Resummation of threshold logarithms in DVCS

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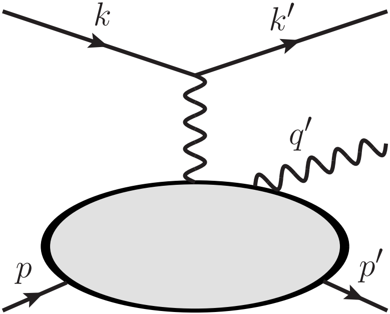
Based on [J.S., JHEP02(2023)207]

QCD Evolution Workshop 2023

25 May 2023

DVCS

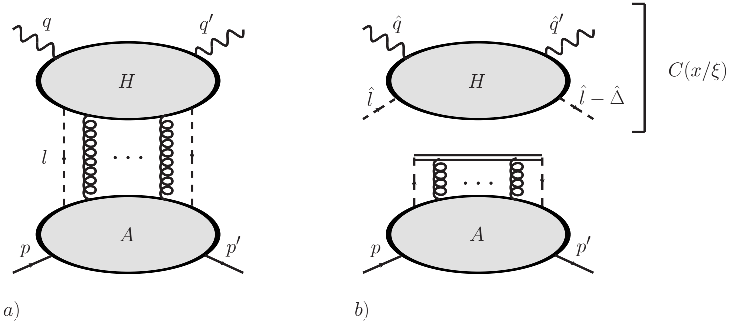
$$\gamma^*(q)\ N(p) \longrightarrow \gamma(q')\ N(p')$$



Kinematical parameters

$$P = \frac{p+p'}{2}, \quad t = (p'-p)^2, \quad Q^2 = -q^2, \quad M^2 = p^2 = p'^2, \quad x_B = \frac{Q^2}{2p\cdot q},$$

$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+} \approx \frac{x_B}{2-x_B}$$



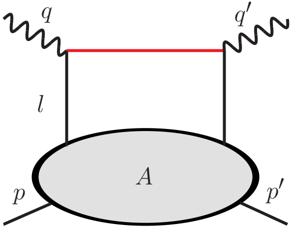
- a) Leading region to DVCS amplitude, b) Factorized form after performing expansion.
  Dashed lines can be quarks or transversely polarized gluons.

- Vector contribution to the amplitude

$$V(\xi,t) = \sum_q \int_{-1}^{1} \frac{dx}{\xi} C_q(x/\xi) F_q(x,\xi,t) + \int_{-1}^{1} \frac{dx}{\xi^2} C_g(x/\xi) F_g(x,\xi,t),$$

where F_q, F_g are GPDs.

- A subtlety: Parton lines connecting H and A are assumed collinear to target. This order of size assumption fails (naively) at "breakpoints" $x = \pm\xi$, where one of the two lines becomes soft.



- Example: hard propagator in handbag diagram

$$(l+q)^2 = 2(x-\xi)(l^- + q^-)P^+ + l_\perp^2 + O(-t, m^2)$$

Canonical p.c.: $|x-\xi| \gg \max(|t|, m^2, |l_\perp^2|)/Q^2$.
Collinear approximation: Set $l_\perp^2, t, m^2, l^-$ to zero.

- Region $x \sim \xi$ is inside the integration region. **But:** Deformation of the integration contour is possible [Collins & Freund, 98], restoring the validity of the collinear approximation.

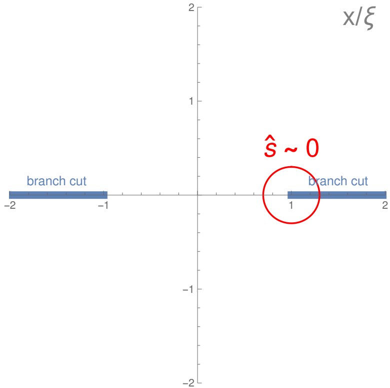
- Leading terms of the quark CF for $x \to \xi$ [Braun, Manashov, Moch, Schoenleber, 20]

$$C_q = \frac{1}{1 - x/\xi}\left[1 + \frac{\alpha_s C_F}{4\pi}\log^2\left(\frac{\xi - x}{2\xi}\right) + \frac{1}{2}\left(\frac{\alpha_s C_F}{4\pi}\log^2\left(\frac{\xi - x}{2\xi}\right)\right)^2 + ...\right],$$

- Question: Does this exponentiate to all orders? Is it possible to resum terms of the form $\frac{\alpha_s^n}{1 \pm x/\xi}\log^k(x \pm \xi)$ to all orders? $\to$ Yes.
- Although these logs are not large, one can
  $\to$ predict higher order terms in the CF
  $\to$ get contributions that might be relevant for precision physics

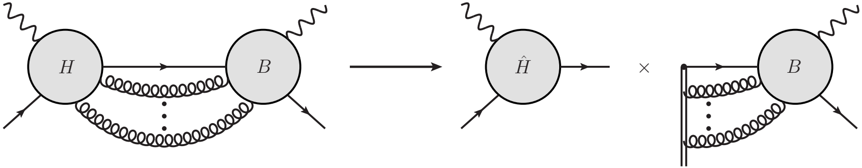
- Consider leading contributions to the CF near the partonic thresholds $\hat{s} = \frac{x-\xi}{2\xi} Q^2 = 0$ (i.e. $x = \xi$) and $\hat{u} = -\frac{x+\xi}{2\xi} Q^2 = 0$ (i.e. $x = -\xi$)

- CF is antisymmetric under $x/\xi \to -x/\xi$



- **Strategy:** Derive factorization for the CF near the partonic thresholds ($\hat{s} = 0$ and $\hat{u} = 0$) and use RG eqs to resum logarithms that are singular at the threshold points

- Key observation: quark CF itself factorizes near $\hat{s} = 0$ (i.e. $x = \xi$). The outgoing parton leg becomes soft and the intermediate quark propagator (previously hard) becomes collinear to outgoing photon.



- Get factorization theorem

$$C_q(x/\xi, \mu = Q) = \frac{h(Q^2, \nu^2) f(-\hat{s}, \nu^2)}{1 - x/\xi} + O((x-\xi)^0 \times \text{logs})$$

- Evolution equations

$$\frac{df(-\hat{s},\nu^2)}{d\log\nu} = \Big[-\Gamma_{\mathsf{cusp}}(\alpha_s(\nu))\log\frac{-\hat{s}}{\nu^2} + \bar{\gamma}_f(\alpha_s(\nu))\Big] f(-\hat{s},\nu^2)$$

$$\frac{dh(Q^2,\nu^2)}{d\log\nu} = \Big[\Gamma_{\mathsf{cusp}}(\alpha_s(\nu))\log\frac{Q^2}{\nu^2} + \bar{\gamma}_h(\alpha_s(\nu))\Big] h(Q^2,\nu^2)$$

- Solve for f

$$f(-\hat{s},\nu^2) = U(\nu_0^2/\nu^2) f(-\hat{s},\nu_0^2).$$

- Set $\nu^2 = Q^2$ to get

$$C_q(x/\xi,\mu=Q) = \frac{h(Q^2,Q^2)}{1-x/\xi} \times \underbrace{U(-\hat{s}/Q^2)}_{\text{threshold logs resummed here}} \times \underbrace{f(-\hat{s},-\hat{s})}_{\text{also some logs here}} + O((x-\xi)^0).$$

- Resummed result:

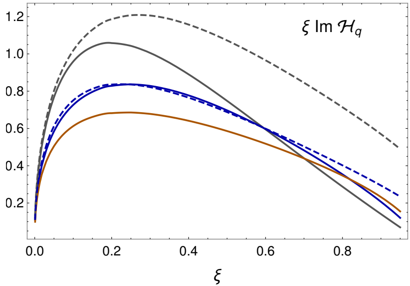
$$
\begin{aligned}
&C_q(x/\xi, Q, \mu = Q) \\
&\sim \frac{\bar{h}(\alpha_s(Q))\bar{f}(\alpha_s(\sqrt{-\hat{s}}))}{1-x/\xi} \exp\left\{\frac{1}{2}\int_{-\hat{s}}^{Q^2} \frac{d\nu^2}{\nu^2}\Big[-\Gamma_{\text{cusp}}(\alpha_s(\nu))\log\Big(\frac{-\hat{s}}{\nu^2}\Big) + \bar{\gamma}_f(\alpha_s(\nu))\Big]\right\} \\
&\overset{\text{LL}}{\sim} \frac{1}{1-x/\xi}\exp\left\{\frac{8\pi C_F}{\alpha_s(Q)\beta_0^2}(1-r+r\log r)\right\} \\
&\sim \frac{1}{1-x/\xi}\exp\left\{\frac{\alpha_s(Q)}{\pi}C_F\log^2\Big(\frac{\xi-x}{2\xi}\Big)\right\},
\end{aligned}
$$

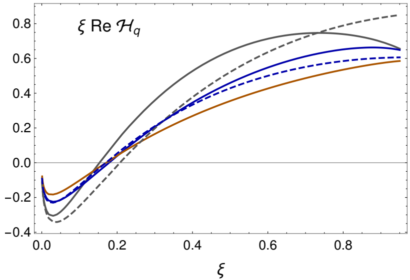
where $r = 1 + \frac{\alpha_s(Q)}{4\pi}\beta_0 \log(\frac{\xi-x}{2\xi})$.

- All ingredients for NNLL ($\alpha_s^n \log^k(\frac{\xi-x}{2\xi})$ with $n-1 \leq k \leq 2n$ in the exponent) can be obtained from the NNLO expression for C_q

- Integration over running coupling $\alpha_s(\sqrt{-\hat{s}})$. **But:** Convolution integral is *defined* on a deformed contour $\Rightarrow$ The Landau pole is naturally avoided.

- Preliminary numerical study. Simple model for non-singlet quark GPD





Gray: LO/NLL, Blue: NLO/NNLL, Brown: NNLO
Straight lines: Fixed order, Dashed lines: with resummation.

- Small corrections at small ξ. Get larger for increasing ξ

- Same resummation formula applies for the quark axial-vector case, i.e. also to the CF of the pion-photon transition form factor
- Numerical analysis was performed for a very simple model for the non-singlet quark GPD $\Rightarrow$ Want to consider impact for more realistic models
- For gluon CF a similar factorization theorem probably holds