Sum rules and factorization of double parton distributions

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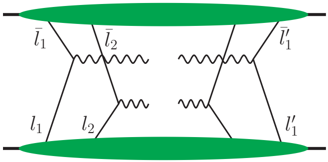
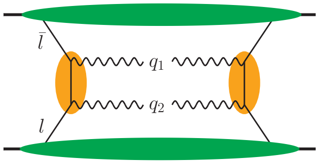
Double parton scattering - DPS

- With increasing $\sqrt{s}$ multiparton interactions become increasing important.
- The first step - DPS in addition to SPS





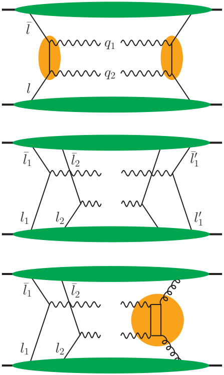
- Collinear factorization with double parton distribution functions (DPDFs):

$$\frac{d\sigma^{AB}_{DPS}}{dx_1\,dx_2\,d\bar{x}_1\,d\bar{x}_2} = \sum_{f_i\bar{f}_i}\int d^2\Delta\; D_{f_1 f_2}(x_1, x_2, \Delta)\,\sigma^{A}_{f_1\bar{f}_1}(q_1)\,\sigma^{B}_{f_2\bar{f}_2}(q_2)\,D_{\bar{f}_1\bar{f}_2}(\bar{x}_1, \bar{x}_2, -\Delta)$$

- In addition, two hard scale dependence in DPDFs: q_1, q_2

(M. Diehl, D. Ostermeier, A. Schäfer, 1111.0910/JHEP)

DPS versus SPS



$\dfrac{s\,d\sigma}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i\, d^2\boldsymbol{q}_i}$	$\dfrac{s\,d\sigma}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i}$
$\dfrac{1}{\Lambda^2 Q^2}$	1
$\dfrac{1}{\Lambda^2 Q^2}$	$\dfrac{\Lambda^2}{Q^2}$
$\dfrac{1}{\Lambda^2 Q^2}$	$\dfrac{\Lambda^2}{Q^2}$

- Inclusive DPS is enhanced due to rising guon density for $x \to 0$

$$d\sigma^{AB}_{DPS} \sim g^2(x) \qquad\qquad d\sigma^{AB}_{\mathrm{SPS}} \sim g(x)$$

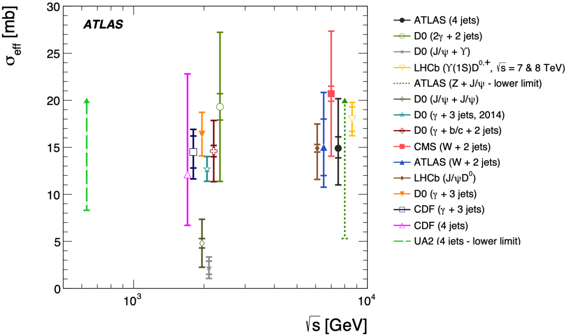
Standard approach to DPS

- In the first approximation:

$$D_{f_1 f_2}(x_1, x_2, \Delta) = D_{f_1}(x_1)\, D_{f_2}(x_2)\, F(\Delta)$$

- Pocket formula:

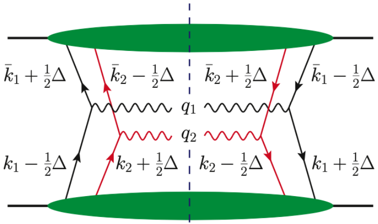
$$\sigma^{AB}_{DPS} = \sigma^{A}_{SPS}\, \sigma^{B}_{SPS} \int d^2\Delta\, F(\Delta) = \frac{\sigma^{A}_{SPS}\, \sigma^{B}_{SPS}}{\sigma_{eff}}$$



- Goal: to go beyond this approximation.

DPDFs

- Double parton distributions are basic objects in DPS:



- DPDFs integrated over transverse momenta:

$$D_{f_1 f_2}(x_1, x_2, \Delta) = \int d^2\mathbf{k}_1\, d^2\mathbf{k}_2\, F_{f_1 f_2}(x_1, x_2, \mathbf{k}_1, \mathbf{k}_2, \Delta)$$

- Spectral condition:

$$0 < x_1 + x_2 \leqslant 1$$

DPDFs in y space

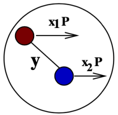
- Introducing Fourier transformed DPDFs:

$$\tilde{D}_{f_1 f_2}(x_1, x_2, \mathbf{y}) = \int d^2\Delta\, \mathrm{e}^{-i\Delta\cdot\mathbf{y}}\, D_{f_1 f_2}(x_1, x_2, \Delta)$$

we have definition through twist-2 partonic operators:

$$\tilde{D}_{f_1 f_2}(x_1, x_2, \mathbf{y}) = \int dy^- \frac{dz_1^-}{2\pi} \frac{dz_2^-}{2\pi}\, \mathrm{e}^{i(z_1^- x_1 + z_2^- x_2)P^+} \langle P|\, \mathcal{O}_{f_1}(0, z_1)\, \mathcal{O}_{f_2}(y, z_2)\, |P\rangle$$

where $z_1 = (z_1^-, \mathbf{0})$, $z_2 = (z_2^-, \mathbf{0})$ and $y = (y^-, \mathbf{y})$. Interpretation:



- Probabilistic interpretation only for $\mathbf{y}$ averaged DDPFs:

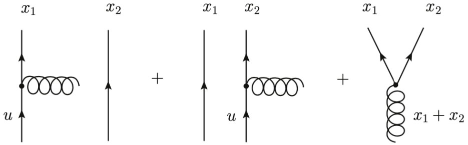
$$D_{f_1 f_2}(x_1, x_2, \Delta = 0) = \int d^2\mathbf{y}\, \tilde{D}_{f_1 f_2}(x_1, x_2, \mathbf{y})$$

Evolution equations for DPDFs

- DGLAP-like equations for $D_{f_1 f_2}(x_1, x_2, Q) \equiv D_{f_1 f_2}(x_1, x_2, \Delta = 0; Q, Q)$

$$\frac{\partial D_{f_1 f_2}(x_1, x_2, Q)}{\partial \ln Q^2} = \frac{\alpha_s(Q)}{2\pi} \sum_f \left\{ \int_{x_1}^{1-x_2} \frac{du}{u} P_{f_1 f}\left(\frac{x_1}{u}\right) D_{f f_2}(u, x_2, Q) \right.$$
$$+ \int_{x_2}^{1-x_1} \frac{du}{u} P_{f_2 f}\left(\frac{x_2}{u}\right) D_{f_1 f}(, x_1, u, Q)$$
$$\left. + \frac{1}{x_1 + x_2} P^R_{f \to f_1 f_2}\left(\frac{x_1}{x_1 + x_2}\right) D_f(x_1 + x_2, Q) \right\}$$

- In non-homogenous term single PDFs, evolved with DGLAP equations:



(A. Snigirev 2003, J. Gaunt and W.J. Stirling 2009, F.A. Ceccopieri 2011, . . .)

Sum rules

- Evolution equations obey momentum and valence number sum rules:

$$\sum_{f_1} \int_0^{1-x_2} dx_1\, x_1\, D_{f_1 f_2}(x_1, x_2, Q) = (1 - x_2)\, D_{f_2}(x_2, Q)$$

$$\int_0^{1-x_2} dx_1 \left[D_{q f_2}(x_1, x_2, Q) - D_{\bar{q} f_2}(x_1, x_2, Q) \right] = (N_q - \delta_{q f_2} + \delta_{\bar{q} f_2})\, D_{f_2}(x_2, Q)$$

- Analogous with respect to x_2 momentum fraction. Sum rules for PDFs:

$$\sum_{f} \int_0^{1} dx\, x\, D_f(x, Q) = 1$$

$$\int_0^{1} dx_1 \left[D_q(x, Q) - D_{\bar{q}}(x, Q) \right] = N_q$$

- Initial conditions for evolution equations should obey the sum rules:

$$D_{f_1 f_2}(x_1, x_2, Q_0), \qquad\qquad D_f(x, Q_0)$$

- Naive factorized ansatz:

$$D_{f_1 f_2}(x_1, x_2, Q_0) = D_{f_1}(x_1, Q_0)\, D_{f_2}(x_2, Q_0)\, \theta(1 - x_1 - x_2)$$

Sum rules badly violated.

- Gaunt-Stirling prescription (0910.4347/JHEP):

$$D_{f_1 f_2}(x_1, x_2, Q_0) = D_{f_1}(x_1, Q_0)\, D_{f_2}(x_2, Q_0)\, \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^{2+\alpha(f_1)}\,(1 - x_2)^{2+\alpha(f_2)}}$$

Better agreement with sum rules, although still approximate.

- Analysis in pure gluonic case - exact momentum sum rules:

(KGB, E.Lewandowska, M.Serino, Z.Snyder, A Staśto, 1507.08583/PLB)

Pure gluonic case

- MSTW08 fit of single PDFs at initial $Q_0 = 1\ \mathrm{GeV}$:

$$D_g(x, Q_0) = \sum_{k=1}^{3} A_k\, x^{\alpha_k}\, (1-x)^{\beta_k}$$

- Double gluon distribution (Dirichlet distribution form):

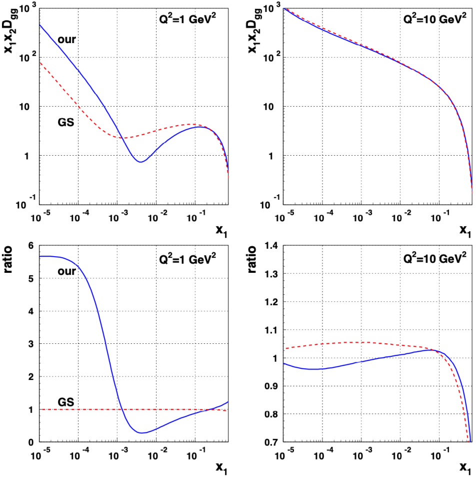
$$D_{gg}(x_1, x_2, Q_0) = \sum_{k=1}^{3} A_k \frac{\Gamma(\beta_k + 1)}{\Gamma(\alpha_k + 1)\Gamma(\beta_k - \alpha_k)}\, (x_1 x_2)^{\alpha_k} (1 - x_1 - x_2)^{\beta_k - \alpha_k - 1}$$

  obeys the momentum sum rule exactly:

$$\int_0^{1-x_2} dx_1\, x_1 D_{gg}(x_1, x_2, Q_0) = (1 - x_2)\, D_g(x_2, Q_0)$$

- Only parameters of single gluon distribution are used.

Numerical results for $x_2 = 10^{-2}$



- No factorization in our ansatz at initial scale.
- Small x factorization restored by evolution: $D_{gg}(x_1, x_2) \approx D_g(x_1)\, D_g(x_2)$

Momentum sum rule and small x factorization

- Momentum sum rules are necessary for small x factorization.

- Step 1: $\alpha_g = -1$ in the initial conditions at $Q_0 = 1\ \mathrm{GeV}$

$$D_g(x, Q_0) = A_g\, x^{\alpha_g} (1-x)^{\beta_g}$$
$$D_{gg}(x_1, x_2, Q_0) = A_g^2\, (x_1 x_2)^{\alpha_g} (1 - x_1 - x_2)^{\beta_g - \alpha_g - 1}$$

Both before and after evolution small x factorization holds

- Step 2: modification of large x behaviour of D_{gg}:

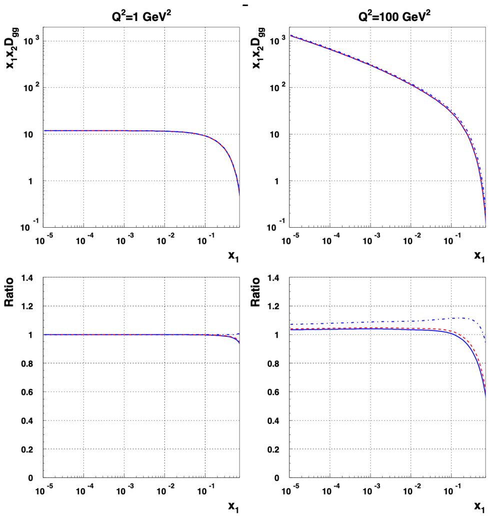
$$\beta_g - \alpha_g - 1 = 2.5 \quad \rightarrow \quad 5$$

Momentum sum rule violated but small x factorization holds:

$$\int_0^{1-x_2} dx_1\, x_1 D_{gg}(x_1, x_2, Q_0) \;\neq\; (1-x_2)\, D_g(x_2, Q_0)$$

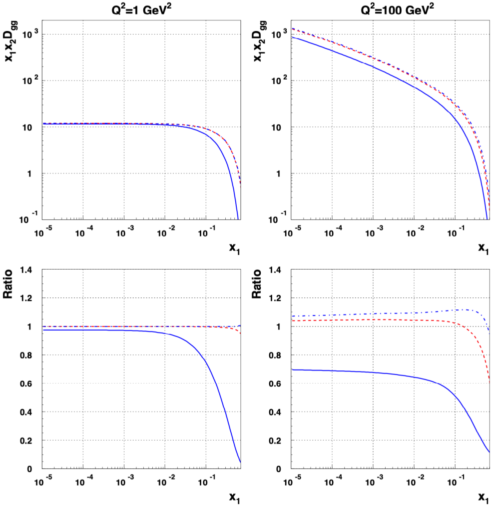
- Evolution strongly violates small x factorization.

Numerical results for $x_2 = 10^{-2}$ with momentum sum rule



- Small x factorization is kept by evolution: $D_{gg}(x_1, x_2) \approx D_g(x_1)D_g(x_2)$.

Numerical results for $x_2 = 10^{-2}$ without momentum sum rule



- Small x fact. is violated by evolution: $D_{gg}(x_1, x_2) \neq D_g(x_1) D_g(x_2)$.

Analytical insight

- Mellin moments:

$$\tilde{D}_g(n,Q) = \int_0^1 dx\, x^{n-1} D_g(x,Q)$$
$$\tilde{D}_{gg}(n_1,n_2,Q) = \int_0^1 dx_1 \int_0^1 dx_2\, x_1^{n_1-1} x_2^{n_2-1}\, \theta(1-x_1-x_2)\, D_{gg}(x_1,x_2,Q)$$

- Momentum sum rules for single and double PDFs:

$$\tilde{D}_g(2,Q) = 1 \tag{1}$$
$$\tilde{D}_{gg}(n_1,2,Q) = \tilde{D}_g(n_1,Q) - \tilde{D}_g(n_1+1,Q) \tag{2}$$

- Solution of evolution equations for $Q\to\infty$ compared to factorized form:

$$\tilde{D}_{gg}(n_1,n_2,Q) \;\approx\; \mathrm{e}^{[\gamma(n_1)+\gamma(n_2)]t}\left\{\tilde{D}_{gg}(n_1,n_2,Q_0) + \tilde{D}_g(n_1+n_2-1,Q_0)\right\}$$
$$\tilde{D}_{gg}(n_1,n_2,Q) \;\approx\; \mathrm{e}^{[\gamma(n_1)+\gamma(n_2)]t}\left\{\tilde{D}_g(n_1,Q_0)\,\tilde{D}_g(n_2,Q_0)\right\}$$

when $(n_1-1)\to 0$ and $(n_2-1) = \mathit{finite}$.

Analytical insight

- Small x factorization when initial conditions obey:

$$\tilde{D}_{gg}(n_1, n_2, Q_0) + \tilde{D}_g(n_1 + n_2 - 1, Q_0) \approx \tilde{D}_g(n_1, Q_0)\, \tilde{D}_g(n_2, Q_0) \qquad (3)$$

- Setting $n_2 = 2$ in (3) and assuming sum rule (1):

$$\tilde{D}_g(2, Q_0) = 1$$

  we obtain sum rule (2)

$$\tilde{D}_{gg}(n_1, 2, Q_0) \approx \tilde{D}_g(n_1, Q_0) - \tilde{D}_g(n_1 + 1, Q_0)$$

- Sum rules (1) and (2) are necessary for small x factorization for $Q \gg Q_0$:

$$D_{gg}(x_1, x_2, Q) \approx D_g(x_1, Q)\, D_g(x_2, Q)$$

Including quarks

- In order to include quarks in our analysis, the parameters of single PDFs must obey unrealistic condition:

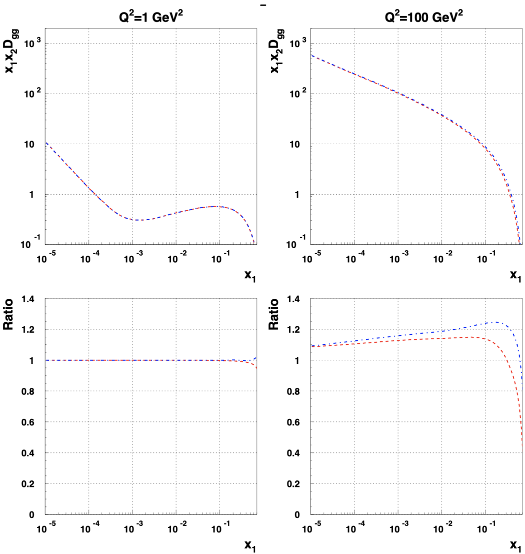
$$\beta_k^{f_2} - \beta_k^{f_1} = \alpha_k^{f_2} - \alpha_k^{f_1}$$

- The program of construcion initial DPDFs out of well known single PDFs with the sum rules exactly fulfilled is unrealistic.
- In practice, only GS ansatz for initial distribution which approximately fulfils sum rules:

$$D_{f_1 f_2}(x_1, x_2, Q_0) = D_{f_1}(x_1, Q_0)\, D_{f_2}(x_2, Q_0)\, \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^{2+\alpha(f_1)}\,(1 - x_2)^{2+\alpha(f_2)}}$$

- Small x factorization after evolution up to $10 - 20\%$ for GS ansatz.

Numerical results for $x_2 = 10^{-2}$ for evolution with quarks



- For GS (red) and fully factorized (blue) initial conditions.

Δ dependence of DPDFs

- For $\Delta \neq 0$ the small x factorization might be broken.
- In M. Diehl, T. Kasemets, S. Keane, 1401.1233/JHEP, the following ansatz was proposed for $x_{1,2} < 0.1$, due to parton correlations in y-space:

$$D_{f_1 f_2}(x_1, x_2, \Delta, Q) = D_{f_1}(x_1, Q)\, D_{f_2}(x_2, Q) \exp\left\{-h_{f_1 f_2}(x_1, x_2)\, \Delta^2\right\}$$

- This has an implications for the pocket formula for DPS cross sections:

$$\sigma_{eff} = \sigma_{eff}(x_1, x_2)$$

- More studies are necessary:

M. Diehl, J.R. Gaunt, D.M. Lang, T. Plößl, A. Schäfer, 2001.10428/EPJC

Summary

- We showed that the momentum sum rules are important for small x factorization of evolved DPDFs.
- This conclusion motivates the importance of the construction of initial DPDFs for evolution equations which fullfil the momentum sum rules.
- The pocket formula for DPS cross sections makes sense although more detailed studies of the effective cross section dependence on the parton momentum fractions are necessary.

Thank you for your attention