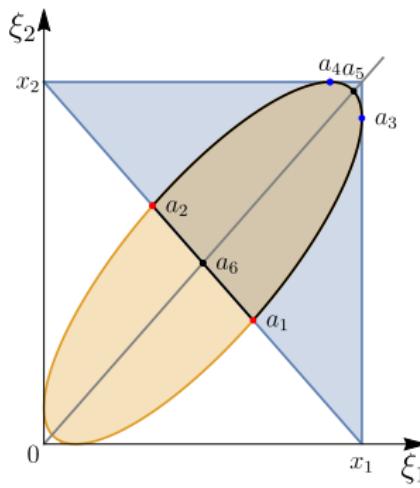
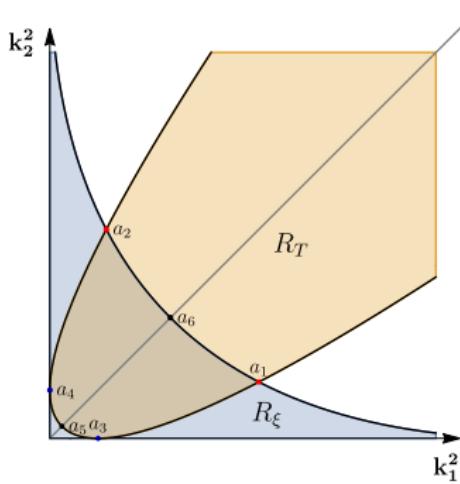
$$[\langle p | \bar{q}(y)\gamma^\mu q(0) | p \rangle]_{\text{tw}2} = 2p^+ \int dx d^4k \delta(k^+ - xp^+) \delta(k^2) e^{i(ky)} k^\mu f_1(x, k_T)$$

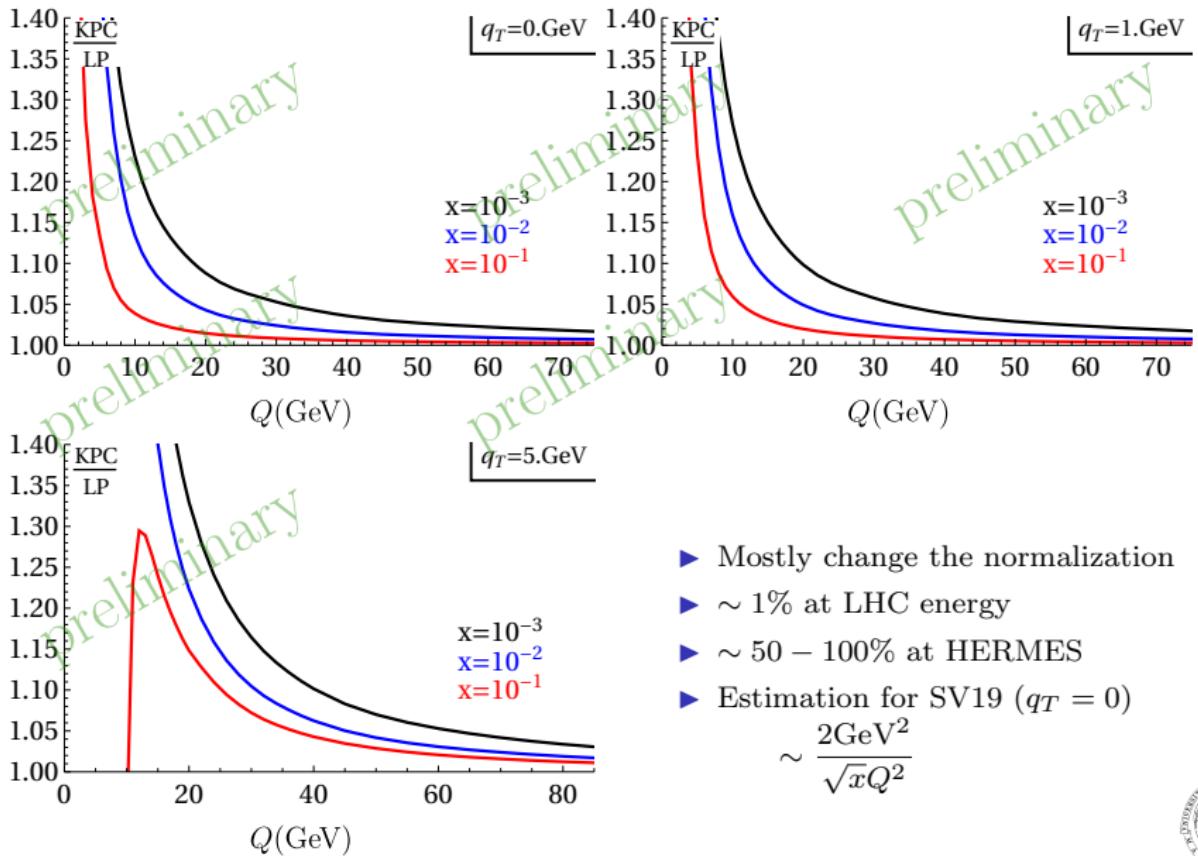
At the leading twist quarks are on mass-shell
alike naive expectations, but now derived in the factorization framework
(respecting evolution, rapidity evolution, etc.)



Resummed KPC

$$\frac{d\sigma}{dq_T} = |C_V(Q)|^2 \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \delta(\mathbf{q}_T - \mathbf{k}_1 - \mathbf{k}_2) \theta((\mathbf{k}_1 - \mathbf{k}_2)^2 < \tau^2) \frac{\rho_{pp}(\text{polarizations})}{\sqrt{\lambda(\mathbf{k}_1^2, \mathbf{k}_2^2, \tau^2)}} F(\xi_1(\mathbf{k}_{1,2}), \mathbf{k}_1^2, Q, Q^2) F(\xi_2(\mathbf{k}_{1,2}), \mathbf{k}_2^2, Q, Q^2)$$

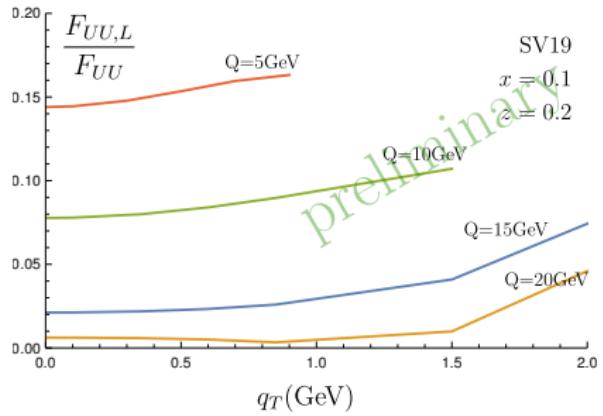
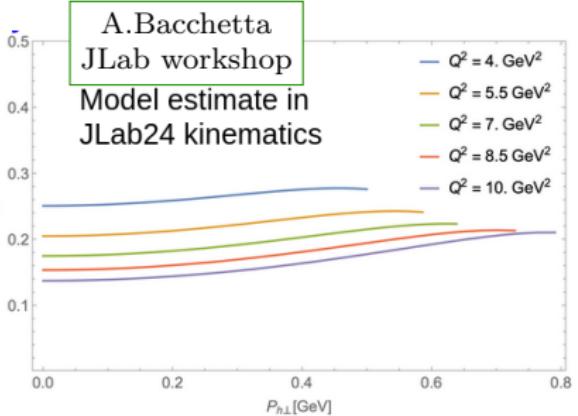




- ▶ Mostly change the normalization
- ▶ $\sim 1\%$ at LHC energy
- ▶ $\sim 50 - 100\%$ at HERMES
- ▶ Estimation for SV19 ($q_T = 0$)

$$\sim \frac{2\text{GeV}^2}{\sqrt{x}Q^2}$$





Conclusion

The future of TMD factorization is after the (resummed) power corrections

Resummed KPCs

- ▶ Restore gauge-invariance
- ▶ Restore frame-invariance
- ▶ Inherit all perturbative part from LP
- ▶ At $Q \rightarrow \infty$ reproduced LP factorization



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Thank you for attention!



Backup



From the (twist-2) *resummation* perspective all types of corrections are alike
 $\sim (bQ)^{-n}$

Kinematic power corrections

$$\partial_\mu f_1(x, b) \longrightarrow \partial_\mu \left(1 + a_s (-2p(x) \ln(b^2 \mu^2) + \dots) + \dots \right) \otimes q(x) = -4b^\mu \frac{a_s}{b^2} [p \otimes q](x) + \dots \quad (1)$$

Genuine power corrections

- ▶ TMD twist-3 matches collinear twist-2

[S.Rodini, AV, 2204.03856]

$$f_\ominus^\perp(x_1, x_2, x_3, b) = \frac{2a_s}{M^2 b^2} \left[-C_F \frac{x_1 - x_3}{x_1} f_1(-x_1) (\theta(x_2, x_3) - \theta(-x_2, -x_3)) \right. \\ \left. + T_F \frac{x_1 - x_3}{x_2} f_g(-x_2) (\theta(x_1, x_3) - \theta(-x_1, -x_3)) \right] + \mathcal{O}\left(b^2, \frac{a_s^2}{b^2}\right),$$



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BUT they do not mix in the properly organized TMD factorization.
 \implies TMD-twist

