Pion and Kaon Fragmentation Functions at Next-to-Next-to-Leading Order

based on Phys.Rev.**D104** (2021) 3, 034007 and Phys.Lett. **B834** (2022) 137456

in collaboration with R. Abdul Khalek, V. Bertone, and A. Khoudli

QCD Evolution Workshop — ICJLab Orsay

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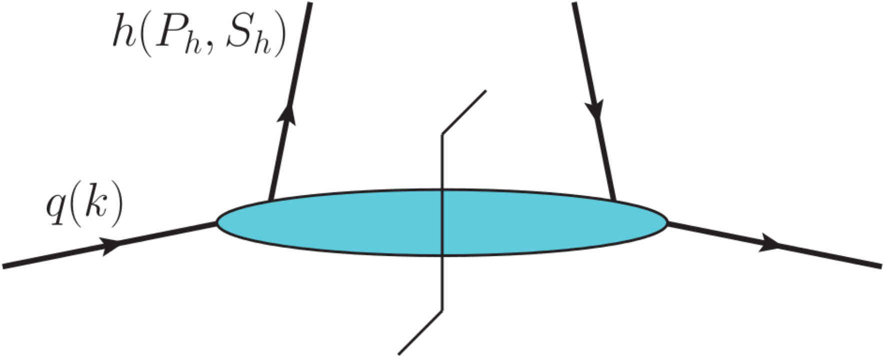
24th May 2023

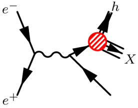
Collinear Fragmentation Functions



$$\Delta^{h/q}(z; P_h, S_h) = \sum\!\!\!\!\!\!\!\int_X \int \frac{d\xi^+}{2\pi} e^{ik^-\xi^+} \langle 0|\mathcal{W}(\infty^+, \xi^+)\psi_q(\xi^+, 0^-, \vec{0}_T|P_h, S_h; X\rangle$$
$$\times \langle P_h, S_h; X|\bar{\psi}_q(0^+, 0^-, \vec{0}_T)\mathcal{W}(0^+, \infty^+)|0\rangle$$

This talk is about a determination of collinear Fragmentation Functions in the vacuum from a global QCD analysis of experimental data

Factorisation of physical observables

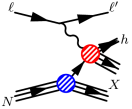


$e^+ + e^- \to h + X$
single-inclusive annihilation (SIA)

$$\frac{d\sigma^h}{dz} = F_T^h(z,Q^2) + F_L^h(z,Q^2) = F_2^h(x,Q^2)$$

$$F^h_{k=T,L,2} = \frac{4\pi\alpha_{\rm em}^2}{Q^2}\langle e^2\rangle\left\{D_\Sigma^h \otimes C^{\rm S}_{k,q} + n_f D_g^h \otimes C^{\rm S}_{k,g} + D^h_{\rm NS}\otimes C^{\rm NS}_{k,q}\right\}$$

up to NNLO [PLB 386 (1996) 422; NPB 487 (1997) 233; PLB 392 (1997) 207]



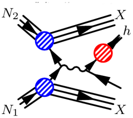
$\ell + N \to \ell' + h + X$
semi-inclusive deep-inelastic scattering (SIDIS)

$$\frac{d\sigma^h}{dxdydz} = \frac{2\pi\alpha_{\rm em}^2}{Q^2}\left[\frac{1+(1-y)^2}{y}2F_1^h + \frac{2(1-y)}{y}F_L^h\right]$$

$$2F_1^h = e_q^2\left\{q\otimes D_q^h + \frac{\alpha_s}{2\pi}\left[q\otimes C^1_{qq}\otimes D_q^h + q\otimes C^1_{gq}\otimes D_g^h + g\otimes C^1_{qg}\otimes D_q^h\right]\right\}$$

$$F_L^h = \frac{\alpha_s}{2\pi}\sum_{q,\bar{q}} e_q^2\left[q\otimes C^L_{qq}\otimes D_q^h + q\otimes C^L_{gq}\otimes D_g^h + g\otimes C^L_{qg}\otimes D_q^h\right]$$

up to NLO [NPB 160 (1979) 301; PRD 57 (1998) 5811]
partial NNLO [PRD 95 (2017) 034027]; approximate NNLO [arXiv:2109.00847]



$N_1 + N_2 \to h + X$
high-p_T hadron production in pp collisions (PP)

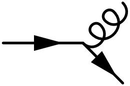
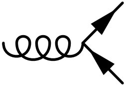
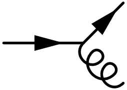
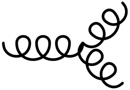
$$E_h\frac{d^3\sigma}{dp^3_{T,h}} = \sum_{a,b,c} f_a\otimes f_b \otimes \hat{\sigma}^c_{ab}\otimes D_c^h$$
$$=$$
$$\sum_{i,j,k}\int\frac{dx_a}{x_a}\int\frac{dx_b}{x_b}\int\frac{dz}{z^2}f^{i/p_a}(x_a)f^{j/p_b}(x_b)D^{h/k}(z)\hat{\sigma}^{ij\to k}\delta(\hat{s}+\hat{t}+\hat{u})$$

up to NLO [PRD 67 (2003) 054004; PRD 67 (2003) 054005]
NNLO in principle possible by extending `NNLOjet` [PRL 118 (2017) 072002]

Evolution of FFs: DGLAP equations

A set of $(2n_f + 1)$ integro-differential equations (n_f=number of active flavours)

$$\frac{\partial}{\partial \ln \mu^2} D_i(x,\mu^2) = \sum_j^{n_f} \int_x^1 \frac{dz}{z} P_{ji}\left(z, \alpha_s(\mu^2)\right) D_j\left(\frac{x}{z}, \mu^2\right)$$

$P_{qq}^{(0)}$  $P_{gq}^{(0)}$  $P_{qg}^{(0)}$  $P_{gg}^{(0)}$ 

LO [Sov. J. Nucl. Phys. 15 (1973) 438; NPB 126 (1977) 298; NPB 136 (1978) 445]
NLO [NPB 175 (1980) 27, PLB 97 (1980) 497, PRD 48 (1993) 116]
NNLO [PLB 638 (2006) 61, PLB 659 (2008) 290, NPB 854 (2012) 133]

Must be careful with fixed-order splitting functions as $z \to 0$ ($m = 1, \dots, 2k+1$)

SPACE-LIKE CASE

$$P_{ji} \propto \frac{\alpha_s^{k+1}}{x} \log^{k+1-m} \frac{1}{x}$$

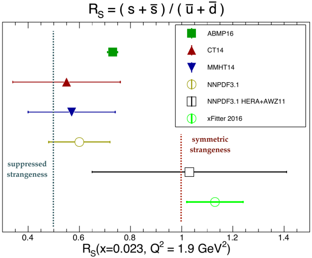
TIME-LIKE CASE

$$P_{ji} \propto \frac{\alpha_s^{k+1}}{z} \log^{2(k+1)-m-1} z$$

Soft gluon logarithms diverge more rapidly in the TL case than in the SL case: as z decreases, the unresummed SGLs spoil the convergence of the FO series for $P(z, \alpha_s)$ if $\log \frac{1}{z} \geq \mathcal{O}\left(\alpha_s^{-1/2}\right)$

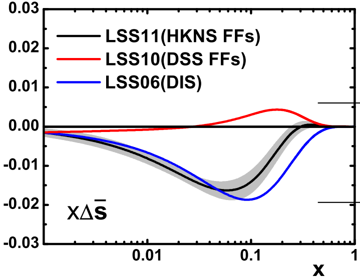
Fragmentation functions at NNLO: why should we bother?

Example 1: The strange (polarised) parton distribution and SIDIS

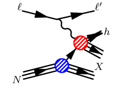


Can SIDIS data be used to determine s?
What is the bias induced by FFs onto PDFs?
$\longrightarrow$ How well do we know kaon FFs?

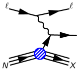
If SIDIS data is used to determine Δs, $K^{\pm}$ FFs for different sets lead to different results
Such results may differ significantly among them and w.r.t. the results obtained from DIS
$\longrightarrow$ How well do we know kaon FFs?



directly from SIDIS Kaon data

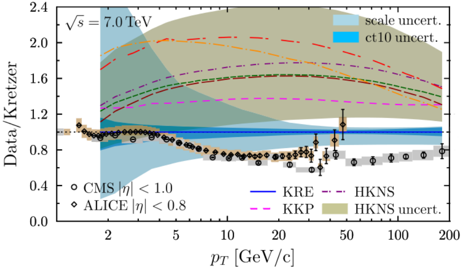


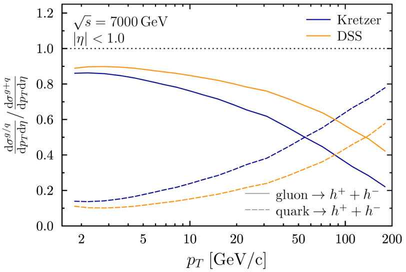
indirectly from DIS + SU(3)

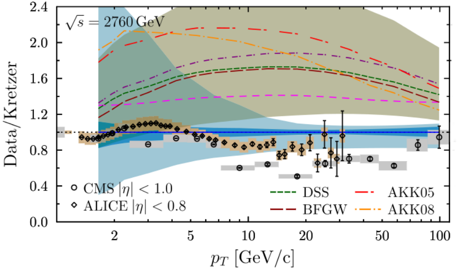


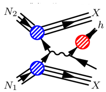
Fragmentation functions at NNLO: why should we bother?

Example 2: Ratio of the inclusive charged-hadron spectra measured by CMS and ALICE









$$E\,\frac{d^3\sigma}{dp_T^3} = \sum_{a,b,c} f_a \otimes f_b \otimes \hat{\sigma}^c_{ab} \otimes D^h_c$$

Figures taken from [NPB 883 (2014) 615]

Predictions from all available FF sets are not compatible with CMS and ALICE data, not even within scale and PDF/FF uncertainties

$\longrightarrow$ How well do we know the gluon FF?

Fragmentation functions at NNLO: why should we bother?

Example 3: Baseline for global analyses of TMDs

Matching coeff. (perturbative calculable)

Collinear PDFs (previous fit)

Perturbative Sudakov evolution factor

$$\hat{f}_1^q(x_B, \mathbf{b}_T; \mu_F, \zeta_F) = [\boxed{C} \otimes \boxed{f_1}](x_B, b_\star; \mu_{b_\star}, \mu_{b_\star}^2) \boxed{\exp\left\{ \int_{\mu_{b_\star}}^{\mu_F} \frac{d\mu'}{\mu'} \gamma(\mu', \zeta_F) \right\}}$$

$$\times \boxed{\left(\frac{\zeta}{\mu_{b_\star}^2}\right)^{K(b_\star,\mu_{b_\star})/2}} \boxed{\left[\frac{\zeta}{Q_0}\right]^{-g_K(\mathbf{b}_T)/2}} \boxed{f_1^{NP}(x, \mathbf{b}_T; \zeta, Q_0)}$$

Collins-Soper kernel

NP part of Collins-Soper Kernel

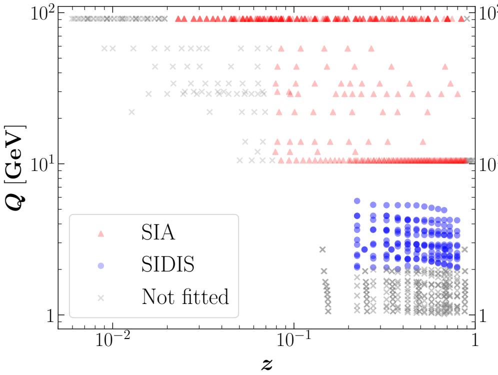
Non perturbative part of TMDs

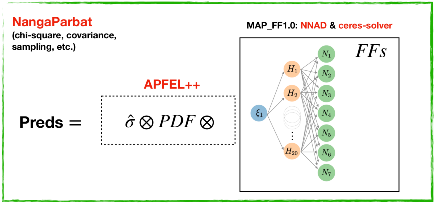
Accuracy $\mathrm{N}^3\mathrm{LL}^-$ (PDFs and FFs at NNLO)

[Figure from Lorenzo Rossi's talk]

1. MAPFF1.0: NLO [Phys.Rev. **D104** (2021) 3, 034007]

MAPFF1.0: pion FFs from SIA and SIDIS



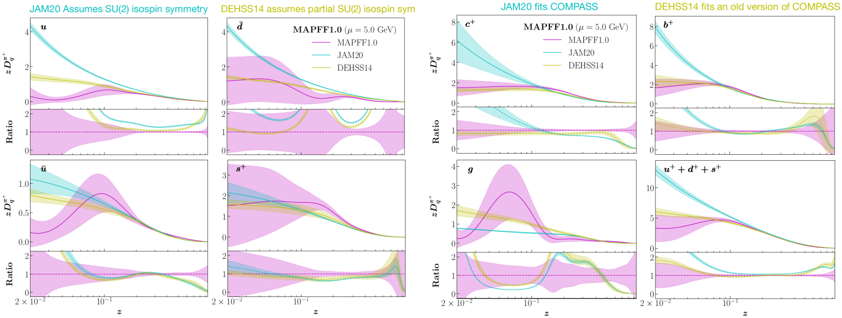


Experiment	$\chi^2/N_{\rm dat}$	$N_{\rm dat}$
BELLE $\pi^\pm$	0.09	70
BABAR prompt $\pi^\pm$	0.90	39
TASSO 12 GeV $\pi^\pm$	0.97	4
TASSO 14 GeV $\pi^\pm$	1.39	9
TASSO 22 GeV $\pi^\pm$	1.85	8
TPC $\pi^\pm$	0.22	13
TASSO 30 GeV $\pi^\pm$	0.34	2
TASSO 34 GeV $\pi^\pm$	1.20	9
TASSO 44 GeV $\pi^\pm$	1.20	6
TOPAZ $\pi^\pm$	0.28	5
ALEPH $\pi^\pm$	1.29	23
DELPHI total $\pi^\pm$	1.29	21
DELPHI uds $\pi^\pm$	2.84	21
DELPHI bottom $\pi^\pm$	1.67	21
OPAL $\pi^\pm$	1.72	24
SLD total $\pi^\pm$	1.14	34
SLD uds $\pi^\pm$	2.05	34
SLD bottom $\pi^\pm$	0.55	34
Total SIA	1.10	377
HERMES π^- deuteron	0.60	2
HERMES π^- proton	0.02	2
HERMES π^+ deuteron	0.30	2
HERMES π^+ proton	0.53	2
COMPASS π^-	0.80	157
COMPASS π^+	1.07	157
Total SIDIS	0.78	322
Total	**0.90**	**699**

Comparing FFs to DEHSS14 and JAM20

basis: $\{D_u^{\pi^+},\ D_{\bar d}^{\pi^+},\ D_d^{\pi^+} = D_{\bar u}^{\pi^+},\ D_s^{\pi^+} = D_{\bar s}^{\pi^+},\ D_c^{\pi^+} = D_{\bar c}^{\pi^+},\ D_b^{\pi^+} = D_{\bar b}^{\pi^+},\ D_g^{\pi^+}\}$

parametric form: $zD_i^{\pi^+}(z, \mu_0 = 5\,\text{GeV}) = \Big(\mathcal{N}_i(z;\boldsymbol{\theta}) - \mathcal{N}_i(1;\boldsymbol{\theta})\Big)^2$



$D_u^{\pi^\pm}$ and $D_{\bar d}^{\pi^\pm}$: suppression of MAPFF1.0 w.r.t. DEHSS14 and JAM 20 at large z

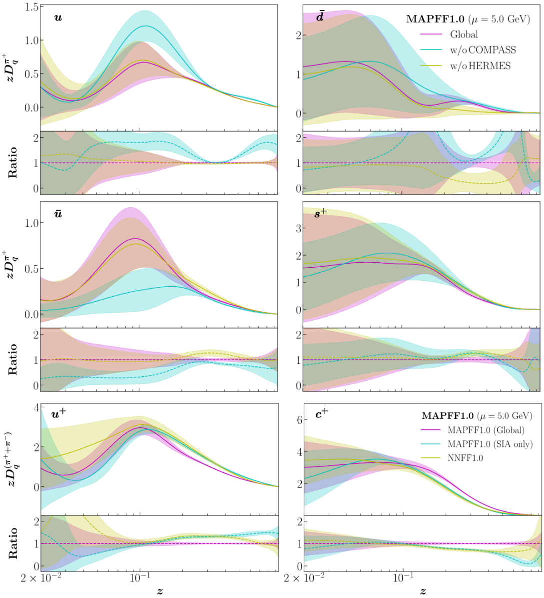
$D_{\bar u}^{\pi^\pm}$ and $D_{s^+}^{\pi^\pm}$: enhancement of MAPFF1.0 w.r.t. DEHSS14 and JAM20

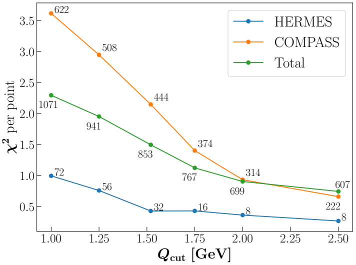
$D_{c^+}^{\pi^\pm}$, $D_{b^+}^{\pi^\pm}$ and $D_{u^+}^{\pi^\pm} + D_{d^+}^{\pi^\pm} + D_{s^+}^{\pi^\pm}$: overall fair agreement across the three sets

$D_g^{\pi^\pm}$: MAPFF1.0 has a different shape and larger uncertainties

DEHSS14 [PRD 105 (2022) L031502] JAM20 [PRD 104 (2021) 016015]

Impact of SIDIS





COMPASS largely more constraining than HERMES

HERMES (8pts) still has non-negligible impact

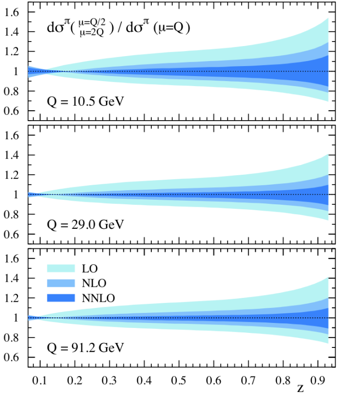
χ^2 decreases as $Q_{\rm cut}$ increases

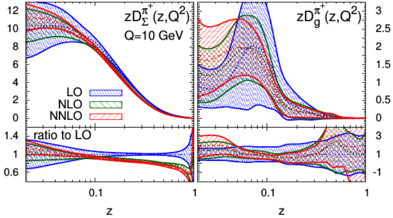
HERMES can be described down to $Q = 1$ GeV, but COMPASS quickly deteriorates

Choose a SIDIS cut $Q > 2$ GeV

2. MAPFF1.0: NNLO [Phys.Lett. **B834** (2022) 137456]

SIA at NNLO [PRD 92 (2015) 114017; EPJC 77 (2017) 516]

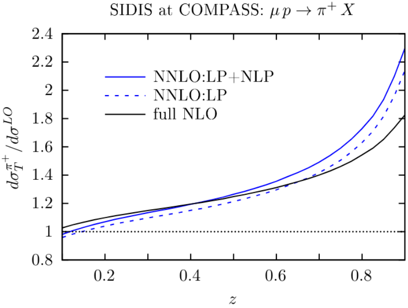


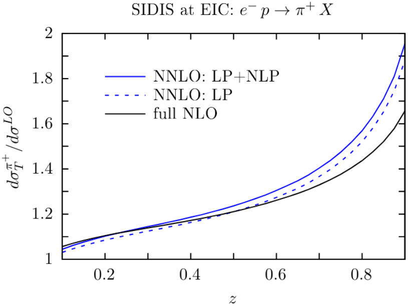


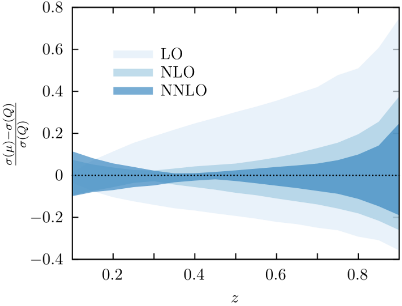
Exp.	$N_{\rm dat}$	LO $\chi^2/N_{\rm dat}$	NLO $\chi^2/N_{\rm dat}$	NNLO $\chi^2/N_{\rm dat}$
BELLE	70	0.60	0.11	0.09
BABAR	40	1.91	1.77	0.78
TASSO12	4	0.70	0.85	0.87
TASSO14	9	1.55	1.67	1.70
TASSO22	8	1.64	1.91	1.91
TPC	13	0.46	0.65	0.85
TPC-UDS	6	0.78	0.55	0.49
TPC-C	6	0.55	0.53	0.52
TPC-B	6	1.44	1.43	1.43
TASSO34	9	1.16	0.98	1.00
TASSO44	6	2.01	2.24	2.34
TOPAZ	5	1.04	0.82	0.80
ALEPH	23	1.68	0.90	0.78
DELPHI	21	1.44	1.79	1.86
DELPHI-UDS	21	1.30	1.48	1.54
DELPHI-B	21	1.21	0.99	0.95
OPAL	24	2.29	1.88	1.84
SLD	34	2.33	1.14	0.83
SLD-UDS	34	0.95	0.65	0.52
SLD-C	34	3.33	1.33	1.06
SLD-B	34	0.45	0.38	0.36
Total	**428**	**1.44**	**1.02**	**0.87**

Excellent perturbative convergence
FFs almost stable from NLO to NNLO
LO FF uncertainties larger than HO
Effects less evident for $K^{\pm}$ and $p/\bar{p}$

SIDIS at NNLO [PRD 104 (2021) 094046; ibid. 106 (2022) 014015]







At k-th order, there are terms of the form:

$$\alpha_s^k \delta(1-x) \left(\frac{\ln^m(1-z)}{(1-z)}\right)_+ \alpha_s^k \delta(1-z) \left(\frac{\ln^m(1-x)}{(1-x)}\right)_+$$
$$\alpha_s^k \left(\frac{\ln^m(1-x)}{(1-x)}\right)_+ \left(\frac{\ln^n(1-z)}{(1-z)}\right)_+$$

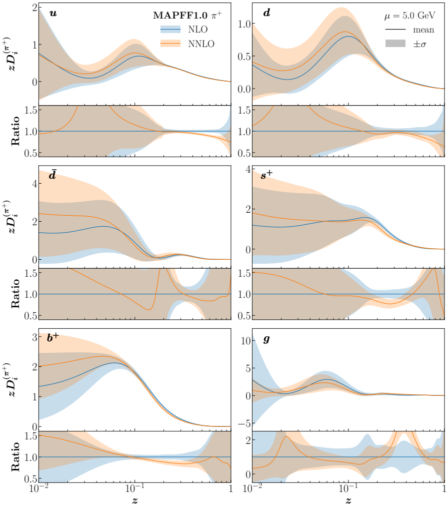
$m \leq 2k-1$ (non-mixed) $m+n \leq 2k-2$ (mixed)

Approximate NNLO corrections obtained from threshold resummation

Use similarity between DY and SIDIS to expand NNLL results to fixed-order NNLO

Combining SIA and SIDIS at NNLO [See also PRL 129 (2022) 012002]

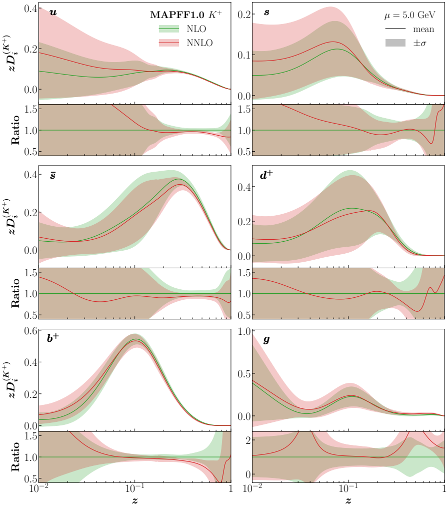
PIONS



Data set	$N_{\rm dat}$	NLO	NNLO
BELLE $h^\pm$	70	0.14	0.13
BABAR $h^\pm$	39	0.91	0.76
TASSO 12 GeV $h^\pm$	4	0.90	0.92
TASSO 14 GeV $h^\pm$	9	1.33	1.35
TASSO 22 GeV $h^\pm$	8	1.65	1.81
TPC $h^\pm$	13	0.23	0.25
TASSO 30 GeV $h^\pm$	2	0.30	0.34
TASSO 34 GeV $h^\pm$	9	1.08	1.48
TASSO 44 GeV $h^\pm$	6	1.13	1.37
TOPAZ $h^\pm$	5	0.24	0.37
ALEPH $h^\pm$	23	1.24	1.46
DELPHI (inclusive) $h^\pm$	21	1.31	1.25
DELPHI (uds tagged) $h^\pm$	21	2.68	2.89
DELPHI (b tagged) $h^\pm$	21	1.58	1.73
OPAL $h^\pm$	24	1.63	1.79
SLD (inclusive) $h^\pm$	34	1.05	1.13
SLD (uds tagged) $h^\pm$	34	1.59	2.16
SLD (b tagged) $h^\pm$	34	0.55	0.68
HERMES h^- d	2	0.41	0.32
HERMES h^+ p	2	0.01	0.02
HERMES h^- d	2	0.17	0.11
HERMES h^+ p	2	0.35	0.32
COMPASS h^-	157	0.48	0.55
COMPASS h^+	157	0.62	0.72
Total	**699**	**0.68**	**0.76**

Combining SIA and SIDIS at NNLO [See also PRL 129 (2022) 012002]

KAONS



Data set	$N_{\rm dat}$	NLO	NNLO
BELLE $h^\pm$	70	0.39	0.41
BABAR $h^\pm$	28	0.36	0.25
TASSO 12 GeV $h^\pm$	3	0.85	0.87
TASSO 14 GeV $h^\pm$	9	1.24	1.22
TASSO 22 GeV $h^\pm$	6	0.89	0.90
TPC $h^\pm$	13	0.38	0.40
TASSO 30 GeV $h^\pm$	—	—	—
TASSO 34 GeV $h^\pm$	5	0.07	0.06
TASSO 44 GeV $h^\pm$	—	—	—
TOPAZ $h^\pm$	3	0.10	0.11
ALEPH $h^\pm$	18	0.49	0.48
DELPHI (inclusive) $h^\pm$	23	0.97	0.99
DELPHI (uds tagged) $h^\pm$	23	0.44	0.38
DELPHI (b tagged) $h^\pm$	23	0.42	0.45
OPAL $h^\pm$	10	0.39	0.36
SLD (inclusive) $h^\pm$	35	0.83	0.67
SLD (uds tagged) $h^\pm$	35	1.37	1.52
SLD (b tagged) $h^\pm$	35	0.75	0.77
HERMES h^- d	2	0.18	0.13
HERMES h^+ p	2	0.05	0.04
HERMES h^- d	2	0.58	0.48
HERMES h^+ p	2	0.56	0.43
COMPASS h^-	156	0.74	0.59
COMPASS h^+	156	0.76	0.67
Total	**659**	**0.62**	**0.55**

Good knowledge of experimental correlations is important

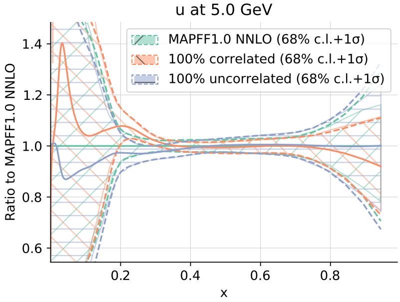
Consider the COMPASS $\pi^{\pm}$ multiplicities

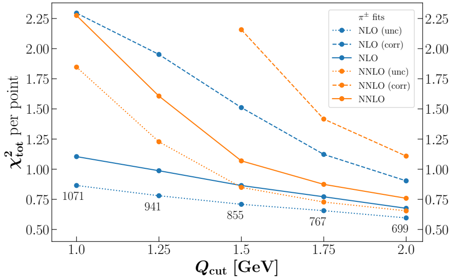
[PLB 764 (2017) 1]

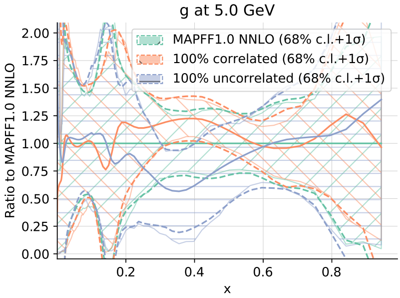
Only 80% of the systematic uncertainty is bin-by-bin correlated

What if you incorporate a different piece of information in a FF fit?

Consider two cases:
full correlation; full decorrelation

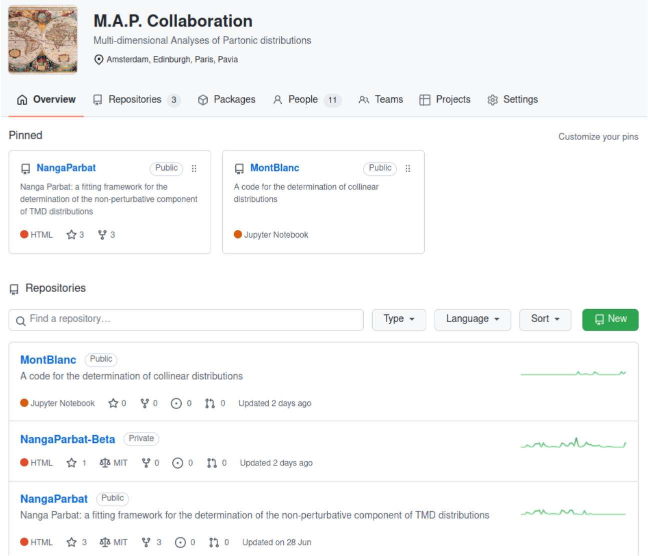


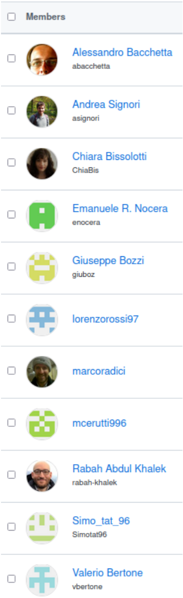




3. Conclusions

MAPFF1.0: making the code open source



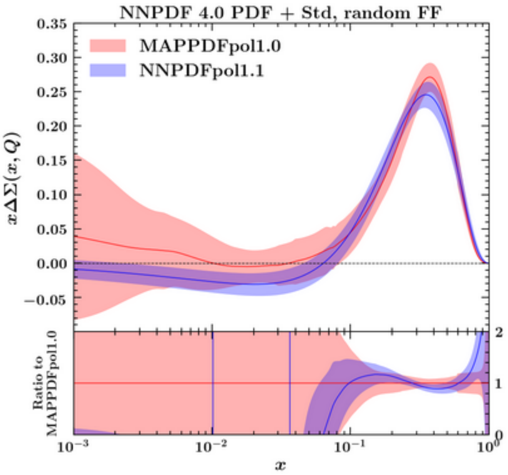


https://github.com/MapCollaboration

Towards MAPPDFPol1.0 (NNLO)

PRELIMINARY

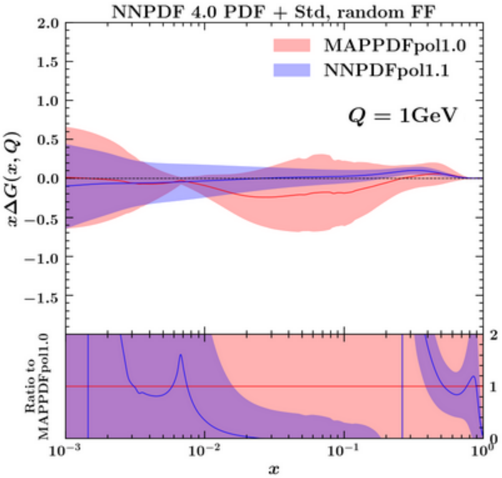
work in progress with A. Bertone and A. Chiefa



Data set		$N_{\rm dat}$	$\chi^2/N_{\rm dat}$
EMC	g_1^p	10	0.51
SMC	g_1^p	12	0.36
	g_1^d	12	1.20
E142	g_1^n	7	0.69
E143	g_1^p	25	0.74
	g_1^d	25	1.25
E154	g_1^n	11	0.26
E155	g_1^p	22	0.80
	g_1^n	22	0.80
COMPASS	g_1^p	15	0.98
	g_1^d	17	1.49
HERMES	g_1^n	8	0.23
	g_1^p	14	0.55
	g_1^d	14	0.77
COMPASS	$A_{1,p,d}^{\pi^+}$	22	1.79
	$A_{1,p,d}^{\pi^-}$	22	0.95
	$A_{1,p,d}^{K^+}$	22	1.00
	$A_{1,p,d}^{K^-}$	22	1.10
HERMES	$A_{1,p,d}^{\pi^+}$	18	1.45
	$A_{1,p,d}^{\pi^-}$	18	1.72
	$A_{1,d}^{K^+}$	9	0.90
	$A_{1,d}^{K^-}$	9	0.93
Total		**356**	0.79

Towards MAPPDFPol1.0 (NNLO)

PRELIMINARY
work in progress with A. Bertone and A. Chiefa



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Summary

1. A number of hard-scattering processes require an appropriate knowledge of FFs
   - probing nucleon momentum, spin and flavour
   - understanding hadronisation
   - understanding spatial distributions and the dynamics of nuclear matter
2. Significant role of new data, combined with increasing theoretical sofistication
   - increased accuracy of fragmentation functions
   - increased precision of fragmentation functions
3. Fragmentation functions as tools
   - global fits of TMDs
   - global fits of polarised (collinear) PDFs

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Thank you